



HAMDARD
Imtihan

MATHE

QUESTIONS DEVELOPED ACCORDING TO
THE NEW EXAMINATION TECHNIQUES:
KNOWLEDGE, UNDERSTANDING,
ANALYTICAL AND CONCEPTUAL



HAMDARD *f*

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- (i) If $x = 0$, the complex number is said to be pure imaginary.
- (ii) If $y = 0$, the complex number is said to be real.
- (iii) It is customary to denote the standard rectangular form of a complex number $x + iy$ as z and we write $x = \text{Re}(z)$ and $y = \text{Im}(z)$.

For example, $\text{Re}(5 - 7i) = 5$ and $\text{Im}(5 - 7i) = -7$

Equality of Complex Numbers

Two complex numbers $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ are said to be equal if and only if $\text{Re}(z_1) = \text{Re}(z_2)$ and $\text{Im}(z_1) = \text{Im}(z_2)$. i.e. $x_1 = x_2$ and $y_1 = y_2$.

For example, if $\alpha + i\beta = -2 + 5i$, then $\alpha = -2, \beta = 5$.

Example 2: If $(2x - 1) + (y + 4)i = 5 + 7i$, then find the values of x and y .

Solution:

Given that $(2x - 1) + (y + 4)i = 5 + 7i$

Since the two complex numbers are equal, their real and imaginary parts must be equal.

$$\begin{aligned} \therefore 2x - 1 &= 5 & , & & y + 4 &= 7 \\ 2x &= 6 & , & & y &= 7 - 4 \\ x &= 3 & , & & y &= 3 \end{aligned}$$

Solved Exercise 1.1

1. Simplify the following:

(ii) $(3x + 2) - (4 - y)i = 5 + 3i$

Sol. $(3x + 2) - (4 - y)i = 5 + 3i$

Since the two complex numbers are equal,
their real and imaginary parts must be
equal.

$$3x + 2 = 5 \quad , \quad -(4 - y) = 3$$

$$3x = 5 - 2 \quad , \quad -4 + y = 3$$

$$3x = 3 \quad , \quad y = 3 + 4$$

$$x = \frac{3}{3} \quad , \quad y = 7$$

$$x = 1$$

(iii) $(2 + i)x + (1 - 2i)y = 3 + 4i$

Sol. $(2 + i)x + (1 - 2i)y = 3 + 4i$

$$2x + xi + y - 2yi = 3 + 4i$$

$$2x + y + xi - 2yi = 3 + 4i$$

$$(2x + y) + (x - 2y)i = 3 + 4i$$

Since the two complex numbers are equal,
their real and imaginary parts must be
equal.

$$2x + y = 3 \quad , \quad x - 2y = 4$$

$$2x + y = 3 \quad (i)$$

So, $x = 2, y = -1$

(iv) $(1 - i)x + (2 + i)y = 4 - i$

Sol. $(1 - i)x + (2 + i)y = 4 - i$

$$x - xi + 2y + yi = 4 - i$$

$$x + 2y - xi + yi = 4 - i$$

$$(x + 2y) + (-x + y)i = 4 - i$$

Since the two complex numbers are equal,
their real and imaginary parts must be
equal.

$$x + 2y = 4 \quad \text{--- (i)} \quad -x + y = -1 \quad \text{--- (ii)}$$

By adding eq (i) and eq (ii), we get

$$\begin{array}{r} x + 2y = 4 \\ -x + y = -1 \\ \hline 3y = 3 \end{array}$$

$$y = \frac{3}{3}$$

$$y = 1$$

Put $y = 1$ in eq. (i)

$$x + 2(1) = 4$$

Multiplication of Two Complex Numbers

Let $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$

$$\begin{aligned}\text{then } z_1 z_2 &= (x_1 + iy_1)(x_2 + iy_2) \\ &= x_1(x_2 + iy_2) + iy_1(x_2 + iy_2) \\ &= x_1x_2 + x_1y_2i + y_1x_2i + i^2y_1y_2 = (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2)\end{aligned}$$

Example 5

- (i) Simplify: $(3 - 4i)(5 - 6i)$
 (ii) If $z_1 = 2 + 3i$ and $z_2 = 4 + 7i$, then find $z_1 z_2$.

Solution:

$$\begin{aligned}\text{(i) } (3 - 4i)(5 - 6i) &= 3(5 - 6i) - 4i(5 - 6i) \\ &= 15 - 18i - 20i + 24i^2 \\ &= 15 - (18 + 20)i - 24 \\ &= 15 - 38i - 24 \\ &= (15 - 24) - 38i = -9 - 38i\end{aligned}$$

$$\text{(ii) } z_1 z_2 = (2 + 3i)(4 + 7i)$$

Using

$$\begin{aligned}(x_1 + iy_1)(x_2 + iy_2) &= (x_1x_2 - y_1y_2) + i(x_1y_2 + y_1x_2) \\ z_1 z_2 &= (2 \times 4 - 3 \times 7) + i(2 \times 7 + 3 \times 4) \\ &= (8 - 21) + (14 + 12)i = -13 + 26i\end{aligned}$$

Division of Two Complex Numbers

Let $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$

$$\text{Now, } \frac{z_1}{z_2} = \frac{x_1 + iy_1}{x_2 + iy_2}, z_2 \neq 0$$

(v) The Additive Inverse

For every complex number z there exists a complex number $-z$ such that,

$$z + (-z) = (-z) + z = 0.$$

$-z$ is called the additive inverse of z .

(v) The Multiplicative Inverse

For any non-zero complex number z , there exists a complex number w such that,

$$zw = wz = 1$$

w is called the multiplicative inverse of z and it is denoted by z^{-1} .

(vi) Distributive Property of Multiplication Over Addition

For any three complex numbers z_1, z_2 , and z_3

$$z_1(z_2 + z_3) = z_1 z_2 + z_1 z_3 \text{ and } (z_1 + z_2) z_3 = z_1 z_3 + z_2 z_3$$

Example 7: If $z = -7 - 4i$, then show that $z+0 = z$.

Solution:

$$\begin{aligned} z + 0 &= (-7 - 4i) + (0 + 0i) \\ &= -7 - 4i + 0 + 0i \\ &= -7 - 4i = z \end{aligned}$$

Shows $z + 0 = z$ (0 is the additive identity)

Example 8: Verify the multiplicative

Example 9: Find the additive inverse of $z = 7 - 10i$.

Solution: $z = 7 - 10i$

Additive inverse of $z = -7 + 10i$

$$\begin{aligned} \text{because } z + (-z) &= 7 - 10i + (-7 + 10i) \\ &= 7 - 10i - 7 + 10i = 0 \end{aligned}$$

Example 10: Find multiplicative inverse of $4 - 3i$.

Solution: Let $z = 4 - 3i$ (given)

3. Find multiplicative inverse for each complex number:

(i) $4 + 5i$

Sol. Let $z = 4 + 5i$

The multiplicative inverse of $z = z^{-1}$

$$z^{-1} = \frac{1}{z} = \frac{1}{4+5i}$$

$$= \frac{1}{4+5i} \times \frac{4-5i}{4-5i}$$

$$= \frac{4-5i}{(4)^2 - (5i)^2}$$

$$= \frac{4-5i}{16-25i^2}$$

$$= \frac{4-5i}{16 - (25 \times (-1))}$$

$$= \frac{4-5i}{16+25}$$

$$= \frac{4-5i}{41} = \frac{4}{41} - \frac{5i}{41}$$

$$\because i^2 = -1$$

(ii) $6 + 2i$

Sol. Let $z = 6 + 2i$

The multiplicative inverse of $z = z^{-1}$

$$z^{-1} = \frac{1}{z} = \frac{1}{6+2i}$$

$$= \frac{1}{6+2i} \times \frac{6-2i}{6-2i}$$

$$= \frac{6-2i}{(6)^2 - (2i)^2} = \frac{6-2i}{36-4i^2}$$

$$= \frac{6-2i}{36 - (4 \times (-1))} \quad \because i^2 = -1$$

$$= \frac{6-2i}{36+4}$$

$$= \frac{6-2i}{40} = \frac{6}{40} - \frac{2i}{40}$$

$$= \frac{3}{20} - \frac{1}{20}i$$

R.H.S

$$\begin{aligned} z_1 (z_2 z_3) &= (2 + 5i) [(1 - 3i)(2 + i)] \\ &= (2 + 5i) [1(2 + i) - 3i(2 + i)] \\ &= (2 + 5i) (2 + i - 6i - 3i^2) \\ &= (2 + 5i) (2 + i - 6i - 3(-1)) \\ &= (2 + 5i) (2 + i - 6i + 3) \\ &= (2 + 5i) (2 + 3 + i - 6i) \\ &= (2 + 5i) (5 - 5i) \\ &= 2(5 - 5i) + 5i(5 - 5i) \end{aligned}$$

(v) $z_1 + (-z_1) = (-z_1) + z_1 = 0$

Sol:

$$\begin{aligned} z_1 &= 2 + 5i \\ -z_1 &= -(2 + 5i) = -2 - 5i \\ z_1 + (-z_1) &= (2 + 5i) + (-2 - 5i) \\ &= 2 + 5i - 2 - 5i \\ &= 2 - 2 + 5i - 5i = 0 + 0 = 0 \\ (-z_1) + z_1 &= (-2 - 5i) + (2 + 5i) \\ &= -2 - 5i + 2 + 5i \\ &= -2 + 2 - 5i + 5i = 0 + 0 = 0 \end{aligned}$$

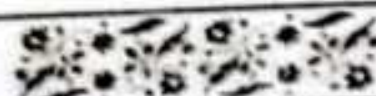
Hence, verified that $z_1 + (-z_1) = (-z_1) + z_1 = 0$

$$\begin{aligned} &= 10 - 10i + 25i - 25i^2 \\ &= 10 - 10i + 25i - 25(-1) \\ &= 10 - 10i + 25i + 25 \\ &= 10 + 25 - 10i + 25i \\ &= 35 + 15i \quad \text{_____ (2)} \end{aligned}$$

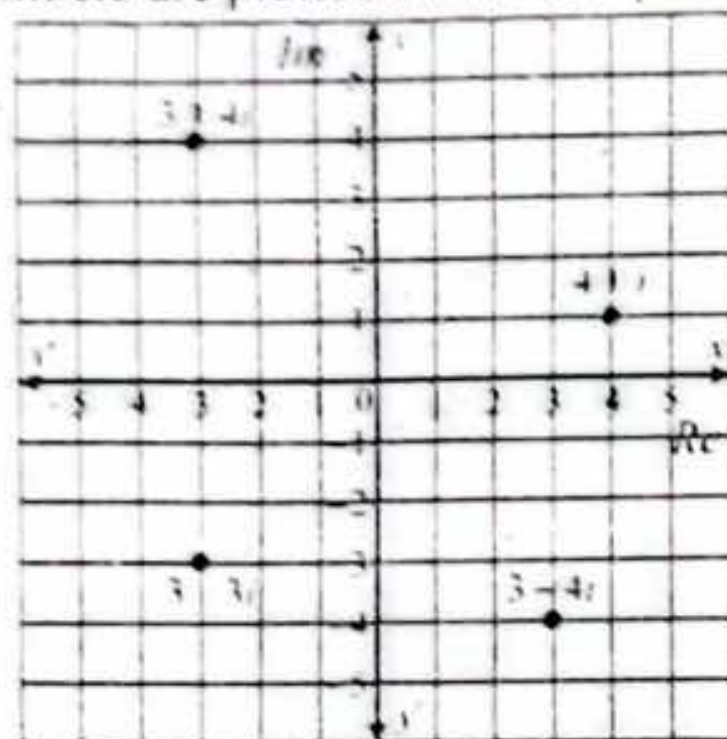
From (1) and (2) we get

L.H.S = R.H.S

Hence, verified that $(z_1 z_2) z_3 = z_1 (z_2 z_3)$



Here some complex numbers are plotted on the complex plane.



Conjugate of a Complex Number

The conjugate of the complex number $x + iy$ is defined as the complex number $x - iy$.

The complex conjugate of z is denoted by $\bar{z}$. To get the conjugate of the complex number z , simply change i by $-i$ in z . For instance $3 - 8i$ is the conjugate of $3 + 8i$. The product of a complex number with its conjugate is a real number.

For example, $(1 + 2i)(1 - 2i) = (1)^2 - (2i)^2 = 1 + 4 = 5$

Geometrical Representation of Conjugate of a Complex Number

Geometrically, the conjugate of z is obtained by reflecting z on the real axis.



Property 3

$$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}, \quad z_2 \neq 0$$

Proof: Let $z_1 = x_1 + iy_1$

and $z_2 = x_2 + iy_2$

$$\frac{z_1}{z_2} = \frac{x_1 + iy_1}{x_2 + iy_2} = \frac{x_1 + iy_1}{x_2 + iy_2} \times \frac{x_2 - iy_2}{x_2 - iy_2}$$

$$\frac{z_1}{z_2} = \frac{(x_1x_2 + y_1y_2) - i(x_1y_2 - y_1x_2)}{(x_2^2 + y_2^2)}$$

$$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{(x_1x_2 + y_1y_2) - i(x_1y_2 - y_1x_2)}{(x_2^2 + y_2^2)} \quad \text{—————(i)}$$

Now, $\frac{\bar{z}_1}{\bar{z}_2} = \frac{x_1 - iy_1}{x_2 - iy_2}$

$$= \frac{(x_1 - iy_1)(x_2 + iy_2)}{(x_2 - iy_2)(x_2 + iy_2)}$$

$$= \frac{x_1x_2 + ix_1y_2 - iy_1x_2 - i^2y_1y_2}{x_2^2 + y_2^2}$$



$$\frac{z_1}{z_2} = \frac{23+2i}{13} = \frac{23}{13} + \frac{2}{13}i$$

$$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{23}{13} - \frac{2}{13}i \quad \text{--- (i)}$$

$$\frac{\bar{z}_1}{\bar{z}_2} = \frac{5-4i}{3-2i} = \frac{(5-4i)(3+2i)}{(3-2i)(3+2i)}$$

$$= \frac{15+10i-12i-8i^2}{9+4} = \frac{15-2i+8}{9+4}$$

$$\frac{\bar{z}_1}{\bar{z}_2} = \frac{5-4i}{3-2i} = \frac{(5-4i)(3+2i)}{(3-2i)(3+2i)}$$

$$= \frac{15+10i-12i-8i^2}{9+4} = \frac{15-2i+8}{9+4}$$

$$\frac{\bar{z}_1}{\bar{z}_2} = \frac{23-2i}{13} = \frac{23}{13} - \frac{2}{13}i \quad \text{--- (ii)}$$

Therefore,

$$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2} \quad \text{(From (i) and (ii))}$$

Property 4

$$\overline{\overline{z}} = z$$

Proof: Let $z = x + iy$

$$\text{Then } \bar{z} = x - iy$$

$$\overline{\bar{z}} = x + iy = z$$

Hence, proved that $\overline{\bar{z}} = z$

Example 14

If $z = 7 + 3i$, then prove that $\overline{\overline{z}} = z$

Sol. $z = 7 + 3i$

$$\text{Then } \bar{z} = 7 - 3i$$

$$\overline{\bar{z}} = 7 + 3i = z$$

Hence, proved that $\overline{\bar{z}} = z$



(iii) $\frac{3}{5} - \frac{4}{5}i$

Sol. $\left| \frac{3}{5} - \frac{4}{5}i \right| = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = \sqrt{\frac{9}{25} + \frac{16}{25}} = \sqrt{\frac{25}{25}} = \sqrt{1} = 1$

(iv) $-\sqrt{2} - \sqrt{3}i$

Sol. $|\sqrt{2} - \sqrt{3}i| = \sqrt{(\sqrt{2})^2 + (\sqrt{3})^2} = \sqrt{2+3} = \sqrt{5}$

2. If $z_1 = 2 + 7i$ and $z_2 = 4 - 3i$, then verify that:

(i) $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$

Sol. L.H.S

$$\begin{aligned} z_1 + z_2 &= (2 + 7i) + (4 - 3i) \\ &= 2 + 7i + 4 - 3i \\ &= 2 + 4 + 7i - 3i \\ &= 6 + 4i \end{aligned}$$

$$\overline{z_1 + z_2} = \overline{6 + 4i} = 6 - 4i$$

R.H.S

$$z_1 = 2 + 7i$$

(ii) $\overline{z_1 z_2} = \overline{z_1} \overline{z_2}$

Sol. L.H.S

$$\begin{aligned} z_1 z_2 &= (2 + 7i)(4 - 3i) \\ &= 2(4 - 3i) + 7i(4 - 3i) \\ &= 8 - 6i + 28i - 21i^2 \\ &= 8 - 6i + 28i - 21(-1) \\ &= 8 - 6i + 28i + 21 \\ &= 8 + 21 - 6i + 28i \\ &= 29 + 22i \end{aligned}$$

$$\overline{z_1 z_2} = \overline{29 + 22i} = 29 - 22i$$

R.H.S

(iv) $z\bar{z} = |z|^2$

Sol. $z = 5 - 2i$

$\bar{z} = 5 + 2i$

$z\bar{z} = (5 - 2i)(5 + 2i)$

$= (5)^2 - (2i)^2$

$= 25 - 4i^2$

$= 25 - (4 \times -1)$

$= 25 + 4$

$= 29$ _____ (i)

$|z| = |5 - 2i| = \sqrt{(5)^2 + (2)^2}$

$= \sqrt{25 + 4} = \sqrt{29}$

$|z|^2 = (\sqrt{29})^2$

$|z|^2 = 29$ _____ (ii)

Hence, from (i) and (ii) verified that $z\bar{z} = |z|^2$

4. If $z = 4 - 3i$, then verify that $|z| = |-z| = |\bar{z}| = |-\bar{z}|$.

Sol. Given that $z = 4 - 3i$

(v) $|z| = |-\bar{z}|$

Sol. $z = 4 - 3i$

$\bar{z} = 4 + 3i$

$-\bar{z} = -(4 + 3i) = -4 - 3i$

$|z| = |4 - 3i|$

$= \sqrt{(4)^2 + (3)^2}$

$= \sqrt{16 + 9} = \sqrt{25}$ _____ (i)

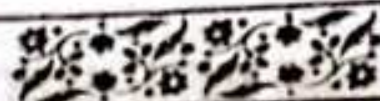
$|-\bar{z}| = |-4 - 3i|$

$= \sqrt{(-4)^2 + (-3)^2}$ _____ (ii)

$= \sqrt{16 + 9} = \sqrt{25}$

Hence, from (i) and (ii) verified that $|z| = |-\bar{z}|$

$= \sqrt{(-4)^2 + (-3)^2} = \sqrt{16 + 9} = 5$



$$= \frac{x^2 - 2xyi + y^2i^2}{(x^2 + y^2)^2} = \frac{x^2 - 2xyi - y^2}{(x^2 + y^2)^2} = \frac{(x^2 - y^2) - 2xyi}{(x^2 + y^2)^2}$$

$$= \frac{x^2 - y^2}{(x^2 + y^2)^2} - \frac{2xyi}{(x^2 + y^2)^2}$$

$$\text{So, Real part} = \frac{x^2 - y^2}{(x^2 + y^2)^2}, \text{ imaginary part} = \frac{-2xy}{(x^2 + y^2)^2}$$

Example 17: Find real and imaginary parts of $(4 + 3i)^{-1}$.

Sol. Let $z = (4 + 3i)^{-1}$

$$= \frac{1}{(4 + 3i)}$$

$$= \frac{1(4 - 3i)}{(4 + 3i)(4 - 3i)} = \frac{4 - 3i}{16 - 9i^2}$$

$$= \frac{4 - 3i}{16 + 9} = \frac{4 - 3i}{25} = \frac{4}{25} - \frac{3}{25}i$$

$$\therefore \text{Re}(z) = \frac{4}{25}, \text{Im}(z) = -\frac{3}{25}$$

Sol. $5z - (3 + i)w = 7 - i$ _____ (i)

$(2 - i)z + 2iw = 4$ _____ (ii)

Step I From equation (i), we solve for w in terms of z .

$$5z - (3 + i)w = 7 - i$$

$$(3 + i)w = 5z - 7 + i$$

$$w = \frac{5z - 7 + i}{3 + i} \text{ _____ (iii)}$$

Step II Put the expression of w in equation (ii)

$$(2 - i)z + 2i \left(\frac{5z - 7 + i}{3 + i} \right) = 4$$

$$(2 - i)(3 + i)z + 10iz - 14i + 2i^2 = 4(3 + i)$$

$$(6 - 2i - 3i - i^2)z + 10iz - 14i + 2i^2 = 12 + 4i$$

$$(6 - 4i - 1)z + 10iz - 14i - 2 = 12 + 4i$$

$$(7 - 4i)z + 10iz - 14i - 2 = 12 + 4i$$

$$7z - 4iz + 10iz - 14i - 2 = 12 + 4i$$

$$7z + 9iz - 12 - 4i + 14i + 2$$

$$7z + 9iz - 14 + 18i$$



$$= \frac{12 - 8i + 9i + 6}{13}$$

$$= \frac{18 + i}{13} = \frac{18}{13} + \frac{i}{13}$$

$$\therefore \text{Real part} = \frac{18}{13}, \text{Imaginary part} = \frac{1}{13}$$

$$\text{(vi)} \left(\frac{2-i}{2+i} \right)^{-2}$$

$$\text{Sol.} \left(\frac{2-i}{2+i} \right)^{-2} = \left(\frac{2+i}{2-i} \right)^2$$

$$= \left(\frac{2+i}{2-i} \times \frac{2+i}{2+i} \right)^2$$

$$= \left(\frac{2(2+i) + i(2+i)}{(2)^2 - (i)^2} \right)^2$$

$$= \left(\frac{4 + 2i + 2i + i^2}{4 - (-1)} \right)^2$$

$$\therefore \text{Real part} = \frac{-7}{25}, \text{Imaginary part} = \frac{24}{25}$$

$$\text{(vii)} \left(\frac{1-2i}{1+i} \right)^2$$

$$\text{Sol.} \left(\frac{1-2i}{1+i} \right)^2$$

$$= \left(\frac{1-2i}{1+i} \times \frac{1-i}{1-i} \right)^2$$

$$= \left(\frac{1(1-i) - 2i(1-i)}{(1)^2 - (i)^2} \right)^2$$

$$= \left(\frac{1-i-2i+2i^2}{1-(-1)} \right)^2$$

$$= \left(\frac{1-i-2i+2(-1)}{1+1} \right)^2$$

$$(1-i-2i-2)^2$$

$$-i + w + wi = 11$$

$$w + wi - i = 11$$

$$w + wi = 11 + i$$

$$w(1 + i) = 11 + i$$

$$w = \frac{11 + i}{1 + i}$$

$$w = \frac{11 + i}{1 + i} \times \frac{1 - i}{1 - i}$$

$$w = \frac{11 - 11i + i - i^2}{(1)^2 - i^2}$$

$$w = \frac{11 - 10i - (-1)}{1 - (-1)}$$

$$w = \frac{11 + 1 - 10i}{1 + 1}$$

$$w = \frac{12 - 10i}{2}$$

$$w = \frac{2(6 - 5i)}{2} = 6 - 5i$$

$$\text{Put } w = \frac{-3i + z}{4} \text{ in (ii)}$$

$$2z + 3\left(\frac{-3i + z}{4}\right) = 11 - 5i$$

$$\frac{4(2z) + 3(-3i + z)}{4} = 11 - 5i$$

$$8z - 9i + 3z = 4(11 - 5i)$$

$$8z - 9i + 3z = 44 - 20i$$

$$11z = 44 - 20i + 9i$$

$$11z = 44 - 11i$$

$$z = \frac{44 - 11i}{11}$$

$$z = \frac{11(4 - i)}{11}$$

$$z = 4 - i$$

$$\text{Put } z = 4 - i \text{ in (iii)}$$

$$w = \frac{-3i + 4 - i}{4}$$

$$4i + 4$$



(iii) Imaginary part of $(2 - i)(2 + i)$ is:

(a) 0

(b) 1

(c) 7

(d) 9

(iv) $x + iy$ will be pure imaginary number, when:

(a) $y = 0$

(b) $x = 0$

(c) $i = 0$

(d) $x = 0, y = 0$

(v) What is additive inverse of $5 - 2i$?

(a) $5 + 2i$

(b) $-5 - 2i$

(c) $5 - 2i$

(d) $-5 + 2i$

(vi) What is multiplicative inverse of $z = 1 + i$?

(a) $1 - i$

(b) i

(c) $\frac{1}{2} - \frac{1}{2}i$

(d) $-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$

(vii) If $z = 4 - 3i$, then $z\bar{z} =$

(a) 3

(b) 9

(c) 16

(d) 25

(viii) Conjugate of $9 - 4i$ is:

(a) $-9 - 4i$

(b) $9 + 4i$

(c) $9 + 9i$

(d) $4 - 9i$

(ix) If $z = 4 + 4i$, then $z + \bar{z} =$

(a) 8

(b) $8 + 8i$

(c) $8i$

(d) 0

(x) If $z = 5 + 4i$, then $|z| =$

(a) 9

(b) 25

(c) 41

(d) $\sqrt{41}$

ANSWERS:

(i) 0

(ii) 13

(iii) 0

(iv) $x = 0$

(v) $-5 + 2i$

1 1



L.H.S

$$\begin{aligned} z_1 z_2 &= (3 + 4i)(2 + 3i) \\ &= 3(2 + 3i) + 4i(2 + 3i) \\ &= 6 + 9i + 8i + 12i^2 \\ &= 6 + 17i + 12(-1) \\ &= 6 - 12 + 17i \\ &= -6 + 17i \end{aligned}$$

$$\overline{z_1 z_2} = \overline{-6 + 17i} = -6 - 17i$$

$$(iii) \left(\frac{z_1}{z_2} \right) = \frac{\overline{z_2}}{\overline{z_1}}$$

Sol. $z_1 = 3 + 4i, \overline{z_2} = 2 + 3i$
 $\overline{z_1} = 3 - 4i, z_2 = 2 - 3i$

L.H.S

$$\begin{aligned} \frac{z_1}{z_2} &= \frac{3 + 4i}{2 + 3i} \\ &= \frac{3 + 4i}{2 + 3i} \times \frac{2 - 3i}{2 - 3i} \\ &= \frac{3(2 - 3i) + 4i(2 - 3i)}{(2)^2 - (3i)^2} \end{aligned}$$

R.H.S

$$\begin{aligned} \overline{z_1} \overline{z_2} &= (3 - 4i)(2 - 3i) \\ &= 3(2 - 3i) - 4i(2 - 3i) \\ &= 6 - 9i - 8i + 12i^2 \\ &= 6 - 17i + 12(-1) \\ &= 6 - 12 - 17i \\ &= -6 - 17i \end{aligned}$$

Hence, verified that $\overline{z_1 z_2} = \overline{z_1} \overline{z_2}$

R.H.S

$$\begin{aligned} \frac{\overline{z_1}}{\overline{z_2}} &= \frac{3 - 4i}{2 - 3i} \\ &= \frac{3 - 4i}{2 - 3i} \times \frac{2 + 3i}{2 + 3i} \\ &= \frac{3(2 + 3i) - 4i(2 + 3i)}{(2)^2 - (3i)^2} \end{aligned}$$

7. Find real and imaginary parts of $z = (2 + 7i)^{-1}$

Sol. $z = (2 + 7i)^{-1}$

$$= \frac{1}{2 + 7i}$$

$$= \frac{1}{2 + 7i} \times \frac{2 - 7i}{2 - 7i}$$

$$= \frac{2 - 7i}{(2)^2 - (7i)^2} = \frac{2 - 7i}{4 - 49i^2} = \frac{2 - 7i}{4 - (49 \times -1)}$$

$$= \frac{2 - 7i}{4 + 49} = \frac{2 - 7i}{53} = \frac{2}{53} - \frac{7}{53}i$$

so, real part = $\frac{2}{53}$, imaginary part = $-\frac{7}{53}$

8. Solve the given simultaneous linear equations with complex coefficients for z and w :

$$iz + (2 - i)w = 4 + i$$

$$iz + (3 + i)w = 3 + 3i$$

Sol. $iz + (2 - i)w = 4 + i$ _____ (i)

$$iz + (3 + i)w = 3 + 3i$$
 _____ (ii)



$$b = \frac{4}{25}$$

Put $b = \frac{4}{25}$ in (i)

$$3a + 4\left(\frac{4}{25}\right) = 1$$

$$3a + \frac{16}{25} = 1$$

$$25 \times 3a + 25 \times \frac{16}{25} = 25 \times 1$$

$$75a + 16 = 25$$

$$75a = 25 - 16 = 9$$

$$a = \frac{9}{75} = \frac{3}{25}$$

$$a = \frac{3}{25}$$

so, $a = \frac{3}{25}$, $b = \frac{4}{25}$

10. Solve the equation for x and y : $(3 - 2i)(x + yi) = 2(x - 2yi) + 2i - 1$

16. If $(3x + 2) + 4i = 14 - yi$ then x and y will be:
 (a) $x = 4, y = 3$ (b) $x = -4, y = 4$ (c) $x = 3, y = 4$ (d) $x = 4, y = -4$
17. If $-5x + yi = -10$ then $y =$ _____
 (a) 10 (b) -5 (c) 0 (d) -2

1.2 Algebraic Operations on Complex Numbers

18. If $z = 3 + 5i$ then $3z =$ _____
 (a) $9 + 15i$ (b) $6 + 8i$ (c) $3 - 5i$ (d) $9 + 25i^2$
19. If $z = x + yi$ then $-5z =$ _____
 (a) $5z = -5x + yi$ (b) $-5x - 5yi$ (c) $x - 5yi$ (d) $5x + 5yi$
20. By adding $(4 + 5i)$ and $(-6 + 7i)$, we get:
 (a) $-6 + 12i$ (b) $-10 + 12i$ (c) $10 + 12i$ (d) $-2 + 12i$
21. $(4 - 3i) - (2 - 5i) =$ _____
 (a) $2 + 2i$ (b) $2 - 3i$ (c) $6 - 5i$ (d) $-2 - 2i$
22. $(5 + 2i) + (3 - 7i) =$ _____
 (a) $2 - 5i$ (b) $15 - 9i$ (c) $8 - 5i$ (d) $8 + 9i$
23. $(4 + i)(4 - i) =$ _____
 (a) 20 (b) 17 (c) 15 (d) 16
24. $\frac{1+i}{1-i}$ in the form of $a + bi$ is:

48. If $z = -\sqrt{3} - \sqrt{5}i$ then $|z| =$ _____

(a) 8

(b) $\sqrt{34}$

(c) $\sqrt{3} + \sqrt{5}$

(d) $\sqrt{8}i$

1.5
Finding Real and imaginary Parts of Complex Number of $(x + iy)^n$ and

$$\left(\frac{x_1 + iy_1}{x_2 + iy_2} \right)^n$$

 , where $x^2 + iy_2 \neq 0$ and $n = \underline{+1}$ and $n = \underline{+2}$

49. $(4 + 3i)^{-2} =$ _____

(a) $(4 + 3i)^2$

(b) $(-4 - 3i)^{-2}$

(c) $\sqrt{(4)^2 + (3)^2}$

(d) $\frac{1}{(4 - 3i)^2}$

50. If z is a complex number then $\text{Re}(z) =$ _____

(a) $\frac{z - \bar{z}}{2i}$

(b) $\frac{z + \bar{z}}{2}$

(c) $\frac{z + \bar{z}}{2}$

(d) $z \cdot \bar{z}$

51. If z is a complex number the $\text{Im}(z) =$ _____

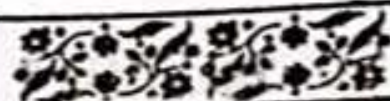
(a) $\frac{z - \bar{z}}{2i}$

(b) $\frac{z - \bar{z}}{2}$

(c) $\frac{z + \bar{z}}{2}$

(d) $\frac{z + \bar{z}}{2i}$

$$(5z - 3i)^{-2}$$



Since the two complex numbers are equal, their real and imaginary parts are also equal.

$$3x + 3 = 7, \quad 5 - y = 3$$

$$3x = 7 - 3, \quad 5 - 3 = y$$

$$3x = 4, \quad 2 = y \Rightarrow y = 2$$

$$x = \frac{4}{3}$$

$$\text{So, } x = \frac{4}{3}, y = 2$$

1.2**Algebraic Operations on Complex Numbers**

12. If $z = 1 - 2yi$ then find $-3z$.

Sol. $z = 1 - 2yi$

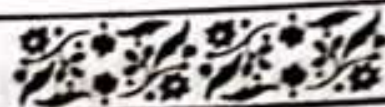
$$\begin{aligned} -3z &= -3(1 - 2yi) \\ &= -3 + 6yi \end{aligned}$$

13. Simplify: $(17 - 7i) + (19 - 7i)$

Sol. $(17 - 7i) + (19 - 7i)$
$$\begin{aligned} &= 17 - 7i + 19 - 7i \\ &= 17 + 19 - 7i - 7i \\ &= 36 - 14i \end{aligned}$$

17. Express $(2 + \sqrt{3}i)(-7 + 2i)$ in the form of $a + bi$.

Sol. $(2 + \sqrt{3}i)(-7 + 2i)$
$$\begin{aligned} &= 2(-7 + 2i) + \sqrt{3}i(-7 + 2i) \\ &= -14 + 4i - 7\sqrt{3}i + 2\sqrt{3}i^2 \end{aligned}$$



26. If $z_1 = 2 + 5i$, $z_2 = 7 - 3i$ then verify that $z_1 + z_2 = z_2 + z_1$

Sol. $z_1 = 2 + 5i$, $z_2 = 7 - 3i$

L.H.S

$$\begin{aligned} z_1 + z_2 &= (2 + 5i) + (7 - 3i) \\ &= 2 + 5i + 7 - 3i \\ &= 2 + 7 + 5i - 3i \\ &= 9 + 2i \end{aligned}$$

R.H.S

$$\begin{aligned} z_2 + z_1 &= (7 - 3i) + (2 + 5i) \\ &= 7 - 3i + 2 + 5i \\ &= 7 + 2 - 3i + 5i \\ &= 9 + 2i \end{aligned}$$

Hence verified that $z_1 + z_2 = z_2 + z_1$

1.3

Complex or Argand Plane

27. What is an Argand Plane?

Ans. An Argand plane is a coordinate plane where the horizontal axis(x-axis) represent the real part and the vertical axis(y-axis) represent the imaginary part of a complex number.

28. Represent the complex number $-7i$ as an ordered pair.

Ans. Since there is no real part, the real value is 0. Therefore, the order pair is $(0, -7)$

29. Represent the ordered pair $(-7, -3)$ as a complex number.

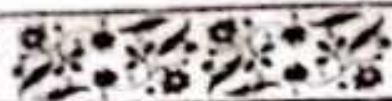
Ans. The complex number of $(-7, -3)$ is $-7 - 3i$.

1.3.1

Conjugate of a Complex Number

30. Define the conjugate of a complex number.

Ans. The conjugate of a complex number $x + yi$ is defined as the complex number $x - yi$.



1.5

Finding Real and imaginary Parts of Complex Number of $(x + iy)^n$ and

$$\left(\frac{x_1 + iy_1}{x_2 + iy_2} \right)^n$$

, where $x^2 + y^2 \neq 0$ and $n = \underline{+1}$ and $n = \underline{+2}$ 39. Find the real and imaginary parts of $(3 - 5i)^{-1}$ Sol. Let $z = (3 - 5i)^{-1}$

$$= \frac{1}{(3 - 5i)}$$

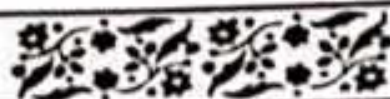
$$= \frac{1}{3 - 5i} \times \frac{3 + 5i}{3 + 5i}$$

$$= \frac{3 + 5i}{(3)^2 + (5i)^2} = \frac{3 + 5i}{9 - (25 \times -1)} = \frac{3 + 5i}{9 + 25}$$

$$= \frac{3 + 5i}{34} = \frac{3}{34} + \frac{5}{34}i$$

$$\operatorname{Re}(z) = \frac{3}{34}, \operatorname{Im}(z) = \frac{5}{34}$$

40. Find the real and imaginary parts of $(1 - i)^{-2}$ Sol. Let $z = (1 - i)^{-2}$



$$3x^2 + 7x = 6$$

$$3x^2 + 7x - 6 = 0$$

$$3x^2 + 9x - 2x - 6 = 0$$

$$3x(x + 3) - 2(x + 3) = 0$$

$$(x + 3)(3x - 2) = 0$$

Now if either of the factor $x + 3$ or $3x - 2$ is zero, then their product is zero. So,

$$x + 3 = 0 \text{ or } 3x - 2 = 0$$

$$x = -3 \text{ or } x = \frac{2}{3}$$

Hence, solution set is $\left\{-3, \frac{2}{3}\right\}$

[multiplying both sides by 3]

Do you know?

Every quadratic equation has at most two roots.

Note

An equation in which variable appears in the denominator of one or more terms is called a **fractional equation**.

$$(i) \quad \frac{2x}{5} - \frac{6}{x} = \frac{3}{7}, x \neq 0$$

$$(ii) \quad \frac{x+3}{x-1} = \frac{2x-1}{x+5}, x \neq 1, -5 \text{ are fractional equations.}$$

Example 2: Solve by factorization method: $\frac{2x-3}{2} = \frac{4x-6}{x}, x \neq 0$

Sol. $\frac{2x-3}{2} = \frac{4x-6}{x}$

$$2x(2x-3) = (4x-6)(2x)$$

Multiplying both sides by LCM of the denominators.

i.e., $2x$ to eliminate denominator



We add square of half of the coefficient of x , i.e., $\left(\frac{1}{5}\right)^2$ on both sides of the equation.

$$x^2 - \frac{2}{5}x + \left(\frac{1}{5}\right)^2 = \frac{18}{5} + \left(\frac{1}{5}\right)^2$$

$$\left(x - \frac{1}{5}\right)^2 = \frac{18}{5} + \frac{1}{25}$$

$$\left(x - \frac{1}{5}\right)^2 = \frac{90+1}{25} = \frac{91}{25}$$

$$\sqrt{\left(x - \frac{1}{5}\right)^2} = \sqrt{\frac{91}{25}}$$

$$\left(x - \frac{1}{5}\right) = \pm \frac{\sqrt{91}}{5}$$

$$x = \frac{1}{5} \pm \frac{\sqrt{91}}{5} = \frac{1 \pm \sqrt{91}}{5}$$

$$\text{S.S} = \left\{ \frac{1 \pm \sqrt{91}}{5} \right\}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Step IV: Taking square root of both the sides.

$$\pm \left(x + \frac{b}{2a}\right) = \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

or $x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$ (Transposing)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This is the quadratic formula.

Remember!

Moving a term from one side of an equation to the other by changing its sign is called transposition

Compare it with $y = a(x - h)^2 + k$. So, the vertex is $(0.25, -10.13)$. To sketch the graph of quadratic function we take some more points.

x	-2.5	-1	1	2
y	5	-7	-9	-4

By plotting the points and vertex, graph is sketched above. The graph of the given quadratic function $y = 2x^2 - x - 10$ opens upwards and it intersects the x-axis at $(-2, 0)$ and $(2.5, 0)$.

(ii) The graph intersects the x-axis at $x = 2.5$ and $x = -2$. Therefore, roots of the quadratic equation $2x^2 - x - 10 = 0$ are 2.5 and -2. Hence, solution set is $\{2.5, -2\}$.

Remember!

For finding the solution of quadratic equation $ax^2 + bx + c = 0$ graphically, first of all we draw the graph of quadratic function $y = ax^2 + bx + c$.

Solved Exercise 2.1

1. Write the following quadratic equations in standard form:

(i) $3x - 1 = 2x^2$

Sol. $3x - 1 = 2x^2$

$$-2x^2 + 3x - 1 = 0$$

$$2x^2 - 3x + 1 = 0$$

(iii) $2x^2 - 4x = 4x + 7$

Sol. $2x^2 - 4x = 4x + 7$

(ii) $2x(x + 1) = 4(2x + 3)$

Sol. $2x(x + 1) = 4(2x + 3)$

$$2x^2 + 2x = 8x + 12$$

$$2x^2 + 2x - 8x - 12 = 0$$

$$2x^2 - 6x - 12 = 0$$

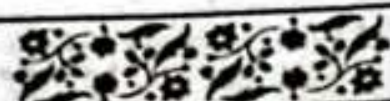
$$2(x^2 - 3x - 6) = 0$$

$$x^2 - 3x - 6 = 0$$

(iv) $4(3x - 2) = 9x^2$

Sol. $4(3x - 2) = 9x^2$

$$12x - 8 = 9x^2$$



(i) $2x^2 + 5x + 2 = 0$

Sol. $2x^2 + 5x + 2 = 0$

Dividing by 2 (the coefficient of x^2) on both sides of the equation.

$$\frac{2x^2}{2} + \frac{5}{2}x + \frac{2}{2} = \frac{0}{2}$$

$$x^2 + \frac{5}{2}x + 1 = 0$$

$$x^2 + \frac{5}{2}x = -1$$

We add square of half of the coefficient of x , i.e., $\left(\frac{5}{4}\right)^2$ on both sides of the equation.

$$x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2 = -1 + \left(\frac{5}{4}\right)^2$$

$$\left(x + \frac{5}{4}\right)^2 = -1 + \frac{25}{16}$$

$$\left(x + \frac{5}{4}\right)^2 = \frac{16 - 25}{16}$$

Solution Set = $\left\{-\frac{1}{2}, -2\right\}$

(ii) $x^2 + x = 42$

Sol. $x^2 + x = 42$

We add square of half of the co-efficient of x , i.e., $\left(\frac{1}{2}\right)^2$ on both sides of the equation.

$$x^2 + x + \left(\frac{1}{2}\right)^2 = 42 + \left(\frac{1}{2}\right)^2$$

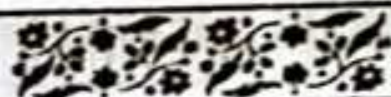
$$\left(x + \frac{1}{2}\right)^2 = 42 + \frac{1}{4}$$

$$\left(x + \frac{1}{2}\right)^2 = \frac{168 + 1}{4}$$

$$\left(x + \frac{1}{2}\right)^2 = \frac{169}{4}$$

Taking square root of both the sides.

$$\sqrt{\left(x + \frac{1}{2}\right)^2} = \sqrt{\frac{169}{4}}$$



Solution set is $\left\{4, \frac{4}{3}\right\}$

$$(v) \frac{1}{1+x} - \frac{1}{3-x} = \frac{6}{35}, x \neq -1, 3$$

$$\text{Sol. } \frac{1}{1+x} - \frac{1}{3-x} = \frac{6}{35}$$

$$\frac{(3-x)-(1+x)}{(1+x)(3-x)} = \frac{6}{35}$$

$$\frac{3-x-1-x}{3-x+3x-x^2} = \frac{6}{35}$$

$$\frac{2-2x}{-x^2+2x+3} = \frac{6}{35}$$

$$35(2-2x) = 6(-x^2+2x+3)$$

(By cross multiplication)

$$70 - 70x = -6x^2 + 12x + 18$$

$$6x^2 - 70x - 12x + 70 - 18 = 0$$

$$6x^2 - 82x + 52 = 0$$

$$2(3x^2 - 41x + 26) = 0$$

(Taking 2 as a common)

$$\left(x - \frac{41}{6}\right)^2 = \frac{1369}{36}$$

Taking square root of both the sides,

$$\sqrt{\left(x - \frac{41}{6}\right)^2} = \sqrt{\frac{1369}{36}}$$

$$x - \frac{41}{6} = \pm \frac{37}{6}$$

$$x - \frac{41}{6} = \frac{37}{6}$$

$$x = \frac{37}{6} + \frac{41}{6}$$

$$x = \frac{78}{6}$$

$$x = 13$$

$$\text{or } x - \frac{41}{6} = -\frac{37}{6}$$

$$\text{or } x = -\frac{37}{6} + \frac{41}{6}$$

$$\text{or } x = \frac{4}{6}$$

$$\text{or } x = \frac{2}{3}$$

$$\text{Solution Set} = \left\{13, \frac{2}{3}\right\}$$

(iii) $2x^2 + 7x = 15$

Sol. $2x^2 + 7x = 15$ (Given)

$2x^2 + 7x - 15 = 0$

$ax^2 + bx + c = 0$ (Standard form)

Comparing the given equation with the standard form.

$a = 2, b = 7, c = -15$

Substituting the values of a, b, c in the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{(7)^2 - 4(2)(-15)}}{2(2)}$$

$$x = \frac{-7 \pm \sqrt{49 + 120}}{4}$$

$$x = \frac{-7 \pm \sqrt{169}}{4}$$

$$x = \frac{-7 \pm 13}{4}$$

$$x = \frac{7 \pm \sqrt{49 - 44}}{2}$$

$$x = \frac{7 \pm \sqrt{5}}{2}$$

Solution set: $\left\{ \frac{7 \pm \sqrt{5}}{2} \right\}$

(v) $\frac{x+4}{x-4} + \frac{x-2}{x-3} = 6\frac{1}{3}, x \neq 4, 3$

Sol. $\frac{x+4}{x-4} + \frac{x-2}{x-3} = 6\frac{1}{3}$

$$\frac{(x-3)(x+4) + (x-4)(x-2)}{(x-4)(x-3)} = \frac{19}{3}$$

$$\frac{x^2 + 4x - 3x - 12 + x^2 - 2x - 4x + 8}{x^2 - 3x - 4x + 12} = \frac{19}{3}$$

$$\frac{x^2 + x^2 + 4x - 3x - 2x - 4x - 12 + 8}{x^2 - 7x + 12} = \frac{19}{3}$$

$$\frac{2x^2 - 5x - 4}{x^2 - 7x + 12} = \frac{19}{3}$$

The graph intersects the x -axis at $x = 6$ and $x = -3$. Therefore, roots of the quadratic equation $x^2 - 3x - 18 = 0$ are -3 and 6 .

Hence, Solution set is $\{-3, 6\}$

(ii) $x^2 - 5x - 14 = 0$

Sol. $x^2 - 5x - 14 = 0$

Let

$$y = x^2 - 5x - 14$$

$$y = x^2 - 5x + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 - 14$$

$$y = \left(x - \frac{5}{2}\right)^2 - \frac{25}{4} - 14$$

$$y = \left(x - \frac{5}{2}\right)^2 - 6.25 - 14$$

$$y = (x - 2.5)^2 - 20.25$$

$$y = (x - h)^2 + k$$

[the vertex form of quadratic function]

By comparing the vertex is $(2.5, -20.25)$

To sketch the graph of quadratic

By plotting the points and vertex, graph is sketched above. The graph of the given quadratic function $y = x^2 - 5x - 14 = 0$ opens upwards and it intersects the x -axis at $(-2, 0)$ and $(7, 0)$.

The graph intersects the x -axis at $x = 7$ and $x = -2$. Therefore, roots of the quadratic equation $x^2 - 5x - 14 = 0$ are 7 and -2 .

Hence, solution set is $\{-2, 7\}$

(iii) $2x^2 + 13x + 6 = 0$

Sol. $2x^2 + 13x + 6 = 0$

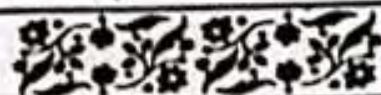
Let

$$y = 2x^2 + 13x + 6$$

$$y = 2 \left[x^2 + \frac{13}{2}x + \frac{6}{2} \right]$$

$$y = 2 \left[x^2 + \frac{13}{2}x + 3 \right]$$

$$y = 2 \left[x^2 + \frac{13}{2}x + \left(\frac{13}{4}\right)^2 - \left(\frac{13}{4}\right)^2 + 3 \right]$$



2.2

Finding Intersection Point(s) Graphically

Example 6: Find the points of intersection of $y = 2x + 4$ with coordinate axes graphically.

Solution: $y = 2x + 4$ is an equation of a line. Now, we draw graph of $y = 2x + 4$.

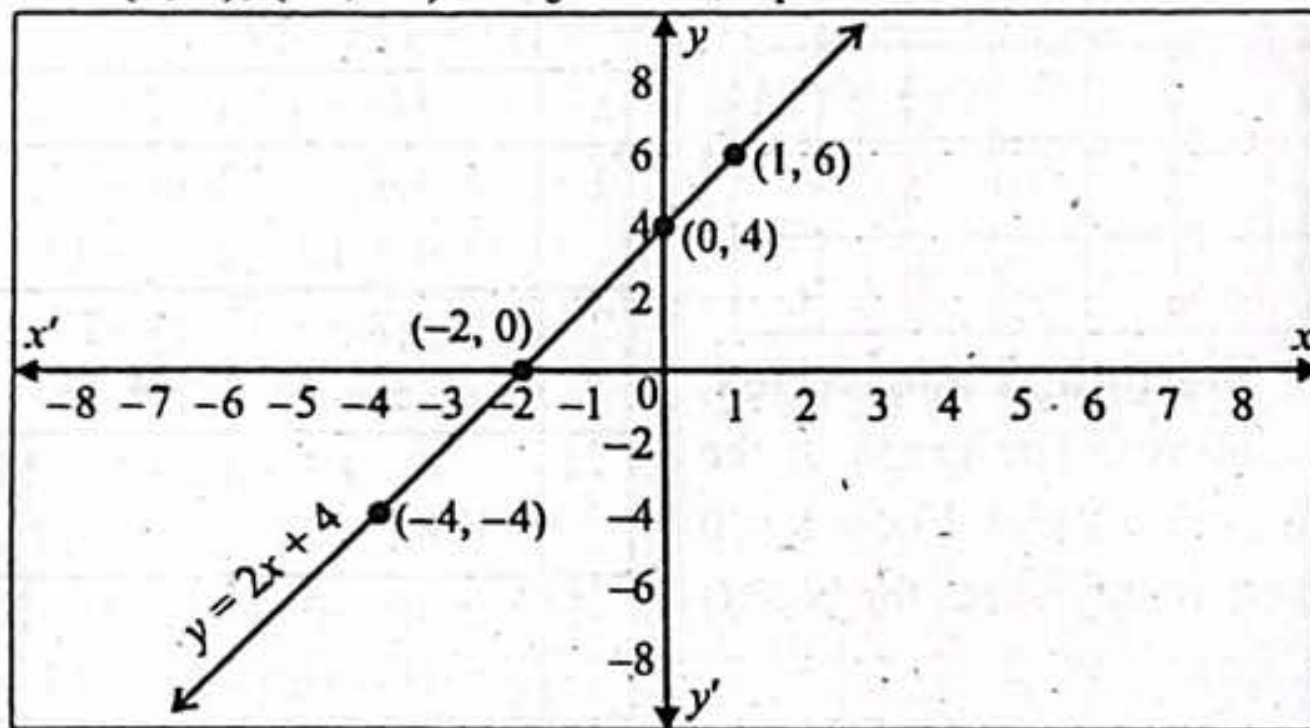
Remember!

To draw a straight line, it is essential to have at least two distinct points.

Table of values

x	1	-4
y	6	-4

We plot points (1, 6), (-4, -4) and join these points.



(i) $x + y = 8$

 Sol. $x + y = 8$ is an equation of a line.

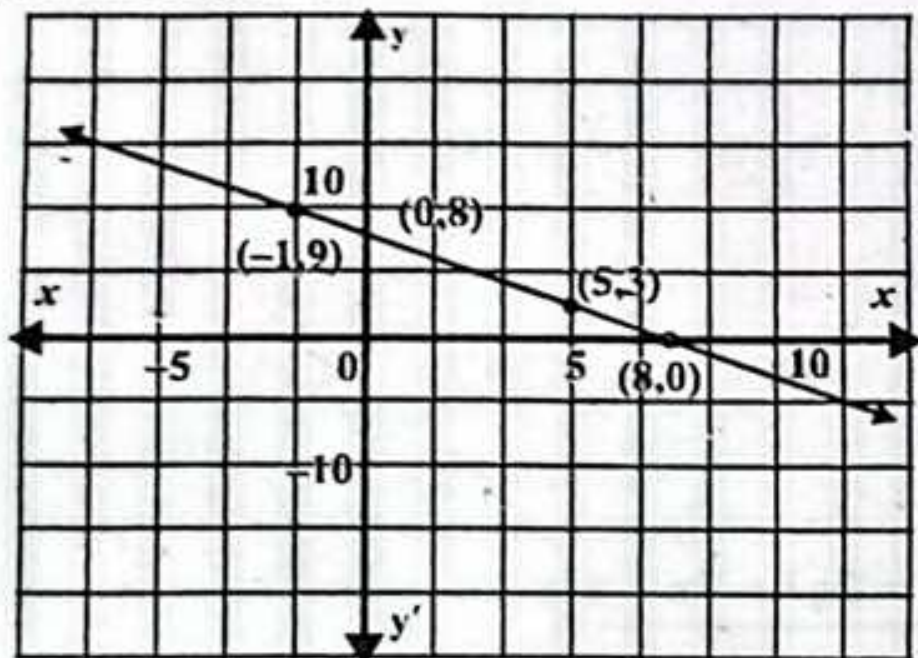
$$y = 8 - x$$

 Now we draw the graph of $x + y = 8$.

 Table of values for $y = 8 - x$

x	$y = 8 - x$	(x, y)
5	$y = 8 - 5 = 3$	(5, 3)
-1	$y = 8 - (-1) = 9$	(-1, 9)

We plot points (5, 3) and (-1, 9) and join these points.



Point of intersection on x-axis is (8, 0) and the point of intersection on y-axis is (0, 8).

Point of intersection on x-axis is (1, 0) and the point of intersection on y-axis is (0, -1)

(iii) $x - 2y = 1$

 Sol. $x - 2y = 1$ is an equation of line.

$$-2y = -x + 1$$

$$2y = x - 1$$

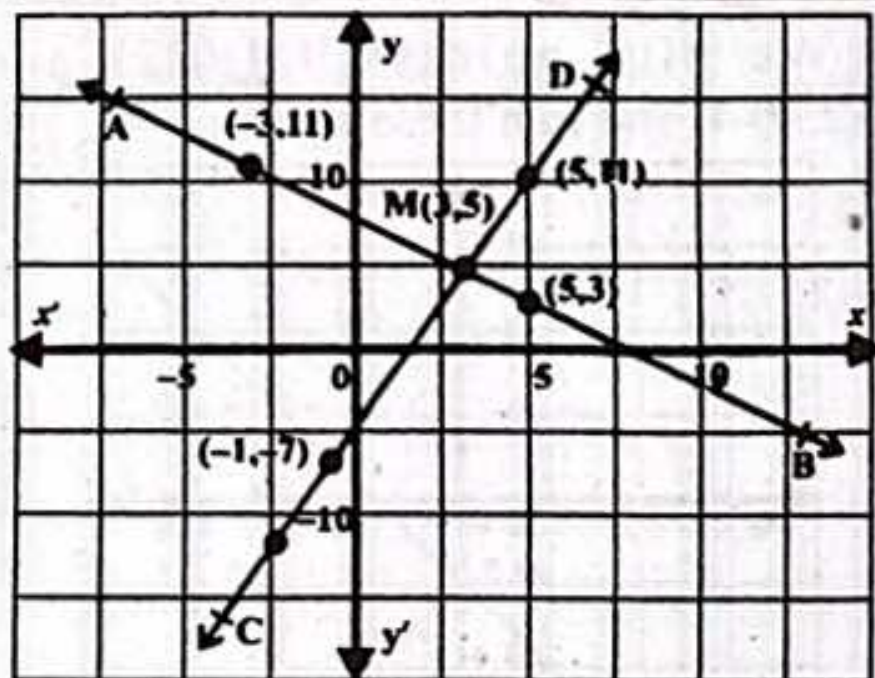
$$y = \frac{x - 1}{2}$$

 Now we draw the graph of $x - 2y = 1$.

 Table of the values for $y = \frac{x - 1}{2}$

x	$y = \frac{x - 1}{2}$	(x, y)
3	$y = \frac{3 - 1}{2} = 1$	(3, 1)
-5	$y = \frac{-5 - 1}{2} = -3$	(-5, -3)

We plot points (3, 1) and (-5, -3) and join these points.



$\overline{AB}$ is the graph of $x + y = 8$

$\overline{CD}$ is the graph of $3x - y = 4$

Hence, the point of intersection is

$M(3, 5)$

(ii) $x - y = 1$

$x + 2y = 7$

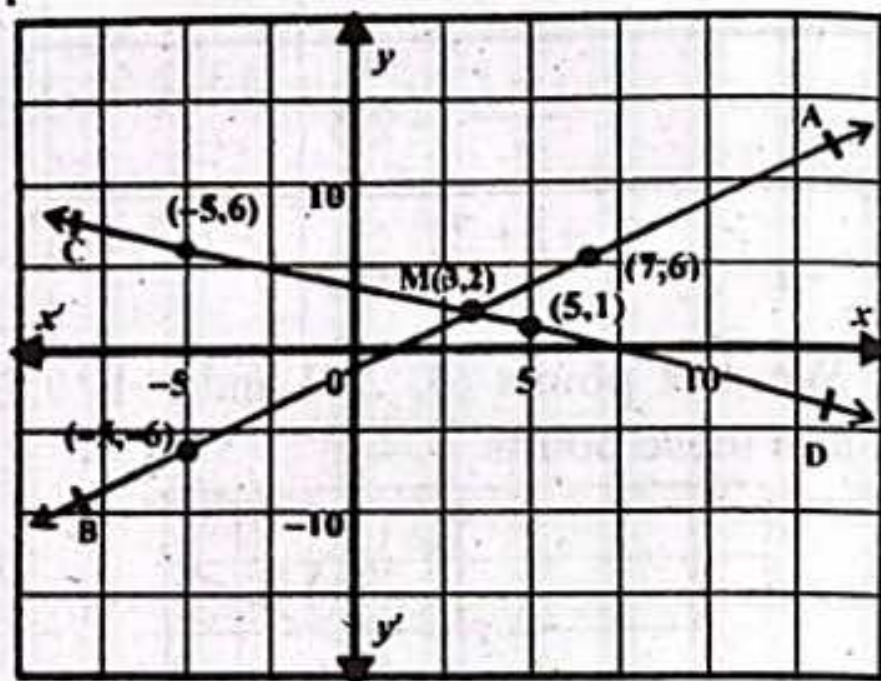
Sol. $x - y = 1$ _____ (i)

$x + 2y = 7$ _____ (ii)

We draw the graph of these two equations.

From (i)

Now we plot points for both of these equations.



$\overline{AB}$ is the graph of $x - y = 1$

$\overline{CD}$ is the graph of $x + 2y = 7$

Hence, the point of intersection is $M(3, 2)$.

(iii) $x - 2y = 1$

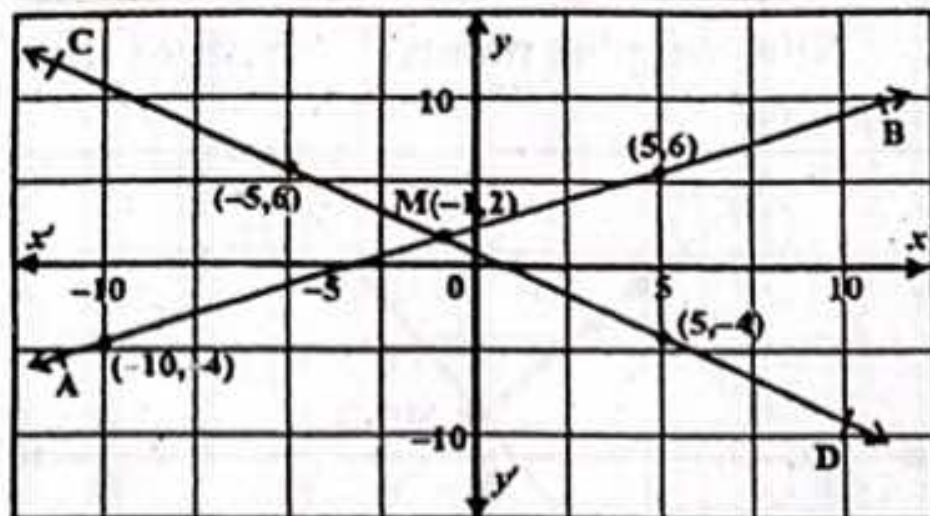
$2x + y = 2$

Sol. $x - 2y = 1$ _____ (i)

$2x + y = 2$ _____ (ii)

We draw the graph of these two equations.

From (i)



$\overline{AB}$ is the graph of $3y = 2x + 8$

$\overline{CD}$ is the graph of $x + y = 1$

Hence, the point of intersection is $M(-1, 2)$

3. Solve the following equations graphically.

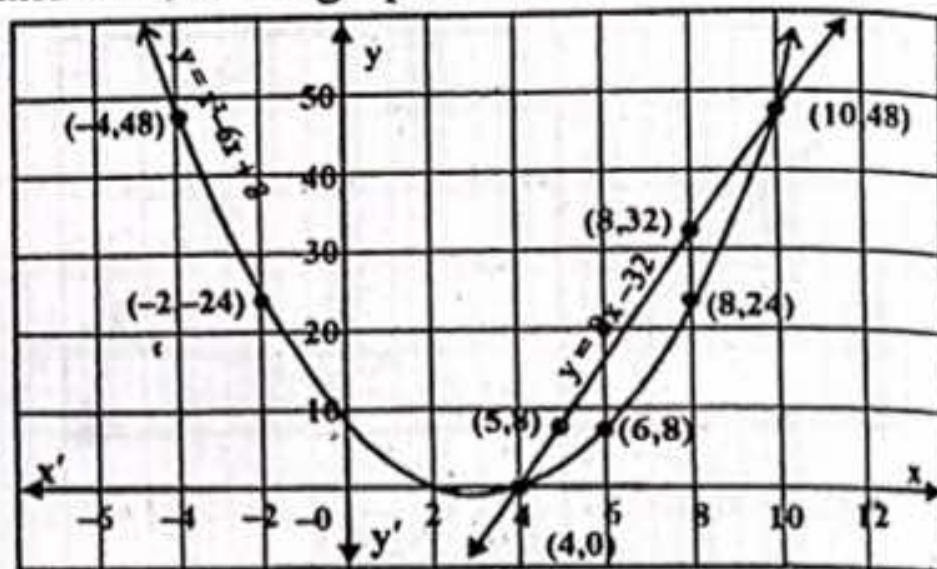
(i) $y = 8x - 32$
 $y = x^2 - 6x + 8$

Sol. $y = 8x - 32$ _____ (i)
 $y = x^2 - 6x + 8$ _____ (ii)

We draw the graph of both of these two equations.

From (i)

Now we plot points of both equations and draw their graph.



Points of intersection are: $(4, 0)$ and $(10, 48)$.

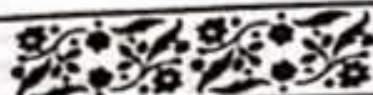
(ii) $y + x = 2$
 $y = 2x^2 + x - 10$

Sol. $y + x = 2$ _____ (i)
 $y = 2x^2 + x - 10$ _____ (ii)

We draw the graph of both of these two equations.

From (i)

$y + x = 2$
 $y = 2 - x$



$$= \alpha\beta = \left(\frac{-b + \sqrt{b^2 - 4ac}}{2a} \right) \left(\frac{-b - \sqrt{b^2 - 4ac}}{2a} \right)$$

$$= \frac{(-b)^2 - (\sqrt{b^2 - 4ac})^2}{(2a)(2a)} = \frac{b^2 - (b^2 - 4ac)}{4a^2}$$

$$= \frac{b^2 - b^2 + 4ac}{4a^2} = \frac{4ac}{4a^2} = \frac{c}{a}$$

$$P = \text{Product of the roots} = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Important Note!

- (i) The standard form of a quadratic equation is $ax^2 + bx + c = 0$ where $a \neq 0$ and $a, b, c \in \mathbb{R}$
- (ii) Sum of the roots $= -\frac{b}{a}$, Product of the roots $= \frac{c}{a}$
- (iii) When the coefficient of x^2 in a quadratic equation is unity (one).
- Sum of roots is equal to the coefficient of x with sign changed.
 - Product of the roots is equal to the constant term.
- (iv) If the constant term of $ax^2 + bx + c = 0$ is zero, then sum of its roots $= -\frac{b}{a}$ and

$$x^2 - \frac{17}{12}x + \frac{1}{2} = 0 \quad [\text{From (i) and (ii)}]$$

So, $12x^2 - 17x + 6 = 0$

Example 12: If α, β are roots of the equation $x^2 - 7x + 10 = 0$, then form an equation whose roots are $2\alpha + 1$ and $2\beta + 1$.

Sol. As α, β are roots of $x^2 - 7x + 10 = 0$, therefore

$$\alpha + \beta = -\frac{-7}{1} = 7 \quad (\text{i}), \quad \alpha\beta = \frac{10}{1} = 10 \quad (\text{ii})$$

$$\begin{aligned} S &= \text{Sum of roots of the required equation} = 2\alpha + 1 + 2\beta + 1 \\ &= 2(\alpha + \beta) + 2 \\ &= 2(7) + 2 = 16 \end{aligned}$$

$$\begin{aligned} P &= \text{Product of roots of the required equation} = (2\alpha + 1)(2\beta + 1) \\ &= 4\alpha\beta + 2(\alpha + \beta) + 1 \\ &= 4(10) + 2(7) + 1 = 55 \end{aligned}$$

The required equation will be

$$x^2 - Sx + P = 0$$

$$x^2 - 16x + 55 = 0 \quad [\text{Putting values of } S \text{ and } P]$$

Example 13: Find the condition that the roots of $ax^2 + bx + c = 0$ may be equal in magnitude but opposite in sign, where $a \neq 0$.

Sol. $ax^2 + bx + c = 0$ (given equation)

$$= \frac{-7-6}{5} = \frac{-13}{5} \text{ _____ (i)}$$

$$P = \text{Product of the roots} = \left(\frac{-7}{5}\right)\left(\frac{-6}{5}\right) = \frac{42}{25} \text{ _____ (ii)}$$

Required equation is $x^2 - Sx + P = 0$

Putting values of S and P in the above equation

$$x^2 - \left(\frac{-13}{5}x\right) + \frac{42}{25} = 0 \text{ (from (i), and (ii))}$$

$$x^2 + \frac{13}{5}x + \frac{42}{25} = 0$$

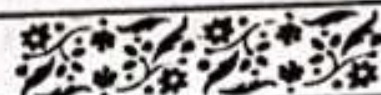
Multiplying throughout by 25, we get

$$25(x^2) + 25\left(\frac{13}{5}x\right) + 25\left(\frac{42}{25}\right) = 25(0)$$

$$25x^2 + 65x + 42 = 0$$

$$\text{(iv)} \quad \frac{-3}{2}, \frac{7}{2}$$

Sol. Roots of the required equation are $\frac{-3}{2}, \frac{7}{2}$



Required equation $x^2 - Sx + P = 0$

Putting the values of S and P in the above equation

$$x^2 - (\alpha + \beta + 4)x + \alpha\beta + 2(\alpha + \beta) + 4 = 0 \quad \text{_____ (v)}$$

Putting the values of $\alpha + \beta$ and $\alpha\beta$ from (i) and (ii) in equation (v), we get

$$x^2 - (-9 + 4)x + 20 + 2(-9) + 4 = 0$$

$$x^2 - (-5)x + 20 - 18 + 4 = 0$$

$$x^2 + 5x + 6 = 0$$

3. Find the equation whose roots are double the roots of $x^2 - px + q = 0$.

Sol. $x^2 - px + q = 0$ (Given)

$$a = 1, b = -p, c = q$$

Let the roots of the given equation are α and β

$$S = \alpha + \beta = \frac{-b}{a} = \frac{-(-p)}{1} = p \quad \text{_____ (i)}$$

$$P = \alpha\beta = \frac{c}{a} = \frac{q}{1} = q \quad \text{_____ (ii)}$$

New roots $2\alpha, 2\beta$

$$S' = \text{Sum of new roots} = 2\alpha + 2\beta = 2(\alpha + \beta) \quad \text{_____ (iii)}$$

$$P' = \text{Product of new roots} = (2\alpha)(2\beta) = 4\alpha\beta \quad \text{_____ (iv)}$$



$$= 2(\alpha + \beta) - \left(\frac{\alpha + \beta}{\alpha\beta} \right)$$

Putting the values of $\alpha + \beta$ and $\alpha\beta$ from (i) and (ii)

$$\begin{aligned} &= 2(-2) - \left(-\frac{2}{4} \right) \\ &= -4 + \frac{1}{2} \\ &= \frac{-8+1}{2} = \frac{-7}{2} \quad \text{_____ (A)} \end{aligned}$$

P = Product of the given roots

$$\begin{aligned} &= \left(2\alpha - \frac{1}{\beta} \right) \left(2\beta - \frac{1}{\alpha} \right) \\ &= 4\alpha\beta - \frac{2\alpha}{\alpha} - \frac{2\beta}{\beta} + \frac{1}{\alpha\beta} \\ &= 4\alpha\beta - 2 - 2 + \frac{1}{\alpha\beta} \end{aligned}$$

$$4x^2 + 4\left(\frac{7}{2}\right)x + 4\left(\frac{49}{4}\right) = 0$$

$$4x^2 + 14x + 49 = 0$$

(iv) α^2, β^2

Given roots are α^2 and β^2

S = Sum of the given roots

$$= \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

Putting the values of $\alpha + \beta$ and $\alpha\beta$ from (i) and (ii)

$$\begin{aligned} &= (-2)^2 - 2(4) \\ &= 4 - 8 = -4 \quad \text{_____ (A)} \end{aligned}$$

$$\begin{aligned} P &= \text{Product of the given roots} = \alpha^2 \beta^2 \\ &= (\alpha\beta)^2 \end{aligned}$$

Putting the value of $\alpha\beta$ from (ii)

$$= (4)^2 = 16 \quad \text{_____ (B)}$$

The required equation is $x^2 - Sx + P = 0$

Putting the values of S and P from A and B

$$x^2 - (-4)x + 16 = 0$$

$$x^2 + 4x + 16 = 0$$

$$A = 2b - a^2 \text{ (Proved)}$$

$$\alpha^2 \beta^2 = \frac{c}{a}$$

$$\alpha^2 \beta^2 = \frac{B}{1}$$

$$\alpha^2 \beta^2 = B$$

$$(\alpha\beta)^2 = B$$

Putting value of $\alpha\beta$ from (ii)

$$b^2 = B$$

$$B = b^2 \text{ (Proved)}$$

9. If α, β are the roots of $x^2 + px + q = 0$, then find the condition that

(i) $\alpha = \beta$

Sol. $x^2 + px + q = 0$ (given equation)

Root of equation are α, β

$$\alpha + \beta = -\frac{b}{a} = \frac{-p}{1} = -p$$

$$\alpha\beta = \frac{c}{a} = \frac{q}{1} = q$$

If $\alpha = \beta$ then

$$\alpha + \beta = -\frac{b}{a} = \frac{-p}{1} = -p \text{ (i)}$$

$$\alpha\beta = \frac{c}{a} = \frac{q}{1} = q \text{ (ii)}$$

$$\alpha = \frac{1}{\beta} \text{ (condition)}$$

If one root is reciprocal of other then their product will be

$$\alpha\beta = q \text{ from (i)}$$

$$\frac{1}{\beta}(\beta) = q$$

$$1 = q$$

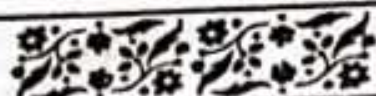
$$q = 1$$

So, the required equation is $q = 1$

2.4 Discriminant of a Quadratic Equation

The quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$ and $a, b, c \in \mathbb{R}$ has two roots.

Let these roots be α and β .



Here, $a = 3$, $b = -9$, $c = -2$

$$\begin{aligned}\text{Disc.} &= b^2 - 4ac \\ &= (-9)^2 - 4(3)(-2) \\ &= 81 + 24 \\ &= 105 > 0\end{aligned}$$

As the discriminant is positive and not a perfect square. So the roots are irrational and unequal.

(ii) $x^2 + 6x + 9 = 0$

Sol. $x^2 + 6x + 9 = 0$

Here, $a = 1$, $b = 6$, $c = 9$

$$\begin{aligned}\text{Disc} &= b^2 - 4ac \\ &= (6)^2 - 4(1)(9) \\ &= 36 - 36 \\ &= 0\end{aligned}$$

As the discriminant is zero so, the roots are real and equal.

(iii) $2x^2 + 4x + 5 = 0$

Sol. $2x^2 + 4x + 5 = 0$

Here, $a = 2$, $b = 4$, $c = 5$

$$\text{Disc.} = b^2 - 4ac$$

As the discriminant is negative. So, the roots are imaginary.

(vi) $x^2 - 8x + 16 = 0$

Sol. $x^2 - 8x + 16 = 0$

Here, $a = 1$, $b = -8$, $c = 16$

$$\begin{aligned}\text{Disc} &= b^2 - 4ac \\ &= (-8)^2 - 4(1)(16) \\ &= 64 - 64 \\ &= 0\end{aligned}$$

As the discriminant is zero. So, the roots are real and equal.

2. For what values of t , the roots of $3x^2 + x + 9t = 0$ are real and unequal?

Sol. $3x^2 + x + 9t = 0$

Here, $a = 3$, $b = 1$, $c = 9t$

Disc. , $= b^2 - 4ac > 0$ (roots are real and unequal)

$$(-1)^2 - 4(3)(9t) > 0$$

$$1 - 108t > 0$$

$$-108t > -1$$



Step IV: Conclusion:

The solution of this inequality $x^2 - 5x + 6 > 0$

is $x \in (-\infty, 2) \cup (3, \infty)$

Example:17 Solve the inequality $2x^2 - 5x + 2 \leq 0$.

Solution: $2x^2 - 5x + 2 \leq 0$.

Step I: Solve the corresponding or associated equation.

$$2x^2 - 5x + 2 = 0$$

Factorizing the quadratic equation

$$2x^2 - 4x - x + 2 = 0$$

$$2x(x-2) - 1(x-2) = 0$$

$$(x-2)(2x-1) = 0$$

$$(2x-1)(x-2) = 0$$

Now, $2x-1=0$ or $x-2=0$

So, the solutions are $x = \frac{1}{2}$ and $x = 2$.

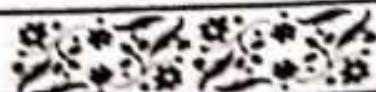
Note

For solution, we use open interval (a, b) when equation has $<$ or $>$ sign. We use closed interval $[a, b]$ when equation has $\leq$ or $\geq$ sign.

Here, critical points are $\left(\frac{1}{2}, 0\right)$ and $(2, 0)$.

Step II: Determine Intervals

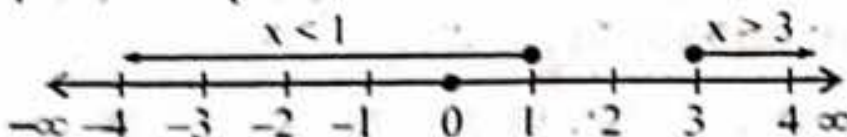
The critical points $\left(\frac{1}{2}, 0\right)$ and $(2, 0)$ divide the number line into three intervals.



So, the solutions are $x = 1, x = 3$.

Hence, the critical points are $(1, 0)$ and $(3, 0)$

These critical points $(1, 0)$ and $(3, 0)$ divides the number line into three intervals.



Interval 1: $x < 1 \Rightarrow x \in (-\infty, 1)$

Interval 2: $1 < x < 3 \Rightarrow x \in (1, 3)$

Interval 3: $x > 3 \Rightarrow x \in (3, \infty)$

Test points in each interval.

$$\begin{aligned} \text{For } x = 0 [\text{in the interval } (-\infty, 1)] : (0 - 3)(2(0) - 2) \\ = (-3)(-2) = 6 > 0 \end{aligned}$$

So, this interval satisfies the inequality.

$$\text{For } x = 2 [\text{in the interval } (1, 3)] : (2 - 3)(2(2) - 2) = (-1)(4 - 2) = (-1)(2) = -2 < 0$$

So, this interval does not satisfy the inequality.

$$\text{For } x = 4 [\text{interval } (3, \infty)] : (4 - 3)(2(4) - 2) = (1)(8 - 2) = (1)(6) = 6 > 0$$

So, this interval satisfies the inequality.

Conclusion:

The solution of this inequality is $(-\infty, 1) \cup (3, \infty)$

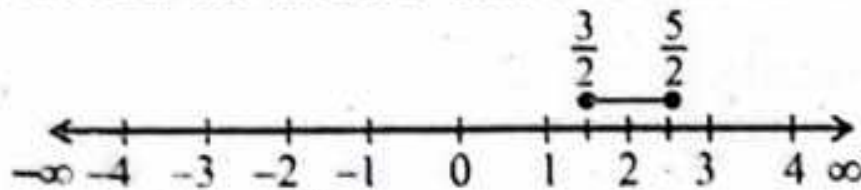
(iii) $x^2 + x - 6 < 0$

Sol. $x^2 + x - 6 < 0$

The corresponding equation

The critical points are $\left(\frac{3}{2}, 0\right)$ and $\left(\frac{5}{2}, 0\right)$

These critical points divide the number line into three intervals.



Interval 1: $x < \frac{3}{2} \Rightarrow x \in \left(-\infty, \frac{3}{2}\right]$,

Interval 2: $\frac{3}{2} < x < \frac{5}{2} \Rightarrow x \in \left[\frac{3}{2}, \frac{5}{2}\right]$

Interval 3: $x > \frac{5}{2} \Rightarrow x \in \left[\frac{5}{2}, \infty\right)$

Test points in each interval.

For $x = 0$ [interval $\left(-\infty, \frac{3}{2}\right]$]: $(2(0) - 5)(2(0) - 3) = (-5)(-3) = 15 > 0$

So, this interval does not satisfy the inequality.

For $x = 2$ [interval $\left[\frac{3}{2}, \frac{5}{2}\right]$]: $(2(2) - 5)(2(2) - 3) = (4 - 5)(4 - 3) = (-1)(1) = -1 < 0$



Solved Exercise 2.6

1. Make F the subject of the formula,

$$C^{\circ} = \frac{5}{9} (F^{\circ} - 32).$$

Sol. $C = \frac{5}{9} (F^{\circ} - 32)$

or $\frac{5}{9} (F^{\circ} - 32) = C$

$$\frac{9}{5} \times \frac{5}{9} (F^{\circ} - 32) = \frac{9}{5} C$$

$$F^{\circ} - 32 = \frac{9}{5} C$$

$$F^{\circ} = \frac{9}{5} C + 32$$

2. The formula for finding simple interest is $I = PRT$.

(a) Make P the subject of the formula

4. The volume of a cylinder is given by the formula, $V = \pi r^2 h$. Make ' h ' the subject of the formula.

Sol. $V = \pi r^2 h$

or $\pi r^2 h = V$

$$h = \frac{V}{\pi r^2}$$

5. The area of a trapezoid is $A = \frac{1}{2} h (b_1 + b_2)$, make h the subject of the formula.

Sol. $A = \frac{1}{2} h (b_1 + b_2)$

or $\frac{1}{2} h (b_1 + b_2) = A$

$$h (b_1 + b_2) = 2A$$

$$h = \frac{2A}{b_1 + b_2}$$

6. If $y = mx + c$, then make ' x ' the

$$x = \frac{-b}{2a} = \frac{-80}{2(-2)} = \frac{-80}{-4} = 20$$

Hence, the price per item that gives maximum profit is Rs. 20.

$$\begin{aligned} P(20) &= -2(20)^2 + 80(20) - 500 \\ &= -2(400) + 1600 - 500 \\ &= 300 \text{ (in thousands)} \end{aligned}$$

Hence, maximum profit is Rs. 300,000.

Note:

We can also find vertex by using

$$x = \frac{-b}{2a}, y = f\left(\frac{-b}{2a}\right)$$

Example 21: A ball is thrown upwards from the ground. Its height after t seconds is modeled by:

$$h(t) = -5t^2 + 20t \text{ (in metres)}$$

For what time interval is the ball at least 15 metres above the ground?

Solved Exercise 2.7

1. A town's population is modeled by $P(t) = -2t^2 + 40t + 800$, where t is years since 2020. Find the year when the population will be at least 1000.

Sol. We need to find t when $P(t) \geq 1000$.

Here,

$$\begin{aligned} -2t^2 + 40t + 800 &\geq 1000. \\ -2t^2 + 40t + 800 - 1000 &\geq 0. \\ -2t^2 + 40t - 200 &\geq 0. \end{aligned}$$

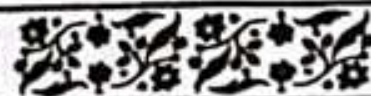
Dividing both sides by -2 , we get

$$\begin{aligned} t^2 - 20t + 100 &\leq 0 \\ (t)^2 - 2(t)(10) + (10)^2 &\leq 0. \\ (t-10)^2 &\leq 0. \end{aligned}$$

By taking square root

$$t - 10 = 0 \text{ (A squared number can never be negative)}$$

$$t = 10$$



$$s = \frac{1600}{20}$$

$$s = 80 \text{ m}$$

So, the maximum height that the ball reached above the ground level is 80m.

6. A freelancer's earnings follow the model $E(h) = -2h^2 + 40h$, where E is earning in rupees and h is hours worked per week. What is the maximum number of hours he should work to maximize earnings?

Sol. $E(h) = -2h^2 + 40h$

We need to maximize the function $E(h)$. This is finding the vertex of the parabola. Identify coefficients:

$$a = -2, b = 40$$

Use vertex Formula for h :

$$h = \frac{-b}{2a}$$

$$h = \frac{-40}{2(-2)} = \frac{-40}{-4} = 10$$

So, he should work 10 hours per week to maximize the earning.

Solved Review Exercise 2

1. For possible answers are given for the following questions. Choose the correct answer.
- (i) The type of the equation $2x^2 - x + 1 = 0$ is:
- (a) quadratic (b) linear (c) third degree (d) pure quadratic



$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(8)(-7)}}{2(8)}$$

$$x = \frac{1 \pm \sqrt{1 + 224}}{16}$$

$$x = \frac{1 \pm \sqrt{225}}{16} = \frac{1 \pm 15}{16}$$

$$x = \frac{1+15}{16}, \quad x = \frac{1-15}{16}$$

$$x = \frac{16}{16} = 1, \quad x = \frac{-14}{16} = -\frac{7}{8}$$

$$\text{S.S} = \left\{ 1, -\frac{7}{8} \right\}$$

(ii) $2x^2 - x - 10 = 0$

Sol. By factorization

$$2x^2 - x - 10 = 0$$

$$2x^2 - 5x + 4x - 10 = 0$$

$$x(2x - 5) + 2(2x - 5) = 0$$

$$(2x - 5)(x + 2) = 0$$

$$2x - 5 = 0 \quad \text{or} \quad x + 2 = 0$$

$$\left(x - \frac{1}{4}\right)^2 = 5 + \frac{1}{16}$$

$$\left(x - \frac{1}{4}\right)^2 = \frac{80+1}{16}$$

$$\sqrt{\left(x - \frac{1}{4}\right)^2} = \sqrt{\frac{81}{16}}$$

$$x - \frac{1}{4} = \pm \frac{9}{4}$$

$$x - \frac{1}{4} = \frac{9}{4} \quad \text{or} \quad x - \frac{1}{4} = -\frac{9}{4}$$

$$x = \frac{9}{4} + \frac{1}{4} \quad \text{or} \quad x = -\frac{9}{4} + \frac{1}{4}$$

$$x = \frac{10}{4} = \frac{5}{2} \quad \text{or} \quad x = -\frac{8}{4} = -2$$

$$\text{So, S.S} = \left\{ \frac{5}{2}, -2 \right\}$$

By quadratic formula



Objective Type Questions



Additional MCQ's

Multiple Choice Questions prepared in the light of new Examination Techniques
(Knowledge, Understanding, Application, Analytical & Conceptual)

2.1

Quadratic Equation

○ Encircle the correct option.

- What is the degree of polynomial $-x + 5x^2 - 7$?
(a) 1 (b) 2 (c) 5 (d) -7
- A quadratic, equation is a polynomial equation of degree.
(a) one (b) two (c) three (d) zero
- What is the standard form of a quadratic equation in one variable x ?
(a) $ax + b = 0$ (b) $ax^3 + bx^2 + c = 0$ (c) $ax^2 + bx + c = 0$ (d) $a^2 + x = c$
- If $a = 0$ in the equation $ax^2 + bx + c = 0$, the equation becomes:
(a) a linear equation (b) a quadratic equation
(c) a pure quadratic equation (d) a cubic equation
- An equation of the form $ax^2 + bx + c = 0$ becomes pure quadratic equation when:



24. Which equation has roots 4 and 5?

- (a) $x^2 - 9x + 20 = 0$ (b) $x^2 + 9x - 20 = 0$ (c) $x^2 - 9x - 20 = 0$ (d) $x^2 + 20x + 9 = 0$

25. What are the roots of $(x - 5)(x + 7) = 0$?

- (a) -5, -7 (b) -5, 7 (c) 1, 7 (d) 5, -7

2.4

Discriminant of a Quadratic Equation

26. The expression $b^2 - 4ac$ in a quadratic equation is known as:

- (a) variable (b) constant (c) coefficient (d) discriminant

27. if $b^2 - 4ac > 0$ and is a perfect square, the roots are:

- (a) rational, real and unequal (b) irrational and equal
(c) imaginary and unequal (d) rational and equal

28. When $b^2 - 4ac = 0$, the roots are:

- (a) imaginary (b) irrational and unequal
(c) real and equal (d) real and unequal

29. Which of the following is irrational and real root according to discriminant?

- (a) $\sqrt{25}$ (b) 5 (c) $\sqrt{7}$ (d) $\frac{4}{3}$

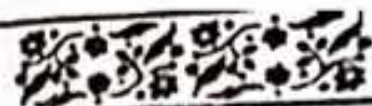
30. If the roots are "complex conjugates" the discriminant must be:

- (a) positive (b) zero (c) negative (d) a perfect square

31. The discriminant of $4x^2 + 4x + 1 = 0$ is:

ANSWERS:

1. 2 2. two 3. $ax^2+bx+c=0$ 4. a linear equation 5. $b=0$ 6. Either $a=0$ or $b=0$
7. two 8. $\frac{2}{5}x + \frac{6}{x} + 4 = 0$ 9. $\left\{\frac{3}{2}, 4\right\}$ 10. divide every term by 3 11. $x + \frac{5}{3} = \pm \frac{11}{3}$
12. $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$ 13. $a=3, b=7, c=-6$ 14. $a(x-h)^2+k$ 15. parabola 16. $(-2,0)$ 17. $M(3, 3)$
18. points 19. $-\frac{b}{a}$ 20. $-\frac{7}{2}$ 21. $\frac{6}{5}$ 22. $\frac{4}{7}$ 23. $x^2-Sx+P=0$ 24. $x^2-9x+20=0$
25. 5, -7 26. Discriminant 27. rational, real and unequal 28. real and equal 29. $\sqrt{7}$ 30. negative
31. 0 32. -23 33. perfect square 34. = 35. roots 36. $(2,0)$ and $(3,0)$
37. $< \text{ or } >$ 38. $\leq \text{ or } \geq$ 39. $3x^2 - 5x + 7 = 0$ 40. $\frac{I}{PR}$ 41. $\frac{P-2\ell}{2}$ 42. $\frac{S-2a}{n-1}$
43. $\frac{y^2}{4a}$ 44. $\frac{2P - m(v_1^2 - v_2^2)}{2mg}$ 45. $\frac{V-u}{a}$ 46. $\frac{2A}{b}$ 47. $\frac{y+5}{3}$ 48. $vi + at$
49. $v_i t + \frac{1}{2}at^2$ 50. 15 51. $x = \frac{-b}{2a}, y = f\left(\frac{-b}{2a}\right)$



$$(x - 2)^2 = 9$$

Taking square root of both sides, we get

$$\sqrt{(x - 2)^2} = \sqrt{9}$$

$$x - 2 = \pm 3$$

$$x - 2 = -3$$

$$x - 2 = 3 \text{ or } x = -3 + 2$$

$$x = 3 + 2 \text{ or } x = -1$$

$$x = 5$$

$$S.S = \{5, -1\}$$

10. Solve $x^2 - 5x + 2 = 0$ by using the quadratic formula.

Sol. $x^2 - 5x + 2 = 0$ (given)

$ax^2 + bx + c = 0$ (standard form)

Comparing the given equation with the standard form.

$$a = 1, b = -5, c = 2$$

Substituting the values of a, b, c in the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(2)}}{2(1)}$$



Point of intersection on x-axis is $(-\frac{2}{3}, 0)$ and the point of intersection on y-axis is $(0, 2)$

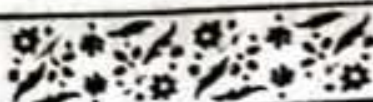
13. Find the point of intersecting of the linear equation $2x+y=5$ with coordinate axes.

Sol. $2x + y = 5$ is an equation of line. Now we draw the graph of $2x + y = 5$

Table of the values for $y = 5 - 2x$

x	$y = 5 - 2x$	Points (x,y)
-2	$y = 5 - 2(-2)$ $= 5 + 4 = 9$	$(-2, 9)$
0	$y = 5 - 2(0)$ $= 5 - 0 = 5$	$(0, 5)$
$\frac{5}{2}$	$y = 5 - 2(\frac{5}{2})$ $= 5 - 5 = 0$	$(\frac{5}{2}, 0)$
3	$y = 5 - 2(3)$ $= 5 - 6 = -1$	$(3, -1)$

We plot the points $(-2, 9), (0, 5), (\frac{5}{2}, 0)$ and $(3, -1)$ and join these points.



Here, $a = 3, b = -3, c = 5$

As α, β are the roots of the given equation $3x^2 - 3x + 5 = 0$,

therefore $S: \alpha + \beta = \frac{-b}{a} = \frac{-(-3)}{3} = 1$

$P: \alpha\beta = \frac{c}{a} = \frac{5}{3}$

Now, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$$= (1)^2 - 2\left(\frac{5}{3}\right)$$

$$= 1 - \frac{10}{3}$$

$$= \frac{3 - 10}{3} = -\frac{7}{3}$$

19. If α, β are the roots of equation $x^2 - 3x - 11 = 0$, then find $\frac{1}{\alpha} + \frac{1}{\beta}$

Sol. Standard form of the quadratic equation is $ax^2 + bx + c = 0$, where $a \neq 0$ and $a, b, c \in \mathbb{R}$

The given equation is

$$x^2 - 3x - 11 = 0$$



28. Examine the nature of the roots

$$25x^2 - 30x + 9 = 0$$

Sol. $25x^2 - 30x + 9 = 0$

$$a = 25, b = -30, c = 9$$

$$\text{Disc} = b^2 - 4ac$$

$$= (-30)^2 - 4(25)(9)$$

$$= 900 - 900 = 0$$

As the discriminant is zero. Therefore the roots are real and equal.

29. Examine the nature of the roots of

$$5x^2 - x - 2 = 0$$

Sol. $5x^2 - x - 2 = 0$

$$a = 5, b = -1, c = -2$$

$$\text{Disc} = b^2 - 4ac$$

$$= (-1)^2 - 4(5)(-2)$$

$$= 1 + 40 = 41 > 0 \text{ and is not a}$$

perfect square.

So, the roots are irrational and unequal.

30. Examine the nature of the roots of

32. If a quadratic equation has roots $x = 2$ and $x = 3$ and the critical points are $(2,0)$ and $(3,0)$. What are the resulting intervals on number line?

Ans. As a quadratic equation has roots $x = 2$ and $x = 3$. Therefore, the critical points $(2,0)$ and $(3,0)$ divide the number line into three intervals $(-\infty, 2)$, $(2, 3)$, $(3, \infty)$

33. What are the inequality symbols?

Ans. These are the inequalities symbols $<$ (less than), $>$ (greater than), $\leq$ (less than or equal to) and $\geq$ (greater than or equal to)

2.6 Changing the Subject of Formula

34. If $y^2 = 4ax$, make 'a' as the subject of the equation.

Sol. $y^2 = 4ax$

**UNIT
03****Matrices and Determinants****Students Learning Outcomes****After completing this unit the students will be able to:**

- ▶ Display information in the form of matrix of order 2
- ▶ Solve situations involving sum, difference and product of two matrices
- ▶ Calculate the product of the scalar quantity and a matrix.
- ▶ Evaluate the determinant and inverse of a matrix of order 2-by-2
- ▶ Solve the simultaneous linear equations in two variables using matrix inversion method and Cramer's rule.
- ▶ Explain, with examples, how mathematics plays a key role in the development of new scientific theories and technologies. [e.g., Mathematical models and simulations are used to design and optimize new materials and drugs, and to understand the behaviours of complex systems such as the human brain,
- ▶ Apply concepts of matrices to real world problems (such as engineering, economics, computer graphics and physics)

2. Write the order of each matrix.

$$A = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, B = [3 \ 4], C = \begin{bmatrix} 3 & 5 \\ 5 & 6 \end{bmatrix}, D = \begin{bmatrix} 0 & 3 & 4 \\ 5 & 0 & 9 \end{bmatrix}, E = \begin{bmatrix} 0 & 1 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}, F = [5]$$

Sol. $A = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Order of the matrix A is 2-by-1

$$C = \begin{bmatrix} 3 & 5 \\ 5 & 6 \end{bmatrix}$$

Order of the matrix C is 2-by-2

$$E = \begin{bmatrix} 0 & 1 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

Order of the matrix E is 3-by-2

$$B = [3 \ 4]$$

Order of the matrix B is 1-by-2

$$D = \begin{bmatrix} 0 & 3 & 4 \\ 5 & 0 & 9 \end{bmatrix}$$

Order of the matrix D is 2-by-3

$$F = [5]$$

Order of the matrix F is 1-by-1

3. Which of the following matrices are equal?

$$A = \begin{bmatrix} 0 & 3 \\ 4 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 9 \\ -7 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 \times 3 & 2 - 1 \\ 2 \times 2 & 4 - 2 \\ 4 + 4 & 3 + 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 5 + 4 \\ 8 + 1 \end{bmatrix}$$

$$E = \begin{bmatrix} 6 & 1 \\ 4 & 2 \end{bmatrix}$$

$$F = \begin{bmatrix} 3 - 3 & 3 \\ 3 + 1 & 1 \end{bmatrix}$$



7. If $\begin{bmatrix} p+q & 5 \\ 11 & p-2q \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 11 & 0 \end{bmatrix}$, then find the values of p and q .

Sol. Given that $\begin{bmatrix} p+q & 5 \\ 11 & p-2q \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 11 & 0 \end{bmatrix}$

As both matrices are equal, so their corresponding elements are same

$$p+q = 6 \quad \text{--- (i)}$$

$$p-2q = 0 \quad \text{--- (ii)}$$

From (ii)

$$p-2q = 0$$

$$p = 2q$$

Put $p = 2q$ in (i)

$$p+q = 6$$

$$2q+q = 6$$

$$3q = 6$$

$$q = \frac{6}{3} = 2$$

$$q = 2 \Rightarrow q = 2$$

Putting $q = 2$ into (i)

$$p+q = 6$$

$$p+2 = 6$$

$$p = 6-2 = 4 \Rightarrow p = 4$$

Hence, $p = 4, q = 2$

3.2

Types of Matrices

(i) **Row Matrix:** A matrix having only one row is called a row matrix.

For example: $A = [0 \ 1 \ 4]$, $B = [a \ b]$, $C = [a]$ are row matrices of order 1×3 , 1×2 and 1×1 respectively.



Thus, B is a skew-symmetric matrix.

(vii) **Negative of a Matrix:** The negative of a matrix A is obtained by multiplying each of its elements by -1 . It is denoted by $-A$.

For example: If $A = \begin{bmatrix} 3 & -6 \end{bmatrix}$, then $-A = \begin{bmatrix} -3 & 6 \end{bmatrix}$

Solved Exercise 3.2

1. From the following matrices identify unit matrices, row matrices, column matrices and null matrices.

$$A = \begin{bmatrix} 5 & 7 & 8 \end{bmatrix}, B = \begin{bmatrix} 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, E = \begin{bmatrix} 6 \\ 0 \\ 8 \end{bmatrix}, F = \begin{bmatrix} 7 \\ 1 \\ 9 \end{bmatrix}$$

Sol. Matrix A is a row matrix of order 1-by-3

Matrix B is a null matrix of order 1-by-1

It is also a row matrix and a column matrix.

Matrix C is a unit matrix of order 2-by-2

Matrix D is a null matrix of order 2-by-2

Matrix E is a column matrix of order 3-by-1

Matrix F is a column matrix of order 3-by-1

2. Identify type of the given matrices as row, column, square and rectangular matrices.

Multiplication of a Matrix by a Scalar

For a given matrix $A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ and a scalar k , we define a new matrix as $kA = \begin{bmatrix} ka & kb & kc \\ kd & ke & kf \end{bmatrix}$.

that is each element has been multiplied by a scalar k . In particular if $k = 1$, then

$$1A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

Addition of Matrices

If A and B are two matrices of the same order, these can be added, then their sum denoted by $A + B$, is again a matrix of the same order. They are said to be **conformable for addition**. $A + B$ is obtained by adding the corresponding entries of A and B .

Example 1: If $A = \begin{bmatrix} 3 & 8 & 2 \\ 5 & 3 & 1 \\ 6 & 0 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 & 2 \\ 3 & 0 & 6 \\ 2 & 3 & 4 \end{bmatrix}$, then find $A + B$ and $B + A$.

Solution : Given that $A = \begin{bmatrix} 3 & 8 & 2 \\ 5 & 3 & 1 \\ 6 & 0 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 & 2 \\ 3 & 0 & 6 \\ 2 & 3 & 4 \end{bmatrix}$

Order of $A = 3$ -by- 3 and order of $B = 3$ -by- 3 . A and B are conformable for addition.

$$= \begin{bmatrix} 1+3 & 2+1 & 3+3 \\ 4+4 & 5+2 & 6+2 \\ 7+0 & 8-1 & 9+0 \end{bmatrix} + \begin{bmatrix} 0 & 2 & 0 \\ 3 & 0 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 3 & 6 \\ 8 & 7 & 8 \\ 7 & 7 & 9 \end{bmatrix} + \begin{bmatrix} 0 & 2 & 0 \\ 3 & 0 & 3 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 5 & 6 \\ 11 & 7 & 11 \\ 8 & 8 & 10 \end{bmatrix} \quad \dots(i)$$

$$\text{and } A + (B + C) = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} + \left(\begin{bmatrix} 3 & 1 & 3 \\ 4 & 2 & 2 \\ 0 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 2 & 0 \\ 3 & 0 & 3 \\ 1 & 1 & 1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} + \begin{bmatrix} 3+0 & 1+2 & 3+0 \\ 4+3 & 2+0 & 2+3 \\ 0+1 & -1+1 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 3 & 3 \\ 7 & 2 & 5 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 5 & 6 \\ 11 & 7 & 11 \\ 8 & 8 & 10 \end{bmatrix} \quad \dots(ii)$$

Hence, from (i) and (ii) $(A + B) + C = A + (B + C)$

Associative law of addition of matrices holds.

Additive Identity of a Matrix

The additive identity of a matrix is a matrix that, when added to any matrix, leaves it unchanged.

(iii) $4X - Z$

$$\begin{aligned} 4X - Z &= 4 \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 4 & -4 \\ -8 & 8 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 4-3 & -4-0 \\ -8-0 & 8+2 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ -8 & 10 \end{bmatrix} \end{aligned}$$

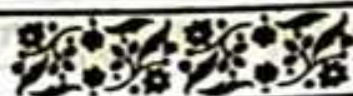
(iv) $X + 2Y + 3Z$

$$\begin{aligned} X + 2Y + 3Z &= \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix} + 2 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + 3 \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} + \begin{bmatrix} 9 & 0 \\ 0 & -6 \end{bmatrix} \\ &= \begin{bmatrix} 1+2+9 & -1+4+0 \\ -2+6+0 & 2+8-6 \end{bmatrix} = \begin{bmatrix} 12 & 3 \\ 4 & 4 \end{bmatrix} \end{aligned}$$

(v) $X - 4Y + Z$

$$\begin{aligned} X - 4Y + Z &= \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix} - 4 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix} - \begin{bmatrix} 4 & 8 \\ 12 & 16 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix} \end{aligned}$$

$$\begin{bmatrix} 1-4+3 & -1-8+0 \\ -2-12+0 & 2-16-2 \end{bmatrix} = \begin{bmatrix} 0 & -9 \\ -14 & -16 \end{bmatrix}$$



$$= \begin{bmatrix} 7+1 & 10-2 \\ -1+0 & 10+5 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ -1 & 15 \end{bmatrix} \text{---(i)}$$

$$2A + (B+C) = 2 \begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix} + \left[\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix} \right]$$

$$= \begin{bmatrix} 4 & 6 \\ -6 & 4 \end{bmatrix} + \begin{bmatrix} 3+1 & 4-2 \\ 5+0 & 6+5 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ -6 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 2 \\ 5 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 4+4 & 6+2 \\ -6+5 & 4+11 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ -1 & 15 \end{bmatrix} \text{---(ii)}$$

Hence, From (i) and (ii) $(2A+B) + C = 2A + (B+C)$

(iv) $3(A + B) = 3A + 3B$

Sol. $3(A + B) = 3 \left[\begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix} \right]$

$$= 3 \begin{bmatrix} 2+3 & 3+4 \\ -3+5 & 2+6 \end{bmatrix} = 3 \begin{bmatrix} 5 & 7 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} 15 & 21 \\ 6 & 24 \end{bmatrix} \text{---(i)}$$

$$3A + 3B = 3 \begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix} + 3 \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 9 \\ -9 & 6 \end{bmatrix} + \begin{bmatrix} 9 & 12 \\ 15 & 18 \end{bmatrix} = \begin{bmatrix} 6+9 & 9+12 \\ -9+15 & 6+18 \end{bmatrix} = \begin{bmatrix} 15 & 21 \\ 6 & 24 \end{bmatrix}$$

3.4

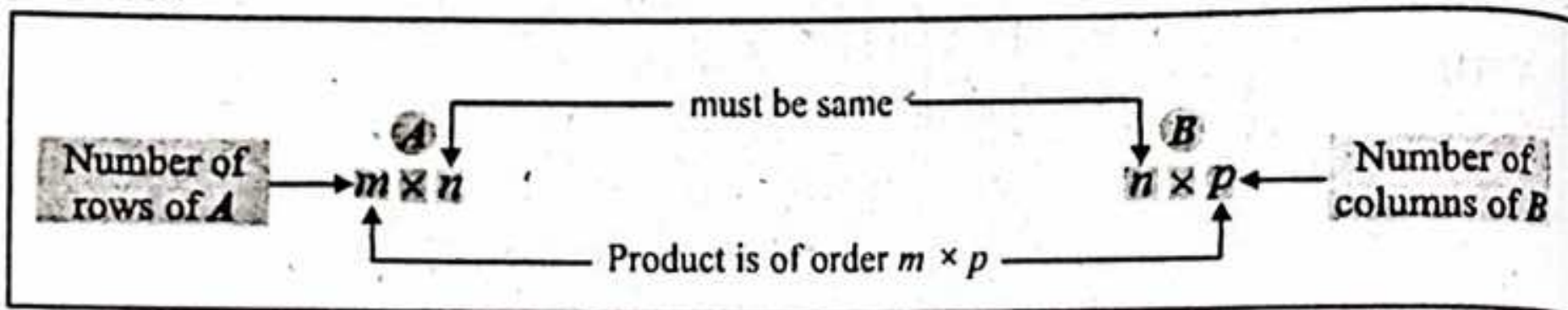
Multiplication of Matrices

A matrix A is said to be conformable for multiplication with a matrix B if the number of columns of A is equal to the number rows of B.

Thus, if A and B are given two matrices, then the product of matrices A and B is denoted by AB and its order will be m-by-p

The order of AB is m-by-p = number of rows of A by number of columns of B

Remember!



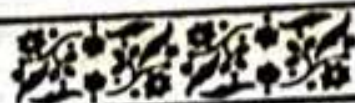
Remember!

For Multiplication

If A and B are two matrices two and we multiply A with B when:

- Number of columns in A = Number of rows in B.
- Multiplication of two matrices A and B is written as AB.
- To find AB, we start with the first row of A and multiply its each element with the corresponding elements of the first column of B and add the products. This gives the first element of the first row of the product matrix AB.

Next, we multiply each element of the first row of A with the corresponding element of second



Now

$$\begin{aligned}
 BA &= \begin{bmatrix} 1 & 5 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 5 & 2 \\ 3 & 4 \end{bmatrix} \\
 &= \begin{bmatrix} 1 \times 5 + 5 \times 3 & 1 \times 2 + 5 \times 4 \\ 0 \times 5 + (-2) \times 3 & 0 \times 2 + (-2) \times 4 \end{bmatrix} \\
 &= \begin{bmatrix} 5 + 15 & 2 + 20 \\ 0 - 6 & 0 - 8 \end{bmatrix} = \begin{bmatrix} 20 & 22 \\ -6 & -8 \end{bmatrix} \quad \dots(ii)
 \end{aligned}$$

From (i) and (ii), it is verified that $AB \neq BA$

Thus, commutative law under multiplication does not hold in general in the multiplication of matrices.

Associative Law of Multiplication of Matrices

If A, B and C are any three matrices, then $(AB)C = A(BC)$ is called associative law of multiplication of matrices.

Let $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 \\ 4 & 0 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1(2) + 0(0) & 1(1) + 0(3) \\ 2(2) + 1(0) & 2(1) + 1(3) \end{bmatrix} = \begin{bmatrix} 2 + 0 & 1 + 0 \\ 4 + 0 & 2 + 3 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & 0 \end{bmatrix}$$



$$AB = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1(4)+2(1) & 1(3)+2(2) \\ 0(4)+1(1) & 0(3)+1(2) \end{bmatrix}$$

$$= \begin{bmatrix} 4+2 & 3+4 \\ 0+1 & 0+2 \end{bmatrix} = \begin{bmatrix} 6 & 7 \\ 1 & 2 \end{bmatrix}$$

$$AC = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1(2)+2(0) & 1(1)+2(1) \\ 0(2)+1(0) & 0(1)+1(1) \end{bmatrix}$$

$$= \begin{bmatrix} 2+0 & 1+2 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$$

$$AB - AC = \begin{bmatrix} 6 & 7 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 6-2 & 7-3 \\ 1-0 & 2-1 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 1 & 1 \end{bmatrix} \quad \dots(ii)$$

Hence from (i) and (ii)

$$A(B - C) = AB - AC$$

Similarly, we can verify right distributive law over subtraction.

Multiplicative Identity of a Matrix

The multiplicative Identity of a matrix is a matrix that, when multiplied with any matrix A, does not change it. i.e., $AI = IA = A$

$$\begin{bmatrix} 2 & 3 \end{bmatrix}$$

$$(iv) A = \begin{bmatrix} 1 & 2 \\ -1 & 1 \\ 3 & 0 \end{bmatrix}, B = \begin{bmatrix} -1 & 4 & 1 \\ 2 & 3 & 1 \end{bmatrix}$$

Sol. Given that $A = \begin{bmatrix} 1 & 2 \\ -1 & 1 \\ 3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 4 & 1 \\ 2 & 3 & 1 \end{bmatrix}$

Number of columns in matrix A = 2

Number of rows in matrix B = 2

Thus, AB is possible.

$$\begin{aligned} AB &= \begin{bmatrix} 1 & 2 \\ -1 & 1 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} -1 & 4 & 1 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1(-1)+2(2) & 1(4)+2(3) & 1(1)+2(1) \\ -1(-1)+1(2) & -1(4)+1(3) & -1(1)+1(1) \\ 3(-1)+0(2) & 3(4)+0(3) & 3(1)+0(1) \end{bmatrix} \\ &= \begin{bmatrix} -1+4 & 4+6 & 1+2 \\ 1+2 & -4+3 & -1+1 \\ -3+0 & 12+0 & 3+0 \end{bmatrix} = \begin{bmatrix} 3 & 10 & 3 \\ 3 & -1 & 0 \\ -3 & 12 & 3 \end{bmatrix} \end{aligned}$$

And number of columns in matrix B = 3



$$= \begin{bmatrix} 5(-3)+1(-1) & 5(-2)+1(-1) \\ -1(-3)+4(-1) & -1(-2)+4(-1) \end{bmatrix} = \begin{bmatrix} -15-1 & -10-1 \\ 3-4 & +2-4 \end{bmatrix}$$

$$= \begin{bmatrix} -16 & -11 \\ -1 & -2 \end{bmatrix} \quad \dots(i)$$

$$AB = \begin{bmatrix} 5 & 1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 5(2)+1(0) & 5(-1)+1(3) \\ -1(2)+4(0) & -1(-1)+4(3) \end{bmatrix}$$

$$= \begin{bmatrix} 10+0 & -5+3 \\ -2+0 & 1+12 \end{bmatrix} = \begin{bmatrix} 10 & -2 \\ -2 & 13 \end{bmatrix}$$

$$AC = \begin{bmatrix} 5 & 1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 5 & 1 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 5(5)+1(1) & 5(1)+1(4) \\ -1(5)+4(1) & -1(1)+4(4) \end{bmatrix}$$

$$= \begin{bmatrix} 25+1 & 5+4 \\ -5+4 & -1+16 \end{bmatrix} = \begin{bmatrix} 26 & 9 \\ -1 & 15 \end{bmatrix}$$

$$AB - AC = \begin{bmatrix} 10 & -2 \\ -2 & 13 \end{bmatrix} - \begin{bmatrix} 26 & 9 \\ -1 & 15 \end{bmatrix} = \begin{bmatrix} 10-26 & -2-9 \\ -2-(-1) & 13-15 \end{bmatrix}$$

$$= \begin{bmatrix} -16 & -11 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} -16 & -11 \\ -1 & -2 \end{bmatrix} \quad \dots(ii)$$



$$= \begin{bmatrix} 10+1 & 2-4 \\ 0-3 & 0+12 \end{bmatrix} = \begin{bmatrix} 11 & -2 \\ -3 & 12 \end{bmatrix}$$

$$CA = \begin{bmatrix} 5 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 5 & 1 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 5(5)+1(-1) & 5(1)+1(4) \\ 1(5)+4(-1) & 1(1)+4(4) \end{bmatrix}$$

$$= \begin{bmatrix} 25-1 & 5+4 \\ 5-4 & 1+16 \end{bmatrix} = \begin{bmatrix} 24 & 9 \\ 1 & 17 \end{bmatrix}$$

$$BA + CA = \begin{bmatrix} 11 & -2 \\ -3 & 12 \end{bmatrix} + \begin{bmatrix} 24 & 9 \\ 1 & 17 \end{bmatrix}$$

$$= \begin{bmatrix} 11+24 & -2+9 \\ -3+1 & 12+17 \end{bmatrix} = \begin{bmatrix} 35 & 7 \\ -2 & 29 \end{bmatrix} \dots(ii)$$

Hence, verified from (i) and (ii) that $(B + C) A = BA + CA$

3. If $\begin{bmatrix} 4 & a \\ b & 3 \end{bmatrix} \begin{bmatrix} 6 \\ 6 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$, then find the values of a and b .

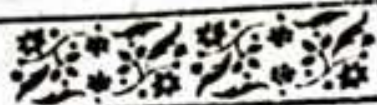
Sol. $\begin{bmatrix} 4 & a \\ b & 3 \end{bmatrix} \begin{bmatrix} 6 \\ 6 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$

$$\begin{bmatrix} 4(6) + a(6) \\ 6b + 18 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

$$a = -\frac{18}{6}$$

$$a = -3$$

$$6b + 18 = 3$$



diagonal elements and changing the signs of other elements. It is denoted as $\text{Adj } A$.

$$\text{If } A = \begin{bmatrix} 4 & 2 \\ -6 & 1 \end{bmatrix}, \text{ then } \text{Adj } A = \begin{bmatrix} 1 & -2 \\ 6 & 4 \end{bmatrix}$$

Multiplicative Inverse of a Non-Singular Matrix

We know $\frac{3}{4}$ and $\frac{4}{3}$ are multiplicative inverse (or reciprocal) as the product $\left(\frac{3}{4} \times \frac{4}{3}\right)$ is 1.

Similarly, if we have two square matrices of same order and their product is a unit matrix, then these matrices are called multiplicative inverse of each other.

Remember!

The inverse of a singular matrix does not exist.

Let A be any square matrix and there exists a square matrix B having the same order as A such that $AB = BA = I$ (a unit matrix). Then A and B are multiplicative inverse of each other. Hence $AA^{-1} = A^{-1}A = I$.

We denote multiplicative inverse of A by A^{-1} .

Adjoint Method to Find Inverse of a square Matrix

$$\text{Let } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ be a non-singular}$$

matrix, then inverse of A is:

$$\text{i.e., } A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$A^{-1} = \begin{bmatrix} \frac{3}{4} & -\frac{5}{4} \\ -1 & 2 \end{bmatrix}$$

Let us verify that A and A^{-1} are multiplicative inverse of each other.



of each other.

Sol. Given that

$$A = \begin{bmatrix} 4 & -2 \\ -6 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{3}{2} & 1 \end{bmatrix}$$

$$\begin{aligned} \text{Now } AB &= \begin{bmatrix} 4 & -2 \\ -6 & 4 \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{3}{2} & 1 \end{bmatrix} \\ &= \begin{bmatrix} 4 \times 1 + (-2) \times \frac{3}{2} & 4 \times \frac{1}{2} + (-2) \times 1 \\ (-6) \times 1 + 4 \times \frac{3}{2} & (-6) \times \frac{1}{2} + 4 \times 1 \end{bmatrix} \\ &= \begin{bmatrix} 4 - 3 & 2 - 2 \\ -6 + 6 & -3 + 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \end{aligned}$$

$$\text{and } BA = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{3}{2} & 1 \end{bmatrix} \begin{bmatrix} 4 & -2 \\ -6 & 4 \end{bmatrix}$$

Example 9: If $A = \begin{bmatrix} x & 8 \\ 5 & 10 \end{bmatrix}$ is a singular matrix, then find the value of x .

Sol. If $A = \begin{bmatrix} x & 8 \\ 5 & 10 \end{bmatrix}$ is a singular matrix, then $|A| = 0$

$$\text{Now } |A| = \begin{vmatrix} x & 8 \\ 5 & 10 \end{vmatrix}$$

Since A is singular, therefore,

$$|A| = 10x - 40 = 0$$

$$\text{i.e. } 10x = 40$$

$$\Rightarrow x = 4$$

Challenge!

Take any 2 by 2 matrix and check whether it is singular or non-singular. Also find its adjoint.

Sol. Let

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$



$$A^{-1} = \frac{1}{|A|} \times \text{Adj } A$$

$$= \frac{1}{25} \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} \frac{5}{25} & \frac{0}{25} \\ \frac{0}{25} & \frac{5}{25} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{5} \end{bmatrix}$$

(ii) $\begin{bmatrix} -4 & 8 \\ 7 & 2 \end{bmatrix}$

Sol. Let

$$A = \begin{bmatrix} -4 & 8 \\ 7 & 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} -4 & 8 \\ 7 & 2 \end{vmatrix}$$

$$= -(4) - 7(8) = -8 - 56 = -64 \neq 0$$

A is non-singular, so A^{-1} is possible

$$A^{-1} = \frac{1}{|A|} \times \text{Adj } A$$

$$\text{Adj } A = \begin{bmatrix} 2 & -8 \\ -5 & 40 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj } A$$

$$= \frac{1}{40} \times \begin{bmatrix} 2 & -8 \\ -5 & 40 \end{bmatrix} = \begin{bmatrix} \frac{2}{40} & \frac{-8}{40} \\ \frac{-5}{40} & \frac{40}{40} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{20} & -\frac{1}{5} \\ -\frac{1}{8} & 1 \end{bmatrix}$$

(iv) $\begin{bmatrix} 3 & 5 \\ 5 & -3 \end{bmatrix}$

Sol. Let

$$A = \begin{bmatrix} 3 & 5 \\ 5 & -3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 5 \\ 5 & -3 \end{vmatrix}$$

$$= 3(-3) - 5(5) = -9 - 25 = -34 \neq 0$$

A is non-singular. So A^{-1} is possible



$$= \begin{bmatrix} 2(2) + (-1)(3) & 2(1) + (-1)(2) \\ -3(2) + 2(3) & -3(1) + 2(2) \end{bmatrix}$$

$$= \begin{bmatrix} 4-3 & 2-2 \\ -6+6 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Since $\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} = I$

So, these are the multiplicative inverses of each other.

(ii) $\begin{bmatrix} 4 & 15 \\ 2 & 8 \end{bmatrix}, \begin{bmatrix} 4 & -\frac{15}{2} \\ -1 & 2 \end{bmatrix}$

Sol. Multiply both matrices

$$\begin{bmatrix} 4 & 15 \\ 2 & 8 \end{bmatrix} \begin{bmatrix} 4 & -\frac{15}{2} \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4(4) + 15(-1) & 4\left(-\frac{15}{2}\right) + 15(2) \\ 2(4) + 8(-1) & 2\left(-\frac{15}{2}\right) + 8(2) \end{bmatrix}$$

$$|AB| = \begin{vmatrix} 0 & -1 \\ -8 & 7 \end{vmatrix}$$

$$= 0(7) - (-8)(-1) = 0 - 8 = -8 \neq 0$$

AB is a non-singular, so $(AB)^{-1}$ is possible.

$$\text{Adj}(AB) = \begin{bmatrix} 7 & 1 \\ 8 & 0 \end{bmatrix}$$

$$(AB)^{-1} = \frac{1}{|AB|} \times \text{Adj}(AB)$$

$$= \frac{1}{-8} \begin{bmatrix} 7 & 1 \\ 8 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{7}{-8} & \frac{1}{-8} \\ \frac{8}{-8} & \frac{0}{-8} \end{bmatrix} = \begin{bmatrix} -\frac{7}{8} & -\frac{1}{8} \\ -1 & 0 \end{bmatrix} \quad \dots(i)$$

$$B = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$$

$$|B| = \begin{vmatrix} 2 & -1 \\ -3 & 2 \end{vmatrix}$$

$$= 2(2) - (-3)(-1) = 4 - 3 = 1 \neq 0$$

B is non singular, so B^{-1} is possible.



$$\text{Adj } A = \begin{bmatrix} 0 & -2 \\ -8 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj } A$$

$$= \frac{1}{-16} \begin{bmatrix} 0 & -2 \\ -8 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{0}{-16} & \frac{-2}{-16} \\ \frac{-8}{-16} & \frac{1}{-16} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{1}{8} \\ \frac{1}{2} & -\frac{1}{16} \end{bmatrix}$$

$$B^{-1}A^{-1} = \begin{bmatrix} \frac{2}{5} & \frac{1}{5} \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & \frac{1}{8} \\ \frac{1}{2} & -\frac{1}{16} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{5}(0) + \left(\frac{1}{5}\right)\left(\frac{1}{2}\right) & \frac{2}{5}\left(\frac{1}{8}\right) + \frac{1}{5}\left(-\frac{1}{16}\right) \\ -1\left(0\right) + 0\left(\frac{1}{2}\right) & -1\left(\frac{1}{8}\right) + 0\left(-\frac{1}{16}\right) \end{bmatrix}$$

Simultaneous linear equations are a set of two or more linear equations with two or more variables that are solved together (simultaneously) to find a common solution. The general form of system of two linear equations in two variables is given below:

$$ax + by = m$$

$$cx + dy = n$$

where a, b, c, d, m and n are real numbers. Here we will find the solution of two simultaneous equations in two variables by the following methods:

- (i) Matrix Inversion Method
- (ii) Cramer's Rule

Matrix Inversion Method

Look at the solution of the following pair of equation.

Let $ax + by = m$

and $cx + dy = n$



$$= \frac{1}{-30} \begin{bmatrix} -2(25) + (-4)(10) \\ -6(25) + 3(10) \end{bmatrix} = \frac{1}{-30} \begin{bmatrix} -50 - 40 \\ -150 + 30 \end{bmatrix}$$

$$= \frac{1}{-30} \begin{bmatrix} -90 \\ -120 \end{bmatrix} = \begin{bmatrix} \frac{-90}{-30} \\ \frac{-120}{-30} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$x = 3, y = 4$$

$$SS = \{(3, 4)\}$$

History!

Gabriel Cramer was a Genevan mathematician. Cramer showed promised in Mathematics from an early age. At 18 he received his doctorate in Mathematics.

In 1750 he published Cramer's rule, giving a general formula for the solution for any unknown in



or

$$IX = A^{-1} B$$

$$X = A^{-1} B \quad [\text{as } IX = X]$$

$$X = \frac{\text{Adj } A}{|A|} \times B$$

$$\left[A^{-1} = \frac{\text{Adj } A}{|A|} \right]$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \begin{bmatrix} m \\ n \end{bmatrix}}{|A|}$$

$$= \frac{\begin{bmatrix} dm - bn \\ -cm + an \end{bmatrix}}{|A|}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{dm - bn}{|A|} \\ \frac{-cm + an}{|A|} \end{bmatrix}$$

$$dm - bn \quad an - cm$$

Example 13: Eight years ago Huria's was $\frac{3}{4}$ of Jannat's age. After four years Huria's age will be $\frac{6}{7}$ of Jannat's age. Find their present ages by using matrices.

Solution: Suppose Huria's present age is x years and Jannat's present age is y years. Eight years ago their ages were $x - 8$ and $y - 8$. According to first condition:

$$x - 8 = \frac{3}{4} (y - 8)$$

$$\text{or } 4x - 32 = 3y - 24$$

$$\text{or } 4x - 3y = -24 + 32$$

$$4x - 3y = 8 \quad (\text{i})$$

After 4 years their ages will be $x + 4$ and $y + 4$.

Applying the second condition:

$$x + 4 = \frac{6}{7} (y + 4)$$

$$= \frac{-48 - 12}{-3} = \frac{-60}{-3} = 20$$

$$= \frac{-60}{-3} = 20$$

$$y = \frac{|A_y|}{|A|} = \frac{\begin{vmatrix} 4 & 8 \\ 7 & -4 \end{vmatrix}}{-3}$$

$$= \frac{-16 - 56}{-3} = \frac{-72}{-3} = 24$$

$\therefore$ Present ages of Huria and Jannat are 20 years and 24 years respectively.

3.8

The Role of Mathematics in Scientific Theories and Technological Advancement

Mathematics is often described as the language of science. It plays a fundamental role in developing scientific theories and advancing modern technologies. From

$$A^{-1} = \frac{1}{|A|} \times \text{Adj} A$$

$$= \frac{1}{-13} \begin{bmatrix} -1 & -2 \\ -5 & 3 \end{bmatrix}$$

Now $X = A^{-1}B$ Can be written as

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-13} \begin{bmatrix} -1 & -2 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} 7 \\ 16 \end{bmatrix}$$

$$= \frac{1}{-13} \begin{bmatrix} -1(7) + (-2)(16) \\ -5(7) + 3(16) \end{bmatrix}$$

$$= \frac{1}{-13} \begin{bmatrix} -7 - 32 \\ -35 + 48 \end{bmatrix}$$

$$= \frac{1}{-13} \begin{bmatrix} -39 \\ 13 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-39}{-13} \\ \frac{13}{-13} \end{bmatrix}$$

or $X = A^{-1}B$

Now $A = \begin{bmatrix} 1 & -2 \\ 2 & 7 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & -2 \\ 2 & 7 \end{vmatrix}$$

$$= 1(7) - 2(-2)$$

$$= 7 + 4$$

$$= 11 \neq 0$$

$$\text{Adj } A = \begin{bmatrix} 7 & 2 \\ -2 & 1 \end{bmatrix}$$

Therefore $A^{-1} = \frac{1}{|A|} \times \text{Adj } A$

$$= \frac{1}{11} \begin{bmatrix} 7 & 2 \\ -2 & 1 \end{bmatrix}$$

Now $X = A^{-1}B$ Can be written as

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11} \begin{bmatrix} 7 & 2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 9 \\ -4 \end{bmatrix}$$



$$|A_r| = \begin{vmatrix} 1 & 4 \\ 2 & 5 \end{vmatrix}$$

$$= 1(5) - 2(4)$$

$$= 5 - 8 = -3$$

$$y = \frac{|A_r|}{|A|}$$

$$= \frac{-3}{-9} = \frac{1}{3}$$

$$\text{Solution set} = \left\{ \left(\frac{8}{3}, \frac{1}{3} \right) \right\}$$

(ii) $x + 2y = 7$
 $3x - 2y = -3$

Sol. Given that $x + 2y = 7$
 $3x - 2y = -3$

Writing the equations in matrix form

$$\begin{bmatrix} x + 2y \\ 3x - 2y \end{bmatrix} = \begin{bmatrix} 7 \\ -3 \end{bmatrix}$$

or $\begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -3 \end{bmatrix}$

(iii) $2x - 5y = -6$
 $4x - 3y = -12$

Sol. Given that $2x - 5y = -6$
 $4x - 3y = -12$

Writing the equations in matrix form

$$\begin{bmatrix} 2x - 5y \\ 4x - 3y \end{bmatrix} = \begin{bmatrix} -6 \\ -12 \end{bmatrix}$$

or $\begin{bmatrix} 2 & -5 \\ 4 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ -12 \end{bmatrix}$

Here, we have

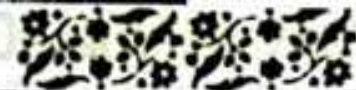
$$A = \begin{bmatrix} 2 & -5 \\ 4 & -3 \end{bmatrix}, A_x = \begin{bmatrix} -6 & -5 \\ -12 & -3 \end{bmatrix},$$

$$A_y = \begin{bmatrix} 2 & -6 \\ 4 & -12 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -5 \\ 4 & -3 \end{vmatrix}$$

$$= 2(-3) - 4(-5)$$

$$= -6 + 20 = 14 \neq 0$$



$$= \begin{bmatrix} -9 \\ -3 \\ -12 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

Thus $x = 3, y = 4$

current in branch A (x) = 3

current in branch B (y) = 4

4. Three forces act on a particle and must be in equilibrium i.e.

$$F_1 + F_2 + F_3 = 0,$$

where $F_1 = \begin{bmatrix} 8 \\ x \end{bmatrix}, F_2 = \begin{bmatrix} -2 \\ -7 \end{bmatrix}, F_3 = \begin{bmatrix} y \\ -1 \end{bmatrix}.$

Find the value of x and y .

Sol. Given

$$F_1 + F_2 + F_3 = 0$$

$$\begin{bmatrix} 8 \\ x \end{bmatrix} + \begin{bmatrix} -2 \\ -7 \end{bmatrix} + \begin{bmatrix} y \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Writing the equations in matrix form

$$\begin{bmatrix} A + B \\ 2A + 3B \end{bmatrix} = \begin{bmatrix} 100 \\ 240 \end{bmatrix}$$

or $\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 100 \\ 240 \end{bmatrix} \quad \text{---(i)}$

Let $C = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}, X = \begin{bmatrix} A \\ B \end{bmatrix}, D = \begin{bmatrix} 100 \\ 240 \end{bmatrix}$

Then (i) can be written as:

$$CX = D$$

or $X = C^{-1}D$

Now $C = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$

$$\begin{aligned} |C| &= \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} \\ &= 1(3) - (2)(1) \\ &= 3 - 2 = 1 \neq 0 \end{aligned}$$

$$\text{Adj } C = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$$

According to first condition:

$$x - 2 = 5(y - 2)$$

$$x - 2 = 5y - 10$$

$$x - 5y = -10 + 2$$

$$x - 5y = -8 \quad (i)$$

After 6 years the age of father will be
 $= x + 6$ years

After 6 years the age of son will
 $= y + 6$ years

According to second condition:

$$x + 6 = 3(y + 6)$$

$$x + 6 = 3y + 18$$

$$x - 3y = 18 - 6$$

$$x - 3y = 12 \quad \text{--- (ii)}$$

Matrix form of (i) and (ii)

$$\begin{bmatrix} x - 5y \\ x - 3y \end{bmatrix} = \begin{bmatrix} -8 \\ 12 \end{bmatrix}$$

$$\text{or } \begin{bmatrix} 1 & -5 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -8 \\ 12 \end{bmatrix} \quad \text{--- (iii)}$$

$$\text{Let } A = \begin{bmatrix} 1 & -5 \\ 1 & -3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} -8 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -3 & 5 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} -8 \\ 12 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} -3(-8 + 5(12)) \\ -1(-8) + 1(12) \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 24 + 60 \\ 8 + 12 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 84 \\ 20 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{84}{2} \\ \frac{20}{2} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 42 \\ 10 \end{bmatrix}$$

$$x = 42, y = 10$$

So, Man,s present age = 42 years

Son's present age = 10 years

Let $A = \begin{bmatrix} 1 & -1 \\ 4 & -3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} -7 \\ -12 \end{bmatrix}$

Then (iii) can be written as

$$A X = B$$

or

$$X = A^{-1} B$$

Now $A = \begin{bmatrix} 1 & -1 \\ 4 & -3 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & -1 \\ 4 & -3 \end{vmatrix}$$

$$= 1(-3) - 4(-1)$$

$$= -3 + 4 = 1 \neq 0$$

$$\text{Adj } A = \begin{bmatrix} -3 & 1 \\ -4 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj } A$$

$$= \frac{1}{1} \begin{bmatrix} -3 & 1 \\ -4 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 1 \end{bmatrix}$$

correct answer.

- (i) If $\begin{bmatrix} a+2 \\ 3 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$, then what is the value of a ?

(a) 3

(b) 5

(c) 6

(d) 7

- (ii) $A = [3 \ 5 \ 0]$ is a _____ matrix.

(a) row

(b) square

(c) column

(d) null

- (iii) $B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ is a _____ matrix.

(a) identity

(b) square

(c) row

(d) column

- (iv) $M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is a/an _____ matrix.

(a) rectangular

(b) identity

(c) column

(d) row

- (v) If $A^t = -A$, then A is _____ matrix.



(ii) $(A + B) + C = A + (B + C)$

Sol. $A + B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 2 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 2+5 & 3+6 \\ 4+2 & 5+1 \end{bmatrix} = \begin{bmatrix} 7 & 9 \\ 6 & 6 \end{bmatrix}$$

$$(A+B)+C = \begin{bmatrix} 7 & 9 \\ 6 & 6 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ 1 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 7+3 & 9+2 \\ 6+1 & 6+7 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 11 \\ 7 & 13 \end{bmatrix} \text{ (i)}$$

$$B+C = \begin{bmatrix} 5 & 6 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ 1 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 5+3 & 6+2 \\ 2+1 & 1+7 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ 3 & 8 \end{bmatrix}$$

$$A + (B + C) = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 8 & 8 \\ 3 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 21+9 & 14+63 \\ 18+6 & 12+42 \end{bmatrix}$$

$$= \begin{bmatrix} 30 & 77 \\ 24 & 54 \end{bmatrix} \text{ (i)}$$

$$AC = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 2(3)+3(1) & 2(2)+3(7) \\ 4(3)+5(1) & 4(2)+5(7) \end{bmatrix}$$

$$= \begin{bmatrix} 6+3 & 4+21 \\ 12+5 & 8+35 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 25 \\ 17 & 43 \end{bmatrix}$$

$$BC = \begin{bmatrix} 5 & 6 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 5(3)+6(1) & 5(2)+6(7) \\ 2(3)+1(1) & 2(2)+1(7) \end{bmatrix}$$

$$= \begin{bmatrix} 15+6 & 10+42 \\ 6 & 11 \end{bmatrix} = \begin{bmatrix} 21 & 52 \\ 6 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{7} & -\frac{6}{7} \\ -\frac{2}{7} & \frac{5}{7} \end{bmatrix} = \begin{bmatrix} -\frac{1}{7} & \frac{6}{7} \\ \frac{2}{7} & -\frac{5}{7} \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

$$\begin{aligned} |A| &= \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} \\ &= 2(5) - 4(3) \\ &= 10 - 12 = -2 \neq 0 \end{aligned}$$

$$\text{Adj } A = \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj } A$$

$$= \frac{1}{-2} \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{29}{14} & -\frac{15}{14} \\ -\frac{15}{7} & \frac{8}{7} \end{bmatrix} \quad \text{--- (ii)}$$

Hence, proved that $(AB)^{-1} = B^{-1}A^{-1}$ from (i) and (ii).

$$(vi) \quad AA^{-1} = A^{-1}A = I$$

$$\text{Sol. } A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}$$

$$\begin{aligned} &= 2(5) - 4(3) \\ &= 10 - 12 = -2 \neq 0 \end{aligned}$$

$$\text{Adj } A = \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj } A$$



$$= \begin{bmatrix} 63 & 137 \\ 119 & 263 \end{bmatrix} \text{ ————— (ii).}$$

Hence, Proved from (i) and (ii) that
(AB) C = A(BC)

4. If $A = \begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix}$, then find

(i) $|B|$

Sol. $B = \begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix}$

$$\begin{aligned} |B| &= \begin{vmatrix} 5 & 3 \\ 2 & 4 \end{vmatrix} \\ &= 5(4) - 2(3) \\ &= 20 - 6 = 14 \end{aligned}$$

(ii) $\text{Adj} B$

Sol. $B = \begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix}$

$$\text{Adj} B = \begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 7 & 3 \\ 2 & 1 \end{vmatrix}$$

$$\begin{aligned} &= 7(1) - 2(3) \\ &= 7 - 6 = 1 \neq 0 \end{aligned}$$

So, A^{-1} is possible

$$\text{Adj} A = \begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj} A$$

$$= \frac{1}{1} \begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix}$$

$$A^{-1} A = \begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix} \begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 7(1) - 3(2) & 1(3) + (-3)(1) \\ -2(7) + 7(2) & -2(3) + 7(1) \end{bmatrix}$$

$$= \begin{bmatrix} 7 - 6 & 3 - 3 \\ -14 + 14 & -6 + 7 \end{bmatrix}$$



$$|A_y| = \begin{vmatrix} 3 & 7 \\ 5 & 2 \end{vmatrix}$$

$$= 3(2) - 5(7)$$

$$= 6 - 35 = -29$$

$$y = \frac{|A_y|}{|A|}$$

$$= \frac{-29}{-23} = \frac{29}{23}$$

(ii) $x - 6y = -15$

$2x + 6y = -3$

Sol. By inversion method

Given that $x - 6y = -15$

$2x + 6y = -3$

Writing the equations in matrix form

$$\begin{bmatrix} x - 6y \\ 2x + 6y \end{bmatrix} = \begin{bmatrix} -15 \\ -3 \end{bmatrix}$$

or $\begin{bmatrix} 1 & -6 \\ 2 & 6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -15 \\ -3 \end{bmatrix}$ ——— (i)

Now $x = A^{-1}B$ can be written as.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{18} \begin{bmatrix} 6 & 6 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} -15 \\ -3 \end{bmatrix}$$

$$= \frac{1}{18} \begin{bmatrix} 6(-15) + 6(-3) \\ -2(-15) + 1(-3) \end{bmatrix}$$

$$= \frac{1}{18} \begin{bmatrix} -90 - 18 \\ +30 - 3 \end{bmatrix}$$

$$= \frac{1}{18} \begin{bmatrix} -108 \\ 27 \end{bmatrix} = \begin{bmatrix} \frac{-108}{18} \\ \frac{27}{18} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ \frac{3}{2} \end{bmatrix}$$

So, $x = -6, y = \frac{3}{2}$

$$\left(\begin{pmatrix} 6 & 3 \end{pmatrix} \right)$$



$$|A| = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix}$$

$$= 2 \times (3) - 1(1) = 6 - 1 = 5 \neq 0$$

$$|A_1| = \begin{vmatrix} 5 & 1 \\ 3 & 3 \end{vmatrix}$$

$$= 5(3) - 3(1) = 15 - 3 = 12$$

$$x = \frac{|A_1|}{|A|}$$

$$= \frac{12}{5}$$

$$|A_2| = \begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix}$$

$$= 2(3) - 1(5) = 6 - 5 = 1$$

$$y = \frac{|A_2|}{|A|}$$

$$= \frac{1}{5}$$

$$12 \quad 1$$

$$\text{Let } A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \quad B = \begin{bmatrix} 21 \\ 27 \end{bmatrix}$$

Then (i) can be written as

$$AX = B$$

$$\text{or } X = A^{-1} B$$

$$\text{Now } A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$$

$$= 2(2) - 1(1)$$

$$= 4 - 1 = 3 \neq 0$$

$$\text{Adj } A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj } A$$

$$= \frac{1}{3} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

Now $x = A^{-1} B$ can be written as



$$\text{Adj } A = \begin{bmatrix} 6 & -3 \\ -2 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{Adj } A$$

$$= \frac{1}{24} \begin{bmatrix} 6 & -3 \\ -2 & 5 \end{bmatrix}$$

Now $X = A^{-1} B$ can be written as

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{24} \begin{bmatrix} 6 & -3 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} 3500 \\ 5000 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{24} \begin{bmatrix} 6(3500) + (-3)(5000) \\ -2(3500) + 5(5000) \end{bmatrix}$$

$$= \frac{1}{24} \begin{bmatrix} 21000 - 15000 \\ -7000 + 25000 \end{bmatrix}$$

$$= \frac{1}{24} \begin{bmatrix} 6000 \\ 18000 \end{bmatrix} = \begin{bmatrix} \frac{6000}{24} \\ \frac{18000}{24} \end{bmatrix}$$

$$= \begin{bmatrix} 250 \\ 750 \end{bmatrix}$$

So, $x = 250$, $y = 750$

Hence, Daily wages of a man = Rs.250

Daily wages of a women = Rs. 750

Objective Type Questions



Additional MCQ's

Multiple Choice Questions prepared in the light of new Examination Techniques
(Knowledge, Understanding, Application, Analytical & Conceptual)



20. Which is not a diagonal Matrix?

(a) $\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix}$

(d) $\begin{bmatrix} -3 & 4 \\ 0 & 2 \end{bmatrix}$

21. What is the type of the matrix $A = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

(a) row

(b) scalar

(c) null

(d) rectangular

22. Identify the unit matrix.

(a) $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

23. Transpose of the matrix $A = \begin{bmatrix} -7 \\ 3 \\ 2 \end{bmatrix}$ is:

(a) $\begin{bmatrix} -7 & 3 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & -7 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ 2 \\ -7 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 3 \\ 3 & -7 \end{bmatrix}$

24. A square matrix A is symmetric if:

(a) $A = A^{-1}$

(b) $A = -A$

(c) $A' = A$

(d) $A' = -A$

25. Which is the example of skew symmetric matrix?

(a) $\begin{bmatrix} 0 & -7 \\ 7 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 5 \\ 5 & 7 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$



41. If $A = \begin{bmatrix} 5 & 7 \\ 0 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then AB is?

(a) $\begin{bmatrix} 7 & 5 \\ 4 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & -4 \\ 5 & 7 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & 7 \\ 0 & -4 \end{bmatrix}$

(d) $\begin{bmatrix} -5 & -7 \\ 0 & 4 \end{bmatrix}$

42. If $A = [-3 \ 7]$ and $B = \begin{bmatrix} -4 \\ -11 \end{bmatrix}$ the order of AB will be:

(a) 1 by 2

(b) 2 by 2

(c) 1 by 1

(d) 2 by 1

43. The associative law of multiplication of matrices is

(a) $AB = BA$

(b) $AB = I$

(c) $AB+C=AB+AC$

(d) $(AB)C = A(BC)$

44. Multiplicative identity of a matrix $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ is

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} -2 & -3 \\ -4 & -5 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix}$

3.5

Multiplicative Inverse of a matrix

45. If $A = \begin{bmatrix} 5 & 3 \\ 3 & 4 \end{bmatrix}$, then $|A|$ will be.

(a) 29

(b) 0

(c) 11

(d) 16

$\begin{vmatrix} 0 & -3 \end{vmatrix}$



38. $\begin{bmatrix} -8 & 12 \\ 0 & 16 \end{bmatrix}$ 39. $\begin{bmatrix} 2 & 3 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \end{bmatrix}$ 40. $[5]$ 41. $\begin{bmatrix} 5 & 7 \\ 0 & -4 \end{bmatrix}$ 42. 1 by 1
43. $(AB)C = A(BC)$ 44. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 45. 11 46. -6
47. $ad - bc$ 48. $\begin{bmatrix} 4 & 10 \\ 2 & 5 \end{bmatrix}$ 49. $\begin{bmatrix} 3 & 7 \\ 4 & 2 \end{bmatrix}$ 50. 2 51. $\begin{bmatrix} 4 & 7 \\ 2 & 9 \end{bmatrix}$
52. $\begin{bmatrix} 7 & 14 \\ 2 & 4 \end{bmatrix}$ 53. $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ 54. $\begin{bmatrix} 7 & 14 \\ 3 & 6 \end{bmatrix}$ 55. $B^{-1} A^{-1}$ 56. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
57. $|B| = 0$

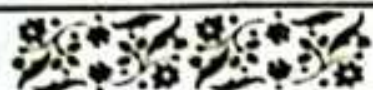
Additional Short Question

Short Answer Questions prepared in the light of new Examination Techniques
(Knowledge, Understanding, Application, Analytical & Conceptual)

3.1.1 Rows and Columns of a Matrix

1. Define a matrix.

Ans. A matrix is a rectangular array in shape, whose elements (entries) are written within square brackets in a specific order.



17. Define "Scalar matrix".

Ans. A diagonal matrix is called a scalar matrix if all the entries in the diagonal are the same.

18. Define "Identity matrix".

Ans. A scalar matrix is called an identity or unit matrix if all main diagonal entries are 1 and the rest are zero.

$$\text{For example } Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

19. Find the transpose of matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$\text{Sol. } A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

20. Find the transpose of matrix

$$-B = \begin{bmatrix} 1 & 0 \\ -4 & -7 \end{bmatrix}$$

23. If $A = \begin{bmatrix} 9 & 8 \\ 8 & 4 \end{bmatrix}$, then show it is a symmetric matrix.

$$\text{Sol. } A = \begin{bmatrix} 9 & 8 \\ 8 & 4 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 9 & 8 \\ 8 & 4 \end{bmatrix} = A$$

Hence, verified that A is a symmetric matrix because $A^t = A$

24. Show that $M = \begin{bmatrix} 0 & 13 \\ -13 & 0 \end{bmatrix}$ is a skew-symmetric matrix.

$$\text{Sol. } M = \begin{bmatrix} 0 & 13 \\ -13 & 0 \end{bmatrix}$$

$$M^t = \begin{bmatrix} 0 & -13 \\ 13 & 0 \end{bmatrix} = -M$$



34. Find the determinat of $C = \begin{bmatrix} -7 & -5 \\ -3 & 0 \end{bmatrix}$

Sol. $C = \begin{bmatrix} -7 & -5 \\ -3 & 0 \end{bmatrix}$

$$\begin{aligned} |C| &= \begin{vmatrix} -7 & -5 \\ -3 & 0 \end{vmatrix} \\ &= -7(0) - (-3)(-5) \\ &= 0 - 15 \\ &= -15 \end{aligned}$$

Singular and Non-Singular Matrices

35. Find whether the matrix $A = \begin{bmatrix} 4 & 7 \\ 12 & 21 \end{bmatrix}$ is singular or non singular

Sol. $A = \begin{bmatrix} 4 & 7 \\ 12 & 21 \end{bmatrix}$

$$\begin{aligned} |A| &= \begin{vmatrix} 4 & 7 \\ 12 & 21 \end{vmatrix} \\ &= 4(21) - 12(7) \end{aligned}$$

$$|B| = \begin{vmatrix} K & 6 \\ 7 & 14 \end{vmatrix} = 0$$

$$K(14) - 7(6) = 0$$

$$14K - 42 = 0$$

$$14K = 42$$

$$K = \frac{42}{14}$$

$$K = 3$$

Adjoint of a Matrix

38. Find the adjoint of matrix

$$A = \begin{bmatrix} 7 & 4 \\ 0 & 3 \end{bmatrix}$$

Sol. $A = \begin{bmatrix} 7 & 4 \\ 0 & 3 \end{bmatrix}$

$$\text{Adj } A = \begin{bmatrix} 3 & -4 \\ 0 & 7 \end{bmatrix}$$

39. Find the adjoint of $M = \begin{bmatrix} s & t \\ u & v \end{bmatrix}$

**UNIT
04****Functions and Graphs****Students' Learning Outcomes.**

After completing this unit, the students will be able to:

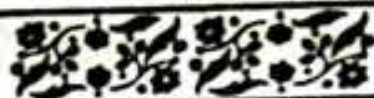
- ▶ Explain operations on functions and compositions of functions.
- ▶ Define inverse functions and demonstrate their domain and range with examples.
- ▶ Formulate composite functions as defined by $(f \circ g)(x) = f(g(x))$.
- ▶ Apply concepts from functions to real-world problems (such as finance, transportation and sales).
- ▶ Plot graphs of absolute valued functions.
- ▶ Solve absolute value linear equations and inequalities in one variable and express the solution as a range of values on a number line.
- ▶ Apply concepts of absolute valued functions to real-world problems (such as to calculate energy wave, magnitude and distance).

4.1**Function**

If X and Y are two non-empty sets, then a function from a set X to a set Y is a rule that assigns each element

History!

Gottfried Wilhelm Leibniz is



Example 6: Let $f(x) = x^2 - 4$, $g(x) = x + 2$. Find $\left(\frac{f}{g}\right)(x)$, where $x \neq -2$.

Solution: $f(x) = x^2 - 4$

$$g(x) = x + 2$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

$$\text{So, } = \frac{x^2 - 4}{x + 2} = \frac{(x + 2)(x - 2)}{(x + 2)} = x - 2$$

Composition of Functions

Composition of function is a process of combining two or more functions to produce a new single function. The composition operator "o" takes two functions f and g and results a new function.

Example 7: If $f(x) = 2x + 3$, $g(x) = x^2$, then find

(i) $(f \circ g)(x)$ (ii) $(g \circ f)(x)$ (iii) $(f \circ f)(x)$ (iv) $(g \circ g)(x)$

Solution: Given $f(x) = 2x + 3$, $g(x) = x^2$

$$\begin{aligned} \text{(i) } (f \circ g)(x) &= f(g(x)) \\ &= f(x^2) \\ &= 2x^2 + 3 \end{aligned}$$

Remember!

In general,

$$(f \circ g)(x) \neq (g \circ f)(x)$$



$$\therefore x = \frac{y-3}{2} \quad [\text{Solve for } x]$$

$$f^{-1}(y) = \frac{y-3}{2} \quad [\because y = f(x) \Rightarrow f^{-1}(y) = x]$$

$$f^{-1}(x) = \frac{x-3}{2} \quad [\text{Replacing } y \text{ by } x]$$

{ Domain $f(x)$: All real numbers
 { Range $f(x)$: All real numbers

{ Domain $f^{-1}(x)$: All real numbers
 { Range $f^{-1}(x)$: All real numbers

Example 9: If $f(x) = x^3$, find $f^{-1}(x)$. Also verify that $f(f^{-1}(x)) = f^{-1}(f(x)) = x$.

Solution: $f(x) = x^3$

$$y = x^3 \quad [\text{Replace } f(x) \text{ with } y]$$

$$x^3 = y$$

$$x = y^{\frac{1}{3}} \quad [\text{Solve for } x]$$

$$f^{-1}(x) = -4y + 8$$

$$f(f^{-1}(x)) = f(-4x + 8) = -\frac{1}{4}(-4x + 8) + 2$$

$$= (x - 2) + 2 = x - 2 + 2 = x$$

$$f^{-1}(f(x)) = f^{-1}\left(-\frac{1}{4}x + 2\right) = -\frac{1}{4}(-4x + 8) + 2$$

$$= (x - 2) + 2 = x - 2 + 2 = x$$

So, verified that $f(f^{-1}(x)) = f^{-1}(f(x)) = x$

Solved Exercise 4.1

1. If $f(x) = 2x + 5$, $g(x) = 3x - 2$, then find

(i) $f(x) + g(x)$

Sol: $f(x) + g(x)$
 $= (2x + 5) + (3x - 2)$
 $= 2x + 5 + 3x - 2 = 2x + 3x + 5 - 2$
 $= 5x + 3$

(ii) $f(x) - g(x)$

Sol: $f(x) - g(x)$
 $= (2x + 5) - (3x - 2)$
 $= 2x + 5 - 3x + 2 = 2x - 3x + 5 + 2$

$$k = -\frac{10}{2}$$

$$k = -5$$

5. Given that $f(x) = 3x + 2$ and $g(x) = 2x + 3$. Find

(i) $g(f(4))$ (ii) $f(f(3))$ (iii) $f(g(-2))$

Sol: Given $f(x) = 3x + 2$ and $g(x) = 2x + 3$

(i) $g(f(4))$

$$f(4) = 3(4) + 2 = 12 + 2 = 14$$

$$g(14) = 2(14) + 3 = 28 + 3 = 31$$

(ii) $f(f(3))$

$$f(3) = 3(3) + 2 = 11$$

$$f(11) = 3(11) + 2 = 33 + 2 = 35$$

(iii) $f(g(-2))$

$$g(-2) = 2(-2) + 3 = -4 + 3 = -1$$

$$f(-1) = 3(-1) + 2 = -3 + 2 = -1$$

6. Find $f^{-1}(x)$ in each of the following.

(i) $f(x) = 2x - 3$

$$\sqrt[3]{x-1} = \sqrt[3]{\frac{y+1}{4}}$$

$$x = \sqrt[3]{\frac{y+1}{4}} \quad [\text{solve for } x]$$

$$f^{-1}(y) = \sqrt[3]{\frac{y+1}{4}} \quad \therefore y = f(x) \Rightarrow f^{-1}(y) = x$$

$$f^{-1}(x) = \sqrt[3]{\frac{x+1}{4}} \quad [\text{Replacing } y \text{ by } x]$$

$$(iii) f(x) = \sqrt{x-1}$$

$$\text{Sol: } f(x) = \sqrt{x-1}$$

$$y = \sqrt{x-1} \quad [\text{Replacing } f(x) \text{ with } y]$$

$$\text{or } \sqrt{x-1} = y$$

$$(\sqrt{x-1})^2 = y^2$$

$$x-1 = y^2$$

$$x = y^2 + 1$$



or $7x - 4 = y$

$$7x = y + 4$$

$$x = \frac{y+4}{7} \quad (\text{Solve for } x)$$

$$f^{-1}(y) = \frac{y+4}{7} \quad [\because y=f(x) \Rightarrow f^{-1}(y)=x]$$

$$f(x) = \frac{x+4}{7} \quad (\text{Replacing } y \text{ by } x)$$

$$\begin{aligned} f(f^{-1}(x)) &= f\left(\frac{x+4}{7}\right) = 7\left(\frac{x+4}{7}\right) - 4 \\ &= x + 4 - 4 = x \end{aligned}$$

$$\begin{aligned} f^{-1}(f(x)) &= f^{-1}\left(\frac{x+4}{7}\right) = 7\left(\frac{x+4}{7}\right) - 4 \\ &= x + 4 - 4 = x \end{aligned}$$

Hence, verified that

$$f(f^{-1}(x)) = f^{-1}(f(x)) = x$$

(iii) $f(x) = \frac{x-3}{7}$

(iv) $f(x) = \frac{x-4}{x+2}$

Sol. $f(x) = \frac{x-4}{x+2}$

$$y = \frac{x-4}{x+2} \quad (\text{Replace } f(x) \text{ with } y)$$

$$y(x+2) = x-4$$

$$xy + 2y = x - 4$$

$$xy - x = -2y - 4$$

$$x(y-1) = -2y-4$$

$$x = \frac{-2y-4}{y-1} \quad (\text{Solve for } x)$$

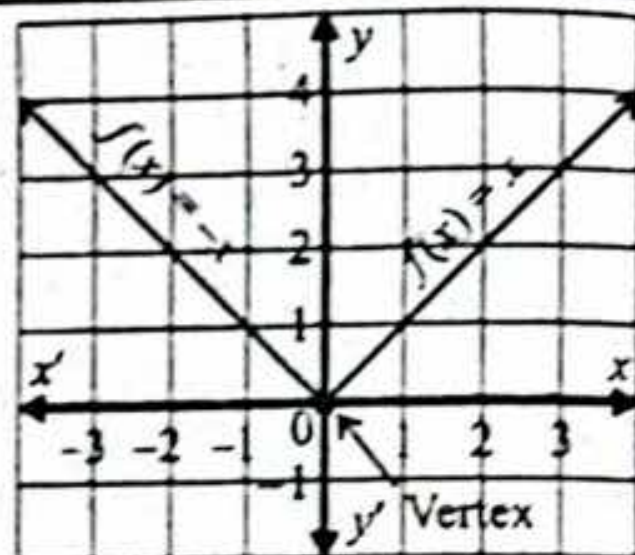
$$f^{-1}(y) = \frac{-2y-4}{y-1} \quad [\because y=f(x) \Rightarrow f^{-1}(y)=x]$$

$$f^{-1}(x) = \frac{-2x-4}{x-1} \quad (\text{Replacing } y \text{ by } x)$$

$$\begin{aligned} f(f^{-1}(x)) &= f\left(\frac{-2x-4}{x-1}\right) = \frac{\frac{-2x-4}{x-1} - 4}{\frac{-2x-4}{x-1} + 2} \end{aligned}$$

The graph of $f(x) = |x|$ V-shape and is symmetric with respect to the y-axis. The point where the graph changes direction is called the vertex $(0, 0)$.

The vertex form of an absolute value function is $g(x) = a|x - h| + k$, where $a \neq 0$ and (h, k) is the vertex. Its graph is symmetric about the line $x = h$.



Example 10: Plot the graph of the following functions:

(i) $f(x) = |x| + 2$

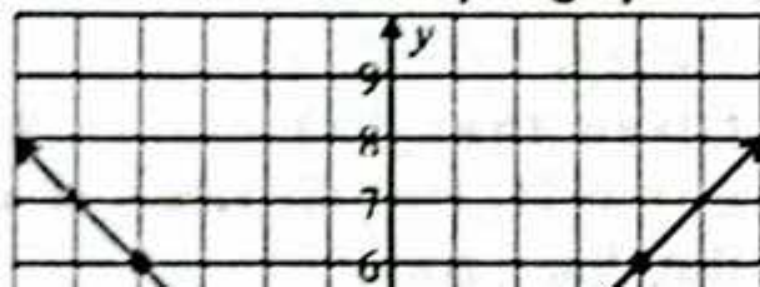
Sol.

Step I Make a table of values.

x	-4	-2	0	2	4...
y	6	4	2	4	6...

Step II Plot the ordered pairs.

Step III Draw the V-shaped graph.



(ii) $g(x) = |x - 3|$

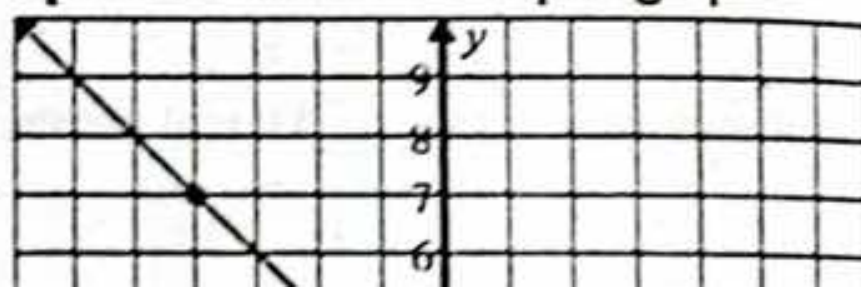
Sol.

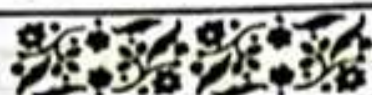
Step I Make a table of values.

x	-4	-2	0	3	5...
y	7	5	3	0	2...

Step II Plot the ordered pairs.

Step III Draw the V-shaped graph.





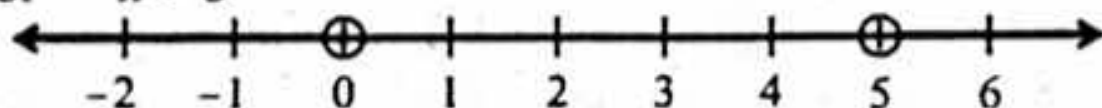
$$|2x - 5| > 5$$

$$\pm(2x - 5) > 5$$

$$-(2x - 5) > 5 \text{ or } 2x - 5 > 5$$

$$2x - 5 < -5 \text{ or } 2x > 10$$

$$x < 0 \text{ or } x > 5$$



The solution is $(-\infty, 0) \cup (5, \infty)$

Solved Exercise 4.2

1. Plot graph for the following absolute valued functions:

(i) $f(x) = |x - 2|$

Sol. $f(x) = |x - 2|$

Table of values

x	$y = f(x)$ $f(x) = x - 2 $	(x, y)
-4	$ -4 - 2 = -6 = 6$	$(-4, 6)$
-2	$ -2 - 2 = -4 = 4$	$(-2, 4)$
0	$ 0 - 2 = -2 = 2$	$(0, 2)$
2	$ 2 - 2 = 0 = 0$	$(2, 0)$

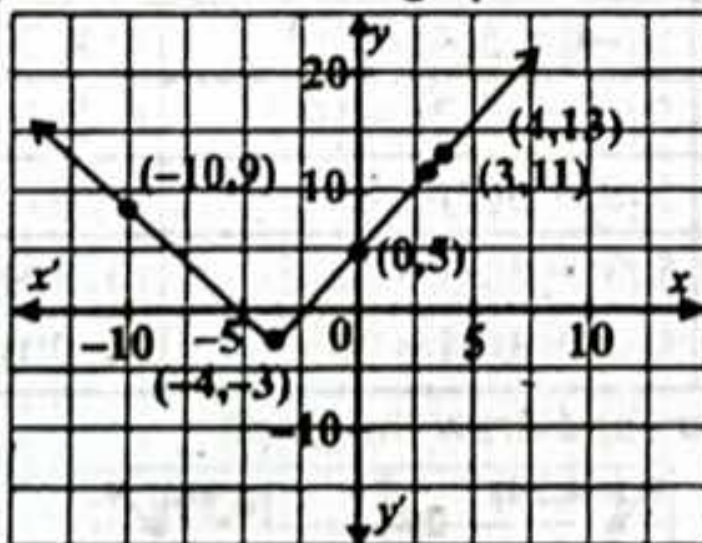


(v) $f(x) = 2|x + 4| - 3$

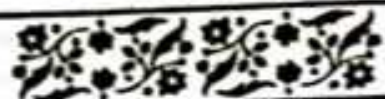
Sol. Table of the values

x	$y = f(x), f(x) = 2 x + 4 - 3$	(x, y)
-10	$2 -10 + 4 - 3 = 2 -6 - 3 = 2(6) - 3 = 9$	$(-10, 9)$
-4	$2 -4 + 4 - 3 = 2 0 - 3 = 2(0) - 3 = -3$	$(-4, -3)$
0	$2 0 + 4 - 3 = 2 4 - 3 = 2(4) - 3 = 5$	$(0, 5)$
3	$2 2 + 4 - 3 = 2 7 - 3 = 2(7) - 3 = 11$	$(3, 11)$
4	$2 4 + 4 - 3 = 2 8 - 3 = 2(8) - 3 = 13$	$(4, 13)$

Plot the above ordered pairs and draw the graph



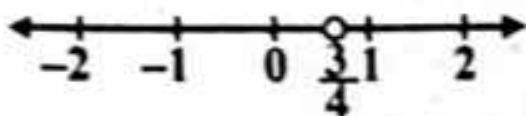
(vi) $f(x) = 2|x + 1| - 6$



$$4x \neq 3$$

$$x \neq \frac{3}{4}$$

$$\text{S.S} = \left(-\infty, \frac{3}{4}\right) \cup \left(\frac{3}{4}, \infty\right)$$



$$\text{(iv)} \quad 1 - 2\left|\frac{3}{2}x - 5\right| > -3$$

$$\text{Sol.} \quad 1 - 2\left|\frac{3}{2}x - 5\right| > -3$$

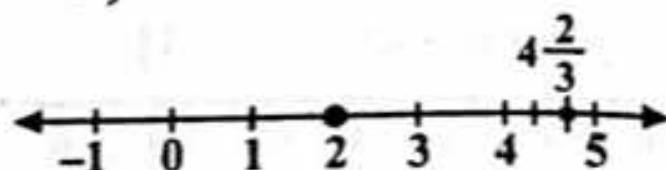
$$-2\left|\frac{3}{2}x - 5\right| > -3 - 1$$

$$-2\left|\frac{3}{2}x - 5\right| > -4$$

$$2\left|\frac{3}{2}x - 5\right| < 4$$

$$|2 \quad | \quad 4$$

$$\text{S.S} \left\{2, 4\frac{2}{3}\right\} \quad 2 < x < \frac{14}{3}$$



$$\text{(v)} \quad |3x - 2| \leq 12$$

$$\text{Sol.} \quad |3x - 2| \leq 12$$

$$3x - 2 \leq \pm 12$$

$$3x - 2 \leq 12 \quad \text{or} \quad 3x - 2 \geq -12$$

$$3x \leq 12 + 2 \quad \text{or} \quad 3 \geq -12 + 2$$

$$3x \leq 14 \quad \text{or} \quad 3x \geq -10$$

$$x \leq \frac{14}{3} \quad \text{or} \quad x \geq -\frac{10}{3}$$

$$x \leq 4\frac{2}{3} \quad \text{or} \quad x \geq -3\frac{1}{3}$$

$$-3\frac{1}{3} \leq x \leq 4\frac{2}{3}$$

$$\text{S.S} = \left[-3\frac{1}{3}, 4\frac{2}{3}\right]$$



For which values of magnitude M will the sensor issue a warning?

Solution:

$$|M - 4.5| > 1.2$$

$$\pm (M - 4.5) > 1.2$$

$$-(M - 4.5) > 1.2 \text{ or } M - 4.5 > 1.2$$

$$M - 4.5 < -1.2 \text{ or } M - 4.5 > 1.2$$

$$M < 3.3 \text{ or } M > 5.7$$

Hence, the sensor issues a warning when $M < 3.3$ or $M > 5.7$..

Solved Exercise 4.3

1. A function $B(t) = 5,000 + 200t$ represents the total balance (in rupees) after t months. What will be the balance after 6 months?

Sol: $B(t) = 5,000 + 200t$ (Given function)

Put $t = 6$ in function

$$B(6) = 5,000 + 200(6)$$

$$B(6) = 5,000 + 1200$$

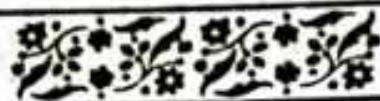
$$B(6) = 6,200$$

Balance = Rs.6,200

2. A function $f(k) = 150 + 20k$ represents the total fare (in rupees) for k kilometers. How much will the fare be for a 12 kilometres ride?

Sol:

$$f(k) = 150 + 20k \text{ (Given function)}$$



$$x > 0.1 \text{ or } x < -0.1$$

So, Rejection occurs for any position greater

Than 0.1mm or less than -0.1 mm

$$x < -0.1\text{mm or } x > 0.1\text{mm}$$

Solved Review Exercise 4

1. Four possible answers are given for the following questions. Choose the correct answer.

(i) If $f(x) = \frac{5x-6}{3}$, then what is the value of $f(3)$?

(a) -1

(b) 3

(c) 9

(d) 15

(ii) A function f from X to Y is represented by:

(a) $f: XY$

(b) $f: Y \rightarrow X$

(c) $f: X \rightarrow Y$

(d) $f: \frac{X}{Y}$

(iii) $(f \circ g)(x) =$

(a) $(f+g)(x)$

(b) $(f-g)(x)$

(c) $f(g(x))$

(d) $f(x) \div g(x)$

(iv) If $f(x) = 2x + 3$, $g(x) = x + 1$, then $f(x) + g(x) =$

(a) $3x$

(b) $3x + 4$

(c) 4

(d) $2x^2 + 3$

(v) If $f(x) = 5x + 2$, $h(x) = 2x - 2$, then what is $f(x) - h(x) =$

(a) $3x$

(b) $5x^2 - 4$

(c) $3x + 4$

(d) $-3x - 4$



$$f^{-1}(y) = \frac{5+y}{y} \quad \therefore y = f(x) \Rightarrow f^{-1}(y) = x$$

$$f^{-1}(x) = \frac{5+x}{x} \quad (\text{Replace } y \text{ by } x)$$

$$(iii) \quad f(x) = \sqrt{x-5}$$

$$\text{Sol. } y = \sqrt{x-5} \quad (\text{Replace } f(x) \text{ with } y)$$

$$y = \sqrt{x-5}$$

$$(\sqrt{x-5})^2 = y^2$$

$$x-5 = y^2$$

$$x = y^2 + 5 \quad (\text{Solve for } x)$$

$$f^{-1}(y) = y^2 + 5 \quad [\because y = f(x) \Rightarrow f^{-1}(y) = x]$$

$$f^{-1}(x) = x^2 + 5 \quad (\text{Replacing } y \text{ by } x)$$

$$(iv) \quad f(x) = \frac{3-x}{2}$$

$$\text{Sol. } y = \frac{3-x}{2} \quad (\text{Replace } f(x) \text{ with } y)$$

$$\text{or } \frac{3-x}{2} = y$$

$$3-x = 2y$$

$$-x = 2y - 3$$

$$x = -2y + 3 \quad (\text{solve for } x)$$

$$f^{-1}(y) = -2y + 3 \quad [\because y = f(x) \Rightarrow f^{-1}(y) = x]$$

$$f^{-1}(x) = -2x + 3 \quad (\text{Replacing } y \text{ by } x)$$

$$f^{-1}(x) = 3 - 2x$$

5. Plot graph for the following absolute valued functions:

(i) $f(x) = 7|x|$

Sol. Table of the values

--	--	--	--	--	--



8. Given that $f(x) = 3x + 7$, find

(i) $f^{-1}(x)$

Sol. $f(x) = 3x + 7$

$$y = 3x + 7 \quad (\text{Replace } f(x) \text{ with } y)$$

$$\text{or } 3x + 7 = y$$

$$3x = y - 7$$

$$x = \frac{y-7}{3} \quad (\text{Solve for } x)$$

$$f^{-1}(y) = \frac{y-7}{3} \quad \because y = f(x) \Rightarrow f^{-1}(y) = x$$

$$f(x) = \frac{x-7}{3} \quad (\text{Replacing } y \text{ by } x)$$

(ii) the value of x for which $f(x) = f^{-1}(x)$

Sol. $f(x) = 3x + 7$

$$y = 3x + 7 \quad (\text{Replace } f(x) \text{ with } y)$$

$$\text{or } 3x + 7 = y$$

$$3x = y - 7$$

$$x = \frac{y-7}{3} \quad (\text{Solve for } x)$$

$$f^{-1}(y) = \frac{y-7}{3} \quad \because y = f(x) \Rightarrow f^{-1}(y) = x$$

$$f^{-1}(x) = \frac{x-7}{3} \quad (\text{Replacing } y \text{ by } x)$$

$$f(x) = f^{-1}(x) \quad (\text{Given})$$

$$3x + 7 = \frac{x-7}{3}$$

$$3(3x + 7) = x - 7$$

$$9x + 21 = x - 7$$

$$9x - x = -7 - 21$$

$$8x = -28$$

$$x = \frac{-28}{8} = \frac{-7}{2}$$



8. If $f(a) = 5$ the 5 is called _____ of the function
(a) f^{-1} (b) domain (c) image (d) power
9. The range of $f(x) = (x+1)$ is:
(a) set of negative numbers (b) set of integers
(c) set of all non-negative real numbers (d) equal to domain

4.1.1**Operations on Functions**

10. If $f(x) = 5x + 1$ and $g(x) = -3x + 7$. Then value of $f(x) + g(x)$ is:
(a) $2x + 8$ (b) $8x + 8$ (c) $2x - 8$ (d) $15x + 8$
11. If $f(x) = x + 4$ and $g(x) = x - 4$, what is $(f \cdot g)(x)$?
(a) $x^2 - 8x - 16$ (b) $2x$ (c) $x^2 + 16$ (d) $x^2 - 16$
12. If $f(x) = x^2$ and $g(x) = 2x - 1$, the value of $(f - g)(x)$ is:
(a) $x^2 + 2x + 1$ (b) $-x^2 + x + 1$ (c) $x^2 - 2x - 1$ (d) $x^2 - 2x + 1$
13. If $f(x) = 3x + 2$ and $g(x) = x - 5$, what is $(f + g)(x)$?
(a) $3x^2 - 13x - 10$ (b) $2x + 7$ (c) $4x - 3$ (d) $4x + 3$
14. If $f(x) = 10x^3$ and $g(x) = 5x$, what is $\frac{f(x)}{g(x)}$?
(a) $2x^2$ (b) $10x^4$ (c) $50x^4$ (d) $10x^2$
15. If $f(x) = x^2 - 1$ and $g(x) = x + 1$, what is value of $(\frac{f}{g})(x)$?
(a) x^2 (b) $x + 1$ (c) $x - 1$ (d) $\frac{3}{x+1}$



38. Which is the vertex form of an absolute value function?

(a) $g(x) = -a|h - x|$

(b) $g(x) = -a|x| + k$

(c) $g(x) = a|x - h| + k$

(d) $g(x) = |x - h| \times k$

39. Which is correct ordered pair for the graph of $f(x) = |x - 5|$

(a) $(-4, 9)$

(b) $(4, -9)$

(c) $(3, 8)$

(d) $(-1, -6)$

40. The correct ordered pair for the graph of $g(x) = |x| + 3$ is:

(a) $(-6, -3)$

(b) $(0, 4)$

(c) $(5, 8)$

(d) $(3, -6)$

41. Which ordered pair is not correct for the graph of function $f(x) = |x - 1|$

(a) $(3, 2)$

(b) $(-5, 6)$

(c) $(-9, 10)$

(d) $(-6, -7)$

4.3

Solution of Absolute Valued Linear Equation in One Variable

42. The solution of equation $|x| = 7$ is:

(a) -7 only

(b) 7 only

(c) 7 and -7

(d) No solution

43. $|x - 10| = 0$ its solution is:

(a) ± 10

(b) -10

(c) 10

(d) No solution

44. The solution of $|3x| = 15$ is:

(a) $x = 5, x = -5$

(b) $x = 15$

(c) $x = 5$

(d) $x = 18, x = 12$

45. Which of the following equations has exactly two solution.

(a) $|x| + 5 = 5$

(b) $|3x| = -1$

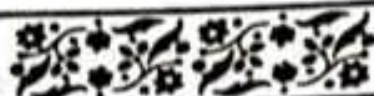
(c) $|x - 8| = 4$

(d) $|x - 2| + 5 = 2$

4.4

Solution of Absolute Value Linear inequality in One Variable

46. $-5 \leq x \leq 5$ is equivalent to:



8. Find the domain and range for functions $f(x) = \sqrt[3]{x}$

Ans. We can take cube root of any number positive or negative. No restriction on what goes in or what comes out.

Domain: All real numbers $(-\infty, \infty)$

Range: All real numbers $(-\infty, \infty)$

4.1.1**Operations on Functions**

9. If $f(x) = 5x^2 - 3$ and $g(x) = x^2 - 5$. Add $f(x)$ and $g(x)$

Sol. $f(x) = 5x^2 - 3$, $g(x) = x^2 - 5$

$$\begin{aligned} f(x) + g(x) &= (5x^2 - 3) + (x^2 - 5) \\ &= 5x^2 - 3 + x^2 - 5 \\ &= 6x^2 - 8 \end{aligned}$$

10. If $f(x) = 3x - 7$ and $g(x) = -x - 3$, then find $f(x) - g(x)$

Sol. $f(x) = 3x - 7$, $g(x) = -x - 3$

$$\begin{aligned} f(x) - g(x) &= (3x - 7) - (-x - 3) \\ &= 3x - 7 + x + 3 \\ &= 4x - 4 \end{aligned}$$

11. If $f(x) = 7x - 3$, $g(x) = 2x^2 + 5$, then find $(f \cdot g)(x)$.

Sol. $f(x) = 7x - 3$, $g(x) = 2x^2 + 5$

$$f(x) \cdot g(x) = (7x - 3)(2x^2 + 5)$$



24. Find the inverse of $f(x) = 5x - 3$

Sol. $f(x) = 5x - 3$
 $y = 5x - 3$ (Replace $f(x)$ with y)

$$5x = y + 3$$

$$x = \frac{y+3}{5} \quad (\text{Solve for } x)$$

$$f^{-1}(y) = \frac{y+3}{5} \quad \because y = f(x) \Rightarrow f^{-1}(y) = x$$

$$f^{-1}(x) = \frac{x+3}{5} \quad (\text{Replacing } y \text{ by } x)$$

25. Find the inverse of $f(x) = 5x^2 + 1$.

Sol. $f(x) = 5x^2 + 1$
 $y = 5x^2 + 1$ (Replace $f(x)$ with y)

$$\text{or } 5x^2 + 1 = y$$

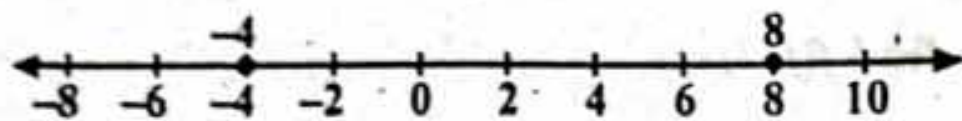
$$5x^2 = y - 1$$

$$x^2 = \frac{y-1}{5}$$

$$\sqrt{x^2} = \sqrt{\frac{y-1}{5}}$$

$$x = 8 \quad \text{or} \quad x = -4$$

$$\text{S.S} = \{8, -4\}$$



34. Solve $|5 - 2x| = 3$, and express solution on number line.

Sol. $5 - 2x = \pm 3$

$$5 - 2x = 3 \quad \text{or} \quad 5 - 2x = -3$$

$$-2x = 3 - 5 \quad \text{or} \quad -2x = -3 - 5$$

$$-2x = -2 \quad \text{or} \quad -2x = -8$$

$$2x = 2 \quad \text{or} \quad 2x = 8$$

$$x = \frac{2}{2} \quad \text{or} \quad x = \frac{8}{2}$$

$$x = 1 \quad \text{or} \quad x = 4$$

$$\text{S.S} = \{1, 4\}$$



4.4

Solution of Absolute Value Linear Inequality in One Variable

35. Solve the inequality $|x - 2| \geq 7$ and express the solution on number line.

Sol. $|x - 2| \geq 7$

$$= \frac{4ac^2}{3a - 2x}$$

Example 2: Reduce to lowest form $\frac{6x^2 - 8xy}{9xy - 12y^2}$

Solution : $\frac{6x^2 - 8xy}{9xy - 12y^2} = \frac{2x(3x - 4y)}{3y(3x - 4y)} = \frac{2x}{3y}$

Solved Exercise 5.1

1. Reduce the following rational expressions to lowest forms:

(i) $\frac{3a^2 - 6ab}{2a^2b - 4ab^2}$

Sol:
$$\begin{aligned} &= \frac{3a^2 - 6ab}{2a^2b - 4ab^2} \\ &= \frac{3a(a - 2b)}{2ab(a - 2b)} \\ &= \frac{3a}{2ab} = \frac{3}{2b} \end{aligned}$$

(ii) $\frac{abx + bx^2}{acx + cx^2}$

Sol:
$$\begin{aligned} &= \frac{abx + bx^2}{acx + cx^2} \\ &= \frac{bx(a + x)}{cx(a + x)} \\ &= \frac{bx}{cx} = \frac{b}{c} \end{aligned}$$

division while maintaining the equality or value of the expression.

Addition and Subtraction of Rational Expressions with Unlike Denominators

Steps:

- (i) Determine the Least Common Multiple (LCM) of the denominators.
- (ii) Rewrite each fraction as an equivalent fraction with the LCM obtained in step (i)
- (iii) Follow the same step for doing addition or subtraction of rational expression with like denominators.

Example 3: Simplify: $\frac{x-2y}{4} + \frac{x+y}{6} - \frac{2x-y}{15}$

Solution : LCM of 4, 6 and 15 = 60

$$\begin{aligned} & \frac{x-2y}{4} + \frac{x+y}{6} - \frac{2x-y}{15} \\ &= \frac{15(x-2y) + 10(x+y) - 4(2x-y)}{60} \\ &= \frac{15x - 30y + 10x + 10y - 8x + 4y}{60} = \frac{17x - 16y}{60} \end{aligned}$$

Example 4: Simplify: $\frac{4}{x^2 - 2xy - 3y^2} + \frac{1}{x^2 + 3xy - 2y^2}$

Solved Exercise 5.2

Simplify:

$$(i) \frac{3}{x-y} + \frac{1}{y-x}$$

$$\text{Sol: } \frac{3}{x-y} + \frac{1}{-x+y}$$

$$= \frac{3}{x-y} + \frac{1}{-(x-y)}$$

$$= \frac{3}{x-y} - \frac{1}{(x-y)}$$

$$(\text{L.C.M} = x-y)$$

$$= \frac{3-1}{x-y}$$

$$= \frac{2}{x-y}$$

$$(ii) \frac{4x}{x^2-3x+2} - \frac{4}{1-x} - \frac{5}{x-2}$$

$$\text{Sol: } \frac{4x}{x^2-3x+2} - \frac{4}{1-x} - \frac{5}{x-2}$$

$$= \frac{4x}{x^2-2x-x+2} - \frac{4}{-x+1} - \frac{5}{x-2}$$

$$= \frac{4x}{x(x-2)-1(x-2)} - \frac{4}{-(x-1)} - \frac{5}{x-2}$$

$$= \frac{4x}{(x-2)(x-1)} + \frac{4}{x-1} - \frac{5}{x-2}$$

$$\text{LCM } (x-2)(x-1)$$

$$= \frac{4x + 4(x-2) - 5(x-1)}{(x-2)(x-1)}$$

$$= \frac{4x + 4x - 8 - 5x + 5}{(x-2)(x-1)}$$

$$= \frac{7}{(x-2)(2x+3)} - \frac{8}{(x-2)(3x+2)}$$

$$\text{LCM} = (x-2)(2x+3)(3x+2)$$

$$= \frac{7(3x+2) - 8(2x+3)}{(x-2)(2x+3)(3x+2)} = \frac{21x+14-16x-24}{(x-2)(2x+3)(3x+2)}$$

$$= \frac{5x-10}{(x-2)(2x+3)(3x+2)} = \frac{5(x-2)}{(x-2)(2x+3)(3x+2)}$$

$$= \frac{5}{(2x+3)(3x+2)}$$

$$(ix) \frac{x+2}{x^2-x-12} - \frac{x}{x^2+6x+9}$$

$$\text{Sol.} \frac{x+2}{x^2-x-12} - \frac{x}{x^2+6x+9}$$

$$= \frac{x+2}{x^2-4x+3x-12} - \frac{x}{x^2+3x+3x+9}$$

$$= \frac{x+2}{x(x-4)+3(x-4)} - \frac{x}{x(x+3)+3(x+3)}$$

Example: $\frac{2a^2 - 2c^2}{15a^2c^2} \div \frac{3a + 3c}{5ac}$

Solution : $\frac{2a^2 - 2c^2}{15a^2c^2} \div \frac{3a + 3c}{5ac}$

$$= \frac{2a^2 - 2c^2}{15a^2c^2} \times \frac{5ac}{3a + 3c} = \frac{2(a^2 - c^2)}{15a^2c^2} \times \frac{5ac}{3(a + c)}$$

$$= \frac{2(a + c)(a - c)}{15a^2c^2} \times \frac{5ac}{3(a + c)} = \frac{2(a - c)}{9ac}$$

Example 9: Simplify: $\frac{x^3 - a^3}{x^2 - 2bx + b^2} \times \frac{x^2 - bx - cx + bc}{ax - x^2} \div \frac{x^2 - cx}{x - b}$

Solution : $\frac{x^3 - a^3}{x^2 - 2bx + b^2} \times \frac{x^2 - bx - cx + bc}{ax - x^2} \div \frac{x^2 - cx}{x - b}$

$$= \frac{x^3 - a^3}{x^2 - 2bx + b^2} \times \frac{x^2 - bx - cx + bc}{ax - x^2} \times \frac{x - b}{x^2 - cx}$$

$$= \frac{(x - a)(x^2 + ax + a^2)}{(x - b)(x - b)} \times \frac{(x - b)(x - c)}{x(a - x)} \times \frac{x - b}{x(x - c)}$$

$$= \frac{(x-2)[(x)^2 + (x)(2) + (2)^2]}{(x+2)(x-2)} \div \frac{x^2 + 2x + 4}{x(x+2) + 2(x+2)}$$

$$= \frac{(x-2)(x^2 + 2x + 4)}{(x+2)(x-2)} \div \frac{x^2 + 2x + 4}{(x+2)(x+2)}$$

$$= \frac{x^2 + 2x + 4}{x+2} \times \frac{(x+2)(x+2)}{x^2 + 2x + 4} = \frac{(x^2 + 2x + 4)(x+2)(x+2)}{(x^2 + 2x + 4)(x+2)}$$

$$= x+2$$

$$(vi) \frac{(a^2 + ab)^2}{(a^2 - ab)^2} \div \left(\frac{a+b}{a-b} \right)^2$$

$$\text{Sol: } \frac{(a^2 + ab)^2}{(a^2 - ab)^2} \div \left(\frac{a+b}{a-b} \right)^2$$

$$= \frac{[a(a+b)]^2}{[a(a-b)]^2} \div \frac{(a+b)^2}{(a-b)^2} = \frac{a^2(a+b)^2}{a^2(a-b)^2} \times \frac{(a-b)^2}{(a+b)^2}$$

$$= \frac{a^2(a+b)^2(a-b)^2}{a^2(a+b)^2(a-b)^2} = 1$$

$$(vii) \frac{x^2 - x - 6}{x^2 - x - 20} \div \left\{ \frac{x^3 - 3x^2}{x^2 + 4x} \times \frac{x^3 - 5x^2 - 14x}{x^2 - 12x + 35} \right\}$$



$$(x) \quad \frac{x^2 - 2x}{2x + 6} \times \frac{x^2 + x - 6}{x^2 - 5x} \times \frac{6x - 30}{x^2 - 2x}$$

$$\text{Sol.} \quad \frac{x^2 - 2x}{2x + 6} \times \frac{x^2 + x - 6}{x^2 - 5x} \times \frac{6x - 30}{x^2 - 2x}$$

$$= \frac{x(x-2)}{2(x+3)} \times \frac{x^2 + 3x - 2x - 6}{x(x-5)} \times \frac{6(x-5)}{x(x-2)}$$

$$= \frac{x(x-2)}{2(x+3)} \times \frac{x(x+3) - 2(x+3)}{x(x-5)} \times \frac{6(x-5)}{x(x-2)}$$

$$= \frac{x(x-2)}{2(x+3)} \times \frac{(x+3)(x-2)}{x(x-5)} \times \frac{6(x-5)}{x(x-2)}$$

$$= \frac{6 \times x(x-2)(x-2)(x+3)(x-5)}{2 \times x \times x(x-2)(x+3)(x-5)}$$

$$= \frac{3(x-2)}{x}$$

5.3

Solution of Equations Reducible to Quadratic Equations

Some algebraic equations are not in standard quadratic form, but they can be reduced or transformed into a quadratic equation by suitable substitution. Such type of



$$3y^2 + 4y - 20 = 0$$

$$3y^2 + 10y - 6y - 20 = 0$$

$$y(3y + 10) - 2(3y + 10) = 0$$

$$(3y + 10)(y - 2) = 0$$

$$3y + 10 = 0, \quad y - 2 = 0$$

$$y = -\frac{10}{3}, \quad y = 2$$

Put $y = -\frac{10}{3}$ in (i), we get

$$x + \frac{1}{x} = -\frac{10}{3} \quad [\text{Multiplying by } 3x]$$

$$3x^2 + 3 = -10x$$

$$3x^2 + 10x + 3 = 0$$

$$3x^2 + 9x + x + 3 = 0$$

$$3x(x + 3) + 1(x + 3) = 0$$

$$(x + 3)(3x + 1) = 0$$

$$x + 3 = 0, \quad 3x + 1 = 0$$

$$x = -3, -\frac{1}{3}$$

Now put $y = 2$ in (i), we get

$$(x^2)^2 = (y)^2$$

$$x^4 = y^2$$

So, that

$$5y^2 - 19y + 12 = 0$$

$$5y^2 - 15y - 4y + 12 = 0$$

$$5y(y-3) - 4(y-3) = 0$$

$$(y-3)(5y-4) = 0$$

$$y-3=0, \quad 5y-4=0$$

$$y=3, \quad 5y=4$$

$$y = \frac{4}{5}$$

Put $x^2 = y$

$$x^2 = 3$$

$$\sqrt{x^2} = \sqrt{3}$$

$$x = \pm\sqrt{3}$$

$$x^2 = \frac{4}{5}$$

$$\sqrt{x^2} = \sqrt{\frac{4}{5}}$$

put $x^2 = y$

$$x^2 = 6$$

$$\sqrt{x^2} = \sqrt{6}$$

$$x = \pm\sqrt{6}$$

$$x^2 = \frac{3}{4}$$

$$\sqrt{x^2} = \sqrt{\frac{3}{4}}$$

$$x = \pm\frac{\sqrt{3}}{2}$$

$$S.S = \left\{ \pm\sqrt{6}, \pm\frac{\sqrt{3}}{2} \right\}$$

(iii) $5x^4 - 22x^2 + 8 = 0$

Sol: $5x^4 - 22x^2 + 8 = 0$

Let $x^2 = y$

$$(x^2)^2 = y^2$$

$$x^4 = y^2$$

So, that

$$5y^2 - 22y + 8 = 0$$

STANDARD

$$(y-9)(y-3)=0$$

$$y-9=0, y-3=0$$

$$y=9, y=3$$

$$\text{put } 3^x=y$$

$$3^x=9$$

$$3^x=3^2$$

$$x=2$$

$$3^x=3$$

$$3^x=3^1$$

$$x=1$$

$$\text{S.S} = \{2, 1\}$$

$$\text{(vii) } 5^{1+3x} + 5^{2-3x} - 126 = 0$$

$$\text{Sol: } 5^{1+3x} + 5^{2-3x} - 126 = 0$$

$$5^1 \cdot 5^{3x} + 5^2 \cdot 5^{-3x} - 126 = 0$$

$$5(5^{3x}) + \frac{25}{5^{3x}} - 126 = 0$$

$$\text{Let } 5^{3x} = y$$

So, that

$$5y + \frac{25}{y} - 126 = 0$$

$$5^{3x} = 5^{-1}$$

$$3x = -1$$

$$x = -\frac{1}{3}$$

$$\text{S.S} = \left\{ \frac{2}{3}, -\frac{1}{3} \right\}$$

$$\text{(viii) } \left(x^2 + \frac{1}{x^2} \right) + 2 \left(x + \frac{1}{x} \right) - 33 = 0$$

$$\text{Sol: Let } x + \frac{1}{x} = y$$

$$\left(x + \frac{1}{x} \right)^2 = y^2$$

$$x^2 + \frac{1}{x^2} + 2 = y^2$$

$$x^2 + \frac{1}{x^2} = y^2 - 2$$

So, that

$$y^2 - 2 + 2y - 33 = 0$$

$$y^2 + 2y - 35 = 0$$

$$y^2 + 7y - 5y - 35 = 0$$

$$2(x^2) + 2\left(\frac{x}{2}\right) - 2(1) = 2(0)$$

$$2x^2 + x - 2 = 0$$

$$a = 2, b = 1, c = -2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(2)(-2)}}{2(2)}$$

$$x = \frac{-1 \pm \sqrt{1+16}}{4}$$

$$x = \frac{-1 \pm \sqrt{17}}{4}$$

$$S.S = \left\{ \frac{13 \pm \sqrt{173}}{2}, \frac{-1 \pm \sqrt{17}}{4} \right\}$$

$$(x) \quad 2x^4 - 5x^3 - 14x^2 - 5x + 2 = 0$$

$$\text{Sol: } 2x^4 - 5x^3 - 14x^2 - 5x + 2 = 0$$

Divide by x^2

$$\frac{2x^4}{x^2} - \frac{5x^3}{x^2} - \frac{14x^2}{x^2} - \frac{5x}{x^2} + \frac{2}{x^2} = \frac{0}{x^2}$$

$$2y^2 - 9y + 4y - 18 = 0$$

$$y(2y-9) + 2(2y-9) = 0$$

$$(2y-9)(y+2) = 0$$

$$2y-9=0, y+2=0$$

$$2y=9, y=-2$$

$$y = \frac{9}{2}$$

$$\text{Put } x + \frac{1}{x} = y$$

$$x + \frac{1}{x} = \frac{9}{2}$$

$$2x(x) + 2x\left(\frac{1}{x}\right) = 2x\left(\frac{9}{2}\right)$$

$$2x^2 + 2 = 9x$$

$$2x^2 - 9x + 2 = 0$$

$$a = 2, b = -9, c = 2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(2)}}{2(2)}$$

articles did he buy?

Solution: Suppose he buys x articles for Rs. 300

$$\therefore \text{Each cost} = \frac{300}{x} \text{ Rs.}$$

He sold $(x-4)$ articles for Rs. 350

$$\text{Each sold for } \frac{350}{x-4} \text{ Rs.}$$

$$\text{Now, } \frac{350}{x-4} - \frac{300}{x} = 1.25$$

$$\frac{350}{x-4} - \frac{300}{x} = \frac{125}{100} = \frac{5}{4}$$

Multiplying both sides by $4(x-4)$, we get

$$350(4x) - 300(x-4)(4) = 5x(x-4)$$

$$1400x - 1200x + 4800 = 5x^2 - 20x$$

$$5x^2 - 20x + 1200x - 1400x - 4800 = 0$$

$$5x^2 - 220x - 4800 = 0$$

(Dividing by 5)

$$x^2 - 44x - 960 = 0$$

$$mx = 9000$$

$$\text{For grapes } g.x = 6000$$

$$\frac{m.x}{g.x} = \frac{9000}{6000}$$

$$\frac{m}{g} = \frac{3}{2}$$

$$m = 1.5g$$

$$\text{put } m = 1.5g$$

$$m + g = 50$$

$$1.5g + g = 50$$

$$2.5g = 50$$

$$g = \frac{50}{2.5} = 20\text{kg}$$

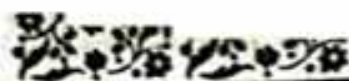
Find the weight of mangoes

$$m = 50 - 20$$

$$m = 30$$

So, Amna bought 30kg of mangoes and 20kg of grapes

Example 15: A train is scheduled to cover a distance of 120 Km at a certain average



$$= \frac{13}{20}$$

$$\begin{aligned} \text{Remaining work} &= 1 - \frac{13}{20} \\ &= \frac{20-13}{20} = \frac{7}{20} \end{aligned}$$

Time taken by pipe A for remaining work

$$\begin{aligned} &\frac{7}{\frac{20}{5}} \\ &= \frac{7}{4} \end{aligned}$$

$$= \frac{7}{20} \times \frac{5}{1} = \frac{7}{4} \text{ hour} = 1 \text{ hr } 45 \text{ minutes}$$

$$\begin{aligned} \text{Total time} &= 2 \text{ hr} + 1 \text{ hr } 45 \text{ min} \\ &= 3 \text{ hours } 45 \text{ minutes} \end{aligned}$$

4. Huria can complete a project in 12 hours by working alone. If Abdul Hadi join her and they finish it

$$\frac{60}{7} = x$$

Hadi will take time = $\frac{60}{7}$ hours = 8 hours, 34 minutes

$$\begin{array}{r} 8 \\ 7 \overline{)60} \\ \underline{56} \\ 4 \end{array} \qquad \begin{array}{r} 34 \\ 7 \overline{)240} \\ \underline{21} \\ 30 \\ \underline{28} \\ 2 \end{array}$$

$$4 \times 60 = 240 \text{ min}$$

5. Two cars start from opposite towns and head towards each other. The distance between them is 240 km. One travels at x km/h and the other at $(x + 10)$ km/h. They meet after 2 hours. Find the speed of each car.



Equation from given condition

$$\frac{30}{x} - \frac{30}{x+5} = 1$$

$$x(x+5) \left(\frac{30}{x} \right) - x(x+5) \left(\frac{30}{x+5} \right) = x(x+5) (1)$$

$$(x+5)(30) - x(30) = x^2 + 5x$$

$$\cancel{30x} + 150 - \cancel{30x} = x^2 + 5x$$

$$-x^2 - 5x + 150 = 0$$

$$x^2 + 5x - 150 = 0$$

$$x^2 + 15x - 10 - 150 = 0$$

$$x(x+15) - 10(x+15) = 0$$

$$(x+15)(x-10) = 0$$

$$x + 15 = 0$$

$$x = -15$$

$$x - 10 = 0$$

$$x = 10$$

Negative ignored

so the speed = 10 km/h

Solved Review Exercise 5

1. Four possible answers are given for the following questions. Choose the correct answer.

(i) What type of expression $\frac{2x-1}{x^2+4}$ is:

(a) a polynomial

(c) a numerical fraction

(b) an algebraic fraction

(d) an equation

(ii) Lowest form of $\frac{18a^2bc^2}{\quad}$ is:

$$\begin{aligned}
 &= \frac{5}{(x+3)(x-2)} - \frac{1}{(x-2)(2x-3)} \\
 &= \frac{5(2x-3) - (x+3)}{(x-2)(x+3)(2x-3)} = \frac{10x-15-x-3}{(x-2)(x+3)(2x-3)} \\
 &= \frac{9x-18}{(x-2)(x+3)(2x-3)} = \frac{9(x-2)}{(x-2)(x+3)(2x-3)} \\
 &= \frac{9}{(x+3)(2x-3)}
 \end{aligned}$$

(ii) $\frac{x^3 - 27}{x^2 - 9} \div \frac{x^2 + 3x + 9}{x^2 + 6x + 9}$

Sol. $\frac{x^3 - 27}{x^2 - 9} \div \frac{x^2 + 3x + 9}{x^2 + 6x + 9}$

$$= \frac{(x)^3 - (3)^3}{(x)^2 - (3)^2} \div \frac{x^2 + 3x + 9}{x^2 + 3x + 3x + 9}$$

$$= \frac{(x-3)[(x)^2 + (x)(3) + (3)^2]}{(x-3)(x+3)} \div \frac{x^2 + 3x + 9}{x(x+3) + 3(x+3)}$$

$$\left(x + \frac{1}{x}\right)^2 = y^2$$

$$x^2 + \frac{1}{x^2} + 2 = y^2$$

$$x^2 + \frac{1}{x^2} = y^2 - 2$$

So, that

$$y^2 - 2 - 3y - 2 = 0$$

$$y^2 - 3y - 4 = 0$$

$$y^2 - 4y + y - 4 = 0$$

$$y(y - 4) + 1(y - 4) = 0$$

$$(y - 4)(y + 1) = 0$$

$$y - 4 = 0, \quad y + 1 = 0$$

$$y = 4, \quad y = -1$$

Put $x + \frac{1}{x} = y$

$$x + \frac{1}{x} = 4$$

(1)

$$x = 2 \pm \sqrt{3}$$

$$x + \frac{1}{x} = -1$$

$$x(x) + x\left(\frac{1}{x}\right) = x(-1)$$

$$x^2 + 1 = -x$$

$$x^2 + x + 1 = 0$$

$$a = 1, b = 1, c = 1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(1) \pm \sqrt{(+1)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{1 - 4}}{2}$$

$$x = \frac{-1 \pm \sqrt{-3}}{2}$$

$$x^2 + 2x = 8$$

$$x^2 + 2x - 8 = 0$$

$$x^2 + 4x - 2x - 8 = 0$$

$$x(x+4) - 2(x+4) = 0$$

$$(x+4)(x-2) = 0$$

$$x+4=0, x-2=0$$

$$x=-4, x=2$$

$$S.S = \{2, -4, -1 \pm \sqrt{71}\}$$

$$(vii) (x-1)(x-2)(x+5)(x-8) = -360$$

$$\text{Sol. } (x-1)(x-2)(x+5)(x-8) = -360$$

$$[(x-1)(x-2)][(x+5)(x-8)] = -360$$

$$(x^2 - 2x - x + 2)(x^2 - 8x + 5x - 40) = -360$$

$$(x^2 - 3x + 2)(x^2 - 3x - 40) = -360$$

$$\text{Let } x^2 - 3x = y$$

$$(y+2)(y-40) = -360$$

$$y^2 - 40y + 2y - 80 = -360$$

$$y^2 - 38y - 80 + 360 = 0$$

$$y^2 - 38y + 280 = 0$$

$$y^2 - 28y - 10y + 280 = 0$$

$$y(y-28) - 10(y-28) = 0$$

$$(y-28)(y-10) = 0$$

$$y-28=0, y-10=0$$

$$y=28, y=10$$

$$\text{Put } x^2 - 3x = y$$

$$x^2 - 3x = 28$$

$$x^2 - 3x - 28 = 0$$

$$x^2 - 7x + 4x - 28 = 0$$

$$x(x-7) + 4(x-7) = 0$$

$$(x-7)(x+4) = 0$$

$$x-7=0, x+4=0$$

$$x=7, x=-4$$

$$x^2 - 3x = 10$$

$$x^2 - 3x - 10 = 0$$

$$x^2 - 5x + 2x - 10 = 0$$

$$x(x-5) + 2(x-5) = 0$$

$$(x-5)(x+2) = 0$$

$$x-5=0, x+2=0$$

$$x=5, x=-2$$

$$S.S = \{-4, -2, 5, 7\}$$

5. Ahmad takes 2 hours to paint 50 glasses. Faiza takes 2 hours to paint 45

2. $\frac{27x^2y}{2x+4y}$ is a type of expression.

- (a) an algebraic fraction (b) polynomial (c) equation (d) inequality

3. $\frac{x+5}{x+2}$ is a:

- (a) polynomial (b) equation
(c) surd (d) rational expression

4. Which is not an rational expression:

- (a) $\frac{3x^2}{y}$ (b) $\frac{2x}{3y}$ (c) $\frac{x+y}{x^2-y^2}$ (d) $\frac{5ab}{10b}$

5. Which is the example of an algebraic fraction?

- (a) $\frac{5}{y}$ (b) $\frac{3x+4}{5x}$ (c) $\frac{a+b}{a-b}$ (d) All of them

5.1.2**Simplification of Rational Expressions**

6. Lowest form of $\frac{3a^2b}{24b}$ is:

- (a) $\frac{8a^2b}{a}$ (b) $\frac{a^2}{8}$ (c) $\frac{15}{16}b$ (d) $\frac{24a}{8b}$

21. Which value of x makes $\frac{x-5}{x^2-5}$ undefined?

- (a) 5 only (b) -5 only (c) 5 and -5 (d) 0

22. $\frac{\frac{1}{x}}{\frac{1}{y}} =$ _____.

- (a) $\frac{y}{x}$ (b) $\frac{1}{xy}$ (c) $\frac{x}{y}$ (d) $\frac{xy}{1}$

23. Simplify $\frac{24x^2y}{5c} \times \frac{10c}{48y}$ is:

- (a) x^2 (b) $240xy$ (c) y^2 (d) $\frac{34}{5xy}$

24. What is the simplified form $\frac{3x}{4y} \div \frac{3x}{8y}$?

- (a) $\frac{9x^2}{32y^2}$ (b) $\frac{3x}{8y}$ (c) $\frac{x}{y}$ (d) 2

25. What is the simplified form of $(a^3 + b^3) \div (a + b)$?

- (a) $a^3 - b^3$ (b) $(a^2 - ab + b^2)$ (c) $(a^2 + ab + b^2)$ (d) $(a^2 - ab - b^2)$

19. $\frac{2f+3c}{cf}$ 20. $\frac{2x+10}{(x-3)(x+1)}$ 21. 5 and -5 22. $\frac{y}{x}$ 23. x^2
24. 2 25. $(a^2 - ab + b^2)$ 26. $4x(a+y)$ 27. $(a^2 + b^2)(a-b)$
28. $\frac{5x-3}{y}$ 29. $\frac{m(x-y)}{z}$ 30. $\frac{1}{20ac^2}$ 31. $3y^2 - 11y + 5 = 0$
32. Reciprocal equation 33. Exponential equation 34. $\pm \frac{\sqrt{6}}{2}$
35. $x^n = y$ 36. 1, 2 37. $\frac{1}{2}$ 38. 0 39. $\frac{1}{x}$
40. $(2y-3)(y-4)$ 41. $(y-16)(y-8)$

Additional Short Question

Short Answer Questions prepared in the light of new Examination Techniques
(Knowledge, Understanding, Application, Analytical & Conceptual)

5.1

Algebraic Fraction

○ Give short answers.

1. Define an algebraic fraction:

Ans. An algebraic fraction is a fraction where the numerator, denominator or both contain



$$= \frac{2x - (x + 3)}{x(x + 3)}$$

$$= \frac{2x - x - 3}{x(x + 3)}$$

$$= \frac{x - 3}{x(x + 3)}$$

14. Simplify $\frac{3}{x^2 - 4} - \frac{1}{x - 2}$

Sol. $\frac{3}{x^2 - 4} - \frac{1}{x - 2}$

$$= \frac{3}{(x + 2)(x - 2)} - \frac{1}{x - 2}$$

$$= \frac{3 - (x + 2)}{(x + 2)(x - 2)}$$

$$= \frac{3 - x - 2}{(x + 2)(x - 2)}$$

17. Simplify $\frac{2x^2 + 5x - 3}{x^2 - 9} \times \frac{x + 3}{2x - 1}$

Sol. $\frac{2x^2 + 5x - 3}{x^2 - 9} \times \frac{x + 3}{2x - 1}$

$$= \frac{2x^2 + 6x - x - 3}{(x)^2 - (3)^2} \times \frac{x + 3}{2x - 1}$$

$$= \frac{2x(x + 3) - 1(x + 3)}{(x + 3)(x - 3)} \times \frac{x + 3}{2x - 1}$$

$$= \frac{(2x - 1)(x + 3)(x + 3)}{(2x - 1)(x + 3)(x - 3)}$$

$$= \frac{x + 3}{x - 3}$$

18. $\frac{x^3 - 8}{x^2 - 4} \times \frac{x + 2}{x^2 + 2x + 4}$

Sol. $= \frac{(x^3) - (2)^3}{(x)^2 - (2)^2} \times \frac{x + 2}{x^2 + 2x + 4}$

5.3

Solution of Equations Reducible to Quadratic Equations

23. For $x^6 - 9x^3 + 7 = 0$, what substitution would you use to reduce it to a quadratic equation.

Sol. Let $x^3 = y$
 $(x^3)^2 = y^2$
 $x^6 = y^2$

The equation becomes $y^2 - 9y + 7 = 0$

24. In the equation $5^{2x} - 6 \cdot 5^x + 5 = 0$, if we let $5^x = y$, what is the resulting quadratic equation?

Sol. If $y = 5^x$, then
 $y^2 = (5^x)^2$
 $y^2 = 5^{2x}$

The equation becomes $y^2 - 6y + 5 = 0$

25. Define exponential equation:

Ans. Equations in which variable occur in exponents are called exponential equations for example $2^x - 3 \cdot 2^{2x} = -56$

26. Define "Reciprocal" equations.

Ans. An equation which remains unchanged when variable is replaced by its reciprocal.

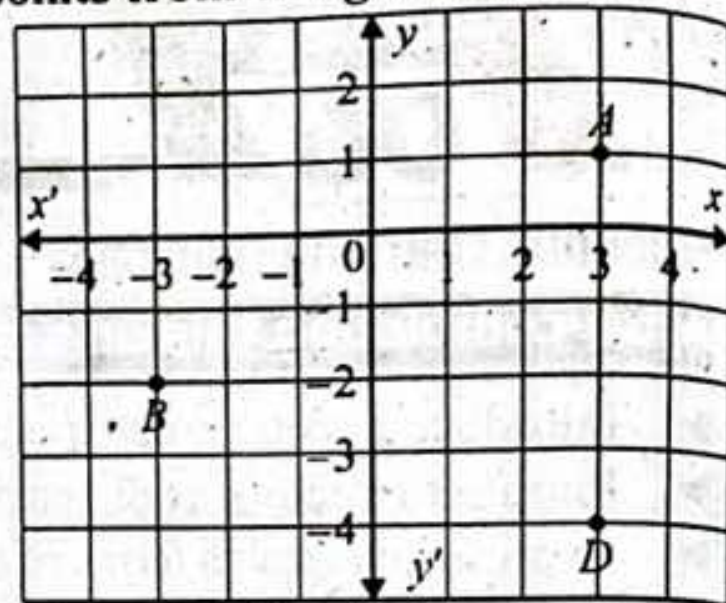
27. The sum of a natural number and its reciprocal is $\frac{10}{3}$. Find the number.

Example 1: Write the coordinates of the following points from the given figure:

- (i) A (ii) B (iii) D

Solution:

- (i) Point A is 3 units to the right of the origin and 1 unit up. So, the x -coordinate is 3 and the y -coordinate is 1. The coordinates of A are $(3, 1)$.
- (ii) Point B is 3 units to the left of the origin and 2 units down. So, the x -coordinate is -3 and the y -coordinate is -2 . The coordinates of B are $(-3, -2)$.
- (iii) Point D is 3 units to the right of the origin and 4 units down. So, the x -coordinate is 3 and the y -coordinate is -4 . The coordinates of D are $(3, -4)$.



6.2

Scalars and Vectors

There are two types of physical quantities that are used in Physics and Mathematics.
Scalar: A scalar is a quantity having magnitude but no direction. Mass, time, volume and power are the examples of scalar quantities.

Vector: A vector is a quantity having both magnitude and direction. Displacement, velocity, acceleration, force and momentum are examples of vector quantities.

Representation of a Vector as Directed Line Segment

A vector has a tail and a tip. Consider the diagram on the right as in Figure
 The tail point A is called the **initial point** and the tip point B is called the



Skilled Practise!

If $P(4, 7)$ and $Q(5, 10)$, then find the magnitude of the vector $\overline{PQ}$.

Sol. Given Points

$$P(4, 7), Q(5, 10)$$

$$\overline{PQ} = (5 - 4)\underline{i} + (10 - 7)\underline{j} \Rightarrow \overline{PQ} = \underline{i} + 3\underline{j}$$

Now magnitude:

$$|\overline{PQ}| = \sqrt{1^2 + 3^2} = \sqrt{1 + 9}$$

$$|\overline{PQ}| = \sqrt{10} \text{ Ans.}$$

Scalar Multiplication of a Vector

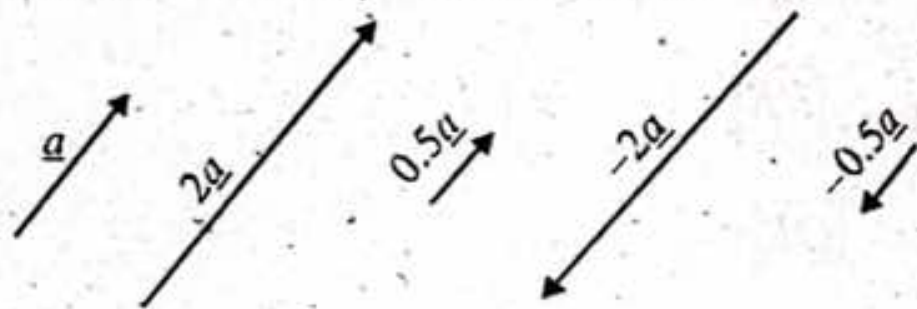
If $\underline{a}$ is a vector and λ is a scalar (a number), then multiplication of a vector $\underline{a}$ with scalar λ is scalar multiplication and it is written as $\lambda \underline{a}$. $\lambda \underline{a}$ is a new vector that has its magnitude scaled by $|\lambda|$.

(i) If $\lambda > 0$, then both $\underline{a}$ and $\lambda \underline{a}$ are in same direction.

(ii) If $\lambda < 0$, then both $\underline{a}$ and $\lambda \underline{a}$ are in opposite direction.

Note

Two vectors are said to be parallel, if they are non-zero scalar multiple of each other.



4. Write the vector $\overrightarrow{AB}$ in the form of $x\mathbf{i} + y\mathbf{j}$:

(i) $A(1, -7), B(-2, 4)$

$$\begin{aligned}\text{Sol. } \therefore \overrightarrow{AB} &= (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} \\ &= (-2 - 1)\mathbf{i} + [4 - (-7)]\mathbf{j} \\ \overrightarrow{AB} &= -3\mathbf{i} + 11\mathbf{j}\end{aligned}$$

(ii) $A(8, 9), B(12, 3)$

$$\begin{aligned}\text{Sol. } \therefore \overrightarrow{AB} &= (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} \\ &= (12 - 8)\mathbf{i} + (3 - 9)\mathbf{j} \\ \overrightarrow{AB} &= 4\mathbf{i} - 6\mathbf{j}\end{aligned}$$

5. Find the magnitude of the $\mathbf{a}$:

(i) $\mathbf{a} = -3\mathbf{i} + 2\mathbf{j}$

$$\begin{aligned}\text{Sol. } \therefore |\mathbf{a}| &= \sqrt{x^2 + y^2} \\ &= \sqrt{(-3)^2 + (2)^2} \\ &= \sqrt{9 + 4} \\ |\mathbf{a}| &= \sqrt{13}\end{aligned}$$

(ii) $\mathbf{a} = 4\mathbf{i} - 3\mathbf{j}$

$$\begin{aligned}\text{Sol. } \therefore |\mathbf{a}| &= \sqrt{x^2 + y^2} \\ &= \sqrt{4^2 + (-3)^2} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} \quad |\mathbf{a}| = 5\end{aligned}$$

(iii) $\mathbf{a} = \frac{1}{2}\mathbf{i} + \frac{3}{2}\mathbf{j}$

$$\text{Sol. } \therefore |\mathbf{a}| = \sqrt{x^2 + y^2}$$

8. If $\underline{a} = 3\underline{i} - \underline{j}$, $\underline{b} = -2\underline{i} + 4\underline{j}$ and $\underline{c} = \underline{i} + 2\underline{j}$, then find unit vector parallel to $3\underline{a} - 2\underline{c} + 4\underline{b}$.

Sol.
$$\begin{aligned} & 3\underline{a} - 2\underline{c} + 4\underline{b} \\ &= 3(3\underline{i} - \underline{j}) - 2(\underline{i} + 2\underline{j}) + 4(-2\underline{i} + 4\underline{j}) \\ &= 9\underline{i} - 3\underline{j} - 2\underline{i} - 4\underline{j} - 8\underline{i} + 16\underline{j} \\ &= (9 - 2 - 8)\underline{i} - (3 + 4 - 16)\underline{j} \\ &= -\underline{i} + 9\underline{j} \end{aligned}$$

Let $\underline{u} = -\underline{i} + 9\underline{j}$

$$\begin{aligned} \Rightarrow |\underline{u}| &= \sqrt{(-1)^2 + 9^2} \\ &= \sqrt{1 + 81} \\ |\underline{u}| &= \sqrt{82} \end{aligned}$$

$$\begin{aligned} \text{Required unit vector} &= \frac{\underline{u}}{|\underline{u}|} \\ &= \frac{-\underline{i} + 9\underline{j}}{\sqrt{82}} \\ &= -\frac{1}{\sqrt{82}}\underline{i} + \frac{9}{\sqrt{82}}\underline{j} \end{aligned}$$

$$= -3(-2\underline{i} + 3\underline{j})$$

$$\underline{b} = -3\underline{a}$$

Which shows that $\underline{a}$ and $\underline{b}$ are parallel,

with $\lambda = -3$

(iii) $\underline{a} = 5\underline{i} - 4\underline{j}$, $\underline{b} = 6\underline{i} - 3\underline{j}$

Sol. We see that there is no such constant λ such that $\underline{b} = \lambda \underline{a}$.

So these vectors are non parallel.

(iv) $\underline{a} = 3\underline{i} - 7\underline{j}$, $\underline{b} = 6\underline{i} - 14\underline{j}$

Sol. Given that

$$\begin{aligned} \underline{b} &= 6\underline{i} - 14\underline{j} \\ &= 2(3\underline{i} - 7\underline{j}) \end{aligned}$$

$$\Rightarrow \underline{b} = 2\underline{a}$$

Which shows that $\underline{a}$ and $\underline{b}$ are parallel

with $\lambda = 2$

10. Find a vector thrice in length of $3\underline{i} - 2\underline{j}$, but opposite in direction.

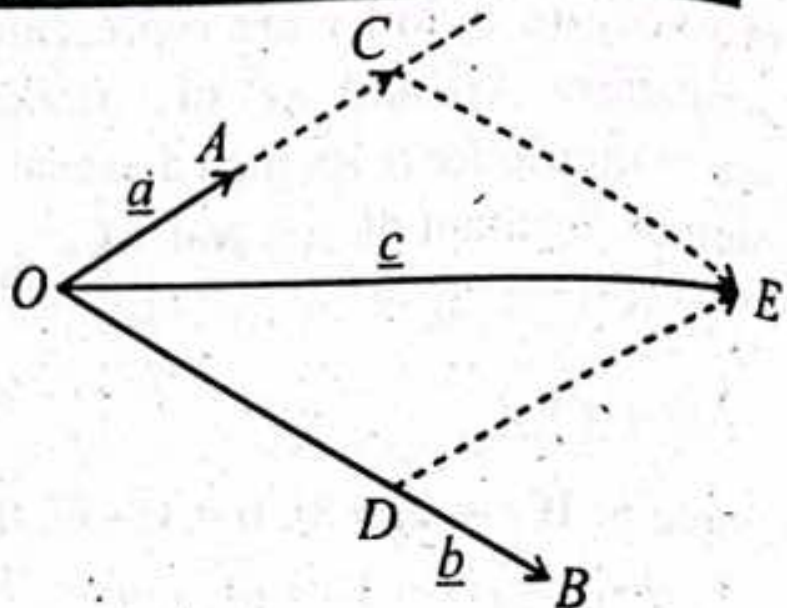
Sol. Required vector
$$\begin{aligned} &= -3(3\underline{i} + 2\underline{j}) \\ &= -9\underline{i} + 6\underline{j} \end{aligned}$$

11. Find two vectors that are double in

6.5 Vectors in Terms of Two Non-Zero and Non-Parallel Coplanar Vectors

Let $\underline{c}$ be any vector lying in the plane of $\underline{a}$ and $\underline{b}$. Draw $\underline{a}$, $\underline{b}$ and $\underline{c}$ such that their initial points coincide.

Construct lines parallel to the vectors $\underline{a}$ and $\underline{b}$ from the terminal point E of $\underline{c}$. Complete the parallelogram ODEC by extension of $\underline{a}$ and $\underline{b}$ if necessary.



$$\overline{OC} = m\overline{OA} = m\underline{a}, \quad \text{where } m \text{ is a scalar.}$$

$$\overline{OD} = n\overline{OB} = n\underline{b},$$

where n is a scalar.

By the parallelogram law of vector addition

$$\overline{OE} = \overline{OC} + \overline{OD} \quad \text{or} \quad \underline{c} = m\underline{a} + n\underline{b}$$

Where $m\underline{a}$ and $n\underline{b}$ are vector components of $\underline{c}$.

Remember!

Two or more vectors are said to be coplanar vectors if they lie in the same plane.

Example 8: If $\underline{a} = 2\underline{i} - 3\underline{j}$ and $\underline{b} = 3\underline{i} + 4\underline{j}$, then find the vector components of $3\underline{a} + 4\underline{b}$.

Sol. $3\underline{a} + 4\underline{b}$

3. Find the values of x and y in each of the following equations:

(i) $(x\underline{i} + y\underline{j}) + (2\underline{i} + 3\underline{j}) = 7\underline{i} + 6\underline{j}$

Sol.

$$\Rightarrow (x+2)\underline{i} + (y+3)\underline{j} = 7\underline{i} + 6\underline{j}$$

Equating the coefficients of $\underline{i}$ & $\underline{j}$ on both sides

$$\begin{aligned} x+2 &= 7 & , & & y+3 &= 6 \\ x &= 7-2 & , & & y &= 6-3. \end{aligned}$$

$$\boxed{x=5}$$

$$\boxed{y=3}$$

(ii) $(x\underline{i} - 5\underline{j}) + (3\underline{i} + 5\underline{j}) = -8\underline{i} + y\underline{j}$

Sol.

$$\Rightarrow (x+3)\underline{i} - (5-5)\underline{j} = -8\underline{i} + y\underline{j}$$

$$(x+3)\underline{i} - 0\underline{j} = -8\underline{i} + y\underline{j}$$

Equating the coefficients of $\underline{i}$ & $\underline{j}$ on both sides

$$\begin{aligned} x+3 &= -8 & , & & 0 &= y \\ x &= -8-3 & , & & \boxed{y=0} \end{aligned}$$

$$\boxed{x=-11}$$

(iii) $(y\underline{i} + 3\underline{j}) + (-5\underline{i} + 2x\underline{j}) = 9\underline{i} + 7\underline{j}$

6.6

Translation by a Vector

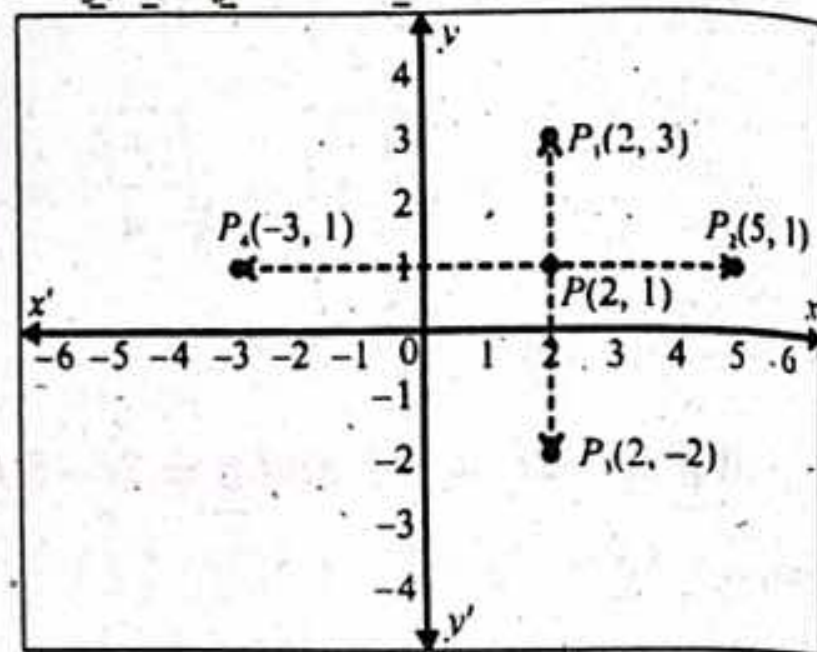
A translation vector is a vector that describes the movement of a point, a body, a system from one position to another without rotation or deformation. The translated figure is called the image of original figure. The image is congruent to the original figure. Let us translate point $P(2, 1)$ by translation vectors $-2\mathbf{j}$, $3\mathbf{i}$, $-3\mathbf{j}$ and $-5\mathbf{i}$.

$$\begin{aligned} P_1 &= (2\mathbf{i} + \mathbf{j}) + (0\mathbf{i} + 2\mathbf{j}) \\ &= 2\mathbf{i} + 3\mathbf{j} \end{aligned}$$

$$\begin{aligned} P_2 &= (2\mathbf{i} + \mathbf{j}) + (3\mathbf{i} + 0\mathbf{j}) \\ &= 5\mathbf{i} + \mathbf{j} \end{aligned}$$

$$\begin{aligned} P_3 &= (2\mathbf{i} + \mathbf{j}) + (0\mathbf{i} - 3\mathbf{j}) \\ &= 2\mathbf{i} - 2\mathbf{j} \end{aligned}$$

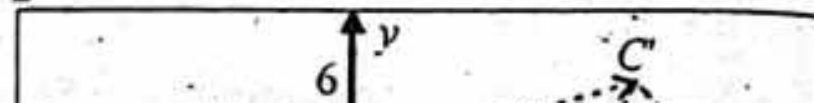
$$\begin{aligned} P_4 &= (2\mathbf{i} + \mathbf{j}) + (-5\mathbf{i} + 0\mathbf{j}) \\ &= -3\mathbf{i} + \mathbf{j} \end{aligned}$$



Example 9: Plot $A(-4, 2)$, $B(-1, 2)$ and $C(-3, 4)$ to form a triangle ABC . Also translate $\triangle ABC$ to $\triangle A'B'C'$ by the translation vector $7\mathbf{i} + 2\mathbf{j}$.

Solution:

New vertex $A' = (-4\mathbf{i} + 2\mathbf{j}) + (7\mathbf{i} + 2\mathbf{j})$



**Example 13:**

A tank fires a shell with velocity vector $\underline{v}_1 = 30\underline{i} + 40\underline{j}$ at a moving target heading east at $\underline{v}_2 = 10\underline{i} + 0\underline{j}$. Find relative velocity and its magnitude.

Solution: Relative velocity = Shell velocity – Target velocity

$$\begin{aligned}\underline{v}_{\text{rel}} &= (30 - 10)\underline{i} + (40 - 0)\underline{j} \\ &= 20\underline{i} + 40\underline{j}\end{aligned}$$

$$|\underline{v}_{\text{rel}}| = \sqrt{20^2 + 40^2} = \sqrt{400 + 1600}$$

$$|\underline{v}_{\text{rel}}| = \sqrt{2000} = 44.72 \text{ m/s}$$

Note

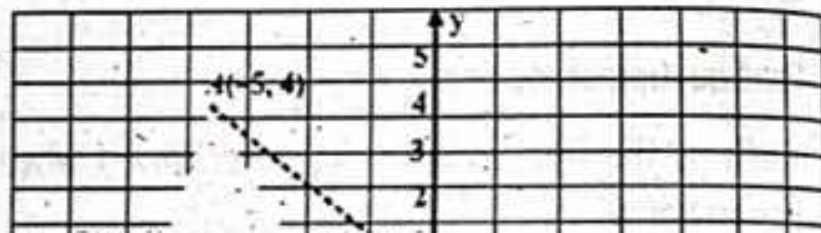
Relative velocity is the velocity of an object as observed from another moving object. It tells us how fast and in what direction one object is moving relative to another.

SOLVED EXERCISE 6.3

1. Plot $A(-5,4)$, $B(-6,1)$ and $C(-3,1)$ to form a $\triangle ABC$. Also translate $\triangle ABC$ to $\triangle A'B'C'$ by translation vector $8\underline{i} - 6\underline{j}$.

Sol. New vertex $A' = (-5\underline{i} + 4\underline{j}) + (8\underline{i} - 6\underline{j})$
 $= (-5 + 8)\underline{i} + (4 - 6)\underline{j}$
 $= 3\underline{i} - 2\underline{j} = (3, -2)$

New vertex $B' = (-6\underline{i} + 1\underline{j}) + (8\underline{i} - 6\underline{j})$





Which shows that opposite sides are equal in magnitude.

$$\begin{aligned}\overline{DC} &= -6\underline{i} - 2\underline{j} \\ &= -(6\underline{i} + 2\underline{j}) \\ \overline{DC} &= -\overline{AB}\end{aligned}$$

Which shows that $\overline{DC}$ and $\overline{AB}$ are scalar multiple of each other. Hence, they are parallel.

6. Points $A(1, 2)$, $B(4, 6)$ and $C(7, 2)$ form a triangle. Check whether triangle ABC is an isosceles by using vector magnitude.

Sol. Given points:

$$A(1, 2), B(4, 6), C(7, 2)$$

Now

$$\overline{AB} = (4-1)\underline{i} + (6-2)\underline{j}$$

$$\overline{AB} = 3\underline{i} + 4\underline{j}$$

and

$$\overline{BC} = (7-4)\underline{i} + (2-6)\underline{j}$$

$$\overline{BC} = 3\underline{i} - 4\underline{j}$$

also

$$\overline{AC} = (7-1)\underline{i} + (2-2)\underline{j}$$

10. A car enters a loop with velocity vector $\underline{v} = 30\underline{j}$ and exits with velocity $\underline{v} = 30\underline{i}$. What is the change in velocity vector?

Sol. initial velocity $\underline{v} = 30\underline{j}$
 final velocity ($\underline{v}'$) $= 30\underline{i}$
 $\Rightarrow$ Change in velocity $= \Delta \underline{v} = \underline{v}' - \underline{v}$
 $= 30\underline{i} - 30\underline{j}$ Ans.

11. An aeroplane has airspeed $\underline{v}_p = 20\underline{i}$ and there is a crosswind $\underline{v}_w = 50\underline{j}$. Find the resultant velocity and its magnitude.

Sol: airspeed $= (\underline{v}_p) = 20\underline{i}$
 cross wind ($\underline{v}_w$) $= 50\underline{j}$
 $\Rightarrow$ Resultant velocity $= \underline{v} = 20\underline{i} + 50\underline{j}$
 Resultant velocity $= |\underline{v}| = \sqrt{(20)^2 + (50)^2}$
 $= \sqrt{400 + 2500}$
 $= \sqrt{2900}$
 $= 10\sqrt{29}$
 $= 53.85$ units

$$\begin{aligned}
 \text{(iii) } \underline{c} + \underline{b} - 4\underline{a} &= 4\underline{i} + \underline{j} + 3\underline{i} + \underline{j} - 4(2\underline{i} - \underline{j}) \\
 &= 4\underline{i} + 3\underline{i} - 8\underline{i} + \underline{j} + \underline{j} + 4\underline{j} \\
 &= (4+3-8)\underline{i} + (1+1+4)\underline{j} \\
 &= -\underline{i} + 6\underline{j} \quad \text{Ans.}
 \end{aligned}$$

5. Find the values of x and y in the following equation.

$$\text{Sol: } (2x\underline{i} + y\underline{j}) + (-\underline{i} + 5\underline{j}) = \frac{1}{4}\underline{i} - 8\underline{j}$$

$$(2x\underline{i} + y\underline{j}) + (-\underline{i} + 5\underline{j}) = \frac{1}{4}\underline{i} - 8\underline{j}$$

$$(2x-1)\underline{i} + (y+5)\underline{j} = \frac{1}{4}\underline{i} - 8\underline{j}$$

Comparing coefficients of $\underline{i}$ and $\underline{j}$ on both sides

$$2x-1 = \frac{1}{4}, \quad y+5 = -8$$

$$2x = \frac{1}{4} + 1, \quad y = -8 - 5$$

$$2x = \frac{1+4}{4}, \quad \boxed{y = -13}$$

$$x = \frac{5}{2 \times 4} \Rightarrow \boxed{x = \frac{5}{8}}$$

6. Plot $A(-5, 3)$, $B(-2, 3)$ and $C(-4, 5)$ to form a triangle. ΔABC Also translate ΔABC to $\Delta A'B'C'$ by translation vectors $5\underline{i} - 2\underline{j}$.

$$\begin{aligned}
 \text{Sol: New vertex } A' &= (-5\underline{i} + 3\underline{j}) + (5\underline{i} - 2\underline{j}) \\
 &= (-5+5)\underline{i} + (3-2)\underline{j} = 0\underline{i} + \underline{j} = (0, 1)
 \end{aligned}$$



10. An aircraft is flying due east with an airspeed of 200 km/h. There is a wind blowing due north at 60 km/h. Find the resultant velocity and its magnitude.

Sol: velocity of aircraft (V_a) = $200\hat{i}$ (East)

velocity of wind (V_w) = $60\hat{j}$ (North)

Resultant velocity = $200\hat{i} + 60\hat{j}$

$$\begin{aligned}\text{Magnitude} = |\underline{V}| &= \sqrt{(200)^2 + (60)^2} = \sqrt{40000 + 3600} \\ &= \sqrt{43600} = 20\sqrt{109} \text{ km/h} \\ &= 208.8 \text{ km/h} \quad \text{Ans.}\end{aligned}$$

Activity

1. Suppose vectors $\underline{a}$ and $\underline{b}$ are equal.

(i) Can we say they originate from the same point? Why or why not?

Ans. No, they do not necessarily originate from the same point. Equal vectors have the same magnitude and direction regardless of the position of their initial points.

(ii) Do they have equal magnitudes? Explain.

Ans. Yes, they must have equal magnitudes. By definition, two vectors are equal if they have the same magnitude.

(iii) Do they have same direction? Why?

Ans. Yes, they have the same direction. Vectors are considered equal only when they have same direction and magnitude.

2. Suppose vectors $\underline{a}$ and $\underline{b}$ are opposite.

(i) Can we assume they begin at the same point? Give a reason.

Ans. No we cannot assume they begin at the same point. The relationship of two vectors is defined

6.2.6

Unit Vector

21. A vector whose magnitude is 1 is called _____ vector.
 (a) Null vector (b) Unit vector
 (c) Position vector (d) Negative vector
22. If $\underline{a}$ is a non-zero vector then its unit vector is represented by:
 (a) $\hat{a}$ (b) $|\underline{a}|$ (c) $|\hat{a}|$ (d) $\underline{a}$
23. If $\underline{a}$ is a non-zero vector, then its unit vector is defined as:
 (a) $\underline{a} = \lambda \underline{a}$ (b) $\hat{a}$ (c) $\hat{a} = \frac{\underline{a}}{|\underline{a}|}$ (d) $\hat{a} = \frac{|\underline{a}|}{\underline{a}}$
24. If $\underline{a}$ is a unit vector then the value of $|\underline{a}|$ is:
 (a) 1 (b) 2 (c) 4 (d) 4

6.3

Addition of Vectors

25. If two sides of triangle represent vectors $\underline{u}$ and $\underline{v}$ in order, the third side represents:
 (a) $\underline{u} - \underline{v}$ (b) $\frac{\underline{u}}{\underline{v}}$ (c) $\underline{u} + \underline{v}$ (d) $|\underline{u} + \underline{v}|$
26. If $\underline{a} = -\underline{i} + \underline{j}$ and $\underline{b} = 2\underline{i} + 2\underline{j}$, then $\underline{a} + \underline{b} =$ _____ ?
 (a) $\underline{i} + 3\underline{j}$ (b) $\underline{i} + 2\underline{j}$ (c) $2\underline{i} + 3\underline{j}$ (d) $-\underline{i} + 3\underline{j}$



$$= \sqrt{\frac{1}{16} + \frac{225}{16}} = \sqrt{\frac{1+225}{16}} = \frac{\sqrt{226}}{16}$$

$$|\underline{u}| = \frac{\sqrt{226}}{4}$$

4. Find unit vector of $\underline{v} = -3\underline{i} + 4\underline{j}$

Sol: $\underline{v} = -3\underline{i} + 4\underline{j}$

OR

$$|\underline{v}| = \sqrt{(-3)^2 + 4^2}$$

$$= \sqrt{9+16}$$

$$= \sqrt{25}$$

$$|\underline{v}| = 5$$

$$\hat{\underline{v}} = \frac{|\underline{v}|}{\underline{v}} = \frac{-3\underline{i} + 4\underline{j}}{5}$$

$$\hat{\underline{v}} = -\frac{3}{5}\underline{i} + \frac{4}{5}\underline{j}$$

$$\hat{\underline{v}} = \frac{\underline{v}}{|\underline{v}|}$$

$$= \frac{-3\underline{i} + 4\underline{j}}{\sqrt{(-3)^2 + 4^2}}$$

$$= \frac{-3\underline{i} + 4\underline{j}}{\sqrt{9+16}}$$

$$= \frac{-3\underline{i} + 4\underline{j}}{\sqrt{25}} = \frac{-3\underline{i} + 4\underline{j}}{5}$$

$$\hat{\underline{v}} = -\frac{3}{5}\underline{i} + \frac{4}{5}\underline{j}$$

5. Check that the vectors $\underline{u} = 8\underline{i} + 4\underline{j}$ and $\underline{v} = 2\underline{i} + \underline{j}$ are parallel or not.

Sol: $\underline{u} = 8\underline{i} + 4\underline{j}$



$$= 6\mathbf{i} - \mathbf{j} - 2\mathbf{i} - 10\mathbf{j}$$

$$= (6-2)\mathbf{i} - (1+10)\mathbf{j}$$

$$\mathbf{v} - 2\mathbf{u} = 4\mathbf{i} - 11\mathbf{j} \quad \text{Ans.}$$

13. If $\mathbf{a} = 7\mathbf{i} - 3\mathbf{j}$, $\mathbf{b} = 3\mathbf{i} + 4\mathbf{j}$ and $\mathbf{c} = 2\mathbf{i} - \mathbf{j}$, then find $2\mathbf{a} - 3\mathbf{b} - \mathbf{c}$.

$$\begin{aligned} \text{Sol: } 2\mathbf{a} - 3\mathbf{b} - \mathbf{c} &= 2(7\mathbf{i} - 3\mathbf{j}) - 3(3\mathbf{i} + 4\mathbf{j}) - (2\mathbf{i} - \mathbf{j}) \\ &= 14\mathbf{i} - 6\mathbf{j} - 9\mathbf{i} - 12\mathbf{j} - 2\mathbf{i} + \mathbf{j} \\ &= (14-9-2)\mathbf{i} - (6+12-1)\mathbf{j} \end{aligned}$$

$$2\mathbf{a} - 3\mathbf{b} - \mathbf{c} = 3\mathbf{j} - 7\mathbf{i} \quad \text{Ans.}$$

14. If $(x\mathbf{i} + y\mathbf{j}) - (2\mathbf{i} - 4\mathbf{j}) = -5\mathbf{i} + 3\mathbf{j}$, then find the values of x and y .

$$\begin{aligned} \text{Sol: } (x\mathbf{i} + y\mathbf{j}) - (2\mathbf{i} - 4\mathbf{j}) &= -5\mathbf{i} + 3\mathbf{j} \\ (x-2)\mathbf{i} + (y+4)\mathbf{j} &= -5\mathbf{i} + 3\mathbf{j} \end{aligned}$$

Comparing coefficients of x and y .

$$\Rightarrow x-2 = -5, \quad y+4 = 3$$

$$x = -5+2, \quad y = 3-4$$

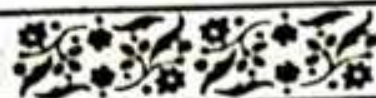
$$\boxed{x = -3}, \quad \boxed{y = -1}$$

6.6

Translation by a vector

15. Define a translation vector.

Ans: A translation vector is a vector that describes the movement of a point, a body, a system from one position to another without rotation or deformation.



22. If $B(4,1)$, $C(-2,3)$ and $D(-8,0)$ are the vertices of a parallelogram ABCD, then find coordinates of the vertex A.

Sol: Let $A(x,y)$ be the required vertex.

Since ABCD is a parallelogram.

So $\overrightarrow{AB} = \overrightarrow{DC}$

$$(4-x)\underline{i} + (1-y)\underline{j} = (-2-(-8))\underline{i} + (3-0)\underline{j}$$

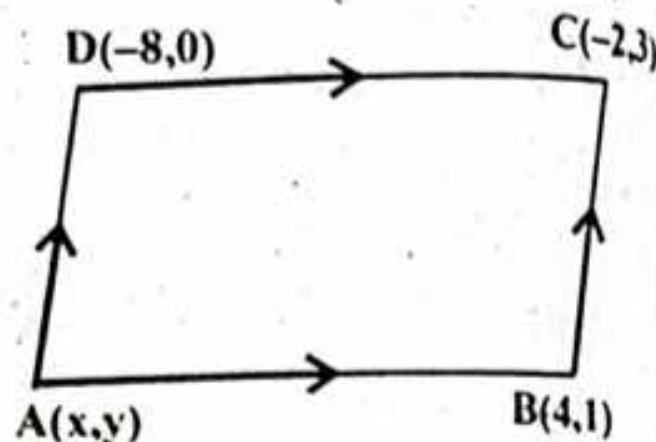
$$(4-x)\underline{i} + (1-y)\underline{j} = 6\underline{i} + 3\underline{j}$$

$$4-x = 6 \quad , \quad 1-y = 3$$

$$-x = 6 - 4 \quad , \quad -y = 3 - 1$$

$$-x = 2 \quad , \quad -y = 2$$

$$\boxed{x = -2} \quad , \quad \boxed{y = -2}$$



So required vertex is $A(-2, -2)$

23. A projectile is launched from ground with initial velocity vector $3\underline{i} + 5\underline{j}$. After how long does it hit a target located 15 meters away horizontally?

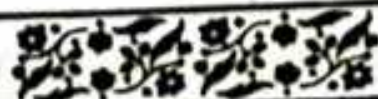
Sol: Given that

$$\underline{v}_i = 3\underline{i} + 5\underline{j}$$

$$\text{Horizontal Velocity} = v_x = 3$$

$$\text{Vertical Velocity} = v_y = 5$$

$$\text{Distance} = x = 15 \text{ m}$$



Signs of Trigonometric Functions

The signs depend on the quadrant in which terminal side of the angle lies.

Consider a circle having radius r and centre at point O .

In 1st Quadrant: $x > 0, y > 0$

$$\sin \theta = \frac{y}{r} > 0, \cos \theta = \frac{x}{r} > 0, \tan \theta = \frac{y}{x} > 0,$$

$$\operatorname{cosec} \theta = \frac{r}{y} > 0, \sec \theta = \frac{r}{x} > 0, \cot \theta = \frac{x}{y} > 0$$

Hence, in the 1st quadrant all trigonometric functions are positive.

In 2nd Quadrant: $x < 0, y > 0$

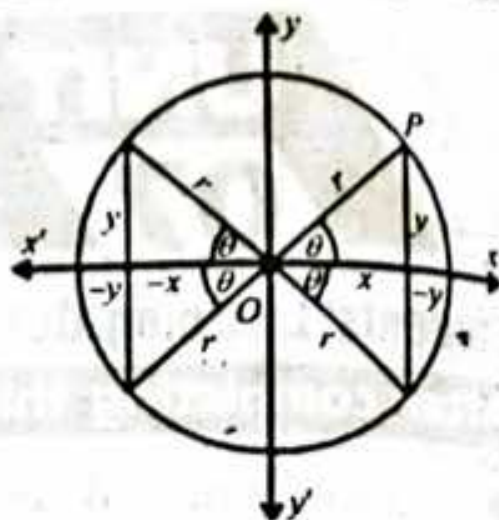
$$\sin \theta = \frac{y}{r} > 0, \cos \theta = \frac{-x}{r} < 0, \tan \theta = \frac{y}{-x} < 0, \operatorname{cosec} \theta = \frac{r}{y} > 0, \sec \theta = \frac{-x}{y} < 0, \cot \theta = \frac{-x}{y} < 0$$

Hence, in the 2nd quadrant $\sin$ and cosec functions are positive and all other trigonometric functions are negative.

In 3rd Quadrant: $x < 0, y < 0$

$$\sin \theta = \frac{-y}{r} < 0, \cos \theta = \frac{-x}{r} < 0, \tan \theta = \frac{-y}{-x} > 0, \operatorname{cosec} \theta = \frac{r}{-y} < 0, \sec \theta = \frac{r}{-x} < 0, \cot \theta = \frac{-x}{-y} > 0$$

Hence, in the 3rd quadrant $\tan$ and $\cot$ functions are positive and all other trigonometric





Extending sine and cosine Functions to Angles Between 90° and 180° .

a. Trigonometric Ratios of an Angle of the Form $(90^\circ + \theta)$

Consider a unit circle having radius 1 with centre at $O(0,0)$ in the Cartesian coordinate system. $P(\cos \theta, \sin \theta)$ be any point in quadrant I on the circumference of the circle,

where $m\angle MOP = \theta = m\angle YOP'$

$m\angle MOP' = (90^\circ + \theta)$ and $m\angle POP' = 90^\circ$

so, we have $m\angle OP'M' = m\angle YOP'$ (Alternate angles)

That is, the angle $(90^\circ + \theta)$ is obtained by rotating the angle θ counterclockwise by 90°

The coordinates of P' will be $(-\sin \theta, \cos \theta)$, i.e. the new x -coordinate of P' becomes the negative of the y -coordinate of the P and new y -coordinate of P' becomes the x -coordinate of the P .

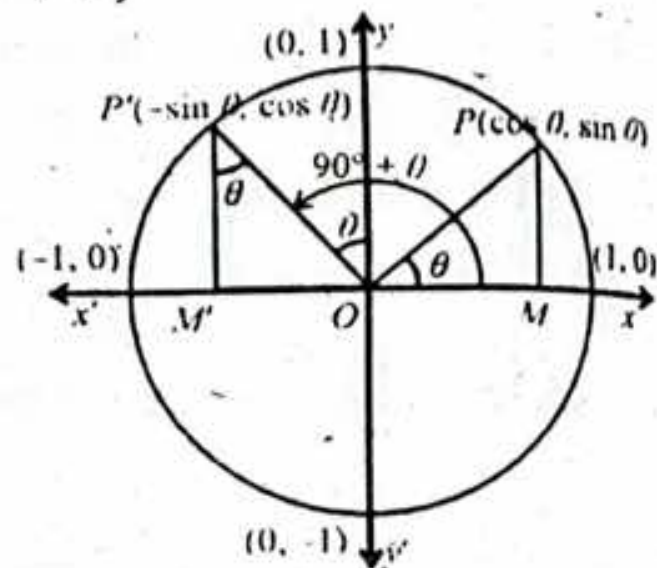
As we know that in unit circle, cosine and sine of any angle are represented by x -coordinate and y -coordinate respectively. Therefore,

$$\cos(90^\circ + \theta) = x\text{-coordinate of } (90^\circ + \theta) = -\sin \theta$$

$$\sin(90^\circ + \theta) = y\text{-coordinate of } (90^\circ + \theta) = \cos \theta$$

$$\text{Hence, } \sin(90^\circ + \theta) = \cos \theta, \text{ cosec}(90^\circ + \theta) = \sec \theta$$

$$\cos(90^\circ + \theta) = -\sin \theta, \sec(90^\circ + \theta) = -\text{cosec } \theta$$

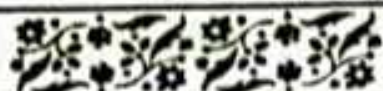


Note

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$



(ii) $\tan 150^\circ$

Sol: $\tan 150^\circ = \tan (180^\circ - 30^\circ) = -\tan 30^\circ = \frac{-1}{\sqrt{3}} \quad \therefore \tan (180^\circ - \theta) = -\tan \theta$

(iii) $\sec 150^\circ$

Sol: $\sec 150^\circ = \sec (180^\circ - 30^\circ) = -\sec 30^\circ = -\frac{2}{\sqrt{3}} \quad \therefore \sec (180^\circ - \theta) = -\sec \theta$

(iv) $\operatorname{cosec} 120^\circ$

Sol: $\operatorname{cosec} 120^\circ = \operatorname{cosec} (180^\circ - 60^\circ) = +\operatorname{cosec} 60^\circ = \frac{2}{\sqrt{3}} \quad \therefore \operatorname{cosec} (180^\circ - \theta) = -\operatorname{cosec} \theta$

(v) $\cos 120^\circ$

Sol: $\cos 120^\circ = \cos (180^\circ - 60^\circ) = -\cos 60^\circ = -\frac{1}{2} \quad \therefore \cos (180^\circ - \theta) = -\cos \theta$

(vi) $\cot 120^\circ$

Sol: $\cot 120^\circ = \cot (180^\circ - 60^\circ) = -\cot 60^\circ = -\frac{1}{\sqrt{3}} \quad \therefore \cot (180^\circ - \theta) = -\cot \theta$

(vii) $\sin 135^\circ$

**Challenge!**

Can we use law of sines, when two sides and included angle or the three sides of a triangle are given?

Sol. No, not directly if you have sides a and b and angle between them. You lack d for side α and γ for side. No pair is complete.

If you have only sides a, b and c you do not have any angle to start the ratio

The Law of cosines

If a, b and c are the sides and α, β and γ are the angles opposite to the sides of any triangle respectively, then the law of cosines is:

$$c^2 = a^2 + b^2 - 2ab \cos \gamma \text{ or } \cos \gamma = \frac{a^2 + b^2 - c^2}{2ab}$$

$$a^2 = b^2 + c^2 - 2bc \cos \alpha \text{ or } \cos \alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

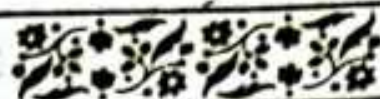
$$b^2 = c^2 + a^2 - 2ca \cos \beta \text{ or } \cos \beta = \frac{c^2 + a^2 - b^2}{2ca}$$

Example 4: In $\triangle ABC$, $a = 3, b = 5, c = 7$. Find the values of α, β and γ .

Solution: Given that $a = 3, b = 5, c = 7$

By using the cosines law, we have

$$\cos \alpha = \frac{b^2 + c^2 - a^2}{2bc} = \frac{5^2 + 7^2 - 3^2}{2 \times 5 \times 7} = \frac{25 + 49 - 9}{70} = \frac{13}{14}$$



$$\begin{aligned}\text{Area of } \triangle ABC &= \frac{a^2 \sin \beta \sin \gamma}{2 \sin \alpha} \\ &= \frac{(2)^2 \sin 65^\circ \sin 80^\circ}{2 \sin 35^\circ} \\ &= \frac{4(0.91)(0.98)}{2 \sin 35^\circ}\end{aligned}$$

Skilled Practice!

Calculate area of $\triangle ABC$, in which

$\beta = 50^\circ$, $\gamma = 40^\circ$, $c = 10\text{cm}$

Sol. $\beta = 50^\circ$, $\gamma = 40^\circ$, $c = 10\text{cm}$

$$\alpha + \beta + \gamma = 180^\circ$$

$$\begin{aligned}\alpha &= 180^\circ - \beta - \gamma \\ &= 180 - 50^\circ - 40^\circ = 90^\circ\end{aligned}$$

$$\begin{aligned}\text{Area} &= \frac{c^2 \sin(\alpha) \sin(\beta)}{2 \sin(\gamma)} \\ &= \frac{10^2 \sin(90^\circ) \sin(50^\circ)}{2 \sin(40^\circ)} \\ &= 100 \times 1 \times 0.766^\circ\end{aligned}$$



$$\alpha + \beta + \gamma = 180^\circ$$

$$\gamma = 180^\circ - \alpha - \beta$$

$$= 180^\circ - 52^\circ - 71^\circ$$

$$= 57^\circ$$

By law of sine

$$\frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

$$\frac{4.8}{\sin 71^\circ} = \frac{c}{\sin 57^\circ}$$

$$\sin 57^\circ \times \frac{4.8}{\sin 71} = c$$

$$\frac{0.8 \times 4.8}{0.9} = c$$

$$4.3 = c$$

$$\text{so, } c = 4.3 \text{ cm, } \alpha = 52^\circ, \gamma = 57^\circ$$

(v) $\beta = 70^\circ, b = 8 \text{ cm}, \alpha = 100$

Sol. $\beta = 70^\circ, b = 8 \text{ cm}, \alpha = 100$

use law of sines

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta}$$

$$0.17 \times \frac{8}{0.94} = c$$

$$1.5 = c$$

$$c = 1.5 \text{ cm}$$

So, $a = 8.4 \text{ cm}, c = 1.5 \text{ cm}, \gamma = 10^\circ$

(vi) $a = 6 \text{ cm}, \alpha = 55^\circ, \gamma = 60^\circ$

Sol. $a = 6 \text{ cm}, \alpha = 55^\circ, \gamma = 60^\circ$

we know

$$\alpha + \beta + \gamma = 180^\circ$$

$$\beta = 180^\circ - \alpha - \gamma$$

$$\beta = 180^\circ - 55 - 60$$

$$\beta = 65^\circ$$

By law of sine

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta}$$

$$\frac{6}{\sin 55^\circ} = \frac{b}{\sin 65^\circ}$$

$$\sin 65^\circ \times \frac{6}{\sin 55^\circ} = b$$



$$= \frac{1}{2} (3) (9) (\sin 78^\circ)$$

$$= 13.5 (0.9781)$$

$$= 13.20 \text{ cm}^2$$

(iv) $a = 10 \text{ cm}$, $\alpha = 62^\circ$, $\beta = 69^\circ$

Sol. $a = 10 \text{ cm}$, $\alpha = 62^\circ$, $\beta = 69^\circ$

We know

$$\alpha + \beta + \gamma = 180^\circ$$

$$\gamma = 180 - \alpha - \beta$$

$$= 180 - 62^\circ - 69^\circ = 49^\circ$$

$$\begin{aligned} \text{Area} &= \frac{a^2 \sin \beta \sin \gamma}{2 \sin \alpha} \\ &= \frac{(10)^2 (\sin 69^\circ) (\sin 49^\circ)}{2 \sin 62^\circ} \end{aligned}$$

$$= \frac{100(0.9336)(0.7547)}{2(0.8829)}$$

$$= \frac{70.4588}{1.7658} = 39.90 \text{ cm}^2$$

$$= \frac{8.4530}{1.9696} = 4.29 \text{ cm}^2$$

(vi) $c = 6.6 \text{ cm}$, $\alpha = 23^\circ$, $\gamma = 89^\circ$

Sol. $c = 6.6 \text{ cm}$, $\alpha = 23^\circ$, $\gamma = 89^\circ$

We know

$$\alpha + \beta + \gamma = 180^\circ$$

$$\beta = 180^\circ - \alpha - \gamma$$

$$= 180^\circ - 23^\circ - 89^\circ = 68^\circ$$

$$\begin{aligned} \text{Area} &= \frac{c^2 \sin \alpha \sin \beta}{2 \sin \gamma} \\ &= \frac{(6.6)^2 (\sin 23^\circ) (\sin 68^\circ)}{2 (\sin 89^\circ)} \end{aligned}$$

$$= \frac{43.56(0.3907)(0.9272)}{2(0.9998)}$$

$$= \frac{15.7799}{1.9996} = 7.89 \text{ cm}^2$$

(vii) $a = 5.3 \text{ cm}$, $b = 4.7 \text{ cm}$, $c = 8.2 \text{ cm}$

Sol. $a = 5.3 \text{ cm}$, $b = 4.7 \text{ cm}$, $c = 8.2 \text{ cm}$

$$a = 5.3, b = 4.7, c = 8.2$$



7.4

Concept of Trigonometry in Real Life

Example 10: A plane takes off at an angle of elevation of 12° and travels 2000 metres along this path. How high is the plane after traveling this distance?

Sol. In triangle ABC,

$$\sin 12^\circ = \frac{m\overline{BC}}{2000}$$

$$(0.2079)2000 = m\overline{BC}$$

$$m\overline{BC} = 415.8 \text{ m}$$

The plane is approximately 415.8 m above the ground.

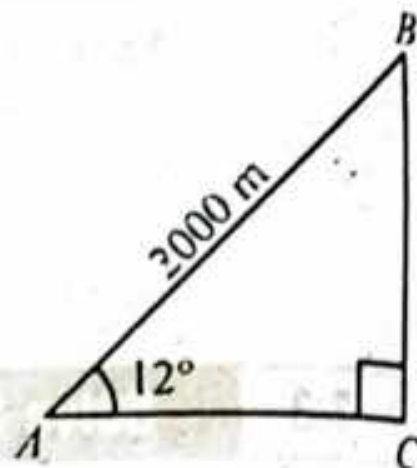
Example 11: In a sound lab, two microphones are placed $c = 3$ m apart and a sound source is $a = 2$ m from one and $b = 2.5$ m from the other. Find the angle between the sound paths a and b .

Sol. Here $a = 2$ m, $b = 2.5$ m, $c = 3$ m

Using law of cosines, we have

$$\cos \gamma = \frac{a^2 + b^2 - c^2}{2ab} = \frac{2^2 + (2.5)^2 - 3^2}{2 \times 2 \times 2.5} = \frac{4 + 6.25 - 9}{10} = 0.125$$

$$\gamma = \cos^{-1}(0.125)$$





$$= \sqrt{200}$$

$$= \sqrt{4 \times 50}$$

$$= 2\sqrt{50}$$

$\overline{PM}$ is the half of $\overline{PR}$

$$\overline{PM} = \frac{\overline{PR}}{2} = \frac{2\sqrt{50}}{2} = \sqrt{50} \text{ cm}$$

In right triangle VMP, use the tangent ratio

$$\tan \theta = \frac{\text{opposite}}{\text{Adjacent}}$$

$$\tan \angle VPM = \frac{\overline{VM}}{\overline{PM}}$$

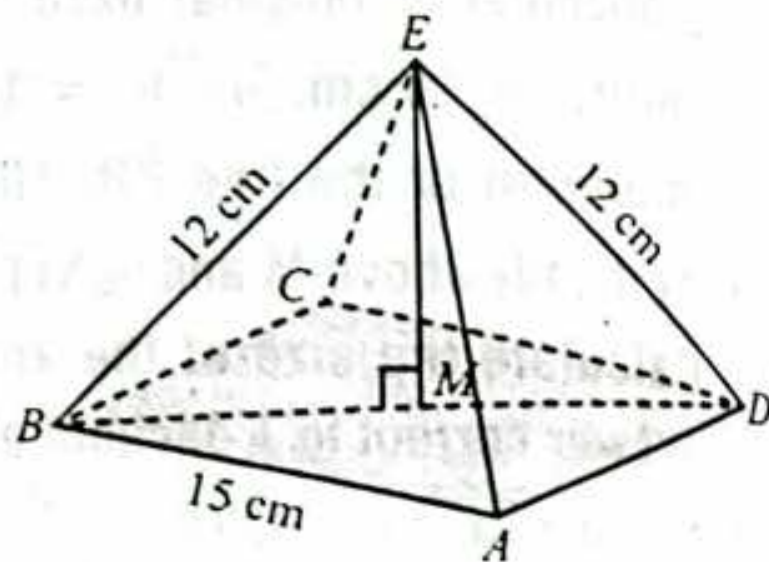
$$\tan \angle VPM = \frac{12}{\sqrt{50}}$$

$$\angle VPM = \tan^{-1} \left(\frac{12}{7.071} \right)$$

$$\angle VPM = 59.5^\circ$$

4. **ABCDE** is a square based pyramid, in which $m\overline{AE} = m\overline{BE} = m\overline{CE} = m\overline{DE} = 12 \text{ cm}$ and $m\overline{AB} = 15 \text{ cm}$. Calculate the size of angle **DEB**. Give your answer to the nearest degree.

Sol. In isosceles triangle **DEB**, we have
 $m\overline{EB} = m\overline{ED} = 12 \text{ cm}$
 $\overline{BD}$ is the diagonal of the square base
 $m\overline{AB} = 15 \text{ cm}$





$$\tan 28^\circ = \frac{x}{60}$$

$$60 \times \tan 28^\circ = x$$

$$60 \times 0.5317 = x$$

$$31.9 = x$$

$$x = 31.9\text{m}$$

So, height of the pole = 31.9m

8. A boat crosses a river 80 m wide, at a 60° angle to the current. What distance does the boat actually travel?

Sol. Width of the river = 80 m

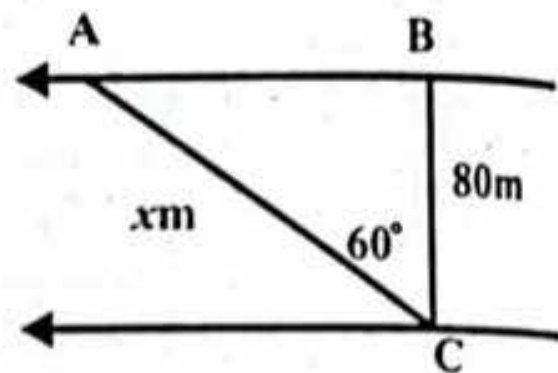
Angle to current = 60°

Let distance travelled by boat = x m (Hypotenuse)

$$\cos \theta = \frac{\text{Adjacent}}{\text{hypotenuse}}$$

$$\cos 60^\circ = \frac{80}{x}$$

$$0.5 = \frac{80}{x}$$





A bearing is a way of describing the direction or angle from one point to another, measured in degrees from the north direction in a clockwise manner. It is always written as a three-digit angle between 000° and 360° .

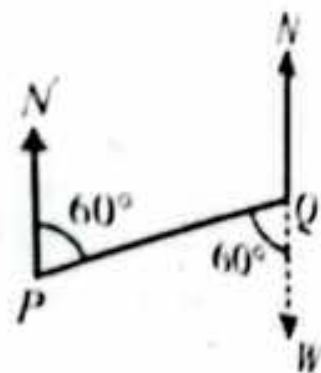
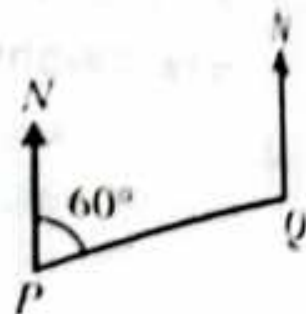
For example, a plane flying east has a bearing of 090° .

Example 12: In the diagram, the bearing of Q from P is 060° . Find the bearing of P from Q .

Solution: Extend the line NQ , so that angle $NQW = 180^\circ$. As alternate angles are equal, therefore

$$060^\circ = m\angle PQW$$

$$\begin{aligned}\text{So, the bearing of } P \text{ from } Q &= 180^\circ + 060^\circ \\ &= 240^\circ\end{aligned}$$

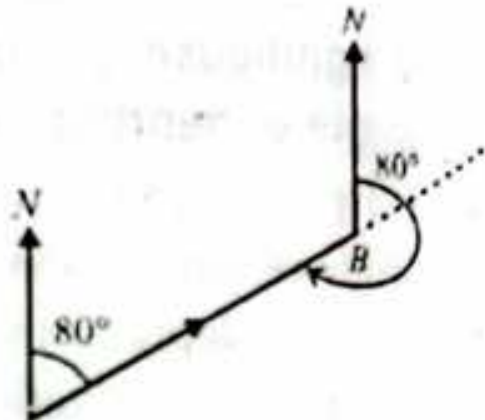


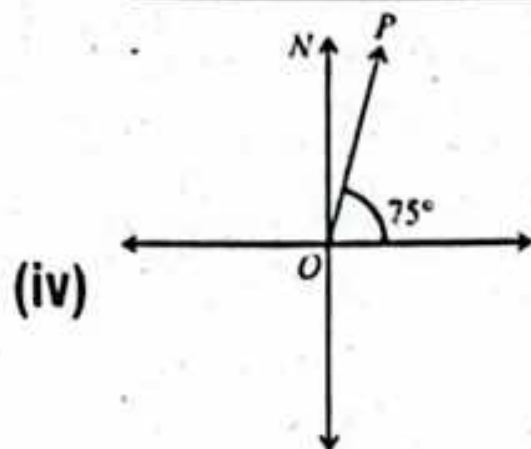
Example 13: Abdullah sails a ship from point A to point B on a bearing of 080° . What bearing should he follow to return from point B back to point A ?

Solution: The diagram shows the journey from A to B .

Extend the line of the journey and we get an angle of 80° at B .

$$\text{Bearing of } A \text{ from } B = 080^\circ + 180^\circ$$





Quadrant of point P = Southwest

Sol. Measure of given angle from the west line = 075°

The angle from North clockwise to the East line = 90°

Since P is 075° back (counter-clockwise) from the east line, we subtract it from 90°

So, the bearing of Point P = $090^\circ - 075^\circ = 015^\circ$

2. The diagram shows the positions of two ships P and Q .

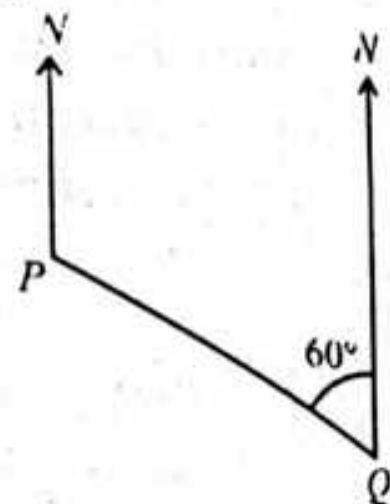
(i) What is the bearing of ship P from ship Q ?

Sol. Measure of angle to the left of North = 060°

Measure of angle of full circle = 360°

Since, we clockwise:

Bearing of Ship P from Ship Q = $360^\circ - 060^\circ$
 = 300°



(ii) What is bearing of ship Q from ship P ?

Sol. Since North lines are parallel = 060°



$$\begin{aligned}\text{South ward distance} &= 20 (\cos 60^\circ) \\ &= 20 \times 0.5 \\ &= 10 \text{ km}\end{aligned}$$

7. Fatima walked south for 5.5 km and then turned west for 1.3 km. Calculate Huria's bearing from her starting point.

Sol. Walk to South = 5.5 km

Walk to West = 1.3 km

Thus quadrant of place = III

Find internal angle (α)

$$\tan(\alpha) = \frac{\text{opposite (West)}}{\text{Adjacent (South)}}$$

$$= \frac{1.3}{5.5}$$

$$\tan(\alpha) = 0.236$$

$$\alpha = \tan^{-1}(0.236)$$

$$\alpha = 13.3$$

$$\begin{aligned}\text{Bearing from her starting point} &= 180^\circ + 13.3^\circ \\ &= 193.3^\circ\end{aligned}$$

8. An aircraft flies 150 km east, then 100 km northeast (45° from east). What is the total displacement?



- (vi) $\frac{b}{\sin \alpha} = \frac{b}{\sin B} = \frac{c}{\sin \gamma}$ (vii) $\cos \gamma = \frac{a^2 + b^2 - c^2}{2ab}$ (viii) $\frac{1}{2} ac \sin \beta$ (ix) North
(x) 3 figures

2. Calculate the area of ΔABC , in which.

- (i) $a = 4$ cm, $b = 6$ cm, $c = 8$ cm

$$\text{Sol: } s = \frac{a+b+c}{2} = \frac{4+6+8}{2} = \frac{18}{2} = 9$$

$$\begin{aligned} \text{Area of } \Delta ABC &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{9(9-4)(9-6)(9-8)} \\ &= \sqrt{9(5)(3)(1)} = \sqrt{135} \\ &= 11.62 \text{ cm}^2 \end{aligned}$$

- (ii) $b = 2.1$ cm, $c = 5$ cm, $\gamma = 45^\circ$

$$\text{Sol: Area of } \Delta ABC = \frac{1}{2} bc \sin \gamma$$

$$= \frac{1}{2} \times 5 \times 2.1 \times \sin 45^\circ$$

$$= 5.25 \times (0.7071)$$

use law of sines

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta}$$

$$\frac{5.4}{\sin 49^\circ} = \frac{3.4}{\sin \beta}$$

$$\sin \beta = \frac{3.4}{5.4} \times \sin 49^\circ$$

$$\sin \beta = 0.629 \times 0.7547$$

$$\sin \beta = 0.475$$

$$\begin{aligned} \beta &= \sin^{-1}(0.475) \\ &= 28.37^\circ \end{aligned}$$

We know

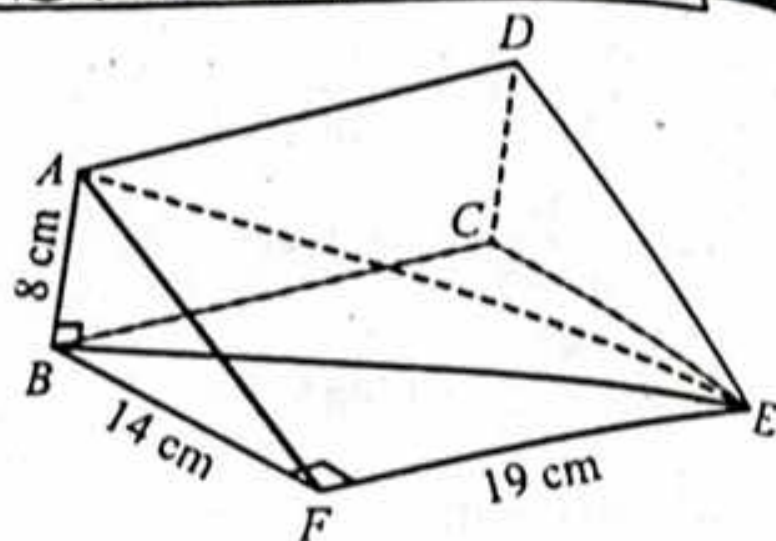
$$\alpha + \beta + \gamma = 180^\circ$$

$$\gamma = 180^\circ - \alpha - \beta$$

$$\gamma = 180^\circ - 49^\circ - 28.37^\circ$$



$$\begin{aligned}
 AF &= \sqrt{(\overline{AB})^2 + (\overline{BF})^2} \\
 &= \sqrt{8^2 + 14^2} \\
 &= \sqrt{64 + 196} \\
 &= \sqrt{260} \\
 &= 16.1 \text{ cm}
 \end{aligned}$$



(ii) Calculate the angle between $\overline{AF}$ and the plane $BCEF$.

Sol: Since a $\overline{AB}$ is vertical to the base plane $BCEF$, the angle is $\angle AFB$

$$\tan \angle AFB = \frac{\overline{AB}}{\overline{BF}}$$

$$\tan \angle AFB = \frac{8}{14}$$

$$\tan \angle AFB = 0.5714$$

$$\angle AFB = \tan^{-1}(0.5714)$$

$$\angle AFB = 29.7^\circ$$

6. Hashim walks 4 km due North, then turns and walks 3 km due East. What is the bearing from the starting point to his final position?



9. What is the value of $\sin 180^\circ$?

(a) $\frac{1}{\sqrt{2}}$

(b) 0

(c) $\sqrt{\frac{3}{2}}$

(d) $\sqrt{2}$

10. $\frac{\sqrt{3}}{2}$ is the value of:

(a) $\sin 60^\circ$

(b) $\cot 60^\circ$

(c) $\cos 45^\circ$

(d) $\tan 180^\circ$

11. For which the value is undefined?

(a) $\tan 45^\circ$

(b) $\tan 90^\circ$

(c) $\tan 0^\circ$

(d) $\tan 30^\circ$

12. 0 is the value of:

(a) $\cos 45^\circ$

(b) $\sin 30^\circ$

(c) $\tan 90^\circ$

(d) $\tan 180^\circ$

13. In 2nd quadrant.

(a) $\sin \theta < 0$ and $\cot \theta > 0$

(b) $\sin \theta > 0$ and $\cos \theta < 0$

(c) $\tan \theta > 0$ and $\cot \theta < 0$

(d) $\cos \theta > 0$ and $\tan \theta < 0$.

14. In 3rd quadrant:

(a) $\cot \theta < 0$

(b) $\sec \theta > 0$

(c) $\tan \theta < 0$

(d) $\sin \theta < 0$

15. In 4th quadrant

(a) $\sin \theta > 0$

(b) $\tan \theta < 0$

(c) $\cot \theta > 0$

(d) $\sec \theta < 0$

16. In which quadrant only $\tan \theta$ and $\cot \theta$ are positive?

(a) I

(b) II

(c) III

(d) IV

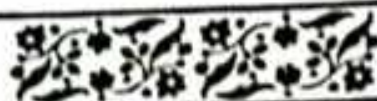
17. In which quadrants is $\cos \theta$ negative

(a) I and III

(b) II and III

(c) I and IV

(d) III and IV



7.2.3

Area of a Triangle

39. Which is correct formula for area of a triangle when two sides and their included angles are given?

- (a) $\frac{1}{2} bc \sin \alpha$ (b) $\frac{1}{2} bc \sin \beta$ (c) $\frac{1}{2} ac \sin \alpha$ (d) $\frac{1}{2} ac \sin \gamma$

40. Which of the following not correct for area of a triangle

- (a) $\frac{1}{2} bc \sin \alpha$ (b) $\frac{1}{2} ac \sin \beta$ (c) $\frac{1}{2} ac \sin \alpha$ (d) $\frac{1}{2} ab \sin \gamma$

41. If a , b and c are the three sides of a triangle, then S for area is:

- (a) $s = \frac{a+b-c}{2}$ (b) $s = \frac{a+b+c}{2}$ (c) $2s = a-b+c$ (d) $\frac{-a+b+c}{2}$

42. Area of $\triangle ABC = -$ is:

- (a) $\sqrt{s(s+a)(s+b)(s-c)}$ (b) $\sqrt{s(s+a)(s+b)(s+c)}$
(c) $\sqrt{s(s-a)(s-b)(s-c)}$ (d) $\sqrt{(a-b)(a-c)(a+b)}$

43. Which is true for area of a triangle?

- (a) $\frac{b^2 \sin \alpha \sin \beta}{2 \sin \beta}$ (b) $\frac{b^2 \sin \alpha \sin \gamma}{2 \sin \beta}$ (c) $\frac{c^2 \sin \alpha \sin \beta}{2 \sin \gamma}$ (d) $\frac{c^2 \sin \alpha \sin \beta}{2 \sin \gamma}$

44. Which is not true for area of a triangle:



3. Write all six trigonometric functions.

Sol. $\sin \theta = \frac{\text{opp.}}{\text{hyp.}}$, $\csc \theta = \frac{\text{hyp.}}{\text{opp.}}$

$\cos \theta = \frac{\text{adj.}}{\text{hyp.}}$; $\sec \theta = \frac{\text{hyp.}}{\text{adj.}}$

$\tan \theta = \frac{\text{opp.}}{\text{adj.}}$, $\cot \theta = \frac{\text{adj.}}{\text{opp.}}$

4. What are the trigonometric values of $\sin 30^\circ$ and $\cos 60^\circ$.

Ans. The value of $\sin 30^\circ$ is $\frac{1}{2}$ and

value of $\cos 60^\circ$ is also $\frac{1}{2}$.

5. Add values of $\sin 90^\circ$, $\cos 0^\circ$ and $\tan 45^\circ$.

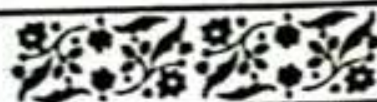
Sol. $\sin 90^\circ + \cos 0^\circ + \tan 45^\circ$
 $= 1 + 1 + 1$
 $= 3$

6. Subtract the value of $\cos 30^\circ$ from the value of $\sin 180^\circ$.

Sol. $\sin 180^\circ - \cos 30^\circ$
 $= 0 - \frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}$

7. Find the product $\sin (45^\circ) \cos (30^\circ) (\tan 45^\circ)$.

Sol. $\sin (45^\circ) \times \cos (30^\circ) \times \tan 45^\circ$
 $= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} \times 1 = \frac{\sqrt{3}}{2\sqrt{2}}$



25. In a triangle if $c = 3$ cm, $\beta = 60^\circ$ and $\gamma = 30^\circ$, then find b .

Sol. By law of sines

$$\frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

$$\frac{b}{\sin 60^\circ} = \frac{3}{\sin 30^\circ}$$

$$\frac{b}{0.886} = \frac{3}{0.5}$$

$$b = \frac{3}{0.5} \times 0.866$$

$$b = 5.20 \text{ cm}$$

7.2.2

The Law of cosines

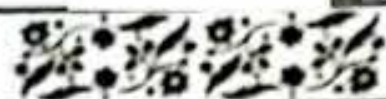
26. How do you arrange the law of cosines to solve for the cos one of angle α .

Sol. The arranged formula is:

$$\cos \alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

27. If $a = 4$, $b = 7$, $c = 9$, then find the value of $\cos \beta$.

Sol. $\cos \beta = \frac{c^2 + a^2 - b^2}{2ca}$



7.5

Three figure Bearing and its Application

34. What are the two primary rules for measuring a bearing?

Ans. The two rules are following:

- i. It must be measured in degrees from the North direction.
- ii. It must be measured in a clockwise manner.

35. How should a bearing always be written numerically?

Ans. It is always written as a three-digit angle between 000° to 360° .

36. What are cardinal directions?

Ans. Cardinal directions are four main joints on a compass North, South, East and West.

37. What are ordinal directions?

Ans. Ordinal directions are the four inter-cardinal points of compass. NE, SE, SW, NW.

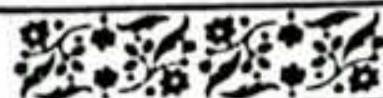
38. In a diagram, the bearing of point y from point x is 045° . Find the bearing of x from y.

Ans. Since the bearing is less than 180° , we add 180° to find the return direction.

$$\text{Bearing} = 045^\circ + 180^\circ = 225$$

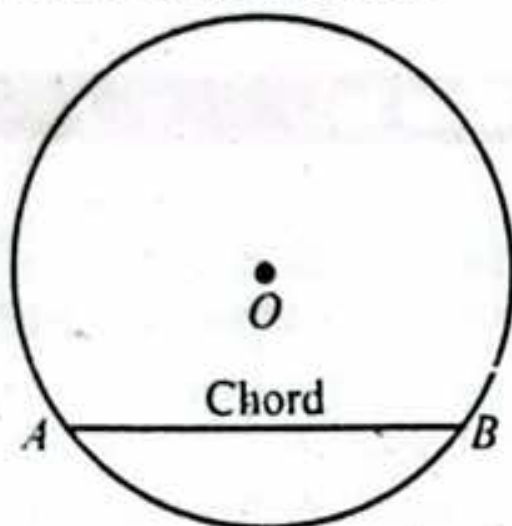
39. A hiker walks from point C to point D on a bearing of 290° . What bearing should the hiker follow to return from D to C.

Sol. Because the bearing is greater than 180° , we subtract 180° to find the reverse bearing.



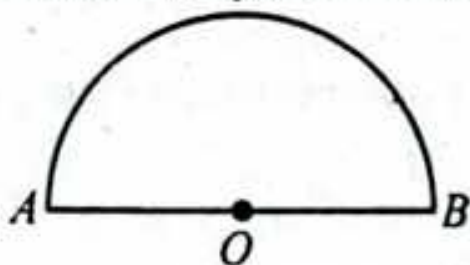
Chord

A line segment joining any two distinct points of a circle. In the below figure, $\overline{AB}$ is a chord of the circle.



Semi-Circle

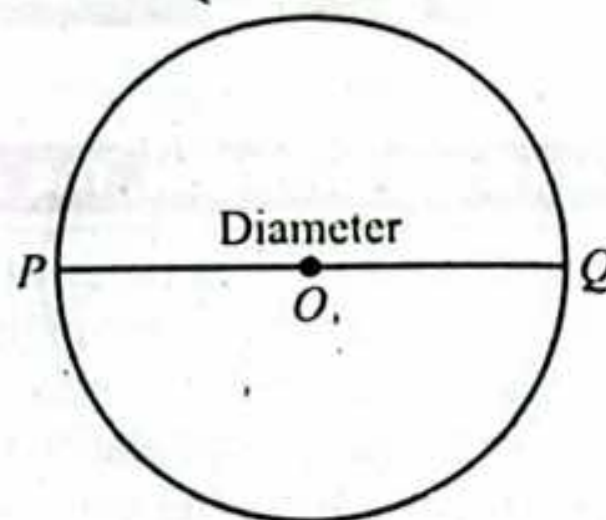
A semi-circle is half of the circle, that is a figure bounded by a diameter and circumference cut by the diameter.



Minor Arc

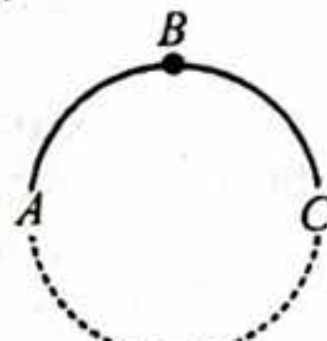
Diameter

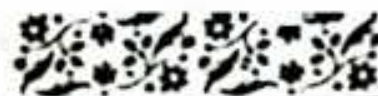
A chord passing through the centre of circle is called diameter. $\overline{PQ}$ is a diameter.



Arc

Any part of the circumference of a circle is called an arc. ABC is an arc of the circle. Similarly, AB and BC are also arcs.





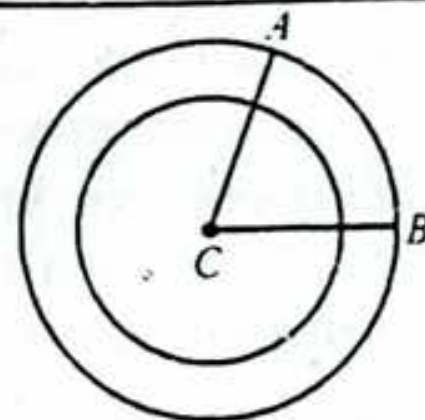
Challenge!

What is the difference between chord and secant?

Ans. Chord is a line segment joining any two distinct points of a line circle. While secant is a straight line cutting the circumference of a circle at two distinct points.

Do you know?

- (i) Circles with same centre are called concentric circles.
- (ii) Any two circles are similar.
- (iii) A circle whose radius is zero is called point circle.



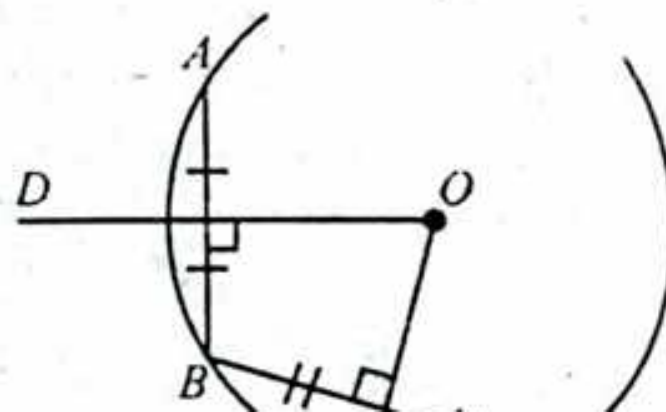
8.1 Theorems (Properties) of a Circle

1. One and Only One Circle Can Pass Through Any Three Non-Collinear Points.

If A, B and C are three points not in the same straight line, then one and only one circle can pass through A, B and C.

Note

- A circle cannot be drawn through more than two points in a straight line.





Now, $(m\overline{OM})^2 = (m\overline{OB})^2 - (m\overline{MB})^2$ [By Pythagoras theorem]

$$(m\overline{OM})^2 = (13)^2 - (12)^2$$

$$m\overline{OM} = \sqrt{(13)^2 - (12)^2}$$

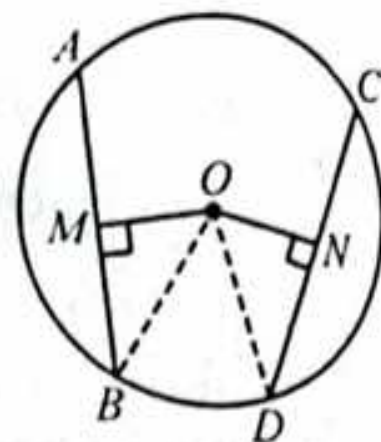
$$= \sqrt{169 - 144}$$

$$= \sqrt{25} = 5\text{ m}$$

4. **If Two Chords of a Circle Are Congruent, then they will be Equidistant from the Centre.**

If $\overline{AB}$ and $\overline{CD}$ are congruent (equal in measures) chords of a circle with centre O . i.e., $m\overline{AB} = m\overline{CD}$, $\overline{OM} \perp \overline{AB}$ and $\overline{ON} \perp \overline{CD}$, then $m\overline{OM} = m\overline{ON}$

Example 5: Two equal-length ropes (chords) are tied in a circular playground. One is 8 m from the centre. Where should the second be placed?



Sol. Since chords are congruent, they must be equidistant from the centre. So, the second chord must be 8 m from the centre.

5. **Two Chords of a Circle which are Equidistant from the Centre of a Circle, are Congruent**

$\overline{AM}$ is the half of $\overline{AB}$ as $\overline{OM} \perp \overline{AB}$
 $m(\overline{AM}) = 7 \text{ m}$

OMA is a right angled triangle

So, by Pythagoras theorem

$$(m\overline{OA})^2 = (m\overline{AM})^2 + m(\overline{OM})^2$$

$$(10)^2 = (7)^2 + (m\overline{OM})^2$$

$$100 = 49 + (m\overline{OM})^2$$

$$100 - 49 = (m\overline{OM})^2$$

$$51 = (m\overline{OM})^2$$

$$\sqrt{(m\overline{OM})^2} = \sqrt{51}$$

$$m\overline{OM} = 7.14 \text{ m}$$

So, the distance of the chord from the centre of the park = 7.14 m

4. In a circle, chords $\overline{AB}$ and $\overline{CD}$ both have length 10 cm. If the distance from the centre to $\overline{AB}$ is 6 cm, what is the distance from centre to $\overline{CD}$?

Sol. In a circle $\overline{AB}$, and $\overline{CD}$ are two chords O is the centre of the circle.

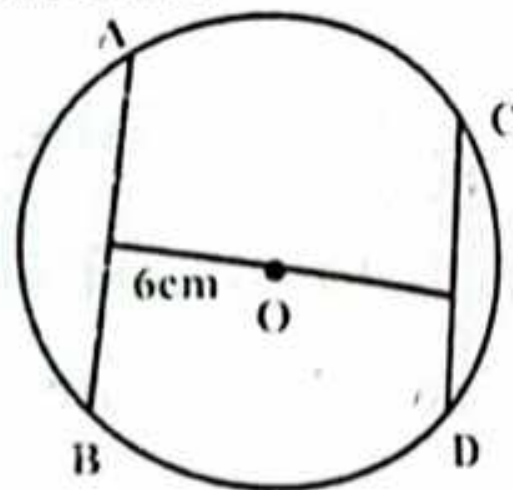
$$m(\overline{AB}) = 10 \text{ cm}$$

$$m(\overline{CD}) = 10 \text{ cm}$$

$$m(\overline{AB}) = m(\overline{CD})$$

If two chords of a circle are congruent, then they will be equidistant from the centre.

Distance of $\overline{AB}$ from the centre = 6 cm



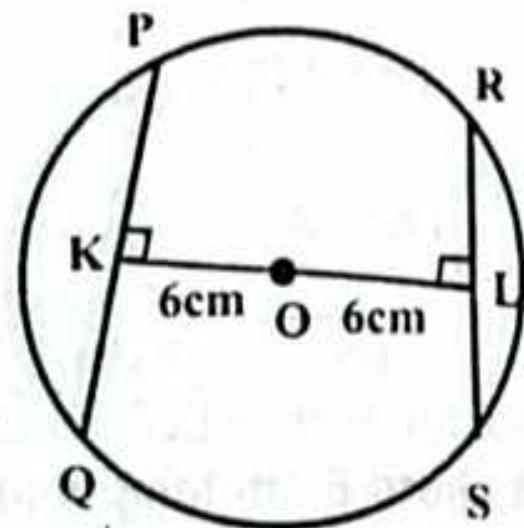
8. In a circle with centre at O, the perpendicular distances of the each chords PQ and RS from the centre are 6 cm. If the length of chord PQ is 18 cm, find the length of the other chord.

Sol. Let O is the centre of the circle $\overline{OK}$ and $\overline{OL}$ are perpendicular distances of the chords PQ and RS from centre respectively

$$\begin{aligned} m\overline{OK} &= m\overline{OL} = 6 \text{ cm} \\ m\overline{PQ} \text{ chord} &= 18 \text{ cm} \end{aligned}$$

Two chords of a circle which are equidistant from the centre of a circle, are congruent

$$\begin{aligned} \text{So } m\overline{PQ} &= m\overline{RS} \\ \text{If } m\overline{PQ} &= 18 \text{ cm} \\ \text{then } m\overline{RS} &= 18 \text{ cm} \end{aligned}$$



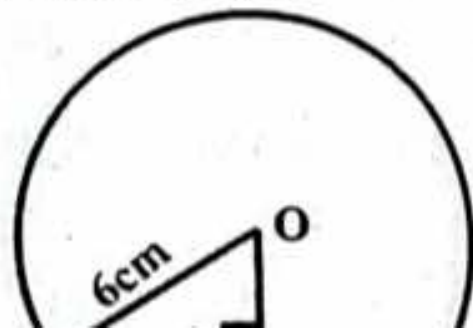
9. A line from the centre of a circle cuts a 10 cm chord at right angle where radius of the circle is 6cm. What is the length from the centre to the chord?

Sol.

$\overline{AB}$ as the chord of a circle
with O as centre

$$m(\overline{AB}) \text{ chord} = 10 \text{ cm}$$

$$m(\overline{OA}) \text{ radius} = 6 \text{ cm}$$



As, we know that equal chords have equal arcs.

$$\therefore \text{Arc } PQ \cong \text{Arc } QR \cong \text{Arc } RS$$

But $\text{Arc } PQ = 3 \text{ cm}$

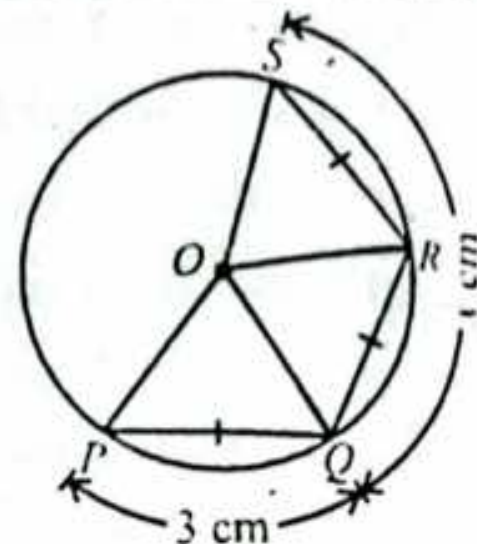
$$\therefore \text{Arc } QR = 3 \text{ cm and Arc } RS = 3 \text{ cm}$$

From figure, we have

$$\text{Arc } QR + \text{Arc } RS = x$$

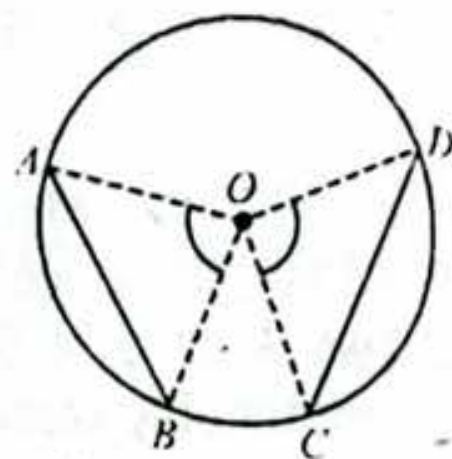
$$3 \text{ cm} + 3 \text{ cm} = x$$

$$x = 6 \text{ cm}$$



13. Equal Chords of a Circle (or of Congruent Circles) Subtend Equal Angles at the Centre (at the corresponding Centres)

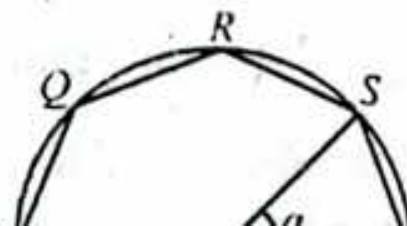
In the circle with centre O , $m\overline{AB} = m\overline{CD}$, chord AB and chord CD subtended $\angle AOB$ and respectively at the centre O , then $m\angle AOB = m\angle COD$



Example 10: In the given figure, $\overline{PT}$ is a diameter of circle.

If $m\overline{PQ} = m\overline{QR} = m\overline{RS} = m\overline{ST}$, then find a .

Solution: $m\overline{PQ} = m\overline{QR} = m\overline{RS} = m\overline{ST}$ (given)



$$\text{put } \theta = x \text{ (Given)}$$

$$x = 77^\circ$$

Now

$\widehat{AB} = \widehat{BC} = \widehat{CD}$ (If chords of circle are congruent then corresponding arc are congruent)

Let the length of each arc = x cm

Add these arcs

$$\widehat{AB} + \widehat{BC} + \widehat{CD} = \widehat{AD}$$

$$x + x + x = 3x$$

$$x + 2x = 3x$$

$$2x = y$$

$$x + y = 3x$$

$$y = 3x - x$$

$$y = 2x$$

$$\text{put } x = 3\text{cm}$$

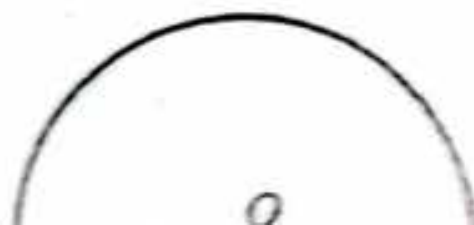
$$y = 2(3\text{cm})$$

$$y = 6\text{cm}$$

So,

$$x = 77^\circ, y = 6\text{cm}$$

3. Find the values of x and y in the given figure, such that $m\widehat{AB} = m\widehat{BD}$.





Solved Review Exercise 8

1. Four possible answers are given for the following questions. Choose the correct answer.

(i) Distance of a point of the circumference to the center of the circle is called:

- (a) radius (b) arc (c) chord (d) tangent

(ii) Radii of same circles are:

- (a) all unequal (b) all equal
(c) half of any chord (d) double of the diameter

(iii) The boundary of the circle is called:

- (a) chord (b) segment (c) circumference (d) diameter

(iv) Any part of a circumference is called:

- (a) chord (b) diameter (c) radius (d) arc

(v) A chord passing through the centre of a circle is called:

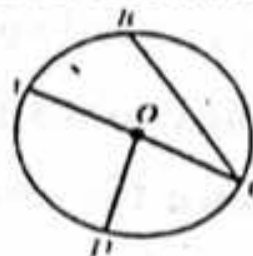
- (a) radius (b) diameter (c) secant (d) circumference

(vi) In the given figure, what is AB?

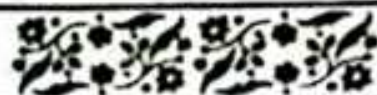
- (a) diameter (b) tangent
(c) chord (d) arc

(vii) In the given figure, major arc is:

- (a) $m\widehat{AB}$ (b) $m\widehat{BC}$ (c) $m\widehat{BDC}$ (d) $m\widehat{AD}$



(viii) circle(s) can pass through the three non-collinear points?



4. In a circular park, two benches are placed so that the chords formed by their positions are both 10 m long. What can you say about the angles subtended at the centre by each bench?

Ans. Since both chords (benches) are 10 m long, they are congruent. Therefore, the angles subtended at the centre by each bench are equal.

(Property: in circles, equal chords subtended equal angles at the centre)

5. Jannat is designing a triangular garden with corners at point A(2, 1), B(4, 4) and C(1, 5). Can she install a circular fountain that touches all three corners?

Sol: Points of corners garden = A(2, 1), B(4, 4), C(1, 5) we will check the points are non-collinear by using slope formula

$$\text{Slope of } \overline{AB} = \frac{4-1}{4-2} = \frac{3}{2}$$

$$\text{Slope of } \overline{BC} = \frac{5-4}{1-4} = \frac{1}{-3} = -\frac{1}{3}$$

$$\text{Slope } \overline{AB} \neq \text{Slope } \overline{BC}$$

Hence, the given point are non-collinear. Therefore, she can install a circular fountain that touches all three corners.

6. Two chords, PQ and RS, each measure 18cm in length. If the distance of PQ from the centre of circle is 7 cm. Find the distance of RS from the centre.



Objective Type Questions



Additional MCQ's

Multiple Choice Questions prepared in the light of new Examination Techniques
(Knowledge, Understanding, Application, Analytical & Conceptual)

Circle and its Components

○ Encircle the correct option.

1. Which term describes the set of all points in a plane that are at the same distance from a fixed point.

- (a) diameter (b) chord (c) circle (d) radial segment

2. Which is the mathematical formula for circumference (c) of a circle?

- (a) $c = \pi r^2$ (b) $2\pi r$ (c) $c = \frac{2}{\pi}$ (d) πr^2

3.  In the figure $\overline{AB}$ is called:

- (a) chord (b) diameter (c) centre (d) radius

4. What is called a figure bounded by a diameter and circumference cut by the diameter?

**ANSWERS:**

1. circle 2. 2π 3. chord 4. semi-circle 5. semi-circle 6. major arc
 7. $\angle ACB$ 8. semi-circle 9. major arc 10. major arc 11. 150° 12. two distinct points
 13. centre 14. concentric circles 15. point circle 16. they coincide 17. two 18. $\frac{y_2 - y_1}{x_2 - x_1}$
 19. -2 20. equal in length 21. 22. 3 cm 23. 5 cm 24. congruent
 25. equal 26. 60° 27. 10 cm

Additional Short Question

Short Answer Questions prepared in the light of new Examination Techniques
 (Knowledge, Understanding, Application, Analytical & Conceptual)

Circle and its components

○ Give short answers.

1. What is circle?

Ans. A circle is the set of all points that are at the same distance from a fixed point.

2. Define the circumference of a circle.

semi-circle.

9. Define a major arc.

Ans. A major arc is an arc greater than the semi-circle.

10. Define a segment of a circle.

Ans. The area (region) enclosed between a chord and the arc which it cuts off is called a segment of the circle.

**UNIT**
09**Tangent and Angles of a Circle****Students' Learning Outcomes****After completing this unit, the students will be able to:**

- ▶ Solve problems by using the tangent and angle properties of a circle:
 - o If a line is drawn perpendicular to a radial segment of a circle at its outer end point, it is tangent to the circle at that point.
 - o The tangent to a circle and the radial segment joining the point of contact and the centre are perpendicular to each other.
 - o The two tangents drawn to a circle from a point outside it, are equal in length.
 - o If two circles touch externally or internally, then the distance between their centres is respectively equal to the sum or difference of their radii.
 - o The measure of a central angle of a minor arc of a circle is double that of the angle subtended by the corresponding major arc.
 - o Any two angles in the same segment of a circle are equal.
 - o The angle in a semi-circle is a right angle, in a segment greater than a semi-circle is less than a right angle, in a segment less than a semi-circle is greater than a right angle.
 - o The opposite angles of any quadrilateral inscribed in a circle are supplementary.
- ▶ Find the arc length and area of sector of a circle using angle in both degrees and

$$(\overline{OP})^2 = (\overline{OA})^2 + (\overline{AP})^2$$

$$12^2 = 5^2 + (\overline{AP})^2$$

$$144 = 25 + (\overline{AP})^2$$

$$(\overline{AP})^2 = 144 - 25 = 119$$

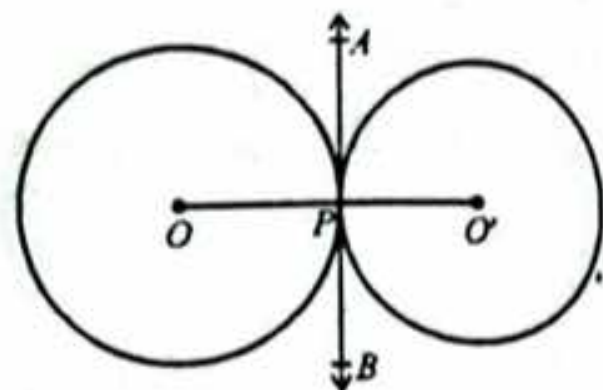
$$\overline{AP} = 10.91 \text{ m}$$

So, the length of each cable is 10.91 m

(a) If Two Circles Touch Externally, then the Distance Between Their Centres is Equal to the Sum of their Radii.

If two circles with centres O and O' touch each other at point P , externally. $\overline{PO}$ and $\overline{PO'}$ are their radii respectively, then

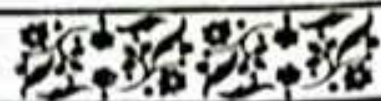
$$\overline{OO'} = \overline{OP} + \overline{O'P}$$



Example 3: The radii of the two circles are 4 cm and 6cm. What is the distance between their centres if they touch externally?

Solution: Since the circles touch externally,

So, the distance between their centres = sum of their radii



perpendicular)

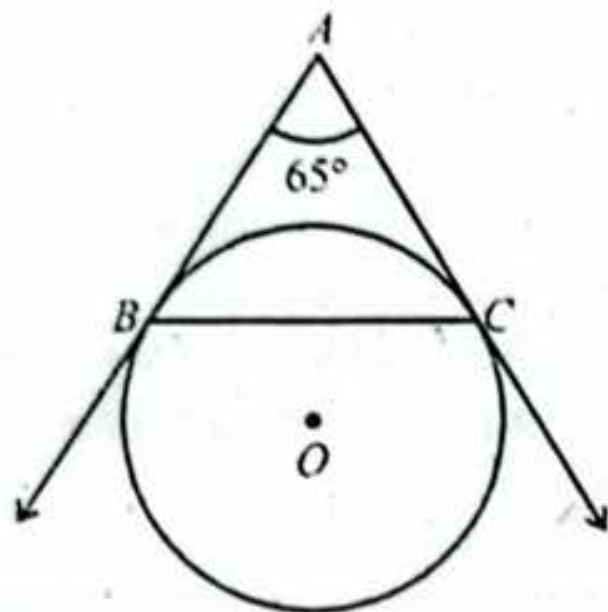
Thus,

$x + 38^\circ + 90^\circ = 180^\circ$ (sum of the three angles of triangle)

$$x + 128^\circ = 180^\circ$$

$$x = 180^\circ - 128^\circ = 52^\circ$$

2. Find the angle ABC .



Sol. Tangents from a point (A) are equal
(Theorem)

$$m\overline{AB} = m\overline{AC}$$

So

$\triangle ABC$ is an isosceles triangle

In $\triangle ABC$, the sum of the angles is 180°

Since $m\overline{AB} = m\overline{AC}$

then

$$m\angle ABC = m\angle ACB$$

Let $m\angle ABC = m\angle ACB = x$

We know

$x + x + 65^\circ = 180^\circ$ (Sum of the angles of a triangle)

$$2x + 65^\circ = 180^\circ$$

$$2x = 180^\circ - 65^\circ$$

$$2x = 115^\circ$$

$$x = \frac{115}{2}$$

$$x = 57.5^\circ$$

So, $m\angle ABC = 57.5^\circ$



Sol. In the circle with centre O, $\widehat{ZX}$ is the minor arc.

It subtends an angle ZO at the centre. and angle ZYX is subtended by the major arc
As by Theorem, the measure of a central angle of a minor arc of circle is double that of the angle subtended by the corresponding major arc.

$$m\angle ZOX = 2m\angle XYZ$$

$$x = 2(85^\circ)$$

$$x = 170^\circ$$

6. In a historical monument, a circular fountain with a radius of 3 m is built. A flagpole is erected 7 m away from the centre of the fountain. Two ropes from the pole are tied to the edge of the fountain, just touching it. Find the length of each rope.

Sol. Radius of circular fountain (r) = 3m

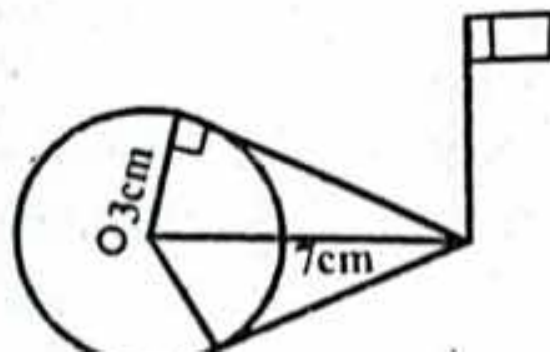
Distance of the pole from centre of fountain (d) = 7m

length of the rope (l) = ?

When a line from an external point touches a circle's edge (tangent) it forms a right angle with the radius at that point

We can use the pythagoras the orem

$$d^2 = l^2 + r^2$$

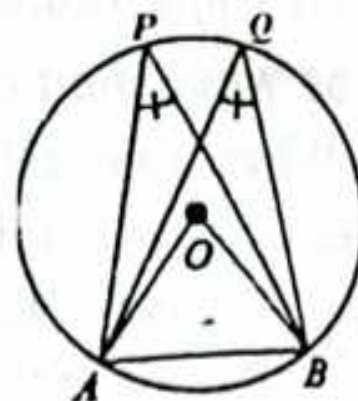




As two tangents to a circle from a point outside are equal So, the length of two wires is equal. So, the length of each wire = 24m

Any Two Angles in the Same Segment of a Circle are Equal.

If $\angle APB$ and $\angle AQB$ be angles in the same segment $APQB$, of a circle whose centre is O , then $m\angle APB = m\angle AQB$



Example 6 In a large circular stained-glass window of a library, two decorative beams are drawn from the same chord AB to two points P and Q on the arc opposite to $\widehat{AB}$ such that $\angle APB$ and $\angle AQB$ are formed. If $\angle APB$ is measured to be 40° , what is $m\angle AQB$?

Sol. According to the article 9.1.5, angles in the same segment are equal.

Since $m\angle APB = 40^\circ$

Therefore, $m\angle AQB = 40^\circ$

Skilled practice!

A projector casts an image over an arc on a circular wall. Two observers on the arc segment report one observing a 50° angle. What is the other angle in the same segment?

Sol. Observer 1 reports an angle of 50°

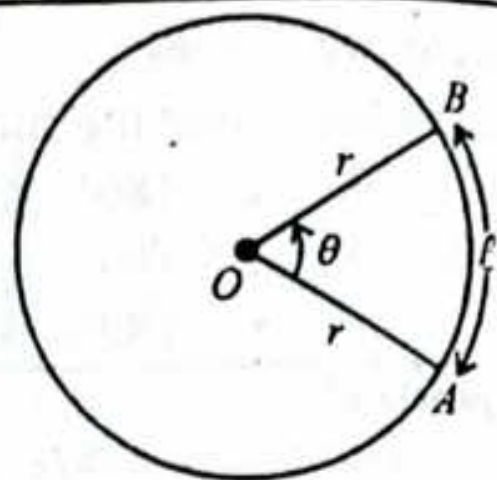
Since observer 2 is in the same

or $\ell : 2\pi r = \theta : 2\pi$

or $\frac{\ell}{2\pi r} = \frac{\theta}{2\pi}$

$$\ell = \frac{\theta}{2\pi} \times 2\pi r$$

$\ell = r\theta$, where θ is in radian.



Area of a Sector

A sector of a circle is a region bounded by two radii and corresponding arc.

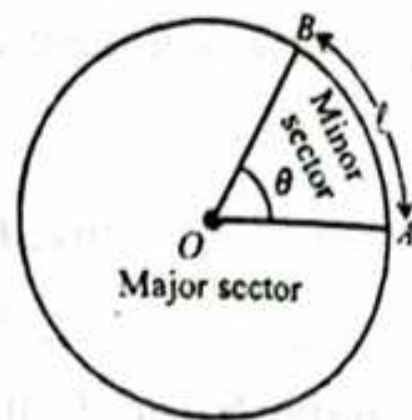
A slice of pizza and region between the hands of a watch are examples of a sector of circle.

A sector divides the circle into two regions namely major sector and minor sector.

The area of a sector of a circular region of radius r is $\frac{1}{2} r^2 \theta$

where θ is the central angle and it is measured in radians.

Proof: As shown in adjoining figure,



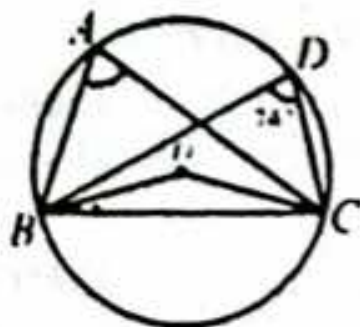
$$\frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\text{Central angle of sector}}{2\pi}$$

Note

$$A = \frac{1}{2} r^2 \theta$$



2. In the adjoining figure $m\angle BDC = 74^\circ$, find $m\angle BAC$, $m\angle BOC$ and $m\angle OBC$.



Sol. Given $m\angle BDC = 74^\circ$

$$m\angle BAC = m\angle BDC$$

$$m\angle BAC = 74^\circ \quad \text{--- (i)}$$

$$m\angle BOC = 2(m\angle BDC)$$

$$m\angle BOC = 2(74^\circ)$$

$$m\angle BOC = 148^\circ \quad \text{--- (ii)}$$

$$m\angle AOB = m\angle BOC$$

$$m\angle AOB = m\angle BOC$$

$$m\angle OBC = m\angle OCB$$

$$m\angle OBC + m\angle OCB + m\angle BOC = 180^\circ$$

$$m\angle OBC + m\angle OBC + m\angle BOC = 180^\circ$$

$$2(m\angle OBC) + 148^\circ = 180^\circ$$

$$2(m\angle OBC) = 180^\circ - 148^\circ$$

$$2(m\angle OBC) = 32^\circ$$

[any two angles in the same segment of a circle are equal]

[The angle subtended by an arc at the centre is double the angle subtended at the circumference]

(Radius of the same circle)

(Base angles of isosceles triangles)

(Sum of the angles of a triangle)

(as $m\angle OBC = m\angle OCB$)



$$m\angle BCA = 180^\circ - 90^\circ - 70^\circ = 20^\circ$$

$$\triangle ACD \cong \triangle ABC (\text{R.H.S})$$

$$m\angle ACD = m\angle BCA$$

$$x = 20^\circ$$

(corresponding angle of congruent triangles)

- 5** In a circular arch over a monument entrance, two spotlight fixtures are placed such that they each shine from two different points on the arch to the same chord PQ on the base of the arch. If the angle formed at one light is 55° , what is the angle at the second light on the same side of chord PQ ?

Sol. Angles subtended by the same chord at the circumference in the same segment are equal.

$$\text{Angle formed by one light} = 55^\circ$$

$$\text{Angle formed by other light} = 55^\circ \quad (\text{Property stated above})$$

- 6.** A circular garden has a walking path forming a quadrilateral inscribed in it. If one angle is 87° , what is opposite angle?

Sol. Measure of one angle in a inscribed quadrilatera = 87°

$$\text{Sum of the opposite angles of inscribed quadrilateral} = 180^\circ$$

(opposite angles are supplementary in a cyclid quadrilateral)

$$\text{Sum of the opposite angle} = 180^\circ - 87 = 93^\circ$$

- 7.** A ferris wheel has a radius of 12m. If a passenger travels a distance of 18 m along the circumference of the ferris wheel, what is the angle (in radians) swept by the passenger's position from the starting point?

Sol.

$$\text{Radius of the ferris wheel } (r) = 12 \text{ m}$$

$$\text{Distance traveled by a passenger } (\ell) = 18 \text{ m}$$

$$\text{Central angle of each slice } (\theta) = \frac{2\pi}{8}$$

$$= \frac{\pi}{4} \text{ rad}$$

Area of one slice (sector) (A) = ?

$$A = \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} (14)^2 \left(\frac{\pi}{4} \right)$$

$$= \frac{1}{2} (196) \left(\frac{\pi}{4} \right)$$

$$= 76.97 \text{ cm}^2$$

12. The perimeter and area of a sector are 14 cm and 10 cm^2 respectively. Find the radius of the circle and the central angle of the sector.

Divide by 2

$$r^2 - 7r + 10 = 0$$

$$r^2 - 5r - 2r + 10 = 0$$

$$r(r - 5) - 2(r - 5) = 0$$

$$(r - 5)(r - 2) = 0$$

$$r - 5 = 0$$

$$r = 5$$

$$\theta = \frac{l}{r}$$

$$\theta = \frac{14 - 2r}{r}$$

$$\theta = \frac{14 - 2(5)}{5}$$

$$\theta = \frac{14 - 10}{5}$$

$$\theta = \frac{4}{5}$$

$$\theta = 0.8 \text{ rad}$$



Because of that 90° angle, ΔOPQ is a right angled triangle.

By pythagorean theorem

$$(m\overline{OQ})^2 = (m\overline{PO})^2 + (m\overline{OP})^2$$

$$(25 \text{ cm})^2 = (24 \text{ cm})^2 + (m\overline{OP})^2$$

$$625 \text{ cm}^2 = 576 + (m\overline{OP})^2$$

$$(m\overline{OP})^2 = 625 - 576$$

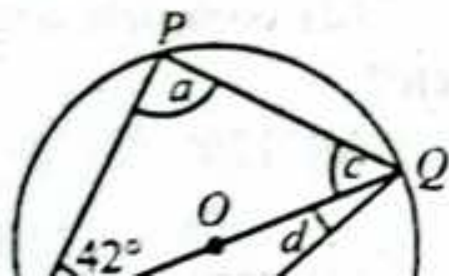
$$(m\overline{OP})^2 = 49$$

$$\sqrt{(m\overline{OP})^2} = \sqrt{49}$$

$$(m\overline{OP}) = 7$$

This the radius of circle is 7 cm

3. Find the angles a , b , c and d in the given figure, if O is the centre of the circle.



$$72^\circ + 90^\circ + m\angle d = 180^\circ$$

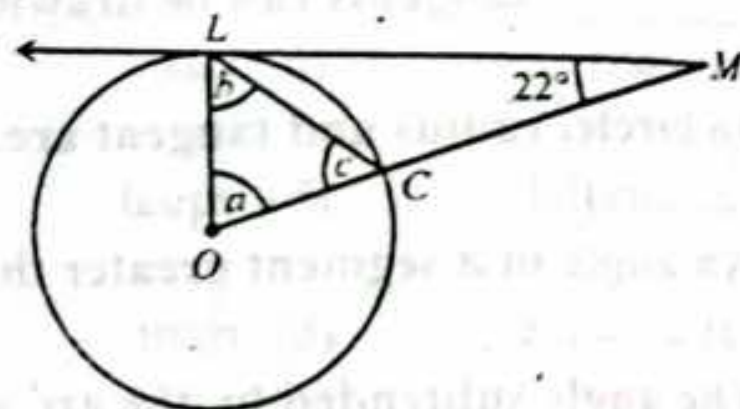
$$162^\circ + m\angle d = 180^\circ$$

$$m\angle d = 180^\circ - 162^\circ$$

$$m\angle d = 18^\circ$$

So, $m\angle a = 90^\circ$, $m\angle b = 90^\circ - m\angle c = 48^\circ$,
 $m\angle d = 18^\circ$

4. Find the angles a , b and c , if $\overline{ML}$ is a tangent line and O is the centre of the circle.



Sol: In ΔOLM

$$m\angle M = 22^\circ \text{ (Given)}$$

$$m\angle OLM = 90^\circ \text{ (radius } \overline{OL} \perp \text{ tangent } \overline{ML})$$

$$m\angle a + m\angle M + m\angle OLM = 180^\circ \text{ [sum of}$$



$$\begin{aligned} &= \frac{1}{2} \times 15 \times 10 \\ &= 75 \text{ cm}^2 \end{aligned}$$

10. A sector cut from a circle of radius 5cm has a perimeter of 16cm. Find area of this sector.

Sol: Radius of the circle (r) = 5 cm

Perimeter of the sector (p) = 16 cm

$$P = r + r\ell + \ell$$

$$16 = 5 + 5 + \ell$$

$$16 = 10 + \ell$$

$$16 - 10 = \ell$$

$$6 = \ell$$

$$\ell = 6 \text{ cm}$$

$$\begin{aligned} \text{Area of the sector} &= \frac{1}{2} r \ell \\ &= \frac{1}{2} \times 5 \times 6 \\ &= 15 \text{ cm}^2 \end{aligned}$$

Objective Type Questions



Additional MCQ's

Multiple Choice Questions prepared in the light of new Examination Techniques
(Knowledge, Understanding, Application, Analytical & Conceptual)

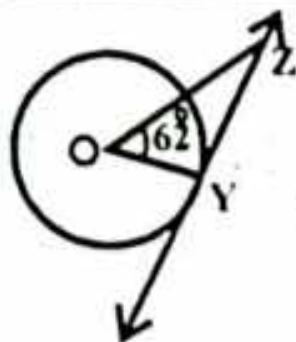
9.1

Tangent to a Circle

- Encircle the correct option.



18. In the figure



$m\angle OZY$ is:

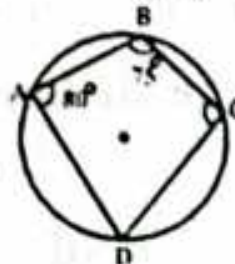
(a) 22°

(b) 27°

(c) 28°

(d) 26°

19. In the figure



then $m\angle BCD$ is:

(a) 100°

(b) 180°

(c) 175°

(d) 360°

9.2

Area of Sector and Arc Length of a Circle.

20. Find the arc length if radius is 10cm and central angle is 2 radians.

(a) 5cm

(b) 10cm

(c) 20cm

(d) 20π cm

21. What is the radius of a circle when an arc of length 6π is subtended by a central angle of π radians.

(a) 3

(b) 6

(c) 12

(d) 6π

22. Find the central angle if $\ell = 15\text{m}$ and $r = 5\text{m}$.

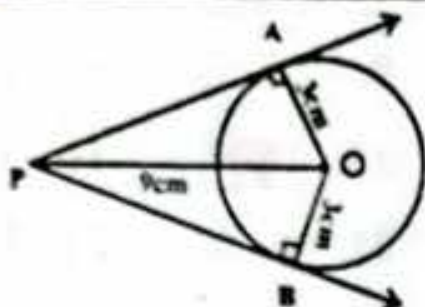
(a) 3rad

(b) 0.33rad

(c) 3π rad

(d) 30 rad

23. If arc length is 10π and the $\theta = 120^\circ$ what is radius?



4. In figure

Find the length of $\overline{PA}$.

Sol. ΔAOP is a right angled triangle because

$\overline{OA} \perp$ tangent $\overline{PA}$ thus $\angle OAP = 90^\circ$

By Pythagoras theorem

$$(m \overline{PO})^2 = (m \overline{PA})^2 + (m \overline{OA})^2$$

$$(9\text{cm})^2 = (m \overline{PA})^2 + (3\text{ cm})^2$$

$$81\text{cm}^2 = (m \overline{PA})^2 + 9\text{cm}^2$$

$$81\text{cm}^2 - 9\text{cm}^2 = (m \overline{PA})^2$$

$$72\text{cm}^2 = (m \overline{PA})^2$$

$$\sqrt{(m \overline{PA})^2} = \sqrt{72\text{cm}^2}$$

$$m \overline{PA} = 8.5\text{cm}$$

5. The radii of two circles are 7cm, 9cm. What is the distance between their centres if they touch externally.

Sol. Since the circles touch externally

So, the distance between their centres = sum of their radii



13. What is the measure of the angle in a segment greater than a semi-circle.

Ans. The measure in a segment greater than a semi-circle is less than a right angle ($<90^\circ$).



14. In figure Why is the angle x equal to 90° .

Sol. Because PQ is a diameter, making x an angle in a semi-circle. The angle in a semi-circle is a right angle of 90° .

9.2

Area of sector and Arc Length of a Circle

15. A circle has a radius of 10cm and a central angle of 60° . Find the arc length.

Sol.

$$r = 10\text{cm}$$

$$\theta = 60^\circ$$

$$\ell =$$

$$\theta = \frac{60}{180} \times \frac{\pi}{3} = \frac{\pi}{3}$$

we know

$$\ell = \theta r$$

$$\ell = \frac{\pi}{3} \times 10$$

$$\ell = 10.47\text{ cm}$$

16. Calculate the area of sector with a radius of 8cm and $\theta = 90^\circ$

Sol.

$$r = 8\text{ cm}$$

$$\theta = 90^\circ$$

**UNIT
10****Practical Geometry of Circles****Student's Learning Outcomes****After completing this unit, the students will be able to:**

- ▶ Locate the centre of a given circle.
- ▶ Draw a circle passing through three given non-collinear points.
- ▶ Complete the circle:
 - by finding the centre,
 - without finding the centre,when a part of its circumference is given.
- ▶ Draw a tangent to a given arc, without using the centre, through a given point P when P is
 - the middle point of the arc.
 - at the end of the arc.
 - outside the arc.
- ▶ Draw a tangent to a given circle from a point P when P lies
 - on the circumference.

Solved Exercise 10.1

1. Construct a circle with the help of given radius and locate its centre by construction:

(i) $r = 1.5$ cm (ii) $r = 1.7$ cm (iii) $r = 2$ cm

(i) $r = 1.5$ cm

Sol. Given : A circle of the radius of 1.5 cm.

Required: To locate the centre of the circle

Steps of construction:

1. Take any two chords $\overline{AB}$ and $\overline{CD}$ of the circle.
2. Draw $\overline{PQ}$ right bisector of $\overline{AB}$.
3. Draw $\overline{RS}$ right bisector of $\overline{CD}$.
4. $\overline{PQ}$ and $\overline{RS}$ intersect each other at point O .
5. Point O is the centre of the circle.

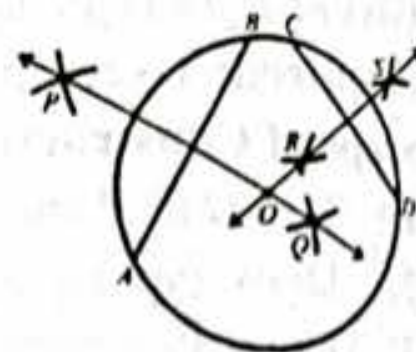
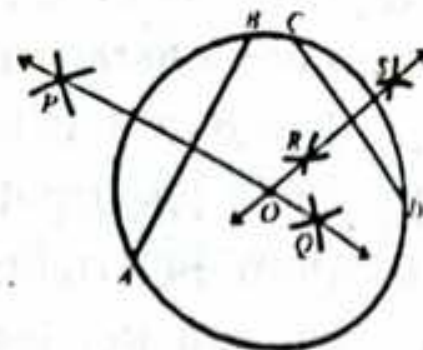
(ii) $r = 1.7$ cm

Sol. Given: A circle of the radius of 1.7 cm.

Required: To locate the centre of the circle.

Steps of Construction:

1. Take any two chords $\overline{AB}$ and $\overline{CD}$ of the circle.
2. Draw $\overline{PQ}$ right bisector of $\overline{AB}$.



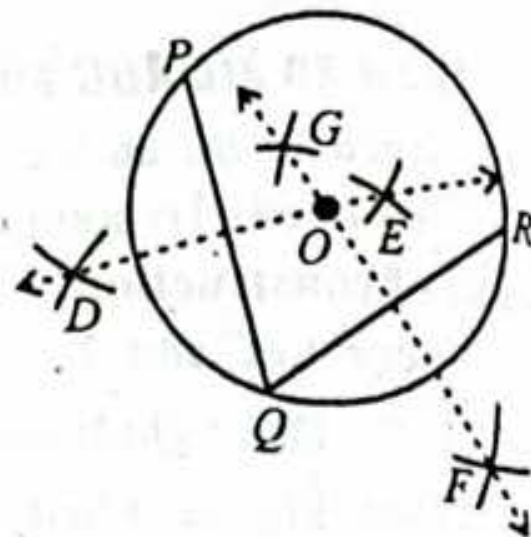
6. A part of the Ferris wheel rim is visible as an arc. Using any three points on the arc, construct the circle by finding its centre.

Sol. Given: Three points P , Q and R on arc

Required: Construct the circle by finding the centre.

Steps of constructions:

1. Join Q to P and R .
2. Draw $\overline{DE}$ right bisector of $\overline{PQ}$.
3. Draw $\overline{FG}$ right bisector of $\overline{QR}$.
4. $\overline{DE}$ and $\overline{FG}$ intersect each other at point O . Now, O is the centre of the required circle.
5. Take O as centre and draw a circle of radii $m\overline{OP} = m\overline{OQ} = m\overline{OR}$, which is the required circle.

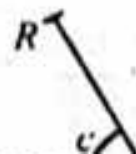


Thus, this circle passes through points P , Q and R on the given part of the circumference.

7. $\overline{ABC}$ an arc of a fountain, complete a circle without finding its centre.

Sol. Given: $\overline{ABC}$ is an arc.

Required: Complete a circle of which $\overline{ABC}$ is





Draw a Tangent to given Circle from a Point P when P Lies on the Circumference.

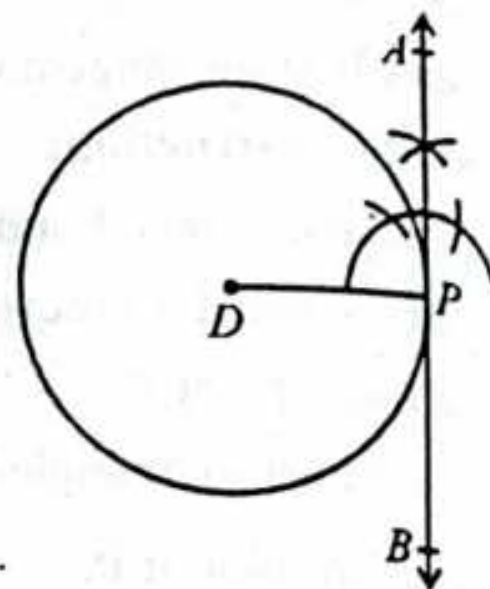
Given: A circle with centre D and a point P on its circumference.

Required: Draw a tangent to the circle at point P .

Steps of Construction:

1. Join centre D with the point P . Now, $\overline{DP}$ is the radial segment.
2. Draw a perpendicular $\overline{AB}$ at point P to the radial segment $\overline{DP}$.

So, $\overline{APB}$ is the required tangent.



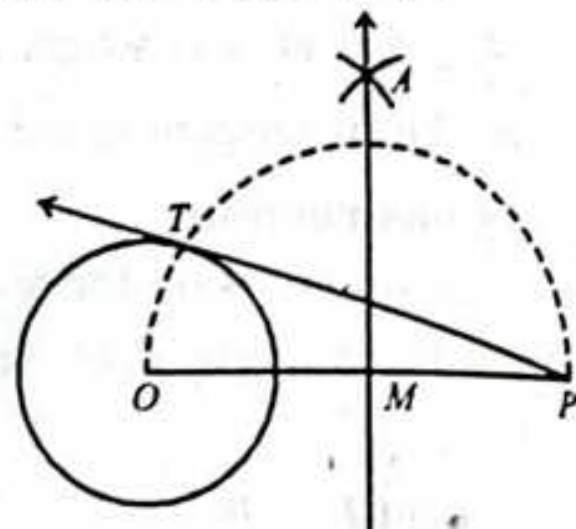
Draw a tangent to the circle from a point P when P lies outside the circle

Given: A circle with centre O and a point P that lies outside the circle..

Required: To draw a tangent to the circle from point P .

Steps of Construction:

1. Join centre O to point P .



Steps of Construction:

1. Join A to P , $\overline{AP}$ cuts the arc at Q .
2. Find the midpoint M of $\overline{AP}$.
3. Take $\overline{AMP}$ as diameter and draw a semi-circle from M .
4. Draw perpendicular $\overline{QS}$ on $\overline{AP}$ that cuts the semi-circle at R .
5. Take $m\overline{PR}$ as radius and draw an arc that cuts $\widehat{ABC}$ at D .
6. Join P to D and produce towards D .

So, $\overline{PD}$ is the required tangent at D .

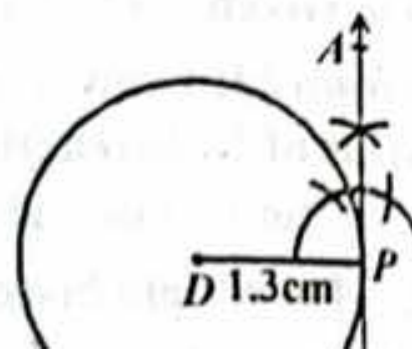
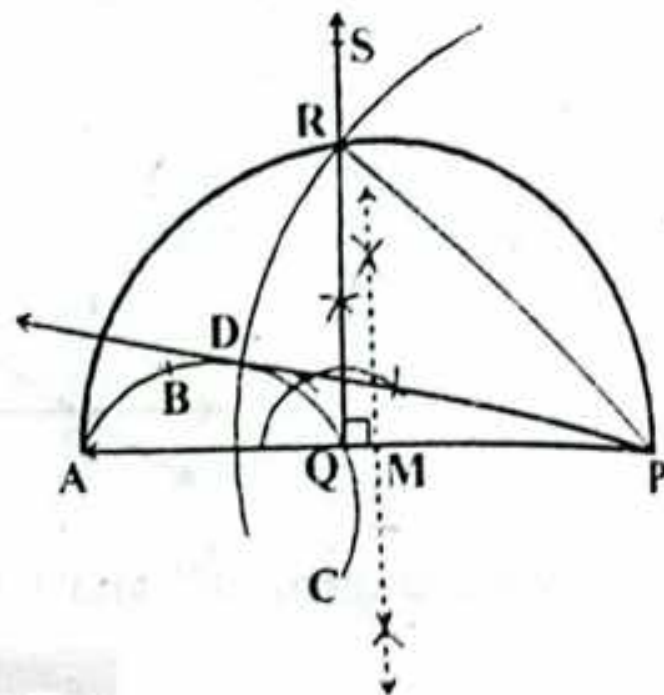
4. Draw a circle of radius 1.3 cm and draw a tangent at point P , when P lies on its circumference.

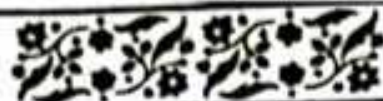
Sol. Given: A circle with centre D of radius 1.3 cm, and a point P on its circumference.

Required: Draw a tangent to the circle at point P .

Steps of Construction:

1. Join centre D with the point P . Now $\overline{DP}$ is the radial





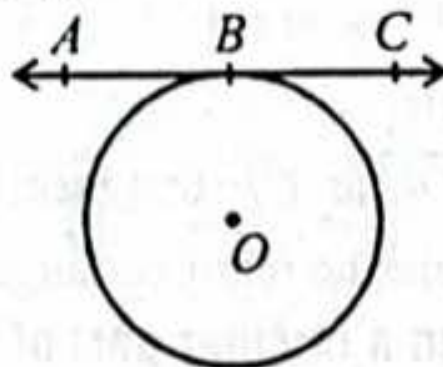
Steps of Construction:

1. Draw a diameter $\overline{AOB}$.
2. Draw a perpendicular at B on $\overline{AOB}$, this is one tangent.
3. Draw an angle of 30° at O , as $m\angle AOC = 30^\circ$ where C is a point on the circumference of the given circle.
4. Draw a perpendicular at C on OC as $\overline{CP}$ that meets the already drawn tangent at P .
Hence, tangents $\overline{BP}$ and $\overline{CP}$ are the required tangents that meet at $m\angle CPB = 30^\circ$.

Solved Review Exercise 10

1. Four possible answers are given for the following questions. Choose the correct answer.

- (i) In the adjacent figure, $\overline{ABC}$ is:

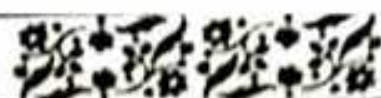


(a) a chord

(b) an arc

(c) a tangent

(d) a secant



Steps of construction

1. Draw a diameter $\overline{AOB}$.
2. Draw a perpendicular at B on $\overline{AOB}$, this is one tangent.
3. Draw an angle of 30° at O, as $m\angle AOC = 30^\circ$, where C is a point on the circumference of the given circle.
4. Draw a perpendicular at C on $\overline{OC}$ as $\overline{CP}$ that meets the already drawn tangent at P. Hence tangents $\overline{BP}$ and $\overline{CP}$ are the required tangents meet at $m\angle CPB = 30^\circ$.
5. Take a part of the circular track, use chords and perpendicular bisectors to complete a circular track.

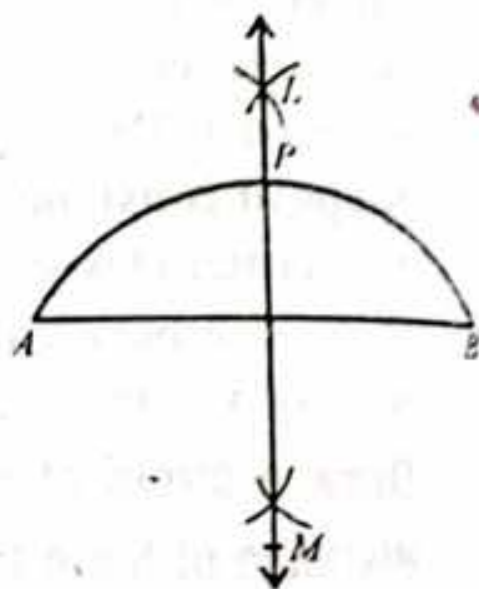
Sol: $\widehat{AB}$ is a part of circumference of a circular track.

Required: Locate the centre of the track's arc.

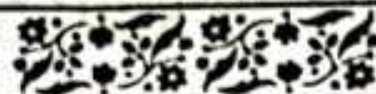
Steps of construction

1. Join and points A and B of the arc.
2. Draw a right bisector $\overline{LM}$ of $\overline{AB}$.

It passes through point P at arc. So, this point P is the center of the track's arc.



6. Two decorative fences touch the circular flower bed of radius 2.1 cm and meet



6. In figure (C), which is equal to $\overline{OR}$:
- (a) $\overline{OD}$ (b) $\overline{OF}$ (c) $\overline{PD}$ (d) $\overline{OQ}$
7. In figure (C), chords of arc $\widehat{PQR}$ are:
- (a) $\overline{PQ}$, $\overline{PR}$ (b) $\overline{PD}$, $\overline{OR}$ (c) $\overline{GF}$ (d) $\overline{DE}$
8. We can complete a circle without finding centre, when a part of _____ is given.
- (a) centre (b) chord (c) diameter (d) circumference

10.2

Tangent to a circle

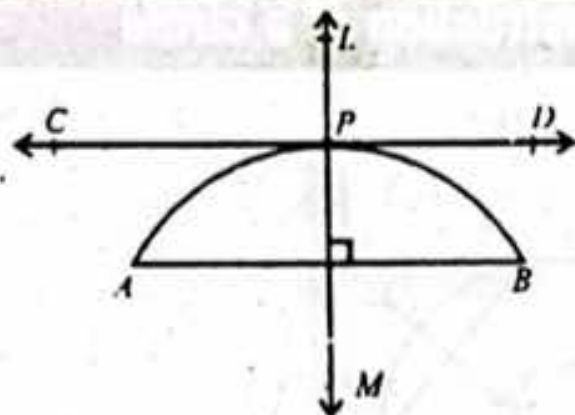
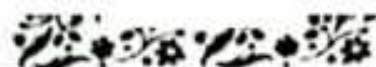


Figure (A)

9. In fig (A), the tangent of arc AB is:
- (a) $\overline{CPD}$ (b) $\overline{LPM}$ (c) $\overline{AB}$ (d) $\overline{LPD}$
10. In fig (A) the midpoint of arc AB is:
- (a) c (b) D (c) L (d) P





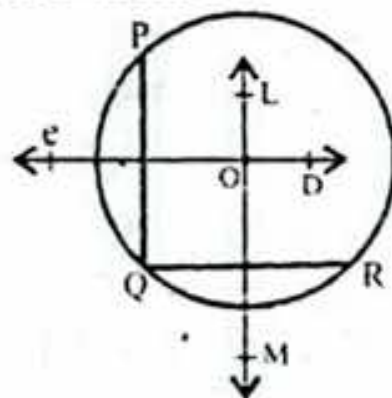
2. How many points on the circumference are required to find the center, and why?

Ans. You need at least three distinct points. These points allow you to draw two chords.

3. What is the significance of the point where the two right bisectors of the chords of a circle meet?

Ans. That point is the centre of the circle. The perpendicular bisector of any chord passes through the centre.

4. In fig how is radius of the circle defined?



Ans. The radius is defined that it is a distance from the centre of the circle to any point on the circumference. Mathematically $r = \overline{OP} = \overline{OQ} = \overline{OR}$

5. What constraint is placed on the length of chords to complete a circle without finding its centre?

Ans. The chords must be equal in length to complete a circle without finding its centre.

6. What are the two ways to complete a circle, when a part of its circumference is given.

Ans. We can complete a circle by finding its centre and without finding its centre, when a part of its circumference is given.

**UNIT
11****Information Handling****Students' Learning Outcomes**

After completing this unit, the students will be able to:

- ▶ Construct cumulative frequency table and cumulative frequency polygon or Ogive.
- ▶ Interpret the median, quartiles, deciles, percentiles and inter quartile range from cumulative frequency curve.
- ▶ Interpret and analyze box and whisker plots.
- ▶ Construct and interpret data from scatter diagrams and also draw lines of best fit.
- ▶ Measure correlation using scatter diagram.
- ▶ Calculate the range, standard deviation and variance for grouped data.
- ▶ Use the mean and standard deviation to compare two sets of data.
- ▶ Solve real life situations involving variance and standard deviation for grouped data.
- ▶ Apply concepts from measures of dispersion to solve real life situations (determining the consistency of data, checking variability in forecasting, manufacturing, finance and economics).



Deciles:

Deciles divide an arranged data set into ten equal parts, each representing 10% of the data. In other words, there are nine decile values from D_1 to D_9 .

For example, D_3 is the value below which 30% of the data lies and D_7 is the value below which 70% of the data falls.

Remember!

P_{25} is the same as Q_1 , P_{50} is the same as the median (Q_2), and P_{75} is the same as Q_3 .

Percentiles:

Percentiles divide an arranged data set into 100 equal parts, each containing 1% of the data. There are ninety nine percentile values from P_1 to P_{99} .

For example, P_{90} means 90% of the data lies below that value.

Interquartile Range (IQR)

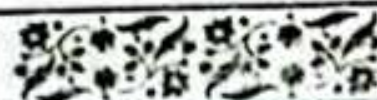
The interquartile range is a measure of spread that represents the distance between the first quartile Q_1 and the third quartile Q_3 . It is calculated as $Q_3 - Q_1$.

Example 2: The table provides the information about the speeds, in km/h of 90 vehicles passing a speed checkpoint. Construct the cumulative frequency polygon by less than method and interpret median, Q_1 , Q_3 , D_4 , P_{95} and IQR.

Speed (km/h)	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
f	10	18	30	17	9	6

Solution:

Speed (km/h)	f	cf
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The position of Q_2 is $\frac{n}{2} = \frac{200}{2} = 100$. So, from the curve $Q_2 = 47$

11.2

Box-and-Whisker Plot

A box-and-whisker plot or box plot is a graphical representation of a data set five-number summary (minimum, first quartile, median, third quartile and maximum). It is useful for visualizing data distribution and comparing data sets.

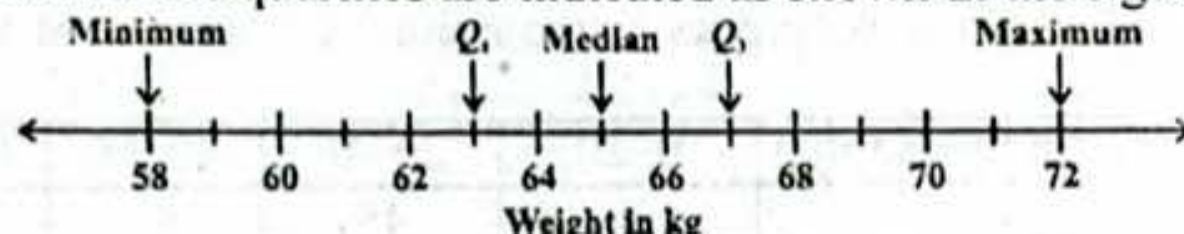
There are following key components of box plot:

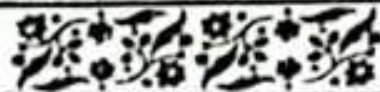
The Box: It represents the interquartile range, which is middle 50% of the data with the box edges marking the first Q_1 and third Q_3 quartiles.

The median: A line within the box representing the second quartile (Q_2) or the middle of the data set.

Whiskers: Lines extending from the box to the minimum and maximum values, showing the overall spread of the data.

We can show this information on a box-and-whisker plot. To begin with, we draw a horizontal number line using a suitable scale. On the number line, the minimum score, the maximum score and the quartiles are indicated as shown in the figure given below:





Negative Correlation

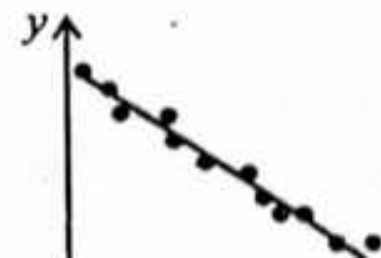
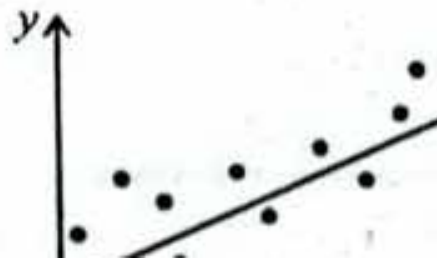
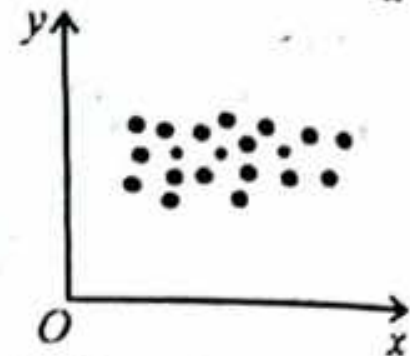
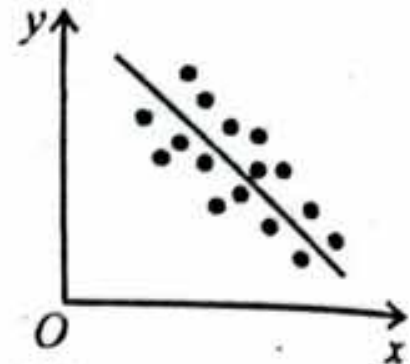
If one variable increases as the other decreases, then it is called negative correlation. If you draw a straight line along the data points, the slope of the line will go down. For example, speed and travel time.

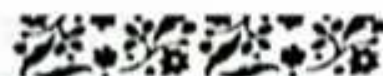
Zero Correlation

If there is not consistent pattern between the two variables, then it is called no correlation. For example, shoe size and test scores.

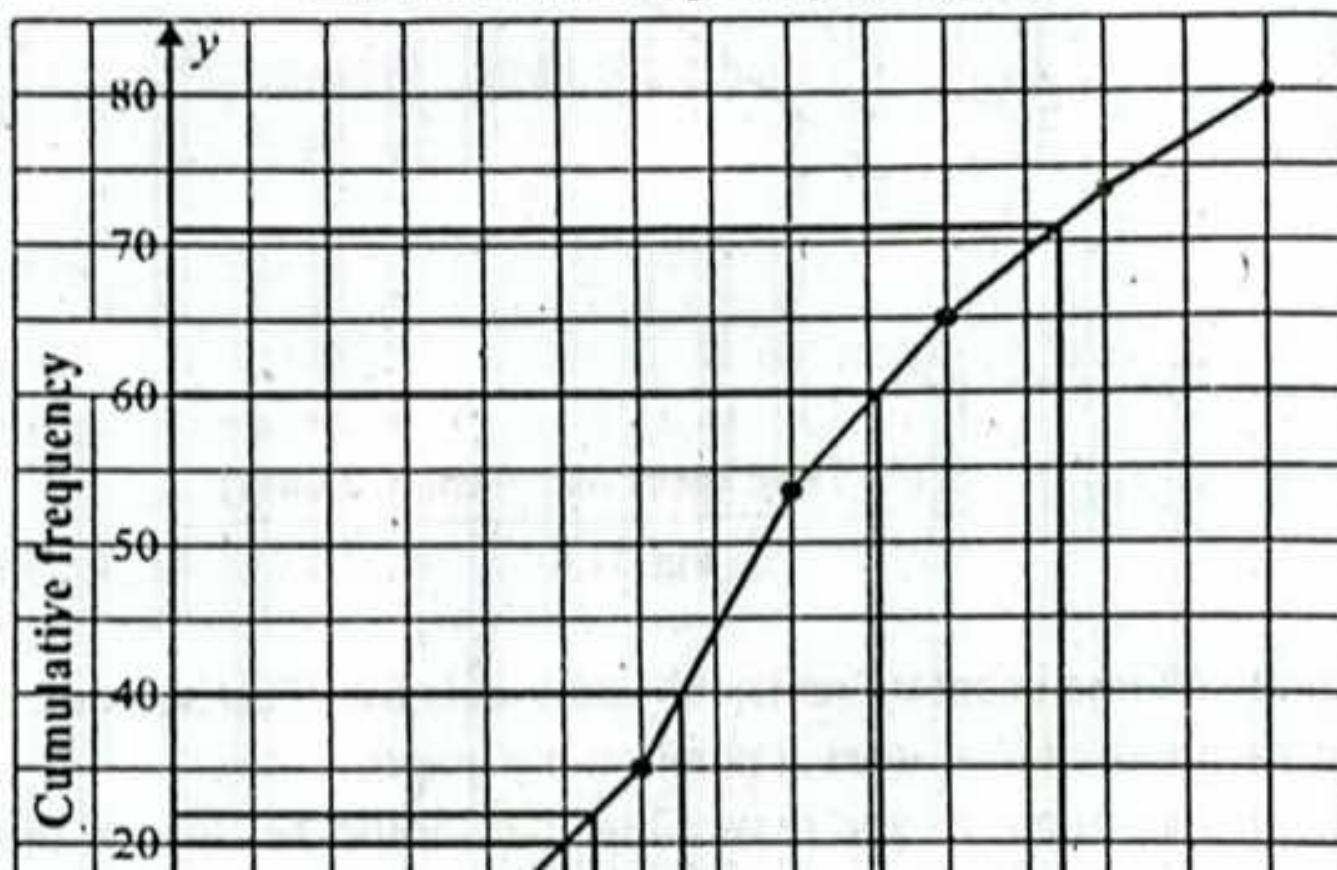
Correlation Pattern of Scatter Diagram

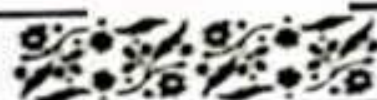
- When the value of Y increases as the X value increases, it is called **strong positive correlation pattern**.
- When the value of Y decreases as the X value increases, it is called **strong negative correlation pattern**.
- When the value of Y increases slightly as the X value increases, it is called **weak positive correlation pattern**.



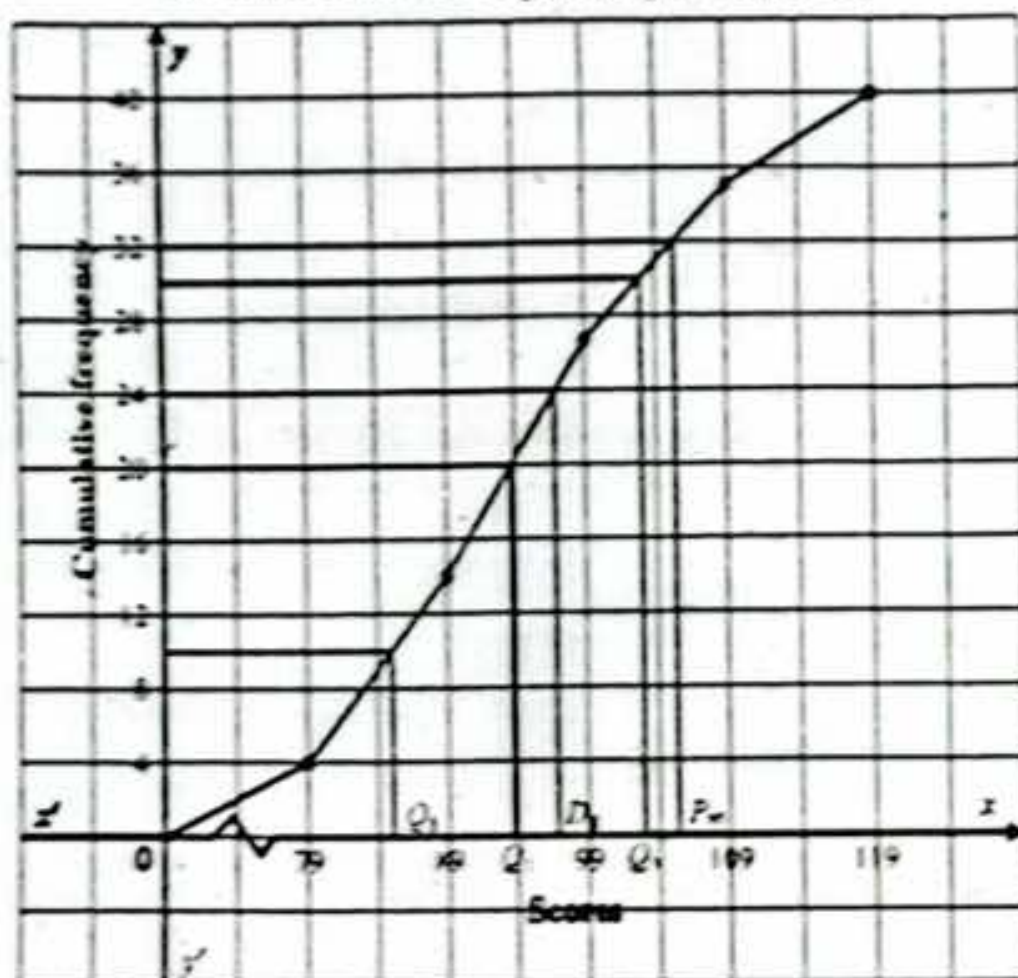


54 – 58	15	$15 + 15 = 30$
59 – 63	23	$30 + 23 = 53$
64 – 68	12	$53 + 12 = 65$
69 – 73	8	$65 + 8 = 73$
74 – 78	7	$73 + 7 = 80$
Total	$n = \sum f = 80$	

Cumulative Frequency Polygon




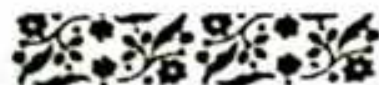
Cumulative Frequency Polygon



Median: The position of median is $\frac{n}{2} = \frac{40}{2} = 20$ (20th value)

Draw a horizontal line from 20 on Y-axis until it meets the curve and then draw vertical line from this point to X-axis. So,

$$\text{Median} = 94$$



Here

Total no. of values = $n = 15$

Median

$$\begin{aligned}\text{Median} &= \left(\frac{n+1}{2}\right)^{\text{th}} \text{ Value} \\ &= \left(\frac{15+1}{2}\right)^{\text{th}} \text{ Value} \\ &= \left(\frac{16}{2}\right)^{\text{th}} \text{ Value}\end{aligned}$$

$\Rightarrow$ 8th Value $\Rightarrow$ Median = 100

Lower Quartile (Q_1)

$$\begin{aligned}Q_1 &= \left(\frac{n+1}{4}\right)^{\text{th}} \text{ Value} \\ &= \left(\frac{15+1}{4}\right)^{\text{th}} \text{ Value}\end{aligned}$$

$$= \left(\frac{16}{4}\right)^{\text{th}} \text{ Value}$$

$$= 4^{\text{th}} \text{ Value} \Rightarrow Q_1 = 96$$

Upper Quartile (Q_3)

$$\begin{aligned}Q_3 &= 3 \left(\frac{n+1}{4}\right)^{\text{th}} \text{ Value} \\ &= 3 \left(\frac{15+1}{4}\right)^{\text{th}} \text{ Value} \\ &= 3 \left(\frac{16}{4}\right)^{\text{th}} \text{ Value} \\ &= [3(4)]^{\text{th}} \text{ Value} \\ &= 12^{\text{th}} \text{ Value} \Rightarrow Q_3 = 105\end{aligned}$$

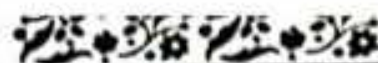
Range

$$\begin{aligned}\text{Range} &= \text{Highest value} - \text{lowest value} \\ &= 110 - 92\end{aligned}$$

$$\Rightarrow \text{Range} = 18$$

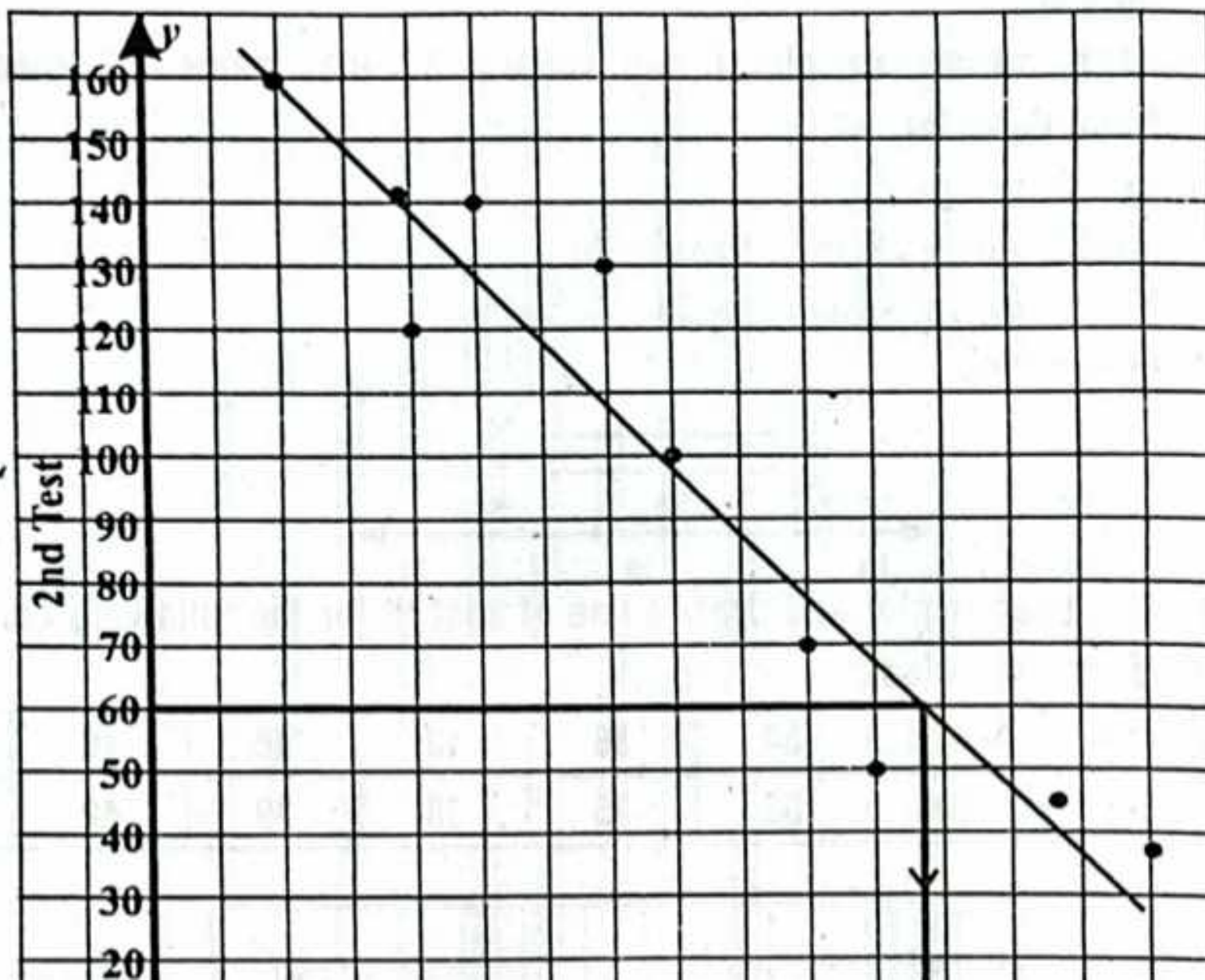
Box-and-whisker plot

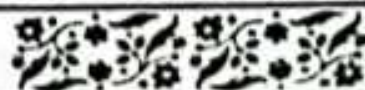
Min. value	Q_1	Median	Q_3	Max. value
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(i) Construct a scatter plot and draw a line of best fit.

Sol.





Although both batsmen have the same mean score, batsman B's runs are closely clustered around the mean, while batsman A's scores are widely spread from 0 to 100. Therefore, to understand how the data values are spread out, we need more statistical information. For this purpose, we will study measures of dispersion.

Measures of dispersion are statistical tool used to describe how spread out or scattered the values in a data set are around a central value.

Different measures of dispersion are

- | | | |
|-----------------------------|-------------------|-----------------------|
| 1. Range | 2. Variance | 3. Standard deviation |
| 4. Interquartile range | 5. Mean deviation | 6. Quartile deviation |
| 7. Coefficient of variation | | |

In this unit, we will discuss only few measures of dispersion.

Range

The difference between highest value (H) and the lowest value (L) is called range.

$$\text{Range (R)} = H - L$$

Where H is the highest value and L is the lowest value.

Example 7: Find the range of the following data:

15, 22, 18, 30, 17, 25

Solution Here $H = 30; L = 15$

$$\begin{aligned} \text{Range (R)} &= H - L \\ &= 30 - 15 = 15 \end{aligned}$$

Skilled practice

What is the range of first 20 prime numbers?

Sol: First 20 prime numbers are
2, 3, 5, 7, 11, 13, ..., 61, 67, 71

Here

$$\text{or } \sigma = \frac{\sum_{i=1}^n f_i x_i^2}{\sum_{i=1}^n f_i} - \left[\frac{\sum_{i=1}^n f_i x_i}{\sum_{i=1}^n f_i} \right]^2 \quad (\text{For grouped data})$$

Challenge!

Can variance be negative?

Ans: No, variance cannot be negative, because its evaluation involves squares of values and summing of zero or positive values.

Example 9: A factory produces rods with lengths (in cm):

50.1, 49.9, 50.0, 50.2, 49.8.

Check the consistency using standard deviation.

Solution

$$\bar{x} = \frac{50.1 + 49.9 + 50.0 + 50.2 + 49.8}{5} = \frac{250}{5} = 50$$

x	$x - \bar{x}$	$\therefore (x - \bar{x})^2$
50.1	0.1	0.01
49.9	-0.1	0.01
50.0	0	0.00
50.2	0.2	0.04
49.8	-0.2	0.04
		$\Sigma (x - \bar{x})^2 = 0.10$



$$\bar{x}(\text{Rooma}) = \frac{\sum x}{n} = \frac{72 + 65 + 80 + 68 + 75}{5}$$

$$= \frac{360}{5} = 72$$

$$\bar{x}(\text{Rooma}) = \frac{\sum (x - \bar{x})^2}{n}$$

$$= \frac{(72 - 72)^2 + (65 - 72)^2 + (80 - 72)^2 + (68 - 72)^2 + (75 - 72)^2}{5}$$

$$= \frac{(0)^2 + (-7)^2 + (8)^2 + (-4)^2 + (3)^2}{5}$$

$$= \frac{0 + 49 + 64 + 16 + 9}{5} = \frac{138}{5} = 27.6$$

$$\sigma(\text{Rooma}) = \sqrt{27.6} = 5.25$$

Since $\bar{x}(\text{Ali}) = 72$, $\bar{x}(\text{Rooma}) = 72$. Both performed equally on average.

$$\sigma(\text{Ali}) = 2.83, \sigma(\text{Rooma}) = 5.25$$

Hence, Ali is more consistent.

$$\bar{x} = \frac{650}{5}$$

$$\bar{x} = 130$$

x	(x - $\bar{x}$)	(x - $\bar{x}$) ²
120	120 - 130 = -10	(-10) ² = 100
135	135 - 130 = 5	5 ² = 25
130	130 - 130 = 0	0 ² = 0
125	125 - 130 = -5	(-5) ² = 25
140	140 - 130 = 10	10 ² = 100
		$\Sigma (x - \bar{x})^2$ = 250

$$\text{Variance } (\sigma^2) = \frac{\Sigma (x - \bar{x})^2}{n}$$

$$= \frac{250}{5}$$

$$\sigma^2 = 50$$

$$\text{Standard Deviation } (\sigma) = \sqrt{\frac{\Sigma (x - \bar{x})^2}{n}}$$

$$\sqrt{250}$$



7. The rainfall recorded in various places of five districts in a week is given below. Find its variance and standard deviation.

Rainfall (in mm)	42	51	54	61	63	71
Number of places	5	13	4	9	5	4

Sol:

Rain fall (in mm) x	Number of places f	$f.x$	x^2	$f.x^2$
42	5	210	1764	8820
51	13	663	2601	33813
54	4	216	2916	11664
61	9	549	3721	33489
63	5	315	3969	19845
71	4	284	5041	20164
$\Sigma f = 40$		$\Sigma fx = 2237$	$\Sigma fx^2 = 127795$	

$$\begin{aligned} \text{Variance } (\sigma^2) &= \frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f} \right)^2 \\ &= \frac{127795}{40} - \left(\frac{2237}{40} \right)^2 \end{aligned}$$



9. The monthly sales (rupees in lacs) for two salespersons over 6 months are:

Person A: 5.5, 5.7, 5.4, 5.6, 5.8, 5.6

Person B: 5.5, 6.5, 4.5, 6.0, 5.0, 6.0

Compare their Performance and consistent

Sol.

● Person A

5.5, 5.7, 5.4, 5.6, 5.8, 5.6

$$\text{Now Mean } (P_A) = \bar{x}_A = \frac{\sum x}{n} = \frac{5.5 + 5.7 + 5.4 + 5.6 + 5.8 + 5.6}{6} = \frac{33.6}{6}$$

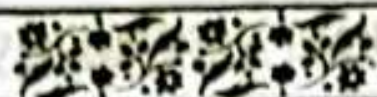
$$\boxed{\bar{x}_A = 5.6}$$

$$\text{And } \sigma^2(P_A) = \frac{\sum (x - \bar{x})^2}{n}$$

$$\sigma^2(P_A) = \frac{(5.5 - 5.6)^2 + (5.7 - 5.6)^2 + (5.4 - 5.6)^2 + (5.6 - 5.6)^2 + (5.8 - 5.6)^2 + (5.6 - 5.6)^2}{6}$$

$$= \frac{(-0.1)^2 + (0.1)^2 + (-0.2)^2 + 0^2 + (0.2)^2 + 0^2}{6}$$

$$= \frac{0.01 + 0.01 + 0.04 + 0 + 0.04 + 0}{6} = \frac{0.1}{6}$$



● Mean

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{31300}{23}$$

$$\boxed{\bar{x} = 1360.87}$$

● Variance

$$\begin{aligned}\sigma^2 &= \frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2 \\ &= \frac{43990000}{23} - \left(\frac{31300}{23} \right)^2\end{aligned}$$

$$\begin{aligned}&= 1912608.70 - (1360.87)^2 \\ &= 1912608.70 - 1851967.16\end{aligned}$$

$$\boxed{\sigma^2 = 60642.72}$$

● Standard Deviation

$$\begin{aligned}\text{S.D} &= \sqrt{\text{Variance}} \\ &= \sqrt{60642.72}\end{aligned}$$

$$\boxed{\text{S.D} = 246.26}$$

11. A company forecasts monthly sales (rupees in millions): 15, 18, 14, 20, 13. Find variability in sales predictions.

Sol: Monthly sales (rupees in millions):

15, 18, 14, 20, 13

To check variability in sales predictions we find mean, variance and S.D of dataset.

● Mean:

$$\bar{x} = \frac{15+18+14+20+13}{5} \quad \because \text{no of values} = n = 5$$

$$= \frac{82}{5} - \left(\frac{24}{5} \right)^2$$

$$= 84.2 - 48$$

$$= 36.2$$

$$\sigma^2 = 3.2$$

$$\text{S.D.}(\sigma) = \sqrt{\text{Variance}}$$

$$= \sqrt{3.2}$$

$$\sigma = 1.79$$

(ii) $\sum fx = 2110$, $\sum fx^2 = 7561$, $\sum f = 6$

Sol. Given that

$$\sum fx = 2110, \sum fx^2 = 7561, \sum f = 6$$

$$\text{Variance } (\sigma^2) = \frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2$$

$$= \frac{7561}{6} - \left(\frac{2110}{6} \right)^2$$

$$\sigma^2 = 1761 - 352^2$$

(iii) $\bar{x} = 18$, $\sum fx^2 = 1670$, $\sum f = 5$

Sol. Given that

$$\bar{x} = 18, \sum fx^2 = 1670, \sum f = 5$$

$$\text{Variance } (\sigma^2) = \frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2$$

$$= \frac{1670}{5} - (\bar{x})^2$$

$$= \frac{1670}{5} - (18)^2$$

$$= 334 - 324$$

$$\sigma^2 = 10$$

$$\text{S.D.}(\sigma) = \sqrt{\text{Variance}}$$

$$= \sqrt{10}$$

$$\sigma = 3.16$$



Median The position of median is $\frac{n}{2} = \frac{40}{2} = 20$ (20th value). Draw a horizontal line from 20 on the y-axis until it meets the curve and then draw vertical line from this point to the x-axis.

Now take the reading on the x-axis, where the vertical line hits the x-axis. So,

$$\text{Median} = 202.5$$

Lower Quartile (Q_1)

The position of Q_1 is $\frac{n}{4} = \frac{40}{4} = 10$ (10th value).

Now from the graph we see that the corresponding value of 10 on y-axis is 192.2 on x-axis.

$$Q_1 = 192.2$$

Upper Quartile (Q_3)

The position of Q_3 is $\frac{3n}{4} = \frac{3(40)}{4} = 30$ (30th value).

Now from the graph we see that the corresponding value of 30 on y-axis is 211.3 on x-axis.

$$Q_3 = 211.3$$

7th Decile (D_7)



Sol:

Class interval	f	x	fx	fx^2
0 - 5	4	$\frac{0+5}{2} = \frac{5}{2} = 2.5$	10	25
5 - 10	12	$\frac{5+10}{2} = \frac{15}{2} = 7.5$	90	675
10 - 15	20	$\frac{10+15}{2} = \frac{25}{2} = 12.5$	250	3125
15 - 20	24	$\frac{15+20}{2} = \frac{35}{2} = 17.5$	420	7350
20 - 25	16	$\frac{20+25}{2} = \frac{45}{2} = 22.5$	360	8100
25 - 30	4	$\frac{25+30}{2} = \frac{55}{2} = 27.5$	110	3025
	$\Sigma f = 80$		$\Sigma fx = 1240$	$\Sigma fx^2 = 22300$

Highest Value (H) = 30

Lowest Value (L) = 0

Range (R) = H - L



- Family B: 38, 52, 40, 50, 42, 49

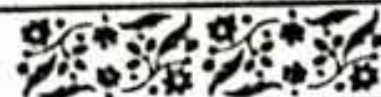
Now

$$\begin{aligned}\text{Mean } (\bar{x}_B) &= \frac{38 + 52 + 40 + 50 + 42 + 49}{6} \\ &= \frac{271}{6}\end{aligned}$$

$$\bar{x}_B = 45.17$$

And Standard Deviation

$$\begin{aligned}\sigma^2_B &= \frac{(38-45.17)^2 + (52-45.17)^2 + (40-45.17)^2 + (50-45.17)^2 + (42-45.17)^2 + (49-45.17)^2}{6} \\ &= \frac{(-7.17)^2 + (6.83)^2 + (-5.17)^2 + (4.83)^2 + (-3.17)^2 + (3.83)^2}{6} \\ &= \frac{51.41 + 46.65 + 26.73 + 23.33 + 10.05 + 14.67}{6} \\ &= \frac{271}{6} \\ \sigma^2_B &= 28.81\end{aligned}$$



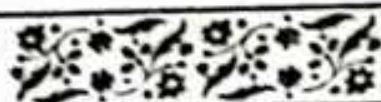
9. Deciles divide an arranged data into _____ equal parts:
(a) 2 (b) 4 (c) 10 (d) 100
10. Percentiles divide an arranged data into _____ equal parts:
(a) 2 (b) 4 (c) 10 (d) 100
11. $IQR =$ _____ ?
(a) $Q_3 - Q_1$ (b) $Q_2 - Q_1$ (c) $Q_1 - Q_3$ (d) $Q_1 - Q_2$
12. The distance between the first quartile Q_1 and the 3rd quartile Q_3 is known as:
(a) IQR (b) Median (c) Mode (d) Mean

11.2

Box-and-Whisker Plot

13. The box in Box-and-Whisker Plot represents:
(a) Q_1 (b) Q_2 (c) Q_3 (d) IQR
14. A line within the box represents:
(a) Q_1 (b) Median (c) Q_3 (d) IQR
15. Lines extending from the box to minimum and maximum values are called:
(a) Box (b) Median (c) Whiskers (d) IQR
16. The median of the dataset 2, 3, 4, 5, 6 is:
(a) 2 (b) 3 (c) 4 (d) 5

17. Median = $\left(\frac{n+1}{2}\right)^{\text{th}}$ value, if n is:



41. The formula $\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2$ is used to calculate:
- (a) Mean (b) Median (c) Variance (d) S.D

42. The formula used to calculate variance of a grouped data:

(a) $\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum n} \right)^2$ (b) $\frac{\sum fx}{\sum f}$ (c) $\sqrt{\frac{\sum fx}{\sum f} - \left(\frac{\sum fx}{\sum n} \right)^2}$ (d) $\frac{\sum (x - \bar{x})^2}{n}$

43. If $\sum fx^2 = 1670$, $\sum fx = 90$ and $\sum f = 5$, then $\sigma^2 =$ _____?
- (a) 5 (b) 10 (c) 15 (d) 20

11.43
Standard Deviation (S.D)

44. Notation used to represent S.D is:

(a) σ (b) σ^2 (c) $\bar{x}$ (d) $\tilde{x}$

45. S.D is the positive square root of _____?

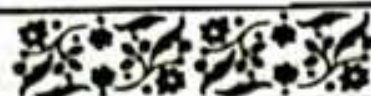
(a) Mean (b) Median (c) Variance (d) Quartiles

46. If variance (σ^2) = 225, then S.D (σ) = _____?

(a) 5 (b) 10 (c) 15 (d) 20

47. The formula to calculate S.D of ungrouped data is:

(a) $\frac{\sum (x - \bar{x})^2}{n}$ (b) $\frac{\sum fx}{\sum f}$ (c) $\sqrt{\frac{\sum (x - \bar{x})^2}{n}}$ (d) $\sqrt{\frac{\sum x^2}{n}}$



6. Write the formula to find median of ungrouped data, when 'n' is odd.

Ans. Formula

$$\text{Median} = \left(\frac{n+1}{2} \right)^{\text{th}} \text{ value}$$

7. Find the median of first 11 whole number.

Ans. First 11 whole numbers are:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

Here $n = 11$

$$\begin{aligned} \text{Median} &= \left(\frac{n+1}{2} \right)^{\text{th}} \text{ value} \\ &= \left(\frac{11+1}{2} \right)^{\text{th}} \text{ value} \\ &= \left(\frac{12}{2} \right)^{\text{th}} \text{ value} \end{aligned}$$

$$\boxed{\text{Median} = 6^{\text{th}} \text{ value} \Rightarrow \text{Median} = 5}$$

8. What is the median of dataset 2.5, 3.5, 4.5, 5.5, 6.5.

Sol. Here

$$n = 5$$

$$\begin{aligned} \text{Median} &= \left(\frac{n+1}{2} \right)^{\text{th}} \text{ value} \\ &= \left(\frac{5+1}{2} \right)^{\text{th}} \text{ value} \\ &= \left(\frac{6}{2} \right)^{\text{th}} \text{ value} = 3^{\text{rd}} \text{ value} \\ \Rightarrow \quad \boxed{\text{Median} = 4.5} \end{aligned}$$

9. What are quartiles?

Ans. Quartiles are values that divide a dataset into four equal parts each containing 25 % of the data.

20. Find IQR Value if $Q_1=73$ and $Q_3=80$.

Ans. Given data

$$Q_3 = 80, Q_1 = 73$$

$$\Rightarrow \text{IQR} = Q_3 - Q_1 \\ = 80 - 73$$

$$\boxed{\text{IQR} = 7}$$

21. What are Deciles?

Sol. The values that divide a dataset into 10 equal parts are called deciles, each representing 10% of the data. There are 9 decile values from D_1 to D_9 .

22. Write the formula for K^{th} decile position.

Sol. Formula: K^{th} decile position = $\frac{K(n+1)}{10}$

23. Find D_3 and D_7 for the following test scores: 12, 15, 20, 22, 25, 28, 30, 32, 35, 40

Sol. Here
 $n = 10$

Now

$$D_3 = \frac{3(10+1)}{10} = \frac{33}{10} = 3.3$$

26. A student scores in 8 subjects are 55, 62, 48, 70, 85, 92, 78, 65. Find the 40th percentile (P_{40}).

Sol. Arrange the data in ascending order:
48, 55, 62, 65, 70, 78, 85, 92

Here

$$n = 8$$

Now

$$P_{40} = \frac{40(8+1)}{100} = \frac{40(9)}{100} = \frac{360}{100} \\ = 3.6 \text{ (3.6}^{\text{th}} \text{ position)}$$

$$\Rightarrow P_{40} = 3^{\text{rd}} \text{ value} + 0.6 (4^{\text{th}} \text{ value} - 3^{\text{rd}} \text{ value})$$

$$\Rightarrow P_{40} = 62 + 0.6 \times (65 - 62) \\ = 62 + 0.6 \times 3 = 62 + 1.8$$

$$\boxed{P_{40} = 63.8}$$

11.3

Scatter Diagram

27. What is a scatter diagram?

Sol. A scatter diagram (or scatter plot) is a graphical tool used to display the relationship between two numerical values (variables).

28. What is the line of best fit?

Ans. A line of best (also called trend line)



- For grouped data

$$\sigma = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}}$$

or

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

- 44. Find variance and standard deviation if**

$$\sum x = 63, \sum x^2 = 623 \text{ } n = 7$$

Sol.

$$\sigma^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2$$

$$\sigma^2 = \frac{623}{7} - \left(\frac{63}{7}\right)^2$$

$$= 89 - 81$$

$$\sigma^2 = 8$$

$$\sqrt{\sigma^2} = \sqrt{8}$$

$$\sigma = 2.89$$

$$= 743.90 - (27.093)^2$$

$$= 743.90 - 734.03065$$

$$\sigma^2 = 9.86935$$

So, variance is 9.86935

- 47. Find standard deviation with given values:**

$$\sum fx^2 = 52690, \sum f = 40 \text{ and } \sum fx = 1300.$$

Sol.

$$\therefore \sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

$$= \sqrt{\frac{52690}{40} - \left(\frac{1300}{40}\right)^2}$$

$$= \sqrt{1317.25 - (32.5)^2}$$

$$= \sqrt{1317.25 - 1056.25}$$

$$\sigma = \sqrt{261}$$

$$\sigma = 16.155$$

So, standard deviation is 9.86935



Events	Explanation	Example
Equally likely events	Two or more events are said to be equally likely if each one of them has an equal chance of occurring.	Numbers 1, 2, 3, 4, 5, 6 are equally likely events, when we roll a dice.
Certain events	In an experiment, the event which surely occurs is called certain event.	When we roll a dice, the event of getting any natural number from 1 to 6 is a certain event.
Impossible events	An impossible event is an event that cannot happen under any circumstances in a given situation.	When we toss two coins, the event of getting three heads is an impossible event.
Mutually exclusive events	Two or more events are said to be mutually exclusive if they don't have common sample points. i.e., events A and B are said to be mutually exclusive if $A \cap B = \phi$	When we roll a dice the events of getting odd numbers and even numbers are mutually exclusive events.
Exhaustive events	The collection of events whose union is the whole sample space is called exhaustive events.	When we toss a coin twice, the collection of events of getting two heads, exactly one head and no head are exhaustive events.
Complementary events	The complement of an event A is	When we toss a coin, the event

When we toss two coins, then the sample space can be written as

$$S = \{HH, HT, TH, TT\}$$

Possibility Diagram

Possibility diagram is a representation in grid or table that shows all possible combinations of two events.

When we toss two coins, then the possibility diagram will look like

Coin 2	Coin 1	
	H	T
	H	T
H	HH	HT
T	TH	TT

Example 2: Hashim rolled a dice and tossed a coin simultaneously. Draw a possibility diagram.

Solution:

Coin	Dice					
	1	2	3	4	5	6
	H	T	H	T	H	T
H	(H,1)	(H,2)	(H,3)	(H,4)	(H,5)	(H,6)
T	(T,1)	(T,2)	(T,3)	(T,4)	(T,5)	(T,6)

Probability of an Event

In a random experiment, let S be the sample space and $E \subseteq S$. Then if E is an event, the probability of occurrence of E is defined as:



Example 7: A game of chance consists of spinning an arrow which is equally likely to come to rest pointing to one of the numbers 1, 2, 3, ..., 12. What is the probability that it will point to:

- (i) 7 (ii) a prime number (iii) a composite number?

Solution: Sample space $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$;
 $n(S) = 12$

- (i) Let A be the event of resting in 7, $n(A) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{12} = \frac{1}{12}$$

- (ii) Let B be the event that the arrow will come to rest in a prime number.

$$B = \{2, 3, 5, 7, 11\}; n(B) = 5$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{5}{12}$$

- (iii) Let C be the event that arrow will come to rest in a composite number.

$$C = \{4, 6, 8, 9, 10, 12\}; n(C) = 6$$

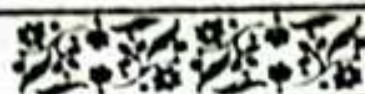
$$P(C) = \frac{n(C)}{n(S)} = \frac{6}{12} = \frac{1}{2}$$



Activity!

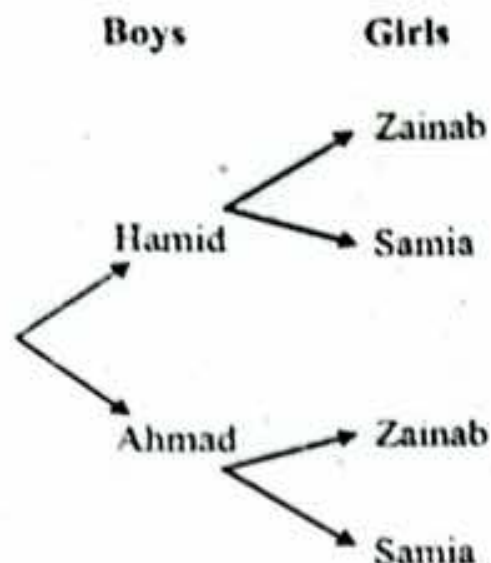
There are four routes R_1, R_2, R_3 , and R_4 from Huria's home to her place of work. There are three parking lots P_1, P_2, P_3 , and two entrances B_1, B_2 into the office building. There are three elevators E_1, E_2, E_3 , to her floor. Using the tree diagram explain how many ways can she reach her office.

Solution:



2. A teacher randomly selects one boy and one girl from a group of 2 boys (Hamid, Ahmad) and 2 girls (Zainab, Samia). Draw a tree diagram and list the sample space for all possible outcomes.

Sol.



Sample Space = {(Hamid, Zainab), (Hamid, Samia), (Ahmad, Zainab), (Ahmad, Samia)}

3. If A is an event of a random experiment such that $P(A) : P(\bar{A}) = 17 : 15$ and $n(S) = 640$ then find (i) $P(\bar{A})$ (ii) $n(A)$

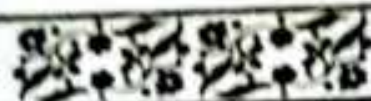
Sol. $P(A) : P(\bar{A}) = 17 : 15$ (Given)

We know

$$P(A) + P(\bar{A}) = 1 \quad (i)$$

(ii) $n(A)$
Sol. $n(S) = 640$ (given)
 $P(A) = 17x$

$$P(\bar{A}) = \frac{1}{17} \text{ from (i)}$$



(ii) Find sample space by drawing possibility diagram.

Sol.

Dice1 \ Dice2	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

(iii) Find the probability of getting

(a) Same number on both dice.

Sol. $S = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)$

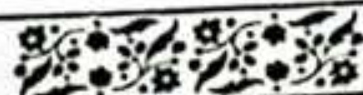
$(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)$

$(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)$

$(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)$

$(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)$

$(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$



(viii) no head.

Sol. $n(S) = 8$

Let H be the event of resulting no head

$$H = \{TTT\}, n(H) = 1$$

$$P(H) = \frac{n(H)}{n(S)} = \frac{1}{8}$$

8. A bag contains 4 red balls, 5 white balls, 6 green balls and 3 black balls. Ali draws a ball at random from the bag. Find the probability that the ball drawn is:

$$\text{Total number of balls} = 4+5+6+3 = 18$$

$$n(S) = 18$$

(i) white

Sol. Let W be the event of resulting a white ball

$$n(W) = 5$$

$$p(W) = \frac{n(W)}{n(S)} = \frac{5}{18}$$

(ii) red

Sol. Let B be the event of resulting red ball.

$$n(B) = 4, n(S) = 18 \text{ (Total number of balls)}$$

$$n(B) = 4$$



$$N = \{ \quad \}, n(N) = 0$$

$$P(N) = \frac{n(N)}{n(S)} = \frac{0}{15} = 0$$

10. If the probability of an event A is $\frac{7}{10}$, then find the probability of the event "not A".

Sol. $P(A) = \frac{7}{10}$ (Given)

$$P(\bar{A}) = 1 - P(A)$$

$$= 1 - \frac{7}{10}$$

$$= \frac{10-7}{10} = \frac{3}{10}$$

11. A dice is rolled twice. Find the probability of having a number greater than 4 on each roll.

Sol. $S = \{1, 2, 3, 4, 5, 6\}$ $n, n(S) = 6$

Number of greater than 4 on a single dice are $\{5, 6\}$, $n = 2$

$$P(\text{First roll} > 4) = \frac{2}{6} = \frac{1}{3}$$



$$\begin{aligned}\therefore P(\text{getting a same number or a total of 10}) &= P(A \cup B) \\ &= P(A) + P(B) - P(A \cap B) \\ &= \frac{6}{36} + \frac{3}{36} - \frac{1}{36} = \frac{8}{36} = \frac{2}{9}\end{aligned}$$

Hence, the required probability is $\frac{2}{9}$.

Solved Exercise 12.2

1. If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$, find $P(A \cup B)$.

Sol. Given that: $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$, $P(A \cap B) = \frac{1}{8}$

(i) **P(A or B)**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8}$$

$$P(A \cup B) = \frac{2+4-1}{8} = \frac{5}{8}$$

2. In an apartment, in selecting a house from door numbers 1 to 50 randomly, find



5. Find the probability of getting a sum of 5 or 7 in a throw of two dice.

Sol. Total out comes $n(S) = 6 \times 6 = 36$

Let A be the event of getting a sum of 5 and B be the event of getting a sum of 7.

$$A = \{(1,4), (2,3), (3,2), (4,1)\}, n(A) = 4$$

$$B = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}, n(B) = 6$$

$$A \cap B = \phi$$

So, A and B are mutually exclusive events

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{1}{9} + \frac{1}{6}$$

$$= \frac{2+3}{18} = \frac{5}{18}$$

6. A card is taken out at random from a standard pack of 52 cards. Find the probability of taking out.

(i) A king or a jack



$$P(B) = \frac{n(B)}{n(S)} = \frac{1}{4}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{4}$$

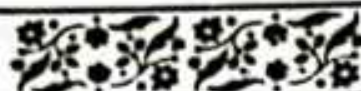
$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= \frac{3}{4} + \frac{1}{4} - \frac{1}{4} \\ &= \frac{3+1-1}{4} = \frac{3}{4} \end{aligned}$$

10. At a busy intersection, 50% of vehicles turn right, 30% turn left and 20% go straight. What is the probability that a randomly selected vehicle turn left or right?

Sol. Let R be event of turning right, L be the event of turning left and T be the event of going straight.

$$P(R) = 50\% = \frac{50}{100} = 0.50$$

$$P(L) = 30\% = \frac{30}{100} = 0.30$$



12.3

Multiplication Law of probability

(i) If A and B are independent events, then $p(A \text{ and } B) = P(A \cap B) = P(A) \times P(B)$

(ii) **Conditional probability**

If A and B are dependent events, then $P(A \text{ and } B) P(A \cap B) = P(A) \times P(B|A)$

Where $P(B|A)$ represents the probability of event B occurring after it is assumed that event A has already occurred (read $B|A$ as " B given A ").

Example 11: A bag contains 4 red and 3 green balls. Kabeer draw two balls one after the other with replacement (the second is drawn after the first is replaced). Find the probability that the first ball is red and the second ball is also red.

Solution: Let A be the event that first ball is red and B be the event that second ball is red.

Then $P(A) = \frac{4}{7}$ and $P(B) = \frac{4}{7}$ (Since the ball is replaced, the sample space is not affected)

$$p(A \text{ and } B) = P(A \cap B) = P(A) \times P(B)$$

$$= \frac{4}{7} \times \frac{4}{7} = \frac{16}{49}$$

$P(A \text{ and } B)$
Multiplication rule

Are
 A and B
Independent?

Yes

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

No

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$



$$P(\overline{A \cap B}) = 1 - P(A \cap B)$$

$$= 1 - \frac{1}{4}$$

$$= \frac{4-1}{4} = \frac{3}{4}$$

2. One letter is chosen from the word PUNJAB and another from LAHORE. What is the probability that:

(i) both are vowels (ii) one is consonant and other is vowel

Sol. $S = \{P, U, N, J, A, B\}$, $n(S) = 6$

(i) both are vowels

Let A be the event for letter vowel

$A = \{U, A\}$, $n(A) = 2$

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

$S = \{L, A, H, O, R, E\}$, $n(S) = 6$

Let B be the event for letter vowel

$B = \{A, O, E\}$, $n(B) = 3$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$



$$n(A) = 13$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

Since, the card is replaced, the sample space is not affected $n(s) = 52$

$$n(B) = 4$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

$$\begin{aligned} P(A \cap B) &= P(A) \times P(B) \\ &= \frac{1}{4} \times \frac{1}{13} = \frac{1}{52} \end{aligned}$$

6. If two cards are selected from a standard deck of 52 cards without replacement, find the probability that.

- (i) Both are black. (ii) Both are queens. (iii) Both are spades.
(iv) Both are diamonds.

Sol: Total cards $n(S) = 52$

- (i) Both are black

Let A be the event of selecting first black card and B be the event of selecting second black card.

$$n(A) = 26$$



$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

Since the card is not replaced the sample space is affected

$$n(s) = 51$$

$$n(B|A) = 4$$

$$P(B|A) = \frac{n(B|A)}{n(s)} = \frac{4}{51}$$

$$\begin{aligned} P(A \text{ and } B) &= P(A) \times P(B|A) \\ &= \frac{1}{13} \times \frac{4}{51} = \frac{4}{663} \end{aligned}$$

Solved Review Exercise 12

1. Four possible answers are given for the following questions. Choose the correct option.
 - (i) The probability of an impossible event is:
(a) 0 (b) 1 (c) not defined (d) -1
 - (ii) What is the probability of getting a head when a fair coin is tossed once?
(a) 0 (b) 0.25 (c) 0.5 (d) 1
 - (iii) The sum of probabilities of all possible outcomes of an experiment is:
(a) 0.5 (b) 0.25 (c) 1 (d) 0.4



$$P(A \text{ and } B) = P(A) \times P(B|A)$$

$$= \frac{1}{2} \times \frac{25}{51} = \frac{25}{102}$$

5. If two dice are rolled, then find the probability of getting the product of face value 6 or the difference of face values 5.

Sol. Total outcomes $n(S) = 6 \times 6 = 36$

Let A be the event of getting product of face value 6 and B be the event of getting difference of face values 5.

$$A = \{(1,6), (2,3), (3,2), (6,1)\}, n(A) = 4$$

$$B = \{(1,6), (6,1)\}, n(B) = 2$$

$$A \cap B = \{(1,6), (6,1)\}, n(A \cap B) = 2$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{36} = \frac{1}{18}$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{2}{36} = \frac{1}{18}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{9} + \frac{1}{18} - \frac{1}{18}$$



12. The complement event of E is:

(a) $\underline{E}$

(b) $P(E)$

(c) $\bar{E}$

(d) $f^{-1}E$

13. The event is certain if:

(a) $P(E) = 1$

(b) $P(E) = 0$

(c) $P(E) > 1$

(d) $P(E) < 0$

14. What is the complement event of an impossible event:

(a) certain event

(b) greater event

(c) mutually event

(d) no event

15. If you toss a coin twice, how many sample points are in the sample space S?

(a) 2

(b) 8

(c) 4

(d) 6

16. Which equation is always true?

(a) $P(A) = P(A')$

(b) $P(A) + P(A') = 0$

(c) $P(A) \times P(A') = 1$

(d) $P(A) + P(A') = 1$

17. Rolling a dice, What is the probability of rolling a number greater than 4:

(a) $\frac{3}{6}$

(b) $\frac{2}{6}$

(c) $\frac{4}{6}$

(d) $\frac{1}{6}$

18. If the probability of rain is 0.3, what is the probability that it will no rain.

(a) 0.7

(b) 0.0

(c) 0.1

(d) 1



12.3

Multiplication Law of Probability

36. Which formula is correct for multiplication Law of probability of independent events A and B:

(a) $P(A|B) = P(A) \times P(B)$

(b) $P(A \cup B) = P(A) + P(B)$

(c) $P(A \cap B) = P(A) + P(B)$

(d) $P(A \cap B) = P(A) \times P(B)$

37. If events A and B are dependent, What is correct for multiplication law:

(a) $P(A \cap B) = P(A) \times P(B/A)$

(b) $P(A \cap B) = \frac{P(A)}{P(B)}$

(c) $P(A \cap B) = P(A) \times P(B)$

(d) $P\left(\frac{A}{B}\right) = P(A) \times P(B)$

38. A fair coin is flipped twice, What is the probability of getting heads on the first flip and heads on second flip?

(a) $\frac{3}{4}$

(b) $\frac{1}{4}$

(c) 1

(d) $\frac{1}{2}$

39. If $P(A) = 0.6$ and $P(B) = 0.5$ What is $P(A \cap B)$?

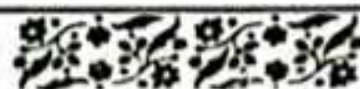
(a) 0.3

(b) 0.83

(c) 0.1

(d) 1.1

40. Two fair dice are rolled. What is the probability of getting 6 on the first dice



9. Define complementary event:

Ans. The complement of an event A is the event representing a collection of a sample points not in A.

10. Define an elementary event.

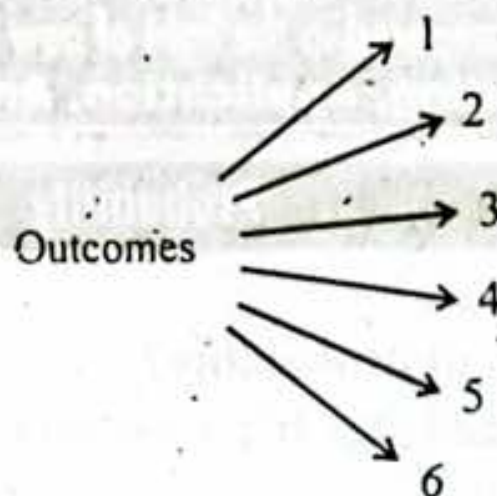
Ans. If an event E consists of only one outcome, then it is called an elementary event.

11. What are mutually exclusive events?

Ans. Two or more events are said to be mutually exclusive if they do not have common sample points.

12. Find the sample space for rolling a dice using tree diagram:

Sol:



$$S = \{1, 2, 3, 4, 5, 6\}$$

13. Find the sample space tossing two coins using tree diagram.



$$\frac{7}{10} = 1 - (PSA)$$

$$P(A) = 1 - \frac{7}{10}$$

$$P(A) = \frac{10-7}{10} = \frac{3}{10}$$

20. A dice is rolled twice. Find the probability to having a number greater than 6 on each roll.

Sol: Total outcomes $n(s) = 6 \times 6 = 36$

Let A be event of having a number greater than 6 on each roll.

Since there is not simple point in sample space greater the six of each roll.

$$A = \{\}, n(A) = 0$$

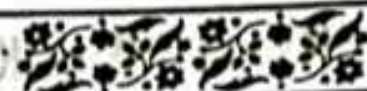
$$P(A) = \frac{n(A)}{n(S)} = \frac{0}{36} = 0$$

12.2**Addition Law of probability**

21. Write the addition law of probability for two non-mutually exclusive events.

Sol: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

22. Write the addition law of probability for two mutually exclusive events.



31. A bag contains 5 red and 3 marbles. Minhah picks two marbel one after other replacing the first what is the probability both are red.

Sol: $n(S) = 8$ (Total of marbles)

Let A be the event of first picking and B be the event of second picking.

$$n(A) = 5, n(B) = 5$$

$$P(A) = \frac{5}{8}, P(B) = \frac{5}{8}$$

$$\begin{aligned} P(A \cap B) &= P(A) \times P(P) \\ &= \frac{5}{8} \times \frac{5}{8} = \frac{25}{64} \end{aligned}$$

32. A box has 6 black balls and 4 white balls. Ajwa takes two ball at random (without replacement). What is the probability that both are white balls.

Sol: $n(S) = 10$ (Total of balls)

Let A be the event that first ball is white and B be the event that second ball is also white
(Since first ball is not replaced, the sample space is affected)

$$n(A) = 4, n(B) = 3$$

$$P(A) = \frac{4}{10}, P(B|A) = \frac{3}{9}$$

Model Paper of Mathematics For Grade-10

Objective Type

Time Allowed: 20 Min.

Max. Marks: 15.

1-Note: Four possible answers A, B, C and D to each question are given. The choice which you think is correct, fill that circle in front of that question with marker or ink pen in the answer-book. Cutting or filling two or more circles will result in zero mark in that question.

Sr.No	Questions	A	B	C	D
i.	$i^2 + i^4$	-1	0✓	1	2
ii.	What is the discriminant of $x^2 + 5x - 5 = 0$?	-20	20	25	45✓
iii.	If $A' = -A$, then A is _____ matrix.	symmetric	row	rectangular	skew symmetric✓
iv.	A function f from X to Y is represented by:	$f: XY$	$f: Y \rightarrow X$	$f: X \rightarrow Y$ ✓	$f: \frac{X}{Y}$
v.	Lowest form of $\frac{15xyz}{10}$ is:	$\frac{3xyz}{2}$ ✓	$\frac{3}{2}$	xyz	$\frac{xyz}{2}$

- (iv) Construct a circle with the help of radius.
- (v) Draw a circle of radius 1.3cm draw a tangent to the circumference.
- (vi) Find median for the given data. 102, 99, 105, 107
- (vii) Find the range of the given data: 15, 2
- (viii) Two coins are tossed together, what are the possible outcomes?
- (ix) Two numbers are randomly chosen from 1 to 10, find the probability that both numbers are prime.

(Pa

Note: Attempt any two (02) questions.

- Q.5: (a)** Solve $3z + (2 + i)w = 11 - i$, (2) for w and z .
- (b)** If the equation $x^2 + 2(1 + k)x + k = 0$ has equal roots, find the value of k .



سکیتب خانہ

لے مطابق مستند و معیاری امدادی کتب
نی کتب برائے میڈیٹرک کلاسز

ہارڈ کاپ امتحانی اسلامیات (لازمی)	
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