CLASS 9

HAMDARD IMTIHANI

Mathematics

GUARANTEED SUCCESS WITH GRADE A+

WITH CONCEPTUAL QUESTIONS



ACCORDING TO THE NEW SYLLABUS AND PAPER PATTERN OF ALL BOARDS OF INTERMEDIATE AND SECONDARY EDUCATION LAHORE, MULTAN, GUJRANWALA, SAHIWAL, D.G.KHAN, SARGODHA, BAHAWALPUR, FAISALABAD AND RAWALPINDI

WITH OBJECTIVE TYPE QUESTIONS BASED ON NEW EXAMINATION TECHNIQUES:

KNOWLEDGE, UNDERSTANDING, APPLICATION, ANALYTICAL & CONCEPTUAL

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Unit 1

Real Numbers

Real Number:

The set of rational numbers is defined as $Q = \left\{\frac{p}{q} : p, q \in Z \wedge q \neq 0\right\}$ and set of irrational numbers (Q') contains those elements which can not be expressed as quotient of integers. The set of Real numbers is the union of the set of rational numbers and irrational numbers i.e., $R = Q \cup Q'$

Decimal Representation of Rational Numbers

(i) Terminating Decimal Numbers:

A decimal number with a finite number of digits after the decimal point is called a terminating decimal number.

For example $\frac{1}{4} = 0.25$, $\frac{8}{25} = 0.32$, $\frac{3}{8} = 0.375$ are all terminating decimals numbers.

(ii) Non-Terminating and Recurring Decimal Numbers

The decimal numbers with an infinitely repeating pattern of digits after the decimal point are called non-terminating and recurring decimal numbers.

Here are some examples.

$\frac{1}{3} = 0.333\ldots = 0.\overline{3}$ (the 3 repeats infinitely)

$\frac{1}{6} = 0.1666\ldots = 0.1\overline{6}$ (the 6 repeats infinitely)

$\frac{22}{7} = 3.142857142857\ldots = 3.\overline{142857}$ (the pattern 142857 repeats infinitely)

$\frac{4}{9} = 0.44444\ldots = 0.\overline{4}$ (4 repeats infinitely)

Recurring and non-terminating decimals are also rational numbers.

Decimal Representation of Irrational Numbers

Decimal numbers that do not repeat a pattern of digits after the decimal point continue indefinitely without terminating. These are also known as non-repeating decimals.

Remember!
e = 2.7182 ... is called Euler's number

Non-terminating and non-recurring decimal numbers are known as irrational numbers.

For Examples:

- $\pi = 3.14159265358979323\ldots$
- $e = 2.71828182845904\ldots$
- $\sqrt{2} = 1.41421356237309\ldots$

Example 1 **Identify the following decimal numbers as rational or irrational numbers:**

(i) 0.35 (ii) 0.444... (iii) $3.\overline{5}$ (iv) 3.36788542... (v) 1.709975947...

Solution: **(i)** 0.35 is a terminating decimal number, therefore it is a rational number.

(ii) 0.444... is a non-terminating and recurring decimal number, therefore it is a rational number.

(iii) $3.\overline{5} = 3.5555...$ is a non-terminating and recurring decimal number, therefore it is a rational number.

(iv) 3.3678542... is a non-terminating and non-recurring decimal number. Therefore, it an irrational number.

(v) 1.709975947... is a non-terminating and non-recurring decimal number, therefore it is an irrational number.

Representation of Rational and Irrational Numbers on Number Line

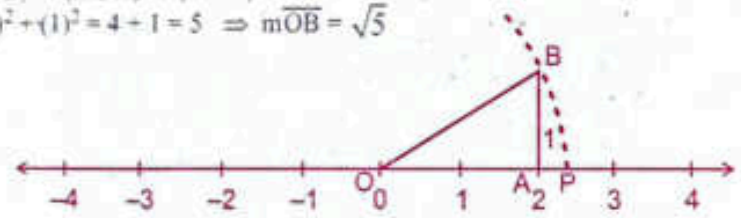
Example 2 **Represent $\sqrt{5}$ on a number line.**

Solution:

$\sqrt{5}$ can be located on the number line by geometric construction. As, $\sqrt{5} = 2.236...$ which is near to 2. Draw a line of $m\overline{AB} = 1$ unit at point A, where $m\overline{OA} = 2$ units, and we have a right-angle triangle OAB. By using Pythagoras theorem

$(m\overline{OB})^2 = (m\overline{OA})^2 + (m\overline{AB})^2$

$= (2)^2 + (1)^2 = 4 + 1 = 5 \Rightarrow m\overline{OB} = \sqrt{5}$



Draw an arc of radius $m\overline{OB} = \sqrt{5}$ taking O as centre, we got point "P" representing $\sqrt{5}$ on the number line.

So, $|\overline{OP}| = \sqrt{5}$

Remember!
(i) Rational number + Irrational number = Irrational number
(ii) Rational number × Irrational number = Irrational number

Example 3 **Express the following recurring decimals as the rational number $\frac{p}{q}$, where p and q are integers. (i) $0.\overline{5}$ (ii) $0.\overline{93}$**

Solution:

(i) $0.\overline{5}$

Let $x = 0.\overline{5}$

$x = 0.55555$ ...(i)

Multiply both sides by 10

$10x = 10(0.5555...)$

$10x = 5.55555$ ...(ii)

Subtracting eq. (i) from eq. (ii)

$10x - x = (5.55555...) - (0.55555...)$

$9x = 5$

$\Rightarrow \quad x = \frac{5}{9}$

Which shows the decimal number in the form of $\frac{p}{q}$.

(ii) $0.\overline{93}$

Solution:

Let $x = 0.\overline{93}$

$x = 0.939393$ ...(i)

Multiply by 100 on both sides

$100x = 100\,(0.939393...)$

$100x = 93.939393...$ ...(ii)

Subtracting (i) from (ii)

$100x - x = (93.939393...) - (0.939393...)$

$99x = 93$

$x = \frac{93}{99}$ which is in the form of $\frac{p}{q}$.

Example 4 **Insert two rational numbers between 2 and 3.**

Solution:

There are infinite rational numbers between 2 and 3.

We have to find any two of them.

For this, findthe average of 2 and 3 as $\frac{2+3}{2} = \frac{5}{2}$

So, $\frac{5}{2}$ is a rational number between 2 and 3,to find another rational number between 2and 3 we will again find average of $\frac{5}{2}$ and 3

i.e., $\frac{\frac{5}{2}+3}{2} = \frac{\frac{5+6}{2}}{2} = \frac{\frac{11}{2}}{2} = \frac{11}{4}$

Hence two rational numbers between 2 and 3 are $\frac{5}{2}$ and $\frac{11}{4}$

Properties of Real Numbers

All calculations involving addition, subtraction, multiplication, and division of real numbers are based on their properties.

Additive properties

Name of the property	$\forall\, a, b, c \in R$	Examples
Closure	$a+b \in R$	$2+3=5 \in R$

Commutative	$a+b=b+a$	$2+5=5+2$ $7=7$
Associative	$a+(b+c)=(a+b)+c$	$2+(3+5)=(2+3)+5$ $2+8=5+5$ $10=10$
Identity	$a+0=a=0+a$	$5+0=5=0+5$
Inverse	$a+(-a)=-a+a=0$	$6+(-6)=(-6)+6=0$

Multiplicative properties

Name of the property	$\forall a,b,c \in R$	Examples
Closure	$abc \in R$	$2\times5=10\in R$
Commutative	$ab=ba$	$2\times3=3\times2=6\in R$
Associative	$a(bc)=(ab)c$	$2\times(3\times5)=(2\times3)\times5$ $2\times15=6\times5$ $30=30$
Identity	$a\times1=1\times a=a$	$5\times1=1\times5=5$
Inverse	$a^{-1}=\frac{1}{a}=\frac{1}{a}\times a=1$	$7^{-1}=\frac{1}{7}=\frac{1}{7}\times7=1$

Distributive Properties

For all real numbers a,b,c

(i) $a(b+c)=ab+ac$ is called left distributive property of multiplication over addition.

(ii) $a(b-c)=ab-ac$ is called left distributive property of multiplication over subtraction.

(iii) $(a+b)c=ac+bc$ is called right distributive property of multiplication over addition.

(iv) $(a-b)c=ac-bc$ is called right distributive property of multiplication over subtraction.

Do you know
0 and 1 are the additive and multiplicative identities of real numbers respectively.

Remember!
0 has no multiplicative inverse.

Properties of Equality of Real numbers

I.	Reflexive property	$\forall a\in R,\ a=a$
II.	Symmetric property	$\forall a,b\in R,\ a=b\Rightarrow b=a$
III.	Transitive property	$\forall a,b,c\in R,\ a=b\wedge b=c\Rightarrow a=c$
IV.	Additive property	$\forall a,b,c\in R,\ a=b\Rightarrow a+c=b+c$
V.	Multiplicative property	$\forall a,b,c\in R,\ a=b\Rightarrow ac=bc$
VI.	Cancellation property w.r.t addition	$\forall a,b,c\in R,\ a+c=b+c\Rightarrow a=b$
VII.	Cancellation property w.r.t multiplication	$\forall a,b,c\in R$ and $c\neq0,\ ac=bc\Rightarrow a=b$

Order Properties

I.	Trichotomy property	$\forall a,b\in R$, either $a=b$ or $a>b$ or $a<b$
II.	Transitive Property	$\forall a,b,c\in R$ • $a>b\wedge b>c\Rightarrow a>c$ • $a<b\wedge b<c\Rightarrow a<c$
III.	Additive property	$\forall a,b,c\in R$ • $a>b\Rightarrow a+c>b+c$ • $a<b\Rightarrow a+c<b+c$
IV.	Multiplicative property	$\forall a,b,c\in R$ • $a>b\Rightarrow ac>bc$ if $c>0$ • $a<b\Rightarrow ac>bc$ if $c>0$ • $a>b\Rightarrow ac<bc$ if $c<0$ • $a<b\Rightarrow ac>bc$ if $c<0$ • $a>b\wedge c>d\Rightarrow ac>bd$ • $a<b\wedge c<d\Rightarrow ac<bd$
V.	Division property	$\forall a,b,c\in R$ • $a<b\Rightarrow \frac{a}{c}<\frac{b}{c}$ if $c>0$ • $a<b\Rightarrow \frac{a}{c}>\frac{b}{c}$ if $c<0$ • $a>b\Rightarrow \frac{a}{c}>\frac{b}{c}$ if $c>0$ • $a>b\Rightarrow \frac{a}{c}<\frac{b}{c}$ if $c<0$
VI.	Reciprocal property	$\forall a,b\in R$ and have same sign • $a<b\Rightarrow \frac{1}{a}>\frac{1}{b}$ • $a>b\Rightarrow \frac{1}{a}<\frac{1}{b}$

Example 5 $a = \frac{2}{3}, b = \frac{3}{2}, c = \frac{5}{3}$ **then verify the distributive properties over addition.**

Solution:

(i) Left distributive property

$a(b + c) = ab + ac$

L.H.S = a(b + c)	R.H.S = ab + ac
$=\frac{2}{3}\left(\frac{3}{2}+\frac{5}{3}\right)$	$=\left(\frac{2}{3}\right)\left(\frac{3}{2}\right)+\left(\frac{2}{3}\right)\left(\frac{5}{3}\right)$
$=\frac{2}{3}\left(\frac{9+10}{6}\right)$	$=1+\frac{10}{9}$
$=\frac{2}{3}\left(\frac{19}{6}\right)$	$=\frac{9+10}{9}$
$=\frac{19}{9}$	$=\frac{19}{9}$

L.H.S = R.H.S

Hence, it is verified that $a(b+c)=ab+ac$

(ii) Right distributive property

$(a + b)c = ac + bc$

L.H.S = (a + b)c	R.H.S = ac + bc
$=\left(\frac{2}{3}+\frac{3}{2}\right)\frac{5}{3}$	$=\left(\frac{2}{3}\right)\left(\frac{5}{3}\right)+\left(\frac{3}{2}\right)\left(\frac{5}{3}\right)$
$=\left(\frac{4+9}{6}\right)\frac{5}{3}$	$=\frac{10}{9}+\frac{15}{6}$
$=\left(\frac{13}{6}\right)\left(\frac{5}{3}\right)$	$=\frac{20+45}{18}$
$=\frac{65}{18}$	$=\frac{65}{18}$

L.H.S = R.H.S

Hence, it is verified that $(a+b)c=ac+bc$

Example 6 **Identify the property that justifies the statement.**

(i) **If $a > 13$ then $a + 2 > 15$**

(ii) **If $3 < 9$ and $6 < 12$ then $9 < 21$**

(iii) **If $7 > 4$ and $5 > 3$ then $35 > 12$**

(iv) **If $-5 < -4 \Rightarrow 20 > 16$**

Solution:

(i) If $a > 13$

Add 2 on both sides

$a + 2 > 13 + 2$

$a + 2 > 15$ (order property w.r.t addition)

(ii) As $3 < 9$ and $6 < 12$

$3 + 6 < 9 + 12$

$9 < 21$

(iii) $7 > 4$ and $5 > 3$ (order property w.r.t addition)

$\Rightarrow 7 \times 5 > 4 \times 3$

$\Rightarrow 35 > 12$ (order property w.r.t multiplication)

(iv) As $-5 < -4$

Multiply on both sides by -4

$(-5) \times (-4) > (-4) \times (-4)$

$\Rightarrow 20 > 16$ (order property w.r.t multiplication)

Solved Exercise 1.1

1. Identify each of the following as a rational or irrational number:

(i) **2.353535**

Sol. 2.353535 is a terminating decimal number, therefore it is a rational number.

(ii) **$0.\overline{6}$**

Sol. $0.\overline{6} = 0.6666\ldots$ is a recurring and non terminating decimal number, therefore it is a rational number.

(iii) **2.236067....**

Sol. 2.236067... is a non-terminating and non-recurring decimal number, therefore it is a an irrational number.

(iv) **$\sqrt{7}$**

Sol: $\sqrt{7}$ is a square root of 7. 7 is not a perfect square, therefore it is an irrational number.

(v) **e**

Sol. $e = 2.71828182845904\ldots$ is a non-terminating and non-recurring decimal number. Therefore it is an irrational number.

(vi) **π**

Sol. $\pi = 3.141592653589\ldots$ is a non-terminating and non recurring decimal number, therefore it is an irrational number.

(vii) **$5 + \sqrt{11}$**

Sol. 5 is a rational number and $\sqrt{11}$ is an irrational number. The sum of a rational number and irrational number is a irrational number, therefore $5+\sqrt{11}$ is an irrational number.

(viii) **$\sqrt{3} + \sqrt{13}$**

Sol. $\sqrt{3}$ and $\sqrt{13}$ are irrational numbers therefore, their sum $\sqrt{3}+\sqrt{13}$ is also a irrational number.

(ix) **$\frac{15}{4}$**

Sol: $\frac{15}{4}$ is in the form of $\frac{p}{q}$, therefore, it is a rational number.

(x) **$(2-\sqrt{2})(2+\sqrt{2})$**

Sol. $(2-\sqrt{2})$ and $(2+\sqrt{2})$ are conjugate surds of each other. The product of two conjugate surds is a rational number. Therefore $(2-\sqrt{2})(2+\sqrt{2})$ is a rational number.

2. Represent the following numbers on number line:

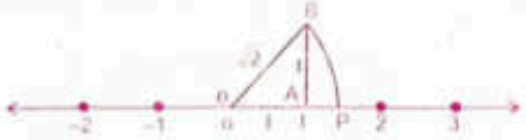
(i) **$\sqrt{2}$**

Sol. The point corresponding to $\sqrt{2}$ may be constructed by forming a right ΔOAB with both sides each of length 1.

By Pythagoras theorem

$m\overline{OB} = \sqrt{(1)^2 + (1)^2} = \sqrt{1+1} = \sqrt{2}$

By drawing an arc with centre at O and radius $m\overline{OB} = \sqrt{2}$, we get the point P represent $\sqrt{2}$, on the number line.



(ii) $\sqrt{3}$

Sol. The following steps will be taken to represent the $\sqrt{3}$ on the number line. First of all, the point corresponding to $\sqrt{2}$ may be constructed by forming a right triangle OAB with two sides each of length 1.

By Pythagoras theorem

$m\overline{OB} = \sqrt{(1)^2 + (1)^2} = \sqrt{1+1} = \sqrt{2}$

By drawing an arc with centre at 0 and radius $m\overline{OB} = \sqrt{2}$, we get the point C on the number line. Then we shall make another right triangle OEC with two sides of lengths $\sqrt{2}$ and 1.

By Pythagoras theorem

$m\overline{OE} = \sqrt{(\sqrt{2})^2 + (1)^2} = \sqrt{2+1} = \sqrt{3}$

By drawing an arc with centre O and radius $m\overline{OE} = \sqrt{3}$, we get the point P represent $\sqrt{3}$ on the number line.



(iii) $4\frac{1}{3}$

Sol. Divide the unit length between 4 and 5 into three equal parts and take the end of the first part from 4 to its right side. The point P represents the number $4\frac{1}{3}$ on the number line.



(iv) $-2\frac{1}{7}$

Sol. Divide the unit length between –2 and –3 into seven equal parts and take the end of the first part from –2 to its left. The point P is the point that represents $-2\frac{1}{7}$ on the

number line.



(v) $\frac{5}{8}$

Sol. Divide the unit length between 0 and 1 into eight equal parts and take th end of the fifth part from 0 to its right side. The point P represents the $\frac{5}{8}$ on number line.



(vi) $2\frac{3}{4}$

Sol. Divide the unit length between 2 and 3 into four equal parts and take the end of third part from 2 to its right side. The point P represents the $2\frac{3}{4}$ on number line.



3. Express the following as a rational number $\frac{p}{q}$ where p and q are integers and $q \neq 0$.

(i) $0.\overline{4}$

Sol. Let $x = 0.\overline{4}$

$x = 0.44444.....$ _____ (i)

Multiply both sides by 10

$10x = 10(0.44444.....)$

$10x = 4.4444....$ _____ (ii)

Subtracting eq. (i) from eq. (ii)

$10x - x = (4.4444...) - (0.44444....)$

$9x = 4$

$x = \frac{4}{9}$

(ii) $0.\overline{37}$

Sol. Let $x = 0.\overline{37}$

$x = 0.373737..... $ _____ (i)

Multiply both sides by 100

$100x = 100\ (0.373737.....)$

$100x = 37.373737..... $ _____ (ii)

Subtracting eq. (i) from eq (ii)

$100x - x = (37.373737...) - (0.373737.....)$

$99x = 37$

$x = \frac{37}{99}$

(iii) $0.\overline{21}$

Sol. Let $x = 0.\overline{21}$

$x = 0.212121..... $ _____ (i)

Multiplying by 100 on both sides

$100x = 100\ (0.212121.....)$

$100x = 21.212121..... $ _____ (ii)

Subtracting eq. (i) from eq. (ii)

$100x - x = (21.212121....) - (0.212121....)$

$99x = 21$

$x = \frac{21}{99}$

4. Name the property used in the following:

(i) $(a+4) + b = a+(4+b)$

Sol. Associative property w.r.t. addition

(ii) $\sqrt{2}+\sqrt{3} = \sqrt{3}+\sqrt{2}$

Sol. Commutative property w.r.t addition.

(iii) $x-x = 0$

Sol. Additive inverse

(iv) $a(b+c) = ab + ac$

Sol. Left distributive property of multiplication over addition

(v) $16+0 = 16$

Sol. Additive identity.

(vi) $100\times1 = 100$

Sol. Multiplicative identity

(vii) $4\times(5\times8) = (4\times5)\times8$

Sol. Associative property w.r.t multiplication.

(viii) $ab = ba$

Sol. Commutative property w.r.t multiplication.

5. Name the property used in the following:

(i) $-3 < -1 \Rightarrow 0 < 2$

Sol. Additive property of inequality.

(ii) if $a < b$ then $\frac{1}{a} > \frac{1}{b}$

Sol. Reciprocal property of inequality.

(iii) If $a < b$ then $a + c < b + c$.

Sol. Additive propetry of inequality.

(iv) If $ac < bc$ and $c > 0$ then $a < b$.

Sol. Multiplicative property of inequality.

(v) If $ac < bc$ and $c < o$ then $a > b$

Sol. Multiplicative property of inequality.

(vi) Either $a > b$ or $a = b$ or $a < b$

Sol. Trichotomy property.

6. Insert two rational numbers between:

(i) $\frac{1}{3}$ and $\frac{1}{4}$

Sol. The average of any two rational numbers will always be a rational number between them.

So, $\frac{\frac{1}{3}+\frac{1}{4}}{2} = \frac{\frac{4+3}{12}}{2} = \frac{\frac{7}{12}}{2} = \frac{7}{12}\times\frac{1}{2} = \frac{7}{24}$

Thus, $\frac{7}{24}$ is one of the required numbers.

Now we shall find average of $\frac{7}{24}$ and $\frac{1}{4}$

$= \frac{\frac{7}{24}+\frac{1}{4}}{2} = \frac{\frac{7+6}{24}}{2} = \frac{\frac{13}{24}}{2} = \frac{13}{24}\times\frac{1}{2} = \frac{13}{48}$

Thus, $\frac{13}{48}$ is the other required number.

Hence, two rational numbers between $\frac{1}{3}$ and $\frac{1}{4}$ are $\frac{7}{24}$ and $\frac{13}{48}$

(ii) 3 and 4

Sol. The average of any two rational numbers will always be a rational number between them.

So, $\frac{3+4}{2} = \frac{7}{2}$

Thus, $\frac{7}{2}$ is one rational number between 3 and 4.

Now, we shall find the average of $\frac{7}{2}$ and 4.

$\frac{\frac{7}{2}+4}{2} = \frac{7+8}{2} = \frac{15}{2}$

Thus, $\frac{15}{2}$ is the other required number.

Hence, two rational numbers between 3 and 4 are $\frac{7}{2}$ and $\frac{15}{2}$.

(iii) $\frac{3}{5}$ **and** $\frac{4}{5}$

Sol. The average of any two rational numbers will always be a rational number between them.

So,

$\frac{\frac{3}{5}+\frac{4}{5}}{2} = \frac{\frac{3+4}{5}}{2} = \frac{\frac{7}{5}}{2} = \frac{7}{5}\times\frac{1}{2} = \frac{7}{10}$

Thus, $\frac{7}{10}$ is one rational number between $\frac{3}{5}$ and $\frac{4}{5}$.

Now, we shall find the average of $\frac{7}{10}$ and $\frac{4}{5}$.

$\frac{\frac{7}{10}+\frac{4}{5}}{2} = \frac{\frac{7+8}{10}}{2} = \frac{\frac{15}{10}}{2} = \frac{15}{10}\times\frac{1}{2} = \frac{15}{20}$

Hence, two rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$ are $\frac{7}{10}$ and $\frac{15}{20}$

Radical Expressions

If n is a positive integer greater than 1 and a is a real number, then any real number x such that $x = \sqrt[n]{a}$ is called n^{th} root of a.

Here $\sqrt{\ }$ is called radical and n is the index of radical. A real number under the radical sign is called a radicand. $\sqrt[3]{5}$, $\sqrt[3]{7}$ are examples of radical form.

Exponential form of $x = \sqrt[n]{a}$ is $x = (a)^{\frac{1}{n}}$.

Laws of Radical sand Indices

Laws of Radical	Laws of Indices
(i) $\sqrt[n]{ab} = \sqrt[n]{a}.\sqrt[n]{b}$	(i) $a^m.a^n = a^{m+n}$ (ii) $(a^m)^n = a^{mn}$
(ii) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$	(iii) $(ab)^n = a^n b^n$ (iv) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
(iii) $\sqrt[n]{a^m} = (\sqrt[n]{a})^m$	(v) $\frac{a^m}{a^n} = a^{m-n}$
(iv) $(\sqrt[n]{a})^n = (a^{\frac{1}{n}})^n = a$	(vi) $a^0 = 1$

Example 7 Simplify the following:

(i) $\sqrt[4]{16x^4y^8}$ (ii) $\sqrt[3]{27x^6y^9z^3}$ (iii) $(64)^{-\frac{4}{3}}$

Solution:

(i) $\sqrt[4]{16x^4y^8}$

$= (16x^4y^8)^{\frac{1}{4}}$ $\because (\sqrt[n]{a} = a^{\frac{1}{n}})$

$= (16)^{\frac{1}{4}}(x^4)^{\frac{1}{4}}(y^8)^{\frac{1}{4}}$ $\because (ab)^m = a^m b^m$

$= 2^{4\times\frac{1}{4}} \times x^{4\times\frac{1}{4}} \times y^{8\times\frac{1}{4}}$ $\because (a^m)^n = a^{mn}$

$= 2xy^2$

(ii) $\sqrt[3]{27x^6y^9z^3}$

$= (27x^6y^9z^3)^{\frac{1}{3}}$ $\because \sqrt[n]{a} = a^{\frac{1}{n}}$

$= (27)^{\frac{1}{3}}(x^6)^{\frac{1}{3}}(y^9)^{\frac{1}{3}}(z^3)^{\frac{1}{3}}$ $\because (ab)^m = a^m b^m$

$= (3^3)^{\frac{1}{3}}(x^6)^{\frac{1}{3}}(y^9)^{\frac{1}{3}}(z^3)^{\frac{1}{3}}$

$= 3^{3\times\frac{1}{3}} \times x^{6\times\frac{1}{3}} \times y^{9\times\frac{1}{3}} . z^{3\times\frac{1}{3}}$ $\because (a^m)^n = a^{mn}$

$= 3x^2y^3z$

(iii) $(64)^{-\frac{4}{3}}$

$= \frac{1}{(64)^{\frac{4}{3}}} = \frac{1}{4^{3\times\frac{4}{3}}}$

$= \frac{1}{4^4} = \frac{1}{256}$

Surds

An irrational radical with rational radicand is called a surd.

For example, if we take the n^{th} root of any rational number a then $\sqrt[n]{a}$ is a surd. $\sqrt{5}$ is a surd because the square root of 5 does not give a whole number but $\sqrt{9}$ is not a surd because it simplifies to a whole number 3 and our result is not an irrational number. Therefore, the radical $\sqrt[n]{a}$ is irrational $\sqrt{7}, \sqrt{2}, \sqrt[3]{11}$ are surds but $\sqrt{\pi}, \sqrt{e}$ are not surds.

Remember!
Every surd is an irrational number but every irrational number is not a surd e.g., $\sqrt{\pi}$ is not a surd.

(i) A surd that contains a single term is called a monomial e.g., $\sqrt{5}, \sqrt{7}$

(ii) A surd that contains the sum of two monomial surds is called a binomial surd e.g., $\sqrt{3}+\sqrt{5}, \sqrt{2}+\sqrt{7}$ etc.

(iii) $\sqrt{a}+\sqrt{b}$ and $\sqrt{a}-\sqrt{b}$ are called conjugate surds of each other.

Remember
The product of two conjugate surds is a rational number.

Rationalization of Denominator

To rationalize a denominator of the form $a + b\sqrt{x}$ or $a - b\sqrt{x}$, we multiply both the numerator and denominator by the conjugate factor.

Example 8: Rationalize the denominator of (i) $\frac{3}{\sqrt{5}+\sqrt{2}}$ **(ii)** $\frac{3}{\sqrt{5}-\sqrt{3}}$

Solution:

(i) $\frac{3}{\sqrt{5}+\sqrt{2}} = \frac{3}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

$= \frac{3(\sqrt{5}-\sqrt{2})}{(\sqrt{5})^2-(\sqrt{2})^2} = \frac{3(\sqrt{5}-\sqrt{2})}{5-2}$

$= \frac{3(\sqrt{5}-\sqrt{2})}{3} = \sqrt{5}-\sqrt{2}$

(ii) $\frac{3}{\sqrt{5}-\sqrt{3}} = \frac{3}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$= \frac{3(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2} = \frac{3(\sqrt{5}+\sqrt{3})}{5-3}$

$= \frac{3(\sqrt{5}+\sqrt{3})}{2}$

Solved Exercise 1.2

1. Rationalize the denominator of following:

(i) $\frac{13}{4+\sqrt{3}}$

Sol. $\frac{13}{4+\sqrt{3}} = \frac{13}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}}$

$= \frac{13(4-\sqrt{3})}{(4)^2-(\sqrt{3})^2}$

$= \frac{13(4-\sqrt{3})}{16-3} = \frac{13(4-\sqrt{3})}{13} = \frac{\cancel{13}^{1}(4-\sqrt{3})}{\cancel{13}^{1}} = 4-\sqrt{3}$

(ii) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$

Sol. $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}} = \frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{3}(\sqrt{2}+\sqrt{5})}{(\sqrt{3})^2}$

$= \frac{\sqrt{3}(\sqrt{2}+\sqrt{5})}{3} = \frac{\sqrt{6}+\sqrt{15}}{3}$

(iii) $\frac{\sqrt{2}-1}{\sqrt{5}}$

Sol. $\frac{\sqrt{2}-1}{\sqrt{5}}$

$= \frac{\sqrt{2}-1}{5} \times \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{\sqrt{5}(\sqrt{2}-1)}{(\sqrt{5})^2}$

$= \frac{\sqrt{5}(\sqrt{2}-1)}{5} = \frac{\sqrt{10}-\sqrt{5}}{5}$

(v) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

Sol. $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$= \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$= \frac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2}$

$= \frac{(\sqrt{3})^2+(\sqrt{2})^2-2(\sqrt{3})(\sqrt{2})}{3-2}$

$= \frac{3+2-2\sqrt{3\times 2}}{1}$

$= 5-2\sqrt{6}$

(iv) $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$

Sol. $\frac{6-4\sqrt{2}}{6+4\sqrt{2}} = \frac{6-4\sqrt{2}}{6+4\sqrt{2}} \times \frac{6-4\sqrt{2}}{6-4\sqrt{2}}$

$= \frac{(6-4\sqrt{2})^2}{(6)^2-(4\sqrt{2})^2} = \frac{(6)^2+(4\sqrt{2})^2-2(6)(4\sqrt{2})}{36-(16\times 2)}$

$= \frac{36+(16\times 2)-12(4\sqrt{2})}{36-32}$

$= \frac{36+32-48\sqrt{2}}{4} = \frac{68-48\sqrt{2}}{4}$

$= \frac{4(17-12\sqrt{2})}{4} = 17-12\sqrt{2}$

(vi) $\frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}}$

Sol. $\frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}} = \frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}}$

$= \frac{4\sqrt{3}(\sqrt{7}-\sqrt{5})}{(\sqrt{7})^2-(\sqrt{5})^2} = \frac{4\sqrt{3}(\sqrt{7}-\sqrt{5})}{7-5}$

$= \frac{\cancel{2}^{1}\cancel{4}\sqrt{3}(\sqrt{7}-\sqrt{5})}{\cancel{2}_{1}}$

$= 2\sqrt{3}(\sqrt{7}-\sqrt{5})$

2. Simplify the following.

(i) $\left(\frac{81}{16}\right)^{-\frac{3}{4}}$

Sol. $\left(\frac{81}{16}\right)^{-\frac{3}{4}} = \left(\frac{16}{81}\right)^{\frac{3}{4}}$

$= \left(\frac{2\times 2\times 2\times 2}{3\times 3\times 3\times 3}\right)^{\frac{3}{4}} = \left(\frac{2^4}{3^4}\right)^{\frac{3}{4}}$

$= \frac{(2^4)^{\frac{3}{4}}}{(3^4)^{\frac{3}{4}}} = \frac{2^{4\times\frac{3}{4}}}{3^{4\times\frac{3}{4}}}$

$= \frac{2^3}{3^3} = \frac{8}{27}$

(ii) $\left(\frac{3}{4}\right)^{-2} - \left(\frac{4}{9}\right)^{\frac{1}{2}} \times \frac{16}{27}$

Sol. $\left(\frac{3}{4}\right)^{-2} - \left(\frac{4}{9}\right)^{\frac{1}{2}} \times \frac{16}{27}$

$= \left(\frac{4}{3}\right)^{2} + \left(\frac{4}{9}\right)^{\frac{1}{2}} \times \frac{2\times 2\times 2\times 2}{3\times 3\times 3} = \frac{4^2}{3^2} + \frac{4^{\frac{1}{2}}}{9^{\frac{1}{2}}} \times \frac{2^4}{3^3}$

$= \frac{(2\times 2)^2}{3^2} + \frac{(2\times 2)^{\frac{1}{2}}}{(3\times 3)^{\frac{1}{2}}} \times \frac{2^4}{3^3} = \frac{(2^2)^2}{3^2} + \frac{(2^2)^{\frac{1}{2}}}{(3^2)^{\frac{1}{2}}} \times \frac{2^4}{3^3}$

$= \frac{2^4}{3^2} + \frac{2^2}{3^2} \times \frac{2^4}{3^3} = \frac{2^4}{3^2} + \frac{2^2}{2^3} \times \frac{2^4}{3^3}$

$= 2^{4-4+2} \times 3^{-2-3} = 2^{2-3} \times 3^{-5}$

$= 2^{2}\times 3^{1} = 4\times 3 = 12$

(iii) $(0.027)^{\frac{1}{3}}$

Sol. $(0.027)^{\frac{1}{3}}$

$= \left(\frac{27}{1000}\right)^{\frac{1}{3}}$

$= \left(\frac{3\times 3\times 3}{10\times 10\times 10}\right)^{\frac{1}{3}} = \left(\frac{3^3}{10^3}\right)^{\frac{1}{3}}$

$= \frac{(3^3)^{\frac{1}{3}}}{(10^3)^{\frac{1}{3}}} = \frac{3^{3\times\frac{1}{3}}}{10^{3\times\frac{1}{3}}} = \frac{3^1}{10^1} = \frac{10}{3}$

(iv) $7\sqrt{\frac{x^{14}\times y^{21}\times z^{35}}{y^{14}\times z^{7}}}$

Sol. $7\sqrt{\frac{x^{14}\times y^{21}\times z^{35}}{y^{14}\times z^{7}}} = \left(\frac{x^{14}\times y^{21}\times z^{35}}{y^{14}\times z^{7}}\right)^{\frac{1}{7}}$

$= (x^{14}\times y^{21-14}\times z^{35-7})^{\frac{1}{7}}$

$= (x^{14}\times y^{7}\times z^{28})^{\frac{1}{7}}$

$= x^{14\times\frac{1}{7}} \times y^{7\times\frac{1}{7}} \times z^{28\times\frac{1}{7}}$

$= x^2 \times y^1 \times z^4 = x^2yz^4$

$= x^2yz^4$

(v) $\frac{5.(25)^{n+1} - 25.(5)^{2n}}{5.(5)^{2n+3} - (25)^{n+1}}$

Sol. $\frac{5.(25)^{n+1}-25.(5)^{2n}}{5.(5)^{2n+3}-(25)^{n+1}} = \frac{5.(5\times 5)^{n+1} - (5\times 5).(5)^{2n}}{5.(5)^{2n+3+1} - (5\times 5)^{n+1}}$

$= \frac{5.(5^2)^{n+1} - 5^2.(5)^{2n}}{5^{2n+3+1} - (5^2)^{n+1}} = \frac{5.5^{2n+2} - 5^{2n+2}}{5^{2n+4} - 5^{2n+2}}$

$= \frac{5.^{2n+2}(5-1)}{5^{2n+2}(5^2-1)}$

$= \frac{5-1}{5^2-1} = \frac{4}{25-1} = \frac{4}{24} = \frac{1}{6}$

(vi) $\dfrac{(16)^{n+1}+20(4^{2n})}{2^{n-3}\times 8^{n+2}}$

Sol. $\dfrac{(16)^{n+1}+20(4^{2n})}{2^{n-3}\times 8^{n+2}}$

$=\dfrac{(2\times2\times2\times2)^{n+1}+20(2\times2)^{2n}}{2^{n-3}\times(2\times2\times2)^{n+2}}$

$=\dfrac{(2^4)^{n+1}+20(2^2)^{2n}}{2^{n-3}\times(2^3)^{n+2}}=\dfrac{2^{4n+4}+20(2)^{4n}}{2^{n-3}\times2^{3n+6}}$

$=\dfrac{2^{4n+4}+20(2)^{4n}}{2^{n-3+3n+6}}=\dfrac{2^{4n+4}+20(2)^{4n}}{2^{4n+3}}$

$=\dfrac{2^{4n}\times2^4+20(2)^{4n}}{2^{4n}\times2^3}=\dfrac{2^{4n}(2^4+20)}{2^{4n}\times2^3}$

$=\dfrac{16+20}{8}=\dfrac{36}{8}=\dfrac{9}{2}$

(viii) $\dfrac{3^n\times9^{n+1}}{3^{n-1}\times9^{n-1}}$

Sol. $\dfrac{3^n\times9^{n+1}}{3^{n-1}\times9^{n-1}}=\dfrac{3^n\times(3\times3)^{n+1}}{3^{n-1}\times(3\times3)^{n-1}}=\dfrac{3^n\times(3^2)^{n+1}}{3^{n-1}\times(3^2)^{n-1}}$

$=\dfrac{3^n\times3^{2n+2}}{3^{n-1}\times3^{2n-2}}=\dfrac{3^{n+2n+2}}{3^{n-1+2n-2}}=\dfrac{3^{3n+2}}{3^{3n-3}}$

$=3^{3n+2-3n+3}=3^5=243$

(vii) $(64)^{\frac{2}{3}}-(9)^{-\frac{3}{2}}$

Sol. $(64)^{\frac{2}{3}}-(9)^{-\frac{3}{2}}$

$=(4\times4\times4)^{\frac{2}{3}}\div(3\times3)^{\frac{3}{2}}$

$=(4^3)^{\frac{2}{3}}\div(3^2)^{\frac{3}{2}}$

$=(4)^{3\times\frac{2}{3}}\div(3)^{2\times\frac{3}{2}}=(4)^2\div(3)^3$

$=\dfrac{(4)^2}{(3)^3}=\dfrac{(3)^3}{(4)^2}=\dfrac{27}{16}$

(ix) $\dfrac{5^{n+3}-6.5^{n+1}}{9\times5^n-2^2\times5^n}$

Sol. $\dfrac{5^{n+3}-6.5^{n+1}}{9\times5^n-2^2\times5^n}=\dfrac{5^n\times5^3-6.5.5^n}{9\times5^n-4\times5^n}$

$=\dfrac{5^n(5^3-6\times5)}{5^n(9-4)}=\dfrac{125-30}{9-4}$

$=\dfrac{95}{9-4}=\dfrac{95}{5}=19$

This question in the book is not correct.

3. If $x=3+\sqrt{8}$ then find the value of:

(i) $x+\dfrac{1}{x}$

Sol. $x=3+\sqrt{8}$

$\dfrac{1}{x}=\dfrac{1}{3+\sqrt{8}}$

$\dfrac{1}{x}=\dfrac{1}{3+\sqrt{8}}\times\dfrac{3-\sqrt{8}}{3-\sqrt{8}}$

$\dfrac{1}{x}=\dfrac{3-\sqrt{8}}{(3)^2-(\sqrt{8})^2}=\dfrac{3-\sqrt{8}}{9-8}=\dfrac{3-\sqrt{8}}{1}$

$\dfrac{1}{x}=3-\sqrt{8}$

$x+\dfrac{1}{x}=3+\sqrt{8}+3-\sqrt{8}\Rightarrow x+\dfrac{1}{x}=6$

(ii) $x-\dfrac{1}{x}$

Sol. $x-\dfrac{1}{x}=(3+\sqrt{8})-(3-\sqrt{8})$

$=3+\sqrt{8}-3+\sqrt{8}=2\sqrt{8}$

(iv) $x^2+\dfrac{1}{x^2}$

Sol. $x+\dfrac{1}{x}=6$

$x-\dfrac{1}{x}=2\sqrt{8}$

$x^2-\dfrac{1}{x^2}=\left(x+\dfrac{1}{x}\right)\left(x-\dfrac{1}{x}\right)$

$x^2-\dfrac{1}{x^2}=6(2\sqrt{8})\Rightarrow x^4-\dfrac{1}{x^2}=12\sqrt{8}$

(v) $x^4+\dfrac{1}{x^4}$

Sol. $x+\dfrac{1}{x}=6$

$\left(x+\dfrac{1}{x}\right)^2=(6)^2$

$(x)^2+2(x)\left(\dfrac{1}{x}\right)+\left(\dfrac{1}{x}\right)^2=36$

$x^2+2+\dfrac{1}{x^2}=36$

$x^2+\dfrac{1}{x^2}=36-2$

$x^2+\dfrac{1}{x^2}=34$

(vi) $\left(x-\dfrac{1}{x}\right)^2$

Sol. $x-\dfrac{1}{x}=2\sqrt{8}\Rightarrow\left(x-\dfrac{1}{x}\right)^2=(2\sqrt{8})^2$

$\left(x-\dfrac{1}{x}\right)^2=4\times8\Rightarrow\left(x-\dfrac{1}{x}\right)^2=32$

(iii) $x^2+\dfrac{1}{x^2}$

Sol. $x+\dfrac{1}{x}=6$

$\left(x+\dfrac{1}{x}\right)^2=(6)^2$

$(x)^2+2(x)\left(\dfrac{1}{x}\right)+\left(\dfrac{1}{x}\right)^2=36$

$x^2+2+\dfrac{1}{x^2}=36$

$x^2+\dfrac{1}{x^2}=36-2$

$x^2+\dfrac{1}{x^2}=34$

By taking square again

$\left(x^2+\dfrac{1}{x^2}\right)^2=(34)^2$

$\left(x^2\right)^2+2(x^2)\left(\dfrac{1}{x^2}\right)+\left(\dfrac{1}{x^2}\right)^2=1156$

$x^4+2+\dfrac{1}{x^4}=1156$

$x^4+\dfrac{1}{x^4}=1156-2$

$x^4+\dfrac{1}{x^4}=1154$

4. Find the rational numbers p and q such that $\frac{8-3\sqrt{2}}{4+3\sqrt{2}} = p+q\sqrt{2}$.

Sol. $\frac{8-3\sqrt{2}}{4+3\sqrt{2}} = p-q\sqrt{2}$

$$\frac{8-3\sqrt{2}}{4+3\sqrt{2}} \times \frac{4-3\sqrt{2}}{4-3\sqrt{2}} = p+q\sqrt{2}$$

$$\frac{8(4-3\sqrt{2})-3\sqrt{2}(4-3\sqrt{2})}{(4)^2-(3\sqrt{2})^2} = p+q\sqrt{2}$$

$$\frac{32-24\sqrt{2}-12\sqrt{2}+9(\sqrt{2})^2}{16-(9\times 2)} = p+q\sqrt{2}$$

$$\frac{32-36\sqrt{2}+9(2)}{16-18} = p+q\sqrt{2}$$

$$\frac{32-36\sqrt{2}+18}{-2} = p+q\sqrt{2}$$

$$\frac{50-36\sqrt{2}}{-2} = p+q\sqrt{2}$$

$$\frac{2(25-18\sqrt{2})}{-2} = p+q\sqrt{2}$$

$$\frac{25-18\sqrt{2}}{-1} = p+q\sqrt{2}$$

$$\frac{-(-25+18\sqrt{2})}{-1} = p+q\sqrt{2}$$

$$-25+18\sqrt{2} = p+q\sqrt{2}$$

By compairing of two sides

$P = -25, q = 18$

5. Simplify the following.

(i) $\frac{(25)^{\frac{3}{2}} \times (243)^{\frac{5}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}}$

Sol. $\frac{(25)^{\frac{3}{2}} \times (243)^{\frac{5}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}}$

$$= \frac{(5\times 5)^{\frac{3}{2}} \times (3\times 3\times 3\times 3\times 3)^{\frac{5}{5}}}{(2\times 2\times 2\times 2\times 2)^{\frac{5}{4}} \times (2\times 2\times 2)^{\frac{4}{3}}}$$

$$= \frac{(5^2)^{\frac{3}{2}} \times (3^5)^{\frac{1}{5}}}{(2^4)^{\frac{5}{4}} \times (2^3)^{\frac{4}{3}}} = \frac{5^{2\times\frac{3}{2}} \times 3^{5\times\frac{1}{5}}}{2^{4\times\frac{5}{4}} \times 2^{3\times\frac{4}{3}}}$$

$$= \frac{5^3 \times 3^1}{2^5 \times 2^4} = \frac{125\times 27}{32\times 16} = \frac{3375}{512}$$

(ii) $\frac{54\times\sqrt[3]{(27)^{2x}}}{9^{x+1}+216(3^{2x-1})}$

Sol. $\frac{54\times\sqrt[3]{(27)^{2x}}}{9^{x+1}+216(3^{2x-1})}$

$$= \frac{2\times 3\times 3\times 3\times\sqrt[3]{(3\times 3\times 3)^{2x}}}{(3\times 3)^{x+1}+2\times 2\times 2\times 3\times 3\times 3(3^{2x-1})}$$

$$= \frac{2\times 3^3\times\sqrt[3]{(3^3)^{2x}}}{(3^2)^{x+1}+2^3\times 3^1(3^{2x-1})}$$

$$= \frac{2\times 3^3\times(3^{2x})^{\frac{1}{3}}}{3^{2x+2}+2^3\times 3^{2x-1+1}}$$

$$= \frac{2\times 3^3\times(3)^{2x\times\frac{1}{3}}}{3^{2x+2}+2^3\times 3^{2x+2}}$$

$$= \frac{2\times 3^3\times 3^{2x}}{3^{2x+2}(1+2^3)}$$

$$= \frac{2\times 3\times 3^2\times 3^{2x}}{3^{2x+2}(1+8)}$$

$$= \frac{2\times 3\times 3^{2x+2}}{3^{2x+2}(9)} = \frac{6}{9^1} = \frac{2}{3}$$

(iii) $\sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{\frac{1}{2}}}}$

Sol. $\sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{\frac{1}{2}}}}$

$$= \sqrt{\frac{(2\times 2\times 2\times 3\times 3\times 3)^{\frac{2}{3}} \times (5\times 5)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{\frac{1}{2}}}}$$

$$= \sqrt{\frac{(2^3\times 3^3)^{\frac{2}{3}} \times (5^2)^{\frac{1}{2}}}{\left(\frac{100}{4}\right)^{\frac{1}{2}}}}$$

$$= \sqrt{\frac{2^{3\times\frac{2}{3}} \times 3^{3\times\frac{2}{3}} \times 5^{2\times\frac{1}{2}}}{(25)^{\frac{1}{2}}}} = \sqrt{\frac{2^2\times 3^2\times 5}{(5^2)^{\frac{1}{2}}}}$$

$$= \sqrt{\frac{2^2\times 3^2\times 5}{5^{2\times\frac{1}{2}}}} = \sqrt{\frac{2^2\times 3^2\times 5}{5^1}}$$

$$= \sqrt{\frac{2^2\times 3^2}{5^{1-1}}} = \sqrt{\frac{2^2\times 3^2}{5^0}}$$

$$= \frac{2^{2\times\frac{1}{2}} \times 3^{2\times\frac{1}{2}}}{5^{2\times\frac{1}{2}}} = \frac{2\times 3}{5} = \frac{6}{5}$$

(iv) $\left(a^{\frac{1}{3}}+b^{\frac{2}{3}}\right)\times\left(a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{2}{3}}+b^{\frac{4}{3}}\right)$

Sol. $\left(a^{\frac{1}{3}}+b^{\frac{2}{3}}\right)\times\left(a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{2}{3}}+b^{\frac{4}{3}}\right)$

$$= \left(a^{\frac{1}{3}}+b^{\frac{2}{3}}\right)\times\left[\left(a^{\frac{1}{3}}\right)^2-\left(a^{\frac{1}{3}}\right)\left(b^{\frac{2}{3}}\right)+\left(b^{\frac{2}{3}}\right)^2\right]$$

$$= \left(a^{\frac{1}{3}}\right)^3+\left(b^{\frac{2}{3}}\right)^3$$

$$= a^{\frac{1}{3}\times 3}+b^{\frac{2}{3}\times 3}$$

$$= a+b^2$$

Applications of Real Numbers in Daily Life.

Real numbers are extremely useful in our daily life. That is probably one of the main reasons we learn how to count, add and subtract from a very young age. We cannot imagine life without numbers.

Real numbers are used various field including

- Medicine and Health
- Environmental science (climate modding, pollution monitoring etc.)
- Computer science (algorithm design, data compression, graphic rendering)
- Navigation and transportation (GPS, flight planning)
- Surveying and architecture
- Statistics and data

Example 9: The sum of two real numbers is 8, and their difference is 2. Find the numbers.

Solution: Let a and b be two real numbers then

$a + b = 8$ ...(i)

$a - b = 2$ ...(ii)

Add (i) and (ii)

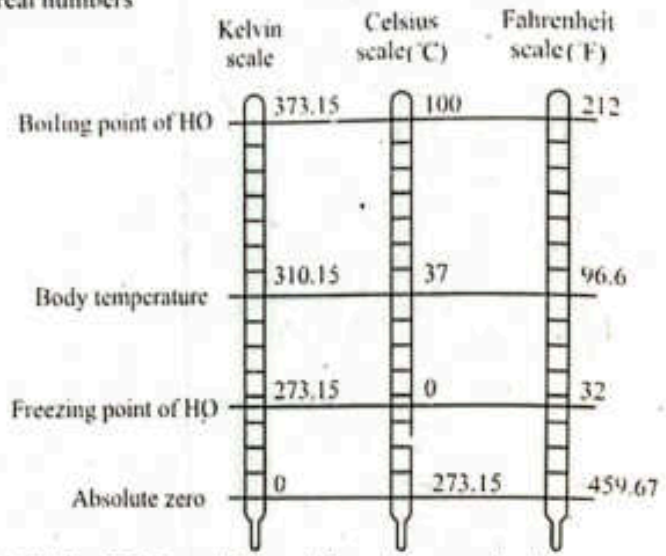
$2a = 10 \Rightarrow a = 5$ put in (ii)

$\Rightarrow 5 - b = 2 \Rightarrow -b = 2 - 5 \Rightarrow -b = -3 \Rightarrow b = 3$

So, 5 and 3 are the required real numbers

Temperature Conversions

In the figure three types of thermometers are shown. We can convert three temperature scales Celsius, Fahrenheit and Kelvin with each other. Conversion formulae are given below:



(i) $K = °C + 273$

(ii) $°C = \frac{5}{9}(F - 32)$

(iii) $F = \frac{9°C}{5} + 32$

Where K, °C and °F show the Kelvin, Celsius and Fahrenheit scales respectively.

Example 10:Normal human body temperature is 98.6 °F. Convert it into Celsius and Kelvin scale.

Solution: Given that °F = 98.6

So Convert it into Celsius scale, we use

$$°C = \frac{5}{9}(F - 32)°$$

$$°C = \frac{5}{9}(98.6 - 32)$$

$$= \frac{5}{9}(66.6)$$

$$= (0.55)(66.6)$$

$$°C = 37°$$

Hence, normal human body temperature at Celsius scale is 37°.

Now, we convert it into Kelvin scale.

$K = C + 273°$

$K = 37° + 273°$

$K = 310\,\text{kelvin}$

Profit and Loss

The traders may earn profit or incur losses.Profit and loss are a part of business.Profit and loss can be calculated by the following formula.

(i) **Profit** = selling price – cost price

P = SP – CP

Profit % $= \left(\frac{profit}{CP} \times 100\right)\%$

(ii) **Loss** = cost price – selling price

Loss = CP – SP

Loss % $= \left(\frac{loss}{CP} \times 100\right)\%$

Example 11: Hamail purchased a bicycle for Rs. 6590 and sold it for Rs. 6850. Find the profit percentage.

Solution:

Cost Price = CP = Rs. 6590

Selling Price = SP = Rs. 6850

Profit = SP – CP = 6850 – 6590

= 6850 – 6590

= Rs 260

Now, we find the profit percentage.

$$\text{Profit \%} = \left(\frac{profit}{CP} \times 100\right)\%$$

$$= \left(\frac{260 \times 100}{6590}\right)\%$$

$$= 3.94\%$$

$$\approx 4\%$$

Example 12: Umair bought a book for Rs. 850 and sold it for Rs.720. What was his loss percentage?

Solution:

Cost price of book = CP = Rs. 850

Selling price of book = SP = Rs. 720

Loss = CP – SP

= 850 – 720 = Rs. 130

= Rs. 130

Loss percentage $= \left(\frac{Loss}{CP} \times 100\right)\%$

$$= \left(\frac{130}{850} \times 100\right)\%$$

$$= 15.29\%$$

Example 13: Saleem, Nadeem, and Tanveer earned a profit of Rs. 4,50,000 from a business. If their investments in the business are in the ratio 4: 7: 14, find each person's profit.

Solution: Profit earned = Rs. 4,50,000
Given ratio = 4: 7: 14
Sum of ratios = 4 + 7 + 14 = 25

$$\text{Saleem earned profit} = \frac{4}{25} \times 4,50,000$$
$$= \text{Rs. } 72,000$$

$$\text{Nadeem earned profit} = \frac{7}{25} \times 4,50,000$$
$$= \text{Rs. } 126,000$$

$$\text{Tanveer earned profit} = \frac{14}{25} \times 4,50,000$$
$$= \text{Rs. } 252,000$$

Example 14: If the simple profit on Rs. 6400 for 12 years is Rs. 3840. Find the rate of profit.

Solution: Principal = Rs. 6400
Simple profit = Rs. 3840
Time = 12 years
To find the rate we use the following formula:

$$\text{Rate} = \frac{\text{amount of profit} \times 100}{\text{time} \times \text{principal}}$$
$$= \frac{3840 \times 100}{12 \times 6400}$$
$$= 5\%$$

Thus, rate of profit is 5%

Solved Exercise 1.3

1. The sum of three consecutive integers is forty-two, find three integers.

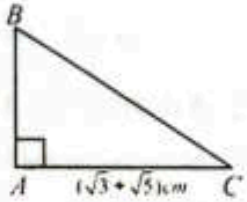
Sol. Let the first integer = x
Second integer = x + 1
Third integer = x + 2
According to the given condition

$$x + x + 1 + x + 2 = 42$$
$$3x + 3 = 42$$
$$3x = 42 - 3$$
$$3x = 39$$
$$x = \frac{39}{3}$$

$$x = 13$$

So, the first integer = 13
Second integer = 13 + 1 = 14
Thrid integer = 13 + 2 = 15

2. The diagram shows right angled ΔABC in which the length of $\overline{AC}$ is $(\sqrt{3}+\sqrt{5})$cm. The area of ΔABC is $(1+\sqrt{15})$cm². Find the length $\overline{AB}$ in the form $(a\sqrt{3}+b\sqrt{5})$cm, where a and b are integers.



Sol. $m\overline{AC} = (\sqrt{3}+\sqrt{5})$ cm

Area of ΔABC = $(1+\sqrt{15})$ cm²
$m\overline{AB} = ?$

$$\text{Area of } \Delta ABC = \frac{1}{2}\, m\overline{AC} \times m\overline{AB}$$
$$(1+\sqrt{15})\,cm^2 = \frac{1}{2} \times (\sqrt{3}+\sqrt{5})\,cm \times m\overline{AB}$$
$$\frac{2\times(1+\sqrt{15})cm^2}{(\sqrt{3}+\sqrt{5})cm} = m\overline{AB}$$
$$\frac{2+2\sqrt{15}}{\sqrt{3}+\sqrt{5}}\ cm = m\overline{AB}$$
$$m(\overline{AB}) = \frac{2+2\sqrt{15}}{\sqrt{3}+\sqrt{5}} \times \frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}-\sqrt{5}}$$
$$= \frac{2(\sqrt{3}-\sqrt{5})+2\sqrt{15}(\sqrt{3}-\sqrt{5})}{(\sqrt{3})^2-(\sqrt{5})^2}$$
$$= \frac{2\sqrt{3}-2\sqrt{5}+2\sqrt{15\times3}-2\sqrt{15\times5}}{3-5}$$
$$= \frac{2\sqrt{3}-2\sqrt{5}+2\sqrt{5\times3\times3}-2\sqrt{3\times5\times5}}{-2}$$
$$= \frac{2\sqrt{3}-2\sqrt{5}+2\times3\sqrt{5}-2\times5\sqrt{3}}{-2}$$
$$= \frac{2\sqrt{3}-2\sqrt{5}+6\sqrt{5}-10\sqrt{3}}{-2}$$
$$= \frac{-8\sqrt{3}+4\sqrt{5}}{-2}$$

$$= \frac{-2(4\sqrt{3} - 2\sqrt{5})}{-2}$$

$$= 4\sqrt{3} - 2\sqrt{5}$$

$$m\overline{AB} = (4\sqrt{3} - 2\sqrt{5})\text{ km}$$

3. A rectangle has sides of length $(2+\sqrt{18})$ m and $(5-\frac{4}{\sqrt{2}})$ m. Express the area of the rectangle in the form $a+b\sqrt{2}$ where a and b are integers.

Sol. Length of one side of rectangle = $(2+\sqrt{18})$m

Length of the other side of rectangle = $(5-\frac{4}{\sqrt{2}})$

Area of rectangle = $(2+\sqrt{18})\text{m} \times (5-\frac{4}{\sqrt{2}})\text{m}$

$$= 2(5-\frac{4}{\sqrt{2}}) + \sqrt{18}\ (5-\frac{4}{\sqrt{2}})\ \text{m}^2$$

$$= 10 - \frac{8}{\sqrt{2}} + 5\sqrt{18} - \frac{4\sqrt{18}}{\sqrt{2}}$$

$$= 10 - \frac{2\times2\times2}{\sqrt{2}} + 5\sqrt{9\times2} - \frac{4\sqrt{9\times2}}{\sqrt{2}}$$

$$= 10 - \frac{4\times\sqrt{2}\times\sqrt{2}}{\sqrt{2}} + 5(3)(\sqrt{2}) - \frac{4(3)(\sqrt{2})}{(\sqrt{2})}$$

$$= 10 - 4\sqrt{2} + 15\sqrt{2} - 12$$

$$= 10 - 12 - 4\sqrt{2} + 15\sqrt{2}$$

$$= -2 + 11\sqrt{2}$$

So, Area of rectangle = $(-2+11\sqrt{2})\text{m}^2$

4. Find two numbers whose sum is 68 and difference is 22.

Sol. Let the first number = x

Let the second number = y

According to first condition

$x + y = 68$ _____(i)

Acoording to second condition

$x - y = 22$ _____(ii)

By adding (i) and (ii)

$x + y = 68$

$x - y = 22$

$2x = 90$

$x = \frac{90}{2} = 45$

By putting $x = 45$ in (i)

$45 + y = 68$

$y = 68 - 45$

$y = 23$

So, the numbers are = 45, 23

5. The weather in Lahore was unusually warm during the summer of 2024. The TV news reported temperature as high as 48°C. By using the formula, $(°F = \frac{9}{5}°C + 32)$ find the temperature as Fahrenheit scale.

Sol. Temperature in Lahore = 48°C

Temperature as Fahrenheit scale = ?

$$°F = \frac{9}{5}°C + 32$$

$$= \frac{9}{5}(48) + 32$$

$$= \frac{432}{5} + 32$$

$$= 86.4 + 32$$

$$= 118.4°F$$

6. The sum of the ages of the father and son is 72 years. Six years ago the father's age was 2 times the age of the son. What was son's age six years ago?

Sol. Let the age of son = x years

Let the age of father = y years

According to the given condition.

$x + y = 72$ _____(i)

Age of son 6 years ago = $x - 6$ years

Age of father 6 years ago = $y - 6$ years

According to the second condition

$2(x-6) = y - 6$

$2x - 12 = y - 6$

$2x - y = -6 + 12$

$2x - y = 6$ _____(ii)

By adding (i) and (ii)

$x + y = 72$

$2x - y = 6$

$3x = 78$

$$x = \frac{78}{3} = 26$$

So, age of son = 26 years

Age of son six years ago = 26 – 6 = 20 years

7. Mirha bought a toy for Rs. 1500 and sold for Rs. 1520. What was her profit percentage?

Sol. cost price of toy = Rs. 1500

sold price of toy = Rs. 1520

profit = 1520 – 1500 = Rs. 20

$$\text{Profit parental} = \frac{\text{profit}}{\text{cost price}} \times 100\%$$

$$= \frac{\cancel{20}^{4}}{\cancel{1500}_{75_3}} \times \cancel{100}^{1}\% = \frac{4}{3}\%$$

$$= 1.33\%$$

8. The annual income of Tayyab is Rs. 9,60,000, while the exempted amount is Rs. 1,30,000. How much tax would he have to pay at the rate of 0.75%.

Sol. The annual income of Tayyab = Rs. 9,60, 000

Exempted amount = Rs. 1,30,000

Remaining taxable amount = 9,60,000 – 1,30,000 = Rs. 8,30,000

Rate of income tax = 0.75%

Amount of tax = $8,30,000 \times 0.75\%$

$$= 8,30,000 \times \frac{75}{100} \times \frac{1}{100}$$

= Rs. 6225

9. Find the compound markup on Rs. 3,75,000 for one year at the rate of 14% compounded annually.

Sol. P = Rs. 3,75,000

t = 1 year

Rate of compound markup = 14%

Compound Markup $= P(1 + r)^t - P$

$= 3,75,000\ (1 + 14\%)^1 - 3,75,000$

$= 3,75,000 \left(1 + \frac{14}{100}\right)^1 - 3,75,000$

$= 3,75,000\ (1 + 0.14)^1 - 3,75,000$

$= 3,75,000\ (1.14)^1 - 3,75,000$

$= 4,27,500 - 3,75,000$

= Rs. 52, 500

Review Exercise 1

1. Four options are given against each statement. Encircle the correct option.

(i) $\sqrt{7}$ **is:**

(a) integer (b) rational number (c) irrational number (d) natural number

(ii) π **and e are:**

(a) natural numbers (b) integers (c) rational numbers (d) irrational numbers

(iii) If n is not a perfect square then $\sqrt{n}$ **is:**

(a) rational number (b) natural number (c) integer (d) irrational number

(iv) $\sqrt{3} + \sqrt{5}$ **is:**

(a) whole number (b) integer (c) rational number (d) irrational number

(v) For all $x \in R$, $x = x$ **is called:**

(a) reflexive property (b) transitive property

(c) symmetric property (d) trichotomy property

(vi) Let a, b, c, $\in$ **R then** $a > b$ **and** $b > c \Rightarrow a > c$ **is called _____property.**

(a) trichotomy (b) transitive (c) additive (d) multiplicative

(vii) $2^x \times 8^x = 64$ **then x =**

(a) $\frac{3}{2}$ (b) $\frac{3}{4}$ (c) $\frac{5}{6}$ (d) $\frac{2}{3}$

(viii) Let a, b, $\in$ **R then a = b and b = a is called _____property.**

(a) reflexive (b) symmetric (c) transitive (d) additive

(ix) $\sqrt{75} + \sqrt{27} =$

(a) $\sqrt{102}$ (b) $9\sqrt{3}$ (c) $5\sqrt{3}$ (d) $8\sqrt{3}$

(x) The product of $(3 + \sqrt{5})(3 - \sqrt{5})$ **is:**

(a) prime number (b) odd number (c) irrational number (d) rational number

Answers:

i. irrational number **ii.** irrational number **iii.** irrational number **iv.** irrational number

v. reflexive property **vi.** transitine **vii.** $\frac{3}{2}$ **viii.** symmetric

ix. $8\sqrt{3}$ **x.** rational number

2. If $a = \frac{3}{2}$, $b = \frac{5}{3}$ **and** $c = \frac{7}{5}$, **then verify that**

(i) $a(b + c) = ab + ac$

Sol.

Given that $a = \frac{3}{2}$, $b = \frac{5}{3}$ and $c = \frac{7}{5}$

L.H.S = a(b + c)

$= \frac{3}{2}\left(\frac{5}{3}+\frac{7}{5}\right)$

$= \frac{3}{2}\left(\frac{25+21}{15}\right)$

$= \frac{23}{5}$

R.H.S = ab + ac

$= \left(\frac{3}{2}\right)\left(\frac{5}{3}\right)+\left(\frac{3}{2}\right)\left(\frac{7}{5}\right)$

$= \frac{15}{6}+\frac{21}{10} = \frac{75+63}{30}$

$= \frac{23}{5}$

L.H.S = R.H.S

Hence it is verified that a(b + c) = ab + ac

(ii) (a + b)c = ac + bc

Sol. Given that $a = \frac{3}{2}$, $b = \frac{5}{3}$ and $c = \frac{7}{5}$

L.H.S = (a + b)c

$= \left(\frac{3}{2}+\frac{5}{3}\right)\frac{7}{5}$

$= \left(\frac{9+10}{6}\right)\frac{7}{5}$

$= \left(\frac{19}{6}\right)\frac{7}{5} = \frac{133}{30}$ ____(i)

R.H.S = ac + bc

$= \left(\frac{3}{2}\right)\left(\frac{7}{5}\right)+\left(\frac{5}{3}\right)\left(\frac{7}{5}\right)$

$= \frac{21}{10}+\frac{35}{15}$

$= \frac{63+70}{30} = \frac{133}{30}$ ____(ii)

From (i) and (ii), we get

L.H.S = R.H.S

Hence it is verified that (a + b)c = ac + bc

3. If $a = \frac{4}{3}, b = \frac{5}{2}, 7 = \frac{7}{4}$, then verify the associative property of real numbers w.r.t addition and multiplication.

Sol. Given that $a = \frac{4}{3}$, $b = \frac{5}{2}$ and $c = \frac{7}{4}$

Associative property of real numbers w.r.t addition

a+(b + c) = (a + b) + c

L.H.S = (a + b) + c

$= \frac{4}{3}+\left(\frac{5}{2}+\frac{7}{4}\right) = \frac{4}{3}+\left(\frac{10+7}{4}\right)$

$= \frac{4}{3}+\frac{17}{4} = \frac{16+51}{12}$

$= \frac{67}{12}$ ____(i)

R.H.S = (a + b) + c

$= \left(\frac{4}{3}+\frac{5}{2}\right)+\frac{7}{4}$

$= \left(\frac{8+15}{6}\right)+\frac{7}{4} = \frac{23}{6}+\frac{7}{4}$

$= \frac{46+21}{12} = \frac{67}{12}$ ____(ii)

From (i) and (ii), we get

L.H.S = R.H.S

Hence, it is verified that a + (b + c) = (a + b) + c

Now, associative property of real numbers w.r.t multiplication

a(bc) = (ab)c

L.H.S = a(bc)

$= \frac{4}{3}\left(\frac{5}{2}\times\frac{7}{4}\right)$

$= \frac{4}{3}\left(\frac{5\times7}{2\times4}\right) = \frac{4}{3}\left(\frac{35}{8}\right)$

$= \frac{140}{24} = \frac{35}{6}$ ____(i)

R.H.S = (ab)c

$= \left(\frac{4}{3}\times\frac{5}{2}\right)\frac{7}{4}$

$= \left(\frac{4\times5}{3\times2}\right)\frac{7}{4} = \left(\frac{20}{6}\right)\left(\frac{7}{4}\right)$

$= \frac{140}{24} = \frac{35}{6}$ ____(ii)

From (i) and (ii), we get

L.H.S = R.H.S

Hence, it is verified that a(bc) = (ab)c

4. Is 0̂ a rational number? Explain.

Sol. Yes, 0 is a rational number because it can be written in the form of $\frac{p}{q}$ where $p, q \in z$ and $q \neq 0$

For example $0 = \frac{0}{7}$ where $0, 7 \in z$ and $7 \neq 0$

5. State trichotomy property of real numbers.

Sol. $\forall$ a, b $\in$ R, either a = b or a > b or a < b.

6. Find two rational numbers between 4 and 5.

Sol. For this, find the average of 4 and 5.

$\frac{4+5}{2} = \frac{9}{2}$

So, $\frac{9}{2}$ is a rational number between 4 and 5

Now, find the average of $\frac{9}{2}$ and 5.

$\frac{\frac{9}{2}+5}{2} = \frac{\frac{9+10}{2}}{2} = \frac{\frac{19}{2}}{2} = \frac{19}{2}\times\frac{1}{2} = \frac{19}{4}$

Hence two rational numbers between 4 and 5 are $\frac{9}{2}$ and $\frac{19}{4}$.

7. Simplify the following:

(i) $\sqrt[4]{\frac{x^{16}y^{32}}{z^{20}}}$

Sol. $\sqrt[4]{\frac{x^{16}y^{32}}{z^{20}}} = \left(\frac{x^{16}y^{32}}{z^{20}}\right)^{\frac{1}{4}} = \frac{x^{16\times\frac{1}{4}}y^{32\times\frac{1}{4}}}{z^{20\times\frac{1}{4}}}$

$= \frac{x^4y^8}{z^5}$

(ii) $\sqrt[3]{(27)^{2x}}$

Sol. $\sqrt[3]{(27)^{2x}} = \sqrt[3]{(3\times3\times3)^{2x}} = \sqrt[3]{(3^3)^{2x}}$

$= \sqrt[3]{3^{6x}} = (3^{6x\times\frac{1}{3}})$

$= 3^{6x\times\frac{1}{3}} = 3^{2x}$

(iii) $\frac{6(3)^{n+2}}{3^{n+1}-3^n}$

Sol. $\frac{6(3)^{n+2}}{3^{n+1}-3^n}$

$= \frac{6(3)^2(3)^n}{3^n.3^1-3^n} = \frac{6(9)3^n}{3^n(3-1)} = \frac{6(9)}{3-1}$

$= \frac{54}{2} = 27$

8. The sum of three consecutive odd integers is 51. Find the three integers.

Sol. Let the first odd integer = x

Second odd integer = x + 2

Third odd integer = x + 4

According to the given condition

$x + x + 2 + x + 4 = 51$

$3x + 6 = 51$

$3x = 51 - 6$

$3x = 45$

$x = \frac{45}{3}$

$x = 15$

So first odd integer = 15

Second odd integer = 15 + 2 = 17

Third odd integer = 17 + 2 = 19

9. Abdullah picked up 96 balls and placed them into two buckets. One bucket has twenty-eight more balls than the other bucket. How many balls were in each bucket?

Sol. Total balls = 96

Let the number of balls put one bucket = x

Number of balls put in other bucket = x + 28

According to given condition

$x + x + 28 = 96$

$2x + 28 = 96$

$2x = 96 - 28$

$2x = 68$

$x = \frac{68}{2} = 34$

So, the number of balls in first bucket = 34

Number of balls in second bucket = 34 + 28

= 62

10. Salma invested Rs. 3,50,000 in a bank which paid simple profit at the rate of $7\frac{1}{4}$% per annum. After 2 years, the rate was increased to 8% per annum. Find the amount she had at the end of 7 years.

Sol. Principal amount (P) = Rs. 3,50,000

Rate for first 2 years $(R_1) = 7\frac{1}{4}\% = \frac{29}{4}\% = 7.25\%$

Time for first rate (T_1) = 2 years

Rate for next 5 years (R_2) = 8%

Interest for first two years

$I_1 = \frac{P\,R_1T_1}{100}$

$= \frac{350000\times7.25\times2}{100}$

= Rs. 50750

Interest for next 5 years

$I_2 = \frac{P\,R_2T_2}{100}$

$= \frac{350000\times8\times5}{100}$

= Rs. 1,40,000

Salma's amount after 7 years = 3,50,000 + 50, 750 + 1,40,000

= Rs. 5,40,750

Objective Type Questions

Additional MCQ's

Multiple Choice Questions prepared in the light of new Examination Techniques (Knowledge, Understanding, Application, Analytical & Conceptual)

1.1 Introduction to Real Numbers

○ Encircle the correct option.

1. Which of them used sexagesimals system for mathematics.
(a) Sumerians (b) Romans (c) Persian (d) Arabs

2. Which of them is sexagesimals System for mathematics.
(a) base 2 (b) base 60 (c) base 5 (d) base 10

3. Which of them used decimal system for mathematics.
(a) Sumerians (b) Romans (c) Egyptians (d) Indians

4. Egyptians used system for mathematics:
(a) base 60 (b) base 5 (c) base 2 (d) base 10

5. Which of them is the symbols used by Egyptians for 10?
(a) ∩ (b) e (c) | (d) ⌋

6. The Egyptians usually wrote numbers _____ starting with the highest denomination.
(a) right to left (b) left to right (c) top to bottom (d) bottom to top

7. ___ used roman Numerals system for mathematics.
(a) Sumerians (b) Egyptians (c) Romans (d) Indian

8. Romans numerals used __ letters to represent different numbers.
(a) 7 (b) 6 (c) 5 (d) 4

9. Which of them developed the concept of zero?
(a) Arabic (b) Romans (c) Egyptians (d) Indian

10. Arabic introduce Arabic numerals to Europe.
(a) 0 — 9 (b) 0 — 1 (c) 0 — 5 (d) 0 — 8

11. _____ developed modern number system
(a) Old era (b) Modern era (c) Indian era (d) None of above

12. Binary number system means:
(a) 1, 0 (b) 0, 1,2, 3, 4 (c) 1, 2 (d) 9, 10

13. Hexadecimal means:
(a) base 2 (b) base 8 (c) base 10 (d) base 16

14. Set of rational numbers is defined as:
(a) $Q=\left\{\frac{p}{q}, p,q \in Z \wedge q \neq 0\right\}$ (b) $Q=\left\{\frac{p}{q}, p,q \in P\right\}$
(c) $Q=\left\{\frac{p}{q}, p, q \in \dot{Q}\right\}$ (d) $Q=\left\{\frac{p}{q}, p, q \in R\right\}$

15. $Q \cup Q' =$ _____
(a) Z (b) R (c) N (d) P

16. $\frac{1}{4}$ **is known as:**
(a) Non Terminating decimal number (b) both (a) and (c)
(c) Terminating decimal number (d) None of these

17. $\frac{1}{6}$ **is known as:**
(a) Terminating decimal number (b) Non-Terminating decimal number
(c) both (a) and (b) (d) None of above

18. 3.5 is known as:
(a) Irrational number (b) Natural number
(c) Rational number (d) Real number

19. 1.709975947 ..... is known as:
(a) Rational number (b) Irrational number
(c) Natural number (d) Real number

20. $0.\overline{5}$ **is also written as:**
(a) 55 (b) $\frac{5}{5}$ (c) $\frac{5}{10}$ (d) $\frac{5}{9}$

21. $0.\overline{93}$ **is also written as:**
(a) $\frac{93}{99}$ (b) $\frac{99}{93}$ (c) 9399 (d) 9993

22. Which of them is closure property w.r.t addition?
(a) $a + b \in R$ (b) $a + 0 \in R$ (c) $a - b \in R$ (d) $\frac{a}{0} = \in R$

23. Which of them is commutative property w.r.t addition?
(a) $a + 0 = 0 + b \in R$ (b) $a + b = b + a \in R$
(c) $a + b = b - a \in R$ (d) $\frac{a}{b} = a + xb \in R$

24. $a + (b + c) = (a + b) + c$ **is known as :**
(a) Closure property (b) Commutative property
(c) Associative property (d) Inverse property

25. $a \times 1 = 1 \times a = a$ **is known as:**
(a) Closure property (b) Commutative property
(c) Identity (d) Associative property

26. $a < b \Rightarrow \frac{1}{a} > \frac{1}{b}$ **is known as :**
(a) Reciprocal property (b) Additive property
(c) Transitive property (d) Trichotomy property

1.2 Radical Expressions

27. $\sqrt[n]{ab}$ = ____

(a) $\sqrt[b]{a}$ (b) $\sqrt[n]{a}\sqrt[n]{b}$ (c) $\sqrt[a]{b}$ (d) $\sqrt[ab]{n}$

28. $\sqrt[n]{a^m}$ = _____

(a) $\sqrt[m]{a}$ (b) $(mn)^a$ (c) $\sqrt[n]{a}$ (d) $(\sqrt[n]{a})^m$

29. $\frac{a^m}{a^n}$ = _____

(a) a^{mn} (b) $a^{\frac{m}{n}}$ (c) a^{m-n} (d) a^{n-m}

30. $(64)^{\frac{1}{8}}$ = _____

(a) $\frac{1}{256}$ (b) 256 (c) 128 (d) 512

31. $\sqrt{5}$ is a _____

(a) Surd (b) Not surd (c) both (a) & (b) (d) None of these

32. $\sqrt{e}$ is a _____

(a) Surd (b) Not surd (c) both (a) & (b) (d) None of these

33. Conjugate of $\sqrt{a}+\sqrt{b}$ is:

(a) $\sqrt{a}-\sqrt{b}$ (b) $\sqrt{ab}$ (c) $\sqrt{a+b}$ (d) $\sqrt{\frac{a}{b}}$

1.3 Applications of real numbers in daily life

34. K = _____

(a) $\frac{C^\circ}{273}$ (b) 273 C° (c) C° − 273 (d) °C + 273

35. °C = _____

(a) $\frac{9}{5}(F-32)^\circ$ (b) $\frac{9}{5}(F+32)^\circ$ (c) $\frac{5}{9}(F-32)^\circ$ (d) $\frac{5}{9}(F+32)^\circ$

36. °F = _____

(a) $\frac{9°C}{5}-32$ (b) $\frac{9°C}{5}+32$ (c) $\left(\frac{9}{5}°C\right)35$ (d) $\left(\frac{5}{9}°C\right)-32$

37. Selling price — Cost price = _____

(a) Profit (b) Loss (c) Interest (d) debit

38. $\left(\frac{\text{loss}}{\text{cp}}\times 100\right)$ %

(a) Profit % (b) Loss % (c) Interest (d) Income

39. Profit % = _____

(a) $\left(\frac{CP}{profit}\times 100\right)\%$ (b) $\left(\frac{loss}{CP}\times 100\right)\%$

(c) $\left(\frac{profit}{CP}\times 100\right)\%$ (d) $\left(\frac{CP}{loss}\times 100\right)\%$

40. Loss = _____

(a) SP + CP (b) $\frac{SP}{CP}$ (c) SP − CP (d) CP − SP

Answers:

1.	Sumerians	2.	base 60	3.	Egyptians	4.	base 10	5.	∩
6.	left to right	7.	Romans	8.	7	9.	Indians	10.	0 — 9
11.	Modern era	12.	1, 0	13.	base 16	14.	$Q=\left\{\frac{p}{q}, p,q\in Z\wedge q\neq 0\right\}$		
15.	R	16.	Terminating decimal number			17.	Non-Terminating decimal number		
18.	Rational number			19.	Irrational number			20.	$\frac{5}{9}$
21.	$\frac{93}{99}$	22.	$a+b\in R$	23.	$a+b=b+a\in R$	24.	Associative property		
25.	Identity	26.	Reciprocal property			27.	$\sqrt[n]{a}\sqrt[n]{b}$	28.	$(\sqrt[n]{a})^m$
29.	a^{m-n}	30.	$\frac{1}{256}$	31.	Surd	32.	Not surd	33.	$\sqrt{a}-\sqrt{b}$
34.	C° + 273	35.	$\frac{5}{9}(F-32)^\circ$	36.	$\frac{9°C}{5}+32$	37.	Profit	38.	Loss %
39.	$\left(\frac{profit}{CP}\times 100\right)\%$			40.	CP − SP				

Conceptual MCQ's

O Encircle the correct option.

1. Rational number + irrational number = _____:
   (a) Irrational number (b) Rational number
   (c) Real number (d) None of above
2. Irrational number × rational number = _____:
   (a) Rational number (b) Irrational number
   (c) Real number (d) None of above
3. What will be the product of two irrational numbers?
   (a) Real numbers (b) Rational numbers (c) Irrational numbers (d) None of above
4. The product/sum of two rational numbers always:
   (a) rational numbers (b) irrational numbers
   (c) real numbers (d) None of above

5. **If $a < b \Rightarrow \frac{1}{a} > \frac{1}{b}$ then it is called:**
   (a) Additive property (b) Transitive property
   (c) Multiplicative property (d) Reciprocal property
6. **Every surd is an irrational number but every irrational number is _____:**
   (a) a surd (b) not a surd
   (c) may or may not surd (d) none of above
7. **Conjugate surd of $\sqrt{x} + \sqrt{y}$ is:**
   (a) $\sqrt{x} - \sqrt{y}$ (b) $\sqrt{x} + \sqrt{y}$ (c) $\sqrt{x}\ \sqrt{y}$ (d) $\sqrt{x} + \sqrt{y}$
8. **To rationalize a denominator of the form or $a + b\sqrt{x}$ or $a - b\sqrt{x}$. We _____ both the numerator and denominator by the conjugate factor.**
   (a) Add (b) Substract (c) Multiply (d) Divide
9. **°C = _____:**
   (a) K – 273 (b) K + 273 (c) (K)(273) (d) $\frac{K}{273}$
10. **Normal human body temperature is:**
   (a) 98.8F (b) 37 °C (c) Both a & b (d) None of these

Answers:

1.	Irrational number	2.	Irrational number	3.	Irrational numbers				
4.	Rational numbers	5.	Reciprocal property						
6.	not a surd	7.	$\sqrt{x} - \sqrt{y}$	8.	Multiply	9.	K – 273	10.	Both a & b

Additional Short Questions

Short Answer Questions prepared in the light of new Examination Techniques (Knowledge, Understanding, Application, Analytical & Conceptual)

1.1 Introduction to Real numbers

1. **Write a brief note on Sumerians.**

Ans. Sumerians used sexagesimals (base 60) system for counting. They used small cones, sphere which corresponded respectively to 1, 10, 60, 600.

2. **Write a brief note on Egyptians.**

Ans. Egyptians used decimal base 10 system for counting. They usually wrote numbers left to right starting with highest denominator.

3. **What do you know about Roman numbers?**

Ans. Roman numerals use 7 letters to represent different numbers. These numbers are I, V, X, L, C, D and M which represent 1, 5, 10, 50, 100, 500 and 1000.

4. **Who developed the concept of zero?**

Ans. Indians developed the concept of zero and made a significant contribution to decimal base 10 system.

5. **What do think about modern era?**

Ans. This era developed modern number system e.g. binary system (base 2) hexadecimal system (base 16).

6. **Define set of rational number.**

Ans. The set of rational number is defined as $Q = \left\{\frac{p}{q}, p, q \in Z \wedge q \neq 0\right\}$

7. **Define set of Real Numbers?**

Ans. The set of Real numbers is the Union of set of rational numbers and irrational numbers $R = Q \cup Q'$

8. **What is terminating decimal number.**

Ans. A decimal number with a finite number of digits after the decimal point is called terminating decimal number. e.g. $\frac{1}{4} = 0.25$.

9. **What is non-terminating and recurring decimal numbers?**

Ans. The decimal numbers with an infinitely repeating pattern of digits after the decimal point are called nonterminating and recurring decimal numbers.

10. **Define irrational numbers.**

Ans. Non-terminating and non-recurring decimal numbers are known as irrational numbers.

11. **Identify 0.444... is rational or irrational number.**

Ans. 0.444... is a recurring decimal number, so it is rational number.

12. **Identify 3.36788542... is rational or irrational number.**

Ans. 3.36788542... is a non-terminating and non-recurring decimal number. Therefore, eit is an irrational number.

13. **Express $0.9\overline{3}$ into $\frac{p}{q}$ form.**

Sol. Let $x = 0.9\overline{3}$

$x = 0.939393\ldots\ldots$ (1)

$100x = 100(0.939393\ldots\ldots)$

$100x = 93.333\ldots\ldots$ (2)

Subtracting eq. (1) from eq. (2)

$100x - x = 93.333\ldots\ldots - 0.939393\ldots\ldots$

$99x = 93 \Rightarrow x = \frac{93}{99}\ \left(\frac{p}{q}\right)$

14. **What is closure property w.r.t additional? Explain with example.**

Ans. $a + b \in R$

$\forall$ a, b, c

Example $2 + 3 = 5 \in R$

15. **Define commutative property under addition with example.**

Ans. $a + b = b + a\ \forall$ a, b $\in R$

Example $2 + 3 = 3 + 2 = 5 \in R$

16. Define Associative property under addition with example..

Ans. $a + (b + c) = (a + b) + c \quad \forall\ a, b, c \in R$

Eample $2 + (3 + 4) = (2 + 3) + 4$

$2 + 7 = 5 + 4$

$9 = 9$

17. What is additive identity property under addition?

Ans. $a + 0 = 0 + a = a \quad \forall a \in R$

$4 + 0 = 0 + 4 = 4$

18. What is additive inverse property under addition?

Ans. $a + (-a) = (-a) + a \quad \forall a \in R$

$a - a = -a + a$

$0 = 0$

$5 + (-5) = (-5) + 5 \quad \forall a \in R$

$0 = 0$

19. What is multiplicative identity under multiplication?

Ans. $a \times 1 = 1 \times a = a \quad \forall a \in R$

$4 \times 1 = 1 \times 4 = 4$

20. What is multiplicative inverse?

Ans. $a \times \frac{1}{a} = \frac{1}{a} \times a = 1 \quad \forall a \in R$

$6 \times \frac{1}{6} = \frac{1}{6} \times 6 = 1$

21. If $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ then verify the associative property over addition.

Sol. Let $a = \frac{1}{2}, \frac{1}{3}$ and $c = \frac{1}{4}$

$a + (b + c) = (a + b) + c$

L.H.S $= a + (b + c)$

$= \frac{1}{2} + \left(\frac{1}{3} + \frac{1}{4}\right)$

$= \frac{1}{2} + \left(\frac{4+3}{12}\right) = \frac{1}{2} + \frac{7}{12}$

$= \frac{6+7}{12}$

$= \frac{13}{12}$ _____(i)

R.H.S $= (a + b) + c$

$= \left(\frac{1}{2} + \frac{1}{3}\right) + \frac{1}{4}$

$= \left(\frac{3+2}{6}\right) + \frac{1}{4} = \frac{5}{6} + \frac{1}{4}$

$= \frac{10+3}{12}$

$= \frac{13}{12}$ _____(ii)

From (i) and (ii), we get

L.H.S. = R.H.S

Hence, it is verified that $a + (b + c) = (a + b) + c$

22. Identify the property that justified the statement if $-5 < -4 \Rightarrow 20 > 16$

Ans. As $-5 < -4$

$-5 \times -4 > -4 \times -4 \Rightarrow 20 > 16$

So, it is multiplicative property of inequality.

1.2 Radical compression

23. What do you know about radical expression?

Ans. If n is a positive integer greater than 1 and a is a real number then any real number x such that $x = \sqrt[n]{a}$ is called nth root of a. Here $\sqrt{}$ is radical and n is the index of the radical.

24. Simplify $(64)^{-\frac{1}{3}}$

Sol. $\frac{1}{(64)^{\frac{1}{3}}} = \frac{1}{(4^3)^{\frac{1}{3}}} = \frac{1}{4^1} = \frac{1}{256}$

25. What is surd?

Ans. An irrational radical with rational radicand is called Surd.

26. Define monomial surd.

Ans. A surd that contains a single term is called monomial surd $\sqrt{5}, \sqrt{7}$ are the examples.

27. Define binomial surd.

Ans. A surd that contains the sum of two monomial surd is called binomial surd $\sqrt{3} + \sqrt{5}$ is the example. of the binomial surd.

28. Rationalize $\frac{3}{\sqrt{5} - \sqrt{3}}$

Ans. $\frac{3}{\sqrt{5} - \sqrt{3}} \times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}} = \frac{3(\sqrt{5} + \sqrt{3})}{(\sqrt{5})^2 - (\sqrt{3})^2} = \frac{3(\sqrt{5} + \sqrt{3})}{5 - 3} = \frac{3(\sqrt{5} + \sqrt{3})}{2}$

1.3 Application of Real numbers in Daily life

29. Write any three applications of real numbers in daily life.

Ans. Real numbers are used in various field including.

(i) Science and engineering physics, mechanical system, electrical circuits.

(ii) Medicine and health algorithm design, data compression, graphic rendering.

(iii) Computer Science.

30. The sum of five real numbers is 8 and difference is 2. Find numbers.

Sol. If a, b are two numbers then

$a + b = 8 - (1)\ a - b = 2 - (2)$

Adding eq. (1) and eq. (2), we get

$a + b = 8$

$\underline{a - b = 2}$

$2a = 10$

$a = 5$

So, $a + b = 8 \Rightarrow 5 + b = 8$

$\Rightarrow b = 8 - 5 = 3$

31. If °F=98.6 then C=?

Sol. $°C = \frac{5}{9}(F-32)$

$°C = \frac{5}{9}(98.6-32)$

$°C = \frac{5}{9}(66.6)$

$°C = 37°$

32. If CP = Rs. 6590, SP = Rs. 6850 Find profit percentage.

Sol. CP = Rs. 6590, SP = Rs. 6850

Profit = SP – CP = 6850 – 6590 = Rs. 260

Now Profit % $= \left(\frac{\text{profit}}{\text{CP}} \times 100\right)\% = \left(\frac{260 \times 100}{6590}\right)\%$

$= 3.94\%$

33. If CP = Rs. 850, SP = Rs. 720 Find loss %.

Sol. CP = Rs. 850, SP = Rs. 720

Loss = CP – SP = 850 – 720 = Rs. 130

Loss % $= \left(\frac{\text{Loss}}{\text{CP}} \times 100\right)\% = \left(\frac{130}{850} \times 100\right)\% = 15.29\%$

Conceptual Short Questions

1. Is $\frac{4}{5}$ is terminating decimal?

Ans. $\frac{4}{5} = 0.8$ Yes it is terminating decimal.

2. In π, e, $\sqrt{2}$ which is called Euler's number?

Ans. e = 2.7182... is called Euler's number.

3. Is zero a rational or irrational number?

Ans. Yes, zero (0) is rational number because it can be written in the form of $\frac{p}{q}$ where p, $q \in z$ and $q \neq 0$.

For example $0 = \frac{0}{7}$ where $0, 7 \in z$ and $7 \neq 0$

4. Write the cancellation property over multiplication.

Ans. $\forall$ a, b, c ∈ R

$ac = bc \Rightarrow a = b$

5. What is the conjugate of 5?

Ans. The conjugate of 5 is 5.

6. What is the product of two conjugate surds?

Ans. The product of two conjugate surds is a rational numbers.

7. What is the human body temperature in Kelvin scale?

Ans. $K = °C + 273°$

$K = 37° + 273°$

K = 310 kelvin.

8. If CP of a thing is Rs. 500 and SP is Rs. 500 then what will be occur?

Ans. When SP and CP of a thing is same then there is no loss and no profit occur.

9. When loss is occurs?

Ans. When selling price is less than cost price then loss will be occurs.

10. When profit is occurs?

Ans. When the selling price is more than the cost price then the profit will be occurs.

Unit 2

Logarithms

Scientific Notation

Scientific notation is a method used to express very large or very small numbers in a more manageable form. It is commonly used in science, engineering, and mathematics to simplify complex calculations.

Remember!
If the number is greater than 1 then n is positive and if the number is less than 1 then n is negative.

A number in scientific notation is written as:

$a \times 10^n$, where $1 \leq a < 10$ and $n \in Z$

Here "a" is called the coefficient or base number.

Conversion of Numbers from Ordinary Notation to Scientific Notation

Example 1: Convert 78,000,000 to scientific notation.

Solution: Step 1: Move the decimal to g. a number between 1 and 10:

7.8

Step 2: Count the number of places you moved the decimal:

7 places

Step 3: Write in scientific notation:

$78,000,000 = 7.8 \times 10^7$

Since we moved the decimal to the **left,** the exponent is **positive.**

Example 2: Convert 0.0000000315 to scientific notation.

Solution: Step 1: Move the decimal to get a number between 1 and 10:

3.15

Step 2: Count the number of places you moved the decimal:

8 places

Step 3: Write in scientific notation:

$0.0000000315 = 3.15 \times 10^{-8}$

Since we moved the decimal to the **right,** the exponent is **negative.**

Conversion of Numbers from Scientific Notation to Ordinary Notation

Example 3: Convert 3.47×10^6 to ordinary notation.

Solution: Step 1: Identify the parts:

Coefficient: 3.47

Exponent: 10^6

Step 2: Since the exponent is positive 6, move the decimal point 6 places to the right.

$3.47 \times 10^6 = 3,470,000$

Example 4: Convert 6.23×10^{-4} to ordinary notation

Solution: Step 1: Identify the parts:

Coefficient: 6.23

Exponent: 10^{-4}

Step 2: Since the exponent is negative 4, move the decimal point 4 places to the left.

$6.23 \times 10^{-4} = 0.000623$

Solved Exercise 2.1

1. Express the following numbers in scientific notation:

(i) 2000000
Sol. $2000000 = 2\times10^{6}$

(ii) 48900
Sol. $48900 = 4.89\times10^{4}$

(iii) 0.0042
Sol. $0.0042 = 4.2\times10^{-3}$

(iv) 0.0000009
Sol. 9×10^{-7}

(v) 73×10^{3}
Sol. $73\times10^{3} = 7.3\times10^{1}\times10^{3} = 7.3\times10^{4}$

(vi) 0.65×10^{2}
Sol. $0.65\times10^{2} = 6.5\times10^{-1}\times10^{2}$
$= 6.5\times10^{-1+2} = 6.5\times10$

2. Express the following numbers in ordinary notation.

(i) 8.04×10^{2}
Sol. $8.04 \times 10^{2} = 804$

(ii) 3×10^{5}
Sol. $3\times10^{5} = 300000$

(iii) 1.5×10^{-2}
Sol. $1.5\times10^{-2} = 0.015$

(iv) 1.77×10^{7}
Sol. $1.77\times10^{7} = 17700000$

(v) 5.5×10^{-6}
Sol. $5.5\times10^{-6} = 0.0000055$

(vi) 4×10^{-5}
Sol. $4\times10^{-5} = 0.00004$

3. The speed of light is approximately 3×10^{8} metres per second. Express it in standard form.

Sol. $3 \times 10^{8} = 300,000,000$ m/sec

4. The circumference of the Earth at the equator is about 40075000 metres. Express this number in scientific notation.

Sol. $40075000 = 4.0075 \times 10^{7}$m

5. The diameter of Mars is 6.7779×10^{3}km. Express this number in standard form.

Sol. $6.7779 \times 10^{3} = 6777.9 = 6778$ km

6. The diameter of Earth is about 1.2756×10^{4}km. Express this number in standard form.

Sol. $1.2756 \times 10^{4} = 12756$km

Logarithm

A logarithm is based on two Greek words: logos and arithmos which means ratio or proportion. John Napier, a Scottish mathematician, introduced the word logarithm. It is a way to simplify complex calculations, especially those involving multiplication and division of large numbers. Today, logarithm remain fundamental in mathematics, with applications in science, finance and technology.

Logarithm of a Real Number

In simple words, the logarithm of a number tells us how many times one number must be multiplied by itself to get another number.

The general form of a logarithm is:

$$\log_b(x) = y$$

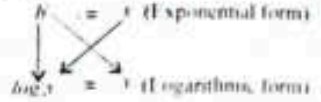
Where:

- b is the **base,**

- x is the **result** or the number whose logarithm is being taken.
- y is the **exponent** or the logarithm of x to the base b.

This means that:

$$b^{y} = x$$



In words, **"the logarithm of x to the base b is y,** means that when b is raised to the power y, it equals x. The relationship between logarithmic form and exponential form is given below:

$\log_b(x) = y \Leftrightarrow b^{y} = x$ where $b > 0$, $x > 0$ and $b \neq 1$

Example 5: Convert $\log_2 8 = 3$ to exponential form.

Solution: $\log_2 8 = 3$
Its exponential form is: $2^3 = 8$

Example 6: Convert $\log_{10} 100 = 2$ to exponential form.

Solution: $\log_{10} 100 = 2$
Its exponential form is: $10^2 = 100$

Example 7: Find the value of x in each case:

(i) $\log_5 25 = x$ (ii) $\log_2 x = 6$

Solution:

(i) $\log_5 25 = x$
Its exponential form is:
$5^x = 25$
$\Rightarrow 5^x = 5^2$
$\Rightarrow x = 2$

(ii) $\log_2 x = 6$
Its exponential form is:
$2^6 = x$
$\Rightarrow x = 64$

Example 8: Convert the following in logarithmic form:

(i) $3^4 = 81$ (ii) $7^0 = 1$

Solution:

(i) $3^4 = 81$
Its logarithmic form is:
$\log_3 81 = 4$

(ii) $7^0 = 1$
Its logarithmic form is:
$\log_7 1 = 0$

Solved Exercise 2.2

1. Express each of the following in logarithmic form:

(i) $10^3 = 1000$
Sol. $10^3 = 1000$
$\log_{10} 1000 = 3$

(ii) $2^8 = 256$
Sol. $2^8 = 256$
$\log_2 256 = 8$

(iii) $3^{-3} = \frac{1}{27}$
Sol. $3^{-3} = \frac{1}{27}$
$\log_3\left(\frac{1}{27}\right) = -3$

(iv) $20^2 = 400$
Sol. $20^2 = 400$
$\log_{20} 400 = 2$

(v) $16^{-\frac{1}{4}} = \frac{1}{2}$
Sol. $16^{-\frac{1}{4}} = \frac{1}{2}$
$\log_{16}\frac{1}{2} = -\frac{1}{4}$

(vi) $11^2 = 121$
Sol. $11^2 = 121$
$\log_{11} 121 = 2$

(vii) $p = q^r$

Sol. $p = q^r$
$\log_q p = r$

(viii) $(32)^{-\frac{1}{5}} = \frac{1}{2}$

Sol.
$(32)^{-\frac{1}{5}} = \frac{1}{2}$
$\log_{32} \frac{1}{2} = -\frac{1}{5}$

2. Express each of the following in exponential form:

(i) $\log_5 125 = 3$

Sol. $\log_5 125 = 3$
$5^3 = 125$

(ii) $\log_2 16 = 4$

Sol. $\log_2 16 = 4$
$2^4 = 16$

(iii) $\log_{23} 1 = 0$

Sol. $\log_{23} 1 = 0$
$23^0 = 1$

(iv) $\log_5 5 = 1$

Sol. $\log_5 5 = 1$
$5^1 = 5$

(v) $\log_2 \frac{1}{8} = -3$

Sol.
$\log_2 \frac{1}{8} = -3$
$2^{-3} = \frac{1}{8}$

(vi) $\frac{1}{2} = \log_9 3$

Sol. $\frac{1}{2} = \log_9 3$
$9^{\frac{1}{2}} = 3$

(vii) $5 = \log_{10} 100000$

Sol. $5 = \log_{10} 100000$
$10^5 = 100000$

(viii) $\log_4 \frac{1}{16} = -2$

Sol.
$\log_4 \frac{1}{16} = -2 \Rightarrow 4^{-2} = \frac{1}{16}$

3. Find the value of x in each of the following:

(i) $\log_x 64 = 3$

Sol. $\log_x 64 = 3$
$x^3 = 64$
$x^3 = 4 \times 4 \times 4$
$x^3 = 4^3 \Rightarrow x = 4$

(ii) $\log_5 1 = x$

Sol. $\log_5 1 = x$
$5^x = 1$
$5^x = 5^0$
$x = 0$

(iii) $\log_x 8 = 1$

Sol. $\log_x 8 = 1$
$x^1 = 8$
$x = 8$

(iv) $\log_{10} x = -3$

Sol. $\log_{10} x = -3$
$\log_{10} x = -3$
$10^{-3} = x$
$\frac{1}{10^3} = x$
$\frac{1}{1000} = x \Rightarrow x = \frac{1}{1000}$

(v) $\log_4 x = \frac{3}{2}$

Sol. $\log_4 x = \frac{3}{2}$
$(4)^{\frac{3}{2}} = x$
$(2^2)^{\frac{3}{2}} = x$
$2^{2 \cdot \frac{3}{2}} = x$
$2^3 = x$
$8 = x \Rightarrow x = 8$

(vi) $\log_2 1024 = x$

Sol. $\log_2 1024 = x$
$2^x = 1024$
$2^x = 2^{10}$
$x = 10$

Common Logarithm

The common logarithm is the logarithm with a base of 10. It is written as $\log_{10}$ or simply as log (when no base is mentioned, it is usually assumed to be base 10).

For Example:

$10^1 = 10 \Leftrightarrow \log 10 = 1$

$10^2 = 100 \Leftrightarrow \log 100 = 2$

$10^3 = 1000 \Leftrightarrow \log 1000 = 3$ and so on.

$10^{-1} = \frac{1}{10} = 0.1 \Leftrightarrow \log 0.1 = -1$

$10^{-2} = \frac{1}{100} = 0.01 \Leftrightarrow \log 0.01 = -2$

$10^{-3} = \frac{1}{1000} = 0.001 \Leftrightarrow \log 0.001 = -3$ and soon.

Characteristic and Mantissa of Logarithms

A logarithm of a number consists of two parts: the characteristic and the mantissa. Here is a simple way to understand them:

(a) Characteristic

The characteristic is the integral part of the logarithm. It tells us how big or small the number is.

Rules for Finding the Characteristic

> **Remember**
> When the characteristic is negative, we write it with bar.

(i) For a number greater than 1:

Characteristic = number of digits to the left of the decimal point – 1

For example, in log 567 the characteristic = 3 – 1 = 2

(ii) For a number less than 1:

Characteristic = –(number of zeros between the decimal point and the first non-zero digit + 1)

For example, in log 0.0123 the characteristic = –(1+1) = –2 or $\bar{2}$

Example 9: Find characteristic of the followings:

(i) log 725 (ii) log 9.87 (iii) log 0.00045 (iv) log 0.54

Solution:

(i) log 725
Characteristic = 3 – 1=2

(ii) log 9.87
Characteristic = 1 – 1=0

(iii) log 0.00045
Characteristic = –(3+1) = $\bar{4}$

(iv) log 0.54
Characteristic= –(0+1)=$\bar{1}$

(b) Mantissa

The mantissa is the decimal part of the logarithm. It represents the "fractional" component and is always positive.

For example, in log 5000 = 3.698 the mantissa is 0.698.

2.3.2 Finding Common Logarithm of a Number

Suppose we want to find the common logarithm of 13.45. The step-by-step procedure to find the logarithm is given below.

> **Remember!**
> log (Number) = Characteristic + Mantissa

Step 1: Separte the integral and decimal parts.
Integral part = 13
Decimal part = 45

Step 2: Find the characteristic of the number
Characteristic = number of digits to the left of the decimal point – 1

Step 3: In common logarithm table (Complete table is given at the end of the book) check the intersection of row number 13 and column number 4 which is 1271.

Step 4: Find mean difference: Check the intersection of row number 13 and column number 5 in the mean difference which is 16.

Logarithm Table

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27

Step 5: Add the numbers found in step 3 and step 4. i.e., 1271 + 16 = 1287 which is the mantissa of given number.

Step 6: Finaly, combine the characteristic and mantissa parts found in step 2 and step 5 respectively. We get 1.1287
So, the value of log 13.45 is 1.1287

Example 10: Find logarithm of the following numbers:
(i) log 345 (ii) log 5.678 (iii) log 0.0036 (iv) log 0.0478

Solution: (i) log 345
Characteristic = 3 – 1 = 2
Mantissa = 0.5378 (Look for 34 in the row and 5 in the column of the log table)
So, log(345) = 2 + 0.5378 = 2.5378

> **Do you know?**
> log (0) = undefined
> log (1) = 0
> $\log_a(a) = 1$

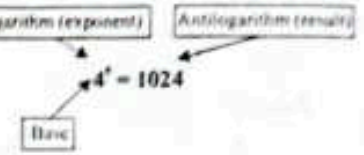
(ii) log 5.678
Characteristic = 1 – 1 = 0
Mantissa = 0.7542 (7536 + 6 = 7542)
So, log(5.678) = 0 + 0.7542 = 0.7542

(iii) log 0.0036
Characteristic = –(2 + 1) = –3
Mantissa = 0.5563 (Look for 36 in the row and 0 in the column of the log table)
So, log(0.0036) = –3 + 0.5563 = $\overline{3}$.4437

(iv) log 0.0478
Characteristic = –(1 + 1) = –2
Mantissa = 0.6794 (Look for 47 in the row and 8 in the column of the log table)
So, log(0.0478) = –2 + 0.6794 = $\overline{2}$.3206

Antilogarithm

An **antilogarithm** is the inverse operation of a logarithm.



An antilogarithm helps to find a number whose logarithmic value is given.

If $\log_b(x) = y \Leftrightarrow b^y = x$ then the process of finding x is called antilogarithm of y.

> **Remember!**
> The word antilogarithm is another word for the number or result. For example, in 4^3=64, the result 64 is the antilogarithm.

Example 11: Find the value of x in the followings:
(i) log x = 0.2568
(ii) logx = –1.4567
(iii) log x = –2.1234

> **Remember!**
> The place between the first non-zero digit from left and its next digit is called reference position. For example, in 1332, the reference position is between 1 and 3 (1 ^ 332)

Solution: (i) log x = 0.2568
Characteristic = 0
Mantissa = 0.2568
Table value of 0.2568 = 1803 + 3
= 1806

So, x = antilog (0.2568) = 1.806 (Insert the decimal point at reference position because characteristic is 0.

(ii) logx = –1.4567
Since mantissa is negative, so we make it positive by adding and subtracting 2
log x = –2 + 2 –1.4567
= –2 + 0.5433
= $\overline{2}$.5433
Here characteristic = $\overline{2}$ and mantissa = 0.5433
Table value of 0.5433 = 3491 + 2
= 3.493
So, x = antilog ($\overline{2}$.5433)
= 0.03493
Since characteristic is $\overline{2}$, therefore decimal point will be before 2 digits left from the reference position

(iii) log x = –2.1234
Since mantissa is negative, so we make it positive by adding and subtracting 3

$\log x = -3 + 3 - 2.1234$

$= -3 + 0.8766 = \bar{3}.8766$

Here characteristic = $\bar{3}$, mantissa = 0.8766

Table value of 0.8766 = 7516 + 10 = 7.526

So, x = antilog $(\bar{3}.8766)$ = 0.007526

(Since characteristic = $\bar{3}$, therefore decimal point will be before 3 digits left from the reference position.)

History

Swiss mathematician and physicist Leonhard Euler introduced 'e' to the base of natural logarithm.

Natural Logarithm

The natural logarithm is the logarithm with base e, where e is a mathematical constant approximately equal to 2.71828. It is denoted as ln.

For example, $\ln e^2 = 2$ i.e., the logarithm of e^2 to the base e is 2.

Difference between common and natural logarithms

Common Logarithm	Natural Logarithm
1. The base of a common logarithm is 10.	1. The base of a natural logarithm is e.
2. It is written as $\log_{10}(x)$ or simply log (x)when no base is specified.	2. It is written as $ln(x)$
3. Common logarithms are widely used in everyday calculations, especially in scientific and engineering applications.	3. Natural logarithms are commonly used in higher level mathematics particularly calculus and applications involving grwoth/decay processes.

Solved Exercise 2.3

1. Find characteristic of the following numbers:

(i) 5287

Sol. 5287

characteristic =4–1=3

(ii) 59.28

Sol. 59.28

characteristic = 2–1=1

(iii) 0.0567

Sol. 0.0567

characteristic=–(1+1)=–2 or $\bar{2}$

(iv) 234.7

Sol. characteristic =3–1=2

(v) 0.000049

Sol. chracteristic

$= -(4+1) = -5$ or $\bar{5}$

(vi) 145000

Sol. characteristic = 6–1 = 5

2. Find logarithm of the following numbers:

(i) 43

Sol. 43

characteristic = 2 – 1 = 1

Mantissa = 0.6335

log 43 = 1 + 0.6335 = 1.6335

(ii) 579

Sol. characteristic = 3 – 1 = 2

Mantissa = 0.7627

log 579 = 2 + 0.7627

= 2.7627

(iii) 1.982

Sol. 1.982

characteristic = 1 – 1 = 0

Mantissa = 0.2971

log 1.982= 0+0.2971 = 0.2971

(iv) 0.0876

Sol. 0.0876

characteristic –(1+1) = –2

Mantissa = 0.9425

log 0.0876 = –2.9425 = –2+0.94525

= 1. 0575

(v) 0.047

Sol. 0.047

characteristic = –(1+1) = –2

Mantissa = 0.6721

log 0.047= $\bar{2}$.6721 = –2 + 0.6721

= $-\bar{1}.3279 = \bar{1}.3279$

(vi) 0.000354

Sol. 0.000354

characteristic = –(3+1) = –4

Mantissa = 0.5490

log 0.000354 = $\bar{4}$.5490 = –4+0.5490

= –3.4510 = $\bar{3}$.4510

3. If log 3.177 = 0.5019, then find:

(i) log 3177

Sol. log 3177

characteristic = 4 – 1 = 3

Mantissa will be same 0.5019

log 3177 = 3.5019

(ii) log 31.77

Sol. log 31.77

characteristic = 2–1 = 1

Mantissa will be same 0.5019

log 31.77 = 1.5019

(iii) log 0.03177

Sol. log 0.03177

characteristic = –(1+1) = –2

Mantissa will be same 0.5019

log 0.03177 = $\bar{2}$.5019 = –2+0.5019 = –1.4981 = $\bar{1}$.4981

4. Find the value of x.

(i) log x = 0. 0065

Sol. logx = 0.0065

characteristics = 0

Mantisa = 0.0065

Anti log table value = 1015

x = anti log 0.0065 ⇒ x = 1.015

(ii) logx = 1.192

Sol. logx = 1.192

characteristic = 1

Mantissa = 0.192

Anti log table value = 1556

x = antilog (1.192) ⇒ x = 15.56

(iii) logx = –3.434

Sol. logx = –3.434

Make mantissa positive by adding and subtracting 4

log x = –4 + 4 – 3.434

logx = –4 + 0.566

logx = $\bar{4}$.566

characteristics = – 4, Mantissa = 0.566

Antilog table value = 3681

x = Antilog ($\bar{4}$. 566)

x = 0. 0003681

(iv) logx = –1.5726

Sol. x = –1.5726

Make mantissa positive by adding and subtracting 2

logx = –2+2–1.5726

logx = –2 + 0.4274

logx = $\bar{2}$.4274

characteristics = 2, Mantissa = 0.4274

Antilog table value = 2675

x = Anti log ($\bar{2}$.4274)

x = 0.02675

(v) logx = 4.3561

Sol. logx = 4.3561

characteristics = 4

Mantisa = 0.3561

Anti log table value = 2271

x = anti log 4.3561

x = 22710

(vi) log x = –2.0184

Sol. logx = –2.0184

Make mantissa positive by adding and subtractings 3.

logx = –3 + 3 –2.0184

logx = –3 + 0.9816

log x= $\bar{3}$.9816

characteristics = $\bar{3}$, Mantissa = 0.9816

x = anti log $\bar{3}$.9816

x = 0.009585

Laws of Logarithm

Laws of logarithm are also known as rules or properties of logarithm. These laws help to simplify logarithmic expressions and solve logarithmic equations.

1. Product Law:

$$\log_b xy = \log_b x + \log_b y$$

The logarithm of a product is the sum of the logarithms of the factors.

Proof: Let

$$m = \log_b x \quad \ldots(i)$$

and $$n = \log_b y \quad \ldots(ii)$$

In exponential form:

$$x = b^m \text{ and } y = b^n$$

Multiply x and y

$$x.y = b^m . b^n = b^{m+n}$$

In logarithmic form:

$$\log_b xy = m + n$$

$$\log_b xy = \log_b x + \log_b y \quad \text{[From (i) and (ii)]}$$

2. Quotient Law:

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

The logarithm of a quotient is the difference between the logarithms of the numerator and the denominator.

Proof:

Let $m = \log_b x \quad \ldots(i)$

and $n = \log_b y \quad \ldots(ii)$

In exponential form:

$$x = b^m \text{ and } y = b^n$$

Divide x and y

$$\frac{x}{y} = \frac{b^m}{b^n} = b^{m-n}$$

In logarithmic form:

$$\log_b\left(\frac{x}{y}\right) = m - n$$

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y \quad \text{[From (i) and (ii)]}$$

3. Power Law:

$$\log_b x^n = n.\log_b x$$

The logarithm of a number raised to a power is the product of the power and the logarithm of the base number.

Proof: Let

$$m = \log_b x \quad \ldots(i)$$

In exponential form:

$$x = b^m$$

Raise both sides to the power n

$$x^n = (b^m)^n = b^{mn}$$

In logarithmic form:

$$\log_b x^n = mn$$

$$\log_b x^n = n.\log_b x \quad \text{[From (i)]}$$

Activity

- Divide the students into small groups.
- Distribute the logarithmic expression cards randomly among the groups.
- Each group will work together to identify which logarithmic law applies to each expression.
- After completing the task, each group will present its findings.

4. Change of Base Law:

$$\log_b x = \frac{\log_a x}{\log_a b}$$

This law allows to change the base of a logarithm from "b" to any other base "a".

Proof: Let

$$m = \log_b x \quad \ldots(i)$$

In exponential form:

$$b^m = x$$

Taking log with base a on both sides, we get

$$\log_a b^m = \log_a x$$

$$m \log_a b = \log_a x$$

$$m = \frac{\log_a x}{\log_a b}$$

$$\log_b x = \frac{\log_a x}{\log_a b} \quad \text{[From (i)]}$$

Applications of Logarithm

Logarithms have a wide range of applications in many fields. Here some examples are given about the applications of logarithms.

Example 12: Expand the following using laws of logarithms:

(i) $\log_3 (20)$ **(ii)** $\log_2 (9)^5$ **(iii)** $\log_{32} 27$

Solution: (i) $\log_2(20)$
$= \log_2(2\times2\times5)$
$= \log_2(2^2\times5)$
$= \log_2(2)^2 + \log_2 5$
$= 2\log_2 2 + \log_2 5$

(ii) $\log_2(9)^5$
$= \log_2(3^2)^5$
$= \log_2(3)^{10}$
$= 10\log_2 3$

(iii) $\log_{32}27 = \frac{\log 27}{\log 32}$
$= \frac{\log 3^3}{\log 2^5}$
$= \frac{3\log 3}{5\log 2} \Rightarrow = \frac{3}{5}\log_2 3$

Example 13: Expand the following using laws of logarithms:

(i) $\log_2\left(\frac{x-y}{z}\right)^3$ **(ii)** $\log_4\left(\frac{xy}{z}\right)^8$

Solution: (i) $\log_2\left(\frac{x-y}{z}\right)^3 = 3\log_2\left(\frac{x-y}{z}\right)$
$= 3[\log_2(x-y) - \log_2 z]$

(ii) $\log_4\left(\frac{xy}{z}\right)^8 = 8\log_4\left(\frac{xy}{z}\right)$
$= 8[\log_4(xy) - \log_4 z]$
$= 8[\log_4 x + \log_4 y - \log_4 z]$

Example 14: Write the following as a single logarithm:

(i) $2\log_3 10 - \log_3 4$ (ii) $6\log_3 x + 2\log_3 11$

Solution: (i) $2\log_3 10 - \log_3 4$
$= \log_3(10)^2 - \log_3 4$
$= \log_3 100 - \log_3 4$
$= \log_3\left(\frac{100}{4}\right)$
$= \log_3 25$

(ii) $6\log_3 x + 2\log_3 11$
$= \log_3 x^6 + \log_3(11)^2$
$= \log_3 x^6 + \log_3(121)$
$= \log_3(121x^6)$

Example 15: The decibel scale measures sound intensity using the formula $L = 40\log_{10}\left(\frac{I}{I_o}\right)$. If a sound has an intensity (I) of 10^5 times the reference intensity (Io). What is the sound level in decibels?

Solution: $L = 40\log_{10}\left(\frac{I}{I_o}\right)$

Put $I = 10^5 I_o$, we get

Do you know?
ln (0) = undefined
ln (1) = 0
ln (e) = 1

$L = 40\log_{10}\left(\frac{10^5 I_o}{I_o}\right)$

$L = 40\log_{10}(10)^5$

$L = 40\times6\log_{10}10$

$L = 40\times6$ $(\because \log_{10}10 = 1)$

$L = 240$ decibels

Solved Exercise 2.4

1. Without using calculator, evaluate the following:

(i) $\log_2 18 - \log_2 9$

Sol. $\log_2 18 - \log_2 9$
$= \log_2\frac{18}{9}$
$= \log_2 2 = 1$

(iii) $\frac{1}{3}\log_2 8 - \log_2 18$

Sol. $\frac{1}{3}\log_2 8 - \log_2 18 = \log_2(8)^{\frac{1}{3}} - \log_2 18$
$= \log_2(2^3)^{\frac{1}{3}} - \log_2 18 = \log_2(2^{3\times\frac{1}{3}}) - \log_2 18$
$= \log_2 2 - \log_2 18 = \log_2\frac{2}{18} = \log_2\frac{1}{9}$
$= \log_2\frac{1}{3^2} = \log_2 3^{-2} = -2\log_2 3 = -2(1) = -2$

(v) $\frac{1}{3}\log_2 64 + 2\log_2 25$

Sol. $\frac{1}{3}\log_2 64 + 2\log_2 25$
$= \log_2(64)^{\frac{1}{3}} + \log_2(25)^2$
$= \log_2(4\times4\times4)^{\frac{1}{3}} + \log_2(5\times5)^2$
$= \log_2 4^{3\times\frac{1}{3}} + \log_2 5^{2\times2}$
$= \log_2 4 + \log_2 5^4$
$= 1 + 4\log_2 5 = 1 + 4(1) = 1 + 4 = 5$

(ii) $\log_2 64 + \log_2 2$

Sol. $\log_2 64 + \log_2 2$
$= \log_2 64\times2$
$= \log_2 2\times2\times2\times2\times2\times2\times2$
$= \log_2 2^7$
$= 7\log_2 2$
$= 7(1)$
$= 7$

(iv) $2\log 2 + \log 25$

Sol. $2\log 2 + \log 25$
$= \log 2^2 + \log 25$
$= \log 4 + \log 25$
$= \log 4\times25$
$= \log 100 = 2$

(vi) $\log_3 12 + \log_3 0.25$

Sol: $\log_3 12 + \log_3 0.25$
$= \log_3(12\times0.25)$
$= \log_3 3 = 1$

2. Write the following as a single logarithm.

(i) $\frac{1}{2}\log 25 + 2\log 3$

sol. $\frac{1}{2}\log 25 + 2\log 3$

$= \log(25)^{\frac{1}{2}} + \log 3^2 = \log(25^{\frac{1}{2}} \times 3^2)$

$= \log(\sqrt{25} \times 9) = \log(5 \times 9)$

$= \log(45)$

(iii) $\log_5 b^2 . \log_a 5^3$

Sol. $\log_5 b^2 . \log_a 5^3$

$= \frac{\log b^2}{\log 5} \times \frac{\log 5^3}{\log a}$

$= \frac{2\log b}{\log 5} \times \frac{3\log 5}{\log a} = \frac{6\log b}{\log a} = \log_a^b$

(v) $4\log_5 x - \log_5 y + \log_5 z$

Sol. $4\log_5 x - \log_5 y + \log_5 z$

$\log_5 x^4 - \log_5 y + \log_5 z$

$= \log_5\left(\frac{x^4 . z}{y}\right)$

(ii) $\log 9 - \log\frac{1}{3}$

Sol. $\log 9 - \log\frac{1}{3}$

$\log\left(\frac{9}{\frac{1}{3}}\right) = \log(9 \times 3)$

$= \log 27$

(iv) $2\log_3 x + \log_3 y$

Sol. $2\log_3 x + \log_3 y$

$= \log_3 x^2 + \log_3 y$

$= \log_3(x^2 . y)$

(vi) $2\ln a + 3\ln b - 4\ln c$

Sol. $2\ln a + 3\ln b - 4\ln c$

$\ln a^2 + \ln b^3 - \ln c^4$

$\ln\left(\frac{a^2 \times b^3}{c^4}\right)$

3. Expand the following using law of logarithms:

(i) $\log\left(\frac{11}{5}\right)$

Sol. $\log\left(\frac{11}{5}\right)$

$= \log 11 - \log 5$

(iii) $\ln\left(\frac{a^2 b}{c}\right)$

Sol. $\ln\left(\frac{a^2 b}{c}\right)$

$= \ln a^2 + \ln b - \ln c$

$2\ln a + \ln b - \ln c$

(ii) $\log_5\sqrt{8a^6}$

Sol. $\log_5\sqrt{8a^6}$

$= \log_5(8a^6)^{\frac{1}{2}} = \frac{1}{2}\log_5(8a^6)$

$= \frac{1}{2}\left[\log_5 8 + \log_5 a^6\right]$

$= \frac{1}{2}\left[\log_5 2^3 + 6\log_5 a\right]$

$= \frac{1}{2}\left[3\log_5 2 + 6\log_5 a\right] = \frac{3}{2}\log_5 2 + \frac{6}{2}\log_5 a$

$= \frac{3}{2}\log_5 + 3\log_5 a$

(v) $\ln\sqrt[3]{16x^3}$ **Sol.** $\ln\sqrt[3]{16x^3}$

$= \ln(16x^3)^{\frac{1}{3}} = \frac{1}{3}\ln(16x^3)$

$= \frac{1}{3}\ln(2^4 x^3) = \frac{1}{3}\left[\ln 2^4 + \ln x^3\right]$

$= \frac{1}{3}\left[4\ln 2 + 3\ln x\right]$

$= \frac{4}{3}\ln 2 + \frac{3}{3}\ln x$

$= \frac{4}{3}\ln 2 + \ln x$

(iv) $\log\left(\frac{xy}{z}\right)^{\frac{1}{9}}$

Sol. $\log\left(\frac{xy}{z}\right)^{\frac{1}{9}} = \frac{1}{9}\log\left(\frac{xy}{z}\right) = \frac{1}{9}\left[\log x + \log y - \log z\right]$

(vi) $\log_2\left(\frac{1-a}{b}\right)^5$

Sol. $\log_2\left(\frac{1-a}{b}\right)^5 = 5\log_2\left(\frac{1-a}{b}\right)$

$= 5\left[\log_2(1-a) - \log_2 b\right]$

4. Find the value of x in the following equations:

(i) $\log 2 + \log x = 1$

Sol. $\log 2 + \log x = 1$

$\log(2 \times x) = 1$

$\log_{10} 2x = 1$

$10^1 = 2x$

$2x = 10$

$x = \frac{10}{2}$

$x = 5$

(iii) $(81)^x = (243)^{x+2}$

Sol. $(81)^x = (243)^{x+2}$

$(3\times3\times3\times3)^x = (3\times3\times3\times3\times3)^{x+2}$

$(3^4)^x = (3^5)^{x+2}$

$3^{4x} = (3)^{5x+10}$

$5x + 10 = 4x$

$5x - 4x = -10$

$x = -10$

(v) $\log(5x - 10) = 2$

Sol. $\log(5x - 10) = 2$

$\log_{10}(5x - 10) = 2$

$10^2 = 5x - 10$

$100 = 5x - 10$

$5x - 10 = 100$

$5x = 100 + 10$

$5x = 110$

$x = \frac{110}{5}$

$x = 22$

(ii) $\log_2 x + \log_2 8 = 5$

Sol. $\log_2 x + \log_2 8 = 5$

$\log_2(8x) = 5$

$2^5 = 8x$

$32 = 8x$

$\frac{32}{8} = x$

$4 = x$

So, $x = 4$

(iv) $\left(\frac{1}{27}\right)^{x-6} = 27$

Sol. $\left(\frac{1}{27}\right)^{x-6} = 27$

$\left(\frac{1}{3\times3\times3}\right)^{x-6} = 3\times3\times3$

$\left(\frac{1}{3^3}\right)^{x-6} = 3^3$

$(3^{-3})^{x-6} = 3^3$

$3^{-3x+18} = 3^3$

$-3x + 18 = 3$

$-3x = 3 - 18$

$-3x = -15$

$3x = 15$

$x = \frac{15}{3}$

$x = 5$

(vi) $\log_2(x+1) - \log_2(x-4) = 2$

Sol. $\log_2(x+1) - \log_2(x-4) = 2$

$$\log_2\left(\frac{x+1}{x-4}\right) = 2$$

$$2^2 = \frac{x+1}{x-4} \Rightarrow 4 = \frac{x+1}{x-4}$$

$4(x-4) = x+1$

$4x - 16 = x + 1$

$4x - x = 1 + 16$

$$3x = 17 \Rightarrow x = \frac{17}{3} = 5\frac{2}{3}$$

5. Find the values of the following with the help of logarithm table:

(i) $\dfrac{3.68 \times 4.21}{5.234}$

Sol. $\dfrac{3.68 \times 4.21}{5.234}$.

let, $x = \dfrac{3.68 \times 4.21}{5.234}$

$\log x = \log\left(\dfrac{3.68 \times 4.21}{5.234}\right)$

$\log x = \log 3.68 + \log 4.21 - \log 5.234$

$\log x = 0.5658 + 0.6243 - 0.7188$

$\log x = 0.4713$

$x = \text{Anti log } 0.4626$

$x = 2.960$

(ii) $4.67 \times 2.11 \times 2.397$

Sol. $4.67 \times 2.11 \times 2.397$

Let, $x = 4.67 \times 2.11 \times 2.397$

$\log x = \log(4.67 \times 2.11 \times 2.397)$

$\log x = \log 4.67 + \log 2.11 + \log 2.397$

$\log x = 0.6693 + 0.3243 + 0.3797$

$\log x = 1.3733$

$x = \text{Antilog } 1.3733$

$x = 23.62$

(iii) $\dfrac{(20.46)^2 \times (2.4122)}{754.3}$

Sol. $\dfrac{(20.46)^2 \times (2.4122)}{754.3}$

Let, $x = \dfrac{(20.46)^2 \times (2.4122)}{754.3}$

$\log x = \log\left(\dfrac{(20.46)^2 \times (2.4122)}{754.3}\right)$

$\log x = \log(20.46)^2 + \log(2.4122) - \log 754.3$

$\log x = 2\log(20.46) + \log(2.4122) - \log 754.3$

$\log x = 2(1.3109) + (0.3824) - (2.8775)$

$\log x = 2.6218 + 0.3824 - 2.8775$

$\log x = 0.1267$

$x = \text{Antilog } 0.1267$

$x = 1.339$

(iv) $\dfrac{\sqrt[3]{9.364} \times 21.64}{3.21}$

Sol. $\dfrac{\sqrt[3]{9.364} \times 21.64}{3.21}$

Let, $x = \dfrac{\sqrt[3]{9.364} \times 21.64}{3.21}$

$\log x = \log\dfrac{\sqrt[3]{9.364} \times 21.64}{3.21}$

$\log x = \log(9.364)^{\frac{1}{3}} + \log 21.64 - \log 3.21$

$\log x = \dfrac{1}{3}\log 9.364 + \log 21.64 - \log 3.21$

$\log x = \dfrac{1}{3}(0.9715) + 1.3353 - 0.5065$

$\log x = 0.3238 + 1.3353 - 0.5065$

$\log x = 1.1526$

$x = \text{Antilog } 1.1526$

$x = 14.21$

6. The formula to measure the magnitude of earthquakes is given by $M = \log_{10}\left(\dfrac{A}{A_0}\right)$. If amplitude (A) is 10,000 and reference amplitude (A_0) is 10. What is the magnitude of the earthquake?

Sol. $A = 10{,}000$

$A_0 = 10$

$M = ?$

$$M = \log_{10}\left(\frac{A}{A_0}\right)$$

$$M = \log_{10}\left(\frac{10000}{10}\right)$$

$M = \log_{10}(1000)$

$M = 3$

Magnitude of earthquakes = 3

7. Abdullah invested Rs. 100,000 in a saving scheme and gains interest at the rate of 5% per annum so that the total value of this investment after t years is.Rs y. This is modelled by an equation $y = 100{,}000\,(1.05)^t$, $t \geq 0$. Find after how many years the investment will be double.

Sol. $y = 100.000\,(1.05)^t$

$2\times10^{5000} = 100000(1.05)^t$

$\log(2\times100000) = t(100000)(1.05)^t$

$\log(2\times10^5) = \log(10^5)(1.05)^t$

$\log 2 + \log 10^5 = \log 10^5 + \log(1.05)^t$

$\log 2 + 5\log 10 = 5\log 10 + t\log(1.05)$

$0.3010 + 5(1) = 5(1) + t(0.0212)$

$0.3010 + 5 = 5 + t(0.0212)$

$0.3010 + 5 - 5 = t(0.0212)$

$0.3010 = t(0.0212)$

$\frac{0.3010}{0.0212} = t$

$t = 14$

So, Investment will be double in 14 years.

8. Huria is hiking up a mountain where the temperature (T) decreases by 3% (or a factor of 0.97) for every 100 metres gained in altitude. The initial temperature (T_i) at sea level is 20°C. Using the formula $T = T_i \times 0.97^{\frac{h}{100}}$, calculate the temperature at an altitude (h) of 500 metres.

Sol. $T_i = 20°c$

$h = 500m$

$T = T_i \times 0.97^{\frac{h}{100}}$

$T = 20 \times (0.97)^{\frac{500}{100}}$

$T = 20 \times (0.97)^5$

$\log T = \log(20\times(0.97)^5)$

$\log T = \log 20 + \log(0.97)^5$

$\log T = \log 20 + 5\log(0.97)$

$\log T = 1.3010 + 5(-0.0132)$

$\log T = 1.3010 - 0.066$

$\log T = 1.235$

$T = \text{Anti log } 1.235$

$T = 17.18$

At an altitude of 500 meters, the temperature would be approximately 17.18°C.

Review Exercise 2

1. Four options are given against each statement. Encircle the correct option.

i. The standard form of 5.2×10^6 is:
(a) 52,000 (b) 520,000 (c) 5,200,000 (d) 52,000,000

ii. Scientific notation of 0.00034 is:
(a) 3.4×10^{-3} (b) 3.4×10^{-4} (c) 3.4×10^{-5} (d) 3.4×10^{-6}

iii. The base of common logarithm is:
(a) 2 (b) 10 (c) 5 (d) e

iv. $\log_2 2^3 =$ _____
(a) 1 (b) 2 (c) 5 (d) 3

v. $\log 100 =$ _____
(a) 2 (b) 3 (c) 10 (d) 1

vi. If $\log 2 = 0.3010$, then $\log 200$ is:
(a) 1.3010 (b) 0.6010 (c) 2.3010 (d) 2.6010

vii. $\log(0) =$
(a) positive (b) negative (c) zero (d) undefined

viii. $\log 10{,}000 =$
(a) 2 (b) 3 (c) 4 (d) 5

ix. $\log 5 + \log 3 =$ _____
(a) $\log 0$ (b) $\log 2$ (c) $\log\left(\frac{5}{3}\right)$ (d) $\log 15$

x. $3^4 = 81$ in logarithmic form is:
(a) $\log_3 4 = 81$ (b) $\log_4 3 = 81$ (c) $\log_3 81 = 4$ (d) $\log_4 81 = 3$

Answers:

i. 5,200,000	ii. 3.4×10^{-4}	iii. 10	iv. 3
v. 2	vi. 2.3010	vii. undefined	viii. 4
ix. $\log 15$	x. $\log_3 81 = 4$		

2. Express the following numbers in scientific notation.

(i) 0.000567
Sol. $0.000567 = 5.67 \times 10^{-4}$

(ii) 734
Sol. $734 = 7.34 \times 10^2$

(iii) 0.33×10^3
Sol. 0.33×10^3
$= 3.3 \times 10^{-1} \times 10^3$
$= 3.3 \times 10^{-1+3}$
$= 3.3 \times 10^2$

3. Express the following numbers in ordinary notation.

(i) 2.6×10^3
Sol. 2.6×10^3
$= 2600$

(ii) 8.794×10^{-4}
Sol. 8.794×10^{-4}
$= 0.0008794$

(iii) 6×10^{-6}
Sol. 6×10^{-6}
$= 0.000006$

4. Express each of the following in logarithmic form.

(i) $3^7 = 2187$
Sol. $3^7 = 2187$
$\log_3 2187 = 7$

(ii) $a^b = c$
Sol. $a^b = c$
$\log_a c = b$

(iii) $(12)^2 = 144$
Sol. $(12)^2 = 144$
$\log_{12} 144 = 2$

5. Express each of the following in exponential for.

(i) $\log_4 8 = x$

Sol. $\log_4 8 = x$

$4^x = 8$

(ii) $\log_9 729 = 3$

Sol. $\log_9 729 = 3$

$9^3 = 729$

(iii) $\log_4 1024 = 5$

Sol. $\log_4 1024 = 5$

$4^5 = 1024$

6. Find value of x in the following:

(i) $\log_9 x = 0.5$

Sol. $\log_9 x = 0.5$

$\log_9 x = \frac{5}{10}$

$\log_9 x = \frac{1}{2}$

$9^{\frac{1}{2}} = x$

$3^{2\times\frac{1}{2}} = x$

$3 = x \Rightarrow x = 3$

(ii) $\left(\frac{1}{9}\right)^{3x} = 27$

Sol. $\left(\frac{1}{9}\right)^{3x} = 27 \Rightarrow \left(\frac{1}{3^2}\right)^{3x} = 3^3$

$(3^{-2})^{3x} = 3^3$

$3^{-6x} = 3^3 \Rightarrow -6x = 3$

$x = -\frac{3}{6} = -\frac{1}{2}$

(iii) $\left(\frac{1}{32}\right)^{2x} = 64$

$\left(\frac{1}{32}\right)^{2x} = 64$

$\left(\frac{1}{2\times2\times2\times2\times2}\right)^{2x} = 2\times2\times2\times2\times2\times2$

$\left(\frac{1}{2^5}\right)^{2x} = 2^6 \Rightarrow (2^{-5})^{2x} = 2^6$

$2^{-10x} = 2^6$

$-10x = 6 \Rightarrow 10x = -6$

$x = -\frac{6}{10} = -\frac{3}{5}$

7. Write the following as a single logarithms.

(i) $7\log x - 3\log y^2$

Sol. $7\log x - 3\log y^2$

$= \log x^7 - \log(y^2)^3$

$= \log x^7 - \log y^6$

$= \log\left(\frac{x^7}{y^6}\right)$

(ii) $3\log 4 - \log 32$

Sol. $3\log 4 - \log 32$

$= \log 4^3 - \log 32$

$= \log\left(\frac{4^3}{32}\right)$

$= \log\left(\frac{64}{32}\right)$

$= \log 2$

(iii) $\frac{1}{3}(\log_3 8 + \log_3 27) - \log_3 3$

Sol. $\frac{1}{3}(\log_3 8 + \log_3 27) - \log_3 3$

$= (\log_3 8 + \log_3 27)^{\frac{1}{3}} - \log_3 3$

$= (\log_3 8\times 27)^{\frac{1}{3}} - \log_3 3$

$= \log_3 \frac{(8\times27)^{\frac{1}{3}}}{3}$

$= \log_3 \frac{\sqrt[3]{8\times27}}{3} = \log_3 \sqrt[3]{\frac{2^3\times3^3}{3^3}} = \log_3 \frac{2\times3}{3} = \log_3 2$

8. Expand the following using laws of logarithms:

(i) $\log(xyz^6)$

Sol. $\log(xyz^6)$

$= \log x + \log y + \log z^6$

$= \log x + \log y + 6\log z$

(ii) $\log_3 \sqrt[5]{m^5n^3}$

Sol. $\log_3 \sqrt[5]{m^5n^3}$

$= \log_3 (m^5n^3)^{\frac{1}{6}}$

$= \frac{1}{6}\log_3 m^5n^3$

$= \frac{1}{6}\left[\log_3 m^5 + \log_3 n^3\right]$

$= \frac{1}{6}\left[5\log_3 m + 3\log_3 n\right]$

(iii) $\log\sqrt{8x^3}$

Sol. $\log\sqrt{8x^3} = \log(8x^3)^{\frac{1}{2}} = \frac{1}{2}\left[\log 8x^3\right]$

$= \frac{1}{2}\left[\log 2^3x^3\right] \Rightarrow \frac{1}{2}\left[\log 2^3 + \log x^3\right]$

$= \frac{1}{2}\left[3\log 2 + 3\log x\right] = \frac{3}{2}\left[\log 2 + \log x\right]$

9. Find the values of the following with the help of logarithm table:

(i) $\sqrt[3]{68.24}$

Sol. $\sqrt[3]{68.24}$

let,

$x = \sqrt[3]{68.24} \Rightarrow \log x = \log\sqrt[3]{68.24}$

$\log x = \log(68.24)^{\frac{1}{3}} \Rightarrow \log x = \frac{1}{3}\log(68.24)$

$\log x = \frac{1}{3}(1.8340) \Rightarrow \log x = 0.6113$

x = antilog 0.6113

$x = 4.0860$

(ii) 319.8×3.543

Sol. Let, $x = 319.8 \times 3.543$

$\log x = \log(319.8 \times 3.543)$

$\log x = \log 319.8 + \log 3.543$

$\log x = 2.5049 + 0.5494$

$\log x = 3.0543$

x = Antilog 3.0543

$x = 1133$

(iii) $\frac{36.12\times750.9}{113.2\times9.98}$

Let $x = \frac{36.12\times750.9}{113.2\times9.98}$

$\log x = \log \frac{36.12\times750.9}{113.2\times9.98}$

$\log x = \log 36.12 + \log 750.9 - \log 113.2 - \log 9.98$

$\log x = 1.5577 + 2.8756 - 2.0538 - 0.9991$

$\log x = 1.3804$

$x = \text{antilog } 1.3804$

$x = 24.01$

10. In the year 2016, the population of a city was 22 millions and was growing a rate of 2.5% per year. The function $p(t) = 22(1.025)^t$ gives the population millions, t years after 2016. Use the model to determine in which year t population will reach 35 millions. Round the answer to the nearest year.

Sol.

$p(t) = 22(1.025)^t$

$35 = 22(1.025)^t$

$\log 35 = \log[22(1.025)^t]$

$\log 35 = \log 22 + \log(1.025)^t$

$\log 35 - \log 22 = \log(1.025)^t$

$\log 35 - \log 22 = t \log(1.025)$

$1.5441 - 1.3424 = t(0.0107)$

$0.2017 = t(0.0107)$

$\frac{0.2017}{0.0107} = t$

$18.9 = t$

$t = 18.9 \approx 19$ years

So, the year when population will be 35 millions = 2016 + 19

= 2035

Objective Type Questions

Additional MCQ's

Multiple Choice Questions prepared in the light of new Examination Techniques (Knowledge, Understanding, Application, Analytical & Conceptual)

2.1 Scientific Notation

Encircle the correct option.

1. **A number in scientific notation is written as:**
(a) $a \times 10^n$ (b) $10 \times a^n$ (c) $10 \times n^a$ (d) $a \times n^{10}$

2. **Scientific notation of 65000000 is:**
(a) 65×10^7 (b) 6.5×10^7 (c) 0.65×10^7 (d) 0.0065×10^7

3. **Ordinary notation of 2.34×10^5 is:**
(a) 234000 (b) 2340000 (c) 2340000 (d) 23400

2.2 Logarithm

4. **The general form of logarithm is:**
(a) $\log_x xy$ (b) $\log_x y = b$ (c) $\log_{xy} = b$ (d) $\log_x(x) = y$

5. **Exponential form of $\log_2 16 = 4$ is:**
(a) $2^4 = 16$ (b) $4^2 = 16$ (c) $16^2 = 4$ (d) $16^4 = 2$

6. **If $\log_x 25 = x$, then x =**
(a) $x = 1$ (b) $x = 2$ (c) $x = 3$ (d) $x = 4$

7. **If $3^4 = 81$ then logarithmic form is:**
(a) $\log_3 4 = 81$ (b) $\log_4 3 = 81$ (c) $\log_3 81 = 4$ (d) $\log_{81} 3 = 4$

2.3 Common Logarithm

8. **A logarithm of a number consists of how many parts?**
(a) Two (b) Three (c) Four (d) Five

9. **The integral part of logarithm is known as:**
(a) mantissa (b) characteristic (c) real (d) natural

10. **In log 725, The Characteristic is:**
(a) 0 (b) 1 (c) 2 (d) 3

11. **In log 0.00045, the characteristic is:**
(a) $\bar{1}$ (b) $\bar{2}$ (c) $\bar{3}$ (d) $\bar{4}$

12. **The decimal part of logarithm is:**
(a) mantissa (b) characteristic (c) real (d) imaginary

13. **What is the logarithm of 345?**
(a) 1.5378 (b) 2.5378 (c) 3.5378 (d) 4.5378

14. **The logarithm of 0.0036 is:**
(a) 0.4437 (b) $\bar{1}$.4437 (c) $\bar{2}$.4437 (d) $\bar{3}$.4437

15. **In $\log x = 0.2568$, the value of x is:**
(a) 4.806 (b) 3.806 (c) 2.806 (d) 1.806

16. **In log x = –2.1234, The value of x is:**
(a) 0.007526 (b) 0.07626 (c) 0.7626 (d) 7626

17. **The natural logarithm is the logarithm with base ___**
(a) 1 (b) 10 (c) e (d) n

18. **The value of e is:**
(a) 71828 (b) 0.71828 (c) 1.71828 (d) 2.71828

2.4 Laws of Logarithm

19. $\log_b x + \log_b y =$ _____
(a) $\log_b xy$ (b) $\log_x by$ (c) $\log_y bx$ (d) $\log_{xy} a$

20. $\log_b x - \log_b y =$ _____
(a) $\log_b \frac{-y}{x}$ (b) $\log_b \frac{x}{y}$ (c) $\log_x \left(\frac{b}{y}\right)$ (d) $\log_y \left(\frac{b}{x}\right)$

21. $\log_b x^n =$ _____
(a) $nx\log_b$ (b) $b \log nx$ (c) $n \log_b x$ (d) $n \log_x n$

22. $\frac{\log_x x}{\log_x b} =$ _____
(a) $\log x^b$ (b) $\log_b x$ (c) $\log \frac{x}{b}$ (d) $\log_b x$

23. $\log_3 20 =$ _____
(a) $2 \log_3 2 + \log_3 5$ (b) $2 \log_3 2 + \log_3 10$
(c) $2 \log_3 10 + \log_3 10$ (d) $2 \log_3 4 + \log \log_3 5$

24. $\log_2 \left(\frac{x-y}{z}\right)^3 =$ _____
(a) $[\log_2 (x-y) - \log y]$ (b) $3 [\log_2 (x-y) - \log_2 z]$
(c) $\log_2 x - \log_2 y - \log_2 2$ (d) $3 \log_2 x - 3\log_2 y$

25. $2 \log_3 10 - \log_3 4 =$ _____
(a) $\log_3 40$ (b) $\log_3 20$ (c) $\log_3 25$ (d) $\log_3 8$

Answers:

1. $a \times 10^n$	2. 6.5×10^7	3. 2340000	4. $\log_b (x) = y$	5. $2^4 = 16$
6. $x = 2$	7. $\log_3 81 = 4$	8. Two	9. characteristic	10. 2
11. $\bar{4}$	12. Mantissa	13. 2.5378	14. $\bar{2}.4437$	15. 1.806
16. 0.007526	17. e	18. 2.71828	19. $\log_b xy$	20. $\log_b \frac{x}{y}$
21. $n \log_b x$	22. $\log_b x$	23. $2 \log_3 2 + \log_3 5$	24. $3 [\log_2 (x-y) - \log_2 z]$	25. $\log_3 25$

Conceptual MCQ's

1. **When we moved the decimal to the _____ the exponent is _____.**
(a) left , positive (b) right , positive (c) left , negative (d) none of these

2. **Convert 5.55×10^{-4} to ordinary notation:**
(a) 0.00555 (b) 0.000555 (c) 0.00500 (d) 0.000555000

3. **Swiss mathematician and physicist Leonhard Euler introduced _____ for the base of natural logarithm.**
(a) e (b) $\log_{10}$ (c) both of a and b (d) None of above

4. **$\log_b xy = \log_b x + \log_b y$ is known as:**
(a) product law (b) quotient law (c) power law (d) change of base law

5. **ℓn (1) = _____:**
(a) 3 (b) 2 (c) 1 (d) 0

6. **$\log_{10} 10 =$ _____**
(a) 0 (b) 1 (c) e (d) 00

7. **log 0 = _____:**
(a) 1 (b) 0 (c) 10 (d) undefined

8. **log (1) = _____**
(a) 1 (b) 0 (c) 10 (d) undefined

9. **$\ell n0 =$ _____:**
(a) undefined (b) 0 (c) 1 (d) 10

10. **ℓn (e) _____:**
(a) undefined (b) 0 (c) 1 (d) 10

Answers:

1. right positive	2. 0.000555	3. e	4. product law	5. 0
6. 1	7. undefined	8. 0	9. undefined	10. 1

Additional Short Questions

Short Answer Questions prepared in the light of new Examination Techniques (Knowledge, Understanding, Application, Analytical & Conceptual)

2.1 Scientific Notation

1. **What do you mean by scientific notation method?**

Ans. Scientific notation is a method used to express very large or small numbers in a more manageable form. It is commonly used in science, engineering and mathematics to simplify complex calculations. A number in scientific notation is written as:
$a \times 10n$, where $1 \leq a < 10$ and $n \in Z$
Here a is called the coefficient or base number.

2. **How we write a number in scientific notation.**

Sol. $a \times 10^n$ where $1 \leq a < 10$ and $n \in Z$

3. **Convert 0.0000000315 into scientific notation.**

Sol. $0.0000000315 = 3.15 \times 10^{-8}$

4. **Convert 6.23×10^{-4} to ordinary notation.**

Ans. 6.23×10^{-4}

$$= \frac{623}{100} \times 10^{-4} = \frac{623}{100 \times 10^4} = \frac{623}{10^6} = \frac{623}{1000000} = 0.000623$$

2.2 Logarithm

5. What do you know about logarithm?

Ans. A logarithm is based on two Greek words, logos and arithmos, which means ratio or proportion.

6. Who introduced word logarithm?

Sol. John Napier, a Scottish mathematician introduced the word logarithm.

7. Why we use logarithm?

Sol. It is a way to simplify complex calculations, especially those involving multiplication and division of large numbers.

8. What is the number of logarithm tells us?

Sol. The logarithm of a number tells us how many times one number must be multiplied by itself to get another number.

9. Write general form of logarithm.

Sol. The general form of logarithm is, $\log_b(x) = y$

Where b is base, x is result and y is exponent.

10. Convert $\log_{10} 100 = 2$ into exponential form:

Sol. $\log_{10} 100 = 2 \Rightarrow 10^2 = 100$

11. If $\log_2 x = 6$ then find x.

Ans. $\log_2 x = 6$

$2^6 = x$

$x = 64$

12. Convert $7^0 = 1$ into logarithm form:

Ans. $7^0 = 1$

$\log_7 1 = 0$

2.3 Common Logarithm

13. How many parts of logarithm? Name them.

Ans. A logarithm of a number consists of two parts. **(i)** Characteristic **(ii)** Mantissa

14. What is characteristic?

Ans. The Characteristic is the integral part of the logarithm. It tells us how big or small the number is.

15. Write the rules for finding the characteristics:

Ans. (i) For a number greater than 1. Characteristic = number of digits to the left of the decimal point – 1. For example in log 567, the characteristics = 3 – 1 = 2 decimal point – I.e. g. In log 567 the characteristic = 3 – 1 = 2

(ii) For a number less than 1. characteristic = – (No. of zeros between the decimal point and the first non-zero digit + 1 e.g. In log 0.0123.

Characteristic = $-(1 + 1) = -2 = \bar{2}$

16. Find the characteristic of log 9.87

Sol. log 9.87

characteristic = 1 – 1 = 0

17. What is mantissa in logarithm?

Ans. The mantissa is the decimal part of logarithm. It represents the fractional component and is always positive e.g in log 5000 = 3.698 the mantissa is 0.698.

18. Find the logarithm of 5.678:

Sol. 5.678

Characteristic = 1 – 1 = 0

mantissa = 0.7542 (7536 + 6=7542)

so, log 5.678 = 0 + 0 .7542 = 0.7542

19. What do you mean by antilogarithm?

Ans. An antilogarithm is the inverse operation of logarithm. An antilogarithm helps to find a number whose logarithmic value is given.

if $\log_b(x) = y \Leftrightarrow b^y = x$ then the process of finding x is called antilogarithm of y.

20. What is reference position?

Ans. The place between the first non-zero digit from left and its next digit is called reference position.

21. Find value of x if logx = –2.1234

Sol. Given that logx = –2.1234

Since mantisa is negative, so we make it positive by adding and subtracting 3.

log x = –3 + 3 – 2.1234

= –3 + 0.8766 = $\bar{3}$.8766

Now characteristic = $\bar{3}$

mantissa = 0.8766

Table value of 0.8766 = 7516 + 10 = 7526

x = antilog ($\bar{3}$.8766)

= 0.007526

22. Define natural logarithm:

Ans. The natural logarithm is the logarithm with base e, where is a mathematical constant whose value is 2.71828

23. Write the differences between common and natural logarithm.

Ans.

Common logarithm	Natural logarithm
(1) The base of common logarithm is 10.	(1) The base of natural logarithm is e.
(2) It is written as $\log_{10}(x)$.	(2) It is written as $\ell n\,(x)$.
(3) It is used widely in everyday calculations especially in scientific and engineering applications.	(3) It is used in higher level mathematics particularly calculus and applications involving growth decay processes.

2.4 Laws of logarithm

24. Prove that $\log_b xy = \log_b x + \log_b y$:

Sol. Let $m = \log_b x$ ____ (1), $n = \log_b y$ ____ (2)

$x = b^m$ and $y = b^n$

$xy = b^m.b^n \Rightarrow xy = b^{m+n}$

$\log_b xy = m + n$

$\log_b xy = \log_b x + \log_b y$

25. Prove $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

Sol. Let,

$m = \log_b x$ __ (1), $n = \log_b y$ ____ (2)

$x = b^m$ and $y = b^n$

$\frac{x}{y} = \frac{b^m}{b^n} \Rightarrow \frac{x}{y} = b^{m-n}$

Its logarithmic form is:

$$\log_b\left(\frac{x}{y}\right) = m-n$$

$$\log_b \frac{x}{y} = \log_b x - \log_b y \text{ (from (1) and (2))}$$

26. Prove $\log_b x^n = n \log_b x$

Sol. Let, $m = \log_b x$

Its exponential from is:

$x = b^m$

Raise both sides to the power n.

$x^n = b^{mn}$

$\log_b x^n = nm$

$\log_b x^n = n \log_b x$

27. Prove $\log_b x = \frac{\log_a x}{\log_a b}$

Sol. Let $m = \log_b x$ ____ (1)

Its exponential from is:

$b^m = x$

$\log_a b^m = \log_a x$

$\log_a^b = \log_a^x$

$m = \frac{\log_a x}{\log_a b}$

$\log_b x = \frac{\log_a x}{\log_a b}$ (from (1)

28. Expand $\log_{32} 27$

Sol. $\log_{32}^{27} = \frac{\log^{27}}{\log^{32}} = \frac{\log^{3^3}}{\log^{2^5}} = \frac{3\log^3}{5\log^2} = \frac{3}{5}\log_2 3$ $\left(\because \log_b x = \frac{\log_a x}{\log_a b}\right)$

29. Expand $\log_5\left(\frac{xy}{z}\right)^8$

Sol. $\log_5\left(\frac{xy}{z}\right)^8 = 8\log_5\left(\frac{xy}{z}\right)$

$= 8(\log_5 (xy) - \log_5 (z))$

$= 8(\log_5 x + \log_5 y - \log_5 z)$

30. Write $6\log_3 x + 2\log_3 11$ in sin logarithm:

Sol. $6\log_3 x + 2\log_3 11$

$= \log_3 x^6 + \log_3 (11)^2$

$= \log_3 x^6 + \log_3 121$

$= \log_3 (121 x^6)$

Conceptual Short Questions

1. Convert 0.000006 into scientific notation.

Sol. 0.000006

$= \frac{6}{1000000} = \frac{6}{10^6} = 6\times10^{-6}$

2. Convert 6.6×10^{-5} into ordinary notation.

Sol. 6.6×10^{-5}

$= \frac{66}{10}\times10^{-5}$

$= \frac{66}{10\times10^5} = \frac{66}{10^6}$

$= \frac{66}{1000000} = 0.000066$

3. Write the general form of logarithm?

Ans. The general form of logarithm is

$\log_b (x) = y$

5. What is the answer of $\log_a (a)$.

Ans. The answer of $\log_a (a)$ is 1.

4. When we write a number with bar?

Ans. When characteristic is negative we write it with bar.

6. What do you mean by reference position.

Ans. The place between the first non-zero digit from left and its next digit is known as reference position.

7. What is the quotient law of logarithm?

Ans. $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

8. Define power law of logarithm.

Ans. The logarithm of a number raised to a power is the product of the power and the logarithm of the base number.

$\log_b x^n = n \log_b x$

9. Write the change of base law of logarithm?

Ans. This law allows to change the base of a logarithm from "b" to any other base a.

$$\log_b x = \frac{\log_a x}{\log_a b}$$

10. What is the value of $\ell n(e)$?

Ans. The value of $\ell n(e)$ is 1.

— ✲✲✲ —

Unit 3

Sets and Functions

Set

A set is described as a well-defined collection of distinct objects, numbers or elements, so that we may be able to decide whether the object belongs to the collection or not.

Capital letters A, B, C, X, Y, Z etc., are generally used as names of sets and small letters a, b, c, x, y, z etc., are used as members or elements of sets.

There are three different ways of describing a set.

(i) **The Descriptive form:** A set may be described in words. For instance, the set of all vowels of the English alphabet.

(ii) **The Tabular form:** A set may be described by listing its elements within brackets. If A is the set mentioned above, then we may write:

$$A = \{a, e, i, o, u\}$$

The tabular form is also known as the Roster form.

(iii) **Set-builder method:** It is sometimes more convenient or useful to employ the method of set-builder notation in specifying sets. This is done by using a symbol or letter for an arbitrary set member and stating the property common to all the members. Thus, the above set may be written as:

$$A = \{x \mid x \text{ is a vowel of the English alphabets}\}$$

This is read as A is the set of all x such that x is a vowel of the English alphabets.

The symbol used for membership of a set is $\in$. Thus, $a \in A$ means a is an element of A or a belongs to A. $c \notin A$ means c does not belong to A or c is not a member of A. Elements of a set can be anything: people, countries, rivers, objects of our thought. In algebra, we usually deal with sets of numbers. Such sets, along with their names are given below: -

N = The set of natural numbers = {1, 2, 3, ...}
W = The set of whole numbers = {0, 1, 2, ...}
Z = The set of integers = {0, ±1, ±2, ...}
O = The set of odd integers = {±1, ±3, ±5,...}
E = The set of even integers = {0, ±2, ±4, ...}
P = The set of prime numbers = {2,3,5,7,11,13,17,...}

Q = The set of all rational numbers $= \left\{x \mid x = \frac{p}{q} \text{ where } p, q \in Z \text{ and } q \neq 0\right\}$

Q' = The set of all irrational numbers $= \left\{x \mid x \neq \frac{p}{q} \text{ where } p, q \in Z \text{ and } q \neq 0\right\}$

R = The set of all real numbers $= Q \cup Q'$

Singleton set: A set with only one element is called a singleton set. For example, {3}, {a}, and {Saturday} are singleton sets.

> **Remember!**
> The set {0} is a singleton set having zero as its only element, and not the empty set.

Empty set: The set with no elements (zero number of elements) is called an empty set, null set, or Void set. The empty set is denoted by the symbol $\varnothing$ or { }.

Equal sets: Two sets A and B are equal if they have exactly the same elements or if every element of set A is an element of set B. If two sets A and B are equal, we write A=B. Thus, the sets {1, 2, 3} and {2, 1,3} are equal.

Equivalent sets: Two sets A and B are equivalent if they have the same number of elements. For example, if A={a,b,c,d,e} and B={1,2,3,4,5}, then A and B are equivalent sets. The symbol ~ is used to represent equivalent sets. Thus, we can write A~B.

Subset: If every element of a set A is an element of set B, then A is a subset of B. Symbolically this is written as $A \subseteq B$ (A is a subset of B).

> **Remember!**
> The subset of a set can also be stated as follows
> $A \subseteq B$ iff $\forall x \in A \Rightarrow A \in B$

Super set: If A is sub set of B. In, such a case, we say B is a super set of A. symbolically this is written.

$$B \supseteq A \text{(B is a superset of A)}.$$

Proper subset: If A is a subset of B and B contains at least one element that is not an element of A, then A is said to be a proper subset of B. In such a case, we write:

$$A \subset B \text{(A is a proper subset of B)}.$$

> **Remember!**
> When we do not want to distinguish between proper and improper subsets, we may use the symbol $\subseteq$ for the relationship. It is easy to see that:
> $N \subset W \subset Z \subset Q \subset R$

Improper subset: If A is a subset of B and A = B, then we say that A is an improper subset of B. From this definition, it also follows that every set A is a subset of itself and is called an improper subset.

For example, let A = {a, b, c}, B = {c, a, b} and C = {a, b, c, d}, then clearly

$$A \subset C, B \subset C \text{ but } A = B.$$

Notice that each of sets A and B is an improper subset of the other because A = B.

Universal set: The set that contains all objects or elements under consideration is called the universal set or the universe of discourse. It is denoted by U.

Power set: The power set of a set S denoted by P(S) is the set containing all the possible subsets of S. For Example:

(i) If C= {a, b, c, d}, then
p(C) = {$\varnothing$, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, {a, b, c,d}}.

(ii) If D = {a}, then P(D) = {$\varnothing$, {a}}

If S is a finite set with n(S) = m representing the number of elements of the set S, then $n\{P(S)\} = 2^m$ is the number of the elements of the power set.

Solved Exercise 3.1

1. Write the following sets in set builder notation:

(i) {1, 4, 9, 16, 25, 36, ..., 484}
Sol. $\{x|x = n^2, n\in N, 1 \le n < 22\}$

(ii) {2, 4, 8, 16, 32, 64, ..., 150}
Sol. $\{x|x = 2^n \wedge 1 \le n < 8\}$

(iii) {0, ± 1, ± 2, ..., ± 1000}
Sol. $\{x|x \in Z, \wedge -1000 \le x \le 1000\}$

(iv) {6, 12, 18, ..., 120}
Sol. $\{x|x = 6n, n\in N\wedge 1 \le n \le 20\}$

(v) {100, 102, 104, ...,400}
Sol. $\{x|x \in E \wedge 100 \le x \le 400\}$

(vi) {1, 3, 9, 27, 81, ...}
Sol. $\{x|x = 3^n \wedge x\in W\}$

(vii) {1, 2, 4, 5, 10, 20, 25, 50, 100}
Sol. {x|x is divisor of 100}

(viii) {5, 10, 15, ..., 100}
Sol. $\{x|x = 5n, n\in N\wedge 1 \le n \le 20\}$

(ix) The set of all integers between –100 and 1000.
Sol. $\{x|x \in Z \wedge -100 < x < 1000\}$

2. Write each of the following sets in tabular form:

(i) {x|x is a multiple of 3∧ x ≤ 35}
Sol. {3, 6, 9, ..., 33}

(ii) $\{x|x \in = R \wedge 2x + 1 = 0\}$
Sol. $\left\{-\frac{1}{2}\right\}$

(iii) $\{x|x \in P \wedge x < 12\}$
Sol. {2, 3, 5, 7, 11}

(iv) {x|x is a divisor of 128}
Sol. {1, 2, 4, 8, 16, 32, 64, 128}

(v) $\{x|x = 2^n, n\in N \wedge n < 8\}$
Sol. {2, 4, 8, 16, 32, 64, 128}

(vi) $\{x|x \in N \wedge x + 4 = 0\}$
Sol. { }

(vii) $\{x|x \in N \wedge x = x\}$
Sol. {1, 2, 3, 4, 5, ...}

(viii) $\{x|x \in Z \wedge 3x + 1 = 0\}$ Sol. { }

3. Write two proper subsets of each of the following sets:

(i) {a, b, c}
Sol. {a},{b}

(ii) {0, 1}
Sol. {0},{1}

(iii) N
Sol. {1},{4}

(iv) Z
Sol. {2},{–2}

(v) Q
Sol. $\left\{\frac{2}{3}\right\}, \left\{\frac{7}{4}\right\}$

(vi) R
Sol. {3}, $\{\sqrt{3}\}$

(vii) $\{x|x \in Q \wedge 0 < x \le 2\}$ Sol. $\{1\}, \left\{\frac{3}{2}\right\}$

4. Is there any set which has no proper subset? If so, name that set
Sol. yes, empty set has no proper subset.

5. What is the difference between {a, b} and {{a,b}}?
Sol. {a,b} has two elements a and b while {{a,b}} has only one element {a,b}

6. What is the number of elements of the power set of each of the following sets?

(i) { }
Sol. $2^n = 2^0 = 1$

(ii) {0,1 }
Sol. $2^n = 2^2 = 4$

(iii) {1, 2, 3, 4, 5, 6, 7}
Sol. $2^n = 2^7 = 128$

(iv) {0, 1, 2, 3, 4, 5, 6, 7}
Sol. $2^n = 2^8 = 256$

(v) {a, {b,c}}
Sol. $2^n = 2^2 = 4$

(vi) {{a,b}, {b,c}, {d,e}}
Sol. $2^n = 2^3 = 8$

7. Write down the power set of each of the following sets:

(i) {9, 11} Sol. Power set of {9,11} = {ϕ, {9}, {11}, {9, 11}}

(ii) {+, –, x, ÷ }
Sol. Power set of {+, –, ×, ÷ } = {ϕ,{+}{–}, {×}, {÷}, {+,–}, {+,×}, {+,÷}, {–,×}, {–,÷}, {×, ÷}, {+, –, ×}, {+, –, ÷}, {+, ×, ÷}, {–, ×, ÷}, {+, –, ×, ÷}}

(iii) {ϕ} Sol. Power set of {ϕ} = {ϕ, {ϕ}}

(iv) {a, {b,c}} Sol. Power set of {a, {b,c}} = {ϕ, {a}, {b,c}, {a, {b,c}}

Operations on Sets

Just as operations of addition, subtraction etc., are performed on numbers, the operations of union, intersection etc., are performed on sets. We are already familiar with them. A review of the main rules is given below:

Union of Two Sets

The union of two sets A and B, denoted by A∪B, is the set of all elements which belong to A or B. Symbolically;

$$A\cup B = \{x|x \in A \vee x \in B\}$$

Thus if A = {1, 2, 3}, B = {2, 3, 4, 5}, then A ∪ B = {1, 2, 3, 4, 5}

Intersection of Two Sets

The intersection of two sets A and B, denoted by A∩B, is the set of all elements that belong to both A and B. Symbolically:

$$A\cap B = \{x|x \in A \wedge x \in B\}.$$

Thus, for the above sets A and B, A∩B = { 2, 3}.

> **Remember!**
> The symbol ∨ means or.
> The symbol ∧ means and.

Disjoint Sets

If the intersection of two sets is the empty set, the sets are said to be disjoint. For example, if

S_1 = The set of odd natural numbers and S_2 = The set of even natural numbers, then S_1 and S_2 are disjoint sets. Similarly, the set of arts students and the set of science students of a college are disjoint sets.

Overlapping Sets

If the intersection of two sets is non-empty but neither is a subset of the other, the sets are called overlapping sets, e.g., if

L = {2, 3, 4, 5, 6} and M= {5, 6, 7, 8, 9, 10}, then L and M are overlapping sets.

Difference of Two Sets

The difference between the sets A and B denoted by A-B, consists of all the elements that belong to A but do not belong to B.

Symbolically, $A-B = \{x|x \in A \wedge x \notin B\}$ and $B-A = \{x|x \in B \wedge x \notin A\}$

For example, if A = {1, 2, 3, 4, 5} and
B ={ 4, 5, 6,7, 8, 9, 10},
then A–B = {1,2,3} and B–A = {6, 7, 8, 9, 10}.

Notice that: A–B ≠ B–A.

Complement of a Set

The complement of a set A, denoted by A′ or A^c relative to the universal set U is the set of all elements of U, which do not belong to A.

> **Note:**
> In view of the definition of complement and difference sets it is evident that for any set A, A′ = U – A

Symbolically:

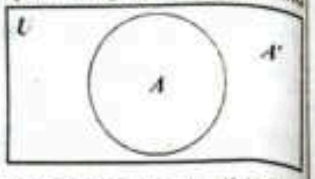
$$A' = \{x | x \in U \wedge x \notin A\}$$

For example, if U = Z, then E' = O and O' = E

For example, If U = Set of alphabets of English language, C = Set of consonants,
W = Set of vowels, then C' = W and W' = C.

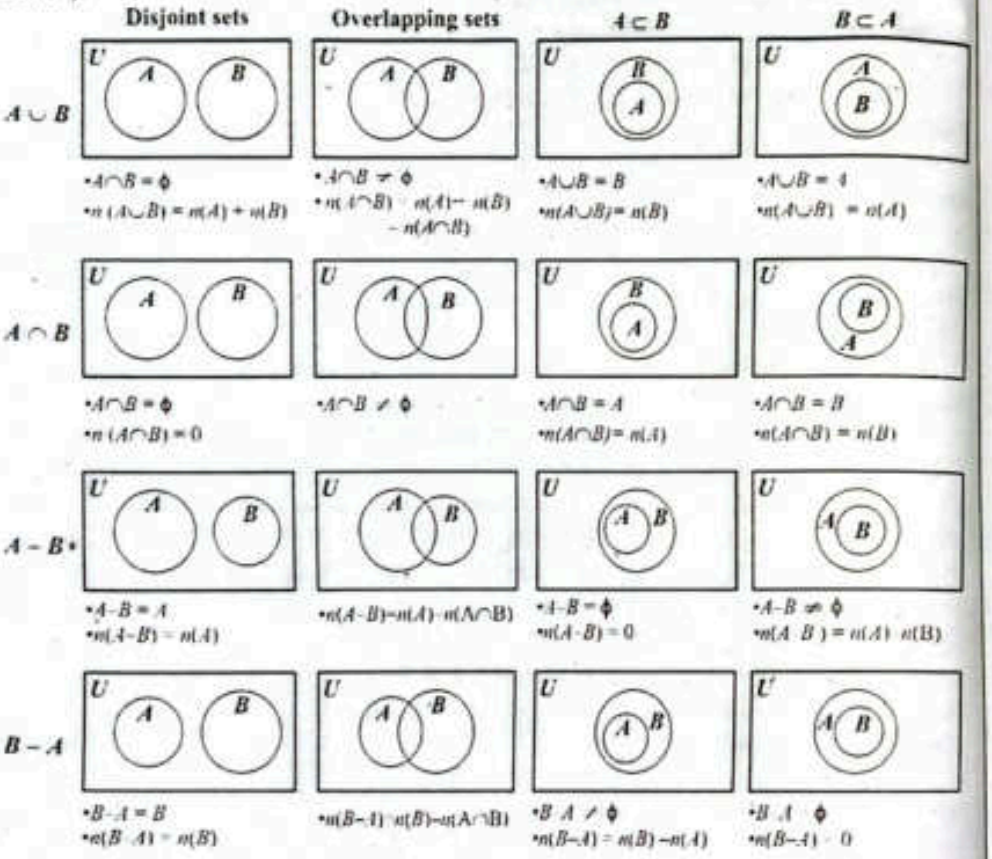
Identification of Sets Using Venn Diagram

Venn diagrams are very useful in depicting visually the basic concepts of sets and relationships between sets. These diagrams were first used by an English logician and mathematician John Venn (1834 to 1883 A.D).



In the adjoining figures, the rectangle represents the universal set U and the lined circular region represents a set A and the remaining portion of the rectangle represents the A' or U–A.

Below are given some more diagrams illustrating basic operations on two sets in different cases (the lined region represents the result of the relevant operation in each case shown below).



Operations on Three Sets

If A, B and C are three given sets, operations of union and intersection can be performed on them in the following ways:

(i) $A \cup (B \cup C)$	(ii) $(A \cup B) \cup C$	(iii) $A \cap (B \cap C)$
(iv) $(A \cap B) \cap C$	(v) $A \cup (B \cap C)$	(vi) $(A \cap C) \cup (B \cap C)$
(vii) $(A \cup B) \cap C$	(viii) $(A \cap B) \cup C$	(ix) $(A \cup C) \cap (B \cup C)$

Properties of union and intersection

We now state the fundamental properties of union and intersection of two or three sets.

Properties

(i) $A \cup B = B \cup A$	(Commutative property of Union)
(ii) $A \cap B = B \cap A$	(Commutative property of Intersection)
(iii) $A \cup (B \cup C) = (A \cup B) \cup C$	(Associative property of Union)
(iv) $A \cap (B \cap C) = (A \cap B) \cap C$	(Associative property of Intersection)
(v) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	(Distributivity of Union over intersection)
(vi) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	(Distributivity of intersection over Union)
(vii) $(A \cup B)' = A' \cap B'$ (viii) $(A \cap B)' = A' \cup B'$	(De Morgan's Laws)

Verification of the Properties Using Sets

Let A = { 1,2, 3}, B = {2, 3, 4, 5} and C= {3, 4, 5, 6, 7, 8}

(i) $A \cup B = \{1,2,3\} \cup \{2,3,4,5\}$; $B \cup A = \{2,3,4,5\} \cup \{1,2,3\}$
$= \{1,2,3,4,5\}$; $= \{1,2,3,4,5\}$
$\therefore A \cup B = B \cup A$

(ii) $A \cap B = \{1,2,3\} \cap \{2,3,4,5\}$; $B \cap A = \{2,3,4,5\} \cap \{1,2,3\}$
$= \{2,3\}$ $= \{2,3\}$
$\therefore A \cap B = B \cap A$

(iii) and (iv) Verify yourself.

(v) $A \cup (B \cap C) = \{1,2,3\} \cup [\{2,3,4,5\} \cap \{3,4,5,6,7,8\}]$
$= \{1,2,3\} \cup \{3,4,5\}$
$= \{1,2,3,4,5\}$ ...(i)

$(A \cup B) \cap (A \cup C) = [\{1,2,3\} \cup \{2,3,4,5\}] \cap [\{1,2,3\} \cup \{3,4,5,6,7,8\}]$
$= \{1,2,3,4,5\} \cap \{1,2,3,4,5,6,7,8\}$
$= \{1,2,3,4,5\}$ ...(ii)

From (i) and (ii), $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(vi) Verify yourself.

(vii) Let the universal set be U= { 1,2,3,4,5,6,7,8,9,10}

$A \cup B = \{1,2,3\} \cup \{2,3,4,5\} = \{1,2,3,4,5\}$
$(A \cup B)' = U - (A \cup B) = \{6,7,8,9,10\}$ ...(i)
$A' = U - A = \{4,5,6,7,8,9,10\}$
$B' = U - B = \{1,6,7,8,9,10\}$

$A' \cap B' = \{4, 5, 6, 7, 8, 9, 10\} \cap \{1, 6, 7, 8, 9, 10\}$

$= \{6, 7, 8, 9, 10\}$ ...(ii)

From (i) and (ii), $(A \cup B)' = A' \cap B'$

(viii) Verify yourself.

Verification of the properties with the help of Venn diagrams

(i) and (ii): Verification is very simple, therefore, do it by yourself.

(iii): In Fig. (1), set A is represented by a vertically lined region and $B \cup C$ is represented by a horizontally lined region. The set $A \cup (B \cup C)$ is represented by the region lined either in one way or both.

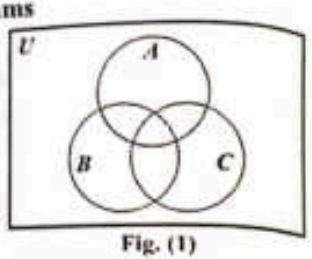


Fig. (1)

In Fig. (2) $A \cup B$ is represented by a horizontally lined region and C by a vertically lined region. $(A \cup B) \cup C$ is represented by the region lined in either one or both ways.

From Fig (1) and (2) we can see that

$A \cup (B \cup C) = (A \cup B) \cup C$

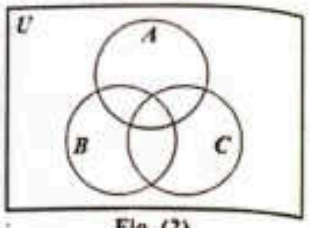


Fig. (2)

(iv) In Fig. (3), the doubly lined region represents

$A \cap (B \cap C)$

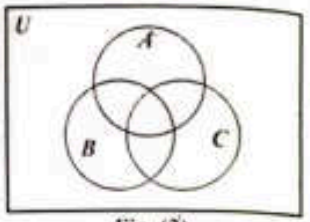


Fig. (3)

In Fig. (4), the doubly lined region represents $(A \cap B) \cap C$ Since in Fig. (3) and Fig. (4), these regions are the same, therefore, $A \cap (B \cap C) = (A \cap B) \cap C$.

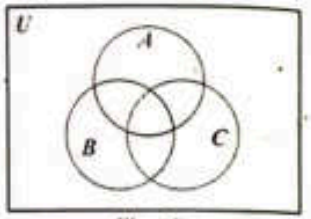


Fig. (4)

(v) In Fig. (5), $A \cup (B \cap C)$ is represented by the region which is lined horizontally or vertically or both ways.

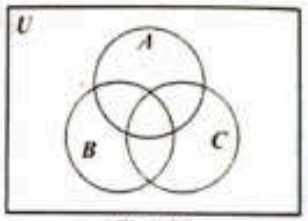


Fig. (5)

In Fig. (6), $(A \cup B) \cap (A \cup C)$ is represented by the doubly lined region.

Since the two regions in Fig (5) and (6) are the same, therefore.

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(vi) Verify yourselves.

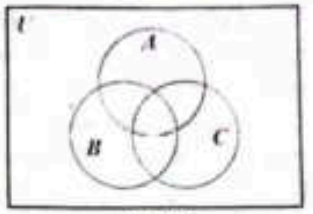


Fig. (6)

(vii) In Fig. (7), $(A \cup B)'$ is represented by a vertically lined region.

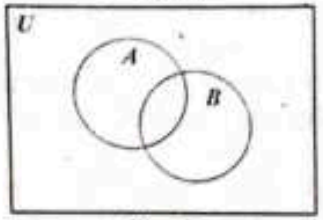


Fig. (7)

In Fig. (8), the doubly lined region represents $A' \cap B'$.

The two regions in Fig. (7) and (8) are the same, therefore,

$(A \cup B)' = A' \cap B'$

(viii) Verify yourselves.

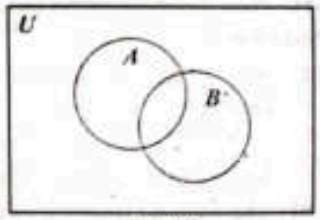


Fig. (8)

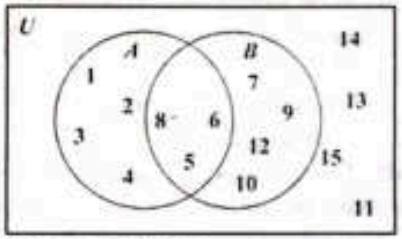
Note:
Only overlapping sets have been considered in the Venn diagrams above. Verification for other cases can be conducted similarly.

Example 1: Consider the adjacent Venn diagram illustrating two non-empty sets, A and B.

(a) Determine the number of elements common to sets A and B.

(b) Identify all the elements exclusively in set B and not in set A.

(c) Calculate the union of sets A and B.



Solution: From the information provided in the Venn diagram, we have:

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

$A = \{1, 2, 3, 4, 5, 6, 8\}$

$B = \{5, 6, 7, 8, 9, 10, 12\}$

(a) The elements in both sets A and B are the intersection of the sets:

$A \cap B = \{5, 6, 8\}$

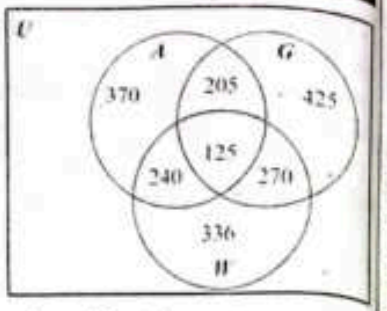
(b) The elements that are only in set B, not in set A, is the sets' differences.

$B - A = \{7, 9, 10, 12\}$

(c) $A \cup B = \{1, 2, 3, 4, 5, 6, 8\} \cup \{5, 6, 7, 8, 9, 10, 12\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12\}$

Example 2: Consider the adjacent Venn diagram representing the students enrolled in different courses in an IT institution.



U = {Students enrolled in IT institutions}
A = {Students enrolled in an Applied Robotics}
G = {Students enrolled in a Game Development}
W = {Students enrolled in a Web Designing}

(a) How many students enrolled in the applied Robotics course?

(b) Determine the total number of Students enrolled in a Game Development.

(c) How many students are enrolled in the Game development and Web designing course?

(d) Identify the students enrolled in Web development but not Applied Robotics.

(e) How many students are enrolled in IT institutions?

(f) How many students enrolled in all three courses?

Solution:

(a) Set A represents the total number of students enrolled in the Applied Robotics program.

Total = 370 + 205 + 125 + 240 = 940

So, the total number of students in the Applied Robotics course is 940.

(b) The total number of students enrolled in a Game Development is represented by the set G.

Total = 205 + 125 + 270 + 425 = 1025

Thus, the Students enrolled in a Game Development is 1025

(c) Total students are enrolled in both the Game development and Web designing The course is the intersection of G and W.

$G \cap W = 125 + 270 = 395$

Therefore, 395 students are enrolled in both the Game development and Web designing Course.

(d) The students who are enrolled in Web development but not in Applied Robotics is the sum of values 336 and 270 in set W.

Total = 336+270 = 606

So, there are 606 students who enrolled in Web development courses but not in Applied Robotics.

(e) The total number of students enrolled in IT Institutions are represented by all the values inside the circles.

Total = 370 + 205 + 125 + 240 + 425 + 270 + 336 = 1971

There are a total of 1971 students enrolled in IT Institutions.

(f) The students who enrolled in all three courses are the intersection of all the circles are represented by the value 125.

Principle of Inclusion and Exclusion for Two Sets

Let A and B be finite sets, then

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$ and $A \cup B$ and $A \cap B$ are also finite.

Principle of Inclusion and Exclusion for Three Sets

If A, B and C are finite sets, then

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$ and $A \cup B \cup C$, $A \cap B$, $A \cap C$, $B \cap C$ and $A \cap B \cap C$ are also finite.

Example 3: There are 98 secondary school students in a sports club. 58 students join the swimming club, and 50 join the tug-of-war club. How many students participated in both games?

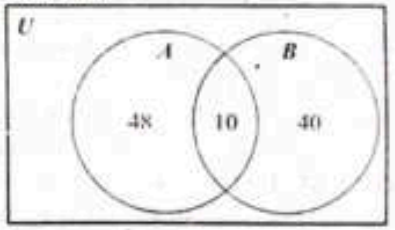
Solution: Let U = {total student in a sports club of school}

A = {students who participated in swimming club}

B = {students who participated in tug-of-war club}

From the statement of problems, we have

$n(U) = n(A \cup B) = 98$, $n(A) = 58$, $n(B) = 50$.



We want to find the total number of student: who participated in both clubs.

$n(A \cap B) = ?$

Using the principles of inclusion and exclusior for two sets:

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 58 + 50 - 98 = 10$

Thus, 10 students participated in both clubs.

The adjacent Venn diagram shows the number of students in each sports club.

Example 4: Mr. Saleem, a school teacher, has a small library in his house containing 150 books. He has two main categories for these books: islamic and science. He categorized 70 books as islamic books and 90 books as science books. There are 15 books that neither belong to the islamic nor science books category. How many books are classified under both the islamic and science categories?

Solution: Let U = {total number of books in library}

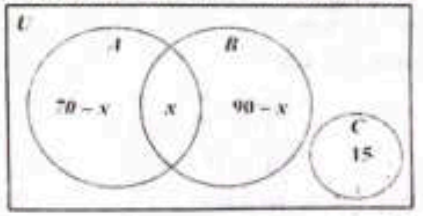
A = {70 books in Islamic category}

B = {90 books in Science category}

C = {15 book that does not belong to any category}

x = number of books that belong to both the categories

The adjacent Venn diagram shows the number of books that are classified under both the islamic and science categories



As, n(U) = 150

So, 70 – x + x + 90 – x + 15 = 150

$\Rightarrow$ 175 – x = 150

$\Rightarrow$ x = 25

Thus, 25 books are classified under both islamic and science categories.

Example 5: In a college, 45 teachers teach mathematics or physics or chemistry. Here is information about teachers who teach different subjects:

- **18 teach mathematics**
- **12 teach physics**
- **8 teach chemistry**
- **6 teach both mathematics and physics**
- **4 teach both physics and chemistry**
- **2 teach both mathematics and chemistry.**
- **How many teachers teach all three subjects?**

Solution: Let U = {total number of teachers in the college}
M = {teachers who teach mathematics}
P = {teachers who teach physics}
C = {teachers who teach chemistry}

From the statement of problems, we have

$n(M \cup P \cup Q) = 45$, $n(M) = 18$, $n(P) = 12$, $n(C) = 8$, $n(M \cap P) = 6$,
$n(P \cap Q) = 4$, $n(M \cap C) = 2$

We want to find the total number of teachers who teach all the subjects.

$n(M \cap P \cap C) = ?$

Using the principle of inclusion and exclusion for three sets:

$$n(M \cup P \cup C) = n(M) + n(P) + n(C) - n(M \cap P) - n(P \cap C) - n(M \cap C) + n(M \cap P \cap C)$$

$$\Rightarrow n(M \cap P \cap C) = n\{M \cup P \cup C\} - n(M) - n(P) - n(C) + n(M \cap P) + n(P \cap C) + n(M \cap C)$$
$$= 45 - 18 - 12 - 8 + 6 + 4 + 2 = 19$$

Therefore 19 teachers teach all three subjects.

Example 6: A survey of 130 customers in a shopping mall was conducted in which they were asked about buying preferences.

The survey result showed the following statistics:

- 57 customers bought garments
- 50 customers bought cosmetics
- 46 customers bought electronics
- 31 customers purchased both garments and cosmetics
- 25 customers purchased both garments and electronics
- 21 customers purchased both cosmetics and electronics
- 12 customers purchased all three products i.e. garments, cosmetics, and electronics.

(a) How many of the customers bought at least one of the products: garments, cosmetics or electronics.

(b) How many of the customers bought only one of the products: garments, Cosmetics or electronics?

(c) How many customers did not buy any of the three products?

Solution: Let U = {total number of customers surveyed in the shopping mall}
G = {Customer who bought garments}
C = {Customer who bought cosmetics}
E = {Customer who bought electronics}

From the statement of problems, we have

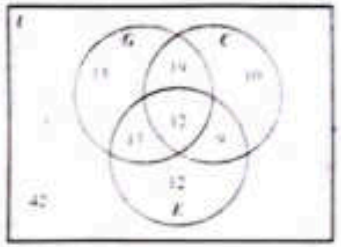
$n(U) = 130$, $n(G) = 57$, $n(C) = 50$, $n(E) = 46$, $n(G \cap C) = 31$, $n(G \cap E) = 25$, $n(C \cap E) = 21$ and $n(G \cap C \cap E) = 12$.

(a) We want to find the total number of customers who have bought at least one of the products: garments, cosmetics, or electronics.
We are to find $n(G \cup C \cup E)$.
Using the principle of inclusion and exclusion for three sets:

$n(G \cup C \cup E) = n(G) + n(C) + n(E) - n(G \cap C) - n(G \cap E) - n(C \cap E) + n(G \cap C \cap E)$
$= 57 + 50 + 46 - 31 - 25 - 21 + 12 = 88$

Thus, 88 customers bought at least one of the products: garments, cosmetics, or electronics.

(b) Customers who bought only garments
$= n(G) - n(G \cap C) - n(G \cap E) + n(G \cap C \cap E)$
$= 57 - 31 - 25 + 12$
$= 13$



Customers who bought only cosmetics
$= n(C) - n(G \cap C) - n(C \cap E) + n(G \cap C \cap E)$
$= 50 - 31 - 21 + 12$
$= 10$

Customers who bought only electronics
$= n(E) - n(G \cap E) - n(C \cap E) + n(G \cap C \cap E)$
$= 46 - 25 - 21 + 12 = 12$

Therefore, the customers bought only one of the products: garments, cosmetics, or electronics $= 13 + 10 + 12 = 35$

Challenge!
The Venn diagram above illustrates the scenario presented in Example 7. Can you provide a justification for each value within the circles?

(c) Since the total number of Customers surveyed was 130, and 88 customers bought at least one of the products: garments, cosmetics, or electronics. The customers who did not buy any of the three products can be calculated as:

$n(G \cup C \cup E)' = n(U) - n(G \cup C \cup E)$
$= 130 - 88 = 42$

So, 42 customers did not buy any of the three products.

Solved Exercise 3.2

1. Consider the universal set $U = \{x: x \text{ is a multiple of 2 and } 0 < x \leq 30\}$, $A = \{x : x \text{ is a multiple of 6}\}$ and $B = \{x : x \text{ is a multiple of 8}\}$

(i) List all elements of sets A and B in tabular form

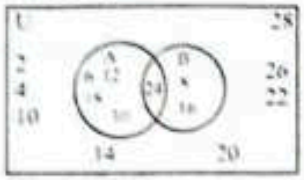
Sol. A = {6,12,18,24,30}
B = {8,16,24}

(ii) Find $A \cap B$

Sol. $A \cap B = \{6, 12, 18, 24, 30\} \cap \{8, 16, 24\}$
$= \{24\}$

(iii) Draw a Venn diagram

Sol.



2. Let, U = {x : x is an integer and 0 < x ≤ 150},
G = {x : x = 2^m for integer m and 0 ≤ m ≤ 12} and
H = {x : x is a square}

(i) List all elements of sets G and H in tabular form

Sol. G = {1, 2, 4, 8, 16, 32, 64, 128}
H = {1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144}

(ii) Find G ∪ H

Sol. G = {1, 2, 4, 8, 16, 32, 64, 128} and H = {1, 4, 9, 16, 25,36,49,64,81,100,121,144}
G∪H = {1, 2, 4, 8, 16, 32, 64, 128}∪ {1,4,9,16,25,36,49,64,81,100,121,144}
= {1, 2, 4, 8, 9,16, 25, 32, 36, 49, 64, 81, 100, 121, 128, 144}

(iii) Find G∩H

Sol. G∩H = {1, 2, 4, 8, 16, 32, 64, 128}∩ {1, 4, 9, 16, 25, 36, 49, 64, 81, 100,121, 144}
= {1, 4, 16, 64}

3. Consider the sets P = {x : x is a prime number and 0 < x ≤ 20} and Q = {x : x is a divisor of 210 and 0 < x ≤ 20}

(i) Find P∩Q

Sol. P∩Q = {2, 3, 5, 7, 11, 13, 17, 19}∩ {1, 2, 3, 5, 6, 7, 10, 14, 15}
= {2, 3, 5, 7}

(ii) Find P∪Q

Sol. P∪Q = {2, 3, 5, 7, 11, 13, 17, 19,} ∪ {1, 2, 3, 5, 6, 7, 10, 14, 15}
= {1, 2, 3, 5, 6, 7, 10, 11, 13, 14, 15, 17, 19}

4 Verify the commutative properties of union and intersection for the following pairs of sets:

(i) A = {1, 2, 3, 4, 5}, B = {4, 6, 8, 10}

Sol. We have to verify A∪B = B∪A, A∩B = B∩A
A∪B = {1, 2, 3, 4, 5,} U {4, 6, 8, 10}
= {1, 2, 3, 4, 5, 6, 8, 10}
B∪A = {4, 6, 8, 10} ∪ {1, 2, 3, 4, 5}
= {1, 2, 3, 4, 5, 6, 8, 10}
So, it is proved that A∪B = B∪A

Now

A∩B = {1, 2, 3, 4, 5} ∩ {4, 6, 8, 10} = {4}
B∩A = {4, 6, 8, 10} ∩ {1, 2, 3, 4, 5} = {4}
So, it is proved that A∩B = B∩A

(ii) N,Z

Sol. We have to prove N∪Z = Z∪N, N∩Z = Z∩N
N∪Z = {1,2,3,4,5,...} ∪ {0, ± 1, ± 2, ± 3, ±4, ...}
= {0, ± 1, ± 2, ± 3, ±, ...}
Z∪N = {0, ± 1, ± 2, ± 3, ± 4, ...} ∪ {1, 2, 3, 4, 5,...}
= {0, ± 1, ± 2, ± 3, ± 4, ...}
So, it is proved that N∪Z = Z∪N
N∩ Z = {1, 2, 3, 4, 5, ...} ∩ {0, ± 1, ± 2,± 3, ±4, ...}
= {1, 2, 3, 4, 5, ...}
Z∩N= {0, ± 1, ± 2,± 3 ...}∩ {1, 2, 3, 4, ...}
= {1, 2, 3, 4, 5, ...}
So, it is proved that N∩ Z = Z∩ N

(iii) A = {x | x ∈ R ∧ x ≥ 0}, B = R.

Sol. We have to verify that → A∪B = B∪A and A∩B = B∩A
A∪B = {x | x ∈ R ∧ x ≥ 0}∪R
= R
B∪A = R ∪ {x | x ∈ R ∧ x ≥ 0}
= R
So, it is verified that A∪B = B∪A

No.

A∩B = {x | x ∈ R ∧ x ≥ 0}, ∩ R
= {x | x ∈ R ∧ x ≥ 0}
B∩A = R∩{x | x ∈ R ∧ x ≥ 0}
= {x | x ∈ R ∧ x ≥ 0}
So, it is verified that A∩B = B∩A

5. Let U = { a, b, c, d, e, f, g, h, i, J}
A = {a, b, c, d, g, h}, B = {c,d,e,f,j},
Verify De Morgan's Laws for these sets. Draw Venn diagram

Sol. We have to verify that
(A∪B)' = A'∩B'
(A∩B)' = A'∪B'
For (A∪B)' = A'∩B'
L.H.S = (A∪B)'
A∪B = {a, b, c, d, g, h}∪{c, d, e, f, j}
= {a, b, c, d, e, f, g, h, j}
(A∪B)' = U – (A∪B)
= {a, b, c, d, e, f, g, h, i, j} – {a, b, c, d, e, f, g, h, i}
= {i}
R.H.S = A'∩B'
A' = U – A

$= \{a, b, c, d, e, f, g, h, i, j\} - \{a, b, c, d, g, h\}$

$= \{e, f, i, j\}$

$B' = U - B$

$= \{a, b, c, d, e, f, g, h, i, j\} - \{c, d, e, f, j\}$

$= \{a, b, g, h, i\}$

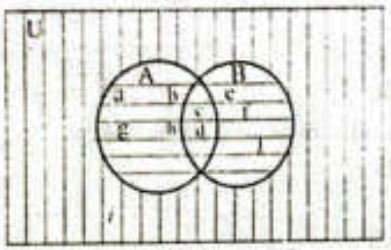
$A' \cap B' = \{e, f, i, j\} \cap \{a, b, g, h, i\}$

$= \{i\}$

So, it is verified that $(A \cup B)' = A' \cap B'$

Venn diagram for $(A \cup B)' = A \cap B'$

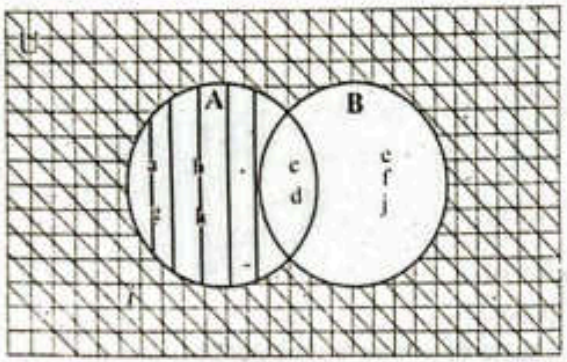
L.H.S



$A \cup B =$ ≡

$(A \cup B)'$ ||||

R.H.S



$A' =$ ≡

$B' =$ ||||

$A \cap B =$ ////

Thus, the area shown by ////// is $A' \cap B'$

Thus, the area ||||| is L.H.S and area show by in ////// R.H.S is same.

For $(A \cap B)^c = A^c \cup B^c$

L.H.S $= (A \cap B)^c$

$A \cap B = \{a, b, c, d, g, h\} \cap \{c, d, e, f, j\}$

$= \{c, d\}$

$(A \cap B)^c = U - (A \cap B)$

$= \{a, b, c, d, e, f, g, h, i, j\} - \{c, d\}$

$= \{a, b, e, f, g, h, i, j\}$

R.H.S $= A^c \cup B^c$

$A^c = U - A = \{a, b, c, d, e, f, g, h, i, j\} - \{a, b, c, d, g, h\}$

$= \{e, f, i, j\}$

$B^c = U - B$

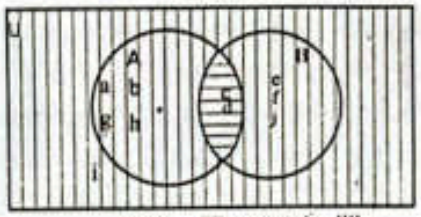
$= \{a, b, c, d, e, f, g, h, i, j\} - \{c, d, e, f, j\}$

$= \{a, b, g, h, i\}$

$A^c \cup B^c = \{e, f, i, j\} \cup \{a, b, g, h, i\} = \{a, b, e, f, g, h, i, j\}$

So, it is verified that L.H.S = R.H.S

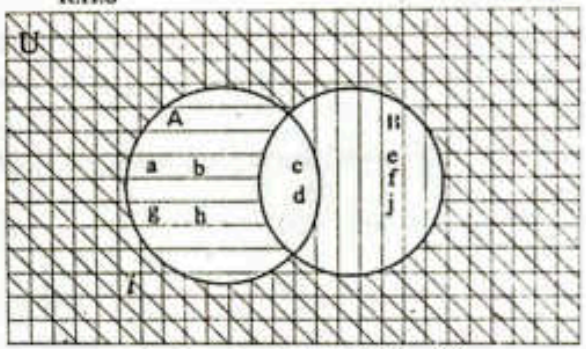
L.H.S



$A \cap B =$ ≡ , $(A \cap B)' =$ ||||

Thus area shown by |||| is $(A \cap B)'$

R.H.S



$B^c =$ ≡ , $A^c =$ |||| , $A^c \cap B^c =$ ////

Thus area slown by ////// |||| and ≡ is $A^c \cap B^c$

So, the area shown by ||||| in L.H.S and area shown by //////, |||| and ≡ is same

6. If U = {1, 2, 3, ... 20} and A = {1, 3, 5, ..., 19}, verify the following:

(i) $A \cup A' = U$ (ii) $(A \cap U) = A$ (iii) $A \cap A' = \phi$

Sol. $A \cup A' = U$

L.H.S $= A \cup A'$

$A' = U - A$

$= \{1, 2, 3, ..., 20\} - \{1, 3, 5, 9, ..., 19\}$

$= \{2, 4, 6, ..., 20\}$

$A \cup A' = \{1, 3, 5, ..., 19\} \cup \{2, 4, 6, ..., 20\}$

$= \{1, 2, 3, 4, ..., 20\}$ = R.H.S

So it is varified that $A \cup A' = U$

(ii) $(A \cap U)$

Sol. $(A \cap U) = A$

L.H.S = $A \cap U$

$\{1, 3, 5, ...19\} \cap \{1, 2, 3, ..., 20\}$

$= \{1, 3, 5, ... 19\}$ = R.H.S

So, it is verified that $A \cap U = A$

(iii) $A \cap A' = \phi$

Sol. $A \cap A' = \phi$

L.H.S = $A \cap A'$

$A' = \cap - A$

$= \{1,2,3,....,20\} - \{1,3,5,....,19\}$

$= \{2, 4, 6, ...20\}$

$A \cap A' = \{1, 3, 5, ..., 19\} \cap \{2, 4, 6, ..., 20\}$

$= \{\ \}$ = R.H.S

So, it is verified that $A \cap B' = \phi$

7. In a class of 55 students, 34 like to play cricket and 30 like to play hockey. Also each student likes to play at least one of the two games. How many students like to play both games?

Sol. Let C denotes the set of students who like to play cricket and H denotes the set who like to play hockey.

Given, $n(C \cup H) = 55$, $n(C) = 34$, $n(H) = 30$

By using

$n(C \cup H) = n(C) + n(H) - n(C \cap H)$

$55 = 34 + 30 - n(C \cap H)$

$55 = 64 - n(C \cap H)$

$n(C \cap H) = 64 - 55$

$n(C \cap H) = 9$

So, 9 students like to play both games.

8. In a group of 500 employees, 250 can speak Urdu, 150 can speak English, 50 can speak Punjabi, 40 can speak Urdu and English, 30 can speak both English and Punjabi, and 10 can speak Urdu and Punjabi. How many can speak all three languages?

Sol. Let A be denotes the set of employes who can speak urdu, B be donotes the set of employes who can speak English and C be denotes the set of employes who can speak punjabi.

Given $n(A \cup B \cup C) = 500$, $n(A) = 250$, $n(B) = 150$, $n(C) = 50$,

$n(A \cap B) = 40$, $n(B \cap C) = 30$, $n(C \cap A) = 10$

By using

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(C \cap A) + n(A \cap B \cap C)$

$500 = 250 + 150 + 50 - 40 - 30 - 10 + n(A \cap B \cap C)$

$500 = 450 - 80 + n(A \cap B \cap C)$

$500 = 370 + n(A \cap B \cap C)$

$500 - 370 = n(A \cap B \cap C)$

$130 = n(A \cap B \cap C)$

So 130 employes can speak all three languages.

9. In sports events, 19 people wear blue shirts, 15 wear green shirts, 3 wear blue and green shirts, 4 wear a cap and blue shirts, and 2 wear a cap and green shirts. The total number of people with either a blue or green shirt or cap is 25. How many people are wearing caps?

Sol. Let B be denotes the set of people who wear blue shirts, G be denotes the set of people who wear green shirts, and C be denotes the set of people who wear caps.

Givon data:

$n(B) = 19$, $n(G) = 15$, $n(B \cap G) = 3$, $n(G \cap C) = 2$

$n(B \cap C) = 4$, $n(B \cup G \cup C) = 25$

By using the principle of inclusion and exclusion for three sets.

$n(B \cup G \cup C) = n(B) + n(G) + n(C) - n(B \cap G) - n(B \cap C) - n(G \cap C) + n(B \cap G \cap C)$

$25 = 19 + 15 + n(c) - 3 - 4 - 2 + 0$

$25 = 34 - 9 + n(c)$

$25 = 25 = n(c)$

$25 - 25 = n(c)$

$0 = n(c)$

So, 0 people are wearing caps. So the given data in this question is not correct.

10. In a training session, 17 participants have laptops, 11 have tablets, 9 have laptops and tablets, 6 have laptops and books, and 4 have both tablets and books. Eight participants have all three items. The total number of participants with laptops, tablets, or books is 35. How many participants have books?

Sol. Let" L" be denotes the set of participants who have laptops, "T" be denotes the set of participles who have tablets and "B" be donetes the set of participants who have books.

Given data:

$n(L) = 17$, $n(T) = 11$, $n(L \cap T) = 9$, $n(L \cap B) = 6$, $n(T \cap B) = 4$

$n(L \cap T \cap B) = 8$, $n(L \cup T \cup B) = 35$

using the principle of inclusion and exclusion for three sets.

$n(L \cup T \cup B) = n(L) + n(T) + n(B) - n(L \cap T) - n(L \cap B) - n(T \cap B) + n(L \cap T \cap B)$

$35 = 17 + 11 + n(B) - 9 - 6 - 4 + 8$

$35 = 36 - 19 + n(B)$

$35 = 17 + n(B)$

$35 - 17 = n(B)$

18 = n(B)

So, 18 paticipants have books.

11. A shopping mall has 150 employees labelled 1 to 150, representing the Universal set U. The employees fall into the following categories:

- **Set A: 40 employees with a salary range of 30k – 45k, labelled from 50 to 89.**
- **Set B: 50 employees with a salary range of 50k – 80k, labelled from 101 to 150.**
- **SetC:60 employees with a salary range of 100k – 150k, labelled from 1 to 49 and 90 to 100.**

(a) Find $(A' \cup B') \cap C$ (b) Find n $\{A \cap (B' \cap C')\}$

Sol. The tabular form of the given sets.

U = {1,2,3,....,150}, A = {50, 51, 52, ... 89}, B = {101, 102, 103, ...150}

C = {1, 2, 3... ,49, 90, 91,92, ... 100}

(a) $(A' \cup B') \cap C$

A' = U – A

= {1,2,3,....,150}–{50, 51, 52, ..., 89}

= {1, 2, 3, ..., 49, 90, 91, 92, ..., 150}

B' = U – B

= {1, 2, 3, ..., 150} – {101, 102, 103, ..., 150}

= {1, 2, 3, ..., 100}

$A' \cup B'$ = { 1, 2, 3, ..., 49, 90, 91, 92, ..., 150}∪{1, 2, 3, ..., 100}

= {1, 2, 3, ..., 150}

$(A' \cup B') \cap C$ = {1, 2, 3, ...,'150}∩ {1, 2, 3, ..., 49, 90, 91, 92, ...,100}

= {1, 2, 3,..., 49, 90, 91, 92, ...,100}

(b) B' = U – B

= {1, 2, 3, ..., 150} – {101, 102, 103, ..., 150}

= {1, 2, 3, 4, ...,100}

C' = U – C

= {1, 2, 3, ..., 150} – {1, 2, 3, ..., 49, 90, 91, 92, ..., 100}

={50, 51, 52, ..., 89, 101, 102, 103, ..., 150}

$B' \cap C'$ = {1, 2, 3, 4, ..., 100}∩{50, 51, 52, ..., 89, 101, 102, 103, ..., 150}

= {50, 51, 52, ..., 89}

$A \cap (B' \cap C')$ = {50, 51, 52, ..., 89}∩{50, 51, 52, ..., 89}

= {50, 51, 52, ..., 89}

So, n $A \cap (B' \cap C')$ = 40

12. In a secondary school with 125 students participate in at least one of the following sports: cricket, football, or hockey.

- **60 students play cricket.**
- **70 students play football.**
- **40 students play hockey.**
- **25 students play both cricket and football.**

- **15 students play both football and hockey.**
- **10 students play both cricket and hockey.**

(a) How many students play all three sports?

Sol. Let "C" be denoted the set of the students participate in cricket, "F" be denoted the sets of the students participate in football and "H" be denoted the students participate in hockey

Given data:

n(C∪F∪H) = 125, n(C) = 60, n (F) = 70, n (H) = 40

n(C∩F) = 25,n(F∩H) = 15, n(C∩H) = 10

$n(C \cup F \cup H) = n(C) + n(F) + n(H) - n(C \cap F) - n(F \cap H) - n(C \cap H) + n(C \cap F \cap H)$

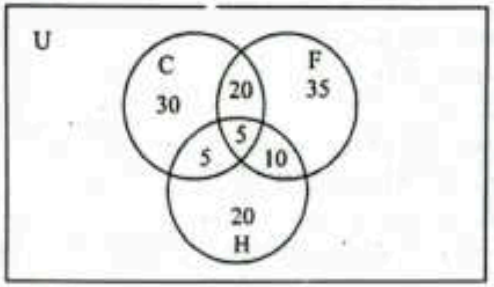
$125 = 60 + 70 + 40 - 25 - 15 - 10 + n(C \cap F \cap H)$

$125 = 170 - 50 + n(C \cap F \cap H)$

$125 = 120 + n(C \cap F \cap H)$

$125 - 120 = n(C \cap F \cap H)$

$5 = n(C \cap F \cap H)$



13. A survey was conducted in which 130 people were asked about their favourite foods. The survey results showed the following information:

- **40 people said they liked nihari.**
- **65 people said they liked biryani.**
- **50 people said they liked korma.**
- **20 people said they liked nihari and biryani.**
- **35 people said they liked biryani and korma.**
- **27 people said they liked nihari and korma.**
- **12 people said they liked all three foods nihari, biryani, and korma**

(a) At least how many people like nihari, biryani or korma?

(b) How many people did not like nihari, biryani, or korma.

(c) How many people like only one of the following foods: nihari, biryani, or korma?

(d) Draw a Venn diagram.

Sol. Let" N" be denoted the set of people who liked nihari, "B" be denoted the set of people who liked biryani and "K" be denoted the set of people who liked korma.

(a) Given data:

$n(N) = 40$, $n(B) = 65$, $n(K) = 50$, $n(N \cap B) = 20$

$n(B \cap K) = 35$, $n(N \cap K) = 27$, $n(N \cap B \cap K) = 12$

using the principle of inclusion and exclusion for three sets.

$n(N \cup B \cup K) = n(N) + n(B) + n(K) - n(N \cap B) - n(B \cap K - n(N \cap K) + n(N \cap B \cap K)$

$n(N \cup B \cup K) = 40 + 65 + 50 - 20 - 35 - 27 + 12$

$n(N \cup B \cup K) = 167 - 82$

$n(N \cup B \cup K) = 85$

Thus, at least 85 people like nihari, biryani and korma.

Number of people did not like nihari, biryani or korma = $n(N \cup B \cup K)'$

$n(N \cup B \cup K)' = n(U) - n(N \cup B \cup K)$

$= 130 - 85 = 45$

Thus, 45 people did not like nihari, biryani or korma.

(c) Number of people who like nihari only.

$= n(N) - n(N \cap B) - n(N \cap K) + n(N \cap B \cap K)$

$= 40 - 20 - 27 + 12 = 5$

Number of people who like biryani only

$= n(B) - n(N \cap B) - n(B \cap K) + n(N \cap B \cap K)$

$= 65 - 20 - 35 + 12 = 22$

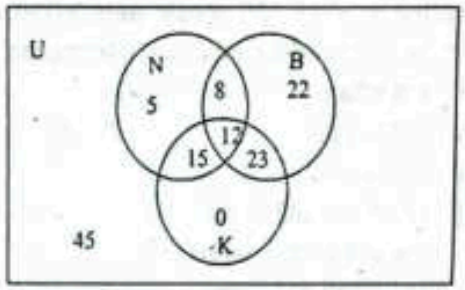
Number of People who like korma only

$= n(K) - n(N \cap K) - n(B \cap K) + n(N \cap B \cap K)$

$= 50 - 27 - 35 + 12 = 0$

So, the people like only one of the food: nihari, birynai or korma = 5 + 22 + 0 = 27

(d)



Binary Relations

In everyday use, relation means an abstract type of connection between two persons or objects, for instance, (teacher, pupil), (mother, son), (husband, wife), (brother, sister), (friend, friend), (house, owner). In mathematics also some operations determine the relationship between two numbers, for example:

$>$: (5, 4) ; square: (25, 5) ; Square root: (2,4) ; Equal: (2 x 2, 4).

In the above examples $>$, square, square root and equal are examples of relations. Mathematically, a relation is any set of ordered pairs. The relationship between the components of an ordered pair may or may not be mentioned.

Cartesion Product: Let A and B be two non-empty sets, then the Cartesian product is the set of all ordered pairs (x, y) such that $x \in A$ and $y \in B$ and is denoted by $A \times B$.

Symbolically we can write it as $A \times B = \{(x,y) | x \in A \text{ and } y \in B\}$.

Binary Relation: Any subset of $A \times B$ is called a binary relation, or simply a relation, from A to B. Ordinarily a relation will be denoted by the letter r.

Domain: The set of the first elements of the ordered pairs forming a relation is called its domain. The domain of any relation r is denoted as Dom r.

Range: The set of the second elements of the ordered pairs forming a relation is called its range. The range of any relation r is denoted as Ran r.

Relation: If A is a non-empty set, any subset of $A \times A$ is called a relation in A.

Example 7: Let c_1, c_2, c_3 be three children and m_1, m_2 be two men such that the father of both c_1, c_2 is m_1 and father of c_3 is m_2. Find the relation {(child, father)}

Solution: C = Set of children = $\{c_1, c_2, c_3\}$ and F = set of fathers = $\{m_1, m_2\}$

The Cartesian product of C and F:

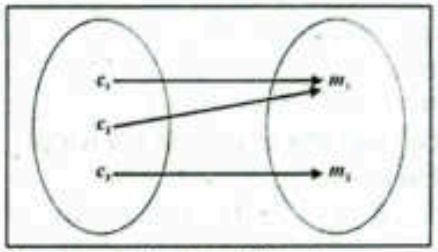
$C \times F = \{(c_1, m_1), (c_1, m_2), (c_2, m_1), (c_2, m_2), (c_3, m_1), (c_3, m_2)\}$

r = set of ordered pairs (child, father).

$= \{(c_1, m_1), (c_2, m_1), (c_3, m_2)\}$

Dom r = $\{c_1, c_2, c_3\}$, Range r = $\{m_1, m_2\}$

The relation is shown diagrammatically in adjacent figure.



Example 8: Let A = {1, 2, 3}. Determine the relation r such that xry iff x < y.

Solution: $A \times A = \{(1,1), (1,2), (1, 3), (2,1), (2,2), (2, 3), (3,1), (3,2), (3, 3)\}$

Clearly, required relation is:

r = {(1, 2), (1, 3), (2, 3)}, Dom r = {1, 2}, Range r = {2, 3}

Relation as Table, Ordered Pair and Graphs

We have learned that a relation in mathematics is any subset of the Cartesian product, which contains all ordered pairs. Each ordered pair consists of two coordinates, x and y. The x coordinate is called abscissa, and the y coordinate is ordinate, often representing an input and an output. Now, we describe the relation in three different ways.

Ordered Pairs: A relation can be represented by a set of ordered pairs. For example, consider a water tank that starts with 1 litre of water already inside. Each minute, 1 additional litre of water is added to the tank. The situation can be represented by the relation $r = \{(x,y) | y = x + 1\}$, where x is the number of minutes (time) that have passed since the filling started andy is the total amount of water (in litres) in the tank.

When x = 0, y = 1 and x= 1, y = 2

In order pair this relation is represented as:

{(0, 1), (1,2), (2, 3), (3,4), (4, 5), (5, 6)}

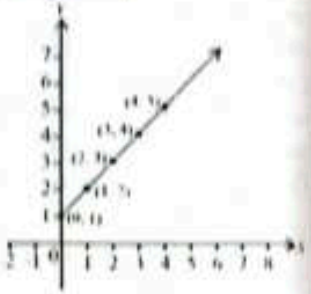
The above relation in table form can be represented as given below:

x (time in minutes)	y = x + 1 (water in litres)
0	y = 0 + 1 = 1
1	y = 1 + 1 = 2
2	y = 2 + 1 = 3
3	y = 3 + 1 = 4
4	y = 4 + 1 = 5
5	y = 5 + 1 = 6

Graph: We can also represent the relations visually by drawing a graph. To draw the diagram, we use ordered pairs. Each ordered pair (x, y) is plotted as a point in the coordinate plane, where x is the first element and y is the second element of the ordered pair.

The relation is represented graphically by the line passing through the points,

{(0,1), (1,2), (2,3), (3,4), (4,5), (5,6)} as shown in the adjacent Figure.



Function and its Domain and Range

Functions

A very important particular type of relation is a function defined as below:

Let A and B be two non-empty sets such that:

(i) f is a relation from A to B, that is, f is a subset of A × B

(ii) Domain f = A

(iii) First element of no two pairs of f are equal, then f is said to be a function from A to B.

The function f is also written as:

$$f : A \to B$$

Which is read as f is a function from A to B. The set of all first elements of each ordered pair represents the domain of f and all second elements represent the range of f. Here, the domain of f is A, and the range of f is B.

If (x, y) is an element of f when regarded as a set of ordered pairs.

We write y = f(x), y is called the value of f for x or the image of x under f.

Example 9: If A = {0, 1, 2, 3, 4} and B = {3, 5, 7, 9, 11}, define a function f : A → B, f={(x, y)|y = 2x +3, x∈A and y∈B}. Find the value of function f, its domain, co-domain and range.

Solution: Given: y = 2x + 3 ; x∈A and y∈B, then value of function,

f = {(0, 3), (1,5), (2, 7), (3, 9), (4, 11)}

Dom f = {0, 1,2, 3, 4} =A

⇒ Co-domain f = B and

⇒ Range f = {3, 5, 7, 9, 11} ⊆ B

Types of functions

In this section we discuss different types of functions:

(i) Into Function

If a function f : A → B is such that Range f ⊂ B i.e., Range f ≠ B, then f is said to be a function from A into B. In Fig. (i), f is clearly a function.

But Range f ≠ B. Therefore, f is a function from A into B.

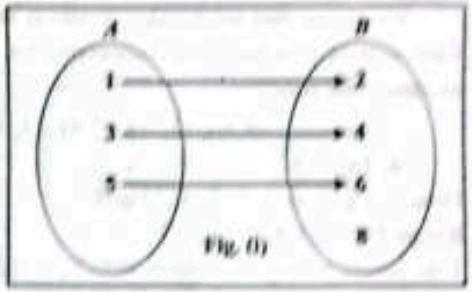


Fig. (i)

$f = \{(1, 3), (3, 4), (5, 6)\}$

(ii) (One - One) Function (or Injective Function)

If a function f from A into B is such that second elements of no two of its ordered pairs are same, then it is called an injective function; the function shown in Fig. (iii) is such a function.

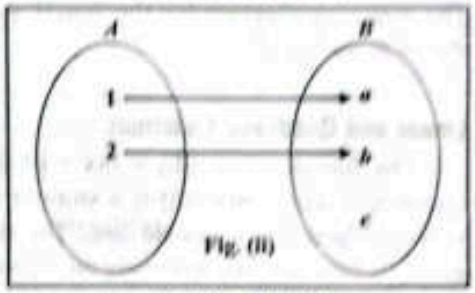


Fig. (ii)

$f = \{(1, a), (2, b)\}$

(iii) Onto Function (or Surjective function)

If a function f: A → B is such that Range f = B i.e., every element of B is the image of some element of A, then f is called an **onto** function or a surjective function.

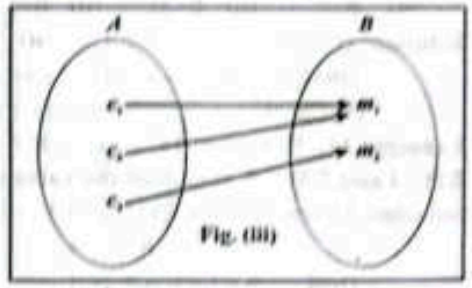


Fig. (iii)

$f = \{(c_1, m_1), (c_2, m_2), (c_3, m_2)\}$

(iv) (One - One) and onto Function (or Bijective Function)

A function f from A to B is said to be a Bijective function if it is both one- one and onto. Such a function is also called (1–1) correspondence between the sets A and B.

(a, z), (b, x) and (c, y) are the pairs of corresponding elements i.e., in this case

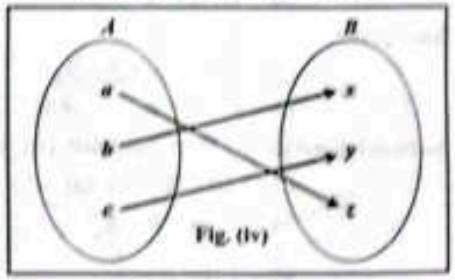


Fig. (iv)

$f = \{(a, z), (b, x), (c, y)\}$

f = {(a, z), (b, x), (c, y)} which is a bijective function or (1 – 1) correspondence between the sets A and B.

Notation of Function

We know that set-builder notation is more suitable for infinite sets. So is the case with respect to a function comprising an infinite number of ordered pairs. Consider for instance, the function

$f = \{(-1,1), (0, 0), (1, 1), (2,4), (3, 9), (4, 16), ...\}$

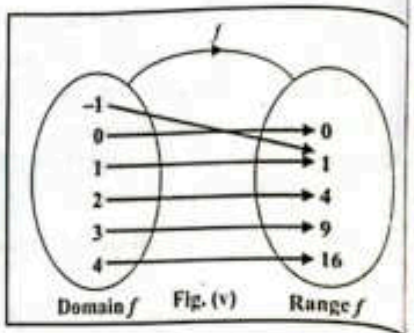
Dom $f = \{-1, 0, 1, 2, 3, 4, ...\}$ and

Range $f = \{0, 1, 4, 9, 16, ...\}$

This function may be written as:

$f = \{(x,y) | y = x^2, x \in N\}$

The mapping diagram for the function is shown in the Fig.(v).



Linear and Quadratic Functions

The function $\{(x, y) | y = mx + c\}$ is called a linear function because its graph (geometric representation) is a straight line. We know that an equation of the form $y=mx+c$ represents a straight line. The function $\{(x, y) | y = ax^2 + bx + c\}$ is called a quadratic function. We will study their geometric representation in the next chapter.

Example 10: If $f(x) = 2x - 1$ and $g(x) = x^2 - 3$, then find:

(i) f(1) (ii) f(–3) (ii) f(7) (iv) g(1) (v) g(–3) (vi) g(4)

Solution: (i) $f(1) = 2 \times 1 - 1 = 1$ (ii) $f(-3) = 2 \times (-3) - 1 = -7$

(iii) $f(7) = 2 \times 7 - 1 = 13$ (iv) $g(1) = (1)^2 - 3 = -3$

(v) $g(-3) = (-3)^2 - 3 = 6$ (vi) $g(4) = (4)^2 - 3 = 13$

Example 11: Consider $f(x) = ax + b + 3$, where a and b are constant numbers. If $f(1) = 4$ and $f(5) = 9$, then find the value of a and b.

Solution: Given function $f(x) = ax + b + 3$

If $f(1) = 4$

Then $a \times 1 + b + 3 = 4$

$\Rightarrow a + b = 1$ ...(i)

Similarly, $f(5) = 9$

$\Rightarrow a \times 5 + b + 3 = 9$

$\Rightarrow a + b = 6$ ...(ii)

Subtract equation (i) from equation (ii), we get.

$(5a + b) - (a + b) = 6 - 1$

$5a + b - a - b = 5$

$4a = 5 \Rightarrow a = \frac{5}{4}$

Substitute $a = \frac{5}{4}$ in the equation (i)

$\frac{5}{4} + b = 1$

$b = 1 - \frac{5}{4}$

$\Rightarrow b = 1 - \frac{1}{4}$

Thus, $a = \frac{5}{4}$ and $b = -\frac{1}{4}$

Solved Exercise 3.3

1. For A = {1, 2, 3, 4}, find the following relations in A. State the domain and range of each relation.

Sol. $A = \{1, 2, 3, 4\}$

$A \times A = \{1, 2, 3, 4\} \times \{1, 2, 3, 4\}$

$= \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)\}$

(i) $\{(x,y) | y = x\}$

$R = \{(1,1), (2,2), (3,3), (4,4)\}$

Domain $R = \{1, 2, 3, 4\}$

Range $R = \{1, 2, 3, 4\}$

(ii) $\{(x,y) | y + x = 5\}$

$R = \{(1, 4), (2, 3), (3, 2), (4, 1)\}$

Domain $R = \{1, 2, 3, 4\}$

Range $R = \{4, 3, 2, 1\}$

(iii) $\{(x,y) | x + y < 5\}$

$R = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1)\}$

Domain $R = \{1, 2, 3\}$

Range $R = \{1, 2, 3\}$

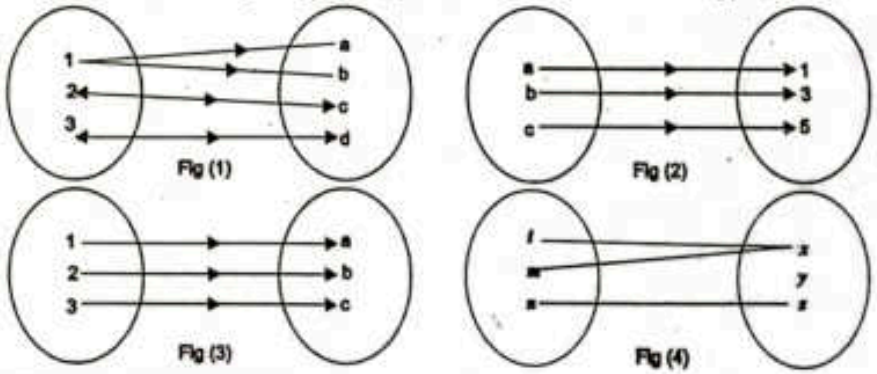
(iv) $\{(x,y) | x + y > 5\}$

$R = \{(2, 4), (3, 3), (3, 4), (4, 2), (4, 3), (4, 4)\}$

Domain $R = \{2, 3, 4\}$

Range $R = \{2, 3, 4\}$

2. Which of the following diagrams represent functions of which type?



Sol. **Fig (1)** does not represent a function becaus there is repetion is domain.

Fig (2) represents a function. It is an one to one and onto function. So, it is a bijective function.

Fig (3) represents a function. It is an one to one and onto function. So, it is a bijective function.

Fig (4) represents a function, which is an into function.

3. If $g(x) = 3x + 2$ and $h(x) = x^2 + 1$, then find:

(i) g(0)

Sol. $g(x) = 3x + 2$

$g(0) = 3(0) + 2 = 0 + 2 = 2$

(ii) g(–3)

Sol. $g(x) = 3x + 2$

$g(-3) = 3(-3) + 2 = -9 + 2 = -7$

(iii) $g\left(\frac{2}{3}\right)$

Sol. $g(x) = 3x + 2$

$g\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right) + 2 = 2 + 2 = 4$

(iv) h(1)

Sol. $h(x) = x^2 + 1$

$h(1) = (1)^2 + 1 = 1 + 1 = 2$

(v) h (–4)

Sol. $h(x) = x^2 + 1$

$h(-4) = (-4)^2 + 1 = 16 + 1 = 17$

(vi) $h\left(-\frac{1}{2}\right)$

Sol. $h(x) = x^2 + 1$

$$h\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^2 + 1 = \frac{1}{4} + 1 = \frac{1+4}{4} = \frac{5}{4}$$

4. Given that $f(x) = ax + b + 1$, where a and b are constant numbers. If $f(3) = 8$ and $f(6) = 14$, then find the values of a and b.

Sol. Given function: $f(x) = ax + b + 1$

If $f(3) = 8$

then $a(3) + b + 1 = 8$

$3a + b + 1 = 8$

$3a + b = 8 - 1$

$3a + b = 7$ .....(i)

Similarly, if $f(6) = 14$

then $a(6) + b + 1 = 14$

$6a + b + 1 = 14$

$6a + b = 14 - 1$

$6a + b = 13$ ..... (ii)

Subtract (i) from (ii)

$(6a + b) - (3a + b) = 13 - 7$

$6a + b - 3a - b = 6$

$3a = 6$

$$a = \frac{6^2}{3_1} \Rightarrow a = 2$$

Putting $a = 2$ in (i)

$3(2) + b = 7$

$6 + b = 7$

$b = 7 - 6 \Rightarrow b = 1$

So, $a = 2, b = 1$

5. Given that $g(x) = ax + b + 5$, where a and b are constant numbers. If $g(-1) = 0$ and $g(2) = 10$, find the values of a and b.

Sol. Given function $g(x) = ax + b + 5$

if $g(-1) = 0$

then $a(-1) + b + 5 = 0$

$-a + b + 5 = 0$

$-a + b = -5$ ..... (i)

Similarly, if

$g(2) = 10$

then

$a(2) + b + 5 = 10$

$2a + b + 5 = 10$

$2a + b = 10 - 5$

$2a + b = 5$

subtract (i) From (ii)

$(2a + b) - (-a + b) = 5 - (-5)$

$2a + b + a - b = 5 + 5$

$3a = 10$

$a = \frac{10}{3}$

putting $a = \frac{10}{3}$ in (i)

$-\frac{10}{3} + b = -5$

$b = -5 + \frac{10}{3}$

$b = \frac{-15+10}{3} = -\frac{5}{3}$

So, $a = \frac{10}{3}, b = -\frac{5}{3}$

6. Consider the function defined by $f(x) = 5x + 1$. If $f(x) = 32$, find the x value.

Sol. Given function

$f(x) = 5x + 1$

if $f(x) = 32$

Then

$5x + 1 = 32$

$5x = 32 - 1$

$5x = 31$

$x = \frac{31}{5}$

7. Consider the function $f(x) = cx^2 + d$, where c and d are constant numbers. If $f(1) = 6$ and $f(-2) = 10$, then find the values of c and d.

Sol. Given function $f(x) = cx^2 + d$

If $(1) = 6$

Then

$c(1)^2 + d = 6$

$c(1) + d = 6$

$c + d = 6$ _____ (i)

If $(-2) = 10$

$c(-2)^2 + d = 10$

$c(4) + d = 10$

$4c + d = 10$ _____(ii)

Subtract (i) from (ii)

$(4c + d) - (c+d) = 10 - 6$

$4c + d - c - d = 4 \Rightarrow 3c = 4$

$$c = \frac{4}{3}$$

Putting $c = \frac{4}{3}$ in (i)

$$\frac{4}{3} + d = 6$$

$$d = 6 - \frac{4}{3}$$

$$= \frac{18-4}{3} = \frac{14}{3}$$

So, $C = \frac{4}{3}, d = \frac{14}{3}$

Review Exercise 3

1. Four options are given against each statement. Encircle the correct option.

(i) The set builder from of the set $\left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots\right\}$ is:

(a) $\left\{x \mid x = \frac{1}{n}, n \in W\right\}$ (b) $\left\{x \mid x = \frac{1}{2n+1}, n \in W\right\}$

(c) $\left\{x \mid x = \frac{1}{n+1}, n \in W\right\}$ (d) $\{x \mid x = 2n+1, n \in W\}$

(ii) If A = { }, then P(A) is:

(a) { } (b) {1} (c) {{ }} (d) ϕ

(iii) If U = {1,2,3,4,5}, A={1,2,3} and B = {3,4,5}, then U – (A∩B) is:

(a) {1,2,4,5} (b) {2,3} (c) {1,3,4,5} (d) {1,2,3}

(iv) If A and B are overlapping sets, then n(A–B) is equal to:

(a) n(A) (b) n(B) (c) A∩B (d) n(A) – n(A∩B)

(v) If $A \subseteq B$ and $B - A \neq \phi$, then n(B–A) is equal to:

(a) 0 (b) n(B) (c) n(A) (d) n(B) – n(A)

(vi) If n(A∪B) = 50, n(A) = 30 and n(B) = 35, then n(A∩B) = :

(a) 23 (b) 15 (c) 9 (d) 40

(vii) If A = {1,2,3,4} and B = {x,y,z}, then cartesian product of A and B contains exactly _____ elements.

(a) 13 (b) 12 (c) 10 (d) 6

(viii) if $f(x) = x^2 - 3x + 2$, then the value of $f(a + 1)$ is equal to:

(a) a + 1 (b) $a^2 + 1$ (c) $a^2 + 2a + 1$ (d) $a^2 - a$

(ix) Given that $f(x) = 3x + 1$, if $f(x) = 28$, then the value of x is:

(a) 9 (b) 27 (c) 3 (d) 18

(x) Let A = {1,2,3} and B = {a, b} two non-empty sets and $f: A \to B$ be a function defined as f = {1,a}, (2,b), (3,b)}, then which of the following statement is true?

(a) f is injective (b) f is surjective (c) f is bijective (d) f is into only

Answers:

i. $\left\{x \mid x = \frac{1}{2n+1}, n \in W\right\}$ ii. {{ }} iii. {1,2,4,5} iv. n(A) – n(A∩B)

v. n(B) – n(A) vi. 15 vii. 12 viii. $a^2 - a$

ix. 9 x. f is surjective

2. Write each of the following sets in tabular forms:

(i) {x|x = 2n, n ∈ N}

Sol. {2,4,6,8,...}

(ii) {x|x = 2m + 1, m ∈ N}

Sol. {3,5,7,9,...}

(iii) {x|x = 11n, n ∈ W ∧ n < 11}

Sol. {0,11,22,33,44,55,66,77,88,99,110}

(iv) {x|x ∈ E ∧ 4 < x < 6}

Sol. { }

(v) {x|x ∈ O ∧ 5≤x < 7}

Sol. {5}

(vi) {x|x ∈ Q ∧ x^2=2}

Sol. { }

(vii) {x|x ∈ Q ∧ x = –x}

Sol. {0}

(viii) {x|x ∈ R ∧ x ∉ Q'}

Sol. {Q}

3. Let U = {1,2,3,4,5,6,7,8,9,10}, A = {2,4,6,8,10}, B = {1,2,3,4,5} and C = {1,3,5,7,9}

List the members of each of the following sets.

(i) A'

Sol. A'

$A' = U - A = \{1,2,3,4, \ldots 10\} - \{2,4,6,8,10\}$

$= \{1,3,5,7,9\}$

(ii) B'

$B' = U - B = \{1,2,3,4, \ldots,10\}\ \{1,2,3,4,5\}$

$= \{6,7,8,9,10\}$

(iii) A∪B

Sol. $A \cup B = \{2,4,6,8,10\} \cup \{1,2,3,4,5\}$

$= \{1,2,3,4,5,6,8,10\}$

(iv) A – B

Sol. $A - B = \{2,4,6,8,10\} - \{1,2,3,4,5\}$

$= \{6,8,10\}$

(v) A∩C

Sol. $A \cap C = \{2,4,6,8,10\} \cap \{1,3,5,7,9\} = \{\ \}$

(vi) A'∪C'

Sol. $A' = U - A = \{1,2,3,4, \ldots, 10\} - \{2,4,6,8,10\}$

$= \{1,3,5,7,9\}$

$C' = U - C = \{1,2,3,4, \ldots, 10\} - \{1,3,5,7,9\}$

$= \{2,4,6,8,10\}$

$A' \cup C' = \{1,3,5,7,9\} \cup \{2,4,6,8,10\}$

$= \{1,2,3,4 \ldots,10\}$

(vii) A'UC

Sol. $A' = U - A = \{1,2,3,4,\ldots,10\} - \{2,4,6,8,10\}$

$= \{1,3,5,7,9\}$

$A' \cup C = \{1,3,5,7,9\} \cup \{1,3,5,7,9\}$

$= \{1,3,5,7,9\}$

(viii) U'

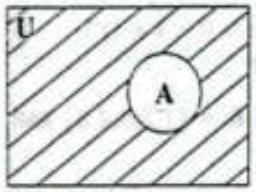
Sol. $U' = U - U = \{1,2,3,4,\ldots,10\} - \{1,2,3,4,\ldots,10\}$

$= \{\ \}$

4. Using the Venn diagrams, if necessary, find the single sets equal to the following:

(i) A'

Sol. A'



(ii) A∩U

Sol. $A \cap U = A$

(iii) A∪U

Sol. $A \cup U = U$

(iv) A∪ϕ

Sol. $A \cup \phi = A$

(v) $\phi \cap \phi$

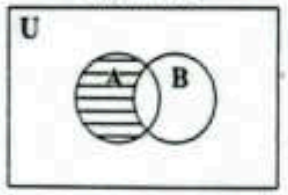
Sol. $\phi \cap \phi = \phi$

5. Use Venn diagrams to verify the following:

(i) A–B = A∩B'

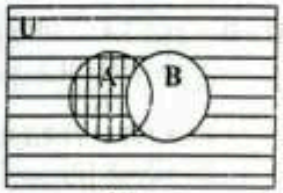
Sol. $A - B = A \cap B'$

L.H.S A–B



$A - B =$ ≡

R.H.S $A \cap B'$



$B' =$ ≡

$A \cap B' =$ ▥

So $A \cap B' =$ ▦

The area shown by ≡ in L.H.S and the area shown by ▦ in R.H.S represent same area.

Note. The statement in the book is wrong.

(ii) $(A - B)' \cap B = B$

Sol. $(A - B)' \cap B = B$

L.H.S $= (A-B)' \cap B$



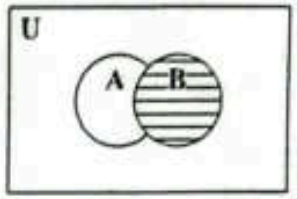
$A - B =$ ≡

$(A-B)' =$ |||

$(A-B)' \cap B =$ ///

So $(A-B)' \cap B =$ ▦

R.H.S B



$B =$ ≡

The area show by ▦ in L.H.S and area show by ≡ in R.H.S represent the same area.

6. Verify the properties for the sets A,B and C given below:

(i) Associativity of Union (ii) Associativity of intersection.

(iii) Distributivity of Union over intersection

(iv) Distributivity of intersection over union.

(a) A = {1, 2, 3, 4}, B = {3, 4, 5, 6, 7, 8}, C = {5, 6, 7, 9, 10}

(b) A = ∅, B = {0}, C = {0, 1, 2}

(c) A = N, B = Z, C = Q

Sol. (i) Associativity of Union

(a) $A \cup (B \cup C) = (A \cup B) \cup C$

L.H.S

$A \cup (B \cup C)$

$B \cup C = \{3,4,5,6,7,8\} \cup \{5,6,7,9,10\} = \{3,4,5,6,7,8,9,10\}$

$A \cup (B \cup C) = \{1,2,3,4\} \cup \{3,4,5,6,7,8,9,10\} = \{1,2,3,4,5,6,7,8,9,10\}$ ...(i)

R.H.S

$(A \cup B) \cup C$

$A \cup B = \{1,2,3,4\} \cup \{3,4,5,6,7,8\} = \{1,2,3,4,5,6,7,8\}$

$(A \cup B) \cup C = \{1,2,3,4,5,6,7,8\} \cup \{5,6,7,9,10\}$

$= \{1,2,3,4,5,6,7,8,9,10\}$ ....(ii)

From (i) and (ii), we get

L.H.S = R. H. S

Hence, associatively of union is verified.

(b) $A \cup (B \cup C) = (A \cup B) \cup C$

L.H.S

$A \cup (B \cup C)$

$B \cup C = \{0\} \cup \{0,1,2\}$

$= \{0,1,2\}$

$A \cup (B \cup C) = \phi \cup \{0,1,2\}$

$= \{0,1,2\}$ __ (i)

R.H.S

$(A \cup B) \cup C$

$A \cup B = \phi \cup \{0\}$

$= \{0\}$

$(A \cup B) \cup C = \{0\} \cup \{0,1,2\}$

$= \{0,1,2\}$ __ (ii)

From (i) and (ii), we get

L.H.S = R. H. S

Hence, associatively of union is verified.

(c) $A \cup (B \cup C) = (A \cup B) \cup C$

L.H.S

$A \cup (B \cup C)$

$B \cup C = Z \cup Q$

$= Q$

$A \cup (B \cup C) = N \cup Q$

$= Q$ ___ (i)

R.H.S

$(A \cup B) \cup C$

$A \cup B = N \cup Z$

$= Z$

$(A \cup B) \cup C = Z \cup Q$

$= Q$ ___ (ii)

From (i) and (ii), we get

L.H.S = R. H. S

Hence, associatively of union is verified.

Sol. (ii) Associative Property of intersection

(a) $A \cap (B \cap C) = (A \cap B) \cap C$

L.H.S $A \cap (B \cap C)$

$B \cap C = \{3,4,5,6,7,8\} \cap \{5,6,7,9,10\}$

$= \{5,6,7\}$

$A \cap (B \cap C) = \{1,2,3,4\} \cap \{5,6,7\}$

$= \{\}$ ___ (i)

R.H.S $(A \cap B) \cap C$

$A \cap B = \{1,2,3,4\} \cap \{3,4,5,6,7,8\}$

$= \{3,4\}$

$(A \cap B) \cap C = \{3,4\} \cap \{5,6,7,9,10\}$

$= \{\}$ ___ (ii)

from (i) and (ii)

Hence, proved L.H.S = R.H.S

(b) $A \cap (B \cap C) = (A \cap B) \cap C$

L.H.S $A \cap (B \cap C)$

$B \cap C = \{0\} \cap \{0,1,2\}$

$= \{0\}$

$A \cap (B \cap C) = \phi \cap \{0\}$

$= \phi$

R.H.S

$(A \cap B) = \phi \cap \{0\}$

$= \phi$

$(A \cap B) \cap C = \phi \cap \{0,1,2\}$

$= \phi$

from (i) and (ii)

Hence, proved L.H.S = R.H.S

(c) $A \cap (B \cap C) = (A \cap B) \cap C$

L.H.S

$(A \cap B) \cap C$

$B \cap C = Z \cap Q = Z$

$A \cap (B \cap C) = N \cap Z$

$= N$

R.H.S

$(A \cap B) \cap C$

$A \cap B = N \cap Z = N$

$(A \cap B) \cap C = N \cap Q$

$= N$

from (i) and (ii)

Hence, proved L.H.S = R.H.S

(iii) Distributivity of union over intersection

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S $A \cup (B \cap C)$

$B \cap C = \{3,4,5,6,7,8\} \cap \{5,6,7,9,10\} = \{5,6,7\}$

$A \cup (B \cap C) = \{1,2,3,4\} \cup \{5,6,7\} = \{1,2,3,4,5,6,7\}$ ..... (i)

R.H.S $(A \cup B) \cap (A \cup C)$

$A \cup B = \{1,2,3,4\} \cup \{3,4,5,6,7,8\} = \{1,2,3,4,5,6,7,8\}$

$A \cup C = \{1,2,3,4\} \cup \{5,6,7,9,10\}$

$= \{1,2,3,4,5,6,7,9,10\}$

$(A \cup B) \cap (A \cup C) = \{1,2,3,4,5,6,7,8\} \cap \{1,2,3,4,5,6,7,9,10\}$

$= \{1,2,3,4,5,6,7\}$ ..... (ii)

From (i) and (ii),

L.H.S = R. H. S

Hence, distributivity of union over intersection is verified.

(b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S $A \cup (B \cap C)$

$B \cap C = \{0\} \cap \{0,1,2\}$

$= \{0\}$

$A \cup (B \cap C) = \phi \cup \{0\}$

$= \{0\}$ ..... (i)

R.H.S

$(A \cup B) = \phi \cup \{0\}$

$= \{0\}$

$A \cup C = \phi \cup \{0,1,2\} = \{0,1,2\}$

$(A \cup B) \cap (A \cup C) = \{0\} \cap \{0,1,2\} = \{0\}$ ...(ii)

(c) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S

$A \cup (B \cap C)$

$B \cap C = Z \cap Q$

$= Z$

$A \cup (B \cap C) = N \cup Z$

$= Z$ .... (i)

R.H.S

$(A \cup B) \cap (A \cup C)$

$A \cup B = N \cup Z = Z$

$A \cup C = N \cup Q = Q$

$(A \cup B) \cap (A \cup C) = Z \cap Q$

$= Z$ .... (ii)

(iv) Distributity of intersection over union

(a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S

$A \cap (B \cup C)$

$B \cup C = \{3,4,5,6,7,8\} \cup \{5,6,7,9,10\}$

$= \{3,4,5,7,8,9,10\}$

$A \cap (B \cup C) = \{1,2,3,4\} \cap \{3,4,5,7,8,9,10\}$

$= \{3,4\}$ .... (i)

R.H.S $(A \cap B) \cup (A \cap C)$

$A \cap B = \{1,2,3,4\} \cap \{3,4,5,6,7\}$

$= \{3,4\}$

$A \cap C = \{1,2,3,4\} \cap \{5,6,7,9,10\}$

$= \{\}$

$(A \cap B) \cup (A \cap C) = \{3,4\} \cup \{\} = \{3,4\}$..(ii)

From (i) and (ii),

L.H.S. = R. H. S

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Hence, distributives of intersection over union is verified.

(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S

$A \cap (B \cup C)$

$B \cup C = \{0\} \cup \{0,1,2\}$

$= \{0\}$

$A \cap (B \cup C) = \phi \cap \{0\}$ .... (i)

$= \phi$

R.H.S $(A \cap B) \cup (A \cap C)$

$A \cap B = \phi \cap \{0\}$

$= \phi$

$A \cap C = \phi \cap \{0,1,2\} = \phi$

$(A \cap B) \cup (A \cap C) = \phi \cup \phi$

$= \phi$ .....(ii)

(c) $A\cap(B\cup C) = (A\cap B)\cup(A\cap C)$

L.H.S

$A\cap(B\cup C)$

$B\cup C = Z\cup Q$

$= Q$

$A\cap(B\cup C) = N\cap Q$

$= N$ ..... (i)

R.H.S

$(A\cap B)\cup(A\cap C)$

$A\cap B = N\cap Z = N$

$A\cap C = N\cap Q = N$

$(A\cap B)\cup(A\cap C) = N\cup N$

$= N$ .....(ii)

From(i) and (ii),

LHS = RHS

$A\cap(B\cap C) = (A\cap B)\cup(A\cap C)$

Hence, distributivity of intersection over union is verified.

7. Verify De Morgan's Laws for the following sets:

$U = \{1,2,3,...,20\}$, $A = \{2,4,6,...,20\}$ and $B = \{1,3,5,...,19\}$

(i) $(A\cup C)' = A'\cap B'$

Sol. L.H.S $(A\cup B)'$

$A\cup B = \{2,4,6,...,20\}\cup\{1,3,5,...,19\}$

$= \{1,2,3,4,5,6,...,19,20\}$

$(A\cup B)' = U-(A\cup B) = \{1,2,3,...,20\}-\{1,2,3,4,5,6,...,19,20\}$

$= \{\ \}$

R.H.S $A'\cap B'$

$A' = U-A = \{1,2,3,...,20\}-\{2,4,6,...,20\} = \{1,3,5,...,19\}$

$B' = U-B = \{1,2,3,...,20\}-\{1,3,5,...,19\} = \{2,4,6,...,20\}$

$A'\cap B' = \{1,3,5,...,19\}\cap\{2,4,6,...,20\} = \{\ \}$

From (i) and (ii),

LHR = RHS

Thus, $(A\cup B)' = A'\cap B'$

(ii) $(A\cap B)' = A'\cup B'$

L.H.S $(A\cap B)'$

$A\cap B = \{2,4,6,...,20\}\cap\{1,3,5,...,19\} = \{\ \}$

$(A\cap B)' = U-(A\cap B) = \{1,2,3,4,...,20\}-\{\ \} = \{1,2,3,4,...,20\}$ ... (i)

R.H.S $A'UB'$

$A'=U-A = \{1,2,3,...,20\}-\{2,4,6,...,20\} = \{1,3,5,...,19\}$

$B'=U-B = \{1,2,3,...,20\}-\{1,3,5,7,...,19\} = \{2,4,6...,20\}$

$A'\cup B' = \{1,3,5,...,19\}\cup\{2,4,6,...,20\}$

$= \{1,2,3,4,...,20\}$ ... (ii)

From (i) and (ii),

LHS = RHS

$(A\cap B)' = A'\cup B$ Hence, Demogan laws is verified.

8. Consider the set $P = \{x|x = 5m, m\in N\}$ and $Q=\{x|x=2m, m\in N\}$. Find $P\cap Q$.

Sol. $P = \{5,10,15,20,25,...\}$

$Q = \{2,4,6,8,10,12,...\}$

$P\cap Q = \{5,10,15,20,25,...\}\cap\{2,4,6,8,10,12,...\}$

$= \{10,20,30,40,...\}$

9. From suitable properties of union and intersection, deduce the following results:

(i) $A\cap(A\cup B) = A\cup(A\cap B)$

Sol. L.H.S $A\cap(A\cup B)$

Using the distributive property

$= (A\cap A)\cup(A\cap B)$

$= A\cup(A\cap B)$ as: $A\cap A = A$

$= R.H.S$

Thus, L.H.S = R.H.S

(ii) $A\cup(A\cap B) = A\cap(A\cup B)$

Sol. L.H.S $= A\cup(A\cap B)$

Using the distributive property

$= (A\cup A)\cap(A\cup B)$

$= A\cap(A\cup B)$ as: $A\cup A = A$

$= R.H.S$

Thus L.H.S = R.H.S

10. If $g(x) = 7x-2$ and $s(x) = 8x^2-3$ find:

(i) g(0)

Sol. $g(x) = 7x-2$

$g(0) = 7(0)-2$

$= 0-2$

$= -2$

(ii) g(−1)

Sol. $g(x) = 7x-2$

$g(-1) = 7(-1)-2$

$= -7-2$

$= -9$

(iii) $g\left(-\frac{5}{3}\right)$

Sol. $g(x) = 7x-2$

$g\left(\frac{-5}{3}\right) = 7\left(\frac{-5}{3}\right)-2$

$= \frac{-35}{3}-2 = \frac{-35-6}{3} = \frac{-41}{3}$

(iv) s(1)

Sol. $S(x) = 8x^2-3$

$S(1) = 8(1)^2-3$

$= 8(1)-3$

$= 8-3$

$= 5$

(v) s(−9)

Sol. $S(x) = 8x^2-3$

$S(-9) = 8(-9)^2-3$

$= 8(81)-3$

$= 648-3$

$= 645$

(vi) $s\left(\frac{7}{2}\right)$

Sol. $s(x) = 8x^2-3$

$s\left(\frac{7}{2}\right) = 8\left(\frac{7}{2}\right)^2-3 = 8^{2}\left(\frac{49}{4_1}\right)-3$

$= 98-3$

$= 95$

11. Given that $f(x) = ax + b$, where a and b are constant numbers. If $f(-2) = 3$ and $f(4) = 10$, then find the values of a and b.

Sol. Given function $f(x) = ax + b$

If

$f(-2) = 3$

Then

$a(-2)+b = 3$

$-2a+b = 3$ _____(i)

If

$f(4) = 10$

$a(4)+b = 10$

$4a+b = 10$ _____ (ii)

Subtract (i) from (ii)

$(4a + b) - (-2a + b) = 10 - 3$

$4a + b + 2a - b = 7$

$6a = 7$

$a = \frac{7}{6}$

Putting $a = \frac{7}{6}$ in ____ (i)

$-2\left(\frac{7}{6}\right) + b = 3$

$-\frac{7}{3} + b = 3$

$b = 3 + \frac{7}{3} = \frac{9+7}{3} = \frac{16}{3}$

So, $a = \frac{7}{6}, b = \frac{16}{3}$

12. Consider the function defined by $k(x) = 7x - 5$. If $k(x) = 100$, find the value o

Sol. Given function $k(x) = 7x - 5$

If $k(x) = 100$,

$k(x) = 7x - 5$

Then

$100 = 7x - 5$

$100 + 5 = 7x$

$105 = 7x$

$\frac{105}{7} = x \Rightarrow 15 = x$

So, $x = 15$

13. Consider function $g(x) = mx^2 + n$, where m and n are constant numbers. If $g(4) = 20$ and $g(0) = 5$, find the values of m and n.

Sol. Given function: $g(x) = mx^2 + n$

If $g(x) = 20$

then $m(4)^2 + n = 20$

$m(16) + n = 20$

$16m + n = 20$ ____(i)

Put $n = 5$ in ____(i)

$16m + 5 = 20$

$16m = 20 - 5$

$16m = 15$

$m = \frac{15}{16}$

So, $m = \frac{15}{16}, n = 5$

If

$g(0) = 5$

Then

$m(0)^2 + n = 5$

$0 + n = 5$

$n = 5$ ____ (ii)

14. A shopping mall has 100 producuts from various categories labeled 1 to 100, representing the universal set U. The products are categorized as follows:

- **Set A: Electronics, consisting of 30 products labeled from 1 to 30.**
- **Set B: Clothing comprises 25 products labeled from 31 to 55.**
- **Set C: Beauty products, comprising 25 products labeled from 76 to 100.**

Write each set in tabular form, and find the union of all three sets.

Sol. Tabula form of the given sets

$A = \{1,2,3,....,30\}$, $B = \{31,32,33,....,55\}$, $C = \{76,77,78,....,100\}$

$A \cup B \cup C = ?$

$A \cup (B \cup C) = \{1,2,3,....,30\} \cup (\{31,32,33,....,55\} \cup \{76,77,78,...100\})$

$A \cup (B \cup C) = \{1,2,3,....,30\} \cup \{31,32,33,....,55,76,77,78,....,100\}$

$A \cup (B \cup C) = \{1,2,3,....,55,76,77,78,....,100\}$

15. Out of the 180 students who appeared in the annual examination, 120 passed the math test, 90 passed the science test, and 60 passed both the math and science tests.

(a) How many passed either the math or science test?

(b) How many did not pass either of the two tests?

(c) How many passed the science test but not the math test?

(d) How many failed the science test?

Sol. Let M be denoted the set for those students who passed the math test. Let S be denoted the set for Those students who passed the science test.

$n(M) = 120, n(S) = 90, n(M \cap S) = 60$

(a) Number of studetns who passed either the Math or science test

$n(M \cup S) = n(M) + n(S) - n(M \cap S) = 120 + 90 - 60 = 150$

So, 150 studetns passed either the math or science test.

(b) Number of students who did not pass either of these two subjects.

$n(M \cup S)' = n(U) - n(M \cup S) = 180 - 150 = 30$

So, 30 students did not pass either of these two subjects.

(c) Number of students who passed the science test but not the math test.

$= n(s) - n(M \cap S) = 90 - 60 = 30$

So, 30 students passed the science test but not the math test.

(d) Number of students who failed the science test.

$n(S)' = n(U) - n(S) = 180 - 90 = 90$

So, 90 students failed the science test.

16. In a software house of a city with 300 software developers, a survey was conducted to determine which programming languages are liked more. The survery revealed the following satistics:

- **150 developers like Python.**
- **130 developers like Java.**
- **120 developers like PHP.**
- **70 developers like both Python and Java.**

- **60 developers like both Python and PHP.**
- **50 developers like both Java and PHP.**
- **40 developers like all three languages: Python, Java and PHP.**

(a) How many developers use at least one of these languages?
(b) How many developers use only one of these languages?
(c) How many developers do not use any of these languages?
(d) How many developers use only PHP?

Sol. Let P be denoted the set of developers who like Python programming language, Let J be denoted set of developers who like Java language and let be denoted set of developers who like PHP language.

$n(P) = 150, n(J) = 130, n(H) = 120, n(P \cap J) = 70$
$n(P \cap H) = 60, n(J \cap H) = 50, n(P \cap J \cap H) = 40$

(a) $n(P \cup J \cup H) = (n) P + n(J) + n(H) - n(P \cap J) - n(P \cap H)$
$- n(J \cap H) + n(P \cap J \cap H$
$= 150 + 130 + 120 - 70 - 60 - 50 + 40$
$= 260$

So, 160 developers use at least one of the languages.

(b) Developer using only Python
$n(P \text{ only}) = n(P) - n(P \cap J) - n(P \cap H) + n(P \cap J \cap H)$
$= 150 - 70 - 60 + 40$
$= 60$

Developers use only Java
$n(\text{only } J) = n(J) - n(P \cap J) - n(J \cap H) - n(P \cap J \cap H)$
$= 130 - 70 - 50 + 40$
$= 50$

Developers use only PHP
$n(\text{only } H) = n(H) - n(P \cap H) - n(J \cap H) - n(P \cap J \cap H)$
$= 120 - 60 - 50 + 40$
$= 50$

Number of developers do not use only on language $= 60 + 50 + 50 = 160$

(c) $n(P \cup J \cup H)' = n(U) - n(P \cup J \cup H)$
$= 300 - 260$
$= 40$

So, 40 developer do not use any language.

(d) Number of developers use only PHP
$n(\text{only } H) = n(H) - n(P \cap H) - n(J \cap H)$
$= 120 - 60 - 50 + 40$
$= 50$

So, 50 developer use only PHP.

Objective Type Question

Additional MCQ's

Multiple Choice Questions prepared in the light of new Examination Techniques (Knowledge, Understanding, Application, Analytical & Conceptual)

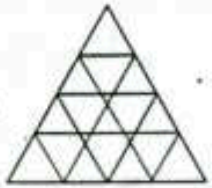
3.1 Mathematics as the study of patterns structures and Relationships

O Encircle the correct option.

1. **The sequence 0,1,1,2,3,5,8,13,21,..... is known as**
(a) Fibonacci sequence (b) odd sequence
(c) even sequence (d) New tanian sequence

2. **The formula of Fibonacci sequence is.**
(a) $F_n = F_{n-1} + F_n + 2$ (b) $F_n = F_{n-1} + F_{n-2}$
(c) $F_n = F_{n-1} + F_n - 2$ (b) $F_n = F_{n-1} + F_{n+2}$

3. **In the given figure Harwinton triangles.**
(a) 15 (b) 16
(c) 17 (b) 18



4. **The study of _____ helps in understanding the concept of relations, functions and especially in statistics.**
(a) Matrix (b) Geometry
(c) Algebra (b) Sets

5. **Which of them is described as a well - define collection of distinct objects, number or elements.**
(a) Sets (b) algebra (c) Matrix (d) Geometry

6. **Names of sets are denoted by**
(a) A, B, C..... (b) a, b, c,..... (c) 1, 2, 3..... (d) α, β, γ

7. **Which of them is used as elements of sets.**
(a) 1, 2, 3 ..... (b) a, b, c, ... (c) both (a) and (b) (d) None of above

8. **How many ways to describes a set?**
(a) two (b) three (c) four (d) five

9. **In which way a set may be described in words?**
(a) Descriptive form (b) Tabular form (c) Set builder form (d) None of above

10. **A = {x/x is a vowel of English alphabets} it is.**
(a) Descriptive form (b) Tabular form (c) Set builder form (d) None of above

11. **The symbol used for membership of a set is**
(a) $\in$ (b) $\notin$ (c) $\forall$ (d) /

12. **The symbol used for does not member of a set is.**
(a) $\in$ (b) $\notin$ (c) $\forall$ (d) /

13. **Set of natural number is:**
(a) {1, 3, 5, 7, ...} (b) {0, 1, 2, 3, ...} (c) {2, 4, 6, ...} (d) {1, 2, 3, 4, ...}

14. **The set of whole number is:**
(a) {1, 3, 5, 7, ...} (b) {0, 1, 2, 3, ...} (c) {2, 4, 6, ...} (d) {1, 2, 3, 4, ...}

15. **The set of integers is as.**
(a) {0, ± 1, ± , 2....} (b) {± 1, ± 3, ± , 5...}
(c) {0, ± 2, ± 4, ...} (d) {2,3, 5, 7, 11, 13, 17, ...}

16. **The set of odd numbers is.**
(a) {0, ± 1, ± , 2....} (b) {± 1, ± 3, ± , 5...}
(c) {0, ± 2, ± 4, ...} (d) {2,3, 5, 7, 11, 13, 17, ...}

17. **The set of even integers is.**
(a) {0, ± 1, ± , 2....} (b) {± 1, ± 3, ± , 5...}
(c) {0, 2, 4, ...} (d) {2,3, 5, 7, 11, 13, 17, ...}

18. **The set of prime number is.**
(a) {0, ± 1, ± , 2....} (b) {± 1, ± 3, ± , 5...}
(c) {0, ± 2, ± 4, ...} (d) {2,3, 5, 7, 11, 13, 17, ...}

19. **The set of rational numbers is:**
(a) $\left\{x / x=\frac{p}{q}, p,q, \in Z \text{ and } q \neq o\right\}$ (b) $\left\{x / x \neq \frac{p}{q}, p,q, \in z, q \neq o\right\}$
(c) $\{\sqrt{2}, \sqrt{4}, \sqrt{6}, \sqrt{8} \ldots\ldots\}$ (d) $\{\sqrt{1}, \sqrt{3}, \sqrt{5}, \sqrt{7} \ldots\ldots\}$

20. **The set of irrational numbers is:**
(a) $\left\{x/x=\frac{p}{q}, p,q, \in Z \text{ and } q \neq o\right\}$ (b) $A'\left\{x/x \neq \frac{p}{q}, p,q, \in Z, \text{ and } q \neq o\right\}$
(c) $\{\sqrt{2}, \sqrt{4}, \sqrt{6}, \sqrt{8} \ldots\ldots\}$ (d) $\{\sqrt{1}, \sqrt{3}, \sqrt{5}, \sqrt{7} \ldots\ldots\}$

21. **$Q \cup Q' =$**
(a) N (b) P (c) E (d) R

22. **The set with _____ element is called singleton set.**
(a) one (b) two (c) three (d) four

23. **The set with _____ element (s) is called an empty set.**
(a) No (b) one (c) two (d) three

24. **The symbol of empty set is.**
(a) $\in$ (b) ϕ (c) $\notin$ (d) $\neq$

25. **Two sets A and B are _____ if they have exactly the same elements.**
(a) equal set (b) Equivalent set (c) Subset (d) Proper set

26. **Two sets A and B are _____ if they have the same number of elements.**
(a) equal set (b) Equivalent set (c) Subset (d) Proper set

27. **If every element of a set is an element of set B then A is ..... of B**
(a) equal set (b) Equivalent set (c) Subset (d) Proper set

28. **If A is a subset of B and B contains at least one element that is not an element of A it is called.**
(a) equal set (b) Equivalent set (c) Subset (d) Proper sub set

29. **If A is a subset of B and A = B then A is an _____ of B**
(a) Improper subset (b) Universal set (c) Power set (d) None of obove

30. **The set that contains all objects or elements under consideration is**
(a) Improper subset (b) Universal set (c) Power set (d) None of obove

31. **Which of them containing all the possible subsets?**
(a) Improper subset (b) Universal set (c) Power set (d) None of obove

32. **Which of them is the set of all elements which belongs to A and B.**
(a) Union of A and B (b) Intersection of A and B
(c) Universal of A and B (d) Difference of A and B

33. **Symbol of Union set is**
(a) ϕ (b) $\cup$ (c) $\cap$ (d) ~

3.2 Operations on Sets

34. **Which of them is the set of all elements that belongs to both A and B**
(a) Power set (b) Union of two sets
(c) Intersection of two sets (d) Universal set

35. **Symbol of Intersection of two sets is**
(a) ϕ (b) $\cup$ (c) $\cap$ (d) ~

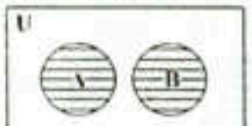
36. **If the intersection of two sets is empty, the sets are said to be.**
(a) Disjoint sets (b) Difference of two sets
(c) Complement set (d) Overlapping sets

37. **If the intersection of two sets is non empty but neither is a subset of the other the sety are called.**
(a) Disjoint set (b) Difference sets (c) Complement set (d) Overlapping sets

38. **Which of them consists of all elements which belongs to A but do not belongs to B called.**
(a) Disjoint set (b) Difference sets (c) Complement set (d) Overlapping sets

39. **Which of them is the set of all element of U which do not belongs to A**
(a) Disjoint set (b) Difference of a sets
(c) Complement of a set (d) Overlapping sets

40. **Which of them is known as disjoints sets?**

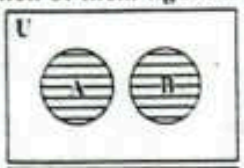
(a) 

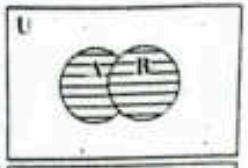
(b) 

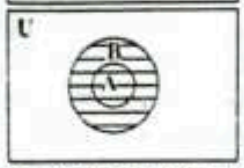
(c) 

(d) 

41. Which of them figure is for overlapping sets?

(a) 

(b) 

(c) 

(d) 

42. AUB = BUA is known as
(a) commutative property of union (b) commutative property of intersection
(c) associative property of union (d) associative property of intersection.

43. A∩B = B ∩ A is known as.
(a) commutative property of union (b) commutative property of intersection
(c) associative property of union (d) associative property of intersection.

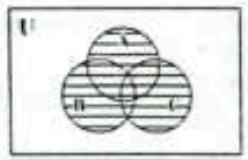
44. A∪(B∪C) = (A∪B)∪C is known as.
(a) commutative property of union (b) commutative property of intersection
(c) associative property of union (d) associative property of intersection.

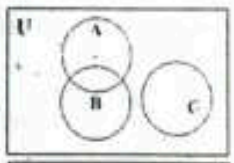
45. A∩(B∩C) = (A∩B)∩C is known as.
(a) commutative property of union (b) commutative property of intersection
(c) associative property of union (d) associative property of intersection.

46. A∪(B∩C) = (A∪B)∩ (A∪C) is known as.
(a) distributive of union (b) distributive of union over intersection
(c) De morgan's law (d) none of above

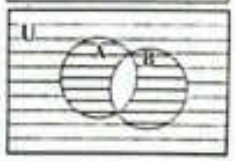
47. A∩(B∪C) = (A∩B)∪(A∩C) is known as.
(a) distributive of union (b) distributives of intersection over union
(c) De morgan's law (d) none of above

48. Which of them is true for A∪ (B∪C)?

(a) 

(b) 

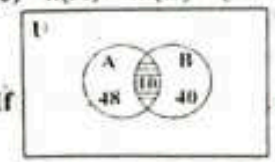
(c) 

(d) 

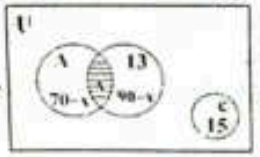
3.2 Real world Applications

49. If A = {1,3,5,7,9,11} then n(A) = _____
(a) 2 (b) 4 (c) 6 (d) 8

50. n(A∪B) = _____
(a) n(A) + n (B) – n(A∩B) (b) n(A) + n (B) + n(A∩B)
(c) n(A) + n (B) – (A∩B) (d) n(A) – n (B) + n(A∩B)

51. If 

then n(A∩B) = _____
(a) 10 (b) 48 (c) 40 (d) 98

52. If 

then x = _____
(a) 70 (b) 25 (c) 90 (d) 20

3.3. Binary Relations

53. Ordered pairs is written as
(a) x (b) (x,y) (c) y (d) None of these

54. A × B = _____
(a) {(x,y) | x ∈A, y∈B} (b) {(x,y) | x ∈A, x∈B}
(c) {(x,y) | y ∈A, y ∈B} (d) None of these

55. If A = {1, 2, 3,}, B = {4, 5} then A × B = _____
(a) {(1, 4), (2, 5)} (b) {(2, 4), (2, 5)}
(c) {(3, 4), (3, 5)} (d) {(1, 4), (1, 5), (2, 4), (2,5) (3,4), (3,5)}

56. If A × B = {(1,2), (3, 4), (5, 6), (7, 8)}then domain is
(a) {1, 3, 5, 7} (b) {1, 4, 6, 8} (c) {1, 2, 3, 4, 5} (d) {6, 7, 8}

57. If A × B = {(1, 2), (3, 4), (5, 6), (7, 8)} then Range is
(a) {1, 3, 5, 7} (b) {2, 4, 6, 8} (c) {1, 2, 3, 4, 5} (d) {6, 7, 8}

3.3.1 Function and its Domain and Range function.

58. Each ordered pair consists of _____ coordinates.
(a) two (b) three (c) four (d) five

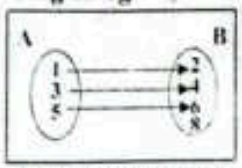
59. In (x,y), x is known as
(a) Abscissa (b) Ordinate (c) Subset (d) None of above

60. In (x,y) y is known as
(a) Abscissa (b) Ordinate (c) Subset (d) None of above

61. If A = {0, 1, 2, 3, 4}, B = {3, 5, 7, 9, 11} and f = {(x, y)| y = 2x + 3, x ∈A, y ∈ B} then f = _____
(a) {0, 1, 2, 3, 4} (b) {3, 5, 7, 9, 11}
(c) {(0, 3), (1,5), (2,7), (3,9) (d) {(0, 1) (2,3), (4,7) (0,9), (1,11)}

62. How many types of functions?
(a) two (b) three (c) Four (d) five

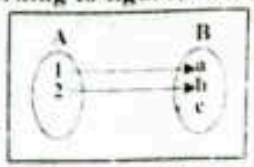
63. According to figure, if is known as



f = {(1,2), (3,4), (5,6)}

(a) Into function
(b) One - one function
(c) onto function
(d) Bijective function

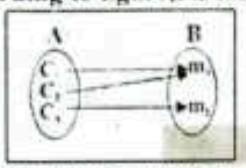
64. According to figure.f is known as



f = {(1,a), (2,b)}

(a) Into function
(b) One - one function
(c) onto function
(d) Bijective function

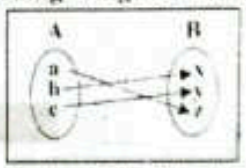
65. According to figure,f is known as



$f = \{(c_1, m_1), (C_2, m_1), (C_3, m_2)\}$

(a) Into function
(b) One - one function
(c) onto function
(d) Bijective function

66. According to figure is known as



f = {(a, z), (b, x), (c, y)}

(a) In to function
(b) One - one function
(c) on to function
(d) Bijective function

67. If f = {(–1, 1), (0, 0), (1, 1), (2, 4), (3, 9), (4, 16)} then Domain f is

(a) {–1, 0, 1, 2, 3, 4} (b) {0, 1, 4, 9, 16}
(d) {– 1, 1, 2, 4, 9, 16} (c) None of above

68. If f = {(-1, 1), (0, 0), (1, 1), (2, 4), (3, 9), (4, 16)}then range f is

(a) {–1, 0, 1, 2, 3, 4} (b) {0, 1, 4, 9, 16}
(d) {– 1, 1, 2, 4, 9, 16} (c) None of above

69. If f (x) = 2x – 1 then f (1) = _____

(a) 0 (b) 1 (c) 2 (d) 3

70. If f (x) = 2x – 1, then f (7) = _____

(a) 10 (b) 11 (c) 12 (d) 13

71. If g(x) = x^2 – 3 then f (– 3) = _____

(a) 2 (b) 4 (c) 6 (d) 8

72. If g(x) = x^2 – 3 then g (4) = _____

(a) 13 (b) 11 (c) 10 (d) 9

Answers:

1.	Fibonacci sequence			2.	$F_n = F_{n-1} + F_{n-2}$	3.	17
4.	Sets	5.	Sets	6.	A, B, C.....	7.	both (a) and
8.	three	9.	Descriptive form	10.	Set builder form	11.	$\in$
12.	$\notin$	13.	{1, 2, 3, 4, ...}	14.	{0, 1, 2, 3, ...}	15.	{0,±1,± , 2....}
16.	{± 1, ± 3, ± ,5...}	17.	{0, 2, 4, ...}	18.	{2,3, 5, 7, 11, 13, 17, ...}		

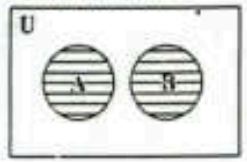
19. $\left\{x / x = \frac{p}{q}, p,q, \in Z \text{ and } q \neq o\right\}$

20. $\left\{x / x \neq \frac{p}{q}, p,q, \in z, \text{ and } q \neq o\right\}$

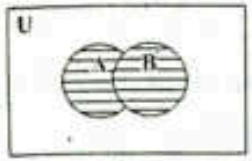
21. R	22. one	23. No	24. ϕ
25. equal set	26. Equivalent set	27. Subset	28. Proper sub set
29.Improper subset	30. Universal set	31. Power set	32.Union of Aand B
33. $\cup$	34. Intersection of two sets		35. $\cap$
36. Disjoint sets	37. Overlappingsets	38. Difference sets	

39. Complement of a set

40.



41.

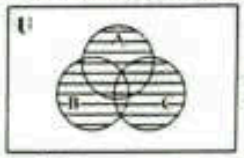


42. commutative property of union

43. commutative property of intersection
44. associative property of union
45. associative property of intersection.
46. distributive of union over intersection

47. distributive of intersection over union

48.



49. 6

50. $n(A) + n(B) - n(A \cap B)$		51. 10	52. 25
53. (x,y)	54. $\{(x,y) \mid x \in A, y \in B\}$	55. {(1, 4), (1, 5), (2, 4), (2,5) (3,4), (3,5)}	
56. {1, 3, 5, 7}	57. {2, 4, 6, 8}	58. two	59. Abscissa
60. Ordinate	61. {(0, 3), (1,5), (2,7), (3,9)		62. Four
63. Into function	64. One - one function	65.onto function	
66. Bijective function	67. {–1, 0, 1, 2, 3, 4}	68.{0, 1, 4, 9, 16}	69. 1
70. 13	71. 6	72. 13	

Conceptual MCQ's

Encircle the correct option.

1. George cantor was a mathematicians:

(a) German (b) Pakistani (c) American (d) Sweden

2. The set {0} is a _____ set.

(a) empty (b) singleton (c) American (d) Sweden

3. The subset of a set can also be stated as?

(a) $A \subseteq B$ iff $\forall x \in A \Rightarrow x \in B$ (b) $A \subseteq A$ iff $\forall x \in A \Rightarrow x \in B$
(c) $A \subseteq A$ iff $\forall x \in A \Rightarrow x \in B$ (d) None of above

4. Which of them is the correct sequence.

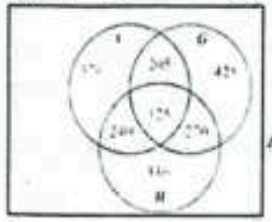
(a) $A \subset W \subset Q \subset \geq \subset R$ (b) $N \subset W \subset Z \subset Q \subset R$
(c) $R \subset W \subset N \subset Z \subset Q$ (d) $W \subset R \subset N \subset Q \subset 7$

5. Symbol of union is
(a) ∩ (b) ∨ (c) ∪ (d) ∧

6. The symbol _____ means or.
(a) ∩ (b) ∨ (c) ∪ (d) ∧

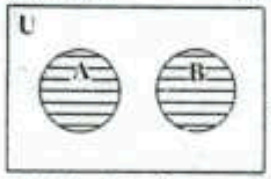
7. The symbol ____ means and.
(a) ∩ (b) ∨ (c) ∪ (d) ∧

8. Which is correct for $A' =$ _____
(a) U – A (b) U – U (c) A – A (d) A – U

9.  According to the figure, the total number of elements is:
(a) 1940 (b) 1950 (c) 1970 (d) 1971

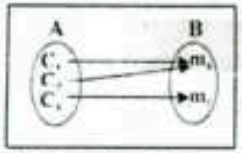
10. $A \times B \neq B \times A$ it is as tatexreut
(a) False (b) true (c) both (a) and (b) (d) None of obove

11. $(2, 3) \neq (3, 2)$ it is statement
(a) False (b) true (c) both (a) and (b) (d) None of obove

12. 

it is known as
(a) Overlapping sets (b) $A \subseteq B$ (c) Disjoint sets (d) None of these

13. $n(A \cap C) + n(A \cap B \cap C) =$ _____
(a) $n(A \cup B \cup C$ (b) $n(A \cap B \cap C$ (c) $n(A \cup B \cap C)$ (d) $n(A \cap B \cup C)$

14. 

,then f is function.
$f = \{(c_1, m_1), (C_2, m_1), (C_3, m_2)\}$
(a) in to (b) on to (c) one – one (d) None of these

Answers:

1. German	2. singleton	3. $A \subseteq B$ iff $\forall x \in A \Rightarrow x \in B$	
4. $N \subset W Z \geq \subset Q \subset R$	5. ∪	6. ∨	7. ∧
8. U – A	9. 1971	10. true	11. true
12. Disjoint sets	13. $n(A \cup B \cap C)$	14. on to	

Additional Short Questions

Short Answer Questions prepared in the light of new Examination Techniques (Knowledge, Understanding, Application, Analytical & Conceptual)

3.1 Mathematics as the study of patterns, structures and Relationships

1. **What is Fibonacci sequence?**
Ans. The sequence 0, 1, 1, 2, 3, 5, 8, 13, 21,..... is known as Fibonacci sequence.

2. **What is the formula of Fibonacci sequence?**
Ans. $F_n = F_{n-1} + F_{n-2}$
Where $F_0 = 0$ and $F_1 = 1$

3. **Define structure**
Ans. A structure is a collection of items or objects, along with particular relationships defined among them.

4. **How study of sets helps us?**
Ans. The study of sets helps in understanding the concept of relations, functions and especially in statistics we use sets to understand probability and other important ideas.

5. **Define sets.**
Ans. A set is described as well defined collection of distinct objects, numbers or elements.

6. **How we represent the sets and elements of sets?**
Ans. Capital letters A,B,C,X,Y,Z etc are generally used as names of sets and small letters a,b,c,x,y,z etc are used as members or elements of sets.

7. **How many ways to described a set?**
Ans. There are three ways of describing a set.

8. **What is descriptive form of a set?**
Ans. A set may be described in words For instance, the set of all vowels of the English alphabet

9. **What is tabular form of sets?**
Ans. A set maybe described by listing its elements within brackets.

10. **What is the set builder method.**
Ans. Set builder method is done by using a symbol or letter for an arbitrary set member and stating the property common to all the members. A = [x | x is a vowel of English alphabet]
This is read as A is the set of all x such that x is a vowel of English alphabets.

11. **Which is symbol of belongs to?**
Ans. The symbol of belongs to is ∈.

12. **Which is symbol of "does not belongs to"?**
Ans. The symbol of does not belongs to is ∉.

13. **What is symbol of rational number set**
Ans. Q is symbol of rational number set

14. **What is the symbol of real numbers set**
Ans. R is the symbol of real numbers.

15. Define singleton set.

Ans. A set with only one element is called a singleton set.e.g.{1} is singleton set.

16. What is empty set?

Ans. The set with no element(s) called an empty set, null set or void set. It is denoted by ϕ or $\{\ \}$

17. What are equal sets?

Ans. Two sets A and B are equal if they have exactly the same elements or if every element of set A is an element of set B If two sets A and B are equal, we write $A = B$

18. Define equivalent sets.

Ans. Two sets A and B are equivalent if they have same number of elements The symbal~ is used to represent equivalent sets.

19. Define subset

Ans. If every element of a set A is an element of set B, then A is a subset of B. It is denoted by $A \subseteq B$.

20. What is proper subset?

Ans. If A is a subset of B and B contains at least one element that is not an element of A, then A is said to be proper subset of B we write it as. $A \subset B$

21. What do you mean by improper subset?

Ans. If A is a subset of B and $A = B$ then A is an improper subset of B.

22. What is Universal set?

Ans. The set that contains all objects or elements under consideration is called the Universal set. It is denoted by U.

23. Define power set.

Ans. The power set of a set S denoted by P (S) is the set containing all the possible subsets of S.

3.2 Operations on Sets.

24. What are the Union of two sets?

Ans. The union of two sets A, B denoted by $A \cup B$, is the set of all elements which belongs to A or B symbolically,

$A \cup B = \{ x \mid x \mid \in A \vee x \in B\}$

25. What do you know about intersection of two sets?

Ans. The intersection of two sets A and B denoted by $A \cap B$ is the set of all elements that belongs to both A and B.

$A \cap B = \{ x \mid x \in A \wedge x \in B\}$

26. Define disjoint sets.

Ans. If the intersection of two sets is the empty set, the sets are said to be disjoint.

27. What are overlapping sets?

Ans. If the intersection of two sets is non- empty but neither is a subset of other, the sets are called overlapping sets.

28. What do you mean by difference of two sets?

Ans. The difference between the sets A and B denoted by A-B, consists of all the elements that belong to A but do not belong to B.

$A-B = \{ x \mid x \in A \wedge x \notin B\}$

$B-A = \{x \mid x \in B \wedge x \notin A\}$

29. What do you mean by complement of a set?

Ans. The complement of a set A denoted by A' or A^c related to the Universal set U is the set of all elements of U, which do not belongs to A.

$A' = \{x \mid x \in U \wedge x \notin A\}$

30. What do you know about Venn diagram?

Ans. Venn diagram is very useful in depicting visually the basic concepts of sets and relationships between sets. These diagrams were first used by an English logician and Mathematician ciam John Venn (1834 to 1883 A.D)

3.2 Operations on three sets

31. Write the commutative property of union.

Ans. If A and B are two sets then $A \cup B = B \cup A$

32. Write the commutative property of Intersection.

Ans. If A and B are two sets then $A \cap B = B \cap A$

33. What is the associative property of union?

Ans. $A \cup (B \cup C) = (A \cup B) \cup C$

35. What is the associative property of Intersection?

Ans. $A \cap (B \cap C) = (A \cap B) \cap C$

36. What is distributivity of Intersection over Union.

Ans. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

37. Write the Morgan's laws

Ans. (i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

38. If A = {1, 2, 3}, B = {4, 5, 6} then find $A \cup B$

Ans. $A \cup B = \{1, 2, 3\} \cup \{4, 5, 6\} = \{1, 2, 3, 4, 5, 6\}$

39. If A = {1, 2, 3}, B = {4, 5, 6} then find $A \cap B$

Ans. $A \cap B = \{1, 2, 3\} \cap \{4,5,6\} = \{\ \} = \phi$

40. Define cardinality of a set.

Ans. The cardinality of a set is defined as the total number of elements. It is basically the size of set. It is devoted by e.g: $A = \{1, 3, 5, 7, 9, 11\}$ then $n(A) = 6$

41. What is the principle of inclusion and exclusion for three sets.

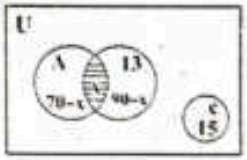
Ans. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

42. If n (A) = 58, n(B) = 50, and $n(A \cup B) = 98$, then find $n(A \cap B)$

Sol. $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 58 + 50 - 98 = 10$

43. Look at the figure and find value of x if n (U) = 150



Sol. $= 70 - x + x + 90 - x + 15 = 150$

$175 - x = 150$

$x = 25$

3.3 Binary Relation

44. What do you mean by relation?

Ans. The Relation means an abstract type of connection between two persons or objects for instance (teacher, pupil) (husband, wife). In mathematics : (5, 4); square (25, 5)

45. What is Cartesian product?

Ans. Let A, B be two non-empty sets, then the Cartesian product is the set of all ordered pairs (x,y) such that $x \in A$, $y \in B$ and is denoted by $A \times B$.

$$A \times B = \{(x,y) \mid x \in A \text{ and } y \in B\}$$

46. What is Domain?

Ans. The set of the first elements of the ordered pairs forming a relation is called domain. It is denoted as Dom.

47. What is Range of Binary relation?

Ans. The set of the second elements of the ordered pairs forming a relation is called range. It is denoted by Ran.

48. How many co-ordinates of ordered pairs?

Ans. Each ordered pair consists of two co-ordinates x and y. The x coordinate is called abscissa and y coordinate is ordinate.

49. What do you mean by ordered pairs?

Ans. A relation can be represented by a set of ordered pairs:

50. Define function.

Ans. Let A and B be two non- empty sets such that

(i) f is a relation from A to B, f is subset of $A \times B$

(ii) Domain f = A

(iii) First element of no two pairs of f are equal then f is said to be a function from A to B. Tis function f is also writter as.

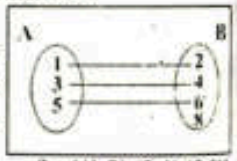
51. How many types of functions?

Ans. There are four types of functions.

(i) In to (ii) on to (iii) one to one and (iv) one to one and on to function.

52. What do you mean by into function?

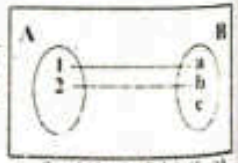
Ans. If a function $f : A \to B$ is such that Range f B I, e, Range f $\subset$ B then f said to be a function from A in to b. Range f$\subset$ B i. e, Range f$\subset$B i. e, Range f $\neq$ B then f is said to be a function from A into B. In figure, f is clearly a function. But range f $\neq$ B. therefore f is a function fromainto B.



f= {(1,2),(3,4),(5,6)}

53. What is one - one function or Injective function?

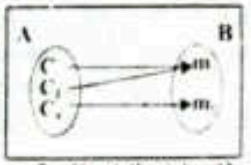
Ans. If a function f from A into B is such that second elements of no two of its ordered pairs are same then it is called an injective function.



f= {(1,a),(2,b),(l,c)}

54. What is onto function?

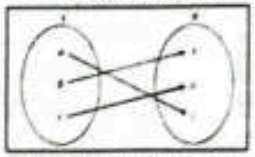
Ans. If a function $f : A \to B$ is such that Range f = B i,e every element of B is the image of some element of A then f is called an onto or surjective function.



f = {(a,z),(b,x),(c,y)}

55. What is one - one and on to or bijective function?

Ans. A function f from A to B is said to be a bijective function if it is both one - one and onto.


f {(a, z), (b, x), (c, y)}

56. If f = {(–1, 1), (0, 0), (1, 1), (2, 4), (3, 9), (4, 16)} then find Domain and range

Ans. Dom f = {– 1, 0, 1, 2, 3, 4}

Range f = {0, 1, 4, 9, 16}

57. Define linear function and quadratic function.

Ans. The function $\{(x,y) \mid y = mx + c\}$ is called linear function because its graph is a straight line and the function $\{(x,y) \mid y = ax^2 + bx + c\}$ is called quadratic function

58. If f(x) = 2x – 1 then find f (–3)

Ans. $f(x) = 2x - 1$

$f(-3) = 2(-3) - 1 = -6 - 1 = -7$

59. If g (x) = x^2 – 3 then find g (–5)

Ans. $g(x) = x^2 - 3$

$g(-5) = (-5)^2 - 3 = 25 - 3 = 22$

60. If A = {3, 6, 9, 12}, B = {1, 3, 6} in binary relation R: A→B such that x–y=6 then find range R:

Ans. As $x - y = 6 \Rightarrow y = x - 6$

R = {(9, 3), (12, 6)}

then Range R = {3, 6}

61. If A = {3, 6, 9, 12}, B = {1, 3, 6} in binary relation R:A→B such that x-y=6 then find DomR.

Sol. Given where $x \in A$ and $y \in B$

$\Rightarrow y = x - 6$ ____ (i)

Put x = 9 in equation (i), we get

$y = 9 - 6 = 3 \Rightarrow y = 3$

Put x = 12 in equation (i), we get

$y = 12 - 6 = 6 \Rightarrow y = 6$

R = {(9, 3), (12, 6)}

then DomR = {9, 12}

Conceptual Short Questions

1. Define sets.

Ans. A set is well, defined collection of distinct objects.

2. How many ways of describing a set?

Ans. There are three different ways of describing a set

(i) Descriptive method (ii) Tabular method (iii) Builder metho

3. What do you mean by size of set?

Ans. A set has a finite or infinite of elements and the number of elements in a se called cardinality or size of the set.

4. What is power set?

Ans. The set of all possible subsets of set X is represented by the power set P(X).

5. Write the power set of A = {1, 2}:

Ans. P(A) = {ϕ, {1}, {2}, {1, 2}}

6. How many operations on sets name them.

Ans. There are four operations as sets. Union, intersection, difference and complemen

7. What do you mean by disjoint and overlapping?

Ans. Sets are considered disjoint if they have no common elements. If they have a common elements they are overlapping.

8. If U {1, 2, 3,---- 20}, B = {2, 4, 6, 8, 10} C = {1, 3, 5, 7, 9, 11}. Find $B^c \cap C^c$

Ans. B^c = U–B = {1, 3, 5, 7, 9, 11, 12, ...20}

C^c = U – C = {2, 4, 6, 8, 9, 11, 12, 13---20}

$B^c \cap C^c$ = {12, 13---20}

9. If A = {2, 4, 6, 8, 10}, B = {1, 3, 5, 7, 9, 11}, C = {1, 2, 3, 5, 7, 10}

Find (A∩C) ∪ (B ∩C)

Ans. A∩C = {2, 10}, B∩C = {1, 3, 5}

(A∩C)∪(B∩C) = {1, 2, 3, 5, 7, 10} = C

10. If U = {5, 10, 15, 20, 25, 30, 35, 40}

A = {5, 10}, B = {10, 20, 30} then Find $(A \cup B)^c = A^c \cap B^c$

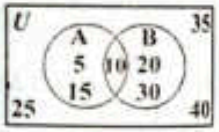
Ans. $(A \cup B)^c$ = [{5, 10,}∪{10, 20, 30}]^c

= [{5, 10, 20, 30}]^c

= {15, 25, 35, 40}

Now $A^c \cap B^c$ = {15, 20, 25, 30, 35, 40}∩{5, 15, 25, 35, 40}

= {15, 25, 35, 40}



11. Write distributive property of union over intersection.

Ans. A∪(B∩C) = (A∪B)∩(A∪C)

12. Write distributive property of intersection over union.

Ans. A∩(B∪C) = (A∩B)∪(A∩C)

13. If A {3, 6, 7, 8, 9}, B = {1, 2, 3, 4, 6} C = {2, 4, 5, 6, 8} find A∪(B∪C)

Ans. A ∪(B∪C) = {3, 6, 7, 8, 9}[∪{1, 2, 3, 4, 6}∪{2, 4, 5, 6, 8}]

= {3, 6, 7, 8, 9}∪{1, 2, 3, 4, 5, 6, 8}

= {1, 2, 3, 4, 5, 6, 7, 8, 9}

14. If A = {3, 6, 7, 8, 9}, B = {1, 2, 3, 4, 5, 6}

C = {2, 4, 5, 6, 8} find A∪ (B∩C)

Sol. A ∪ (B∩C) = {3, 6, 7, 8, 9}∪[{1, 2, 3, 4, 6}∩{2, 4, 5, 6, 8}]

= {3, 6, 7, 8, 9}∪{2, 4, 6} = {2, 3, 4, 6, 7, 8, 9}

15. If A = {3, 6, 7, 8, 9}, B = {1, 2, 3, 4, 6}

C = {2, 4, 5, 6, 8} find A∩(B∪C)

Sol. = {3, 6, 7, 8, 9}∩[{1, 2, 3, 4, 5, 6}∪{2, 4, 5, 6, 8}]

= {3, 6, 7, 8, 9}∩{1, 2, 3, 4, 5, 6, 8} = {3, 6, 8}

16. If n(M∪P∪C) = 45, n(M) = 10, n(P) = 10

n(C) = 5, n(M∩P) = 6, n(P∩C) = 4,

n(M∩C) = 1 find n(M∩P∩C)

Sol. Given that n(M∪P∪C) = 45, n(M) = 18, n(P) = 12,n(C) = 8, n(M∩P) = 6, n(P∩C) = 4 and n (M∩C) = 2

using the principle of Inclusion and exclusion for three sets.

n(M∪P∪C) = n(M) + n(P) + n(C) – n(M∩P) – n(P∩C) – (M∩C) + n(M∩P∩C)

⇒n (M∩P∩C)– n(M) – n(P) – n(C) + n(M∩P) + n(P∩C) + n(M∩C)

= 45 – 10 – 10 – 5 + 6 + 4 + 1 = 31

17. If n(M∪P∪C) = 45, n(M) = 10, n(P) = 10

n(C) = 5, n(M∩P) = 6, n(P∩C) = 4, n(M∩C) = 1

find n(M∩P∩C)

Ans. n(M∪P∪C) = n(M) + n(P) + n(C) – n(M∩P)– n(P∩C) – n(M∩C) + n(M∩P∩C)

n(M∩P∩C) = n(M∪P∪C) – n(M) – n(P) – n(C)+n(M∩P) + n(P∩C) + n(M∩C)

= 45 – 10 – 10 – 5 + 6 + 4 + 1 = 31

18. If U = {1, 2, 3,---- 100}, A = {1, 2, 3,----30}

B = {31, 32, 33, ---- 55}, C = {76, 77, 78, ---- 100} then find A∪$(B \cup C)^c$

Ans. $(B \cup C)^c$ = U –(B∪C) = {1, 2,3----100}–({31, 32, ---55}∪{76, 77, ---100})

= {1, 2, 3,----100}–{31, 32----55, 76, 77----100}

= {1, 2, 3,---- 30, 56, 57, ----75}

A∪$(B \cup C)^c$ = {1, 2, 3,---- 30, 56, 57, 58,----75}

19. What do you mean by Binary relation?

Ans. The product of two non empty sets A and B is denoted by A×B. The elements of A×B are represented by ordered pairs (x, y) such that x∈A and y∈B. A relation R: Any subset of A × B is called a binary relation.

20. What do you mean by domain?

Ans. The sets of first element of each ordered pair is called domain of relation R denoted by Dom R.

21. What do you mean by Range?

Ans. The set of second elements of each ordered pair is called range of a relation R denoted by Range R.

22. If A = {2, 4, 6}, B = {3, 5} find A×B.

Ans. A×B = {2, 4, 6}×{3, 5}

A×B = {(2, 3), (2, 5), (4, 3), (4, 5), (6, 3), (6, 5)}

23. If A = {2, 4, 6}, B = {3, 5} find B×A

Sol. B×A = {3, 5} × {2, 4, 6}

= {(3, 2), (3, 4), (3, 6), (5, 2), (5, 4), (5, 6)}

24. If A = {2, 3, 4, 5}, B = {9, 12, 13, 14} In binary relation R: A→B = {y = 3x} then find the domain R.

Ans. y = 3x, R = {(3, 9), (4, 12)}
DomR = {3, 4}

25. If A = {2, 3, 4, 5}, B = {9, 12, 13, 14} in binary relation R: A→B = {y=3x} then find Range R.

Ans. y = 3x, R = {(3, 9), (4, 12)}
Range R = {9, 12}

26. Define relation.

Ans. A relation in mathematics represent relationship between elements of two or more sets.

27. How many ways to represent relation?

Ans. There are three ways to represent relation.

28. What do you mean by Table in relation?

Ans. A Table is a simple and straight forward way to represent a relation.

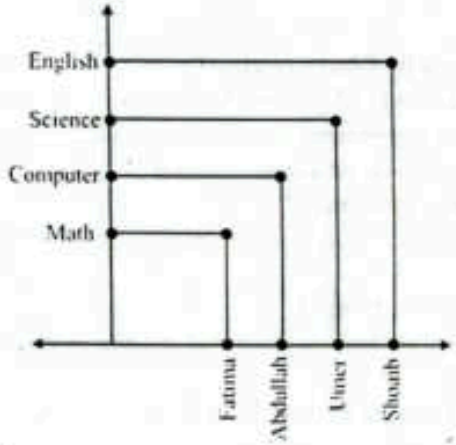
Students	Subjects
Fatima	mathematics
Abdullah	Computer
Umer	Science
Shoaib	English

29. What do you mean by order pairs in relation?

Ans. As ordered pair in a pair of elements written in a specific order (Fatima, Math) (Abdullah, Computer) (Umer, Science) (Shoaib, English)

30. What do you mean by graph in relation?

Ans. It is visual representation of a relatin.



31. Define Functions:

Ans. Let A and B be two non-empty sets a relation f:A→B is called function if it satisfies following conditions.

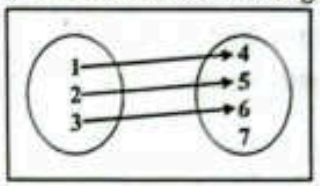
(a) Dom f = A (b) Every x∈A appears in one and only one ordered pair in f.

32. How many types of functions?

Ans. There are four types of functions.

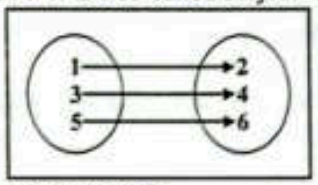
80. What is into function?

Ans. A function f:A→B is said to be into function if Range f⊂B i.e. range f≠B.



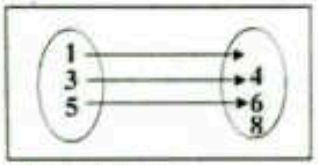
33. What is onto function?

Ans. A function f:A→B is onto function if Range f=B. i.e, every element of B is the simage of some element of A. It is also called surjective functions.



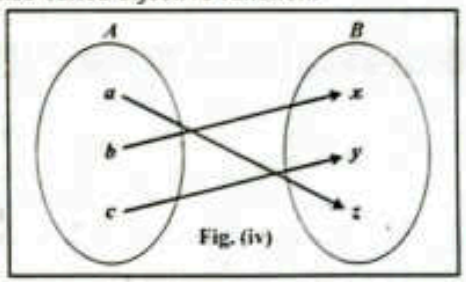
34. What is one to one and into function?

Ans. If a function f from A into B is such that second elements of no to one and into function.



35. What is one to one and on to function.

Ans. Function f from A to B is said to be a one to one and onto function if it is both one - one and onto. It is also called bijective function.



$f = \{(a, z), (b, x), (c, y)\}$

— ✲✲✲ —

Unit 4 Factorization and Algebraic Manipulation

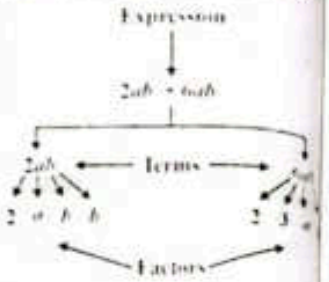
Identifying Common Factors and Trinomials Concretely, Pictorially and Symbolically

Common Factors

In algebra, a common factor is an expression that divides two or more expressions exactly. For example,

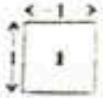
$$2x - 6 = 2(x - 3)$$

Here 2 is the common factor which is exactly divides by both terms 2x and 6.



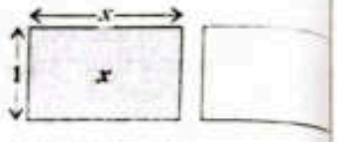
Unit Tiles

Here one grey unit tile represent 1 and one white unit tile represents –1. Both grey and white unit tiles form a zero pair.



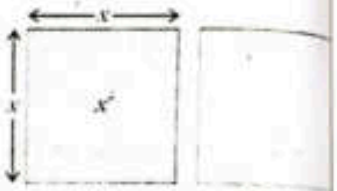
Rectangular Tiles

The grey rectangular tile represents x and the white rectangular tile represents –x. Both grey and white rectangular tiles also form a zero pair.



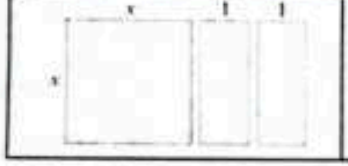
Squared Tiles

The grey squared tile measure x units on each side and it has an area of $x \times x = x^2$ units. This tile is labelled as x^2 tile. The white squared tile represents-x^2. Both grey and white squared tiles form a zero pair.



Example 1: Find common factor of $x^2 + 2x$ concretely, pictorially and symbolically

Solution: We arrange one x^2 tile and two x tiles into a rectangle.

Concretely	Pictorially	Symbolically
		$x^2 + 2x = x(x + 2)$

Trinomial Factoring

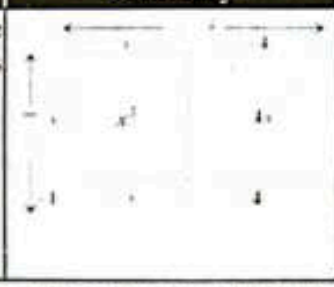
Trinomial factoring is converting trinomial expression as a product of two binomia expressions. A trinomial is an expression with three terms and binomial is an expressio with two terms.

For example, x^2+4x+4 and $3x^2-x-2$ are trinomials whereas $x+2$ and $3x-1$ are binomials.

Teacher's Note
Algebraic tiles of different sizes can easily be made with different coloured chart papers.

Example 2: Factorize $x^2 - 5x + 4$ concretely, pictorially and symbolically.

Solution:

Concretely	Pictorially	Symbolically
We arrange one x^2 tile, five –x tiles and four unit tiles into a rectangle. 		$x^2 - 5x + 4$ $= (x - 1)(x - 4)$

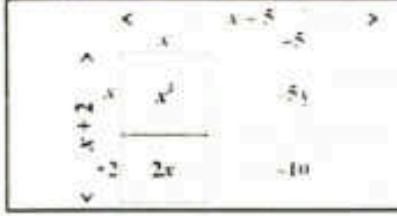
Example 3: Factorize $x^2 - 3x - 10$ concretely, pictorially and symbolically.

Solution:

Concretely we arrange one x^2 tile, three –x tiles and ten –1 tiles into rectangle.



We see that there are not enough rectangular tiles to make a larger rectangle. To fix this issue, we add zero pair. Adding two x tiles and two –x tiles does not change the given expression because $2x - 2x = 0$.

Pictorially	Symbolically
	$x^2 - 3x - 10 = (x - 2)(x - 5)$

Factorizing Quadratic and Cubic Algebraic Expressions

Type – I: Factorization of expression of the types $x^2 + px + q$ and $ax^2 + bx + c$

The procedure is explained in the following examples to factorize the above type of expressions:

Example 4: Factorize: $x^2 + 9x + 14$

Solution: Two numbers whose product is +14 and their sum is 9 are + 2, +7.

Product of factors	Sum of factors
$14 \times 1 = 14$	$14 + 1 = 15$
$7 \times 2 = 14$	$7 + 2 = 9$

So, $x^2 + 9x + 14$

$= x^2 + 2x + 7x + 14$

$= x(x + 2) + 7(x + 2)$

$= (x + 2)(x + 7)$

Example 5: Factorize: $x^2 - 11x + 24$

Solution: Two numbers whose product is +24 and their sum is −11 are −8, −3.

So, $x^2 - 11x + 24$

$= x^2 - 8x - 3x + 24$

$= x(x-8) - 3(x-8)$

$= (x-8)(x-3)$

Product of factors	Sums of factors
$24 \times 1 = 24$	$24 + 1 = 25$
$8 \times 3 = 24$	$8 + 3 = 11$
$(-8) \times (-3) = 24$	$-8 - 3 = -11$
$6 \times 4 = 24$	$6 + 4 = 10$
$12 \times 2 = 24$	$12 + 2 = 14$

Example 6: Factorize: $p^2 + 11p + 18$

Solution: $p^2 + 11p + 18$

$= p^2 + 9p + 2p + 18$ $\quad \because 9+2=11,\ 9\times 2=18$

$= p(p+9) + 2(p+9)$

$= (p+9)(p+2)$

In all quadratic trinomials factorized so far, the coefficient of x^2 was 1. We will now consider cases where the coefficient of x^2 is not 1.

Example 7: Factorize: $2x^2 + 17x + 26$

Solution:

Step - I: Multiply the coefficient of x^2 with constant term.i.e.,

$2 \times 26 = 52$

Step - II: List all the factors of 52:

1, 52	−1, −52
2, 26	−2, −26
4, 13	−4, −13

Remember!
An expression having degree 2 is called a quadratic expression.

Step - II: List all the factors of 52:

Step - III: Sum of factors equals middle term (17)

$1 + 52 = 53$ $\quad -1, -52 = -53$

$2 + 26 = 28$ $\quad -2, -26 = -28$

$\boxed{4 + 13 = 17}$ $\quad -4, -13 = -17$

Step - IV: Change the middle term in the given expression

$2x^2 + 17x + 26$

$= 2x^2 + 4x + 13x + 26$

Step - V: Take commons from first two terms and last two terms

$= 2x(x+2) + 13(x+2)$

Step - VI: Again, take common from both terms

$= (x+2)(2x+13)$

Example 8: Factorize: $3x^2 - 4x - 4$

Solution: $3x^2 - 4x - 4$

$= 3x^2 + 2x - 6x - 4$ $\quad \because 2\times(-6) = -12,\ +2-6 = -4$

$= x(3x+2) - 2(3x+2)$

$= (3x+2)(x-2)$

Solved Exercise 4.1

1. Factorize by identifying common factors.

(i) $6x + 12$

Sol. $6x + 12$

Take out common factor 6

$= 6(x+2)$

(ii) $15y^3 + 20y$

Sol. $15y^2 + 20y$

Take out commonfactor 5y

$= 5y(3y+4)$

(iii) $-12x^2 - 3x$

Sol. $-12x^2 - 3x$

Take out common factor −3x

$= -3x(4x+1)$

(iv) $4a^2b + 8ab^2$

Sol. $4a^2b + 8ab^2$

Take out common factor 4ab

$= 4ab(a+2b)$

(v) $xy - 3x^2 + 2x$

Sol. $xy - 3x^2 + 2x$

Take out common factor x

$= x(y - 3x + 2)$

(vi) $3a^2b - 9ab^2 + 15ab$

Sol. $3a^2b - 9ab^2 + 15ab$

Take out common factor 3ab

$= 3ab(a - 3b + 5)$

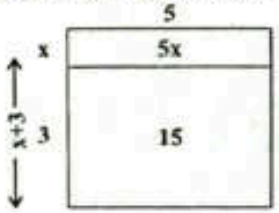
2. Factorize and represent pictorially:

(i) $5x + 15$

Sol. $5x + 15$

$= 5(x+3)$

Pictorially Representation



(ii) $x^2 + 4x + 3$

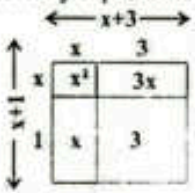
Sol. $x^2 + 4x + 3$

$= x^2 + 3x + x + 3$

$= x(x+3) + 1(x+3)$

$= (x+3)(x+1)$

Pictorially representation



(iii) $x^2 + 6x + 8$

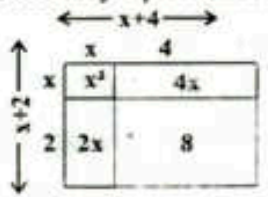
Sol. $x^2 + 6x + 8$

$= x^2 + 4x + 2x + 8$

$= x(x+4) + 2(x+4)$

$= (x+4)(x+2)$

Pictorially representation



(iv) $x^2 + 4x + 4$

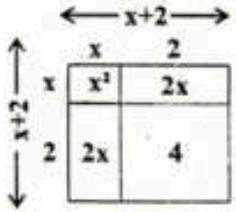
Sol. $x^2 + 4x + 4$

$= (x)^2 + 2(x)(2) + (2)^2$ $\quad$ as $a^2 + 2ab + b^2 = (a+b)^2$

$= (x+2)^2$

$= (x+2)(x+2)$

Pictorically representation



3. Factorize:

(i) $x^2 + x - 12$

Sol. $x^2 + x - 12$
$= x^2 + 4x - 3x - 12$
$= x(x+4) - 3(x+4)$
$= (x+4)(x-3)$

(ii) $x^2 + 7x + 10$

Sol. $x^2 + 7x + 10$
$= x^2 + 5x + 2x + 10$
$= x(x+5) + 2(x+5)$
$= (x+5)(x+2)$

(iii) $x^2 - 6x + 8$

Sol. $x^2 - 6x + 8$
$= x^2 - 4x - 2x + 8$
$= x(x-4) - 2(x-4)$
$= (x-4)(x-2)$

(iv) $x^2 - x - 56$

Sol. $x^2 - x - 56$
$= x^2 - 8x + 7x - 56$
$= x(x-8) + 7(x-8)$
$= (x-8)(x+7)$

(v) $x^2 - 10x - 24$

Sol. $x^2 - 10x - 24$
$= x^2 - 12x + 2x - 24$
$= x(x-12) + 2(x-12)$
$= (x-12)(x+2)$

(vi) $y^2 + 4y - 12$

Sol. $y^2 + 4y - 12$
$= y^2 + 6y - 2y - 12$
$= y(y+6) - 2(y+6)$
$= (y+6)(y-2)$

(vii) $y^2 + 13y + 36$

Sol. $y^2 + 13y + 36$
$= y^2 + 9y + 4y + 36$
$= y(y+9) + 4(y+9)$
$= (y+9)(y+4)$

(viii) $x^2 - x - 2$

Sol. $x^2 - x - 2$
$= x^2 - 2x + x - 2$
$= x(x-2) + 1(x-2)$
$= (x-2)(x+1)$

4. Factorize:

(i) $2x^2 + 7x + 3$

Sol. $2x^2 + 7x + 3$
$= 2x^2 + 6x + x + 3$
$= 2x(x+3) + 1(x+3)$
$= (x+3)(2x+1)$

(ii) $2x^2 + 11x + 15$

Sol. $2x^2 + 11x + 15$
$= 2x^2 + 6x + 5x + 15$
$= 2x(x+3) + 5(x+3)$
$= (x+3)(2x+5)$

(iii) $4x^2 + 13x + 3$

Sol. $4x^2 + 13x + 3$
$= 4x^2 + 12x + x + 3$
$= 4x(x+3) + 1(x+3)$
$= (x+3)(4x+1)$

(iv) $3x^2 + 5x + 2$

Sol. $3x^2 + 5x + 2$
$= 3x^2 + 3x + 2x + 2$
$= 3x(x+1) + 2(x+1)$
$= (x+1)(3x+2)$

(v) $3y^2 - 11y + 6$

Sol. $3y^2 - 11y + 6$
$= 3y^2 - 9y - 2y + 6$
$= 3y(y-3) - 2(y-3)$
$= (y-3)(3y-2)$

(vi) $2y^2 - 5y + 2$

Sol. $2y^2 - 5y + 2$
$= 2y^2 - 4y - y + 2$
$= 2y(y-2) - 1(y-2)$
$= (y-2)(2y-1)$

(vii) $4z^2 - 11z + 6$

Sol. $4z^2 - 11z + 6$
$= 4z^2 - 8z - 3z + 6$
$= 4z(z-2) - 3(z-2)$
$= (z-2)(4z-3)$

(viii) $6 + 7x - 3x^2$

Sol. $6 + 7x - 3x^2$

Rearranging the given terms

$= -3x^2 + 7x + 6$
$= -3x^2 + 9x - 2x + 6$
$= -3x(x-3) - 2(x-3)$
$= (x-3)(-3x-2)$
$= (-)(3-x)(-)(3x+2)$
$= (3-x)(3x+2)$

Type - II: Factorization of the expression of the types $a^4 + a^2b^2 + b^4$ or $a^4 + b^4$

Let's factorize the first expression

$a^4 + a^2b^2 + b^4$
$= a^4 + b^4 + a^2b^2$

$= (a^2)^2 + (b^2)^2 + a^2b^2$
$= (a^2)^2 + (b^2)^2 + 2a^2b^2 - 2a^2b^2 + a^2b^2$ (Adding and subtracting $2a^2b^2$)
$= (a^2 + b^2)^2 - a^2b^2$
$= (a^2 + b^2)^2 - (ab)^2$
$= (a^2 + b^2 - ab)(a^2 + b^2 + ab)$
$= (a^2 - ab + b^2)(a^2 + ab + b^2)$

Remember!
$a^2 - b^2 = (a-b)(a+b)$
$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$

Example 9: Factorize: $x^4 + x^2 + 25$

Solution: $x^4 + x^2 + 25$
$= x^4 + 25 + x^2$
$= (x^2)^2 + (5)^2 + 2(x^2)(5) - 2(x^2)(5) + x^2$ (Adding and subtracting $2(x^2)(5)$)
$= (x^2 + 5)^2 - 10x^2 + x^2$
$= (x^2 + 5)^2 - 9x^2$
$= (x^2 + 5)^2 - (3x)^2$
$= (x^2 + 5 - 3x)(x^2 + 5 + 3x)$
$= (x^2 - 3x + 5)(x^2 + 3x + 5)$

Activity
- Prepare cards by writing several expressions.
- Divide students in small groups.
- Each group will draw a card and factorize the expression.
- The group which completes the most correct factorizations in a set time will win.

Example 10: Factorize: $x^4 + y^4$

Solution: $x^4 + y^4$
$= (x^2)^2 + (y^2)^2$
$= (x^2)^2 + (y^2)^2 + 2(x^2)(y^2) - 2(x^2)(y^2)$ (Adding and subtracting $2x^2y^2$)
$= (x^2 + y^2)^2 - (\sqrt{2}xy)^2$
$= (x^2 + y^2 - \sqrt{2}xy)(x^2 + y^2 + \sqrt{2}xy)$
$= (x^2 - \sqrt{2}xy + y^2)(x^2 + \sqrt{2}xy + y^2)$

Example 11: Factorize: $a^4 + 64$

Solution: $a^4 + 64$
$= (a^2)^2 + (8)^2$
$= (a^2)^2 + (8)^2 + 2(a^2)(8) - 2(a^2)(8)$ (Adding and subtracting $2(a^2)(8)$)
$= (a^2 + 8)^2 - 16a^2$
$= (a^2 + 8)^2 - (4a)^2$
$= (a^2 + 8 - 4a)(a^2 + 8 + 4a)$
$= (a^2 - 4a + 8)(a^2 + 4a + 8)$

Type - III: Factorization of the expression of the types

- **$(ax^2 + bx + c)(ax^2 + bx + d) + k$**
- **$(x + a)(x + b)(x + c)(x + d) + k$**
- **$(x + a)(x + b)(x + c)(x + d) + kx^2$**

For explanation consider the following examples:

Example 12: Factorize: $(x^2 + 5x + 4)(x^2 + 5x + 6) - 3$

Solution: $(x^2 + 5x + 4)(x^2 + 5x + 6) - 3$
$= (y + 4)(y + 6) - 3$ (Let $y = x^2 + 5x$)
$= y^2 + 6y + 4y + 24 - 3$
$= y^2 + 10y + 21$

$= y^2 + 7y + 3y + 21$
$= y(y + 7) + 3(y + 7)$
$= (y + 7)(y + 3)$
$= (x^2 + 5x + 7)(x^2 + 5x + 3)$ $(\because\ y = x^2 + 5x)$

Example 13: Factorize: $(x + 2)(x + 3)(x + 4)(x + 5) - 15$

Solution: $(x + 2)(x + 3)(x + 4)(x + 5) - 15$

Re-arrange the given expression because $2 + 5 = 3 + 4$

$[(x + 2)(x + 5)][(x + 3)(x + 4)] - 15$
$= (x^2 + 5x + 2x + 10)(x^2 + 4x + 3x + 12) - 15$
$= (x^2 + 7x + 10)(x^2 + 7x + 12) - 15$
$= (y + 10)(y + 12) - 15$ Let $y = x^2 + 7x$
$= y^2 + 12y + 10y + 120 - 15$
$= y^2 + 22y + 105$
$= y^2 + 15y + 7y + 105$
$= y(y + 15) + 7(y + 15)$
$= (y + 15)(y + 7)$
$= (x^2 + 7x + 15)(x^2 + 7x + 7)$ $(\because\ y = x^2 + 7x)$

Example 14: Factorize: $(x - 2)(x + 2)(x + 1)(x - 4) + 2x^2$

Solution: $(x - 2)(x + 2)(x + 1)(x - 4) + 2x^2$
$= [(x - 2)(x + 2)][(x + 1)(x - 4)] + 2x^2$ $[\because\ (-2) \times 2 = 1 \times (-4)]$
$= (x^2 - 2^2)(x^2 - 4x + x - 4) + 2x^2$
$= (x^2 - 4)(x^2 - 3x - 4) + 2x^2$
$= y(y - 3x) + 2x^2$ Let $y = x^2 - 4$
$= y^2 - 3xy + 2x^2$
$= y^2 - 2xy - xy + 2x^2$
$= y(y - 2x) - x(y - 2x)$
$= (y - 2x)(y - x)$
$= (x^2 - 4 - 2x)(x^2 - 4 - x)$ $(\because\ y = x^2 - 4)$
$= (x^2 - 2x - 4)(x^2 - x - 4)$

Type - IV: Factorization of the expression of the types

- **$a^3 + 3a^2b + 3ab^2 + b^3$**
- **$a^3 - 3a^2b + 3ab^2 - b^3$**

Factorization of such types of expressions is explained in the following examples:

Example 15: Factorize: $8x^3 + 60x^2 + 150x + 125$

Solution: $8x^3 + 60x^2 + 150x + 125$
$= (2x)^3 + 3(2x)^2(5) + 3(2x)(5)^2 + (5)^3$
$= (2x + 5)^3$
$= (2x + 5)(2x + 5)(2x + 5)$

Remember!
$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Example 16: Factorize: $x^3 - 18x^2 + 108x - 216$

Solution: $x^3 - 18x^2 + 108x - 216$
$= (x)^3 - 3(x)^2(6) + 3(x)(6)^2 - (6)^3$

$= (x - 6)^3$
$= (x - 6)(x - 6)(x - 6)$

Type - V: Factorization of the expression of the types $a^3 \pm b^3$

The expression $a^3 + b^3$ is a sum of cubes and it can be factorized using the following identity:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

The expression $a^3 - b^3$ is a difference of cubes and it can be factorized using the following identity:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Example 17: Factorize: $8x^3 + 27$

Solution: $8x^3 + 27$
$= (2x)^3 + (3)^3$
$= (2x + 3)[(2x)^2 - (2x)(3) + (3)^2]$
$= (2x + 3)(4x^2 - 6x + 9)$

Example 18: Factorize: $x^3 - 27y^3$

Solution: $x^3 - 27y^3$
$= (x)^3 - (3y)^3$
$= (x - 3y)[(x)^2 + (x)(3y) + (3y)^2]$
$= (x - 3y)(x^2 + 3xy + 9y^2)$

Do you know?
$(a + b)^2 \neq a^2 + b^2$
$(a - b)^2 \neq a^2 - b^2$
$(a + b)^3 \neq a^3 + b^3$
$(a - b)^3 \neq a^3 - b^3$

Solved Exercise 4.2

1. Factorize each of the following expressions:

(i) $4x^4 + 81y^4$

Sol. $4x^4 + 81y^4$
$= (2x^2)^2 + (9y^2)^2 + 2(2x^2)(9y^2) - 2(2x^2)(9y)^2$ (Adding and Subtracting) $2(2x^2)(9y^2)$
$= (2x^2 + 9y^2)^2 - 36x^2y^2$
$= (2x^2 + 9y^2)^2 - (6xy^2)$
$= (2x^2 + 9y^2 + 6xy)(2x^2 + 9y^2 - 6xy)$
$= (2x^2 + 6xy + 9y^2)(2x^2 - 6xy + 9y^2)$

(ii) $a^4 + 64b^4$

Sol. $a^4 + 64b^4$
$= (a^2)^2 + (8b^2)^2 + 2(a^2)(8b^2) - 2(a^2)(8b^2)$ (Adding and Subtracting) $2(a^2)(8b^2)$
$= (a^2 + 8b^2)^2 - 16a^2b^2$
$= (a^2 + 8b^2)^2 - (4ab)^2$
$= (a^2 + 8b^2 + 4ab)(a^2 + 8b^2 - 4ab)$
$= (a^2 + 4ab + 8b^2)(a^2 - 4ab + 8b^2)$

(iii) $x^4 + 4x^2 + 16$

Sol. $x^4 + 4x^2 + 16$
$= x^4 + 16 + 4x^2$
$= (x^2)^2 + (4)^2 + 4x^2$

$= (x^2)^2 + (4)^2 + 2(x^2)(4) - 2(x^2)(4) + 4x^2$ (Adding and Subtracting) $2(x^2)(4)$
$= (x^2 + 4)^2 - 8x^2 + 4x^2$
$= (x^2 + 4)^2 - 4x^2$
$= (x^2 + 4)^2 - (2x)^2$
$= (x^2 + 4 + 2x)(x^2 + 4 - 2x)$
$= (x^2 + 2x + 4)(x^2 - 2x + 4)$

(iv) $x^4 - 14x^2 + 1$

Sol. $x^4 - 14x^2 + 1$
$= (x^2)^2 + (1)^2 - 14x^2$
$= (x^2)^2 + (1)^2 + 2(x^2)(1) - 2(x^2)(1) - 14x^2$ (Adding and Subtracting) $2(x^2)(1)$
$= (x^2 + 1)^2 - 2x^2 - 14x^2$
$= (x^2 + 1)^2 - 16x^2$
$= (x^2 + 1)^2 - (4x)^2$
$= (x^2 + 1 + 4x)(x^2 + 1 - 4x)$
$= (x^2 + 4x + 1)(x^2 - 4x + 1)$

(v) $x^4 - 30x^2y^2 + 9y^4$

Sol. $x^4 - 30x^2y^2 + 9y^4$
$= (x^2)^2 + (3y^2)^2 + 2(x^2)(3y^2) - 2(x^2)(3y^2) - 30x^2y^2$ (Adding and subtracting $2(x^2)(3y^2)$)
$= (x^2 + 3y^2)^2 - 6x^2y^2 - 30x^2y^2$
$= (x^2 + 3y^2)^2 - 36x^2y^2$
$= (x^2 + 3y^2)^2 - (6xy)^2$
$= (x^2 + 3y^2 + 6xy)(x^2 + 3y^2 - 6xy)$
$= (x^2 + 6xy + 3y^2)(x^2 - 6xy + 3y^2)$

(vi) $x^4 - 7x^2y^2 + y^4$

Sol. $x^4 - 7x^2y^2 + y^4$
$= (x^2)^2 + (y^2)^2 + 2(x^2)(y^2) - 2(x^2)(y^2) - 7x^2y^2$ (Adding and subtracting $2(x^2)(y^2)$)
$= (x^2 + y^2)^2 - 2x^2y^2 - 7x^2y^2$
$= (x^2 + y^2)^2 - 9x^2y^2$
$= (x^2 + y^2)^2 - (3xy)^2$
$= (x^2 + y^2 + 3xy)(x^2 + y^2 - 3xy)$
$= (x^2 + 3xy + y^2)(x^2 - 3xy + y^2)$

(The statement in the book is not correct)

2. Factorize each of the following expressions:

(i) $(x+1)(x+2)(x+3)(x+4)+1$

Sol. $(x+1)(x+2)(x+3)(x+4)+1$

Re-arrange the given expression because $1 + 4 = 2 + 3$

$[(x+1)(x+4)][(x+2)(x+3)]+1$
$= (x^2 + 4x + x + 4)(x^2 + 3x + 2x + 6) + 1$
$= (x^2 + 5x + 4)(x^2 + 5x + 6) + 1$

Let, $x^2 + 5x = y$

$= (y + 4)(y + 6) + 1$
$= (y^2 + 6y + 4y + 24) + 1$
$= y^2 + 10y + 24 + 1$
$= y^2 + 10y + 25$
$= (y)^2 + 2(y)(5) + (5)^2$
$= (y + 5)^2$
$= (y + 5)(y + 5)$
$= (x^2 + 5x + 5)(2^2 + 5x + 5) = (x^2 + 5x + 5)^2$ $\because$ $y = x^2 + 5x$

(ii) $(x+2)(x-7)(x-4)(x-1)+17$

Sol. $(x+2)(x-7)(x-4)(x-1)+17$

Re-arrange the given expression because $2 - 7 = -4 - 1$

$[(x+2)(x-7)][(x-4)(x-1)]+17$
$= [x(x-7) + 2(x-7)][x(x-1) - 4(x-1)] + 17$
$= [(x^2 - 7x + 2x - 14)[(x^2 - x - 4x + 14)] + 17$
$\Rightarrow (x^2 - 5x - 14)(x^2 - 5x + 4) + 17$

Let, $x^2 - 5x = 7$
$= (y - 14)(y + 4) + 17$
$= [y(y + 4) - 14(y + 14)] + 17$
$= y^2 + 4y - 14y - 56 + 17$
$= y^2 + 4y - 10y - 39$
$= y^2 - 13 + 3y - 39$
$= (y - 13)(y + 3)$
$= (x^2 - 5x - 13)(x^2 - 5x + 3)$ $\because$ $y = x^2 - 5x$

(iii) $(2x^2 + 7x + 3)(2x^2 + 7x + 5) + 1$

Sol. $(2x^2 + 7x + 3)(2x^2 + 7x + 5) + 1$

Let, $2x^2 + 7x = y$
$= (y+3)(y+5) + 1$
$= (y^2 + 5y + 3y + 15) + 1$
$= y^2 + 8y + 16$
$= (y)^2 + 2(y)(4) + (4)^2$
$= (y + 4)^2$
$= (y + 4)(y + 4)$
$= (2x^2 + 7x + 4)(2x^2 + 7x + 4)$ $\because$ $y = 2x^2 + 7x$
$= (2x^2 + 7x + 4)^2$

(iv) $(3x^2 + 5x + 3)(3x^2 + 5x + 5) - 3$

Sol. $(3x^2 + 5x + 3)(3x^2 + 5x + 5) - 3$

Let, $3x^2 + 5x = y$
$= (y + 3)(y + 5) - 3$
$= y^2 + 5y + 3y + 15 - 3$
$= y^2 + 8y + 12$
$= y^2 + 6y + 2y - 12$ $2 \times 6 = 12 | 2 + 6 = 8$

$= y(y + 6) + 2(y + 6)$
$= (y + 6)(y + 2)$
$= (3x^2 + 5x + 6)(3x^2 + 5x + 2)$ $\because$ $y = 3x^2 + 5x$

(v) $(x+1)(x+2)(x+3)(x+6) - 3x^2$

Sol. $(x+1)(x+2)(x+3)(x+6) - 3x^2$

Re-arrange the expression because $1 \times 6 = 2 \times 3$

$= [(x+1)(x+6)][(x+2)(x+3)] - 3x^2$
$= (x^2 + 6x + x + 6)(x^2 + 3x + 2x + 6) - 3x^2$
$= (x^2 + 7x + 6)(x^2 + 5x + 6) - 3x^2$
$= (x^2 + 6 + 7x)(x^2 + 6 + 5x) - 3x^2$

let, $x^2 + 6 = y$

$= (y + 7x)(y + 5x) - 3x^2$
$= (y^2 + 5xy + 7xy + 35x^2) - 3x^2$
$= y^2 + 12xy + 35x^2 - 3x^2$
$= y^2 + 12xy + 32x^2$
$= y^2 + 8xy + 4xy + 32x^2$ $\quad 8 \times 4 = 32$
$= y(y + 8x) + 4x(y + 8x)$ $\quad 8 + 4 = 12$
$= (y + 8x)(y + 4x)$
$= (x^2 + 6 + 8x)(x^2 + 6 + 4x)$
$= (x^2 + 8x + 6)(x^2 + 4x + 6)$ $\because$ $x^2 + 6y = y$

(vi) $(x+1)(x-1)(x+2)(x-2) + x^2$

Sol. $(x+1)(x-1)(x+2)(x-2) + x^2$
$= [(x+1)(x-1)][(x+2)(x-2)] + x^2$
$= [(x)^2 - (1)^2][(x)^2 - (2)^2] + x^2$
$= (x^2 - 1)(x^2 - 4) + x^2$
$= x^4 - 4x^2 - x^2 + 4 + x^2$
$= x^4 - 4x^2 + 4$
$= (x^2)^2 - 2(x^2)(2) + (2)^2$
$= (x^2 - 2)^2$
$= (x^2 - 2)(x^2 - 2)$

(The question in book is wrong)

3. Factorize:

(i) $8x^3 + 12x^2 + 6x + 1$

Sol. $8x^3 + 12x^2 + 6x + 1$
$(2x)^3 + 3(2x)^2(1) + 3(2x)(1)^2 + (1)^3$
$= (2x + 1)^3$

(ii) $27a^3 + 108a^2b + 144ab^2 + 64b^3$

Sol. $27a^3 + 108a^2b + 144ab^2 + 64b^3$
$= (3a)^3 + 3(3a)^2(4b) + 3(3a)(4b)^2 + (4$
$= (3a + 4b)^3$

(iii) $x^3 + 18x^2y + 108xy^2 + 216y^3$

Sol. $x^3 + 18x^2y + 108xy^2 + 216y^3$
$= (x^3) + 3(x)^2(6y) + 3(x)(6y)^2 + (6y)^3 = (x + 6y)^3$

The question in the book is not correct.

(iv) $8x^3 - 125y^3 + 150xy^2 - 60x^2y$

Sol. $8x^3 - 125y^3 + 150xy^2 - 60x^2y$

Re-arrange the given expression

$= 8x^3 - 6x^2y + 150xy^2 - 125y^3$
$= (2x)^3 - 3(2x)^2(5y) + 3(2x)(5y)^2 - (5y)^3$
$= (2x - 5y)^3$
$= (2x - 5y)(2x - 5y)(2x - 5y)$

4. Factorize:

(i) $125a^3 - 1$

Sol. $125a^3 - 1$
$= (5a)^3 - (1)^3$
$= (5a - 1)[(5a)^2 + (5a)(1) + (1)^2]$
$= (5a - 1)(25a^2 + 5a + 1)$

(ii) $64x^3 + 125$

Sol. $64x^3 + 125$
$= (4x)^3 + (5)^3$
$= (4x + 5)[(4x)^2 - (4x)(5) + (5)]$
$= (4x + 5)(16x^2 - 20x + 25)$

(iii) $x^6 - 27$

Sol. $x^6 - 27$
$(x^2)^3 - (3)^3$
$= (x^2 - 3)[(x^2)^2 + (x^2)(3) + (3)^2]$
$= (x^2 - 3)(x^4 + 3x^2 + 9)$

(iv) $1000a^3 + 1$

Sol. $1000a^3 + 1$
$= (10a)^3 + (1)^3$
$= (10a + 1)[(10a)^2 - (10a)(1) + (1)^2]$
$= (10a + 1)(100a^2 - 10a + 1)$

(v) $343x^3 + 216$

Sol. $343x^3 + 216$
$= (7x)^3 + (6)^3$
$= (7x + 6)[(7x)^2 - (7x)(6) + (6)^2]$
$= (7x + 6)(49x^2 - 42x + 36)$

(vi) $27 - 512y^3$

Sol. $27 - 512y^3$
$= (3)^3 - (8y)^3$
$= (3 - 8y)[(3)^2 + (3)(8y) + (8y)^2]$
$= (3 - 8y)(9 + 24y + 64y^2)$

Highest Common Factor (HCF) and Least Common Multiple (LCM) of Algebraic Expressions

Highest Common Factor (HCF)

The HCF of two or more algebraic expressions refers to the greatest algebraic expression that divides then without leaving a remainder.

We can find HCF of given expressions by the following two methods:

(a) By factorization (b) By division

(a) HCF by FactorizationMethod

Example 19: Find the HCF of $6x^2y$, $9xy^2$

Solution: $6x^2y = 2 \times 3 \times x \times x \times y$
$9xy^2 = 3 \times 3 \times x \times y \times y$
$\therefore$ HCF $= 3 \times x \times y$ (Product of common factors)
$= 3xy$

Example 20: Find the HCF by factorization method $x^3 - 27$, $x^2 + 6x - 27$, $x^2 - 9$

Solution: $x^3 - 27 = x^3 - 3^3$
$= (x - 3)[(x)^2 + (3)(x) + (3)^2]$
$= (x - 3)(x^2 + 3x + 9)$

$x^2 + 6x - 27 = x^2 + 9x - 3x - 27$
$= x(x + 9) - 3(x + 9)$
$= (x + 9)(x - 3)$
$x^2 - 9 = x^2 - 3^2$
$= (x - 3)(x + 3)$
Hence, HCF $= x - 3$

(b) HCF by DivisionMethod

Example 21: Find HCF of $6x^3 - 17x^2 - 5x + 6$ and $6x^3 - 5x^2 - 3x + 2$ by using divi method.

Solution:

$$6x^3 - 17x^2 - 5x + 6 \overline{)\ 6x^3 - 5x^2 - 3x + 2}\ (1$$
$$6x^3 - 17x^2 - 5x + 6$$
$$12x^2 - 2x - 4$$

Here, $12x^2 + 2x - 4 = 2(6x^2 + x - 2)$

2 is not common in both the given polynomials, so we ignore it and consider $6x^2 + x - 2$.

$$6x^2 + x - 2 \overline{)\ 6x^3 - 17x^2 - 5x + 6}\ (x - 3$$
$$6x^3 + x^2 - 2x$$
$$-18x^2 - 3x + 6$$
$$-18x^2 - 3x + 6$$
$$0$$

Hence, HCF $= 6x^2 + x - 2$

Least Common Multiple (LCM)

The LCM of two or more algebraic expressions is the smallest expression tha divisible by each of the given expressions.

To find the LCM by factorization, we use the formula.

LCM = Common factors × Non-common factors

Example 22: Find the LCM of $4x^2y$, $8x^3y^2$.

Solution: $4x^2y = 2 \times 2 \times x \times x \times y$
$8x^3y^2 = 2 \times 2 \times 2 \times x \times x \times x \times y \times y$
Common factors $= 2 \times 2 \times x \times x \times y = 4x^2y$
Non-common factors $= 2 \times x \times y = 2xy$
LCM = Common factors × Non-common factors
$= 4x^2y \times 2xy$
$= 8x^3y^2$

Example 23: Find the LCM of the polynomials $x^2 - 3x + 2$, $x^2 - 1$ and $x^2 - 5x + 4$.

Solution: As $x^2 - 3x + 2 = x^2 - 2x - x + 2$
$= x(x - 2) - 1(x - 2)$
$= (x - 2)(x - 1)$

And $x^2 - 1 = (x - 1)(x + 1)$
$x^2 - 5x + 4 = x^2 - 4x - x + 4$
$= x(x - 4) - 1(x - 4)$
$= (x - 4)(x - 1)$
Common factors $= x - 1$
Non-common factors $= (x + 1)(x - 2)(x - 4)$
LCM = Common factors × Non-common factors
$= (x - 1) \times (x + 1)(x - 2)(x - 4)$
$= (x - 1)(x + 1)(x - 2)(x - 4)$

Relationship Between LCM and HCF

The relationship between LCM and HCF can be expressed as follows:

$$LCM \times HCF = p(x) \times q(x)$$

Where, $p(x) = 1^{st}$ polynomial
$q(x) = 2^{nd}$ polynomial

Example 24: LCM and HCF of two polynomials are $x^3 - 10x^2 + 11x + 70$ and $x - 7$. If one of the polynomials is $x^2 - 12x + 35$, find the other polynomial.

Solution:Given that:LCM $= x^3 - 10x^2 + 11x + 70$
HCF $= x - 7$
$p(x) = x^2 - 12x + 35$

As we know that: $q(x) = \dfrac{LCM \times HCF}{p(x)}$

$$= \frac{(x^3 - 10x^2 + 11x + 70)(x - 7)}{x^2 - 12x + 35}$$

$$x^2 - 12x + 35 \overline{)\ x^3 - 10x^2 + 11x + 70}\ (x + 2$$
$$x^3 - 12x^2 + 35x$$
$$2x^2 - 24x + 70$$
$$2x^2 - 24x + 70$$
$$0$$

So, $q(x) = (x + 2)(x - 7)$
$= x^2 - 7x + 2x - 14$
$= x^2 - 5x - 14$

Example 25: The LCM of $x^2y + xy^2$ and $x^2 + xy$ is $xy(x + y)$. Find the HCF.

Solution: Given that: LCM $= xy(x + y)$
HCF = ?
1^{st} polynomial $= x^2y + xy^2$
2^{nd} polynomial $= x^2 + xy$

As we know that:
LCM × HCF = Product of two polynomials

$$\text{HCF} = \frac{\text{Product of two polynomials}}{\text{LCM}}$$

$$= \frac{(x^2y + xy^2)(x^2 + xy)}{xy(x+y)}$$

$$= \frac{xy(x+y)x(x+y)}{xy(x+y)}$$

$$= x(x+y)$$

Solved Exercise 4.3

1. Find HCF by factorization method.

(i) $21x^2y, 35xy^2$

Sol. $21x^2y, 35xy^2$

Factorization of $21x^2y = 3 \times 7 \times x \times x \times y$

Factorization of $35xy^2 = 5 \times 7 \times x \times y \times y$

HCF = Product of common factors

$= 7 \times x \times y$

$= 7xy$

(ii) $4x^2 - 9y^2, 2x^2 - 3xy$

Sol. $4x^2 - 9y^2, 2x^2 - 3xy$

Factorization of $4x^2 - 9y^2 = (2x)^2 - (3y)^2$

$= (2x+3y)(2x-3y)$

Factorization of $2x^2 - 3xy = x(2x-3y)$

HCF = Product of common factors

$= (2x - 3y)$

(iii) $x^3 - 1, x^2 + x + 1$

Sol. $x^3 - 1, x^2 + x + 1$

Factorization of $x^3 - 1 = (x)^3 - (1)^3$

$= (x-1)[(x^2) + (x)(1) + (1)^2]$

$= (x-1)(x^2 + x + 1)$

Factorization $x^2 + x + 1 = x^2 + x + 1$

HCF = Product of common factors

$= x^2 + x + 1$

(iv) $a^3 + 2a^2 - 3a, 2a^3 + 5a^2 - 3a$

Sol. $a^3 + 2a^2 - 3a, 2a^3 + 5a^2 - 3a$

Factorization of $a^3 + 2a^2 - 3a = a(a^2 + 2a - 3)$

$= a[a^2 + 3a - a - 3]$

$= a[a(a+3) - 1(a+3)]$

$= a(a+3)(a-1)$

Factorization of $2a^3 + 5a^2 - 3a = a(2a^2 + 5a - 3)$

$= a[2a^2 + 6a - a - 3]$

$= a[2a(a+3) - 1(a+3)]$

$= a(a+3)(2a-1)$

H.C.F = Product of common factors

$= a(a+3)$

(v) $t^2 - 3t - 4, t^2 + 5t + 4, t^2 - 1$

Sol. $t^2 - 3t - 4, t^2 + 5t + 4, t^2 - 1$

Factorization of $t^2 - 3t - 4 = t^2 - 4t + t - 1$

$= t(t-4) + 1(t-4)$

$= (t-4)(t+1)$

Factorization of $t^2 + 5t + 4 = t^2 + 4t + t + 4$

$= t(t+4) + 1(t+4) = (t+4)(t+1)$

Factorization of $t^2 - 1 = (t)^2 - (1)^2$

$= (t+1)(t-1)$

HCF = Product of common factors

$= (t+1)$

The question in book is not correct.

(vi) $x^2 + 15x + 56, x^2 + 5x - 24, x^2 + 8x$

Ans. $x^2 + 15x + 56, x^2 + 5x - 24, x^2 + 8x$

Factorization of $x^2 + 15x + 56 = x^2 + 8x + 7x + 56$

$= x(x+8) + 7(x+8)$

$= (x+8)(x+7)$

Factorization of $x^2 + 5x - 24 = x^2 + 8x - 3x - 24$

$= x(x+8) - 3(x+8) = (x+8)(x-3)$

Factorization of $x^2 + 8x = x(x+8)$

HCF = Product of common factors

$= (x+8)$

2. Find HCF of the following expressions by using division method:

(i) $27x^3 + 9x^2 - 3x - 10, 3x - 2$

Sol. $27x^3 + 9x^2 - 3x - 10, 3x - 2$

$$\begin{array}{r|l} & 9x^2+9x+5 \\ \hline 3x-2 & 27x^3-9x^2-3x-10 \\ & \pm 27x^3 \mp 18x^2 \\ \hline & +27x^2-3x-10 \\ & \pm 27x^2 \mp 18x \\ \hline & 15x-10 \\ & \pm 15x \mp 10 \\ \hline & 0 \end{array}$$

So, HCF = $3x - 2$

The question in book is not correct.

(ii) $x^3 - 9x^2 + 21x - 9, x^2 - 4x + 3$

Sol. $x^3 - 9x^2 + 21x - 9, x^2 - 4x + 3$

$$x^2-4x+3 \overline{)\, x^3-9x^2+21x-9 \,(\, x-5}$$
$$\pm x^3 \mp 4x^2 \pm 3x$$
$$-5x^2+18x-9$$
$$-5x^2 \pm 20x \mp 15$$
$$-2x+6$$

$$x-3 \overline{)\, x^2-4x+3 \,(\, x-1}$$
$$x^2 \mp 3x$$
$$-x+3$$
$$\mp x \pm 3$$
$$0$$

H.C.F = $x - 3$

The question in the book is not correct.

(iii) $2x^3 + 2x^2 + 2x + 2, 6x^3 + 12x^2 + 6x + 12$

Sol. $2x^3 + 2x^2 + 2x + 2, 6x^3 + 12x^2 + 6x + 12$

$$2x^3+2x^2+2x+2 \overline{)\, 6x^3+12x^2+6x+12 \,(\, 3}$$
$$-6x^3 \pm 6x^2 \pm 6x \pm 6$$
$$6x^2+6$$

Here, $6x^2 + 6 = 6(x^2 + 1)$

6 is not common so we ignore it and consider only $x^2 + 1$.

$$x^2+1 \overline{)\, 2x^3+2x^2+2x+2 \,(\, 2x+2}$$
$$\pm 2x^3 \quad \pm 2x$$
$$2x^2+2$$
$$\pm 2x^2 \pm 2$$
$$0$$

H.C.F = $x^2 + 1$

(iv) $2x^3 - 4x^2 + 6x, x^3 - 2x, 3x^2 - 6x$

Sol. First we find the HCF of $2x^3 - 4x^2 + 6x$ and $x^3 - 2x$

$$x^3-2x \overline{)\, 2x^3-4x^2+6x \,(\, 2}$$
$$-2x^3 \quad \mp 4x$$
$$-4x^2+10x$$

Here, $-4x^2+10x = -2x(2x^2-5x)$ $-2x$ so we ignore it and consider only $2x^2 - 5x$

$$2x^2-5x \overline{)\, x^3-2x \,(\, x}$$
$$\times 2$$
$$2x^3-4x$$
$$\pm 2x^3 \quad \mp 5x^2$$
$$5x^2-4x$$

$$2x^2-5x \overline{)\, 5x^2-4x \,(\, 2}$$
$$\times 5$$
$$10x^2-25x$$
$$\pm 10x^2 \mp 8x$$
$$17x$$

$$17x \overline{)\, 5x^2-4x \,(\, \frac{5x}{17}}$$
$$\pm 5x^2$$
$$-1)\ -4x$$

Take -1 as common

$$4x \overline{)\, 17x \,(\, 4}$$
$$\pm 16x$$
$$x \overline{)\, 4x \,(\, 4}$$
$$\pm 4x$$
$$0$$

No, we shall find HCF of $3x^2 - 6x$ and x

$$x \overline{)\, 3x^2-6x \,(\, 3x-6}$$
$$\pm 3x^2$$
$$-6x$$
$$\mp 6x$$
$$0$$

So, Final HCF = x

3. Find LCM of the following expressions by using prime factorization method.

(i) $2a^2b, 4ab^2, 6ab$

Sol. $2a^2b, 4ab^2, 6ab$

Factorization of $2a^2b = 2 \times a \times a \times b$

Factorization of $4ab^2 = 2 \times 2 \times a \times b \times b$

Factorization of $6ab = 2 \times 3 \times a \times b$

Product of common factors $= 2 \times a \times b = 2ab$

Product of non-common factors $= 2 \times 3 \times a \times b = 6ab$

LCM = Product of common factors • Product of non-common factors

$= 2ab \times 6ab$

$= 12a^2b^2$

(ii) $x^2 + x, x^3 + x^2$

Sol. $x^2 + x, x^3 + x^2$

Factorization of $x^2 + x = x(x+1)$

Factorization of $x^3 + x^2 = x^2(x+1)$

$= x \times x(x+1)$

Product of common factors = $x(x+1)$

Product of non common factors = x

LCM = Product of common factors × Product of non-common factors

$= x(x+1) \times x$

$= x^2(x+1)$

(iii) $a^2 - 4a + 4, a^2 - 2a$

Sol. $a^2 - 4a + 4, a^2 - 2a$

Factorization of $a^2 - 4a + 4 = a^2 - 2a - 2a + 4$

$= a(a-2) - 2(a-2)$

$= (a-2)(a-2)$

Factorization of $a^2 - 2a = a(a-2)$

Common factor $= (a-2)$

Non-common factors $= a(a-2)$

LCM = Product of common factors × Product of non-common factors

$= (a-2) \times a(a-2) = a(a-2)^2$

(iv) $x^4 - 16, x^3 - 4x$

Sol. $x^4 - 16, x^3 - 4x$

Factorization of $x^4 - 16 = (x^2)^2 - (4)^2$

$= (x^2 + 4)(x^2 - 4)$

$= (x^2 + 4)[(x)^2 - (2)^2]$

$= (x^2 + 4)(x + 2)(x - 2)$

Factorization of $x^3 - 4x = x(x^2 - 4)$

$= x[(x)^2 - (2)^2]$

$= x(x + 2)(x - 2)$

Product of common factors $= (x + 2)(x - 2) = x^2 - 4$

Product of non-common factors $= x(x^2+4)$

LCM = Product of common factors × Product of non-common factors

$= (x^2-4) \times x(x^2+4)$

$= x(x^2+4)(x^2-4)$

$= x[(x^2)^2-(4)^2]$

$= x(x^4-16)$

(v) $16-4x^2, x^2+x-6, 4-x^2$

Sol. $16-4x^2, x^2+x-6, 4-x^2$

Factorization of $16-4x^2 = 4(4-x^2)$

$= 4[(2)^2-(x)^2]$

$= 4(2+x)(2-x)$

Factorization of $x^2+x-6 = x^2+3x-2x-6$

$= x(x+3)-2(x+3)$

$= (x+3)(x-2)$

Factorization of $4-x^2 = (2)^2-(x)^2$

$= (2+x)(2-x)$

Product of common factors $= (2+x)(2-x) = 4-x^2$

Product of non-common factors $= 4(x+3)(x-2)$

LCM = Product of common factors × Product of non-common factors

$= (4-x^2) \times 4(x+3)(x-2)$

$= 4(4-x^2)(x+3)(x-2)$

4. The HCF of two polynomials is $y-7$ and their LCM is $y^3-10y^2+11y+70$, one of the polynomials is $y^2-5y-14$, find the other.

Sol. HCF $= y-7$

LCM $= y^3-10y^2+11y+70$

$P(x) = y^2-5y-14$

$q(x) = ?$

$p(x) \times q(x) = \text{HCF} \times \text{LCM}$

$y^2-5y-14 \times q(x) = (y-7)(y^3-10y^2+11y+70)$

$$q(x) = \frac{(y-7)(y^3-10y^2+11y+70)}{y^2-5y-14}$$

$q(x) = (y-7)(y-5)$

$= y^2-5y-7y+35$

$= y^2-12y+35$

$$\begin{array}{r|l} & y-5 \\ \hline y^2-5y-14 & y^3-10y^2+11y+70 \\ & \pm y^3 \mp 5y^2 \mp 14y \\ \hline & -5y^2+25y+70 \\ & \mp 5y^2 \pm 25y \pm 70 \\ \hline & 0 \end{array}$$

5. The LCM and HCF of two polynomial $p(x)$ and $q(x)$ are $36x^3(x+a)(x^3-a^3)$ a $x^2(x-a)$ respectively. If $p(x) = 4x^2(x^2-a^2)$, find $q(x)$.

Sol. LCM $= 36x^3(x+a)(x^3-a^3)$

HCF $= x^2(x-a)$

$p(x) = 4x^2(x^2-a^2)$

$q(x) = ?$

$p(x)q(x) = \text{LCM} \times \text{HCF}$

$$q(x) = \frac{\text{LCM} \times \text{HCF}}{p(x)}$$

$$q(x) = \frac{36x^3(x+a)(x^3-a^3) \times x^2(x-a)}{4x^2(x^2-a^2)}$$

$$= \frac{36\, x^3 (x+a)(x^3-a^3) \times x^2 (x-a)}{4\, x^2 (x+a)(x-a)}$$

$= 9x^3(x^3-a^3)$

6. The HCF and LCM of two polynomials is $(x+a)$ and $12x^2(x+a)(x^2-a^2)$ respectively. Find the product of the two polynomials.

Sol. HCF $= (x+a)$

LCM $= 12x^2(x+a)(x^2-a^2)$

Product of the two polynomial $= ?$

Product of the two polynomial $= \text{HCF} \times \text{LCM}$

$= 12x^2(x+a)(x^2-a^2) \times (x+a)$

$= 12x^2(x+a)(x+a)(x^2-a^2)$

$= 12x^2(x+a)^2(x^2-a^2)$

Square Root of an Algebraic Expression

The square root of an algebraic expression refers to a value that, when multiplied by itself, gives the original expression. Just like finding the square root of a number, taking the square root of an algebraic expression involves determining what expression, when squared, results in the given expression.

For example, square root of $4a^2$ is $\pm 2ab$ because $2a \times 2a = 4a^2$ and $(-2a) \times (-2a) = 4a^2$

There are following two methods for finding the square root of an algebraic expression:

(a) By factorization method (b) By division method

(a) Square Root by Method

Example 26: Find the square root of the expression $36x^4 - 36x^2 + 9$

Solution: $36x^4 - 36x^2 + 9$

$= 9(4x^4 - 4x^2 + 1$

$= 9[(2x^2)^2 - 2(2x^2)(1) + (1)^2]$

$= 3^2(2x^2-1)^2$

Taking square root on both sides

$\sqrt{36x^4-36x^2+9} = \sqrt{3^2(2x^2-1)^2}$

$= \sqrt{3^2} \cdot \sqrt{(2x^2-1)^2}$

$= \pm 3(2x^2-1)$

(b) Square Root by Division Method

When the degree of the polynomial is higher, division method in finding the square root is very useful.

Example 27: Find the square root of the polynomial $x^4 - 12x^3 + 42x^2 - 36x + 9$.

Solution: Multiply x^2 by x^2 to get x^4

Multiply the quotient (x^2) by 2, so we get $2x^2$. By dividing $-12x^3$ by $2x^2$, we get $-6x$. By continuing in this way, we get the remainder.

Hence, square root of $x^4 - 12x^3 + 42x^2 - 36x + 9$ is $\pm(x^2 - 6x + 3)$

$$
\begin{array}{r|l}
x^4 - 12x^3 + 42x^2 - 36x + 9 & x^2 - 6x + 3 \\
\hline
x^4 & \\
\hline
-12x^3 + 42x^2 & 2x^2 - 6x \\
-12x^3 + 36x^2 & \\
\hline
6x^2 - 36x + 9 & 2x^2 - 12x + 3 \\
6x^2 - 36x + 9 & \\
\hline
0 &
\end{array}
$$

Real Wordls Problems of Factorization.

Example 28: Cost function for producing a part is modeled by:

$$C(x) = 5x^2 - 25x + 30$$

Where x is the width of the component and C(x) is the cost. Find the value of x when C(x) is minimum.

Solution: $C(x) = 5x^2 - 25x + 30$

$= 5(x^2 - 5x + 6)$

$= 5(x^2 - 2x - 3x + 6)$

$= 5[x(x-2) - 3(x-2)]$

$= 5(x-2)(x-3)$

Thus, the minimum cost occurs when x = 2 or x =

Example 29: The potential energy U(x) of a particle moving in a cubic potential is expressed as:

$$U(x) = x^3 - 6x^2 + 12x - 8$$

Factorize the expression to find the points where the energy in minimized.

Solution: $U(x) = x^3 - 6x^2 + 12x - 8$

$= (x)^3 - 3(x)^2(2) + 3(x)(2)^2 - (2)^3$

$= (x-2)^3$

$= (x-2)(x-2)(x-2)$

The factorized form of the potential energy function shows that the energy is minimized at x = 2.

Example 30: A company's profit p(x) is modeled by the quadratic equation:

$$P(x) = -5x^2 + 50x - 120$$

Where x represents the number of units produced and P(x) represents the profit in dollars. Find how many units should be produced to maximize profit.

Solution: $P(x) = -5x^2 + 50x - 120$

$= -5(x^2 - 10x + 24)$

$= -5[x^2 - 4x - 6x + 24]$

$= -5[x(x-4) - 6(x-4)]$

$= -5(x-4)(x-6)$

We can see that profit will be 0 when x = 4 or x = 6. As coefficients of x^2 is negative, the maximum profit occurs at the midpoint between 4 and 6.

Which is: $x = \frac{4+6}{2} = \frac{10}{2} = 5$

Thus, the company should produce 5 units to maximize profit.

Solved Exercise 4.4

1. Find the square root of the following polynomials by factorization method:

(i) $x^2 - 8x + 16$

Sol. $x^2 - 8x + 16$

Factorization of $x^2 - 8x + 16 = (x)^2 - 2(x)(4) + (4)^2$

$= (x-4)^2$

Now, take the square root of both sides

$\sqrt{x^2 - 8x + 16} = \sqrt{(x-4)^2}$

$= \pm(x-4)$

(ii) $9x^2 + 12x + 4$

Sol. $9x^2 + 12x + 4$

Factorization of $9x^2 + 12x + 4 = (3x)^2 + 2(3x)(2) + (2)^2$

$= (3x+2)^2$

Take the square root of both sides

$\sqrt{9x^2 + 12x + 4} = \sqrt{(3x+2)^2}$

$= \pm(3x+2)$

(iii) $36a^2 + 84a + 49$

Sol. $36a^2 + 84a + 49$

$36a^2 + 84a + 49 = (6a)^2 + 2(6a)(7) + (7)^2 = (6a+7)^2$

Take square root of both sides

$\sqrt{36a^2 + 84a + 49} = \sqrt{(6a+7)^2} = \pm(6a+7)$

(iv) $64y^2 - 32y + 4$

Sol. $64y^2 - 32y + 4$

$64y^2 - 32y + 4 = 4(16y^2 - 8y + 1)$

$= 4[(4y)^2 - 2(4y)(1) + (1)^2] = 4(4y-1)^2$

By taking square root of both sides

$\sqrt{64y^2 - 32y + 4} = \sqrt{4(4y-1)^2}$

$= \pm 2(4y-1) = \pm(8y-2)$

(v) $200t^2 - 120t + 18$

Sol. $200t^2 - 120t + 18$

$200t^2 - 120t + 18 = 2(100t^2 - 60t + 9)$

$= 2[(10t)^2 - 2(10t)(3) + (3)^2] = 2(10t-3)^2$

By taking square root of both sides

$\sqrt{200t^2 - 120t + 18} = \sqrt{2(10t-3)^2} = \pm\sqrt{2}(10t-3)$

(vi) $40x^2 + 120x + 90$

Sol. $40x^2 + 120x + 90$

Factorization of $40x^2 + 120x + 90 = 10(4x^2 + 12x + 9)$

$= 10[(2x)^2 + 2(2x)(3) + (3)^2] = 10(2x+3)^2$

Take the square root of both sides

$$\sqrt{40x^2 + 120x + 90} = \sqrt{10(2x+3)^2} = \pm\sqrt{10}(2x+3)$$

2. Find the square root of the following polynomials by division method:

(i) $4x^4 - 28x^3 + 37x^2 + 42x + 9$

Sol. $4x^4 - 28x^3 + 37x^2 + 42x + 9$

$$\begin{array}{r|l} & 2x^2 - 7x - 3 \\ \hline 2x^2 & 4x^4 - 28x^3 + 37x^2 + 42x + 9 \\ & -4x^4 \\ \hline 4x^2 - 7x & -28x^3 + 37x^2 \\ & \pm 28x^3 \pm 49x^2 \\ \hline 4x^2 - 14x - 3 & -12x^2 + 42x + 3 \\ & \mp 12x^2 \pm 42x \pm 3 \\ \hline & 0 \end{array}$$

Thus $\sqrt{4x^4 - 28x^3 + 37x^2 + 42x + 9} = \pm(2x^2 - 7x - 3)$

(ii) $121x^4 - 198x^3 - 183x^2 + 216x + 144$

Sol. $121x^4 - 198x^3 - 183x^2 + 216x + 144$

$$\begin{array}{r|l} & 11x^2 - 9x - 12 \\ \hline 11x^2 & 121x^4 - 198x^3 - 183x^2 + 216x + 144 \\ & -121x^4 \\ \hline 22x^2 - 9x & -198x^3 - 183x^2 \\ & \mp 198x^3 \pm 81x^2 \\ \hline 22x^2 - 18x - 12 & -264x^2 + 216x + 144 \\ & \mp 264x^2 \pm 216x \pm 144 \\ \hline & 0 \end{array}$$

Thus, $\sqrt{121x^4 - 198x^3 - 183x^2 + 216x + 144} = \pm(11x^2 - 9x - 12)$

(iii) $x^4 - 10x^3y + 27x^2y^2 - 10xy^3 + y^4$

Replace $-10x^3y$ inetead of $-10x^2y$

Sol. $x^4 - 10x^3y + 27x^2y^2 - 10xy^3 + y^4$

$$\begin{array}{r|l} & x^2 - 5xy + y^2 \\ \hline x^2 & x^4 - 10x^3y + 27x^2y^2 - 10xy^3 + y^4 \\ & -x^4 \\ \hline 2x^2 - 5xy & -10x^3y + 27x^2y^2 \\ & \mp 10x^3y \pm 25x^2y^2 \\ \hline 2x^2 - 10xy + y^2 & 2x^2y^2 - 10xy^3 + y^4 \\ & \mp 2x^2y^2 \mp 10xy^3 \pm y^4 \\ \hline & 0 \end{array}$$

Thus, $\sqrt{x^4 - 10x^3y + 27x^2y^2 - 10xy^3 + y^4} = \pm(x^2 - 5xy + y^2)$

(iv) $4x^4 - 12x^3 + 37x^2 - 42x + 49$

Sol. $4x^4 - 12x^3 + 37x^2 - 42x + 49$

$$\begin{array}{r|l} & 2x^2 - 3x + 7 \\ \hline 2x^2 & 4x^4 - 12x^3 + 37x^2 - 42x + 49 \\ & -4x^4 \\ \hline 4x^2 - 3x & -12x^3 + 37x^2 \\ & \mp 12x^3 \pm 9x^2 \\ \hline 4x^2 - 6x + 7 & +28x^2 - 42x + 49 \\ & \pm 28x^2 \mp 42x \pm 49 \\ \hline & 0 \end{array}$$

Thus, $\sqrt{4x^4 - 12x^3 + 37x^2 - 42x + 49} = \pm(2x^2 - 3x + 7)$

3. An investor's return R(x) in rupees after investing x thousand rupees is given by quadratic expression: $R(x) = -x^2 + 6x - 8$

Factorization the expression d find d the investment levels that result in zero return.

Sol. To find the investment levels result in zero return, set $R(x) = 0$

$$\begin{aligned} -x^2 + 6x - 8 &= 0 \\ -(x^2 - 6x + 8) &= 0 \\ x^2 - 6x + 8 &= 0 \\ x^2 - 4x - 2x + 8 &= 0 \\ x(x-4) - 2(x-4) &= 0 \\ (x-4)(x-2) &= 0 \end{aligned}$$

$x - 4 = 0$ | $x - 2 = 0$

$x = 4$ | $x = 2$

to find the investment levels in rupees, we need to multiply the value of x by 1000, since x is in thousands.

$x = 2 \Rightarrow$ Investment levels that result in zero return $= 2 \times 1000$

$x = 4 \Rightarrow$ Investment levels that result in zero return = Rs. 2000

return 4×1000 = Rs. 4000

So, the investment levels that result in zero return is Rs 4000 and Rs. 2000.

4. A company's profit P(x) in rupees from selling x units of a product is modeled by the cubic expression:

$$P(x) = x^3 - 15x^2 + 75x - 125$$

Find the break-even point(s), where the profit is zero.

Sol. To find the break-even point where profit is zero set, $p(x) = 0$

$$\begin{aligned} x^3 - 15x^2 + 75x - 125 &= 0 \\ (x)^3 - 3(x)^2(5) + 3(x)(5)^2 - (5)^3 &= 0 \\ (x-5)^3 &= 0 \\ (x-5)(x-5)(x-5) &= 0 \\ x - 5 &= 0 \end{aligned}$$

$x = 5$

Therefore, the company breaks even when they sell 5 units of the product.

5. The potential energy V(x) in an electric field varies as a cubic function of distance x, given by:

$$V(x) = 2x^3 - 6x^2 + 4x$$

Determine where the potential energy is zero.

Sol. To find the potential energy zero set, $V(x) = 0$

$$2x^3 - 6x^2 + 4x = 0$$
$$2x(x^2 - 3x + 2) = 0$$
$$2x[x^2 - 2x - x + 3] = 0$$
$$2x[x(x-2) - 1(x-2)] = 0$$
$$2x(x-2)(x-1) = 0$$

$2x = 0$ | $x - 2 = 0$ | $x - 1 = 0$

$x = \frac{0}{2}$ | $x = 2$ | $x = 1$

$x = 0$

So, the points where potential energy is zero are 0, 1 and 2

6. In structural engineering, the deflection Y(x) of a beam is given by:

$$Y(x) = 2x^2 - 8x + 6$$

This equation gives the vertical deflection at any point x along the beam. Find the points of zero deflection.

Sol. To find the points of zero deflection set, $Y(x) = 0$

$$2x^2 - 8x + 6 = 0$$
$$2(x^2 - 4x + 3) = 0$$
$$x^2 - 4x + 3 = 0$$
$$x^2 - 3x - x + 3 = 0$$
$$x(x-3) - 1(x-3) = 0$$
$$(x-3)(x-1) = 0$$

$x - 3 = 0$ | $x - 1 = 0$

$x = 3$ | $x = 1$

So, points of zero deflection are 3 and 1.

Review Exercise 4

1. Four options are given against each statement. Encircle the correct option.

i. The factorization of $12x + 36$ is:

(a) $12(x + 3)$ (b) $12(3x)$ (c) $12(3x + 1)$ (d) $x(12 + 36x)$

ii. The factors of $4x^2 - 12x + 9$ are:

(a) $(2x + 3)^2$ (b) $(2x - 3)^2$

(c) $(2x - 3)(2x + 3)$ (d) $(2 + 3x)(2 - 3x)^2$

iii. The HCF of a^3b^2 and ab^2 is:

(a) a^3b^2 (b) ab^2 (c) a^2b^2 (d) a^2b

iv. The LCM of $16x^2$, $4x$ and $30xy$ is:

(a) $480x^2y$ (b) $240xy$ (c) $240x^2y$ (d) $120x^4y$

v. Product of LCM and HCF = _____ of two polynomials.

(a) sum (b) difference (c) product (d) quotient

vi. The square root of $x^2 - 6x + 9$ is:

(a) $\pm(x - 3)$ (b) $\pm(x + 3)$ (c) $x - 3$ (d) $x + 3$

vii. The LCM of $(a - b)^2$ and $(a - b)^4$ is:

(a) $(a - b)^2$ (b) $(a - b)^3$ (c) $(a - b)^4$ (d) $(a - b)^6$

viii. Factorization of $x^3 + 3x^2 + 3x + 1$ is:

(a) $(x + 1)^3$ (b) $(x - 1)^3$

(c) $(x + 1)(x^2 + x + 1)$ (d) $(x - 1)(x^2 - x + 1)$

ix. Cubic polynomial has degree:

(a) 1 (b) 2 (c) 3 (d) 4

x. One of the factors of $x^3 - 27$ is:

(a) $x - 3$ (b) $x + 3$ (c) $x^2 - 3x + 9$ (d) Both a and c

Answers:

i. $12(x + 3)$ ii. $(2x - 3)^2$ iii. ab^2 iv. $240x^2y$ v. product
vi. $\pm(x - 3)$ vii. $(a - b)^4$ viii. $(x + 1)^3$ ix. 3 x. $x - 3$

2. Factorize the following expressions:

(i) $4x^3 + 18x^2 - 12x$

Sol. $4x^3 + 18x^2 - 12x$
$= 2x(2x^2 + 9x - 6)$

(ii) $x^3 + 64y^3$

Sol. $x^3 + 64y^3$
$= (x)^3 + (4y)^3$
$= (x + 4y)[(x)^2 - (x)(4y) + (4y)^2]$
$= (x + 4y)(x^2 - 4xy + 16y^2)$

(iii) $x^3y^3 - 8$

Sol. $x^3y^3 - 8 = (xy)^3 - 8$
$= (xy - 2)[(xy)^2 + (xy)(2) + (2)^2]$
$= (xy - 2)(x^2y^2 + 2xy + 4)$

(iv) $-x^2 - 23x - 60$

Sol. $-x^2 - 23x - 60$
$= -(x^2 + 23x + 60)$
$= -[x^2 + 20x + 3x + 60]$
$= -[x(x + 20) + 3(x + 20)]$
$= -(x + 20)(x + 3)$

(v) $2x^2 + 7x + 3$

Sol. $2x^2 + 7x + 3$
$= 2x^2 + 6x + x + 3$
$= 2x(x + 3) + 1(x + 3)$
$= (x + 3)(2x + 1)$

(vi) $x^4 + 64$

Sol. $x^4 + 64$
$= (x^2)^2 + (8)^2 + 2(x^2)(8) - 2(x^2)(8)$ (Adding and Subtracting $-2(x^2)(8)$)
$= (x^2 + 8)^2 - 16x^2$
$= (x^2 + 8)^2 - (4x)^2$
$= (x^2 + 8 + 4x)(x^2 + 8 - 4x)$
$= (x^2 + 4x + 8)(x^2 - 4x + 8)$

(vii) $x^4 + 2x^2 + 9$

Sol. $x^4 + 2x^2 + 9$

$= (x^2)^2 + (3)^2 + 2(x^2)(3) - 2(x^2)(3) + 2x^2$ (Adding and Subtracting $2(x^2)(3)$)

$= (x^2 + 3)^2 - 6x^2 + 2x^2$

$= (x^2 + 3)^2 - 4x^2$

$= (x^2 + 3)^2 - (2x)^2$

$= (x^2 + 3 + 2x)(x^2 + 3x - 2x)$

$= (x^2 + 2x + 3)(x^2 - 2x + 3)$

(viii) $(x + 3)(x + 4)(x + 5)(x + 6) - 360$

Sol. $(x + 3)(x + 4)(x + 5)(x + 6) - 360$

Re-arrange as $3 + 6 = 4 + 5$

$= (x + 3)(x + 6)(x + 4)(x + 5) - 360$

$= [(x+3)(x+6)][(x+4)(x+5)] - 360$

$= (x^2+6x+3x+18)(x^2+5x+4x+20) - 360$

$= (x^2 + 9x + 18)(x^2 + 9x + 20) - 360$

Let, $x^2 + 9x = y$

$(y+18)(y+20) - 360$

$= y^2 + 20y + 18y + 360 - 360$

$= y^2 + 38y = y(y + 38)$

$= (x^2 + 9x)(x^2+9x+38)$

$= x(x + 9)(x^2 + 9x + 38)$

(ix) $(x^2 + 6x + 3)(x^2 + 6x - 9) + 36$

Sol. $(x^2 + 6x + 3)(x^2 + 6x - 9) + 36$

Let $x^2 + 6x = y$

$= (y + 3)(y - 9) + 36$

$= (y^2 - 9y + 3y - 2y) + 36$

$= (y^2 - 6y - 27) + 36$

$= y^2 - 6y - 27 + 36$

$= y^2 - 6y + 9$

$= (y)^2 - 2(y)(3) + (13)^2$

$= (y - 3)^2$

$= (x^2 + 6x - 3)^2$

3. Find LCM and HCF by prime factorization method:

(i) $4x^3 + 12x^2$, $8x^2 + 16x$

Sol. $4x^3 + 12x^2$, $8x^2 + 16x$

Factorization of $4x^3 + 12x^2$ $= 4x^2(x + 3)$

$= 2 \times 2 \times x \times x(x + 3)$

Factorization of $8x^2 + 16x$ $= 8x(x + 2$

$= 2 \times 2 \times 2 \times x(x + 2)$

Product of common factors $= 2 \times 2 \times x = 4x$

Product of non-common factors $= 2 \times x(x + 2)(x + 3)$

$= 2x(x + 2)(x + 3)$

HCF = Product of common factors = $4x$

LCM = Product of common factors × Product of non-common factors

$= 4x \times 2x(x + 2)(x + 3)$

$= 8x^2(x + 2)(x + 3)$

(ii) $x^3 + 3x^2 - 4x$, $x^2 - 4x + 3$

Sol. $x^3 + 3x^2 - 4x$, $x^2 - 4x + 3$

Factorization of $x^3 + 3x^2 - 4x$ $= x(x^2 + 3x - 4)$

$= x(x^2 + 4x - x - 4)$

$= x[x(x + 4) - 1(x + 4)]$

$= x(x + 4)(x - 1)$

Factorization of $x^2 - 4x + 3$ $= x^2 - 3x - x + 3$

$= x(x - 3) - 1(x - 3)$

$= (x - 3)(x - 1)$

Product of common factors $= (x - 1)$

Product of non-common factors $= x(x - 3)(x + 4)$

HCF = Product of common factors = $(x - 1)$

LCM = Product of common factors × Product of non-common factors

$= (x - 1) \times (x - 3)(x + 4)$

$= x(x - 1)(x - 3)(x + 4)$

(iii) $x^3 + 8x + 16$, $x^3 - 16$

Sol. $x^3 + 8x + 16$, $x^2 - 16$

Factorization of $x^2+8x+16$ $= x^2 + 4x + 4x + 16$

$= x(x + 4) + x(x + 4)$

$= (x + 4)(x + 4)$

Factorization of $x^2 - 16$ $= (x)^2 - (4)^2$

$= (x + 4)(x - 4)$

Product of common factors $= (x + 4)$

Product of non-common factors $= (x + 4)(x - 4)$

HCF = Product of common factors = $(x + 4)$

LCM = Product of common factors × Product of non-common factors

$= (x + 4) \times (x + 4)(x - 4)$

$= (x - 4)(x + 4)^2$

(iv) $x^3 - 9x$, $x^2 - 4x + 3$

Sol. $x^3 - 9x$, $x^2 - 4x + 3$

Factorization of $x^3 - 9x$ $= x(x^2 - 9)$

$= x[(x)^2 - (3)^2]$

$= x(x + 3)(x - 3)$

Factorization of $x^2 - x - 6$ $= x^2 - 3x + 2x - 6$

$= x(x - 3) + 2(x - 3)$

$= (x + 2)(x - 3)$

Product of common factors $= (x - 3)$

Product of non-common factors $= x(x + 2)(x + 3)$

HCF = Product of common factors = $(x - 3)$

LCM = Product of common factors × Product of non-common factors

$= (x - 3) \times x(x + 2)(x + 3)$

$= x(x - 3)(x + 2)(x + 3)$

$= x(x - 2)(x - 3)(x + 3)$

$= x(x + 2)[(x)^2 - (3)^2]$

$= x(x + 2)(x^2 - 9)$

4. Find square root by factorization and division method of the expression. $16x^4 + 8x^2 - + 1$.

Sol. By factorization method

Factorization of $16x^4 + 8x^2 + 1 = (4x^2)^2 + 2(4x^2)(1) + 1^2$

$= (4x^2 + 1)^2$

Take square root of both sides

$$\sqrt{16x^4 + 8x^2 + 1} = \sqrt{(4x^2 + 1)^2}$$

$$= \pm(4x^2 + 1)$$

$$\begin{array}{r|l} & 4x^2 + 1 \\ \hline 4x^2 & 16x^4 + 8x^2 + 1 \\ & -16x^4 \\ \hline 8x^2 + 1 & 8x^2 + 1 \\ & \pm 8x^2 \pm 1 \\ \hline & 0 \end{array}$$

$$\sqrt{6x^4 + 8x^2 + 1} = \pm(4x^2 + 1)$$

5. Huria is analyzing the total cost of her loan, is modeled by the expression $C(x) = x^2 - 8x + 15$, where x represents the number of years. What is the optimal repayment period for Huria's loan?

Sol. To find the optimal repayment period, set $C(x) = 0$

$x^2 - 8x + 15 = 0$

$x^2 - 5x - 3x + 15 = 0$ $\quad -5 \times -3 = 15$

$x(x - 5) - 3(x - 5) = 0$ $\quad -5 + (-3) = -8$

$(x - 5)(x - 3) = 0$

$x - 5 = 0$	$x - 3 = 0$
$x = 5$	$x = 3$

So, the optimal repayment period is 5 years or 3 years.

Objective Type Questions

Additional MCQ's

Multiple Choice Questions prepared in the light of new Examination Techniques (Knowledge, Understanding, Application, Analytical & Conceptual)

4.1 Identifying common factors and trinomials Concretely, Pictorially and Symbolically

Encircle the correct option.

1. **It is an expression that divides two or more expression exactly.**
(a) common factor (b) mutual factor (c) co-factor (d) none of above

2. **In $x^2 + 3x$, the common factor is:**
(a) 1 (b) x (c) 3 (d) none of above

3. **It is an expression with three terms.**
(a) Monomial (b) Binomial (c) Trinomial (d) None of above

4. **It is an expression with two terms.**
(a) Monomial (b) Tetramial (c) Trinomial (d) Binomial

5. **Factorization of $x^2 - 5x + 4$ is:**
(a) $(x - 1)(x - 4)$ (b) $(x + 1)(x + 4)$ (c) $(x + 1)(x - 4)$ (d) $(x - 1)(x + 4)$

6. **Factorization of $x^2 - 3x - 10$ is:**
(a) $(x + 2)(4 + 5)$ (b) $(x + 2)(x - 5)$ (c) $(x - 2)(x - 5)$ (d) $(x - 2)(x + 5)$

7. **In $x^2 + 9x + 14$ if one factor is $(x + 2)$ then other factor is:**
(a) $x + 9$ (b) $x - 2$ (c) $x + 7$ (d) $x - 7$

8. **Factorization of $x^2 - 11x + 24$ is:**
(a) $(x + 8)(x - 3)$ (b) $(x - 8)(x + 3)$ (c) $(x + 8)(x + 3)$ (d) $(x - 8)(x - 3)$

9. **Factorization of $P^2 + 11P + 18$ is:**
(a) $(P + 9)(P + 2)$ (b) $(P - 9)(P - 2)$ (c) $(P + 9)(P - 2)$ (d) $(P - 9)(P + 2)$

10. **Factors of $3x^2 - 4x - 4$ is:**
(a) $(3x - 2)(x + 2)$ (b) $(3x + 2)(x - 2)$ (c) $(3x - 2)(x - 2)$ (d) $(3x + 2)(x + 2)$

11. **Factorization of $x^4 + x^2 + 25$ is:**
(a) $(x^2 + 3x + 5)(x^2 + 3x + 5)$ (b) $(x^2 - 34 - 5)(x^2 - 3x - 5)$
(c) $(x^2 - 3x + 5)(x^2 + 3x + 5)$ (d) $(x^2 + 3x - 5)(x^2 + 3x - 5)$

12. $\left(x^2 - \sqrt{2}xy + y^2\right)\left(x^2 + \sqrt{2}xy + y^2\right) =$ _____.
(a) $-x^4 + y^4$ (b) $-x^4 - y^4$ (c) $x^4 - y^4$ (d) $x^4 + y^4$

13. **Factors of $(x^2 + 5x + 4)(x^2 + 5x + 6) - 3$ is:**
(a) $(x^2 + 5x + 7)(x^2 + 5x + 3)$ (b) $(x^2 - 5x - 7)(x^2 - 5x - 3)$
(c) $(x^2 + 5x + 7)(x^2 - 5x - 3)$ (d) $(x^2 - 5x - 7)(x^2 + 5x + 3)$

14. **Factors of $8x^3 + 60x^2 + 150x + 125$ is:**
(a) $(2x - 5)(2x - 5)(2x - 5)$ (b) $(2x + 5)(2x + 5)(2x + 5)$
(c) $(2x - 5)(2x + 5)(2x + 5)$ (d) None of these

15. **Factorization of $x^3 - 27y^3$ is:**
(a) $(x - 3)(x^2 - 3x\,y - 9y^2)$ (b) $(x + 3y)(x^2 - 3x\,y - 9y^2)$
(c) $(x + 3y)(x^2 + 3x\,y + 9y^2)$ (d) $(x - 3y)(x^2 + 3x\,y + 9y^2)$

4.2 Highest common factors (HCF) and least common multiple (LCM) of algebraic expression

16. **HCF of $6x^2y, 9xy^2$ is:**
(a) 3xy (b) 2xy (c) xy (d) 4xy

17. **HCf of $6x^3 - 17x^2 - 5x + 6$ and $6x^3 - 5x^2 - 3x + 2$ is:**
(a) $6x^2 - x - 2$ (b) $6x^2 + x - 2$ (c) $6x^2 + x + 2$ (d) $6x^2 - x + 2$

18. **LCM of $4x^2y, 8x^3y^2$ is:**
(a) $4x^3y^2$ (b) $6x^3y^2$ (c) $8x^3y^2$ (d) $10x^3y^2$

19. If LCM of $x^2y + xy^2$ and $x^2 + xy$ is $xy(x + y)$ then HCF is:
(a) $(x + y)(x + y)$ (b) $(x - y)(x - y)$ (c) $x(x - y)$ (d) $x(x + y)$

4.3 Square root of an algebraic Expression.

20. The square root of $36x^4 - 36x^3 + 9$ is:
(a) $\pm 3(2x^2 - 1)$ (b) $\pm(2x^2 - 1)$ (c) $\pm 3(2x^2 + 1)$ (d) $\pm(2x^2 + 1)$

21. $C(x) = 5x^2 - 25x + 30 =$ _____.
(a) $5(x + 2)(4 - 3)$ (b) $5(x - 2)(x - 3)$ (c) $5(x + 2)(x - 3)$ (d) $5(x - 2)(x + 3)$

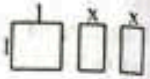
22. How many method to find the square root of an algebraic expression.
(a) two (b) three (c) four (d) five

Answers:

1. common factor 2. x 3. Trinomial 4. Binomial 5. $(x - 1)(x - 4)$
6. $(x+2)(x-5)$ 7. $x + 7$ 8. $(x-8)(x-3)$ 9. $(P+9)(P+2)$ 10. $(3x+2)(x-2)$
11. $(x^2 - 3x + 5)(x^2 + 3x + 5)$ 12. $x^4 + y^4$ 13. $(x^2 + 5x + 7)(x^2 + 5x + 3)$
14. $(2x + 5)(2x + 5)(2x + 5)$ 15. $(x - 3y)(x^2 + 3xy + 9y^2)$ 16. $3xy$
17. $6x^2 + x - 2$ 18. $8x^3y^2$ 19. $x(x + y)$ 20. $\pm 3(2x^2 - 1)$
21. $5(x - 2)(x - 3)$ 22. two

Conceptual MCQ's

○ Encircle the correct option.

1. What is the concerete form of $x^2 + 2x$.
(a)  (b)  (c)  (d) 

2. In $x^2 + 9x + 14$ one factor is $(x + 2)$ other factor is:
(a) $(x + 2)$ (b) $(x + 7)$ (c) $(x - 2)$ (d) $(x - 7)$

3. In $3x^2 - 4x - 4$, one factor is $(x - 2)$ other factor is:
(a) $(x - 2)$ (b) $(x + 2)$ (c) $(3x + 2)$ (d) $(3x - 2)$

4. $a^2 - b^2 =$ _____:
(a) $(a + b)(a + b)$ (b) $(a)(b) + (a)(b)$ (c) $(a - b)(a - b)$ (d) $(a - b)(a + b)$

5. $(a \pm b)^2 =$ _____:
(a) $a^2 + b^2 \pm 2ab$ (b) $a^2 - b^2 - 2ab$ (c) $a^2 + b^2 \mp 2ab$ (d) None of above

6. $x^3 - 27y^3 =$ _____:
(a) $(x + 3y)(x^2 + 3xy + 9y^2)$ (b) $(x - 3y)(x^2 + 3xy + 9y^2)$
(c) $(x - 3y)(x^2 - 3xy - 9y^2)$ (d) None of above

7. $(a - b)^2$ _____ $a^2 - b^2$ which of the following is the best answer.
(a) $\neq$ (b) $=$ (c) $<$ (d) $>$

8. If $16x^4 + y^4$ then factors are:
(a) $4x^2 + y^2 - \sqrt{8}xy$ (b) $4x^2 - y^2 - \sqrt{8}xy$ (c) $4x^2 + y^2 + \sqrt{8}xy$ (d) $4x^2 - y^2 + \sqrt{8}xy$

9. Which of them is correct for HCF?
(a) $\frac{LCM \times P(x)}{q(x)}$ (b) $\frac{P(x) \times q(x)}{LCM}$ (c) $\frac{q(x)}{P(x)(HCF)}$ (d) $\frac{LCM}{P(x)q(x)}$

10. The square root of an algebraic expression refers to a value that when _____ by itself gives the original expression:
(a) multiplied (b) divide (c) add (d) subtract

Answers:

1.  2. $(x + 7)$ 3. $(3x + 2)$ 4. $(a - b)(a + b)$ 5. $a^2 + b^2 \pm 2ab$
6. $(x - 3y)(x^2 + 3xy + 9y^2)$ 7. $\neq$ 8. $4x^2 + y^2 + \sqrt{8}xy$
9. $\frac{P(x) \times q(x)}{LCM}$ 10. multiplied

Additional Short Questions

Short Answer Questions prepared in the light of new Examination Techniques (Knowledge, Understanding, Application, Analytical & Conceptual)

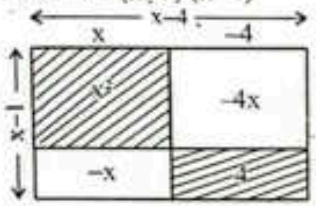
4.1 Identifying common factors and trinomials Concretely, Pictorially and Symbolically

1. What do you mean by common factor?
Ans. A common factor is an expression that divides two or more expressions exactly e.g: $2x - 6 = 2(x - 3)$ here 2 is common factor.

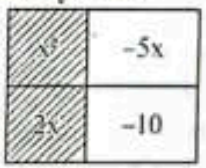
2. Define trinomial. Ans. A trinomial is an expression with three terms.

3. What is binomial? Ans. A binomial is an expression with two terms.

4. Factorize $x^2 - 5x + 4$ concretely.
Sol. $x^2 - 5x + 4 = (x - 1)(x - 4)$



5. Look at the picture and write in symbolically form.



Ans. $(x + 2)(x - 5)$

6. Factorize $x^2 + 9x + 14$
Sol. $x^2 + 9x + 14$
$= x^2 + 2x + 7x + 14$
$= x(x + 2) + 7(x + 2)$
$= (x + 2)(x + 7)$

7. Factorize $2x^2 + 17x + 26$
Sol. $2x^2 + 17x + 26$
$= 2x^2 + 4x + 13x + 26$
$= 2x(x + 2) + 13(x + 2)$
$= (x + 2)(2x + 13)$

8. Factorize $a^4 + 64$.
Sol. $a^4 + 64$
$= (a^2)^2 + (8)^2$

$= (a^2)^2 + (8)^2 + 2(a^2)(8) - 2(a^2)(8)$
$= (a^2 + 8)^2 - 16a^2$
$= (a^2 + 8)^2 - (4a)^2$
$= (a^2 + 8 - 4a)(a^2 + 8 + 4a)$

9. Factorize $(x + 2)(x + 3)(x + 4)(x + 5) - 15$

Sol. $= (x + 2)(x + 3)(x + 4)(x + 5) - 15$
$= [(x+2)(x+5)][(x+3)(x+4)] - 15a$
$= (x^2+7x+10)(x^2+7x+12) - 15$
let $x^2 + 7x = y$
so, $= (y + 10)(y + 12) - 15$
$= y^2 + 12y + 10y + 120 - 15$
$= y^2 + 22y + 105$
$= y^2 + 15y + 7y + 105$
$= y(y + 15) + 7(y + 15)$
$= (y + 15)(y + 7)$
$= (x^2 + 7x + 15)(x^2 + 7x + 7)$ $(\because = x^2 + 7y)$

10. Factorize $x^3 - 18x^2 + 108x - 216$

Sol. $x^3 - 18x^2 + 108x - 216$
$= (x)^3 - 3(x)^2(6) + 3(x)(6)^2 - 6^3$
$= (x - 6)^3$
$= (x - 6)(x - 6)(x - 6)$

11. Factorize $x^3 - 27y^3$

Sol. $x^3 - 27y^3$
$= (x)^3 - (3y)^3$
$= (x - 3y)(x^2 + 9y^2 + 3xy)$

4.2 Highest common factors (HCF) and least common multiple (LCM) of algebraic Expression

12. How many types to find HCF?

Ans. There are two types to find HCF.

13. What is meant by HCF?

Ans. HCF means Highest Common Factor.

14. Define HCF.

Ans. The HCF of algebraic expression refers to the greatest algebraic expression that divides two or more algebraic expressions without leaving a remainder.

15. Find HCF of $x^3 - 27$, $x^2 + 6x - 27$ and $x^2 - 9$ by factorization method.

Sol. $x^3 - 27 = x^3 - 3^3 = (x - 3)(x^2 + 3x + 3^2)$
$= (x - 3)(x^2 + 3x + 9)$
$x^2 + 6x - 27 = x^2 + 9x - 3x - 27$
$= x(x + 9) - 3(x + 9)$
$= (x + 9)(x - 3)$
$x^2 - 9 = x^2 - 3^2 = (x - 3)(x + 3)$
Common factor $= (x - 3)$
So, HCF $= x - 3$

16. Find HCF of $6x^3 - 17x^2 - 5x + 6$ and $6x^3 - 5x^2 - 3x + 2$ division method.

Sol.

$$
\begin{array}{r|l}
 & 1 \\
\hline
6x^3 - 17x^2 - 15x + 6 & 6x^3 - 5x^2 - 3x + 2 \\
 & 6x^3 - 17x^2 - 5x + 6 \\
\hline
 & 12x^2 - 2x - 4
\end{array}
$$

$$
\begin{array}{r|l}
 & x \quad 3 \\
\hline
(6x^2 + x - 2) & 6x^3 - 17x^2 - 5x + 6 \\
 & 6x^3 + x^2 - 2x \\
\hline
 & -18x^2 - 3x + 6 \\
 & -18x^2 - 3x + 6 \\
\hline
 & 0
\end{array}
$$

So, HCF $= 6x^2 + x - 2$

17. What is meant by LCM?

Ans. LCM means least common multiple.

18. Define LCM.

Ans. The LCM of two or more algebraic expressions is the smallest expression that is divisible by each of the given expression.

19. What is the formula to find LCM.

Ans. LCM = Common factors x non common factors.

20. Find the LCM of $x^2 - 3x + 2$, $x^2 - 1$ and $x^2 - 5x + 4$.

Sol. $x^2 - 3x + 2 = x^2 - 2x - x + 2$
$= x(x - 2) - 1(x - 2)$
$= (x - 2)(x - 1)$
and $x^2 - 1 = (x - 1)(x + 1)$
$x^2 - 5x + 4 = x^2 - 4x - x + 4$
$= x(x - 4) - 1(x - 4)$
$= (x - 4)(x - 1)$
Common factors $= x - 1$
Non-common factors $= (x + 1)(x - 2)(x - 4)$
LCM = Common factors × Non-common factors
Now LCM $= (x - 1) \times (x + 1)(x - 2)(x - 4)$
$= (x - 1)(x + 1)(x - 2)(x - 4)$

21. How we express the relationship between LCM and HCF.

Ans. The relationship between LCM and HCF is as:
LCM × HCF = P (x) × q (x)
here p (x) = first polynomial
q(x) = Second polynomial

22. If LCM = $x^3 - 10x^2 + 11x + 70$
HCF = $x - 7$
$p(x) = x^3 - 12x + 35$
find value of q (x)

$$x^2 - 12x + 35 \overline{)\, x^3 - 10x^2 + 11x + 70} \quad (x + 2)$$
$$x^3 - 12x^2 + 35x$$
$$2x^2 - 24x + 70$$
$$2x^2 - 24x + 70$$
$$0$$

Sol. $q(x) = \dfrac{LCM \times HCF}{P(x)}$

$$q(x) = \frac{(x^3 - 10x^2 + 11x + 70)(x - 7)}{x^2 - 12x + 35}$$
$$q(x) = (x+2)(x-7)$$
$$= x^2 - 7x + 2x - 14$$
$$= x^2 - 5x - 14$$

4.3 Square root of an algebraic Expression.

23. What do you mean by square root of algebraic expression?

Sol. The square root of an algebraic expression refers to a value that, when multiplied by itself, gives the original expression.

24. How many methods to find a square root of algebraic expression?

Sol. There are two methods to find the square root of algebraic expression.

25. Find the square root of $36x^4 - 36x^2 + 9$ by factorization method.

Sol. $36x^4 - 36x^2 + 9$
$$= 9(4x^4 - 4x^2 + 1)$$
$$= 9\left[(2x^2)^2 - 2(2x^2)(1) + (1)^2\right]$$
$$= 3^2(2x^2 - 1)^2 \Rightarrow \sqrt{36x^4 - 36x^2 + 9} = \sqrt{3^2(2x^2 - 1)^2} = \pm 3(2x^2 - 1)$$

26. Find square root of $x^4 - 12x^3 + 42x^2 - 36x + 9$ by division method.

Sol.

$$x^2 - 6x + 3 \overline{)\, x^4 - 12x^3 + 42x^2 - 36x + 9}$$
$$x^4$$
$$-12x^3 + 42x^2$$
$$-12x^3 + 36x^2$$
$$6x^2 - 36x + 9$$
$$6x^2 - 36x + 9$$
$$0$$

so, square root is $\pm(x^2 - 6x + 3)$

27. Cost function for producing a part is modeled by: $C(x) = 5x^2 - 25x + 30$
Find value of x if x is width of component and C(x) is the cost

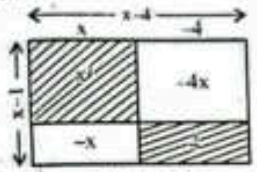
Sol. $C(x) = 5x^2 - 25x + 30$
$$= 5(x^2 - 5x + 6)$$
$$= 5(x^2 - 2x - 3x + 6)$$
$$= 5[x(x-2) - 3(x-2)]$$

$$= 5(x-2)(x-3)$$

Thus, the minimum cost occurs when x = 2 and x = 3.

Conceptual Short Questions

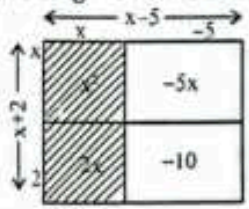
1. The given figure is in Pictorial form.



Write in the symbolic form.

Sol. $x^2 - 5x + 4 = (x-1)(x-4)$.

2. The given figure is in Pictorial form.



Write in symbolic form.

Sol. $x^2 - 3x - 10 = (x+2)(x-5)$

3. Define quadratic equation?

Ans. An expression having degree 2 is called quadratic expression.

4. In $(3x^2 - 4x - 4)$, the one factor is $(x - 2)$. What is the second factor?

Sol. $3x^2 - 4x - 4$
$$= 3x^2 + 2x - 6x - 4$$
$$= x(3x+2) - 2(3x+2) = (3x+2)(x-2)$$
The second factor is $(3x + 2)$.

5. What is the formula of $(a \pm b)^3$

Sol. $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

6. How many method to find the square root of an Algebraic expression? Name them.

Ans. There are two methods find the square root of an algebraic expression.
i. Square root by factorization method. ii. Square root by division method.

7. How many methods to find the HCF? Name them.

Ans. There are two methods to find the HCF.
i. By factorization method. ii. By division method.

8. What is the formula to find the LCM?

Ans. LCM = common factor × Non – common factors.

9. Write the relationship between LCM and HCF.

Sol. LCM × HCF = P(x) × q(x)
P(x) = 1st polynomial
q(x) = 2nd polynomal

10. What is the factorization of $a^3 + b^3$?

Sol. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

— ✲✲✲ —

Unit 5 Linear Equations and Inequalities

Linear Equation

An equation of the form $ax + b = 0$ where 'a' and 'b' are constants, $a \neq 0$ and 'x' is a variable, is called a linear equation in one variable. In linear equation, the highest power of the variable is always 1.

Remember!
ax+b=0and a ≠ 0 is also called the general form of linear equation in one variable.

Solving a Linear Equation in One Variable

Solving a linear equation in one variable means finding the value of the variable that makes the equation true. To solve the equation, the goal is to isolate the variable on one side of the equation and determine its value.

Steps to solve a Linear Equation in One Variable

- Combine like terms on each side of the equation.
- Simplify expressions, including distributing any multiplication over parentheses.

Isolate the Variable Term

- Move all terms containing the variable to one side of the equation and all constant terms numbers to the other side. We can do this by adding or subtracting terms from both sides of the equation.

Solve for the variable

- Once the variable terms is isolated, solve for the variable by dividing or multipying both sides of the equation by the co-efficient of the variable.

Check Your Solution

- Substitute the solution into the original equation to ensure that solution is correct.

Example 1: Solve the following equations and represent their solutions on real line.

(i) $3x - 5 = 7$ (ii) $\frac{x-2}{5} - \frac{x-4}{2} = 2$

Remember!
A linear equation in one variable has only one solution.

Solution:

(i) $3x - 5 = 7$

$3x - 5 + 5 = 7 + 5$

$3x = 12$

$\frac{3x}{3} = \frac{12}{3}$

$x = 4$

Check: Substitute $x = 4$ into the original equation

$3(4) - 5 = 7$

$12 - 5 = 7$

$7 = 7$

So, $x = 4$ is a solution because it makes the original equation true.

Representation of the solution on a number line:

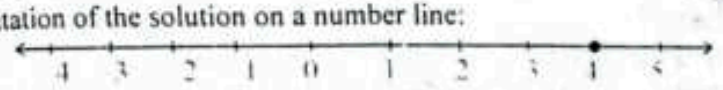


Fig. 5.1

Remember!
We check the solution after solving linear equations to ensure the accuracy of our work.

(ii) $\frac{x-2}{5} - \frac{x-4}{2} = 2$

$\frac{2(x-2) - 5(x-4)}{10} = 2$

$\frac{2x - 4 - 5x + 20}{10} = 2$

$\frac{-3x + 16}{10} = 2$

$-\frac{3x + 16}{10} = 2 \times 10$

$-3x + 16 = 20$

$-3x + 16 - 16 = 20 - 16$

$-3x = 4$

$x = \frac{-4}{3}$

Remember!
Moving terms from one side of the equation to the other to solve for a variable is called transposition. When we move a term, we change its sign: if it is positive, it becomes negative, and if it is negative, it becomes positive.

Check: Substitute $x = -\frac{4}{3}$ back into the original equation

$\frac{-\frac{4}{3} - 2}{5} - \frac{-\frac{4}{3} - 4}{2} = 2$

$\Rightarrow \frac{\frac{-4-6}{3}}{5} - \frac{\frac{-4-12}{3}}{2} = 2 \Rightarrow \frac{-10}{15} - \frac{-16}{6} = 2$

$\Rightarrow -\frac{2}{3} + \frac{8}{3} = 2 \Rightarrow \frac{-2+8}{3} = 2$

$\Rightarrow \frac{6}{3} = 2 \Rightarrow 2 = 2$

So, is $x = -\frac{4}{3}$ the solution of given equation.

Representation of the solution on a number line:

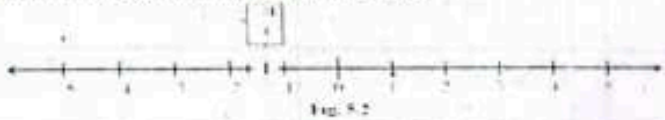


Fig. 5.2

Linear Inequalities

Inequalities are expressed by the following four symbols:

$>$ (greater than), $<$ (less than), $\geq$ (greater than or equal to), $\leq$ less than or equal to)

For example,

(i) $ax < b$ (ii) $ax + b \geq c$ (iii) $ax + by > c$ (iv) $ax + by \leq c$

are inequalities. Inequalities (i) and (ii) are in one variable while inequalities (iii) and (iv) are in two variables.

Do you know?
The order (or sense) of an inequality is change by multiplying or dividing its each side by negative constant.

Example 2: Find solution of $\frac{2}{3}x - 1 < 0$ and also represent it on a real line.

Solution:

$$\frac{2}{3}x - 1 < 0 \quad ...(i)$$

$$\Rightarrow \quad \frac{2}{3}x < 1$$

$$\Rightarrow \quad 2x < 3 \Rightarrow x < \frac{3}{2}$$

History
The inequality signs < and > were introduced by the English mathematician Thomas Harriot in the early 17th century.
For further information, you can use the following link:
https://en.wikipedia.org/wiki/Thomas_Harriot

It means that all real numbers less than $\frac{3}{2}$ are in the solution of (i)

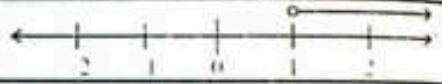
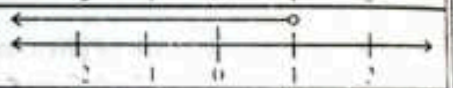
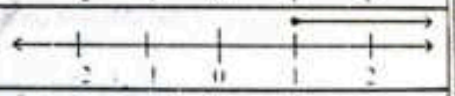
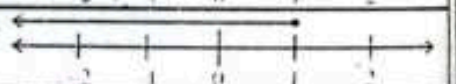
Thus the interval $(-\infty, \frac{3}{2})$ or $-\infty < x < \frac{3}{2}$ is the solution of the given inequality which is shown in figure 5.3



Fig. 5.3

Following are we conclude that the solution of an inequality consists of all solutions of the inequality.

the inequalities and their solutions on a real line:

Inequality	Solution	Representation on real line
$x > 1$	$(1, \infty)$ or $1 < x < \infty$	
$x < 1$	$(-\infty, 1)$ or $-\infty < x < 1$	
$x \geq 1$	$[1, \infty)$ or $1 \leq x < \infty$	
$x \leq 1$	$(-\infty, 1]$ or $-\infty < x \leq 1$	

Solution of a Linear Inequality in Two Variables

Generally a linear inequality in two variables x and y can be one of the following forms: $ax + by < c$; $ax + by > c$; $ax + by \leq c$; $ax + by \geq c$

Where a,b,c are constants and a,b are not both zero.

We know that the graph of linear equation of the form $ax + by = c$ is a line which divides the plane into two disjoint regions as stated below:

(i) The set of ordered pairs (x,y) such that $ax + by < c$

(ii) The set of ordered pairs (x,y) such that $ax + by > c$

The regions (i) and (ii) are called half planes and the line $ax + by = c$ is called the boundary of each half plane.

Note that a vertical line divides the plane into left and right half planes while a non-vertical line divides the plane into upper and lower half planes.

A solution of a linear inequality in x and y is an ordered pair of numbers which satisfies the inequality.

For example, the ordered pair (1, 1) is a solution of the inequality $x + 2y < 6$ because $1+2(1) = 3 < 6$ which is true.

Note that the linear equation $ax + by = c$ is called **"associated or corresponding equation"** of each of the above-mentioned inequalities.

Procedure for Graphing a linear Inequality in two Variables

(i) The corresponding equation of the inequality is first graphed by using 'dashes' if the inequality involves the symbols $>$ or $<$ and a solid line is drawn if the inequality involves the symbols $\geq$ or $\leq$.

(ii) A test point (not on the graph of the corresponding equation) is chosen which determines on which side of the boundary line the half plane line.

Example 3: Solve the inequality $x + 2y < 6$

Solution: The associated equation of the inequality

$x + 2y < 6$ (i)

is $x + 2y = 6$ (ii)

The line (ii) intersects the X-axis and Y-axis at (6, 0) and (0, 3) respectively. As no point of the line (ii) is a solution x of the inequality (i), so the graph of the line (ii) is shown by using dashes. We take O (0, 0) as a test point because it is not on the line (ii).

Substituting $x = 0$, $y = 0$ in the expression $x + 2y$ gives $0 - 2(0) = 0 < 6$. So, the point (0, 0) satisfies the inequality (i). Any other point below the line (ii) satisfies the inequaljty (i), that is all points in the half plane containing the point (0,0) satisfy the inequality (i).

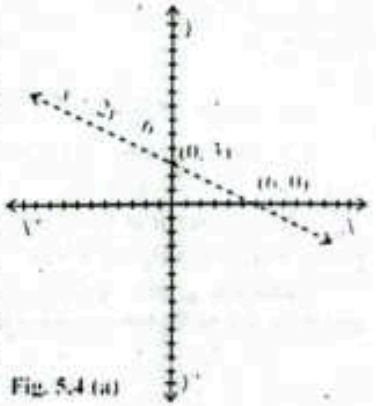


Fig. 5.4 (a)

Thus, the graph of the solution set of inequality (i) is a region on lies the origin-side of the line (ii), that is, the region below the line (ii). A portion of the open half plane below the line (ii) is shown as shaded region in figure 5.4(a)

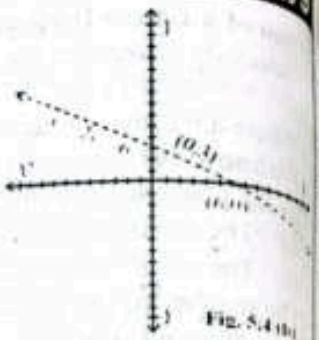


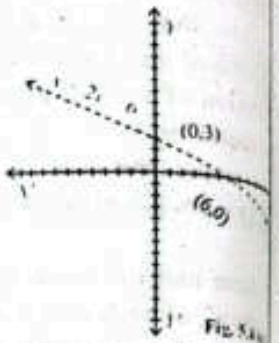
Fig. 5.4 (b)

Note:

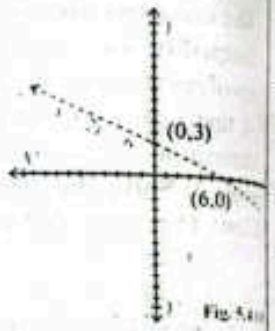
All points above the dashed line satisfy the inequality $x + 2y > 6$ (iii)

A portion of the open half plane above the line (ii) is shown by shading in figure 5.4(b)

Note: 1. The graph of the inequality $x + 2y \leq 6$ ...(iv) includes the graph of the line (ii).' The open half-plane below the line (ii) including the graph of the line (ii) is the graph of the inequality (iv). A portion of the graph of the inequality (iv) is shown by shading in fig. 5.4 (c).



Note: 2 All points on the line (ii) and above the line (ii) satisfy the inequality $x + 2y \geq 6$ .... (v). This means that the solution set of the inequality (v) consists of all points above the line (ii) and all points on the lines (ii). The graph of the inequality (v) is partially shown as shaded region in fig. 5.4 (d).



Note: 3 The graphs of $x + 2y \leq 6$ and $x + 2y \geq 6$ are closed half planes.

Example 4. Graph the following linear inequalities in xy-plane:

(i) $2x \geq -3$ (ii) $y \leq 2$

Solution: (i) The inequality $2x \geq -3$ in xy-plane is considered as $2x + 0y \geq -3$ and its solution set consists of all point (x, y)

such that x, $y \in R$ and $x \geq -\frac{3}{2}$

The corresponding equation of the inequality is

$2x = -3$ ..... (i)

which a vertical line (parallel to the y-axis) and its graph is drawn in figure 5.5(a).

Thus, the graph of $2x \geq -3$ consist of boundary line and the open half-plane to the right of the line (1).

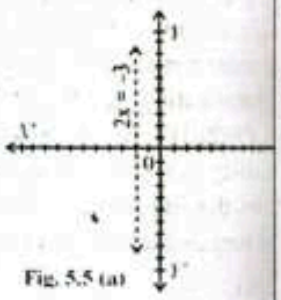
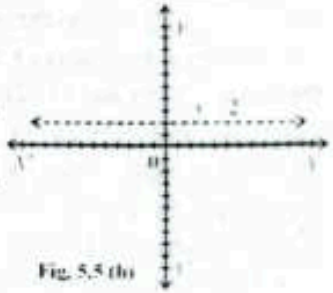


Fig. 5.5 (a)

(ii) The associated equation of the inequality $(y \leq 2)$ is

$y = 2$ ..... (ii)

which is a horizontal line (parallel to the x-axis) and its graph is shown in figure 5.5 (b). Here the solution set of the inequality $y < 2$ is the open half plane below the boundary line $y = 2$. Thus, the graph of $y \leq 2$ consists of the boundary line and the open half plane below it.


Fig. 5.5 (b)

Solution of Two Linear Inequalities in Two Variables

The graph of a system of linear inequalities consists of the set of all ordered pairs (x,y) in the xy-plane which simultaneously satisfies all the inequalities in the system.

Example 5: Find the solution region by drawing the graph of the system of inequalities

$$x - 2y \leq 6$$
$$2x + y \geq 2$$

Solution: $x - 2y \leq 6$ ...(i)

$2x + y \geq 2$ ...(ii)

The associated equation of (i) is

$x - 2y = 6$ ...(iii)

For x-intercept, put $y = 0$ in (iii), we get

$x - 2(0) = 6$

$x - 0 = 6$

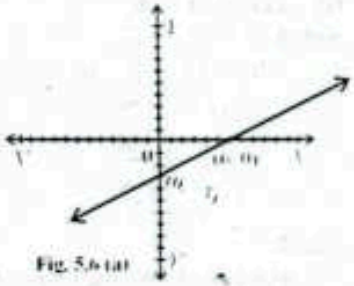
$\Rightarrow \quad x = 6$, so the point is (6, 0)

For y-intercept, put $x = 0$ in (iii), we get

$0 - 2y = 6$

$\Rightarrow \quad -2y = 6$

$\Rightarrow \quad y = \frac{6}{-2} = -3$, so the point is (0, –3)


Fig. 5.6 (a)

The graph of the line $x - 2y = 6$ is drawn by joining the point (6, 0) and (0, –3). The point (0, 0) satisfies the inequality $x - 2y < 6$ because $0 - 2(0) = 0 < 6$. Thus, the graph of $x - 2y \leq 6$ is the upper half-plane including the graph of the line $x - 2y = 6$. The closed half-plane is partially shown by shading in figure 5.6 (a).

The associated equation of (ii) is

$2x + y = 2$ ...(iv)

For x-intercept, put $y = 0$ in (iv), we get

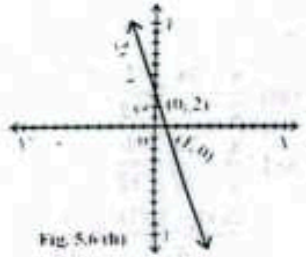
$2x + 0 = 2$

$\Rightarrow \quad 2x = 2$

$\Rightarrow \quad x = 1$, so the point is (1, 0)

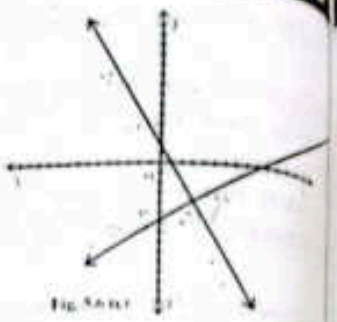
For y-intercept, put $x = 0$ in (iv), we get

$2(0) + y = 2$


Fig. 5.6 (b)

$\Rightarrow$ $y = 2$, so the point is (0, 2)

We draw the graph of the line $2x + y = 2$ joining the points (1, 0) and (0, 2). The point (0, 0) does not satisfy the inequality $2x + y > 2$ because $2(0)+0 = 0 \not> 2$. Thus, the graph of the inequality $2x + y \geq 2$ is the closed half-plane not on the origin-side of the line $2x + y = 2$ and partially shown by shading in figure 5.6 (b).



The solution region of the given system of inequalities is the intersection of the graphs indicated in figures 5.6 (a) and 5.6(b) is shown as shaded region in figure 5.6 (c).

Solved Exercise 5.1

1. Solve and represent the solution on a real line.

(i) $12x + 30 = -6$

Sol. $12x + 30 = -6$

$12x + 30 - 30 = -6 - 30$

$12x = -36$

$\frac{12x}{12} = \frac{-36}{12}$

$x = -3$

Check

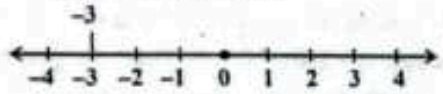
Put $x = -3$ into the original equation

$12(-3) + 30 = -6$

$-36 + 30 = -6$

$-6 = -6$

So, $x = -3$ is solution



(ii) $\frac{x}{3} + 6 = -12$

Sol. $\frac{x}{3} + 6 = -12$

$\frac{x}{3} + 6 - 6 = -12 - 6$

$\frac{x}{3} = -18$

$\frac{x}{3} \times 3 = -18 \times 3$

$x = -54$

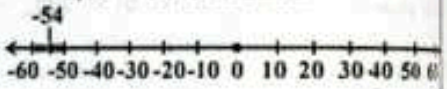
Check: Put $x = -54$ into the orignal equation

$\frac{-54}{3} + 6 = -12$

$-18 + 6 = -12$

$-12 = -12$

So, $x = -54$ is solution



(iii) $\frac{x}{2} - \frac{3x}{4} = \frac{1}{12}$

Sol. $\frac{x}{2} - \frac{3x}{4} = \frac{1}{12}$

$\frac{2(x) - 3x(1)}{4} = \frac{1}{12}$

$\frac{4 \times 2x - 3x}{4} = 4 \times \frac{1}{12}$

$-x = \frac{1}{3}$

$x = -\frac{1}{3}$

Check: Put $x = -\frac{1}{3}$ into the orignal equation.

$\frac{\frac{-1}{3}}{2} - \frac{3\left(-\frac{1}{3}\right)}{4} = \frac{1}{12}$

$\frac{-1}{6} + \frac{1}{4} = \frac{1}{12}$

$\frac{-2+3}{12} = \frac{1}{12}$

$\frac{1}{12} = \frac{1}{12}$

So, $x = -\frac{1}{3}$ is solution.



(iv) $2 = 7(2x + 4) + 12x$

Sol. $2 = 7(2x + 4) + 12x$

$2 = 14x + 28 + 12x$

$2 - 28 = 14x + 12x$

$-26 = 26x$

$\frac{-26}{26} = x$

$-1 = x$

$x = -1$

Check put $x = -1$ into the orignal equation

$2 = 7[2(-1) + 4] + 12(-1)$

$2 = 7[-2 + 4] - 12$

$2 = 7(2) - 12$

$2 = 14 - 12$

$2 = 2$

So, $x = -1$ is solution of the given equation.



(v) $\frac{2x-1}{3} - \frac{3x}{4} = \frac{5}{6}$

Sol. $\frac{4(2x-1) - 3(3x)}{12} = \frac{5}{6}$

$\frac{8x - 4 - 9x}{12} = \frac{5}{6}$

$\frac{8x - 9x - 4}{12} = \frac{5}{6}$

$\frac{-x - 4}{12} = \frac{5}{6}$

$12 \times \frac{-x-4}{12} = {}^{2}\cancel{12} \times \frac{5}{\cancel{6}}$

$-x - 4 = 10$

$-x - 4 = 10$

$-x - 4 + 4 = 10 + 4$

$-x = 14$

$x = -14$

So, $x = -14$ is solution of the equation.

Check: Put $x = -14$ in to orignal equation

$\frac{2(-14)-1}{3} - \frac{3(-14)}{4} = \frac{5}{6}$

$\frac{-28-1}{3} - \frac{-42}{4} = \frac{5}{6}$

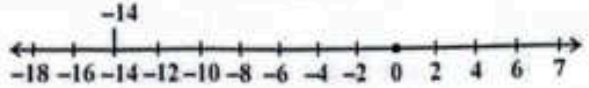
$\frac{-29}{3} + \frac{42}{4} = \frac{5}{6}$

$\frac{4(-29) + 3(42)}{12} = \frac{5}{6}$

$\frac{-116 + 126}{12} = \frac{5}{6}$

$\frac{10}{12} = \frac{5}{6}$

$\frac{5}{6} = \frac{5}{6}$



(vi) $\frac{-5x}{10} = 9 - \frac{10}{5}x$

Sol. $\frac{-5x}{10} = 9 - \frac{10}{5}x$

$\frac{-5x}{10} + \frac{10}{5}x = 9$

$\frac{-5x + 20x}{10} = 9$

$\frac{15x}{10} = 9$

$15x = 9 \times 10$

$15x = 90$

$x = \frac{90}{15}$

$x = 6$

Check: Put x = 6 in orignal equation

$\frac{-5(6)}{10} = 9 - \frac{10}{5}(6)$

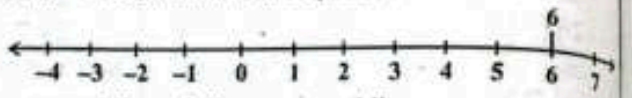
$\frac{-30}{10} = 9 - \frac{60}{5}$

$\frac{-30}{10} = \frac{45 - 60}{5}$

$\frac{-30}{10} = \frac{-15}{5}$

$-3 = -3$

So, x = 6 is solution of the equation



2. Solve each inequality and represent the solution on a real line.

(i) $x - 6 \leq -2$

Sol. $x - 6 \leq -2$

$x \leq -2 + 6$

$x \leq 4$

It mens all real numbers less than and equal to 4 are the solution of this inequality.

$-\infty < x \leq 4$



(ii) $-9 > -16 + x$

Sol. $-9 > -16 + x$

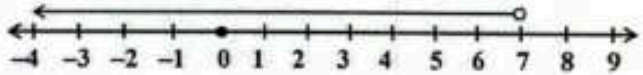
$-9 + 16 > +x$

$7 > x$

$x < 7$

It means all real numbers less than 7 are the solution of this inequality.

$-\infty < x < 7$



(iii) $3 + 2x \geq 3$

Sol. $3 + 2x \geq 3$

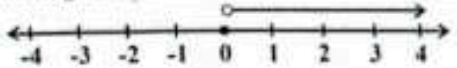
$2x \geq 3 - 3$

$2x \geq 0$

$x \geq \frac{0}{2}$

$x \geq 0$

It means all real numbers greater than or equal to 0 are the solution of this inequality.



(iv) $6(x+10) \leq 0$

Sol. $6(x+10) \leq 0$

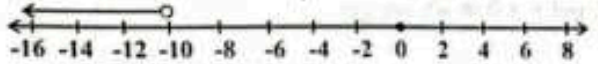
$6x + 60 \leq 0$

$6x \leq -60$

$x \leq -\frac{60}{6}$

$x \leq -10$

It means all real numbers less than or equal to –10 are the solution of this inequality



(v) $\frac{5}{3}x - \frac{3}{4} < -\frac{1}{12}$

Sol. $\frac{5}{3}x - \frac{3}{4} < -\frac{1}{12}$

Multiply on both sides by 12 (the LCM of denominators)

$\cancel{12}^{4} \times \frac{5}{\cancel{3}}x - \cancel{12}^{3} \times \frac{3}{\cancel{4}} < \cancel{12} \times -\frac{1}{\cancel{12}}$

$20x - 9 < -1$

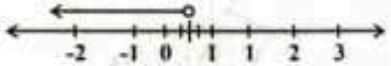
$20x < -1 + 9$

$20x < 8$

$x < \frac{8}{20}$

$x < \frac{2}{5}$

It means all real numbers less than $\frac{2}{5}$ are the solution of this inequality.



(vi) $\frac{1}{4}x - \frac{1}{2} \leq -1 + \frac{1}{2}x$

Sol. $\frac{1}{4}x - \frac{1}{2} \leq -1 + \frac{1}{2}x$

Multiply on both sides by 4 (LCM of denominator)

$4 \times \frac{1}{4}x - 4^2 \times \frac{1}{2} \leq 4 \times -1 + 4^2 \times \frac{1}{2}x$

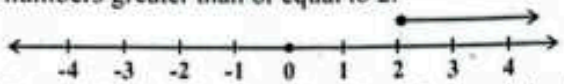
$x - 2 \leq -4 + 2x$

$x - 2x \leq -4 + 2$

$-x \leq -2$

$x \geq 2$

It mean all real numbers greater than or equal to 2.



3. Shade the solution region for the following linear inequalities in xy-plane:

(i) $2x + y \leq 6$

Sol. $2x + y \leq 6$

The associated equation of this inequality is:

$2x + y = 6$ _____ (i)

For x intercept, put $y = 0$ in (i), we get

$2x + 0 = 6$

$2x = 6$

$x = \frac{6}{2}$

$x = 3$

So, the point is (3, 0)

For y-intercept, put $x = 0$ in (i), we get

$2(0)+y = 6$

$0+y = 6$

$y = 6$

So, the point is (0,6)

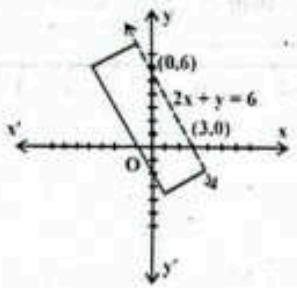
Take (0,0) as a test point and put it in $2x + y \leq 6$

$2(0) + 0 \leq 6$

$0 + 0 \leq 6$

$0 \leq 6$ (True)

Point (0,0) satisfies the inequality, so its solution is towards origin, which is closed half plane grap.



$x = \frac{20}{5}$

$x = 4$

So, the point is (4,0)

For y-intercept, put $x = 0$ in (i), we get

$5(0) - 4y = 20$

$0 - 4y = 20$

$-4y = 20$

$4y = -20$

$y = -\frac{20}{5}$

$y = -5$

So, the point is (0,–5)

Take (0,0) as a test point, put it in inequalty

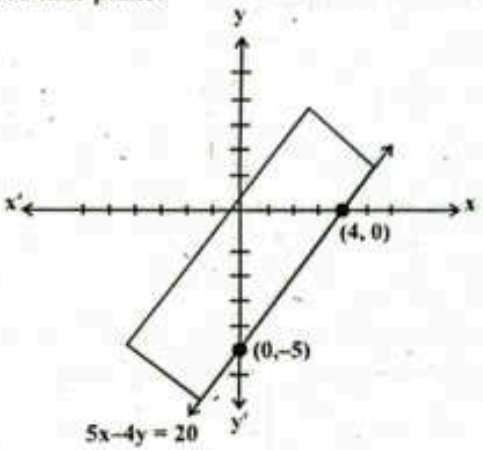
$5x - 4y \leq 20$

$5(0) - 4(0) \leq 20$

$0 - 0 \leq 20$

$0 \leq 20$ (true)

Because point (0,0) satisfies the given inequality, so its solution is towards the origin, which is the closed half plane.



(v) $2x + 1 \geq 0$

Sol. $2x + 1 \geq 0$

The associated equation of this inequality is:

$2x + 1 = 0$ _____ (i)

From (i)

$2x + 1 = 0$

$2x = -1$

$x = -\frac{1}{2}$

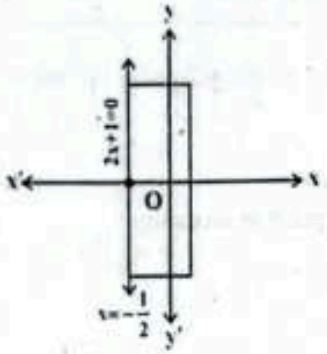
Take (0,0) as a test point, put it in the $2x + 1 > 0$

$2(0) + 1 \geq 0$

$0 + 1 \geq 0$

$1 \geq 0$ (true)

Because test point satisfies the given inequality, so its solution is towards the origin, which is the closed half plane.



(vi) $3y - 4 \leq 0$

Sol. $3y - 4 \leq 0$

The associated equation of this inequality is:

$3y - 4 = 0$

From (i)

$3y - 4 = 0 \quad \Rightarrow \quad 3y = 4$

$y = \frac{4}{3}$

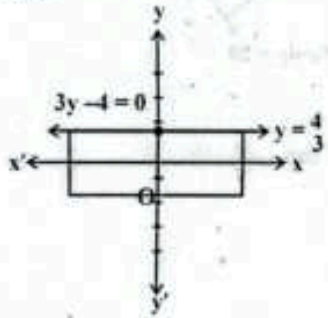
Take (0,0) as a test point, put it in $3y - 4 \leq 0$

$3(0) - 4 \leq 0$

$0 - 4 \leq 0$

$-4 \leq 0$ (true)

Because test point (0,0) satisfies the given inequality, so its solution is towards the origin, which is closed half plane.



) $3x + 7y \geq 21$

ol. $3x + 7y \geq 21$

The associated equation of this inequality is:

$3x + 7y = 21$ _____ (i)

For x-intercept, put $y = 0$ in (i), we get

$3x + 7(0) = 21$

$3x + 0 = 21$

$3x = 21$

$x = \frac{21}{3}$

$x = 7$

So, the point is (7,0)

For y-intercept, put $x = 0$ in (i), we get

$3(0) + 7y = 21$

$0 + 7y = 21$

$7y = 21$

$y = \frac{21}{7}$

$y = 3$

So, the point is (0, 3)

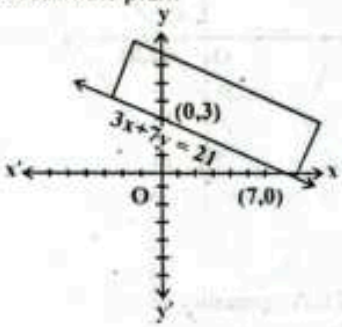
Take (0,0) as a test point, put it in $3x + 7y \geq 21$

$3(0) + 7(0) \geq 21$

$0 + 0 \geq 21$

$0 \geq 21$ (false)

Because point (0,0) does not satisfy the given inequality so, its solution is away from the origin, which is the closed half plan.



iii) $3x - 2y \geq 6$

ol. $3x - 2y \geq 6$

The associated equation of this inequality is:

$3x - 2y = 6$ _____ (i)

For x-intercept, put $y = 0$ in (i), we get

$3x - 2(0) = 6$

$3x - 0 = 6$

$3x = 6$

$x = \frac{6}{3}$

$x = 2$

So, the point is (2, 0)

For y, intercept, put x = 0 in (i), we get

$3(0) - 2y = 6$

$0 - 2y = 6$

$-2y = 6$

$2y = -6$

$y = \frac{-6}{2}$

$y = -3$

So, the point is (0, –3)

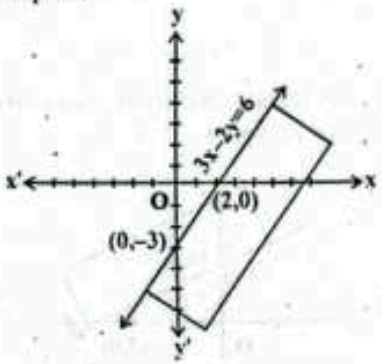
Take (0,0) as a test point, put it in $3x - 2y \geq 6$

$3(0) - 2(0) \geq 6$

$0 - 0 \geq 6$

$0 \geq 6$ (false)

Because point (0,0) does not satisfy the given inequality, so its solution is away from the origin which is closed half plan.



(iv) $5x - 4y \leq 20$

Sol. $5x - 4y \leq 20$

The associated equation of this inequality is:

$5x - 4y = 20$ _____ (i)

For x-intercept, put y = 0 in (i), we get

$5x - 4(0) = 20$

$5x - 0 = 20$

$5x = 20$

So, the point is (7,0)

For y-intercept, put x = 0 in (iii), we get

$3(0) + 7y \geq 21$

$0 + 7y = 21$

$7y = 21$

$y = \frac{21}{7}$

$y = 3$

So, the point is (0, 3)

Take (0, 0) as a test point, put it in (i)

$3(0) + 7(0) \geq 21$

$0 + 0 \geq 21$

$0 \geq (21)$ (false)

Because point (0,0) does not satisfy the given inequality, so its solution is away from the origin.

The associated equation of (ii) is:

$x - y = 2$

For x-intercept, put y = 0 in (iv), we get

$x - 0 = 2$

$x = 2$

So, the point is (2,0)

For y-intercept, put x = 0 in (iv), we get

$0 - y = 2$

$-y = 2$

$y = -2$

So, the point is (0, –2)

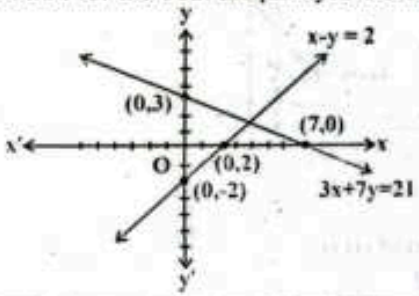
Take (0,0) as a test point, put it in (ii)

$0 - 0 \leq 2$

$0 \leq 2$ (true)

Because point (0,0) satisfies the given inequality, so its solution is towards origin.

Thus, the common solution for the both inequalityes is shaded veglon.



4. Indicate the solution region of the following linear inequalities by shading:

(i) $2x - 3y \le 6$, $2x + 3y < 12$

Sol. $2x - 3y \le 6$ ___ (i)

$2x + 3y \le 12$ ___ (ii)

The associated equation of (i) is

$2x - 3y = 6$ _____ (iii)

For x-intercept, put $y = 0$ in (iii), we get

$2x - 3(0) = 6$

$2x - 0 = 6$

$2x = 6$

$x = \frac{6}{2}$

$x = 3$

So, the point is (3,0)

For y-intercept, put $x = 0$ in (iii), we get

$2(0) - 3y = 6$

$0 - 3y = 6$

$-3y = 6$

$3y = -6$

$y = \frac{6}{3}$

$y = -2$

So, the point is (0, –2)

Take (0,0) as a test point, put it in (i)

$2(0) - 3(0) \le 6$

$0 - 0 \le 6$

$0 \le 6$ (True)

Because point (0,0), satisfies the given inequality, so its solution is towards the origen.

The associated equation of (ii) is

$2x + 3y = 12$ (iv)

For x-intercept, put $y = 0$ in (iv), we get

$2x + 3(0) = 12$

$2x + 0 = 12$

$2x = 12$

$x = \frac{12}{2}$

$x = 6$

So, the point is (6,0)

For y-intercept, put $x = 0$ in (iv), we get

$2(0) + 3y = 12$

$0 + 3y = 12$

$3y = 12$

$y = \frac{12}{3}$

$y = 4$

So, the point is (0,4)

Take (0,0) as a test point, put it (ii)

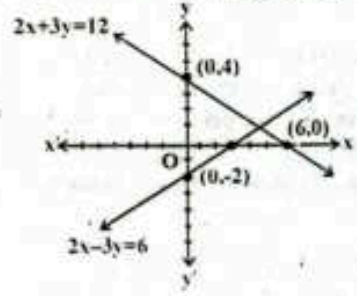
$2(0) + 3(0) \le 12$

$0 + 0 \le 12$

$0 \le 12$ (true)

Because point (0,0) satisfies the given in equality, so its solution is towards the origen.

Thus, the common solution for both the inequalities is shaded region.



(iv) $4x - 3y \le 12$, $x \ge -\frac{3}{2}$

Sol. $4x - 3y \le 12$ _____ (i)

$x \ge -\frac{3}{2}$ _____ (ii)

The associated equation of (i) is:

$4x + 3y = 12$ _____ (iii)

For x-intercept, put $y = 0$ in (iii), we get

$4x - 3(0) = 12$

$4x - 0 = 12$

$4x = 12$

$x = \frac{12}{4}$

$x = 3$

So, the point is (3,0)

For y-intercept, put $x = 0$ in (iii), we get

$4(0) - 3y = 12$

$0 - 3y = 12$

$-3y = 12$

$3y = -12$

$y = \frac{-12}{3}$

$y = -4$

So, the point is (0,–4)

Take (0,0) as a test point, put it in

$4(0) - 3(0) \le 12$

$0 - 0 \le 12$

$0 \le 12$ (ture)

Because, point (0,0), satisfies the given inequality, so its solution is towards origin.

From (ii)

The associated equation of (ii) is

$x = \frac{-3}{2}$

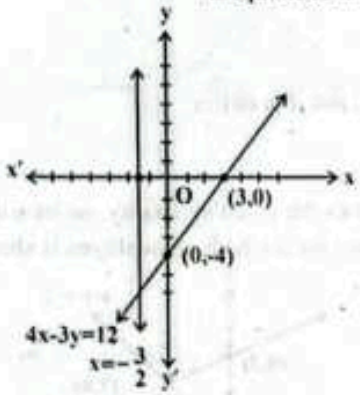
Thus, $x = \frac{-3}{2}$ is a vertical line.

Take (0,0) as a test point, put it in

$0 \ge \frac{-3}{2}$ (true)

Because point (0,0), satisfies the given inequality, so its solution is towards the origin.

So, the common solution for the both inequalities is shaded region.



(v) $3x + 7y \ge 21$, $y \le 4$

Sol. $3x + 7y \ge 21$ ____ (i)

$y \le 4$ _____ (ii)

The associated equation of (i) is:

$3x + 7y = 21$ _____ (ii)

(ii) $x+y \geq 5, -y+x \leq 1$

Sol. $x+y \geq 5$ _____ (i)

$-y+x \leq 1$ _____ (ii)

The associated equation of (i) is:

$x+y=5$ _____ (iii)

For x-intercept, put $y=0$ in (iii), we get

$x+0=5$

$x=+5$.

So, the point is (5,0).

For y-intercept, put $x=0$ in (iii), we get

$0+y=5$

$y=5$

So, the point is (0,5)

Take (0,0) as a test point, put it in (i)

$0+0 \geq 5$

$0 \geq 5$ (false)

Because point (0,0) does not satisfy (i) so, its solution is away from origin.

The assocaited is away from origin

The associated equation of (ii) is:

$-y+x=1$ _____ (iv)

for x-intercept, put $y=0$ in (iv), we get

$-0+x=1$

$x=1$

So, the point is (1,0)

for y-intercept, put $x=0$ in (iv), we get

$-y+0=1$

$-y=1$

$y=-1$

so, the point is (0,–1)

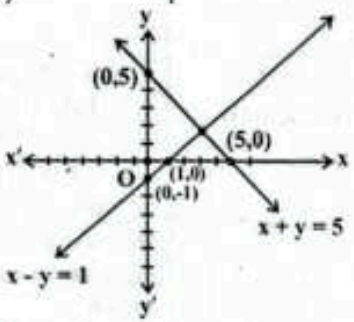
Take (0,0) as a test point, put it in (ii)

$-0+0 \leq 1$

$0 \leq 1$ (true)

Because point (0,0) satisfies the given inequality, so its solution is towards origin.

Thus, the common solution for the both inequalities is shahded region.



(iii) $3x+7y \geq 21, x-y \leq 2$

Sol. $3x+7y \geq 21$ _____ (i)

$x-y \leq 2$ _____ (ii)

The associated equation of (i) is:

$3x+7y=21$ _____ (iii)

For x-intercept, put $y=0$ in (iii), we get

$3x+7(0)=21$

$3x+0=21 = 3x=21$

$x=\frac{21}{3}$

$x=7$

Similarly, line (iv) is graphed by joining the points (6, 0) and (0, 3) by solid line. For $x=0$ and $y=0$, we have:

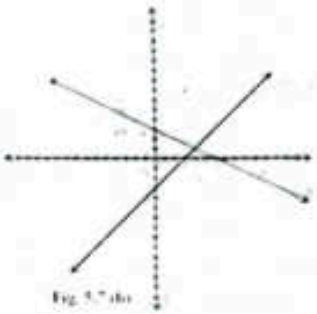
$0-0=0<3$ and $0+2(0)=0<6$

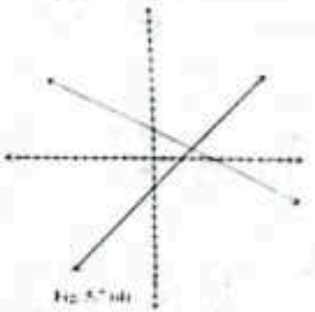
So, both the closed half-planes are on the origin sides of the lines (iii) and (iv). The intersection of these closed half-planes is partially displayed as shaded region in fig. 5.7(a).

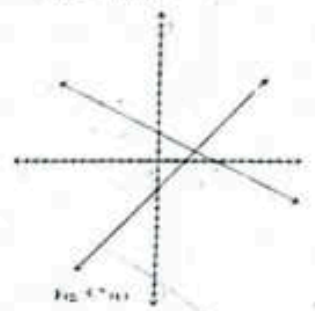
The graph of $y \geq 0$, will be the closed upper half plane.

The intersection of graph shown in figure 5.7(a) and closed upper half plane is partially displayed as shaded region in figure 5.7 (b). The graph of $x \geq 0$ will be closed right half plane.

The intersection of the graph shown in fig. 5.7(a) and closed right half plane is graphed in fig. 5.7 (c).







Finally, the graph of the given system of linear inequalities is displayed in figure 5.7 (d) which is the feasible region for the given system of linear inequalities. The points (0 ,0), (3, 0), (4, 1) and (0, 3) are corner points of the feasible region.

> **Remember!**
> A point of a solution region where two of its boundary lines intersect, is called a **corner point** or **vertex** of the solution region.

Maximum and Minimum Values of a Function in the Feasible Solution

A function which is to be maximized or minimized is called an **objective function.** Note that there are infinitely many feasible solutions in the feasible region. The feasible solution which maximizes or minimizes the objective function is called the **optimal solution.**

Procedure for determining optimal solution

(i) Graph the solution set of linear inequality constraints to determine feasible region.

(ii) Find the corner points of the feasible region.

(iii) Evaluate the objective function at each corner point to find the optimal solution.

Example 7: Find the maximum and minimum values of the function defined as:

$f(x,y)=2x+3y$

subject to the constraints:

$x-y \leq 2$

$x + y \leq 4$

$x \geq 0, y \geq 0$

Solution:

$x - y \leq 2$ ...(i)

$x + y \leq 4$ ...(ii)

The associated equation of (i) is

$x - y = 2$

x-intercept and y-intercept of $x - y = 2$ are (2, 0) and (0,-2) respectively. The graph of the line $x - y = 2$ is drawn by joining the points (2, 0) and (0, –2). The point (0,0) satisfies the inequality $x - y \leq 2$ because $0-0 = 0 < 2$. Thus, the graph of $x - y \leq 2$ is the upper half-plane including the graph of the line $x - y = 2$. The closed half-plane is partially shown by shading in figure 5.8(a).

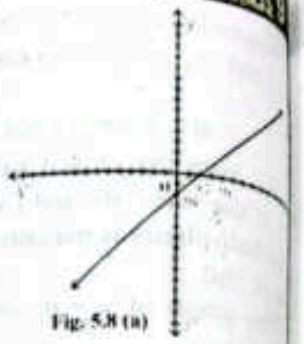


Fig. 5.8 (a)

The associated equation of (ii) is

$x + y = 4$

x-intercept and y-intercept of $x + y = 4$ are (4, 0) and (0,4), respectively the graph of the line $x + y = 4$ is drawn by joining the points (4,0) and (0,4). The point (0,0) satisfies the inequality $x + y \leq 4$. The closed half-plane is partially shown by shading in figure 5.8(b).

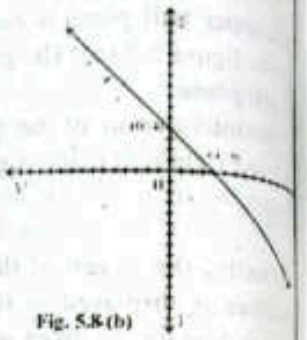


Fig. 5.8 (b)

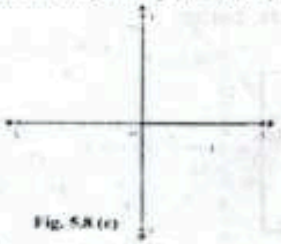


Fig. 5.8 (c)

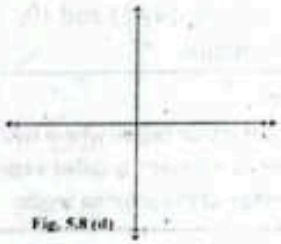


Fig. 5.8 (d)

The graph of $x \geq 0$ and $y \geq 0$ is shown by shading in figures 5.8(c) and 5.8(d) respectively.

The feasible region of the given system of inequalities is the intersection of the graphs indicated in figures 5.8(a), 5.8(b), 5.8(c) and 5.8(d) and is shown as shaded region ABCD in figure 5.9(e).

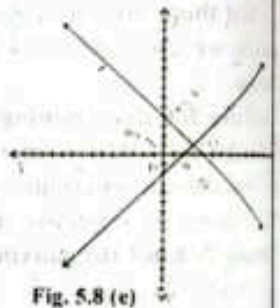


Fig. 5.8 (e)

Corner points of the feasible region are (0,0), (2,0), (3,1) and (0,4). Now, we find values of $f(x,y)=2x+3y$ at the corner points.

$f(0,0) = 2(0) + 3(0) = 0$

$f(2,0) = 2(2) + 3(0) = 4$

$f(3,1) = 2(3) + 3(1) = 9$

$f(0,4) = 2(0) + 3(4) = 12$

For x-intercept, put $y = 0$ in (iii), we get

$3x + 7(0) = 21$

$3x + 0 = 21$

$3x = 21$

$x = \frac{21}{3}$

$x = 7$

So, the point is (7,0)

For y-intercept, put $x = 0$ in (iii), we get.

$3(0) + 7y = 21$

$0 + 7y = 21$

$+7y = 21$

$7y = 21$

$7y = \frac{21}{3}$

$y = 3$

So, the point is (0, –3)

Take (0,0) does not satisfy the given inequality, so its solution is away from origin.

From (ii) associated equation of (ii) is:

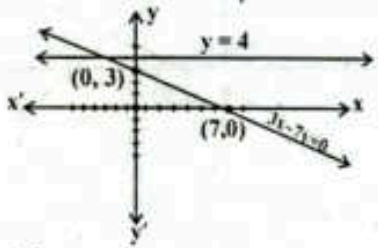
$y = 4$ (It is a horizontal line)

Take (0,0) as a test point, put it in (ii)

$0 \leq 4$ (true)

Because point (0,0) satisfies the given inequality so its solution is towards origin.

Thus, the common solution for the both inequalities is shaded region.



(vi) $5x + 7y \leq 35$, $x - 2y \leq 2$

Sol. $5x + 7y \leq 35$ _____ (i)

$x - 2y \leq 2$ _____ (ii)

The associated equation of (i) is:

$5x + 7y = 35$ _____ (iii)

For x–intercept, put $y = 0$ in (iii), we get

$5x + 7(0) = 35$

$5x + 0 = 35$

$5x = 35$

$x = \frac{35}{5}$

$x = 7$

So, the point is (7,0).

For y-intercept, put $x = 0$ in (iii), we get

$5(0) + 7y = 35$

$0 + 7y = 35$

$7y = 35$

$y = \frac{35}{7}$

$y = 5$

So, the point is (0,5)

Take (0,0) as a test point, put it in (i)

$5(0) + 7(0) \leq 35$

$0 + 0 \leq 35$

$0 \leq 35$ (true)

Because point (0,0) satisfies the given inequality.

So its solution is towards origin

The associated equation of (ii) is:

$x - 2y = 2$ _____ (iv)

For x-intercept, put y = 0 in (iv), we get

$x - 2(0) = 2$

$x - 0 = 2$

$x = 2$

So, the point is (2,0)

For y-intercept, put x = 0 in (iv), we get

$0 - 2y = 2$

$-2y = 2$

$2y = -2$

$y = \frac{-2}{2} \Rightarrow y = -1$

So, the point is (0,–1)

Take (0,0) as a test point, put it in (ii)

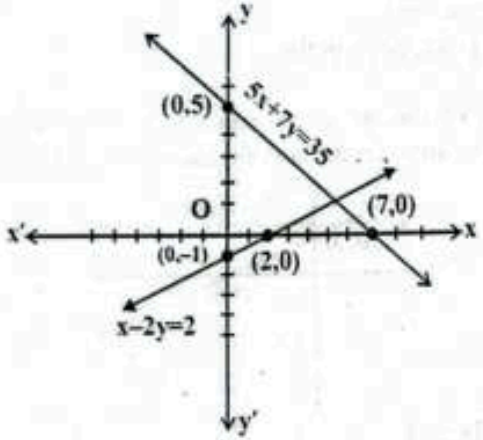
$0 - 2(0) \leq 2$

$0 - 0 \leq 2$

$0 \leq 2$ (true)

because point (0,0) satisfies the given inequality, so its solution is towards origin.

Thus the common solution for the both inequalities is shaded region.



Feasible Solution

A region which is restricted to the first quadrant is referred to as a **feasible region** for the set of given constraints. Each point of the feasible region is called a **feasible solution** of the system of linear inequalities (or for the set of a given constraints).

Example 6: Shade the feasible region and find the corner points for the following system of inequalities (or subject to the following constraints).

$x - y \leq 3$

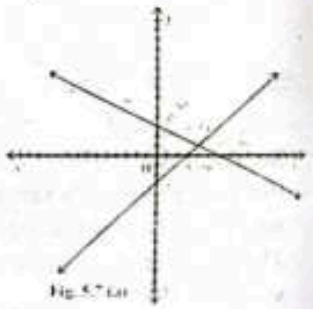
$x + 2y \leq 6, \quad x \geq 0, \quad y \geq 0$

Solution: The associated equations for the inequalities

$x - y \leq 3$ (i) and $x + 2y \leq 6$ (ii)

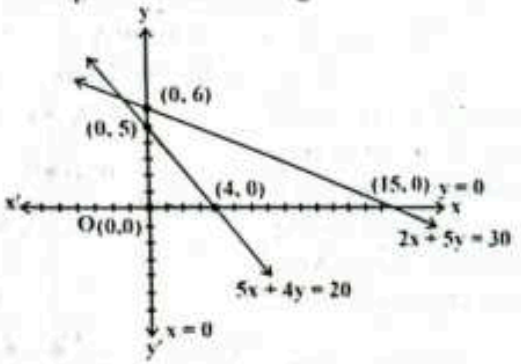
are $x - y = 3$ (iii) and $x + 2y = 6$ (iv)

As the point (3, 0) and (0,–3) are on the line (iii), so the graph of $x - y = 3$ is drawn by joining the points (3, 0) and (0,–3) by solid line.



Because (0,0) satisfies all the give inequalities, so their solution is towards origion. So, the common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x \geq 0, y \geq 0$ so, solution region will be in first quardant.

Now we find the corner points of fesible region.



The corner points of the feasible region for the given system of linear inequalities are (0,0), (4,0) and (0,5).

Now we evaluate the objective function f (x,y) = x + 3y at these corner points.

Corner points	f (x,y) = x + 3y
(0,0)	f (0,0) = 0 + 3(0) = 0
(4,0)	f (4,0) = 4 + 3(0) = 4
(0,5)	f (0,5) = 0 + 3(5) = 15

Hence f is maximum at the corner point (0,5).

3. Maximize z = 2x + 3y; subject to the constraints:

$2x + y \leq 4, \quad 4x - y \leq 4, \quad x \geq 0; y \geq 0$

Sol. $2x + y \leq 4$

$4x - y \leq 4$

$x \geq 0$

$y \geq 0$

The associated equations of the above inequalities are:

$2x + y = 4$ _____ (i)

Put x = 0 in eq (i)

$2(0) + y = 4$

$0 + y = 4$

$y = 4$

∴ (0,4) is a point on the line (i)

Put y = 0 in eq (i)

$2x + (0) = 4$

$2x = 4$

$x = \frac{4}{2}$

$x = 2$

$4x - y = 4$ (ii)

Put x = 0 in eq (ii)

$4(0) - y = 4$

$0 - y = 4$

$-y = 4$

$y = -4$

∴ (0, – 4) is a point on the line (ii)

Put y = 0 in eq (ii)

$4x - (0) = 4$

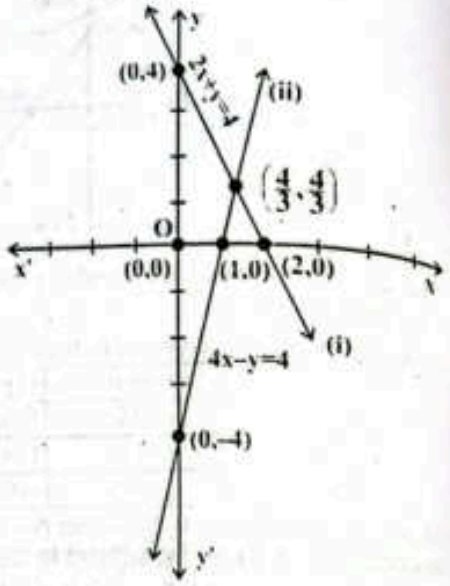
$4x = 4$

$x = \frac{4}{4}$

$x = 1$

$\therefore$ (2,0) is a point on line (i)
Take (0,0) as a test point put it in
$2x + y \leq 4$
$2(0) + 0 \leq 4$
$0 \leq 4$ (True)

$\therefore$ (1,0) is point on line (ii)
Take (0,0) as a test point.
Put it in
$4x - y \leq 4$
$4(0) - 0 \leq 4$
$0 \leq 4$ (True)



Because (0,0) satisfies all given inequalites, so their solution is towards origin. So, the common solution of the given system of inequalites is shown by the shaded region as shown in the figure. As $x \geq 0$, $y \geq 0$ so, solution region will be in first quardrant.

Now we find the corner points of the feasible region.

Solving $2x + y = 4$ and $4x - y = 4$ we get $\left(\frac{4}{3},\frac{4}{3}\right)$ the corner points of the feasible region for the given system of linear inequalites are (0,0), (1,0), (0,4) and $\left(\frac{4}{3},\frac{4}{3}\right)$.

Now we evaluate the objective function $z = f(x,y) = 2x + 3y$ at these corner points.

Corner Points	$z = f(x,y) = 2x + 3y$
(0,0)	$z = f(0,0) = 2(0) + 3(0) = 0$
(1,0)	$z = f(1,0) = 2(1) + 3(0) = 2$
(0,4)	$z = f(0,4) = 2(0) + 3(4) = 12$
$\left(\frac{4}{3},\frac{4}{3}\right)$	$z = f\left(\frac{4}{3},\frac{4}{3}\right) = 2\left(\frac{4}{3}\right) + 3\left(\frac{4}{3}\right) = \frac{20}{3}$
(0,4)	$z = f(0,4) = 2(0) + 3(4) = 0 + 12 = 12$

Hence z is maximum at the corner point (0,4).

4. Minimize $z = 2x + y$; subject to the constraints:

$x + y \geq 3$; $7x + 5y \leq 35$; $x \geq 0$; $y \geq 0$

Sol. $x + y \geq 3$
$7x + 5y \leq 35$
$x \geq 0$
$y \geq 0$
The associated equations of the above inequalities are:

Thus, the minimum value of f is 0 at the corner point (0,0) and maximum value of f is 12 at corner point (0,4)

Solved Exercise 5.2

1. Maximize $f(x, y) = 2x + 5y$; subject to the constraints
$2y - x \leq 8$; $x - y \leq 4$; $x \geq 0$; $y \geq 0$

Sol. $2y - x \leq 8$;
$x - y \leq 4$;
$x \geq 0$; $y \geq 0$
The associated equations of the above inequalities are:

$2y - x = 8$ _____ (i)
Put $x = 0$ in eq (i)
$2y - 0 = 8$
$2y = 8$
$y = \frac{8}{2} = y = 4$

$\therefore$ (0,4) is a point on the line (i)
Put $y = 0$ in eq (i)
$2(0) - x = 8$
$0 - x = 8$
$-x = 8 = x = -8$

$\therefore$ (–8,0) is a point on the line (i) Take (0,0) as a test point.
Put It in
$2y - x \leq 8$
$2(0) - 0 \leq 8$
$0 \leq 8$ (True)

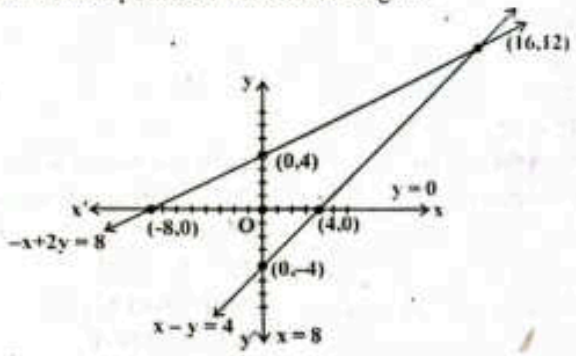
$x - y = 4$ _____ (ii)
Put $x = 0$ in eq (ii)
$0 - y = 4$
$-y = 4$
$y = -4$

$\therefore$ (0,–4) is a point on the line (ii)
Put $y = 0$ in eq (ii)
$x - 0 = 4$
$x = 4$

$\therefore$ (0,4) is a point on the line (ii) Take (0,0) as a test point. Put it in
$x - y \leq 4$
$0 - 0 \leq 4$
$0 \leq 4$ (True)

Because (0,0) satisfies all the given inequalities, so their solution is towards origin. The common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x \geq 0$, $y \geq 0$, Solution region will be in first quadrant.

Now we find the corner points of the feasible region.



Solving $2y - x = 8$ and $x - y = 4$ we get (16,12)

The corner points of the feasible region for the given system of the linear inequa are (0,0), (4,0), (0,4) and (16,12).

Now we evalute the objective function $f(x,y) = 2x + 5y$ at these corner points.

Corner Point	$f(x,y) = 2x + 5y$
(0,0)	$f(0,0) = 2(0) + 5(0) = 0$
(4,0)	$f(4,0) = 2(4) + 5(0) = 8$
(0,4)	$f(0,4) = 2(0) + 5(4) = 20$
(16,12)	$f(16,12) = 2(16) + 5(12) = 92$

Hence, f is maximum at the corner point (16,12).

2. Maximize $f(x, y) = x + 3y$; subject to the constraints

$2x + 5y \le 30$; $5x + 4y \le 20$; $x \ge 0$; $y \ge 0$

Sol. $2x + 5y \le 30$

$5x + 4y \le 20$

$x \ge 0$

$y \ge 0$

The associated equations of the above inequalities are:

$2x + 5y = 30$ _____ (i)

Put $x = 0$ in eq (i)

$2(0) + 5y = 30$

$0 + 5y = 30$

$5y = 30$

$y = \frac{30}{5}$

$y = 6$

$\therefore$ (0,6) is a point on the line (i)

Put $y = 0$ in eq (i)

$2x + 5(0) = 30$

$2x + 0 = 30$

$2x = 30$

$x = \frac{30}{2}$

$x = 15$

$\therefore$ (15,0) is a point on the line (i)

Take (0,0) as a test point.

Put it in

$2x + 5y \le 30$

$2(0) + 5(0) \le 30$

$0 \le 30$ (True)

$5x + 4y = 20$ _____ (ii)

Put $x = 0$ in eq (ii)

$5(0) + 4y = 20$

$0 + 4y = 20$

$4y = 20$

$y = \frac{20}{4}$

$y = 5$

$\therefore$ (0,5) is a point on the line (ii)

Put $y = 0$ in eq (ii)

$5x + 4(0) = 20$

$5x + 0 = 20$

$5x = 20$

$x = \frac{20}{5}$

$x = 4$

$\therefore$ (4,0) is a point on the line (ii)

Take (0,0) as a test point

Put it in

$5x + 4y \le 20$

$5(0) + 4(0) \le 20$

$0 \le 20$ (True)

$x + y = 3$ _____ (i)

Put $x = 0$ in eq (i)

$0 + y = 3$

$y = 3$

$\therefore$ (0,3) is a point on the line (i)

Put $y = 0$ in eq (i)

$x + (0) = 3$

$x = 3$

$\therefore$ (3,0) is a point on the line (i)

Take (0,0) as a test point.

Put in

$x + y \ge 3$

$0 + 0 \ge 3$

$0 \ge 3$ (False)

$7x + 5y = 35$ _____ (ii)

Put $x = 0$ in eq (ii)

$7(0) + 5y = 35$

$0 + 5y = 35$

$5y = 35$

$y = \frac{35}{5}$

$y = 7$

$\therefore$ (0,7) is a point on the line (ii)

Put $y = 0$ in eq (ii)

$7x + 5(0) = 35$

$7x + 0 = 35$

$7x = 35$

$x = \frac{35}{7}$

$x = 5$

$\therefore$ (5,0) is a point on the line (ii)

Take (0,0) as a test point.

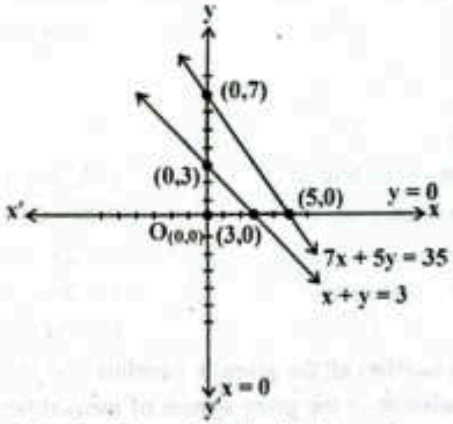
Put it in

$7x + 5y \le 35$

$7(0) + 5(0) \le 35$

$0 \le 35$ (True)

Because (0,0) does not satisfy the inequality (i), so its solution is away from the origin, while (0,0) satisfy the inequality (ii), so its solution is towards origin. As $x \ge 0$, $y \ge 0$ so, solution region will be in first quadrant, Now we find the corner points of the feasible region.



The corner points of the feasible region for the give system of linear inequalities are (0,3), (3,0), (5,0) and (0,7).

Now we evaluate the objective function $z = f(x,y) = 2x + y$ at these corner points

Corner points	$z = f(x,y) = 2x + y$
(0,3)	$z = f(0,3) = 2(0) + (3) = 3$
(3,0)	$z = f(3,0) = 2(3) + (0) = 6$
(5,0)	$z = f(5,0) = 2(5) + (0) = 10$
(0,7)	$z = f(0,7) = 2(0) + (7) = 7$

Hence z is minimum at the corner point (0,3).

5. Maximize the function defined as; $f(x, y) = 2x + 3y$ subject to the constraints

$2x + y \le 8$; $x + 2y \le 14$; $x \ge 0$; $y \ge 0$

Sol. $2x + y \le 8$

$x + 2y \le 14$

$x \ge 0$

$y \ge 0$

The associated equations of the above inequalities are:

$2x + y = 8$ _____ (i)

Put $x = 0$ in eq (i)

$2(0) + y = 8$

$0 + y = 8$

$y = 8$

$\therefore$ (0,8) is a point on the line (i).

Put $y = 0$ in eq (i)

$2x + (0) = 8$

$2x = 8$

$x = \frac{8}{2}$

$x = 4$

$\therefore$ (4,0) is a point on the line (i)

Take, (0,0) as a test point put it in

$2x + y \le 8$

$2(0) + (0) \le 8$

$0 \le 8$ (True)

$x + 2y = 14$ _____ (ii)

Put $x = 0$ in eq (ii)

$0 + 2y = 14$

$2y = 14$

$y = \frac{14}{2}$

$y = 7$

$\therefore$ (0,7) is a point on the line (ii).

Put $y = 0$ in eq (ii)

$x + 2(0) = 14$

$x + 0 = 14$

$x = 14$

$\therefore$ (14,0) is a point on the line (ii).

Take (0,0) as a test point put it in

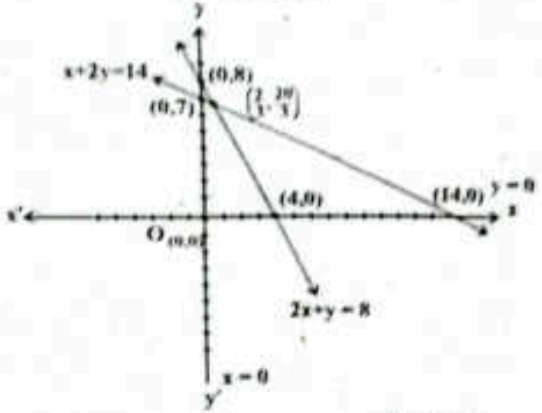
$x + 2y \le 14$

$0 + 2(0) \le 14$

$0 \le 14$ (True)

Because (0,0) satisfies all the given in equalities, so their solution is towards or

So, the common solution of the given system of inequalities is shown by shaded re as shown in figure. A $x \ge 0$, $y \ge 0$ so, solution region will be in first quardant. Now find the corner points of the feasible region.



Solving $2x + y = 8$ and $x + 2y = 14$ we get $\left(\frac{2}{3}, \frac{20}{3}\right)$. The corner points of the feasible region for the given (0,0), (4,0), $\left(\frac{2}{3}, \frac{20}{3}\right)$ and (0,7)

Now we evaluate the objective function $f(x,y) = 2x + 3y$ at these corner points.

Corner points	$f(x,y) = 2x + 3y$
(0,0)	$f(0,0) = 2(0) + 3(0) = 0$
(4,0)	$f(4,0) = 2(4) + 3(0) = 8$
$\left(\frac{2}{3}, \frac{20}{3}\right)$	$f\left(\frac{2}{3}, \frac{20}{3}\right) = 2\left(\frac{2}{3}\right) + 3\left(\frac{20}{3}\right) = \frac{4}{3} + \frac{60}{3} = \frac{64}{3}$
(0, 7)	$f(0, 7) = 2(0) + 3(7) = 0 + 21 = 21$

Henc z is minimum at corner point $\left(\frac{2}{3}, \frac{20}{3}\right)$.

6. Find minimum and maximum values of $z = 3x + y$; subject to the constraints:

$3x + 5y \ge 15$; $x + 6y \ge 9$; $x \ge 0$; $y \ge 0$

Sol. $3x + 5y \ge 15$

$x + 6y \ge 9$

$x \ge 0$

$y \ge 0$

The associated equation of the above inequalities are:

$3x + 5y = 15$ _____ (i)

Put $x = 0$ in eq (i)

$3(0) + 5y = 15$

$0 + 5y = 15$

$5y = 15$

$y = \frac{15}{5}$

$y = 3$

$x + 6 = 9$ _____ (ii)

Put $x = 0$ in eq (ii)

$0 + 6y = 9$

$6y = 9$

$y = \frac{9}{6}$

$y = \frac{3}{2}$

$\therefore$ (0,3) is a point on line (i)

Put y = 0 in eq (i)

$3x + 5(0) = 15$

$3x + 0 = 15$

$3x = 15$

$x = \frac{15}{3}$

$x = 5$

$\therefore$ $\left(0, \frac{3}{2}\right)$ is a point on line (ii)

Put y = 0 in eq (ii)

$x + 6(0) = 9$

$x + 0 = 9$

$x = 9$

$\therefore$ (5,0) is a point on line (i)

Take (0,0) as a test point.

Put it in

$3x + 5y \geq 15$

$3(0) + 5(0) \geq 15$

$0 \geq 15$ (False)

$\therefore$ (9,0) is a point on line (ii)

Take (0,0) as a test point.

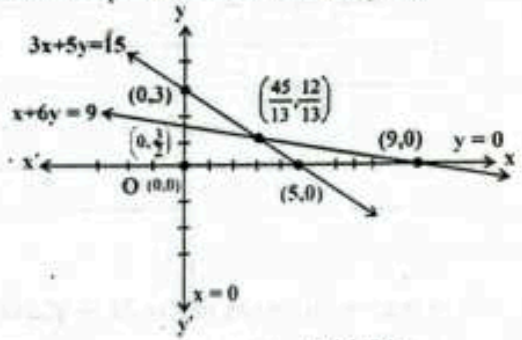
Put it in

$x + 6y \geq 9$

$0 + 6(0) \geq 9$

$0 \geq 9$ (False)

Because (0,0) does not satisfy all the given inequalities, so their solution is from origin. So, the common solution of given system of inequalities is shown by shaded region as shown in the figure. As $x \geq 0$, $y \geq 0$ so, solution region will be in quardant. Now we find corner points of the feasible region.



Solving $3x + 5y = 15$ and $x + 6y = 9$ we get $\left(\frac{45}{13}, \frac{12}{13}\right)$

The corner points of the feasible region for the given system of linear in equ are (0,3), $\left(\frac{45}{13}, \frac{12}{13}\right)$ and (9,0).

Now we evaluate the objective function $z = f(x,y) = 3x + y$ at these corner poin

Corner Points	$z = f(x,y) = 3x+y$
(0,3)	$z = f(0,3) = 3(0) + 3 = 3$
$\left(\frac{45}{13}, \frac{12}{13}\right)$	$z = f\left(\frac{45}{13}, \frac{12}{13}\right) = 3\left(\frac{45}{13}\right) + \frac{12}{13} = \frac{147}{13}$
9,0	$z = f(9,0) = 3(9) + 0 = 27$

Hence z is minimum at the corner point (0,3), and (9, 0) is maximum point.

Review Exercise 5

Four options are given against each statement. Encircle the correct one.

In the following, linear equation is:

(a) $5x > 7$ (b) $4x - 2 < 1$ (c) $2x + 1 = 1$ (d) $4 = 1 + 3$

Solution of $5x - 10 = 10$ is:

(a) 0 (b) 50 (c) 4 (d) –4

If $7x + 4 < 6x + 6$, then x belongs to the interval.

(a) $(2, \infty)$ (b) $[2, \infty)$ (c) $(-\infty, 2)$ (d) $(-\infty, 2]$

A vertical line divides the plane into

(a) left half plane (b) right half plane (c) full plane (d) two half plane

The equation formed the linear inequality is called

(a) linear equation (b) associated equation

(c) quadratic equal (d) none of these

$3x + 4 < 0$ is:

(a) equation (b) inequality (c) not inequality (d) identity

Corner point is also called:

(a) code (b) vertex (c) curve (d) region

(0,0) is solution of inequality:

(a) $4x + 5y > 8$ (b) $3x + y > 6$ (c) $-2x + 3y < 0$ (d) $x + y > 4$

The solution region restricted to the first quadrant is called:

(a) objective region (b) feasible region

(c) solution region (d) constraints region

A function that is to be maximized or minimized is called:

(a) solution function (b) objective function

(c) feasible function (d) none of these

nswers:

2x + 1 = 1	ii. 4	iii. $(-\infty, 2)$	iv. two half plane
associated equation	vi. inequality	vii. vertex	viii. $-2x + 3y < 0$
feasible region	x. objective function		

Solve and represent their solutions on real line.

$\frac{x+5}{3} = 1 - x$

$\frac{x+5}{3} = 1 - x$

$3 \times \frac{(x+5)}{3} = 3(1-x)$

$x + 5 = 3 - 3x$

$x + 3x = 3 - 5$

$4x = -2$

$\frac{-1+10}{\frac{2}{3}} = 1 + \frac{1}{2}$

$\frac{9}{\frac{2}{3}} = \frac{2+1}{2}$

$$x = -\frac{2}{4}$$

$$x = -\frac{1}{2}$$

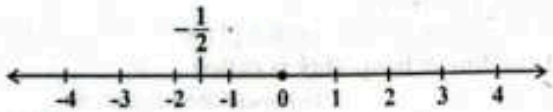
Check put $x = -\frac{1}{2}$ into the original equation

$$\frac{-\frac{1}{2}+5}{3} = 1-\left(-\frac{1}{2}\right)$$

$$\frac{\cancel{9}}{2}\times\frac{1}{\cancel{3}_1} = \frac{3}{2}$$

$$\frac{3}{2} = \frac{3}{2}$$

So, $x = -\frac{1}{2}$ is solution.



(ii) $\frac{2x+1}{3}+\frac{1}{2}=1-\frac{x-1}{3}$

Sol. $\frac{2x+1}{3}+\frac{1}{2}=1-\frac{x-1}{3}$

$$\frac{2x+1}{3}+\frac{1}{2} = 1-\frac{x-1}{3}$$

$$\frac{2(2x+1)+3(1)}{6} = \frac{3-x+1}{3}$$

$$\frac{4x+5}{6} = \frac{4-x}{3}$$

$$3(4x+5) = 6(4-x)$$

$$12x+15 = 24-6x$$

$$12x+6x = 24-15$$

$$18x = 9$$

$$x = \frac{\cancel{9}^{1}}{\cancel{9}^{2}}$$

$$x = \frac{1}{2}$$

Check: Put $x = \frac{1}{2}$ in original equation

$$\frac{2\left(\frac{1}{2}\right)+1}{3}+\frac{1}{2} = 1-\frac{\frac{1}{2}-1}{3}$$

$$\frac{1+1}{3}+\frac{1}{2} = 1-\frac{1-2}{3}$$

$$\frac{2}{3}+\frac{1}{2} = 1-\frac{1-2}{3}$$

$$\frac{2(2)+3(1)}{6} = 1-\frac{-1}{3}$$

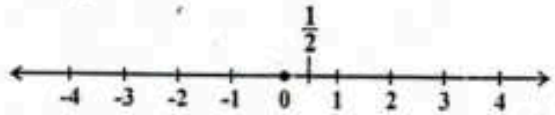
$$\frac{4+3}{6} = 1-\left(-\frac{1}{2}\times\frac{1}{3}\right)$$

$$\frac{7}{6} = 1+\frac{1}{6}$$

$$\frac{7}{6} = \frac{6+1}{6}$$

$$\frac{7}{6} = \frac{7}{6}$$

So, $x = \frac{1}{2}$ is solution



(iii) $3x+7<16$

Sol. $3x+7<16$

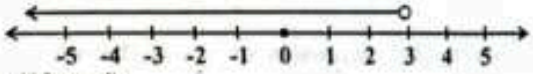
$3x<16-7$

$3x<9$

$x<\frac{9}{3}$

$x<3$

It means all the real numbers less than 3 are the solution of this inequality.



(iv) $5(x-3)\geq 26x-(10x+4)$

Sol. $5(x-3)\geq 26x-(10x+4)$

$5x-15\geq 26x-10x-4$

$5x-15\geq 16x-4$

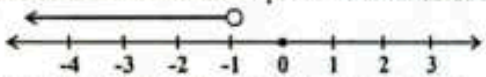
$5x-16x\geq -4+15$

$-11x\geq 11$

$x\leq \frac{-11}{11}$

$x\leq -1$

It mean all the real number less than or equalto -1 is the solution of this inequality.



3. Find the solution region of the following linear equalities.

(i) $3x-4y\leq 12$; $3x+2y\geq 3$

Sol. $3x-4y\leq 12$ _____ (i)

$3x+2y\geq 3$ _____ (ii)

The associated equation of (i) is

$3x-4y=12$ _____ (iii)

For y-intercept, put $x=0$ (iii), we get:

$3(0)-4y=12$

$0-4y=12$

$-4y=12$

$y=\frac{12}{-4}$

$y=-3$

So, the point is $(0,-3)$

For x-intercept, put $y=0$ (iii), we gets

$3x-4(0)=12$

$3x-0=12$

$3x=12$

$$x = \frac{12}{3}$$

$$x = 4$$

So, the point is (4,0)

Take (0,0) as test point, put in (i)

$3(0 - 4(0) \leq 12$

$0 - 0 \leq 12$

$0 \leq 12$ (true)

Because point (0,0), satisfies the given inequality so its solution is towards the origin.

The associated equation of (ii) is:

$3x + 2y = 3$ _____ (iv)

For y-intercept, put $x = 0$ in (iv), we get:

$3(0) + 2y = 3$

For y-intercept, put $x = 0$ in (iv), we get:

$3(0) + 2y = 3$

$0 + 2y = 3$

$2y = 3$

$$y = \frac{3}{2}$$

So, the point is $\left(0, \frac{3}{2}\right)$

For x-intercept, put $y = 0$ in (iv), we get.

$3x + 2(0) = 3$

$3x + 0 = 3$

$3x = 3$

$$x = \frac{3}{3}$$

$x = 1$

So, the point is (1,0).

Take (0,0) as a test point put in (ii)

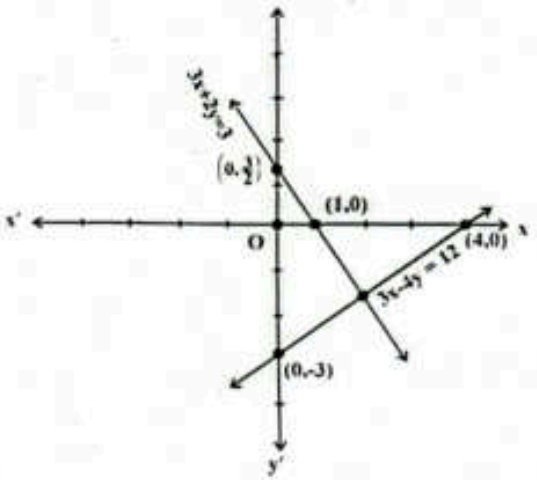
$3(0) + 2(0) \geq 3$

$0 + 0 \geq 3$

$0 \geq 3$ (False)

Because point (0,0) does not satisfy the given inequality so, its solution is away from origin.

The solution of both inequality equations is shown by shaded area in figure.



$2x + y \leq 4$; $x + 2y \leq 6$

$2x + y \leq 4$ _____ (i)

$x + 2y \leq 6$ _____ (ii)

The associated equation of (i) is:

$2x + y = 4$ _____ (iii)

For y-intercept, put $x = 0$ in (iii), we get:

$2(0) + y = 4$

$0 + y = 4$

$y = 4$

So, the point is (0,4)

For x-intercept, put $y = 0$ in (iii), we get:

$2x + (0) = 4$

$2x = 4$

$$x = \frac{4}{2}$$

$x = 2$

So, the point is (2,0).

Take (0,0) as a test point. Put in (i)

$2(0) + (0) \leq 4$

$0 + 0 \leq 4$

$0 \leq 4$ (True)

Because point (0,0) satisfies the given inequality so its solution is towards origin.

The associated equation of (ii) is:

$x + 2y = 6$ _____ (iv)

For y-intercept, put $x = 0$ in eq (iv), we get:

$(0) + 2y = 6$

$2y = 6$

$y = \frac{6}{2}$

$y = 3$

So, the point is (0,3)

For x-intercept, put $y = 0$ in eq (iv), we get:

$x + 2(0) = 6$

$x + 0 = 6$

$x = 6$

So, the point is (6,0)

Take (0,0) as test point. Put in (ii)

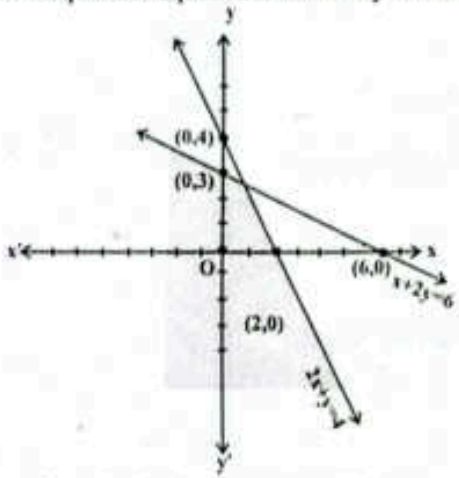
$(0) + 2(0) \leq 6$

$0 + 0 \leq 6$

$0 \leq 6$ (True)

Because point (0,0) satisfies the given inequality so its solution is towards origin.

The solution of both inequalities equation is shown by shaded area in the figure.



4. Find the maximum value of g (x,y) = x + 4y subject to constraints

$x + y \leq 4$, $x \geq 0$ and $y \geq 0$.

Sol. $x + y \leq 4$

$x \geq 0$

$y \geq 0$

The associated equation of the inequality is

$x + y = 4$ _____(i)

Put $x = 0$ in eq (i)

$0 + y = 4$

$y = 4$

So, (0,4) is the point

Put $y = 0$ in eq (i)

$x + 0 = 4$

$x = 4$

So, (4,0) is the point

Take (0,0) as a test point and put in $x+y \leq 4$

$0 + 0 \leq 4$

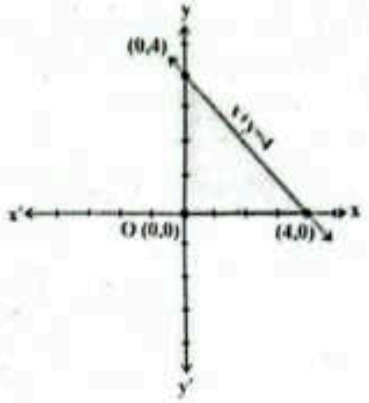
$0 \leq 4$ (True)

As (0,0) satisfies the inequality, so its solution is towards origin. Solution of the given system of inequality is shown in the figure. As $x \geq 0$ and $y \geq 0$, so solution will be in first quardant.

Now we find the corner point of the feasible region.

The corner points of the feasible region for the given system of linear inequlity are (4,0), (0,0) and (0,4)

Now we evaluate the objective function $g(x,y) = x + 4y$ at these corner points:



Corner points	g (x,y) = x + 4y
(0,0)	g (0,0) = (0) + 4(0) = 0
(4,0)	g (4,0) = (4) + 4(0) = 4
(0,4)	g (0,4) = (0) + 4(4) = 16

Hence g is maximum at the corner (0,4).

5. Find the minimum value of f (x,y) = 3x + 5y subject to constraints

$x + 3y \geq 3$, $x + y \geq 2$, $x \geq 0$, $y \geq 0$.

Sol. $x + 3y \geq 3$

$x + y \geq 2$

$x \geq 0$

$y \geq 0$

The associate equation of both inequalities are:

$x + 3y = 3$ ___ (i)

Put $x = 0$ in eq (i)

$(0) + 3y = 3$

$3y = 3$

$y = \frac{3}{3}$

$y = 1$

So (0,1) is a point on the line (i)

$x + 3(0) = 3$

$x + 0 = 3$

$x = 3$

So, (3,0) is a point on the line (i)

Take (0,0) as a test point and put in

$x + 3y \geq 3$

$0 + 3(0) \geq 3$

$0 \geq 3$ (false)

$x + y = 2$ _____ (ii)

Put $x = 0$ in eq (ii)

$0 + y = 2$

$y = 2$

So (0,2) is a point on the line (ii)

Put $y = 0$ in eq (ii)

$x + 0 = 2$

$x = 2$

So, (2,0) is a point on the line (ii)

Take (0,0) as a test point and put in $x+y \geq 2$

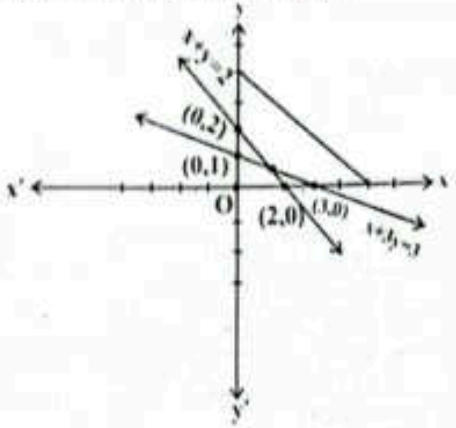
$x + y \geq 2$

$0 + 0 \geq 2$

$0 \geq 2$ (false)

Because (0,0) does not satisfies all the given inequalities, so their solution is away from origin. So, the common solution of the given system of inequalities is shown by the shaded region as shown in the figure. As $x \geq 0$ and $y \geq 0$ so, solution will be in first quardant.

Now we find the corner point of the feasible region.



Solving $x + 3y = 3$ and $x + y = 2$ we get $\left(\frac{3}{2}, \frac{1}{2}\right)$.

The corner points of the feasible region for the given system of linear inequalities are $(0,2), \left(\frac{3}{2}, \frac{1}{2}\right), (3,0)$.

Now we evaluate the objective function $f(x,y) = 3x + 5y$ at these corner points.

Corner points	$f(x,y) = 3x + 5y$
$(0,2)$	$f(0,2) = 3(0) + 5(2)$ $= 10$
$\left(\frac{3}{2}, \frac{1}{2}\right)$	$f\left(\frac{3}{2}, \frac{1}{2}\right) = 3\left(\frac{3}{2}\right) + 5\left(\frac{1}{2}\right)$ $= \frac{9}{2} + \frac{5}{2}$ $= \frac{14}{2} = 7$
$(3,0)$	$f(3,0) = 3(3) + 5(0)$ $= 9$

Hence f is minimum at the corner $\left(\frac{3}{2}, \frac{1}{2}\right)$

Objective Type Questions

Additional MCQ's

Multiple Choice Questions prepared in the light of new Examination Techniques (Knowledge, Understanding, Application, Analytical & Conceptual)

5.1 Linear Equation

Encircle the correct option.

1. **Linear equation is of the form:**
   (a) $ax + b = 0, a \neq 0$ (b) $ax + b = 0\ a = 0$
   (c) $ax^2 + bx = 0, a \neq 0$ (d) $ax + b = 0, a \neq 0$
2. **The highest power of variable in linear equation is:**
   (a) 0 (b) 1 (c) 2 (d) 3
3. **The solution of $3x - 5 = 7$ is:**
   (a) 2 (b) 3 (c) 4 (d) 5
4. **A linear equation in one variable has only _____ solution.**
   (a) one (b) two (c) three (d) four
5. **If $\frac{x-2}{5} - \frac{x-4}{2} = 2$ then x = _____**
   (a) $\frac{-4}{3}$ (b) $\frac{-4}{5}$ (c) $\frac{-4}{7}$ (d) $\frac{-4}{9}$

5.2 Linear Inequalities

6. **How many inequalities symbols are there:**
   (a) two (b) three (c) four (d) five
7. **What is the symbol of "greater than"**
   (a) < (b) > (c) ≈ (d) ≠
8. **What is the symbol of "less than"**
   (a) < (b) > (c) = (d) ≠
9. **What is the symbol of "greater than or equal to"**
   (a) ≠ (b) ≈ (c) ≤ (d) ≥
10. **What is the symbol of "less than or equal to"**
    (a) ≈ (b) ≠ (c) ≤ (d) ≥
11. **The value of x if $\frac{2}{3}x - 1 < 0$**
    (a) $x < \frac{3}{2}$ (b) $x < \frac{2}{3}$ (c) $x < \frac{5}{2}$ (d) $x < \frac{4}{5}$

12. A solution of a linear in equalities in x and y in an ordered pair of nu which _____ the inequality.
(a) not satisfy (b) satisfy (c) both a and b (d) none of abo

13. There are infinitely many ordered pairs that satisfy the inequality $x + 2$, so its graph will
(a) half plane (b) full plane (c) one third plane (d) quarter plan

14. The linear equation _____ is called associated or corresponding equati
(a) $ax + b = 0$ (b) $ax + by = c$ (c) $ax^2 + by^2 = 0$ (d) none of abo

15. The graphs of $x + 2y < 6$ and $x + 2y = 6$ are:
(a) closed half planes (b) open half planes
(c) closed full planes (d) open full planes

5.3 Feasible Solution

16. The system of linear inequalities involved in the problem concerned is call
(a) problem constraints (b) equation
(c) problem solving (d) problem creation

17. A point of a solution region where two of its boundary lines interact is call
(a) half point (b) full point (c) corner point (d) center point

18. A function which is to be maximized is called:
(a) objective function (b) surjective function
(c) null function (d) none of above

19. The feasible solution which maximized the objective function is called:
(a) perfect solution (b) optimal solution (c) null solution (d) false solution

20. A horizontal line divides the plane in:
(a) two half plane (b) two left plane (c) two right plane (d) none of abov

Answers:

1. $ax + b = 0, a \neq 0$	2. 1	3. 4	4. one	
5. $\frac{-4}{3}$	6. four	7. $>$	8. $<$	9. $\geq$
10. $\leq$	11. $x < \frac{3}{2}$	12. satisfy	13. half plane	14. $ax + by =$
15. closed half planes		16. problem constraints		17. corner poi
18. objective function		19. optimal solution		20. two half pl

Conceptual MCQ's

O Encircle the correct option.

1. Linear equation means:
(a) $ax + b = 0$ (b) $ax^2 + by = 0$ (c) $ax^2 + bx = 0$ (d) $ax^2 + by^2 = 0$

2. The highest power of variable is always 1 in which equation:
(a) Linear (b) quadratic (c) Polynomial (d) None of abov

3. Why we check the solution after solving equation?
(a) For contradiction (b) For accuracy
(c) For deletion (d) For extension

4. Associated or corresponding equation is also called:
(a) $ax^2 + by^2 = c^2$ (b) $a + b = c$ (c) $ax + by = c$ (d) $ab + xy = c$

5. Corner point means:
(a) two boundary lines Intersects (b) three points meet
(c) two points are parallel (d) None of above

6. For $x > 1$ which is correct:
(a) $(0, 1)$ (b) (∞, ∞) (c) $(0, \infty)$ (d) $(1, \infty)$

7. For $x < 1$ which is sure?
(a) $(-1, \infty)$ (b) $(10, \infty)$ (c) $(-\infty, 1)$ (d) $(1, \infty)$

8. For $x \geq 1$ which is true?
(a) $1 < x < \infty$ (b) $1 \leq x < \infty$ (c) $-\infty < x < 1$ (d) $-\infty < x \leq 1$

9. For $x \leq 1$ which is interval is true?
(a) $-\infty < x \leq 1$ (b) $1 \leq x < \infty$ (c) $-\infty < x < 1$ (d) $1 \leq x < \infty$

10. A _____ line divides the plane into left and right half planes:
(a) horizontal (b) vertical (c) origin (d) None of above

Answers:

1. $ax + b = 0$ 2. Linear 3. For accuracy 4. $ax + by = c$
5. two boundary lines Intersects 6. $(1, \infty)$ 7. $(-\infty, 1)$ 8. $1 \leq x < \infty$
9. $-\infty < x \leq 1$ 10. vertical

Additional Short Questions

Short Answer Questions prepared in the light of new Examination Techniques (Knowledge, Understanding, Application, Analytical & Conceptual)

5.1 Linear Equation

1. How linear equation and inequalities helps us?

Ans. Linear equations and in equalities are widely used in various field to model and solve real would problem. They help in understanding relationship between variables and making decisions.

2. What do you know about linear equation?

Ans. An equation of the form $ax + b = 0$ where a, b are constant, $a \neq 0$ and x is variable is called linear equation in one variable. In linear equation the highest power of variable is 1.

3. Solve $\frac{x-2}{5} - \frac{x-4}{2} = 2$

Sol. $\frac{x-2}{5} - \frac{x-4}{2} = 2$

$$\frac{2(x-2)-5(x-4)}{10}=2$$

$$2x-4-5x+20=20$$

$$-3x-4=0 \Rightarrow -3x=4 \Rightarrow x=\frac{-4}{3}$$

4. How many solutions of linear equation in one variable?

Ans. A linear equation in one variable has only one solution.

5. Why we check the solution after solving the linear equation.

Ans. We check the solution after solving linear equation to answer the accuracy work.

6. Define transposition.

Ans. Moving terms from one side of the equation to the other to solve for a va called transposition.

5.2 Linear Inequalities

7. In equalities are expressed by how many symbols?

Ans. In equalities are expressed by the following four symbols. $>$, $<$, $\geq$, $\leq$

8. How order of an inequality is changed?

Ans. The order of an inequality is changed by multiplying or dividing it, each sid negative constant.

9. Who introduced inequality signs?

Ans. The inequality signs $<$ and $>$ were introduced by English mathematician T Harriot in the early 17th century.

10. Find value of x if $\frac{2}{3}x-1<0$.

Sol. $\frac{2}{3}x-1<0$

$$\frac{2}{3}x-<1$$

$$2x<3$$

$$x<\frac{3}{2}$$

11. Write the forms of linear inequality in two variables x and y.

Sol. $ax+by<c$, $ax+by>c$

$ax+by\leq c$, $ax+by\geq 0$

12. What do you mean by solution?

Ans. A solution of linear inequality in x and y is an ordered pair of numbers satisfy the inequality,

13. What is associated equation?

Ans. The linear equation $ax+by=c$ is called associated or corresponding equation.

14. What happened when vertical and non-vertical line divide the plane?

Ans. A vertical line divides the plane into left and right half planes while a non-vertical line divides the plane into upper and lower half planes.

15. Write the procedure for graphing a linear inequality is two variables.

Ans. (i) The corresponding equation of the inequality is first graphed by using dashes if the inequality involves the symbols $>$ or $<$ and a solid line is drawn if the in equality involves the symbols $\geq$ or $\leq$.

(ii) A test point is chosen which determines that the half plane is an which side of the boundary line.

16. What do you mean by solution region?

Ans. The graph of system of linear inequality assist of the set of all ordered pairs (x, y) in the xy - plane which simultaneously satisfy all the inequalities in the system. To find the graph of such a system, we draw the graph of each inequality is the system and the same co-ordinate axis and then take intersection of all the graphs. The common region so obtained is called the solution region for the system of inequality.

5.3 Feasible Solution

17. What is problem constraint?

Ans. When we tackle a certain problem from every day life each linear inequality concerning the problem is named as problem constraint. The system of linear inequalities involved in a problem concerned in called problem constraints.

18. What are non-negative constraints?

Ans. The variables used in the system of linear inequalities relating the problems of everyday life are non-negative and are called non-negative constraints.

19. What is decision variables?

Ans. Non-negative constraints play an important role. For taking decision. These variables are called decision variables.

20. What is feasible region?

Ans. A region which is restricted to the first quadrant is referred to as a feasible region for the set of given constraints.

21. What is feasible solution?

Ans. Each point of the feasible region is called feasible solution of the system of linear inequalities.

22. What is corner paint?

Ans. A point of a solution region where two of its boundary lines intersect is called corner point or vertex of the solution region.

23. What is objective function?

Ans. A function which is to be maximized of minimized is called objective function.

24. What is optimal solution?

Ans. The feasible solution which maximizes or minimizes the objective function is called optimal solution.

25. Write the procedure for determining optimal solution.

Ans. (i) Graph the solution set of linear inequalities constraints to determine feasible region.

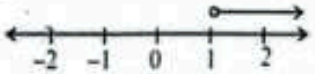
(ii) Find the corner points of the feasible.

(iii) Evaluate the objective function at each corner point to find the optimal solution.

Conceptual Short Questions

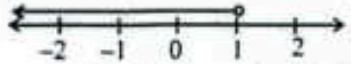
1. According to number line write in in-equailty form.

Ans. $x > 1$



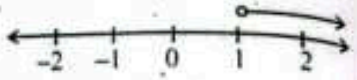
2. Represent $x < 1$ to number line.

Ans. $x < 1$



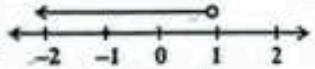
3. Represent $x \geq 1$ to numbers line.

Ans. $x \geq 1$



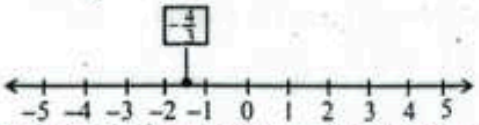
4. Represent $x \leq 1$ to number line.

Ans.



5. Represent $x = -\frac{4}{3}$ to number line.

Ans. $x = -\frac{4}{3}$



6. What is highest power of variable of linear equation?

Ans. The highest power of variable of linear equation is 1.

7. What happened in transposition?

Ans. Moving terms from one side of the equation to the other to solve for the variable.

8. How many feasible solutions in the feasible region?

Ans. There are infinitely many feasible solutions in feasible region.

9. Define optional solution.

Ans. The feasible solution which minimizes the objective function is called optimal solution.

10. What do you mean by finding the value of variable?

Ans. Solving a linear equation in one variable.

— ✲✲✲ —

Unit 6

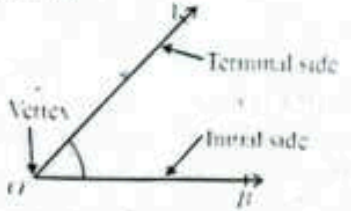
Trigonometry

Trigonometry

Trigonometry is a branch of mathematics that deals with the relationships between angles and sides of triangles, especially right-angled triangles.

Identifying Angles in Standard Position (Degrees and Radians)

Angle: A plane figure which is formed by two rays sharing common end point is called on angle. $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are rays and angle is AOB. Writen as $m\angle AOB$ or AOB



The angle is said to be in standard position if:

(a) Its **vertex** is located at the origin of the coordinate plane.

(b) One of its rays (the **initial side**) lies along the positive x-axis.

(c) The other ray (the **terminal side**) determines the direction of the angle.

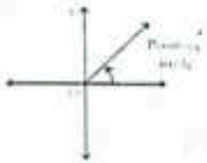
An angle is measured from the initial side to the terminal side. It is usually represented by Greek letters θ, β, γ etc.

Types of angles are:

- Acute angle $0 < \theta < 90°$
- Obtuse angle $90° < \theta < 180°$
- Right angle $\theta = 90°$
- Straight angle $\theta = 180°$
- Reflex angle $180° < \theta < 360°$
- Full rotation $\theta = 360°$

Positive angles

The angle will be positive if the terminal side is rotated counterclockwise from the initial side.



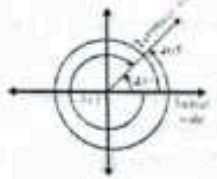
Negative angles

The angle will be negative if the terminal side is rotated clockwise from the initial side.



Co-Terminal Angles

Co-terminal angles are angles that share the same initial side and terminal side in standard position, but they may have different measures. These angles differ by a multiple of 360° or 2π rad. For example, 45°, 405° and −315° are co-terminal angles because $405° = 45° + 360°$ and $-315° = 45° - 360°$.



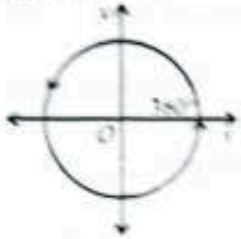
Degree Measurement

Degree

A degree (°) is a unit of measurement of angles. It represents one $\frac{1}{360}$ of a full rotation around a point.

Full Circle

A full rotation around a central point forms an angle of 360°.



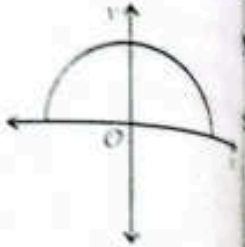
Right Angle

One-quarter of a full rotation, or a angle, is called a right angle.



Half Circle

A straight angle, or half of a full rotation, measures 180°.



Degrees, Minutes, Seconds

The degree measure is further divided into minutes (′) and seconds (″).

1° = 60′ (60 minutes)

1′ = 60″ (60 seconds)

1° = 3600″ (60 × 60 seconds)

Converting Degrees to Minutes and Seconds

To convert decimal degrees to degrees, minutes, and seconds (DMS), follow steps:

- Separate the whole number part (degrees) of the decimal.
- Multiply the decimal part by 60 to get the minutes.
- The whole number part of the result is the minutes. Multiply the decimal pa the minutes by 60 to get the seconds.

Example 1: Convert 73.12° to degrees, minutes, and seconds.

Solution:

Degrees: The whole number part is 73°.

Minutes: Take the decimal part (0.12) and multiply by 60: 0.12 × 60 = 7.2. The wh number part is 7, so it's 7 minutes.

Seconds: Now take the decimal part (0.2) and multiply by 60: 0.2 × 60 = 12. So, t seconds. Final result: 73° 7′ 12″

Example 2: Convert 109.42° to degrees, minutes, and seconds.

Solution:

Degrees: The whole number part is 109°.

Minutes: Take the decimal part (0.42) and multiply by 60: 0.42 × 60 = 25.2. The wh number part is 25, so it's 25 minutes.

Seconds: Now take the decimal part (0.2) and multiply by 60: 0.2 × 60 = 12. So, seconds are 12 seconds.

Final result: 109° 25′ 12″.

Converting from Degrees, Minutes, and Seconds to Decimal Degrees

To convert from degrees, minutes, and seconds (DMS) to decimal degrees, follow steps:

- Keep the degrees as they are.
- Convert minutes to decimal degrees: Divide the number of minutes by 60.
- Convert seconds to decimal degrees: Divide the number of seconds by 3600.
- Add all the values together.

Example 3 Convert 45° 45′ 45″ to decimal degrees.

Solution: Degrees: Keep 45.

Minutes to decimal: $\frac{45}{60} = 0.75$

Seconds to decimal: $\frac{45}{3600} = 0.0125$

Add them together: 45 + 0.75 + 0.0125 = 45.7625

Final result: 45.7625°

Example 4: Convert 94° 27′ 54″ to decimal degrees.

Solution: Degrees: Keep 94.

Minutes to decimal: $\frac{27}{60} = 0.45$

Seconds to decimal: $\frac{54}{3600} = 0.015$

Add them together: 94 + 0.45 + 0.015 = 94.465

Final result: 94.465°

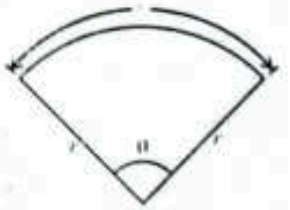
Circular Measure (Radian)

There is another system of angular measurement called circular system.

The **radian**, used in many areas of mathematics.

A **radian** is "the angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle".

In a circle with a radius r, and an arc length is equal to the radius of the circle, then the angle θ subtended by that arc is 1 radian.



$\theta = \frac{r}{r} = 1$ radian $\quad \therefore \quad \theta = \frac{\text{Arc length}}{\text{Radius}} = \frac{r}{r}$

A complete circle has an arc length equal to the circumference ($2\pi r$), so the angle subtended by the entire circle (the full rotation) is 2π radians. This means:

- One full revolution of a circle is 2π radians, or 360°.
- Therefore, 1 radian = $\frac{360^\circ}{2\pi} \approx 57.2958^\circ$ and $1^\circ = \frac{2\pi}{360} = 0.01745$ rad

Conversation between degrees and radians.

Radians to Degrees: 1 rad $= \frac{180^\circ}{\pi}$ degrees

Degrees to Radians: $1^\circ = \frac{\pi}{180^\circ}$ rad

Example 5: Convert radians to degree

(i) $\frac{5\pi}{3}$ rad (ii) $\frac{7\pi}{6}$ rad (iii) $\frac{11\pi}{6}$ rad (iv) 1.2 rad

Solution: (i) $\frac{5\pi}{3}$ rad $= \frac{5\pi}{3} \times \frac{180^\circ}{\pi} = 300^\circ$ $\left(1 \text{ rad } \frac{180^\circ}{\pi} \text{ degrees}\right)$

(ii) $\frac{7\pi}{6}$ rad $= \frac{7\pi}{6} \times \frac{180^\circ}{\pi} = 210^\circ$

(iii) $\frac{11\pi}{6}$ rad $= \frac{11\pi}{6} \times \frac{180^\circ}{\pi} = 330^\circ$

(iv) 1.2 rad $= 1.2 \times \frac{180^\circ}{\pi} = 68.75^\circ$ $(\because \pi = 3.14159)$

Example 6: Convert degree to radian

(i) 15° (ii) 75° (iii) 315° (iv) 15° 15′

(i) $15^\circ = 15 \times \frac{\pi}{180} = \frac{\pi}{12}$ rad or 0.262 rad

(ii) $75^\circ = 75 \times \frac{\pi}{180} = \frac{5\pi}{12}$ rad or 1.309 rad

(iii) $315^\circ = 315 \times \frac{\pi}{180} = \frac{7\pi}{4}$ rad or 5.498 rad

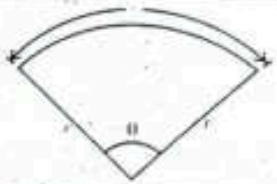
(iv) $15^\circ 15' = 15^\circ + \left(\frac{15}{60}\right)^\circ = 15.25^\circ = 15.25 \times \frac{\pi}{180}$ rad $= 0.266$ rad

Turns	0 turn	$\frac{1}{12}$ turn	$\frac{1}{8}$ turn	$\frac{1}{6}$ turn	$\frac{1}{4}$ turn	$\frac{1}{2}$ turn	1 tu
Radians	0 rad	$\frac{\pi}{6}$ rad	$\frac{\pi}{4}$ rad	$\frac{\pi}{3}$ rad	$\frac{\pi}{2}$ rad	π rad	2π ra
Degrees	0°	30°	45°	60°	90°	180°	360

Arc Length and Area of sector

Arc length of sector $= \ell = r\theta$ and

Area of sector $= A = \frac{1}{2} r^2 \theta$



Proof: We know that:

$\ell = \frac{\theta}{360^\circ} \times 2\pi r$

$= \frac{\theta}{2\pi} \times 2\pi r$ $(2\pi \text{ radius} = 360^\circ)$

$= r\theta$

Proof: We know that:

$A = \frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{\theta}{2\pi} \times \pi r^2$ $(2\pi \text{ radius} = 360^\circ)$

$= \frac{1}{2} r^2 \theta$

Hence arc length, $\ell = r\theta$ and area of sector, $A = \frac{1}{2} r^2 \theta$

Example 7: Find the arc length of a sector with radius r = 10 cm and central angle $\theta = 60^\circ$.

Solution: Convert $\theta = 60^\circ$ to radians: $\theta = 60 \times \frac{\pi}{180} = \frac{\pi}{3}$ radians.

$$\ell = r\theta = 10 \times \frac{\pi}{3} \approx 10.47 \text{ cm}$$

The arc length is approximately 10.47 cm

Example 8: Find the area of a sector with radius r = 8 cm and central angle $\theta = 45^\circ$.

Solution: Convert $\theta = 45^\circ$ to radians: $\theta = 45 \times \frac{\pi}{180} = \frac{\pi}{4}$ radians.

$$A = \frac{1}{2} r^2 \theta = \frac{1}{2} \times 8^2 \times \frac{\pi}{4} = 8\pi \text{ cm}^2 \approx 25.12 \text{cm}^2$$

The area of the sector is approximately 25.12cm^2.

Example 9: If arc length of a sector of radius 5 cm is 11 cm, find the angle subtended by the arc in radians and degrees.

Solution: $r = 5$ cm ; $\ell = 11$ cm, ; $\theta = ?$

$\therefore \ell = r\theta$

$11 = 5\theta \Rightarrow \theta = \frac{11}{5} = 2.2$ rad

$\theta = 2.2 \times \frac{180}{\pi} \approx 126.1^\circ$

Thus, the angle subtended by the arc in radians is 2.2 rad and degrees is 126.1°

Solved Exercise 6.1

1. Find in which quadrant the following angles lie. Write a co-terminal angle for each:

(i) 65°

Sol. 1st quadrant; 425° and –295 are co-terminal angles.

(ii) 135°

Sol. 2nd quadrant; 495° and –225 are co-terminal angles.

(iii) –40°

Sol. 4th quadrant; –400° and 320° are co-terminal angles.

(iv) 210°

Sol. 3rd quadrant; 570° and –150° are co-terminal angles.

(v) –150°

Sol. 3rd quadrant; –510° and 210° are co-terminal angles.

2. Convert the following into degrees, minutes, and seconds.

(i) 123.456° **(ii) 58.7891°** **(iii) 90.5678°**

(i) 123.456°

Sol. 123.456°
$= 123° + 0.456°$
$= 123° + (0.456 \times 60)'$
$= 123° + (27.36)'$
$= 123° + 27' + (0.36)'$
$= 123° + 27' + (0.36 \times 60)''$
$= 123° + 27' + 22''$
$= 123° 27' 22''$

(ii) 58.7891°

Sol. 58.7891°
$= 58° + 0.7891°$
$= 58° + (0.789 \times 60)'$
$= 58° + (47.346)'$
$= 58° + 47' + 0.346'$
$= 58° + 47' + (0.346 \times 60)''$
$= 58° + 47' + 21''$
$= 58° 47' 21''$

(iii) 90.5678°

Sol. 90.5678°
$= 90° + 0.5678°$
$= 90° + (0.5678)°$
$= 90° + (0.5678 \times 60)'$
$= 90° + (34.068)'$
$= 90° + 34' + 0.068'$
$= 90° + 34' + (0.068 \times 60)''$
$= 90° + 34' + 4''$
$= 90° 34' 4''$

3. Convert the following into decimal degrees.

(i) 65° 32' 15'' **(ii) 42° 18' 45''** **(iii) 78° 45' 36''**

(i) 65° 32' 15''

Sol. 65° 32' 15''
$= 65° + \left(\frac{32}{60}\right)° + \left(\frac{15}{3600}\right)°$
$= 65° + 0.533° + 0.0042°$
$= 65.5372°$

(ii) 42° 18' 45''

Sol. 42° 18' 45''
$= 42° + \left(\frac{18}{60}\right)° + \left(\frac{15}{3600}\right)°$
$= 42° + 0.3° + 0.0125°$
$= 42.3125°$

(iii) 78° 45' 36''

Sol. 78° 45' 36''

$= 78° + \left(\frac{45}{60}\right)° + \left(\frac{36}{3600}\right)°$
$= 78° + 0.75° + 0.01°$
$= 78°.76°$

4. Convert the following into radian.

(i) 36°

Sol. 36°
$= 36 \times \frac{\pi}{180}$
$= \frac{\pi}{5}$ rad
$= 0.6283$ rad

(ii) 22.5°

Sol. 22.5°
$= 22.5 \times \frac{\pi}{180}$
$= \frac{225}{10} \times \frac{\pi}{180}$
$= \frac{\pi}{8}$ rad
$= 0.39$ rad

(iii) 67.5°

Sol. 67.5°
$= 67.5 \times \frac{\pi}{180}$
$= \frac{675}{10} \times \frac{\pi}{180}$
$= \frac{3\pi}{8}$ rad
$= 1.18$ rad

5. Convert the following into degrees.

(i) $\frac{\pi}{16}$ rad

Sol. $\frac{\pi}{16}$ rad
$= \frac{\pi}{16} \times \frac{180°}{\pi}$
$= \left(\frac{45}{4}\right)°$
$= 11.25°$

(ii) $\frac{11\pi}{5}$ rad

Sol. $\frac{11\pi}{5}$ rad
$= \frac{11\pi}{5} \times \frac{180°}{\pi}$
$= 396°$

(iii) $\frac{7\pi}{6}$ rad

Sol. $\frac{7\pi}{6}$ rad
$= \frac{7\pi}{6} \times \frac{180°}{\pi}$
$= 210°$

6. Find the arc length and area of a sector if:

(i) r = 6 cm and central angle $\theta = \frac{\pi}{3}$ adians.

(ii) $r = \frac{4.8}{\pi}$ cm and central angle $\theta = \frac{5\pi}{6}$ radians.

Sol. (i) $r = 6$
$\theta = \frac{\pi}{3}$ radians
$\ell = ?$
Area of the sector = ?
$\ell = r\theta$
$= 6 \times \frac{\pi}{3}$
$= 2\pi$
$= 6.28$ cm

Area of the sector $= \frac{1}{2} r^2\theta$
$= \frac{1}{2} \times (6)^2 \times \frac{\pi}{3}$
$= \frac{1}{2} \times 36 \times \frac{\pi}{3}$
$= 18.85$ cm^2

Sol. (ii) $r = \frac{4.8}{\pi}$ cm

$\theta = \frac{5\pi}{6}$ radian

$\ell = ?$

Area of sector = ?

$\ell = r\theta$

$= \frac{4.8}{\pi} \times \frac{5\pi}{6}$

Area of sector $= \frac{1}{2} r^2\theta$

$= \frac{1}{2}\left(\frac{4.8}{\pi}\right)^2 \times \frac{5\pi}{6}$

$= \frac{1}{2} \times \frac{4.8 \times 4.8}{\pi \times \pi} \times \frac{5\pi}{6}$

$= \frac{23.4 \times 5}{12\pi} = \frac{115.2}{12\pi}$

$= 3.06$ cm²

7. If the central angle of a sector is 60° and the radius of the circle is 12 cm, find the area of the sector and the percentage of the total area of the circle it represents.

Sol. $\theta = 60°$, r = 12cm

Area of sector = ?

$\theta = 50°$

$= 60 \times \frac{\pi}{180}$

$= \frac{\pi}{3}$ rad

Area of sector $= \frac{1}{2} r^2\theta$

$= \frac{1}{2}(12)^2 \times \frac{\pi}{3}$

$= \frac{1}{2} \times \frac{24}{144} \times \times \frac{\pi}{3}$

$= 24\pi = 75.4$ cm

Area of circle $= \pi r^2$

$= \pi(12)^2$

$= 144\pi$

$= 452.39$ cm²

Percentage of sector area $= \frac{75.4}{452.39} \times 100\%$

$= 16.67\%$

8. Find the percentage of the area of sector subtending an angle $\frac{\pi}{8}$ radians.

Sol. $\theta = \frac{\pi}{8}$

Area of sector $= \frac{1}{2} r^2\theta$

$= \frac{1}{2} r^2 \times \frac{\pi}{8}$

$= \frac{r^2\pi}{16}$

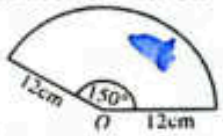
Area of circle $= \pi r^2$

Percentage of the sector area $= \frac{\frac{r^2\pi}{16}}{\pi r^2} \times 100\%$

$= \frac{r^2\pi}{16} \times \frac{1}{\pi r^2} \times 100\%$

$= \frac{100}{16}\% = 6.25\%$

9. A circular sector of radius r = 12cm has an angle of 150°. This sector is cut out and then bent to form a cone. What is the slant height and the radius of the base of this cone?
Hint: Arc length of sector = circumference of cone.



Sol.

radius of circle = 12cm

$\theta = 150°$

slnat height of come = ?

radiaus of the base of cone = ?

Slant height of the cone is same as the radius of the original circle.

So, slant height of cone = 12cm

$\theta = 150°$

$\theta = 150° \times \frac{\pi}{180}$

$= \frac{5\pi}{6}$ rad

length of the arc of sector $= r\theta$

$= 12 \times \frac{5\pi}{6} = 10\pi$

The are length of sector become the circumfrence of the base of the cone

$c = 2\pi r$

$10\pi = 2 \times \pi \times r$

$\frac{10\pi}{2} = r$

$5 = r$

So, radius of the base of cone = 5cm

Trigonometric Ratios

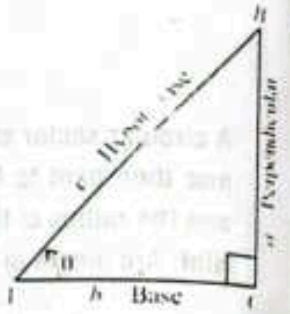
The functions that relate angles to side in a right-angled tiangle are known as trigonometric functions (sine, cosine, tangent etc).

Trigonometric Ratios of an Acute Angle

The trigonometric ratios are applied to acute angle in a right-angled triangle, but the concepts extend to angles greater than 90° and are widely used in many areas of mathematics and science.

Let us consider a right-angled triangle ACB with respect to an angle θ (theta) = m∠CAB with m∠ACB = 90°. In the triangle ACB, the side BC is called perpendicular, which is opposite to an angle 'θ'.

The side AC is called the base and the side AB is called the hypotenuse. Let m $\overline{BC}$ = a, m$\overline{AC}$ = b and m $\overline{AB}$ = c.



For this right angled triangle ACB, the trigonometric ratios of an angle "θ" are defined as:

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{a}{c} \quad ; \quad \text{cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{c}{a}$$

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{b}{c} \quad ; \quad \sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{c}{b}$$

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{a}{b} \quad ; \quad \cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{b}{a}$$

The six trigonometric ratios described with reference to a right-angled triangle ACB are sine (sin), cosine (cos), tangent (tan), cosecant (cosec or csc), secant (sec) and cotangent (cot).

Note that $\tan\theta = \frac{\sin\theta}{\cos\theta}$

and $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Not: (i) $\text{cosec}\,\theta = \frac{1}{\sin\theta}$

(ii) $\sec\theta = \frac{1}{\sin\theta}$

(iii) $\cot\theta = \frac{1}{\tan\theta}$

Trigonometric Ratios of Complementary Angles

We consider a right-angled triangle ACB, in which m∠A = θ, m∠C = 90° then, m∠B = 90° − θ. Using the trigonometric-ratios of ∠B, we get

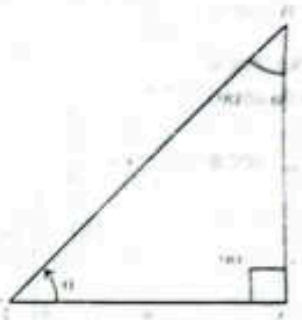
$$\sin\beta = \sin(90^\circ - \theta) = \frac{m\overline{AC}}{m\overline{AB}} = \frac{b}{c} \qquad \ldots(i)$$

Using ratios of A, we get

$$\cos\theta = \frac{m\overline{AC}}{m\overline{AB}} = \frac{b}{c} \qquad \ldots(ii)$$

From (i) and (ii), we get,

$$\boxed{\sin(90^\circ - \theta) = \cos\theta}$$



Similarly, we have

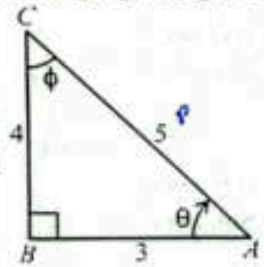
$$\cos(90^\circ - \theta) = \sin\theta \quad ; \quad \tan(90^\circ - \theta) = \cot\theta \quad ; \quad \cot(90^\circ - \theta) = \tan\theta$$

$$\sec(90^\circ - \theta) = \text{cosec}\,\theta \quad ; \quad \text{cosec}(90^\circ - \theta) = \sec\theta$$

Solved Exercise 6.2

1. For each of the following right-angled triangles, find the trigonometric ratios

(a)



For figure (a)

(i) sin θ

Sol. sin θ = ?

$$\sin\theta = \frac{m\overline{BC}}{m\overline{AC}} = \frac{4}{5}$$

(ii) cos θ

Sol. cos θ = ?

$$\cos\theta = \frac{m\overline{AB}}{m\overline{AC}} = \frac{3}{5}$$

(iii) tan θ

Sol. tan θ = ?

$$\tan\theta = \frac{m\overline{BC}}{m\overline{AB}} = \frac{4}{3}$$

(iv) sec θ

Sol. sec θ = ?

$$\sec\theta = \frac{m\overline{AC}}{m\overline{AB}} = \frac{5}{3}$$

(v) cosec θ

Sol. cosec θ = ?

$$\text{cosec}\,\theta = \frac{m\overline{AC}}{m\overline{AB}} = \frac{5}{4}$$

(vi) cot ϕ

Sol. cot ϕ = ?

$$\cot\phi = \frac{m\overline{BC}}{m\overline{AB}} = \frac{4}{3}$$

(vii) tan ϕ

Sol. tan ϕ = ?

$$\tan\phi = \frac{m\overline{AB}}{m\overline{BC}} = \frac{3}{4}$$

(viii) cosec ϕ

Sol. cosec ϕ = ?

$$\text{cosec}\,\phi = \frac{m\overline{AC}}{m\overline{AB}} = \frac{5}{3}$$

(ix) sec ϕ

Sol. sec ϕ = ?

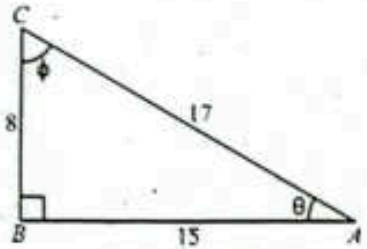
$$\sec\phi = \frac{m\overline{AC}}{m\overline{BC}} = \frac{5}{4}$$

(x) cos ϕ

Sol. cos ϕ = ?

$$\cos\phi = \frac{m\overline{BC}}{m\overline{AC}} = \frac{4}{5}$$

(b)



For figure (b)

(i) sin θ

Sol. sin θ = ?

$$\sin\theta = \frac{m\overline{BC}}{m\overline{AC}} = \frac{8}{17}$$

(ii) cos θ

Sol. cos θ = ?

$$\cos\theta = \frac{m\overline{AB}}{m\overline{AC}} = \frac{15}{17}$$

(iii) tan θ

Sol. tan θ = ?

$$\tan\theta = \frac{m\overline{BC}}{m\overline{AB}} = \frac{8}{15}$$

(iv) sec θ

Sol. sec θ = ?

$$\sec\theta = \frac{m\overline{AC}}{m\overline{AB}} = \frac{17}{15}$$

(v) cosec θ

Sol. cosec θ = ?

$$\text{cosec}\,\theta = \frac{m\overline{AC}}{m\overline{BC}} = \frac{17}{8}$$

(vi) cot ϕ

Sol. cot ϕ = ?

$$\cot\phi = \frac{m\overline{BC}}{m\overline{AB}} = \frac{8}{15}$$

(vii) tan ϕ

Sol. tan ϕ = ?

$$\tan\phi = \frac{m\overline{AB}}{m\overline{BC}} = \frac{15}{8}$$

(viii) cosec ϕ

Sol. cosec ϕ = ?

$$\text{cosec}\,\phi = \frac{m\overline{AC}}{m\overline{AB}} = \frac{17}{15}$$

(ix) sec ϕ

Sol. sec ϕ = ?

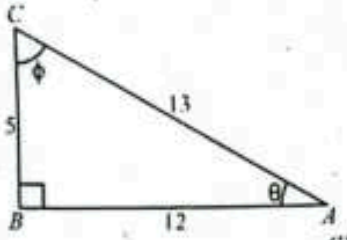
$$\sec\phi = \frac{m\overline{AC}}{m\overline{BC}} = \frac{17}{8}$$

(x) cos ϕ

Sol. cos ϕ = ?

$$\cos\phi = \frac{m\overline{BC}}{m\overline{AC}} = \frac{8}{17}$$

(c)



For figure (c)

(i) sin θ

Sol. sin θ = ?

$$\sin\theta = \frac{m\overline{BC}}{m\overline{AC}} = \frac{5}{13}$$

(ii) cos θ

Sol. cos θ = ?

$$\cos\theta = \frac{m\overline{AB}}{m\overline{AC}} = \frac{12}{13}$$

(iii) tan θ

Sol. tan θ = ?

$$\tan\theta = \frac{m\overline{BC}}{m\overline{AB}} = \frac{5}{12}$$

(iv) sec θ

Sol. sec θ = ?

$$\sec\theta = \frac{m\overline{BC}}{m\overline{AB}} = \frac{5}{12}$$

(v) cosec θ

Sol. cosec θ = ?

$$\text{cosec}\,\theta = \frac{m\overline{AC}}{m\overline{BC}} = \frac{13}{5}$$

(vi) cot ϕ

Sol. cot ϕ = ?

$$\cot\phi = \frac{m\overline{BC}}{m\overline{AB}} = \frac{5}{12}$$

(vii) tan ϕ

Sol. tan ϕ = ?

$$\tan\phi = \frac{m\overline{AB}}{m\overline{BC}} = \frac{12}{5}$$

(viii) cosec ϕ

Sol. cosec ϕ = ?

$$\text{cosec}\,\phi = \frac{m\overline{AC}}{m\overline{AB}} = \frac{13}{12}$$

(ix) sec ϕ

Sol. sec ϕ = ?

$$\sec\phi = \frac{m\overline{AC}}{m\overline{BC}} = \frac{13}{5}$$

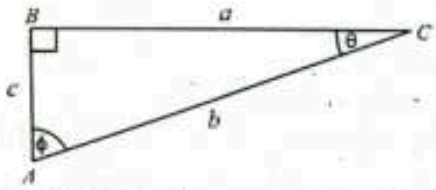
(x) cos ϕ

Sol. cos ϕ = ?

$$\cos\phi = \frac{m\overline{BC}}{m\overline{AC}} = \frac{5}{13}$$

2. For each of the following right-angled triangles ABC find the trigonometric ratios for which $m\angle A = \phi$ and $m\angle C = \theta$?

Sol. For figure (a)



(i) sin θ

Sol: $\sin\theta = \frac{c}{b}$

(ii) cos θ

Sol. $\cos\theta = \frac{a}{b}$

(iii) tan θ

Sol. $\tan\theta = \frac{c}{a}$

(iv) sin ϕ

Sol. $\sin\phi = \frac{a}{b}$

(v) cos ϕ

Sol. $\cos\phi = \frac{c}{b}$

(vi) tan ϕ

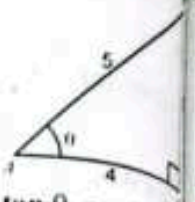
Sol. $\tan\phi = \frac{a}{c}$

3. Considering the adjoining triangle ABC, verify that:

(i) $\sin\theta \operatorname{cosec}\theta = 1$

(ii) $\cos\theta \sec\theta = 1$

(iii) $\tan\theta \cot\theta = 1$



(i) $\sin\theta \operatorname{cosec}\theta = 1$

Sol. L.H.S

$= \sin\theta \operatorname{cosec}\theta$

$= \frac{3}{5} \times \frac{5}{3}$

$= 1 = \text{R.H.S}$

(ii) $\cos\theta \sec\theta = 1$

Sol. L.H.S

$= \cos\theta \sec\theta$

$= \frac{4}{5} \times \frac{5}{4}$

$= 1 = \text{R.H.S}$

(iii) $\tan\theta \cot\theta = 1$

Sol. L.H.S

$= \tan\theta \cot\theta$

$= \frac{3}{4} \times \frac{4}{3}$

$= 1 = \text{R.H.S}$

4. Fill in the blanks.

Sol. (i) $\sin 30° = \sin(90° - 60°) = \cos \underline{60°}$

(ii) $\cos 30° = \cos(90° - 60°) = \sin \underline{60°}$

(iii) $\tan 30° = \tan(90° - 60°) = \cot \underline{60°}$

(iv) $\tan 60° = \tan(90° - 30°) = \cot \underline{30°}$

(v) $\sin 60° = \sin(90° - 30°) = \cos \underline{30°}$

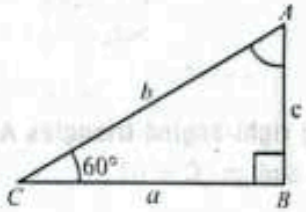
(vi) $\cos 60° = \cos(90° - 30°) = \sin \underline{30°}$

(vii) $\sin 45° = \sin(90° - 45°) = \cos \underline{45°}$

(viii) $\tan 45° = \tan(90° - 45°) = \cot \underline{45°}$

(ix) $\cos 45° = \cos(90° - 45°) = \sin \underline{45°}$

5. If in a right angled triangle ABC, $m\angle B = 90°$ and C is an acute angle of 60°. Also $\sin m\angle A = \frac{a}{b}$, then find the following trigonometric ratios.



$m\angle B = 90°$

$m\angle C = 60°$

$m\angle A = 180° - 90° - 60° = 30°$

(i) $\frac{m\overline{BC}}{m\overline{AB}}$

Sol. $\frac{m\overline{BC}}{m\overline{AB}} = ?$

$\frac{m\overline{BC}}{m\overline{AB}} = \frac{a}{b}$

(ii) $\cos 60°$

Sol. $\cos 60° = ?$

$\cos 60° = \frac{a}{b}$

(iii) $\tan 60°$

Sol. $\tan 60° = ?$

$\tan 60° = \frac{c}{a}$

(iv) $\operatorname{cosec}\frac{\pi}{3}$

Sol. $\operatorname{cosec}\frac{\pi}{3} = ?$

$\operatorname{cosec}\frac{\pi}{3} = \frac{b}{c}$

(v) $\cot 60°$

Sol. $\cot 60° = ?$

$\cot 60° = \frac{a}{c}$

(vi) $\sin 30°$

Sol. $\sin 30° = ?$

$\sin 30° = \frac{a}{b}$

(vii) $\cos 30°$

Sol. $\cos 30° = ?$

$\cos 30° = \frac{c}{b}$

(viii) $\tan\frac{\pi}{6}$

Sol. $\tan\frac{\pi}{6} = ?$

$\tan\frac{\pi}{6} = \frac{c}{a}$

(ix) $\sec 30°$

Sol. $\sec 30° = ?$

$\sec 30° = \frac{b}{c}$

(x) $\cot 30°$

Sol. $\cot 30° = ?$

$\cot 30° = \frac{c}{a}$

Trigonometric Identities

Fundamental Trigonometric Identities

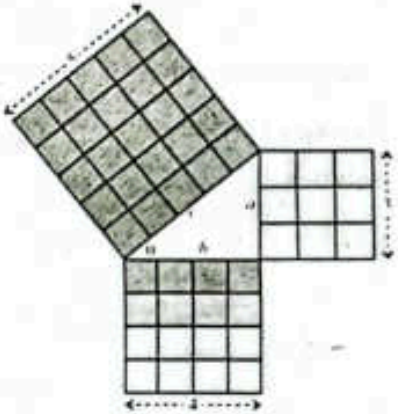
We shall consider some of the fundamental identities used in trigonometry. The key to these basic identities is the Pythagoras theorem in geometry.

"The square of the length of the hypotenuse of a right triangle is equal to the sum of the square of the lengths of the other two sides".

$c^2 = a^2 + b^2$

$5^2 = 3^2 + 4^2$

$25 = 9 + 16$



In the given figure: the perpendicular equals to the length 'a' base equals to the length 'b', and hypotenuse equals to the length 'c'.

By Pythagoras Theorem, we have

$a^2 + b^2 = c^2$ ...(i)

$\frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{c^2}{c^2}$ (Dividing by c^2)

$\boxed{\sin^2\theta + \cos^2\theta = 1}$ ...(ii)

$a^2 + b^2 = c^2$

Dividing equation (i) by b^2, we have

$\frac{a^2}{b^2} + \frac{b^2}{b^2} = \frac{c^2}{b^2}$

$\boxed{\tan^2\theta + 1 = \sec^2\theta}$ ...(iii)

$\frac{a^2}{a^2} + \frac{b^2}{a^2} = \frac{c^2}{a^2}$ Dividing equation (i) by a^2, we have

$\boxed{1 + \cot^2\theta = \operatorname{cosec}^2\theta}$ ...(iv)

The identities (ii), (iii) and (iv) are known as Pythagoras identities.

Example 10: Show that $(\sec^2\theta - 1)\cos^2\theta = \sin^2\theta$

Solution: L.H.S $= (\sec^2\theta - 1)\cos^2\theta$

$= \tan^2\theta \cos 2\theta \quad (\because 1 + \tan^2\theta = \sec^2\theta)$

$= \frac{\sin^2\theta}{\cos^2\theta} \cdot \cos^2\theta \quad \left(\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right)$

$= \sin^2\theta = \text{R.H.S}$

Hence, $(\sec^2\theta - 1)\cos^2\theta = \sin^2\theta$

Example 11: Show that $\tan\theta + \cos\theta = \sec\theta \operatorname{cosec}\theta$

Solution: L.H.S $= \tan\theta + \cot\theta$

$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin\theta \cdot \sin\theta + \cos\theta \cdot \cos\theta}{\cos\theta \cdot \sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta\cos\theta} \quad (\because \sin^2\theta + \cos^2\theta = 1)$

$= \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta} = \sec\theta \cdot \operatorname{cosec}\theta = \text{R.H.S.}$

Hence, $\tan\theta + \cot\theta = \sec\theta \operatorname{cosec}\theta$

Example 12: Show that $\frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}$

Solution:

L.H.S $= \frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta}$

$= \frac{1}{\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}} - \frac{1}{\sin\theta}$

$= \frac{\sin\theta}{1 - \cos\theta} - \frac{1}{\sin\theta}$

$= \frac{\sin\theta(1 + \cos\theta)}{(1 - \cos\theta)(1 + \cos\theta)} - \frac{1}{\sin\theta}$

$= \frac{\sin\theta(1 + \cos\theta)}{1 - \cos^2\theta} - \frac{1}{\sin\theta}$

$= \frac{\sin\theta(1 + \cos\theta)}{\sin^2\theta} - \frac{1}{\sin\theta}$

$= \frac{1 + \cos\theta}{\sin\theta} - \frac{1}{\sin\theta}$

$= \frac{1 + \cos\theta - 1}{\sin\theta} = \frac{\cos\theta}{\sin\theta} = \cot\theta$

R.H.S $= \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}$

$= \frac{1}{\sin\theta} - \frac{1}{\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}}$

$= \frac{1}{\sin\theta} - \frac{\sin\theta}{1 + \cos\theta}$

$= \frac{1}{\sin\theta} - \frac{\sin\theta(1 - \cos\theta)}{(1 + \cos\theta)(1 - \cos\theta)}$

$= \frac{1}{\sin\theta} - \frac{\sin\theta(1 - \cos\theta)}{1 - \cos^2\theta}$

$= \frac{1}{\sin\theta} - \frac{\sin\theta(1 - \cos\theta)}{\sin^2\theta}$

$= \frac{1}{\sin\theta} - \frac{1 - \cos\theta}{\sin\theta}$

$= \frac{1 - 1 + \cos\theta}{\sin\theta} = \frac{\cos\theta}{\sin\theta} = \cot\theta$

Hence, L.H.S. = R.H.S

Example 13: Show that $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

Solution: L.H.S $= \sin^6\theta + \cos^6\theta$

$= (\sin^2)^3\theta + (\cos^2\theta)^3$

$= (\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta)$

$= (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta$

$= 1 - 3\sin^2\theta\cos^2\theta = \text{R.H.S}$

Hence, $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

Example 14: If $\tan\theta = \frac{3}{4}$, find the remaining trigonometric ratios, when θ lies in first quadrant.

Solution: Given: $\tan\theta = \frac{3}{4} = \frac{a}{c}$.

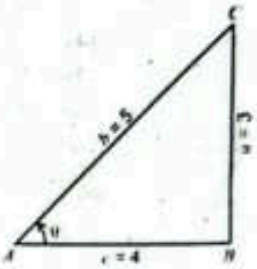
Where, $a = 3, c = 4$

By Pythagoras theorem, we have

$b^2 = a^2 + c^2$

$= 9 + 16 = 25$

$\Rightarrow \quad b = 5$



Therefore, $\sin\theta = \frac{a}{b} = \frac{3}{5}$; $\operatorname{cosec}\theta = \frac{b}{a} = \frac{5}{3}$

$\cot\theta = \frac{c}{b} = \frac{4}{5}$; $\sec\theta = \frac{b}{c} = \frac{5}{4}$

$\cot\theta = \frac{c}{a} = \frac{4}{3}$

Solved Exercise 6.3

If θ lies in first quadrant, find the remaining trigonometric ratios of θ.

i) $\sin\theta = \frac{2}{3}$

Sol. Given: $\sin\theta = \frac{2}{3} = \frac{a}{b}$

Where $a = 2, b = 3$

By Pythagoras theorem, we have

$b^2 = a^2 + c^2$

$3^2 = 2^2 + c^2$

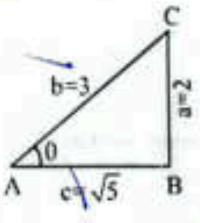
$9 = 4 + c^2$

$9 - 4 = c^2$

$5 = c^2$

$\sqrt{c^2} = \sqrt{5}$

$c = \sqrt{5}$



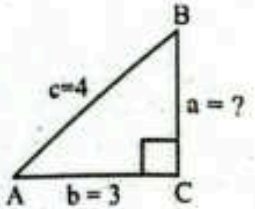
$\sin\theta = \frac{a}{b} = \frac{2}{3}$; $\operatorname{cose}\theta = \frac{b}{c} = \frac{3}{2}$

$\cos\theta = \frac{c}{b} = \frac{\sqrt{5}}{3}$; $\sec\theta = \frac{b}{c} = \frac{3}{\sqrt{5}}$

$\tan\theta = \frac{a}{b} = \frac{2}{\sqrt{5}}$; $\cot\theta = \frac{c}{b} = \frac{\sqrt{5}}{2}$

(ii) $\cos\theta = \frac{3}{4}$

Sol. $\cos\theta = \frac{3}{4} = \frac{b}{c}$



Where b = 3 and c = 4

By Pytagoras theorem, we have

$c^2 = b^2 + a^2$

$(4)^2 = (3)^2 + a^2$

$16 = 9 + a^2$

$a^2 = 16 - 9$

$a^2 = 7$

$\sqrt{a^2} = \sqrt{7}$

$a = \sqrt{7}$

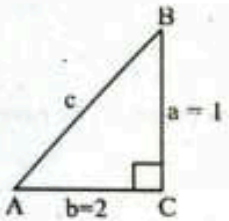
$\sin\theta = \frac{a}{c} = \frac{\sqrt{7}}{4}$, $\operatorname{cose}\theta = \frac{c}{a} = \frac{4}{\sqrt{7}}$

$\cos\theta = \frac{b}{c} = \frac{3}{4}$, $\sec\theta = \frac{c}{b} = \frac{4}{3}$

$\tan\theta = \frac{a}{b} = \frac{\sqrt{7}}{3}$, $\cot\theta = \frac{b}{a} = \frac{3}{\sqrt{7}}$

(iii) $\tan\theta = \frac{1}{2}$

Sol. $\tan\theta = \frac{1}{2} = \frac{a}{b}$



Where a = 1, b = 2

By Pythagoras theorem, we have:

$c^2 = b^2 + a^2$

$c^2 = (2)^2 + (1)^2$

$c^2 = 4 + 1$

$c^2 = 5$

$\sqrt{c^2} = \sqrt{5}$

$c = \sqrt{5}$

$\sin\theta = \frac{a}{c} = \frac{1}{\sqrt{5}}$, $\operatorname{cose}\theta = \frac{c}{a} = \sqrt{5}$

$\cos\theta = \frac{b}{c} = \frac{2}{\sqrt{5}}$, $\sec\theta = \frac{c}{b} = \frac{\sqrt{5}}{2}$

$\tan\theta = \frac{a}{b} = \frac{1}{2}$, $\cot\theta = \frac{b}{a} = \frac{2}{1} = 2$

) $\sec\theta = 3$

l. $\sec\theta = \frac{3}{1} = \frac{b}{c}$

Where b = 3, c = 1

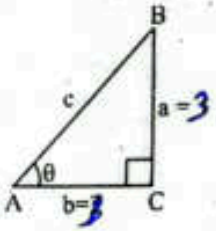
By Pythagoras theorem, we have:

$c^2 = b^2 + a^2$

$(3)^2 = (1)^2 + a^2$

$a^2 = 9 - 1 \Rightarrow a^2 = 8$

$\sqrt{a^2} = \sqrt{8} \Rightarrow a = 2\sqrt{2}$



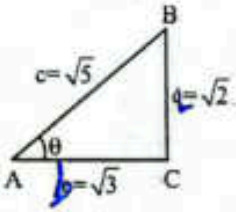
$\sin\theta = \frac{a}{c} = \frac{2\sqrt{2}}{3}$, $\operatorname{cose}\theta = \frac{c}{a} = \frac{3}{2\sqrt{2}}$

$\cos\theta = \frac{b}{c} = \frac{1}{3}$, $\sec\theta = \frac{c}{b} = \frac{3}{1} = 3$

$\tan\theta = \frac{a}{b} = \frac{2\sqrt{2}}{1} = 2\sqrt{2}$, $\cot\theta = \frac{b}{a} = \frac{1}{2\sqrt{2}}$

) $\cot\theta = \sqrt{\frac{3}{2}}$

l. $\cot\theta = \sqrt{\frac{3}{2}} = \frac{b}{a} = \frac{\sqrt{3}}{\sqrt{2}}$



Where a = 1, b = $\sqrt{3}$, a = $\sqrt{2}$

By Pythagoras theorem, we have:

$c^2 = a^2 + b^2$

$c^2 = (\sqrt{2})^2 + (\sqrt{3})^2$

$c^2 = 2 + 3 \Rightarrow c^2 = 5$

$\sqrt{c^2} = \sqrt{5} \Rightarrow c = \sqrt{5}$

$\sin\theta = \frac{a}{c} = \frac{\sqrt{2}}{\sqrt{5}} = \sqrt{\frac{2}{5}}$, $\operatorname{cose}\theta = \frac{c}{a} = \frac{\sqrt{5}}{\sqrt{2}} = \sqrt{\frac{5}{2}}$

$\cos\theta = \frac{b}{c} = \frac{\sqrt{3}}{\sqrt{5}} = \sqrt{\frac{3}{5}}$, $\sec\theta = \frac{c}{b} = \frac{\sqrt{5}}{\sqrt{3}} = \sqrt{\frac{5}{3}}$

$\tan\theta = \frac{a}{b} = \frac{\sqrt{2}}{\sqrt{3}} = \sqrt{\frac{2}{3}}$, $\cot\theta = \frac{b}{a} = \frac{\sqrt{3}}{\sqrt{2}} = \sqrt{\frac{3}{2}}$

Prove the following identities:

2. $(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$

Sol. $(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$

L.H.S $= (\sin\theta + \cos\theta)^2$

$= \sin^2\theta + \cos^2\theta + 2(\sin\theta)(\cos\theta)$

$= 1 + 2\sin\theta\cos\theta \quad (\because \sin^2\theta + \cos^2\theta = 1)$

$=$ R.H.S

3. $\dfrac{\cos\theta}{\sin\theta} = \dfrac{1}{\tan\theta}$

Sol. $\dfrac{\cos\theta}{\sin\theta} = \dfrac{1}{\tan\theta}$

L.H.S $= \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\frac{\cos\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}$ (Divide denominator and numerator by $\cos\theta$)

$= \dfrac{1}{\tan\theta} \quad \left(\because \dfrac{\sin\theta}{\cos\theta} = \tan\theta\right)$ ✓

$=$ R.H.S

Hence, $\dfrac{\cos\theta}{\sin\theta} = \dfrac{1}{\tan\theta}$

4. $\dfrac{\sin\theta}{\operatorname{cosec}\theta} + \dfrac{\cos\theta}{\sec\theta} = 1$

Sol. $\dfrac{\sin\theta}{\operatorname{cosec}\theta} + \dfrac{\cos\theta}{\sec\theta} = 1$

L.H.S $= \dfrac{\sin\theta}{\operatorname{cosec}\theta} + \dfrac{\cos\theta}{\sec\theta}$

$= \sin\theta \times \dfrac{1}{\operatorname{cosec}\theta} + \cos\theta\dfrac{1}{\sec\theta}$

$= \sin\theta \times \sin\theta + \cos\theta \times \cos\theta \quad \because \dfrac{1}{\operatorname{cosec}\theta} = \sin\theta$

$= \sin^2\theta + \cos^2\theta \quad \because \dfrac{1}{\sec\theta} = \cos\theta$

$= 1 \quad \because \sin^2\theta + \cos^2\theta = 1$

$=$ R.H.S

Hence, $\dfrac{\sin\theta}{\operatorname{cosec}\theta} + \dfrac{\cos\theta}{\sec\theta} = 1$

$\cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

$\cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

L.H.S $= \cos^2\theta - \sin^2\theta \quad \because \cos^2\theta + \sin^2\theta = 1$

$= \cos^2\theta - (1 - \cos^2\theta) \quad \sin^2\theta = 1 - \cos^2\theta$

$= \cos^2\theta - 1 + \cos^2\theta$

$= 2\cos^2\theta - 1$

$=$ R.H.S

Hence, $\cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

$\cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$

$\cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta \quad \because \sin^2\theta + \cos^2\theta = 1$

L.H.S $= \cos^2\theta - \sin^2\theta \quad \cos^2\theta = 1 - \sin^2\theta$

$= 1 - \sin^2\theta - \sin^2\theta$

$= 1 - 2\sin^2\theta$

$=$ R.H.S

Hence, $\cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$

$\dfrac{1-\sin\theta}{\cos\theta} = \dfrac{\cos\theta}{1+\sin\theta}$

$\dfrac{1-\sin\theta}{\cos\theta} = \dfrac{\cos\theta}{1+\sin\theta}$

L.H.S $= \dfrac{1-\sin\theta}{\cos\theta}$

$= \dfrac{1-\sin\theta}{\cos\theta} \times \dfrac{1+\sin\theta}{1+\sin\theta} = \dfrac{(1)^2-(\sin\theta)^2}{\cos\theta(1+\sin\theta)}$

$= \dfrac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)}$

$= \dfrac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)} \quad \because \sin^2\theta + \cos^2\theta = 1$

$= \dfrac{\cos^2\theta}{\cos\theta(1+\sin\theta)} \quad \cos^2\theta = 1 - \sin^2\theta$

$= \dfrac{\cos\theta}{1+\sin\theta}$

$=$ R.H.S

Hence, $\dfrac{1-\sin\theta}{\cos\theta} = \dfrac{\cos\theta}{1+\sin\theta}$

8. $(\sec\theta - \tan\theta)^2 = \dfrac{1-\sin\theta}{1+\sin\theta}$

Sol. $(\sec\theta - \tan\theta)^2 = \dfrac{1-\sin\theta}{1+\sin\theta}$

L.H.S = $(\sec\theta - \tan\theta)^2$

$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$ $\quad\because\quad \sec\theta = \frac{1}{\cos\theta}, \tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \left(\frac{1-\sin\theta}{\cos\theta}\right)^2 = \frac{(1-\sin\theta)^2}{\cos^2\theta}$

$= \frac{(1-\sin\theta)^2}{\cos^2\theta} = \frac{(1-\sin\theta)^2}{1-\sin^2\theta}$

$= \frac{(1-\sin\theta)^{\cancel{2}1}}{(1+\sin\theta)\cancel{(1-\sin\theta)}}$

$= \frac{1-\sin\theta}{1+\sin\theta}$ = R.H.S

Hence, $(\sec\theta - \tan\theta)^2 = \frac{1-\sin\theta}{1+\sin\theta}$

9. $(\tan\theta + \cot\theta)^2 = \sec^2\theta \operatorname{cosec}^2\theta$

Sol. $(\tan\theta + \cot\theta)^2 = \sec^2\theta \operatorname{cosec}^2\theta$

L.H.S = $(\tan\theta + \cot\theta)^2$

$= \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)^2$ $\quad\because\quad \tan\theta = \frac{\sin\theta}{\cos\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta}$

$= \left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right)^2$

$= \left(\frac{1}{\cos\theta\sin\theta}\right)^2$ $\quad\because\quad \sin^2\theta + \cos^2\theta = 1$

$= \frac{1}{\cos^2\theta\sin^2\theta} = \frac{1}{\cos^2\theta}\cdot\frac{1}{\sin^2\theta}$

$= \sec^2\theta \operatorname{cosec}^2\theta$ = R.H.S $\quad\because\quad \frac{1}{\cos^2\theta} = \sec^2\theta, \frac{1}{\sin^2\theta} = \operatorname{cosec}^2\theta$

Hence, $(\tan\theta + \cot\theta)^2 = \sec^2\theta \operatorname{cosec}^2\theta$

10. $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \tan\theta + \sec\theta$

Sol. $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \tan\theta + \sec\theta$

L.H.S = $\frac{(\tan\theta + \sec\theta) - 1}{\tan\theta - \sec\theta + 1}$

$= \frac{(\tan\theta + \sec\theta) - (\tan^2\theta - \sec^2\theta)}{\tan\theta - \sec\theta + 1}$ $\quad\because\quad 1 + \tan^2\theta = \sec^2\theta$

$\quad\quad 1 = \sec^2\theta - \tan^2\theta$

$= \frac{(\tan\theta + \sec\theta) - (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)}{\tan\theta - \sec\theta + 1}$

$= \frac{(\tan\theta + \sec\theta)(1 - (\sec\theta - \tan\theta))}{\tan\theta - \sec\theta + 1}$ Taking in common $(\tan\theta + \sec\theta)$

$= \frac{(\tan\theta + \sec\theta)(1 - \sec\theta + \tan\theta)}{\tan\theta - \sec\theta + 1}$

$= \frac{(\tan\theta + \sec\theta)\ \cancel{(\tan\theta - \sec\theta + 1)}}{\cancel{\tan\theta - \sec\theta + 1}}$

$= \tan\theta + \sec\theta$ = R.H.S

Hence, $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \tan\theta + \sec\theta$

$\sin^3\theta - \cos^3\theta = (\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta)$

$\sin^3\theta - \cos^3\theta = (\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta)$

L.H.S = $\sin^3\theta - \cos^3\theta$

$= (\sin\theta)^3 - (\cos\theta)^3$

$= (\sin\theta - \cos\theta)((\sin\theta)^2 + (\sin\theta)(\cos\theta) + (\cos\theta)^2)$

$= (\sin\theta - \cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta)$

$= (\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta)$ = R.H.S $\quad\because\quad \sin^2\theta + \cos^2\theta = 1$

Hence, $\sin^3\theta - \cos^3\theta = (1 + \sin\theta\cos\theta)(\sin\theta - \cos\theta)$

$\sin^6\theta - \cos^6\theta = (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$

$\sin^6\theta - \cos^6\theta = (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$

L.H.S = $\sin^6\theta - \cos^6\theta$

$= (\sin^2\theta - \cos^2\theta)((\sin^2\theta)^2 + (\sin^2\theta)(\cos^2\theta) + (\cos^2\theta)^2$

$= (\sin^2\theta - \cos^2\theta)((\sin^2\theta)^2 + (\cos^2\theta)^2 + \sin^2\theta\cos^2\theta + 2\sin^2\theta\cos^2\theta - 2\sin^2\theta\cos^2\theta$

$= (\sin^2\theta - \cos^2\theta)((\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta\cos^2\theta - 2\sin^2\theta\cos^2\theta)$

$= (\sin^2\theta - \cos^2\theta)((\sin^2\theta + \cos^2\theta)^2 - \sin^2\theta\cos^2\theta)$ $\quad\because\quad \sin^2\theta + \cos^2\theta = 1$

$= (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$ = R.H.S

Hence, $\sin^6\theta - \cos^6\theta = (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$

lues of Trigonometric Ratios of Special Angles

rigonometric ratios of 45° $\left(\frac{\pi}{4}\right)$

onsider a square ACBD of side length 1 hit.

Ve know that the diagonals bisect the ngles. So, in the triangle ABC

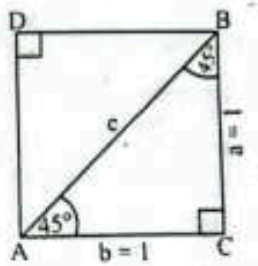
$m\angle A = m\angle B = 45°$ and $m\angle C = 90°$.

Using Pythagoras theorem ΔABC,

$c^2 = a^2 + b^2$

$c^2 = 1 + 1$

$c^2 = 2 \quad\Rightarrow\quad c = \sqrt{2}$



The trigonometric ratio are:

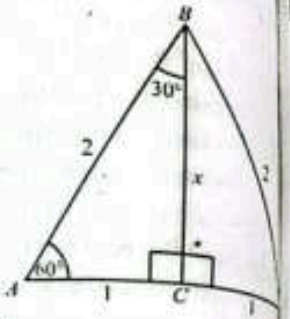
$$\sin 45° = \frac{a}{c} = \frac{1}{\sqrt{2}} \quad ; \quad \operatorname{cosec} 45° = \frac{c}{a} = \sqrt{2}$$

$$\cos 45° = \frac{b}{c} = \frac{1}{\sqrt{2}} \quad ; \quad \sec 45° = \frac{c}{b} = \sqrt{2}$$

$$\tan 45° = \frac{a}{b} = 1 \quad ; \quad \cot 45° = \frac{b}{a} = 1$$

Trigonometric Ratios of 30° $\left(\frac{\pi}{6}\text{radian}\right)$ and 60° $\left(\frac{\pi}{3}\text{radian}\right)$:

Consider an equilateral triangle ABD of side 2 units.
Draw a perpendicular bisector $\overline{BC}$ on $\overline{AD}$. The point C is the midpoint of $\overline{AD}$. So, $m\overline{AC} = m\overline{CD}$ in which $m\angle BAC=60°$, $m\angle ABC=30°$, $m\angle ACB=90°$.
Let $m\overline{BC} = x$ units.



Using Pythagoras theorem in the ΔABC.

$$2^2 = 1^2 + x^2$$

$$x^2 = 4 - 1 \Rightarrow x^2 = 3 \Rightarrow x = \sqrt{3}, \ (m\overline{BC} = \sqrt{3} \text{ units})$$

Trigonometric ratios of 30° $\left(\frac{\pi}{6}\text{radian}\right)$:

In the triangle. ABC with $m\angle ABC = 30°$.

$$\sin 30° = \frac{1}{2} \quad ; \quad \operatorname{cosec} 30° = 2$$

$$\cos 30° = \frac{\sqrt{3}}{2} \quad ; \quad \sin 30° = \frac{2}{\sqrt{3}}$$

$$\tan 30° = \sqrt{3} \quad ; \quad \cot 30° = \frac{1}{\sqrt{3}}$$

Trigonometric Ratios of 60° $\left(\frac{\pi}{3}\text{radian}\right)$: with $m\angle A = 60°$.

In right angled triangle ABC, with $m\angle A = 60°$.

$$\sin 60° = \frac{\sqrt{3}}{2} \quad ; \quad \cos 60° = \frac{1}{2} \quad ; \quad \tan 60° = \sqrt{3}$$

$$\operatorname{cosec} 60° = \frac{2}{\sqrt{3}} \quad ; \quad \sec 60° = 2 \quad ; \quad \cot 60° = \frac{1}{\sqrt{3}}$$

These results in the form of a table can be written as:

θ	0	$30° = \frac{\pi}{6}$	$45° = \frac{\pi}{4}$	$60° = \frac{\pi}{3}$	$90° = \frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

Solved Exercise 6.4

1. Find the value of the following trigonometric ratios without using the calculator.

Sol. Considere an equlateral triangle ABD of sides 2 units. Draw a perpendicular bisector $\overline{AC}$ on $\overline{BD}$. So, $\overline{BC} = \overline{CD} = 1$

In right triangle ABC
$m\angle BCA = 90°$, $m\angle ABC = 60°$, $m\angle BAC = 30°$
By Pythagoras theorem

$$2^2 = 1^2 + x^2$$
$$4 = 1 + x^2$$
$$4 - 1 = x^2$$
$$\sqrt{3} = \sqrt{x^2}$$
$$\sqrt{3} = x$$

So, $m\overline{AC} = \sqrt{3}$

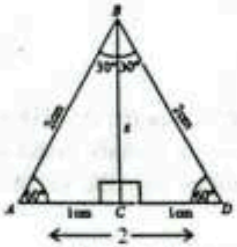


Figure (A)

(i) sin 30°
Sol. $\sin 30° = ?$

$$\sin 30° = \frac{\text{opp}}{\text{hyp}} = \frac{1}{2}$$

(from figure A)

(ii) cos 30°
Sol. $\cos 30° = ?$

$$\cos 30° = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{3}}{2}$$

(from figure A)

(iii) tan $\frac{\pi}{6}$ **Sol.** $\tan \frac{\pi}{6} = ?$

$$\tan \frac{\pi}{6} = \frac{\text{opp}}{\text{adj}} = \frac{1}{\sqrt{3}}$$

(from figure A)

(iv) tan 60°
Sol. $\tan 60° = ?$

$$\tan 60° = \frac{\text{opp}}{\text{adj}} = \sqrt{3}$$

(from figure A)

(v) sec 60°
Sol. $\sec 60° = ?$

$$\sec 60° = \frac{\text{hyp}}{\text{adj}} = \frac{2}{1} = 2$$

(from figure A)

(vi) cos $\frac{\pi}{3}$ **Sol.** $\cos \frac{\pi}{3} = ?$

$$\cos \frac{\pi}{3} = \frac{\text{adj}}{\text{hyp}} = \frac{1}{2}$$

(from figure A)

(vii) cot 60°

Sol. cot 60° = ?

$$\cot 60° = \frac{\text{adj}}{\text{opp}} = \frac{1}{\sqrt{3}}$$

(from figure A)

(ix) sec 30°

Sol. sec 30° = ?

$$\sec 30° = \frac{\text{hyp}}{\text{adj}} = \frac{2}{\sqrt{3}}$$

(from figure A)

(xi) sin 45°

Sol. sin 45° = ?

$$\sin 45° = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{2}}$$

(from figure A)

(viii) sin 60°

Sol. sin 60° = ?

$$\sin 60° = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{3}}{2}$$

(from figure A)

(x) cosec 30°

Sol. cosec 30° = ?

$$\text{cosec } 30° = \frac{\text{hyp}}{\text{adj}} = 2$$

(from figure A)

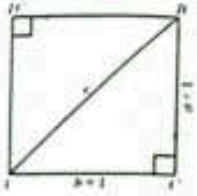
(xii) cos $\frac{\pi}{4}$

Sol. cos $\frac{\pi}{4}$ = ?

$$\cos \frac{\pi}{4} = \frac{\text{adj}}{\text{hyp}} = \frac{1}{\sqrt{2}}$$

(from figure A)

Consider a square ACBD of side length 1 unit. We know diagonals bisect the angles ACB bisects the angle. In ΔBC m∠A = m∠B = 45°, m∠C = 90°



By Pytagoras theorem

2. Evaluate:

(i) 2 sin 60° cos60°

Sol. $= 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$

(iii) 2 sin 45° + 2cos 45°

Sol. 2 sin 45° + 2cos 45°

$$= 2\left(\frac{1}{\sqrt{2}}\right) + 2\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}}$$

$$= \frac{4}{\sqrt{2}} = \frac{4 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$$

$$= \frac{4 \times \sqrt{2}}{(\sqrt{2})^2} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

(ii) 2cos $\frac{\pi}{6}$ sin $\frac{\pi}{6}$

Sol. $= 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$

(iv) sin 60° cos 30° + cos 60° sin 30°

Sol. sin 60° cos 30° + cos 60° sin 30°

$$= \left(\frac{(\sqrt{3})}{2}\right)\left(\frac{(\sqrt{3})}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{(\sqrt{3})^2}{4} + \frac{1}{4}$$

$$= \frac{3}{4} + \frac{1}{4}$$

$$= \frac{3+1}{4} = \frac{4}{4} = 1$$

(v) cos 60° cos 30° - sin 60° sin 30°

Sol. cos 60° cos 30° - sin 60° sin 30°

$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$$

$$= 0$$

(vii) cos 60° cos 30° + sin 60° sin 30°

Sol. cos 60° cos 30° + sin 60° sin 30°

$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$$

$$= \frac{\sqrt{3} + \sqrt{3}}{4}$$

$$= \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$$

(vi) sin 60° cos 30° - cos 60° sin 30°

Sol. sin 60° cos 30° - cos 60° sin 30°

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 - \frac{1}{4} = \frac{3}{4} - \frac{1}{4}$$

$$= \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$$

(viii) tan $\frac{\pi}{6}$ cot $\frac{\pi}{6}$ + 1

Sol. tan $\frac{\pi}{6}$ cot $\frac{\pi}{6}$ + 1

$$= \left(\frac{1}{\sqrt{3}}\right)(\sqrt{3}) + 1$$

$$= \frac{\sqrt{3}}{\sqrt{3}} + 1$$

$$= 1 + 1 = 2$$

3. If sin $\frac{\pi}{4}$ and cos $\frac{\pi}{4}$ equal to $\frac{1}{\sqrt{2}}$ each, then find the value of the followings:

(i) 2 sin 45° – 2 cos 45°

Sol. 2 sin 45° – 2 cos 45°

$$= 2\left(\frac{1}{\sqrt{2}}\right) - 2\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{2}{\sqrt{2}} - \frac{2}{\sqrt{2}} = 0$$

(iii) 5 cos 45° – 3 sin 45°

Sol. 5 cos 45° – 3 sin 45°

$$= 5\left(\frac{1}{\sqrt{2}}\right) - 3\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{5}{\sqrt{2}} - \frac{3}{\sqrt{2}} = \frac{5-3}{\sqrt{2}} = \frac{2}{\sqrt{2}}$$

$$= \frac{2 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$$

$$= \frac{\sqrt{2}}{(\sqrt{2})^2} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

(ii) 3 cos 45° + 4 sin 40°

Sol. 3 sin 45° + 4 cos 45°

$$= 3\left(\frac{1}{\sqrt{2}}\right) + 4\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{3}{\sqrt{2}} + \frac{4}{\sqrt{2}}$$

$$= \frac{3+4}{\sqrt{2}}$$

$$= \frac{7}{\sqrt{2}}$$

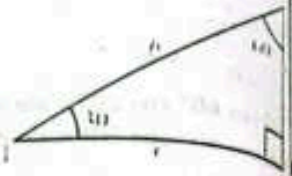
Solution of a Triangle

Finding the measures of the remaining elements is called the solution of a triangle only

Case I: When measures of one side and one angle are given.

Example 15: Solve triangle ABC, in which $m\angle B = 90°$, $m\angle A = 30°$, a = 2cm

Solution: We are required to find b, c and $m\angle C$.



Now $m\angle C = m\angle B - m\angle A$

$= 90° - 30°$

$= 60°$ ...(i)

and $\frac{a}{b} = \sin 30°$

$\Rightarrow \frac{2}{b} = \sin 30° \quad (\because a = 2)$

$\Rightarrow \frac{2}{b} = \frac{1}{2} \quad \left(\because \sin 30° = \frac{1}{2}\right)$

$\Rightarrow b = 4\text{cm}$ ...(ii)

and $\frac{a}{c} = \tan 30°$

$\Rightarrow \frac{2}{c} = \frac{1}{\sqrt{3}} \quad \left(\because a = 2, \tan 30° = \frac{1}{\sqrt{3}}\right)$

thus $c = 2\sqrt{3}\text{cm}$ ...(iii)

(i), (ii) and (iii) are the required results.

Case II: When measure of the hypotenuse and an angle are given.

Example 16: Solve triangle ABC, when $m\angle A = 60°$, b = 5 cm, $m\angle B = 90°$

Solution: We are required to find a, c and $m\angle C$

$m\angle A = 60°$

$m\angle B = 90°$

$m\angle C = m\angle B - m\angle A$

$= 90° - 60°$

$= 30°$ ...(i)



Now $\frac{a}{b} = \sin 60°$

$\frac{a}{5} = \frac{\sqrt{3}}{2} \quad \left(\because b = 5, \sin 60° = \frac{\sqrt{3}}{2}\right)$

$\Rightarrow a = \frac{5\sqrt{3}}{2}$

$\Rightarrow a = 4.33\text{ cm}$ ...(ii)

and $\frac{c}{b} = \cos 60°$

$\frac{c}{5} = \frac{1}{2} \quad \left(\because b = 5, \cos 60° = \frac{1}{2}\right)$

$\Rightarrow c = \frac{5}{2}$

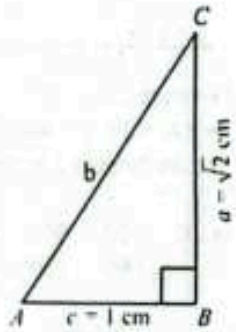
$\Rightarrow c = 2.5\text{ cm}$ ...(iii)

(i), (ii) and (iii) are the required results.

Case III: When measure of two sides are given.

Example 17: Solve triangle ABC, when $a = \sqrt{2}$ cm, c = 1 cm and $m\angle B = 90°$

Solution: We are require to find b, $m\angle A$, $m\angle C$



By Pythagoras theorem, we have

$b^2 = c^2 + a^2$

or $b^2 = (1)^2 + (\sqrt{2})^2$

or $b^2 = 1 + 2$

or $b^2 = 3$

or $b^2 = \sqrt{3}\text{ cm}$ ...(i)

Now $\sin m\angle A = \frac{a}{b} = \frac{\sqrt{2}}{\sqrt{3}} \Rightarrow m\angle A \sin^{-1}\sqrt{\frac{2}{3}} = 54.7°$

$\Rightarrow m\angle A = 54.7°$ ...(ii)

and $m\angle C = m\angle B - m\angle A$

$= 90° - 54.7°$

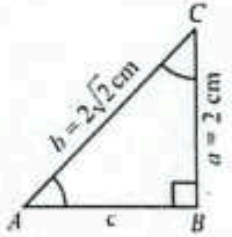
$= 35.3°$ ...(iii)

(i), (ii) and (iii) are the required results.

Case IV: When measure of one side and hypotenuse are given.

Example 18: Solve triangle ABC, when a = 2cm, $b = 2\sqrt{2}$ cm and $m\angle B = 90°$

Solution: We are require to find $m\angle A$, $m\angle C$ and c.



By Pythagoras theorem, we have

$b^2 = a^2 + c^2$

or $c^2 = b^2 - a^2$

$= (2\sqrt{2})^2 - (2)^2$

$= 8 - 4$

$= 4$

or $c = 2\text{cm}$ ...(i)

Now $\frac{c}{b} = \cos m\angle A$

or $\frac{c}{b} = \cos m\angle A = \frac{1}{\sqrt{2}}$

$\Rightarrow m\angle A = 45°$

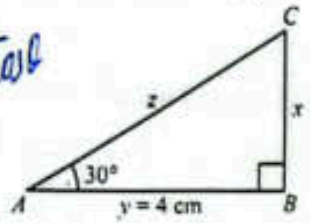
Thus, $m\angle C = m\angle B - m\angle A$
$= 90° - 45°$
$= 45°$ ...(iii)

Hence (i), (ii) and (iii) are the required results.

Solved Exercise 6.5

1. Find the values of x, y and z from the following right angled triangles.

(i)



Sol. Given $m\angle B = 90°$,
$m\angle A = 30$, $y = 4$cm
We are required to find x, z and $m\angle C$
$m\angle C = 180° - 90° - 30°$
$= 180° - 120° = 60°$

Now $\frac{x}{y} = \tan 30°$

$\frac{x}{4} = \frac{1}{\sqrt{3}}$

$x = \frac{4}{\sqrt{3}}$

$\frac{y}{z} = \cos 30°$

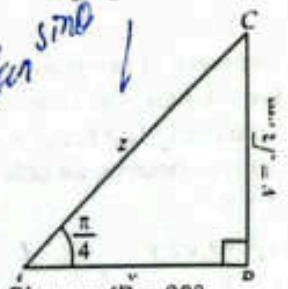
$\frac{4}{z} = \frac{1}{\sqrt{3}}$

$4\sqrt{3} = z$

Hence,

$m\angle C = 60$, $x = \frac{4}{\sqrt{3}}$ cm, $z = 4\sqrt{3}$ cm

(ii)



Sol. Given $m\angle B = 90°$,

$m\angle A = \frac{\pi}{4}$, $y = \sqrt{3}$ cm

We are require to find $m\angle C$, x and z
$m\angle C = 180° - 90° - 45°$
$= 180° - 135° = 45°$

Now $\frac{y}{x} = \tan \frac{\pi}{4}$

$\frac{\sqrt{3}}{x} = 1$

$\sqrt{3} = x$

$x = \sqrt{3}$ cm

$\frac{y}{z} = \sin 45°$

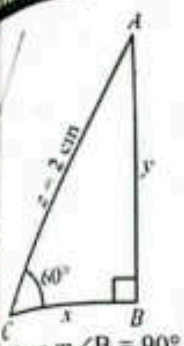
$\frac{\sqrt{3}}{z} = \frac{1}{\sqrt{2}}$

$\sqrt{3} \times \sqrt{2} = z$

$\sqrt{6} = z$

$z = \sqrt{6}$ cm

Hence, $m\angle C = 45°$, $x = \sqrt{3}$ cm, $z = \sqrt{6}$ cm



Given $m\angle B = 90°$,
$m\angle C = 60°$, $z = 2$cm
We are require to find $m\angle A$, x and z.
$m\angle A = 180° - 90° - 60°$
$= 180° - 150° = 30°$

$\frac{x}{z} = \cos 60°$

$\frac{x}{2} = \frac{1}{2}$

$x = \frac{2}{2} = 1$

$x = 1$cm

$\frac{y}{z} = \sin 60°$

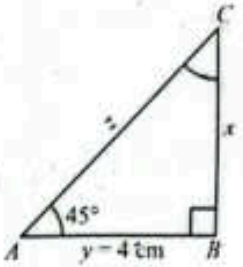
$\frac{y}{2} = \frac{\sqrt{3}}{2}$

$y = \frac{\sqrt{3}}{2} \times 2$

$y = \sqrt{3}$ cm

ence, $m\angle A = 30°$, $x = 1$cm, $y = \sqrt{3}$ cm

(iv)



Sol. Given $m\angle B = 90°$,
$m\angle A = 45°$, $y = 4$cm
We are require to find $m\angle C$, x and z.
$m\angle C = 180° - 90° - 45°$
$= 180° - 135° = 45°$

Now $\frac{x}{y} = \tan 45°$

$\frac{x}{4} = 1$

$x = 4$cm

$\frac{y}{z} = \cos 45°$

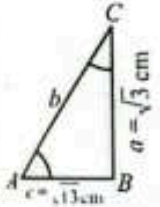
$\frac{4}{z} = \frac{1}{\sqrt{2}}$

$4\sqrt{2} = z$

$z = 4\sqrt{2}$ cm

Hence, $m\angle C = 45°$, $x = 4$cm $z = 4\sqrt{2}$ cm

Find the unknown side and angles of the following triangles.



ol. Given $m\angle B = 90°$, $c = \sqrt{13}$ cm, $a = \sqrt{3}$ cm

We are required to find b, $m\angle A$ and $m\angle C$.

By Pythagoras theorem

$b^2 = a^2 + c^2$

$b^2 = (\sqrt{3})^2 + (\sqrt{13})^2$

$b^2 = 3 + 13$

$b^2 = 16$

$\sqrt{b^2} = \sqrt{16}$

$b = 4cm$

now $\tan\theta = \frac{a}{c}$

$\tan\angle A = \frac{\sqrt{3}}{\sqrt{13}}$

$\angle A = \tan^{-1}\frac{\sqrt{3}}{\sqrt{13}}$

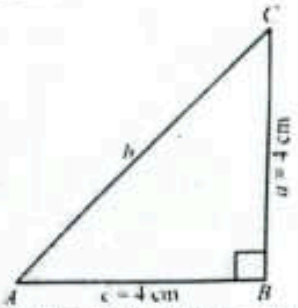
$m\angle A = 25.65°$

$m\angle C = 180° - 90° - 25.65°$

$= 64.35°$

Hence, $b = 4cm$, $m\angle A = 25.65°$, $m\angle C = 64.35°$

(ii)



Sol. Given $m\angle B = 90°$, $c = 4cm$, $a = 4$ cm

We are required to find b, $m\angle A$ and $m\angle C$

By Pythagoras theorem

$b^2 = a^2 + c^2$

$b^2 = (4)^2 + (4)^2$

$b^2 = 16 + 16$

$\frac{b^2}{\sqrt{b^2}} = 32$

$\sqrt{b^2} = \sqrt{32}$

$b = \sqrt{2\times2\times2\times2\times2}$

$b = \sqrt{2^2\times2^2\times2} \Rightarrow b = 2\times2\sqrt{2}$

$b = 4\sqrt{2}$ cm

$\tan\angle A = \frac{4}{4}$

$\tan\angle A = 1$

$\angle A = \tan^{-1}(1)$

$\angle A = 45°$

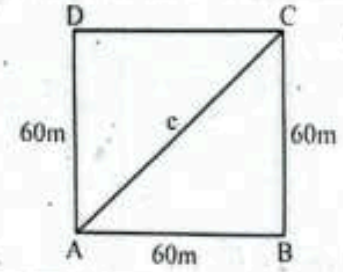
$m\angle A = 180° - 90° - 45°$

$= 180° - 135° = 45°$

Hence, $b = 4\sqrt{2}$ cm, $m\angle A = 45°$, $m\angle C = 45°$

3. Each side of a square field is 60m long. Find the lengths of the diagonals of field.

Sol.



Let ABCD is a square field with each side is 60m long. $\overline{AC}$ is required diagonal.

$m\overline{AC} = ?$

In right triangle ABC, $m\overline{AB} = 60m$, $m\overline{BC} = 60m$

$(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2$

$(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2$

$(AC)^2 = (60)^2 + (6)^2$

$(AC)^2 = 3600 + 3600$

$(\overline{AC})^2 = 7200$

$\sqrt{(AC)^2} = \sqrt{7200}$

$\overline{AC} = 84.85$ cm

So, the length of the diagonal is 84.85 cm

As both diagonal of a square are equal, so the length of other diagonal is also 84.85cm.

4. Solve the following triangles when $m\angle B = 90°$:

(i) $m\angle C = 60°$, $c = 3\sqrt{3}$cm

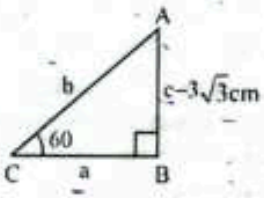
Sol. In the given right triangle

$m\angle B = 90°$, $m\angle C = 60°$ $c = 3\sqrt{3}$ cm

We are required to find $m\angle A$ a and c

$m\angle A = 180° - 90° - 60°$

$= 180° - 150° = 30°$ .....(i)



Now,

$\tan 60° = \frac{c}{a}$

$\sqrt{3} = \frac{3\sqrt{3}}{a}$

$a = \frac{3\sqrt{3}}{\sqrt{3}}$

$a = 3cm$

$b^2 = a^2 + c^2$ .....(ii)

$\sin 60° = \frac{c}{b}$

$\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{b}$

$b = 3\sqrt{3}\times\frac{2}{\sqrt{3}}$

$b = 6cm$ .....(iii)

Thus, (i), (ii) and (iii) are required results.

(ii) $m\angle C = 45°$, $a = 8$ cm

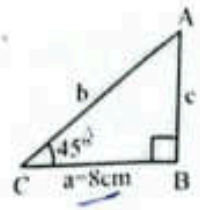
Sol. In the given right triangle

$m\angle B = 90°$, $m\angle C = 45°$, $a = 8$cm

We are required to find $m\angle A$, b and c.

$m\angle A = 180° - 90° - 45°$

$= 180° - 135° = 45°$ .....(i)



No $\cos 45° = \frac{a}{b}$

$\frac{1}{\sqrt{2}} = \frac{8\text{cm}}{b}$

$b = 8 \times \sqrt{2}$

$b = 8\sqrt{2}$ cm .....(ii)

$\frac{c}{a} = \tan 45°$

$\frac{c}{8} = 1$

$c = 1 \times 8$

$c = 8$cm .....(iii)

Thus, (i), (ii) and (iii) are required results.

(iii) $a = 12$cm, $c = 6$ cm

Sol. In the given right triangle

$m\angle B = 90°$, $a = 12$cm, $c = 6$cm

We are required to find b, $m\angle C$, $m\angle A$

By Pythagoras theorem.

$b^2 = a^2 + c^2$

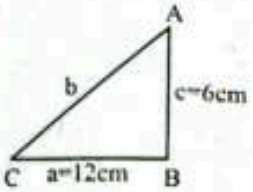
$b^2 = (12)^2 + (6)^2$

$b^2 = 144 + 36$

$b^2 = 180$

$\sqrt{b^2} = \sqrt{180}$

$b = 6\sqrt{5}$ cm .....(i)



Now $\tan\theta = \frac{c}{a}$

$\tan\angle C = \frac{6}{12}$

$\tan\angle C = \frac{1}{2}$

$\angle C = \tan^{-1}\frac{1}{2}$

$\angle C = 26.6°$ .....(ii)

$m\angle A = 180° - 90° - 26.6°$

$= 180° - 116.6°$

$= 63.4°$ .....(iii)

Thus, (i), (ii) and (iii) are required results.

(iv) $m\angle A = 60°$, $c = 4$ cm

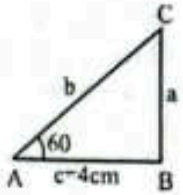
Sol. In the given right triangle

$m\angle B = 90°$, $m\angle A = 60°$, $c = 4$cm

We are required to find $m\angle C$, a and b.

$m\angle C = 180° - 90° - 60°$

$= 180° - 150° = 30°$ .....(i)



$\frac{a}{c} = \tan 60°$

$\frac{a}{4} = \sqrt{3}$

$a = 4\sqrt{3}$ cm .....(ii)

$\frac{c}{b} = \cos 60°$

$\frac{4}{b} = \frac{1}{2}$

$4 \times 2 = b \Rightarrow 8 = b$

$b = 8$cm .....(iii)

Thus, (i), (ii) and (iii) are required results.

(v) $m\angle A = 30°$, $c = 4$ cm

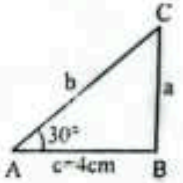
Sol. In the given right triangle

$m\angle B = 90°$, $m\angle A = 30°$, $c = 4$cm

We are required to find $m\angle C$, a and b.

$m\angle C = 180° - 90° - 30°$

$= 180° - 120° = 60°$ .....(i)



$\frac{a}{c} = \tan 30°$

$\frac{a}{4} = \frac{1}{\sqrt{3}}$

$a = \frac{4}{\sqrt{3}}$ cm .....(ii)

$\frac{c}{b} = \cos 30°$

$\frac{4}{b} = \frac{\sqrt{3}}{2}$

$$4 \times \frac{2}{\sqrt{3}} = b$$

$$\frac{8}{\sqrt{3}} = b$$

$$b = \frac{8}{\sqrt{3}} \text{ cm} \quad \text{.....(iii)}$$

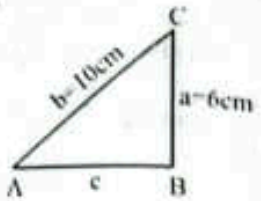
Thus, (i), (ii) and (iii) are required results.

(vi) b = 10cm, a = 6 cm

Sol. In the given right triangle

$m\angle B = 90°$, b = 10cm, a = 6cm

We are required to find $m\angle A$, $m\angle C$ and c.



Now By Pythagoras theorem

$b^2 = a^2 + c^2$

$10^2 = 6^2 + c^2$

$100 = 36 + c^2$

$100 - 36 = c^2$

$64 = c^2$

$\sqrt{c^2} = \sqrt{64}$

$c = 8\text{cm}$

$$\sin\angle A = \frac{a}{b}$$

$$\sin\angle A = \frac{6}{10}$$

$\sin\angle A = \sin^{-1}(0.6)$

$\angle A = 36.9°$

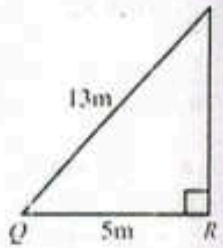
$m\angle C = 180° - 90° - 36.9°$

$= 180° - 126.9°$

$= 53.1°$ .....(ii)

Thus, (i) (ii) and (iii) are required results.

5. Let Q and R be the two points on the same bank of a canal. The point P is placed on the other bank straight to point R. Find the width of the canal and the angle PQR in radians.



Sol. $\overline{PR}$ is the width of the canal

In right triangle PQR

$\overline{PQ} = 13$

$\overline{PQ} = 5$

$\overline{PR} = ?$

By Pytagoras theorem

$(\overline{PQ})^2 = (\overline{PR})^2 + (\overline{QR})^2$

$(13)^2 = (\overline{PR})^2 + (5)^2$

$169 = (\overline{PR})^2 + 25$

$169 - 25 = (\overline{PR})^2$

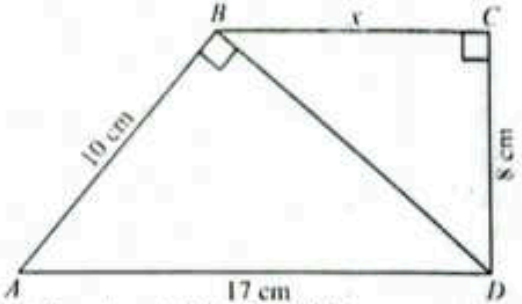
$144 = (\overline{PR})^2$

$\sqrt{(\overline{PR})^2} = \sqrt{144}$

$\overline{PR} = 12\text{m}$

So, the width of canal is 12m

6. Calculate the length x in the adjoining figure.



Sol. There are two right triangles. ΔABD and ΔBCD

From right triangle ABC

$\overline{AB} = 10\text{cm}$

$\overline{AD} = 17\text{cm}$

$\overline{BD} = ?$

By Pytagoras theorem

$(\overline{AD})^2 = (\overline{AB})^2 + (\overline{BD})^2$

$(17)^2 = (10\text{cm})^2 + (\overline{BD})^2$

$289\text{ cm}^2 = 100\text{ cm}^2 + (\overline{BD})^2$

$289\text{ cm}^2 = 1000\text{ cm}^2 + (\overline{BD})^2$

$189\text{ cm}^2 = (\overline{BD})^2$

$\sqrt{(\overline{BD})^2} = \sqrt{189\text{cm}^2}$

$\overline{BD} = 13.75\text{ cm}$

From right triangle BCD

$\overline{CD} = 8\text{ cm}$

$\overline{BD} = 13.75\text{ cm}$

$\overline{BC} = ?$

By Pytagoras theorem

$(\overline{BD})^2 = (\overline{BC})^2 + (\overline{CD})^2$

$(13.75)\text{cm}^2 = x^2 + (8\text{ cm})^2$

$189\text{ cm}^2 = x^2 + 64\text{ cm}^2$

$189\text{ cm}^2 - 164\text{ cm}^2 = x^2$

$125\text{ cm}^2 = x^2$

$\sqrt{x^2} = \sqrt{125\text{ cm}^2}$

$x = 11.18\text{ cm}$

7. If the ladder is placed along the wall such that the foot of the ladder is 2 m away from the wall. If the length of the ladder is 8 m, find the height of the wall?

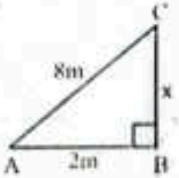
Sol. Let the height of wall = x m

By Pythagoras theorem

$(AC)^2 = (AB)^2 + (BC)^2$

$(8\text{ m})^2 = (2\text{m})^2 + x^2$

$64\text{m}^2 = 4\text{m}^2 + x^2$



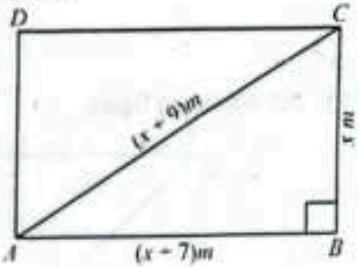
$64m^2 - 4m^2 = x^2 \Rightarrow 60m^2 = x^2$

$\sqrt{x^2} = \sqrt{60\ m^2}$

$x = 7.75m$

Thus, the hight of the wall is 7.75m

8. The diagonal of a rectangular field ABCD is (x+9)m and the sides are (x+7)m and x m. Find the value of x.



Sol. In right triangle ABC

$\overline{AC} = (x + 9)m$

$\overline{AB} = (x + 7)m$

$\overline{BC} = ?$

By Pythagoras theorem

$(AC)^2 = (AB)^2 + (BC)^2$

$(x+9)^2 = (x+7)^2 + x^2$

$(x)^2 + 2(x)(9) + (9)^2 = (x)^2 + 2((x)(7) + (7)^2 + x^2$

$x^2 + 18x + 81 = x^2 + 14x + 49 + x^2$

$18x - 14x + 81 - 49 = x^2 + x^2 - x^2$

$4x + 32 = x^2$

$0 = x^2 - 4x - 32$

$x^2 - 4x - 32 = 0$

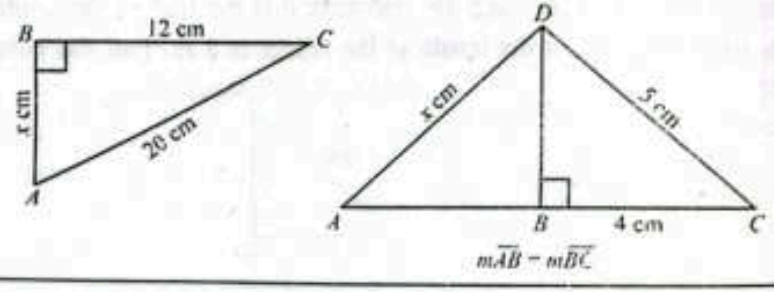
$x^2 - 8x + 4x - 32 = 0$

$(x - 8)(x + 4) = 0$

$x - 8 = 0$	$x + 4 = 0$
$x = 8$	$x = -4$ ignore it

so, $x = 8m$

9. Calculate the value of 'x' in each case.



$m\overline{AB} = m\overline{BC}$

Sol. In right triangle ABC

$\overline{AB} = x$ cm

$\overline{BC} = 12$ cm

$\overline{AC} = 20$ cm

By Pythagoras theorem

$(AC)^2 = (AB)^2 + (BC)^2$

$(20\ cm)^2 = x^2 + (12\ cm)^2$

$400\ cm^2 = x^2 + 144\ cm^2$

$400\ cm^2 - cm^2 = x^2 = x^2$

$256\ cm^2 = x^2$

$\sqrt{x^2} = \sqrt{256\ cm^2}$

$x = 16cm$

Figure (ii)

There are two right triangles ABD and BCD

In triangle BCD

$\overline{DC} = 5cm$, $\overline{BC} = 4cm$, $\overline{BD} = ?$

By Pythagoras theorem

$(\overline{DC})^2 = (\overline{BC})^2 + (\overline{BD})^2$

$(5cm)^2 = (4cm)^2 + (\overline{BD})^2$

$25cm^2 = 16\ cm^2 + (BD)^2$

$25cm^2 - 16cm^2 = (BD)^2 \Rightarrow 9cm^2 = (BD)^2$

$\sqrt{(\overline{BD})^2} = \sqrt{9\ cm^2}$

$\overline{BD} = 3cm$

In right triangle ABD

$\overline{AB} = 4$ cm

$\overline{BD} = 3$ cm

$\overline{AD} = x$ cm $= ?$

By Pythagoras theorem

$(\overline{AD})^2 = (\overline{AB})^2 + (\overline{BD})^2$

$x^2 = (4cm)^2 + (3cm)^2$

$x^2 = 16\ cm^2 + 9\ cm^2$

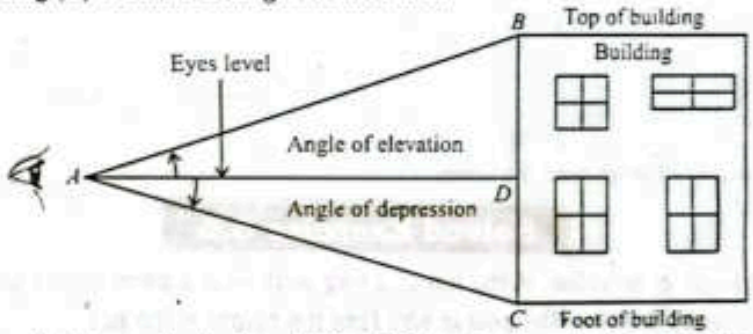
$x^2 = 25\ cm^2$

$\sqrt{x^2} = \sqrt{25\ cm^2}$

$x = 5cm$

The Angle of Elevation and the Angle of Depression

The angle between the horizontal line AD (eye level) and a line from the eye A to the top of building (B) is called an **angle of elevation.**



The angle between the horizontal line AD (eye level) and of line from the eye 'A' to the foot of the building (C) is called the angle of depression.

Example 19: The angle of elevation of the top of a pole 40 m high is 60° when se from a point on the ground level. Find the distance of the point from the foot of t pole.

Solution: In the triangle ABC, we have

$m\overline{BC} = 40$ m

$m\angle A = 60°$

Let $m\overline{AB} = x$ the point B is the foot of pole B(C)

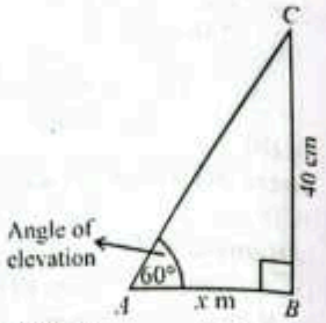
In right angled triangle ABC,

$$\tan 60° = \frac{m\overline{BC}}{m\overline{AB}}$$

$$\sqrt{3} = \frac{40}{x}$$

$$\Rightarrow \quad x = \frac{40}{\sqrt{3}}$$

$$\Rightarrow \quad x = 23.09\,\text{m}$$



Hence, distance of the point from the foot of the pole = 23.09 m

Example 20: From the top of a lookout tower, the angle of depression of a buildin has on the ground level of 45°. How far is a man on the ground from the tower, i the height of the lower is 30 m?

Solution: In the triangle ABC, AB is the tower and point C is the position of man we have

$m\overline{AB} = 30$ m

$m\angle CAD = m\angle C = 45°$

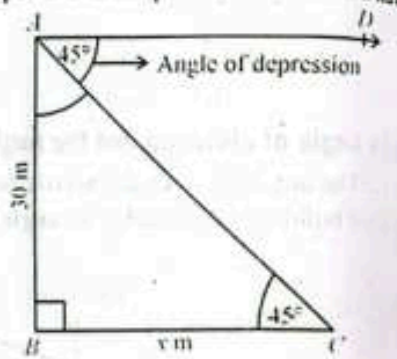
$m\overline{BC} = x\,\text{m} = ?$

Let x be the base of right angled triangle ABC.

$$\tan 45° = \frac{m\overline{AB}}{m\overline{BC}}$$

$$\Rightarrow \quad 1 = \frac{30}{x}$$

$$\Rightarrow \quad x = 30\ \text{m}$$



Hence, man is 30 m far from the tower.

Solved Exercise 6.6

1. The angle of elevation of the top of a flag post from a point on the ground leve 40 m away from the flag post is 60°. Find the height of the post.

Sol. Let $\overline{BC}$ is the hight of the post

In right triangle ABC, we have

$m\overline{AC} = 40$ m

$m\angle A = 60$

$m\overline{BC} = ?$

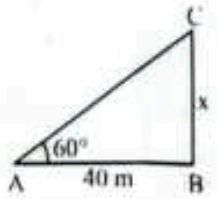
$$\frac{\overline{BC}}{\overline{AB}} = \tan 60°$$

$$\frac{x}{40} = \sqrt{3}$$

$x = 40 \times \sqrt{3}$

$x = 40 \times 1.732$

$x = 69.28$ m



So, the hight of post is 69.28 m

2. An isosceles triangle has a vertical angle of 120° and a base 10 cm long Find the length of its altitude.

Sol. If vertical angle of an isosocels triangle is of 120°, the each base angle will be of 30°.

Let $\overline{CD}$ is the length of its altitude.

In right triangle ACD, we have

$m\overline{AD} = 5$c m

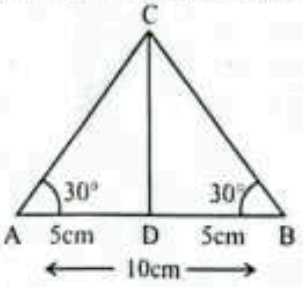
$m\angle A = 30°$

$$\frac{\overline{CD}}{\overline{AD}} = \tan 30°$$

$$\frac{\overline{CD}}{5} = \frac{1}{\sqrt{3}}$$

$$\overline{CD} = \frac{1}{\sqrt{3}} \times 5$$

$$\overline{CD} = \frac{5}{\sqrt{3}} = \frac{5}{1.732} = 2.9\text{cm}$$



3. A tree is 72 m high. Find the angle of elevation of its top from a point 100 m away on the ground level.

Sol. Let the angle of elevation = θ

In triangle ABC, we have

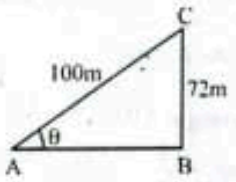
$m\angle A = 90°$

$m\overline{BC} = 72$m

$$\frac{\overline{CD}}{\overline{AD}} = 100\text{m}$$

$$\sin\theta = \frac{m\overline{BC}}{m\overline{AC}}$$

$$\sin\theta = \frac{72}{100}$$



$$\sin\theta = \frac{72}{100}$$

$$\sin\theta = 0.722$$

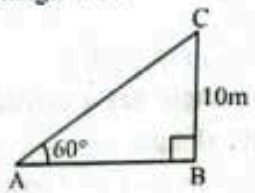
$$\theta = \sin^{-1} 0.72$$

$$\theta = 35.7°$$

Hence, angle of elevation ≅ 35.7°

4. A ladder makes an angle of 60° with the ground and reaches a height of 10m along the wall. Find the length of the ladder.

Sol. Let $m\overline{AC}$ is the length of ladder in right triangle ABC



In triangle ABC, we have

$m\angle A = 90°$

$m\overline{BC} = 10m$

$m\angle A = 60°$

$m\overline{AC} = ?$

then

$$\frac{m\overline{BC}}{m\overline{AC}} = \sin 60°$$

$$\frac{10}{m\overline{AC}} = \frac{\sqrt{3}}{2}$$

$$10 \times \frac{2}{\sqrt{3}} = \overline{AC}$$

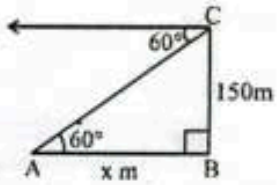
$$\frac{20}{\sqrt{3}} = m\overline{AC}$$

$$11.55 = m\overline{AC}$$

So, the length of ladder ≅ 11.55m

5. A light house tower is 150 m high from the sea level. The angle of depression from the top of the tower to a ship is 60°. Find the distance between the ship and the tower.

Sol. Let the distance between the ship and tour = $m\overline{AB} = x$ m



In triangle ABC, we have

$m\overline{BC} = 150m$, $m\angle A = 60°$

$m\overline{AB} = x = ?$

Then in right triangle ABC

$$\frac{m\overline{BC}}{m\overline{AB}} = \tan 60°$$

$$\frac{150}{x} = \sqrt{3}$$

$$\frac{150}{\sqrt{3}} = x$$

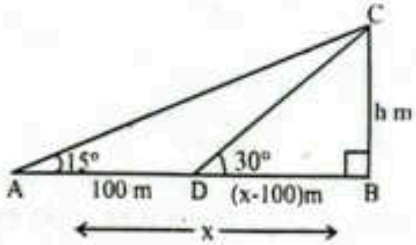
$$\frac{150}{1.732} = x$$

$$86.61 = x \Rightarrow x = 86.6 \text{ m}$$

Thus, the distance between the ship and the tower is 86.6 m.

Measure of an angle of elevation of the top of a pole is 15° from a point on the ground, in walking 100 m towards the pole the measure of angle is found to be 30°. Find the height of the pole.

Let the height of the pole = $m\overline{BC}$ = hm.



In right triangle ABC, we have

$m\angle B = 90°$, $m\angle B = 15°$, $m\overline{BC} = hm$

$$\tan 15° = \frac{m\overline{BC}}{m\overline{AB}}$$

$$\tan 15° = \frac{h}{x}$$

$$x = \frac{h}{\tan 15°} \quad \text{_____(i)}$$

In right triangle BCD, we have

$mBD = (x\text{-}100)m$, $m\angle BD = 30°$, $m\overline{BC} = hm$

$$\tan 30° = \frac{m\overline{BC}}{m\overline{BD}}$$

$$\tan 30° = \frac{h}{\tan 30°}$$

$$\tan 30° = (x\text{-}100) = h$$

$$x = \frac{h}{\tan 15°} \text{ from (i)}$$

$$\tan 30° = \left(\frac{h}{\tan 15°} - 100\right) = h$$

$$\tan 30° = \frac{h}{\tan 15} - \tan 30 \times 100 = h$$

$$0.5774 \times \frac{h}{0.2679} - 05774 \times 100 = h$$

$$2.1553h - 57.74 = h$$

$$2.1553h - h = 57.74$$

$$h(2.1553 - 1) = 57.74 \Rightarrow h(1.1553) = 57.74$$

$$h = \frac{57.74}{1.1553}$$

$$h = 49.98 \text{ m}$$

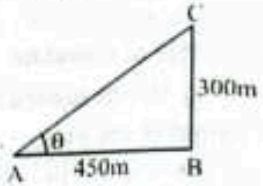
So, the hight of pole is 49.98 m.

7. Find the measure of an angle of elevation of the Sun, if a tower 300 m high casts a shadow 450 m long.

Sol. Let the measure of the angle of elevation of sun is = θ

In right triangle ABC, we have

$m\overline{AB}$ = 450m, 450m, $m\overline{BC}$ = 300m



$$\tan m\angle A = \frac{m\overline{BC}}{m\overline{AB}}$$

$$\tan\theta = \frac{300}{450}$$

$$\tan\theta = 0.667$$

$$\theta = \tan^{-1}(0.667)$$

$$\theta = 33.7°$$

Thus, the measure of angle of elevation of the sum is 33.7°.

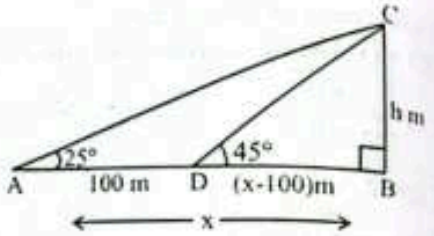
8. Measure of angle of elevation of the top of a cliff is 25°, on walking 100 metres towards the cliff, measure of angle of elevation of the top is 45o. Find the height of the cliff.

Sol. Let the height of cliff = $m\overline{BC}$ = hm

In right triangle ABC, we have

$m\angle A = 90°$, $m\angle A = 25°$

$m\overline{AB}$ = x m



$$\tan 25° = \frac{m\overline{BC}}{m\overline{AB}}$$

$$\tan 25° = \frac{h}{x}$$

$$x = \frac{h}{\tan 25°} \quad \text{_____(i)}$$

In right triangle BCD, we have

$m\overline{BD}$ = (x−100)m,. $m\angle D = 45°$, $m\overline{BC}$ = h m

$$\tan 45° = \frac{m\overline{BC}}{m\overline{BD}}$$

$$1 = \frac{h}{x - 100}$$

$$x - 100 = h$$

pur $$x = \frac{h}{\tan 25°}$$

$$\frac{h}{\tan 25°} - 100 = h$$

$$\frac{h}{\tan 25°} - h = 100$$

$$h\left(\frac{1}{\tan 25°} - 1\right) = 100$$

$$h\left(\frac{1}{0.4663} - 1\right) = 100$$

$$h(2.1445 - 1) = 100$$

$$h(1.1445) = 100$$

$$h = \frac{100}{1.1445}$$

$$h = 87.4 \text{ m}$$

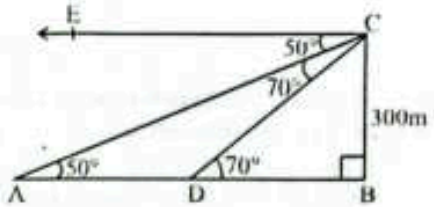
So, the hight of cliff ≅ 87.4 m

From the top of a hill 300 m high, the measure of the angle of depression of a point on the nearer shore of the river is 70° and measure of the angle of depression of a point, directly across the river is 50°. Find the width of the river How far is the river from the foot of the hill?

Sol. Let $m\overline{AD}$ is the width of the river and m $\overline{BD}$ is the distance of river from the foot of the hill

In right triangle ABC, we have

$m\overline{BC}$ = 300m, $m\angle A = 50°$



$$\tan 50° = \frac{m\overline{BC}}{m\overline{AB}}$$

$$1.192 = \frac{300}{m\overline{AB}}$$

$$m\overline{AB} = \frac{300}{1.192}$$

$$m\overline{AB} = 251.68\text{m}$$

In right triangle BCD, we have

$m\overline{BC}$ = 300m, $m\angle D = 70°$

$$\tan 70° = \frac{m\overline{BC}}{m\overline{BD}}$$

$$2.748 = \frac{300}{m\overline{AB}}$$

$$m\overline{BD} = \frac{300}{2.748}$$

$$m\overline{BD} = 10.9.17\text{m}$$

$$m\overline{AD} = m\overline{AB} - m\overline{BD}$$

$$= 251.68\text{m} - 10.917\text{m} = 142.51\text{m}$$

So, the width of river ≅ 142.51m

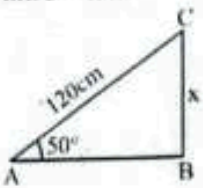
Distance of the river from foot of the hill ≅ 109.17m

10. A kite has 120 m of string attached to it when at an elevation of 50°. How far is it above the hand holding it? (Assume that the string is stretched.

Sol. Let the hight kite from the holding hand = $m\overline{BC} = x$ m

In right triangle ABC, we have

$m\overline{AC} = 120$m, $m\angle A = 50°$



$$\frac{m\overline{BC}}{m\overline{AB}} = \sin 50°$$

$$\frac{x}{120} = 0.766$$

$$x = 120 \times 0.766$$

$$x = 91.92 \text{ m}$$

So, the is 91.92m high bove holding hand kite.

Review Exercise 6

1. Four options are given against each statement. Encircle the correct one.

(i) The value of $\tan^{-1} 2$ in radians is:

(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{2}$ (c) $0.4636\,\pi$ (d) 1.1071

(ii) In a right triangle, the hypotenuse is 13 units and one of the angles is $\theta = 30°$. The length of the opposite side is:

(a) 6.5 units (b) 7.5 units (c) 6 units (d) 5 units

(iii) A person standing 50 m away from a building sees the top of the building at an angle of elevation of 45°. Height of the building is:

(a) 50 m (b) 25 m (c) 35 m (d) 70 m

(iv) $\sec^2\theta - \tan^2\theta =$ _____.

(a) $\sin^2\theta$ (b) 1 (c) $\cos^2\theta$ (d) $\cot^2\theta$

(v) If $\sin\theta = \frac{3}{5}$, and θ is an acute angle, $\cos^2\theta =$ _____:

(a) $\frac{7}{25}$ (b) $\frac{24}{25}$ (c) $\frac{16}{25}$ (d) $\frac{4}{25}$

(vi) $\frac{5\pi}{24}$ rad = _____ degrees.

(a) 30° (b) 37.5° (c) 45° (d) 52.5°

(vii) 292.5° = _____ rad.

(a) $\frac{17\pi}{6}$ (b) $\frac{17\pi}{4}$ (c) $1.6\,\pi$ (d) $1.625\,\pi$

(viii) Which of the following is a valid identity?

(a) $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$ (b) $\cos\left(\frac{\pi}{2} - \theta\right) = \cos\theta$

(c) $\cos\left(\frac{\pi}{2} - \theta\right) = \sec\theta$ (d) $\cos\left(\frac{\pi}{2} - \theta\right) = \operatorname{cosec}\theta$

(ix) $\sin 60° =$ _____.

(a) 1 (b) $\frac{1}{2}$ (c) $\sqrt{3}$ (d) $\frac{\sqrt{3}}{2}$

(x) $\cos^2 100\,\pi + \sin^2 100\,\pi =$ _____.

(a) 1 (b) 2 (c) 3 (d) 4

Answers:

i. 1.1071 ii. 6.5 units iii. 50 m iv. 1 v. $\frac{16}{25}$

vi. 37.5° vii. $1.625\,\pi$ viii. $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$ ix. $\frac{\sqrt{3}}{2}$ x. 1

2. Convert the given angles from:

(a) degrees to radians giving answer in terms of π.

(i) 255°

Sol. $255° = 225 \times \frac{\pi}{180}$

$= \frac{17}{12}\,\pi$ radian

or $1.42\,\pi$ rad

(ii) 142.5°

Sol. $142.5° = 142.5 \times \frac{\pi}{180}$

$= \frac{19}{24}\,\pi$ rador $0.79\,\pi$ rad

(ii) 75° 45′

Sol. $75°\,45' = 75° + \left(\frac{45}{60}\right)°$

$= 75° + 0.75°$

$= (75.75)°$

$= 75.75 \times 5\frac{\pi}{180}$

$= \frac{101}{240}\,\pi$ radian

or 0.42π rad

(b) radians to degrees giving answer in degrees and minutes.

(i) $\frac{17\pi}{24}$

Sol. $\frac{17\pi}{24} \times \frac{180}{\pi}$

$= 127.5° = 127° + 0.5°$

$= 127° + (0.5 \times 60)'$

$= 127° + 30' = 127°\,30'$

(ii) $\frac{7\pi}{12}$

Sol. $\frac{7\pi}{12} \times \frac{180}{\pi}$

$= 105°$

(iii) $\frac{11\pi}{16}$

Sol. $\frac{11\pi}{16} \times \frac{180}{\pi}$

$= 123.75° = 123° + 0.75°$

$= 123° + (0.75 \times 60)'$

$= 123° + 45'$

$= 123°\,45'$

3. Prove the following trigonometric identities:

(i) $\frac{\sin\theta}{1-\cos\theta} = \frac{1+\cos\theta}{\sin\theta}$

Sol. $\frac{\sin\theta}{1-\cos\theta} = \frac{1+\cos\theta}{\sin\theta}$

L.H.S $= \frac{\sin\theta}{1-\cos\theta} = \frac{\sin\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}$

$= \frac{\sin\theta\,(1+\cos\theta)}{(1)^2-(\cos)^2}$

$= \frac{\sin\theta\,(1+\cos\theta)}{1-\cos^2\theta}$

$= \frac{\sin\theta\,(1+\cos\theta)}{\sin^2\theta}$ $\quad\therefore\ \sin\theta + \cos^2\theta = 1$, $\sin\theta = 1-\cos^2\theta$

$= \frac{1+\cos\theta}{\sin\theta} = \text{R.H.S}$

Hence, $\frac{\sin\theta}{1-\cos\theta} = \frac{1+\cos\theta}{\sin\theta}$

(ii) $\sin\theta\,(\text{cosec}\,\theta - \sin\theta) = \frac{1}{\sec^2\theta}$

Sol. $\sin\theta\,(\text{cosec}\,\theta - \sin\theta) = \frac{1}{\sec^2\theta}$

L.H.S $= \sin\theta\,(\text{cosec}\,\theta - \sin\theta)$

$= \sin\theta\left(\frac{1}{\sin\theta} - \sin\theta\right)$ $\quad\because\ \text{cosec}\,\theta = \frac{1}{\sin\theta}$

$= \frac{\sin\theta}{\sin\theta} - \sin^2\theta = 1 - \sin^2\theta$

$= \cos^2\theta$ $\quad\because\ 1-\sin^2\theta = \cos^2\theta$

$= \frac{1}{\sec^2\theta}$ $\quad\because\ \cos^2\theta = \frac{1}{\sec^2\theta}$

$= \text{R.H.S}$

Hence, $\sin\theta\,(\tan\theta + \sin\theta) = \frac{1}{\sec^2\theta}$

(iii) $\frac{\text{cosec}\,\theta - \sec\theta}{\text{cosec}\,\theta + \sec\theta} = \frac{1-\tan\theta}{1+\tan\theta}$

Sol. $\frac{\text{cosec}\,\theta - \sec\theta}{\text{cosec}\,\theta + \sec\theta} = \frac{1-\tan\theta}{1+\tan\theta}$

L.H.S $= \frac{\text{cosec}\,\theta - \sec\theta}{\text{cosec}\,\theta + \sec\theta}$

$= \frac{\frac{\text{cosec}\,\theta}{\text{cosec}\,\theta} - \frac{\sec\theta}{\text{cosec}\,\theta}}{\frac{\text{cosec}\,\theta}{\text{cosec}\,\theta} + \frac{\sec\theta}{\text{cosec}\,\theta}} = \frac{1-\sec\theta\times\frac{1}{\text{cosec}\,\theta}}{1+\sec\theta\times\frac{1}{\text{cosec}\,\theta}}$

$= \frac{1-\frac{1}{\cos\theta}\times\sin\theta}{1+\frac{1}{\cos\theta}\times\sin\theta} = \frac{1-\frac{\sin\theta}{\cos\theta}}{1+\frac{\sin\theta}{\cos\theta}} = \frac{1-\tan\theta}{1+\tan\theta} = \text{R.H.S}$

Hence, $\frac{\text{cosec}\,\theta - \sec\theta}{\text{cosec}\,\theta + \sec\theta} = \frac{1-\tan\theta}{1+\tan\theta}$

(iv) $\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta}$

Sol. $\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta}$

L.H.S $= \tan\theta + \cot\theta$

$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$ $\quad\because\ \tan\theta = \frac{\sin\theta}{\cos\theta},\ \cot\theta = \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta\cos\theta} = \text{R.H.S}$ $\quad\because\ \sin^2\theta + \cos^2\theta = 1$

Hence, $\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta}$

(v) $\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta} + \frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta} = \frac{2}{1-2\sin\theta}$

Sol. $\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta} + \frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta} = \frac{2}{1-2\sin\theta}$

L.H.S $= \frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta} + \frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$

$= \frac{(\cos\theta+\sin\theta)^2 + (\cos\theta-\sin\theta)^2}{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}$

$= \frac{\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta + \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta}{(\cos\theta)^2 - (\sin\theta)}$

$$= \frac{1 + 2\sin\theta\cos\theta + 1 - 2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} \quad \because \sin^2\theta + \cos^2\theta = 1$$

$$= \frac{2}{1 - \sin^2\theta - \sin^2\theta} \quad \because \cos^2\theta = 1 - \sin^2\theta$$

$$= \frac{2}{1 - 2\sin\theta} = \text{R.H.S}$$

Hence, L.H.S = R.H.S

(vi) $\frac{1+\cos\theta}{1-\cos\theta} = (\text{cosec}\,\theta + \cot\theta)^2$

Sol. $\frac{1+\cos\theta}{1-\cos\theta} = (\text{cosec}\,\theta + \cot\theta)^2$

R.H.S $= (\text{cosec}\,\theta + \cot\theta)^2$

$$= \left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{1+\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1+\cos\theta)^2}{\sin\theta^2}$$

L.H.S $= \frac{1+\cos\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}$

$$= \frac{(1+\cos\theta)^2}{(1)^2 - (\cos)^2\theta}$$

$$= \frac{(1+\cos\theta)^2}{1 - \cos^2\theta}$$

$$= \frac{(1+\cos\theta)^2}{\sin^2\theta}$$

Hence, proved L.H.S = R.H.S

4. If $\tan\theta = \frac{3}{\sqrt{2}}$, then find the remaining trigonometric ratios when θ lies in 1st quadrant.

Sol. Given $\tan\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{a}{c}$

Where, $a = 3$, $c = \frac{3}{\sqrt{2}}$

By Pythagoras theorem, we have

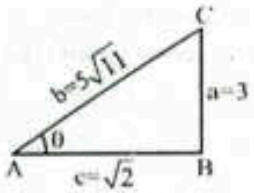
$b^2 = a^2 + c^2$

$b^2 = 3^2 + (\sqrt{2})^2$

$b^2 = 9 + 2 \Rightarrow b^2 = 11$

$\sqrt{b^2} = \sqrt{11}$

$b = \sqrt{11}$



Therefore $\sin\theta = \frac{a}{b} = \frac{3}{\sqrt{11}}$; $\text{cosec}\,\theta = \frac{b}{a} = \frac{\sqrt{11}}{3}$

$\cos\theta = \frac{c}{b} = \frac{2}{\sqrt{11}} = \sqrt{\frac{2}{11}}$; $\sec\theta = \frac{b}{c} = \frac{\sqrt{11}}{\sqrt{2}} = \sqrt{\frac{11}{2}}$

$\tan\theta = \frac{a}{c} = \frac{3}{\sqrt{2}}$; $\cot\theta = \frac{c}{a} = \frac{\sqrt{2}}{3}$

From a point on the ground, the angle of elevation to the top of a 30 m high building is 28°. How far is the point from the base of the building?

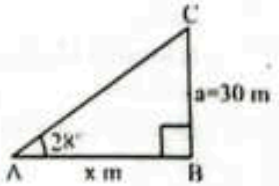
Let the distance from the point to the base of building = $m\overline{AB} = x$m

In the triangle ABC, we have

$m\overline{BC} = 40$m

$m\angle A = 28°$, $m\angle B = 90°$

In right triangle ABC



$\tan 28° = \frac{m\overline{BC}}{m\overline{AB}}$

$0.5317 = \frac{30}{x}$

$x = \frac{30}{0.5317}$

$x = 56.42$m

Thus, the distance from the base of building to the given point is 56.5m.

A ladder leaning against a wall forms an angle of 65° with the ground. If the ladder is 10 meters long, how high does it reach on the wall?

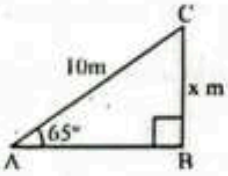
Sol. Let the hight of wall

where the upper end of ladder reached is mBC = xm

In the triangle ABC, we have

$m\angle B = 90°$, $m\angle A = 65°$, $m\overline{AC} = 10$m

In the right triangle ABC



$\frac{m\overline{BC}}{m\overline{AC}} = \sin 65°$

$\frac{x}{10} = 0.9063$

$x = 10 \times 0.9063$

$x = 9.06$ m

So, the hight of wall where the ladder reached is 9.06 m.

Objective Type Questions

Additional MCQ's

Multiple Choice Questions prepared in the light of new Examination Techniques (Knowledge, Understanding, Application, Analytical & Conceptual)

6.1 Identifying Angles in Standard Position (Degrees and Radians)

○ Encircle the correct option.

1. A plane figure which is formed by two rays showing a common end point is:
(a) angle (b) degree (c) side (d) plane
2. The two rays of an angle are known as:
(a) angle (b) sides (c) plane (d) degree
3. An angle is represented by:
(a) > (b) < (c) θ (d) m
4. The angle will be _____ if the terminal side is rotated counter clockwise from initial side.
(a) positive (b) negative (c) neutral (d) noen of above
5. The angle will be ____ if the terminal side is rotated clock wise from the initial side.
(a) positive (b) negative (c) neutral (d) none of above
6. Which of them is unit of measurement:
(a) degree (b) plane (c) side (d) vector
7. A full rotation around central point forms angle of _____.
(a) 90° (b) 180° (c) 270° (d) 360°
8. One quarter of a full rotation is:
(a) 90° (b) 180° (c) 270° (d) 360°
9. A straight angle or half of a full rotation is:
(a) 90° (b) 180° (c) 270° (d) 360°
10. 1° = _____.
(a) 15 minutes (b) 30 minutes (c) 60 minutes (d) 90 minutes
11. 1' = _____.
(a) 15 seconds (b) 30 seconds (c) 90 seconds (d) 60 seconds
12. 1° = _____.
(a) 900 seconds (b) 1800 seconds (c) 3600 seconds (d) 7200 seconds
13. 73.12° is equal to _____ minutes.
(a) 4387.2 (b) 4388.2 (c) 4390 (d) 4373
14. 109.42° = _____.
(a) 109° 25' 12" (b) 109° 10' 10" (c) 109° 25' 25" (d) 109° 12' 12"

15. 45° 45' 45" = _____.
(a) 45.45° (b) 45.7625° (c) 45.25° (d) 45.76°
16. A complete circle has an arc length equal to the circumference
(a) 2r (b) 2π (c) 2πr (d) πr²
17. 1 radian = _____.
(a) 56.2958° (b) 57.2958° (c) 58.2958° (d) 59.2958°
18. $\frac{180°}{\pi}$ degree = _____.
(a) 1 rad (b) 2 rad (c) 3 rad (d) 4 rad
19. $\frac{\pi}{180°}$ rad = _____.
(a) 45° (b) 30° (c) 10° (d) 1°
20. $\frac{5\pi}{3}$ rad = _____.
(a) 100° (b) 200° (c) 300° (d) 400°
21. $\frac{7\pi}{6}$ rad = _____.
(a) 200° (b) 210° (c) 220° (d) 230°
22. 330° = _____.
(a) $\frac{11\pi}{6}$ (b) $\frac{13\pi}{5}$ (c) $\frac{14\pi}{4}$ (d) $\frac{15\pi}{2}$
23. 68.75° = _____.
(a) 1.1 rad (b) 1.2 rad (c) 1.3 rad (d) 1.4 rad
24. Arc length of sector is:
(a) $\ell = r\theta$ (b) $r = \ell\theta$
(c) $\theta = r\ell$ (d) none of above



25. Area of sector given figure is:
(a) $r^2\theta$ (b) $\frac{1}{2}r^2\theta$
(c) $\frac{1}{3}r^2\theta$ (d) $\frac{1}{4}r^2\theta$



6.2 Trigonometric Ratios

26. $\frac{\text{Perpendicular}}{\text{Hypotenuse}}$ = _____.
(a) sin θ (b) cos θ (c) tan θ (d) none of above
27. $\frac{\text{Base}}{\text{Hypotenuse}}$ = _____.
(a) sin θ (b) cos θ (c) tan θ (d) none of above

28. $\frac{\text{Perpendicular}}{\text{Base}}$ = _____.
(a) sin θ (b) cos θ (c) tan θ (d) none of above

29. $\frac{\text{Hypotenuse}}{\text{Perpendicular}}$ = _____.
(a) cosec θ (b) sec θ (c) cot θ (d) none of above

30. $\frac{\text{Hypotenuse}}{\text{Base}}$ = _____.
(a) cosec θ (b) sec θ (c) cot θ (d) tan θ

31. $\frac{\text{Base}}{\text{Perpendicular}}$ = _____.
(a) tan θ (b) sec θ (c) cot θ (d) cosec θ

32. $\frac{1}{\sin\theta}$ = _____.
(a) cot θ (b) tan θ (c) sec θ (d) cosec θ

33. $\frac{1}{\cos\theta}$ = _____.
(a) sec θ (b) cosec θ (c) tan θ (d) cot θ

34. $\frac{1}{\tan\theta}$ = _____.
(a) sec θ (b) cot θ (c) cosec θ (d) sin θ

6.3 Trigonometric Identities

35. $\sin^2\theta + \cos^2\theta$ = _____.
(a) 0 (b) 1 (c) 2 (d) 3

36. $\tan^2 + 1$ = _____.
(a) $\sin^2\theta$ (b) $\cos^2\theta$ (c) $\sec^2\theta$ (d) $\cot^2\theta$

37. $\cot^2 + 1$ = _____.
(a) $\tan^2\theta$ (b) $\cos^2\theta$ (c) $\text{cosec}^2\theta$ (d) $\cot^2\theta$

38. $(\sec^2\theta - 1)\cos^2\theta$ = _____.
(a) $\sin^2\theta$ (b) $\cos^2\theta$ (c) $\tan^2\theta$ (d) $\text{cosec}^2\theta$

6.4 Values of Trigonometric Ratios and Angles

39. $\sin 45° =$ _____.
(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) 1

40. $\cos 45° =$ _____.
(a) 0 (b) 1 (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

41. $\tan 45° =$ _____.
(a) 0 (b) 1 (c) 2 (d) $\sqrt{2}$

42. $\text{cosec } 45° =$ _____.
(a) 0 (b) 1 (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

43. $\sec 45° =$ _____.
(a) 0 (b) 1 (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

44. $\cot 45° =$ _____.
(a) 0 (b) 1 (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

45. $\sin 30° =$ _____.
(a) 1 (b) 0 (c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$

46. $\cos 30° =$ _____.
(a) 0 (b) 1 (c) $\sqrt{2}$ (d) $\frac{\sqrt{3}}{2}$

47. $\tan 30° =$ _____.
(a) 1 (b) $\sqrt{2}$ (c) $\frac{1}{\sqrt{3}}$ (d) 0

48. $\sec 30° =$ _____.
(a) 0 (b) $\frac{2}{\sqrt{3}}$ (c) 2 (d) $\sqrt{2}$

49. $\text{cosec } 30° =$ _____.
(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$ (c) 2 (d) $\frac{2}{\sqrt{3}}$

50. $\cot 30° =$ _____.
(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{2}}$ (c) 0 (d) $\sqrt{3}$

51. $\sin 60° =$ _____.
(a) $\frac{\sqrt{3}}{2}$ (b) $\sqrt{3}$ (c) $\sqrt{2}$ (d) $\frac{2}{\sqrt{3}}$

52. $\sec 60° =$ _____.
(a) 0 (b) 1 (c) 2 (d) $\sqrt{3}$

53. $\text{cosec } 60° =$ _____.
(a) $\sqrt{2}$ (b) $\frac{2}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $\frac{\sqrt{3}}{2}$

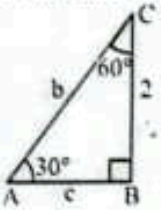
54. $\tan 60° =$ _____.
(a) 1 (b) 0 (c) $\sqrt{2}$ (d) $\sqrt{3}$

55. $\cos 60° =$ _____.
(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{3}}$

56. $\cot 60° =$ _____.
(a) 0 (b) 1 (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{3}}$

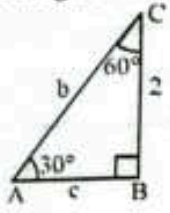
6.5 Solution of Triangle

57. In figure value of b is:



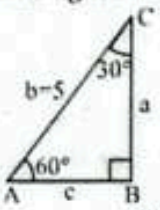
(a) 1 (b) 2
(c) 3 (d) 4

58. In the given figure value of c is:



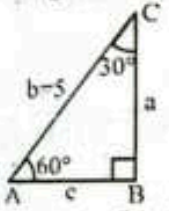
(a) $\sqrt{3}$ (b) $\sqrt{2}$
(c) $2\sqrt{3}$ (d) $3\sqrt{2}$

59. In the given figure value of a is:



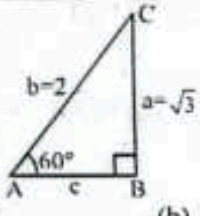
(a) 4.33 (b) 3.33
(c) 2.33 (d) 1.33

60. In the given figure value of c is:



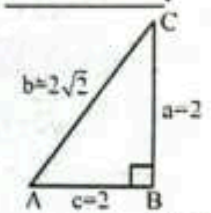
(a) 0.5 (b) 1.5
(c) 2.5 (d) 3.5

61. In the given figure value of m∠C = _____.



(a) 30° (b) 45°
(c) 60° (d) 90°

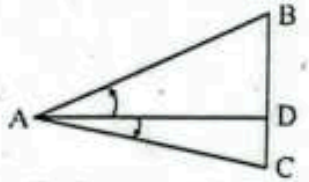
62. In the given figure the value of m∠A= _____.



(a) 30° (b) 45°
(c) 60° (d) 90°

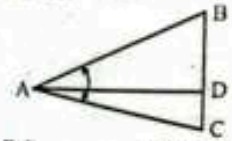
6.6 The angle of elevation and angle of depression

63. According to figure angle of elevation is:



(a) ∠BAD (b) ∠BDA
(c) ∠ABD
(d) none of the above

64. According to figure angle of dpression is:



(a) ∠ABC (b) ∠ADB
(c) ∠ACD (d) ∠CAD

Answers:

1. angle	2. sides	3. θ	4. positive	5. negative
6. degree	7. 360°	8. 90°	9. 180°	10. 60 minutes
11. 60 seconds	12. 3600 seconds	13. 4387.2	14. 109° 25′ 12″	15. 45.7625°
16. 2π	17. 57.2958°	18. 1 rad	19. 1°	20. 300°
21. 210°	22. $\frac{11\pi}{6}$	23. 1.2 rad	24. $\ell = r\theta$	25. $\frac{1}{2}r^2\theta$
26. $\sin\theta$	27. $\cos\theta$	28. $\tan\theta$	29. $\operatorname{cosec}\theta$	30. $\sec\theta$
31. $\cot\theta$	32. $\operatorname{cosec}\theta$	33. $\sec\theta$	34. $\cot\theta$	35. 1
36. $\sec^2\theta$	37. $\operatorname{cosec}^2\theta$	38. $\sin^2\theta$	39. $\frac{1}{\sqrt{2}}$	40. $\frac{1}{\sqrt{2}}$
41. 1	42. $\sqrt{2}$	43. $\sqrt{2}$	44. 1	45. $\frac{1}{2}$
46. $\frac{\sqrt{3}}{2}$	47. $\frac{1}{\sqrt{3}}$	48. $\frac{2}{\sqrt{3}}$	49. 2	50. $\sqrt{3}$
51. $\frac{\sqrt{3}}{2}$	52. 2	53. $\frac{2}{\sqrt{3}}$	54. $\sqrt{3}$	55. $\frac{1}{2}$
56. $\frac{1}{\sqrt{3}}$	57. 4	58. $2\sqrt{3}$	59. 4.33	60. 2.5
61. 30°	62. 45°	63. ∠BAD	64. ∠CAD	

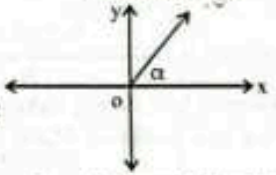
Conceptual MCQ's

Encircle the correct option.

1. **The relationships between angles and sides of triangle is:**
(a) Trigonometry (b) sets (c) matrix (d) Algebra

2. **Sides of the angle means:**
(a) two rays (b) No rays (c) three rays (d) None of above

3. **In the figure angle α known as:**



(a) negative angle (b) positive angle (c) neutral angle (d) None of above

4. **A complete circle has:**
(a) 90° (b) 180° (c) 270° (d) 360°

5. **60′ = _____:**
(a) 50° (b) 10° (c) 1° (d) 0°

6. **The term radian first time used in:**
(a) 1773 (b) 1873 (c) 1973 (d) 2073

7. **15° 15′ is equal to _____**
(a) 0.266 rad (b) 1.266 rad (c) 2.266 rad (d) 3.266 rad

8. **Core trigonometric functions are:**
(a) Sin, cos, tan (b) Cosec, sec, cot (c) Sin, Sec, tan (d) cos, cosec, cot

9. **$1 - \cos^2\theta =$ _____**
(a) $\sec^2\theta$ (b) $\sin^2\theta$ (c) $\tan^2\theta$ (d) $\cot^2\theta$

10. **$\sqrt{2}$ is equal to:**
(a) $\sin 45°$ (b) $\tan 45°$ (c) $\sec 45°$ (d) None of above

Answers:

1.	Trigonometry	2.	two rays	3.	positive angle	4.	360°	5.	1°
6.	1873	7.	0.266 rad	8.	Sin, cos, tan	9.	$\sin^2\theta$	10.	sec 45°

Additional Short Questions

Short Answer Questions prepared in the light of new Examination Techniques (Knowledge, Understanding, Application, Analytical & Conceptual)

6.1 Identifying Angles in Standard Position (Degrees and Radians)

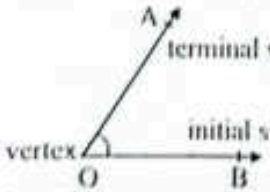
1. Define Trigonometry.

Ans. Trigonometry is a branch of mathematics which deals with the relationship between the angles and sides of triangles especially right angled triangles.

2. Write the importance of trigonometry.

Ans. It plays a vital role in various fields like physics engineering, architecture and astronomy. We can solve problems involving angles and distances which appears in real life situations like calculating the height of buildings distances between objects and angles measurements in navigation.

3. Define angle.



Ans. A plane figure which is formed by two rays showing a common end point is called angle. OA and OB are rays and angle is AOB. The two rays are known as sides vertex is located at origin.

4. How many types of angles? Name them?

Ans. There are six types of angles:

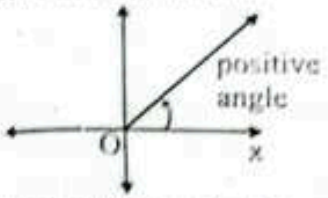
(i) Acute angle $0 < \theta < 90°$ (ii) Obtuse angle $90° < \theta < 180°$
(iii) Right angle $\theta = 90°$ (iv) Straight angle $\theta = 180°$
(v) Reflex angle $180° < \theta < 360°$
(vi) Full rotation $\theta = 360°$

5. How we measures an angle?

Ans. An angle is measured from the initial side to the terminal side. This usually represented by Greek symbol $\theta, \alpha, \beta, \gamma$ etc.

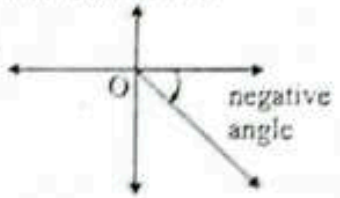
6. What are positive angles?

Ans. The angle will be positive if the terminal side is rotated counter clock wise from the initial side.



7. What are negative angles?

Ans. The angle will be negative if the terminal side is rotated clock wise from the initial side.

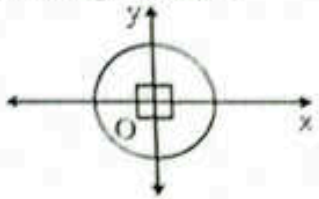


8. What do you mean by Degree?

Ans. A degree (°) is a unit of meaurement of angles. It represents one $\frac{1}{360°}$ of a full rotation around a point or a degree is the meaure of an angle with complete circle being 360°.

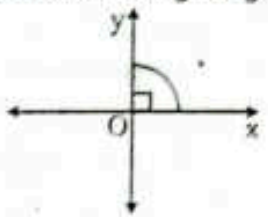
9. What is full cirlce?

Ans. A full rotation around a central point forms an angle of 360°.



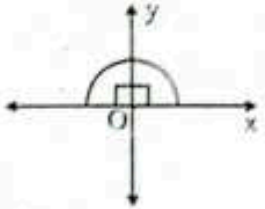
10. What is right angle?

Ans. one quarter of a full rotation or a 90° angle is known as right angle.



11. What is half cirlce?

Ans. A straight angle or half of a full rotation meaures 180°.



12. Write the further division of degree.

Ans. $1° = 60'$ (60 minutes) $1' = 60''$ (60 seconds) $1° = 3600''$ (60×60 seconds)

13. Write the steps to convert degree into minutes and seconds.

Ans. (i) Separate the whole number part (degrees) of the decimal.
(ii) Multiply the decimal part by 60 to get the minutes.
(iii) The whole number part of the result is the minutes. Multiply the decimal part of the minutes by 60 to get the seconds.

14. Convert 109.42° into minutes.

Ans. 109.42°
$(109.42 \times 60)' = 6665.2'$

15. Write steps convert degree minutes and seconds to decimal degrees.

Ans. (i) Keep the degrees as they are,
(ii) Convert minutes to decimal degrees: Divide the number of minutes by 60.
(iii) Convert seconds to decimal degrees: Divide the number of seconds by 3600.
(iv) Add all the values together.

16. Convert 94° 27' 54" to decimal degrees.?

Sol.(i) Degrees: (94) **(ii)** Minutes to decimal: $\frac{27}{60} = 0.45$

(iii) Seconds to decimal: $\frac{54}{3600} = 0.015$ **(iv)** Add all $94+0.45+0.015 = 95$

17. What is radian?

Ans. The radian denoted by symbol "rad" is the unit of angle in SI and standard unit of angular measure used in many areas of mathematics.

18. Define radian.

Ans. The angle subtended at the centre of circle by an arc whose length is equal to radius of the circle.

19. What is circumference?

Ans. A complete circle has an arc length equal to the circumference $2\pi r$. So the angle subtended by the entire circle is 2π radius.

20. How many degrees are there in one radius?And vise verse?

Sol. $1 \text{ rad} = \frac{180°}{\pi}$ degrees

$1° = \frac{\pi}{180°}$ rad

21. Convert $\frac{11\pi}{6}$ into degrees.

Sol. $\frac{11\pi}{6}$ rad $= \frac{11\pi}{6} \times \frac{180°}{\pi} = \frac{11\times180}{6}$

$= 11 \times 30 = 330°$

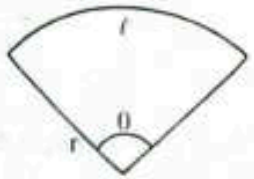
22. Convert 315° into radian. **Sol.** $315° \times \frac{\pi}{180°} = \frac{7\pi}{4}$ rad $= 5.498$ rad

23. Find arc length of sector of circle.

Ans. Arc length =?

$\ell = \frac{\theta}{360°} \times 2\pi r$

$\ell = \frac{\theta}{2\pi} \times 2r = r\theta$

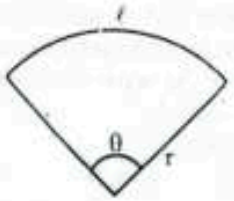


24. Find area of sector of circle?

Sol. $A = \frac{\theta}{360°} \times \pi r^2$

$= \frac{\theta}{2\pi} \times \pi r^2$

$= \frac{1}{2} r^2\theta$



25. If $r = 5$cm, $\ell = 11$cm $\theta = ?$

Sol. $\ell = r\theta$

$11 = 5\theta \Rightarrow \theta = \frac{11}{5} = 2.2$ rad

$\theta = 2.2 \times \frac{180}{\pi} = 126.1°$

6.2 Trigonometric Ratios

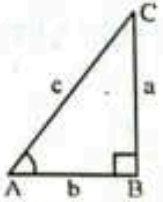
26. What are the core trigonometric functions?

Ans. Sine, cosine and tangent are the core trigonometric functions which are relate angles to side lengths in right-angled triangles.

27. How we can find the sinθ?

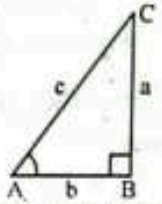
Sol. $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

$\sin\theta = \frac{a}{c}$



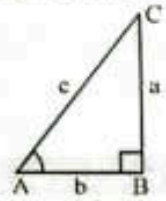
28. How we can find cosθ?

Sol. $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{b}{c}$



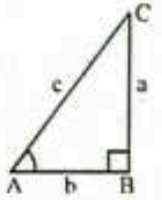
29. How we can find tanθ?

Sol. $\tan\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{a}{b}$



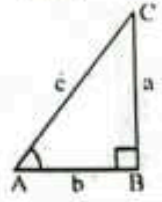
30. According to given find cosecθ?

Sol. $\text{cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{c}{a}$



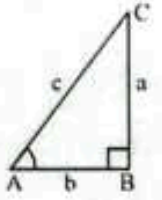
31. According to figure what is value of secθ?

Sol. $\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{c}{b}$



32. According to figure what is the value of cotθ?

Sol. $\cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{b}{a}$



6.3 Trigonometric Identifies

33. What is the value of $\sin^2\theta + \cos^2\theta$.

Sol. $\sin^2\theta + \cos^2\theta = 1$.

34. What is the value of $\tan^2\theta + 1$.

Sol. $\tan^2\theta + 1 = \sec^2\theta$.

35. What is the value of $\text{cosec}^2 - 1$.

Sol. $\text{cosec}^2\theta - 1 = \cot^2\theta$.

36. Show that $\tan\theta+\cot\theta=\sec\theta\,\text{cosec}\theta$.

Sol. L.H.S $= \tan\theta + \cot\theta$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta} \quad \sin^2\theta+\cos^2\theta = 1$$

$$= \frac{1}{\sin\theta\cos\theta} = \frac{1}{\sin\theta} \quad \frac{1}{\cos\theta}$$

$$= \text{cosec}\theta\,\sec\theta$$

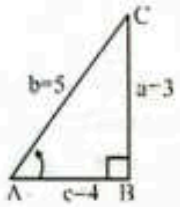
37. If $\tan\theta = \frac{3}{4}$ find the remaining trigonometric ratios when θ lies in I quadrant.

Sol. $\tan\theta = \frac{3}{4} = \frac{a}{c}$

$a = 3,\ c = 4$

$b^2 = a^2 + c^2 = 9 + 16 = 25$

$b = 5$



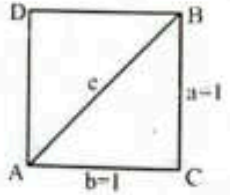
$\sin\theta = \frac{a}{b} = \frac{3}{5}$	$\text{cosec}\theta = \frac{b}{a} = \frac{5}{3}$
$\cos\theta = \frac{c}{b} = \frac{4}{5}$	$\sec\theta = \frac{b}{c} = \frac{5}{4}$
$\tan\theta = \frac{a}{c} = \frac{3}{4}$	$\cot\theta = \frac{c}{a} = \frac{4}{3}$

6.4 Values of Trigonometric Ratios and Angles

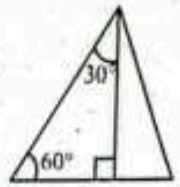
38. According to figure write all trigonometric ratios of $45°\left(\frac{\pi}{4}\right)$.

Sol.

$\sin 45° = \frac{1}{\sqrt{2}}$	$\text{cosec}\theta = \sqrt{2}$
$\cos 45° = \frac{1}{\sqrt{2}}$	$\sec\theta = \sqrt{2}$
$\tan 45° = 1$	$\cot\theta = 1$



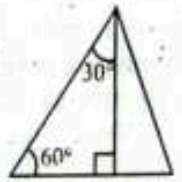
39. According to figure write all trignometric ratios of $30°\left(\frac{\pi}{6}\right)$.



Sol. $\sin 30° = \frac{1}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$, $\tan 30° = \frac{1}{\sqrt{3}}$

$\text{cosec}\ 30° = 2$, $\sec 30° = \frac{2}{\sqrt{3}}$, $\cot 30° = \sqrt{3}$

40. According to figure write all trigonometric ratio of $60°\left(\frac{\pi}{3}\right)$.



Sol. $\sin 60° = \frac{\sqrt{3}}{2}$, $\cos 60° = \frac{1}{2}$, $\tan 60° = \sqrt{3}$

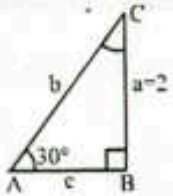
$\text{cosec}60° = \frac{2}{\sqrt{3}}$, $\sec 60° = 2$, $\cot 60° = \frac{1}{\sqrt{3}}$

6.5 Solution of Triangle

41. How many angles and sides of a triangle?

Ans. There are three angles and sides of a triangle.

42. According to figure find the unknown values.



Sol. $m\angle C = m\angle B - m\angle A = 90° - 30° = 60°$

and $\frac{a}{b} = \sin 30° \Rightarrow \frac{2}{b} = \sin 30° \Rightarrow \frac{2}{b} = \frac{1}{2} = b = 4$

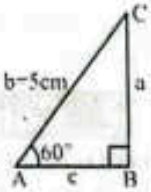
$$\frac{a}{c} = \tan 30 \Rightarrow \frac{2}{c} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow c = 2\sqrt{3}$$

43. According to figure find the unkown values.

Sol. $m\angle A = 60°$, $m\angle B = 90°$

$m\angle C = 90° - 60° = 30°$



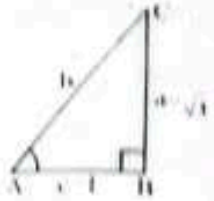
Now $\frac{a}{b} = \sin 60°$

$\frac{a}{5} = \frac{\sqrt{3}}{2} \Rightarrow a = \frac{5\sqrt{3}}{2} = 4.33\text{cm}$ and $\frac{c}{b} = \cos 60°$

$\frac{c}{5} = \frac{1}{2} \Rightarrow c = \frac{5}{2} = 2.5\text{cm}$

44. According to figure find the unkown values.

Sol. According to Pythagoras Theorem



$$b^2 = c^2 + a^2$$

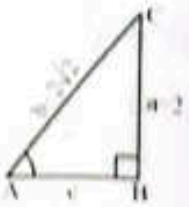
$$b^2 = 1^2 + \left(\sqrt{3}\right)^2 = 1 + 3 = 4 \Rightarrow b = 2$$

Now $\sin m\angle A = \frac{a}{b} = \frac{\sqrt{3}}{2} = 60°$

Now $m\angle C = m\angle B - m\angle A = 90 - 60° = 30°$

45. According to figure find the unkown values.

Sol. By pythagoras theorem



$$b^2 = a^2 + c^2$$

$$c^2 = b^2 - a^2 = \left(2\sqrt{2}\right)^2 - (2)^2 = 8 - 4 = 4 \Rightarrow c = 2$$

Now $\frac{c}{b} = \cos m\angle A = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

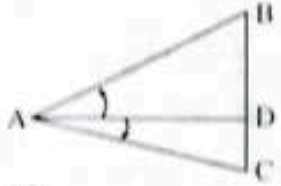
$m\angle A = 45°$

So, $m\angle C = m\angle B - m\angle A = 90° - 45° = 45°$

6.6 The angle of elevation and angle of depression

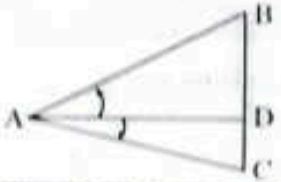
46. What is angle of elevation?

Sol. The angle between the horizontel line AD and a line from the eye A to the top (B) is called angle of elevation.



47. What is angle of depression?

Sol. The angle made the horizontal line AD and of line from the A to the bottom of the C is called angle of elevation.



Conceptual Short Questions

1. How many types of angles?

Sol. There are six types of angles.

2. What is the symbol of degree?

Ans. The symbol of degree is (°).

Define Pythagoras theorem

The square of the length of the hypotenuse of a right triangle is equal to of the square of the lengths of the other two sides.

$c^2 = a^2 + b^2$

What is the value of tan $\left(\frac{\pi}{6}\right)$?

The value of tan $\left(\frac{\pi}{6}\right)$ is $\frac{1}{\sqrt{3}}$.

What is the value of cosec $\left(\frac{\pi}{4}\right)$?

The value of cosec $\left(\frac{\pi}{4}\right)$ is $\sqrt{2}$.

What is value of cot $\left(\frac{\pi}{3}\right)$?

The value of cot $\left(\frac{\pi}{3}\right)$ is $\frac{1}{\sqrt{3}}$.

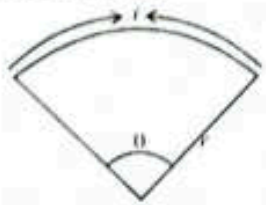
What is the value of cos 0°?

The value of cos 0° is 1.

What is the value of tan $\left(\frac{\pi}{4}\right)$?

The value of tan $\left(\frac{\pi}{4}\right)$ is 1.

Look at the figure. Find the area of sector.



As $A = \frac{\theta}{360}(\pi r^2)$

$= \frac{\theta}{2\pi}(\pi r^2)$

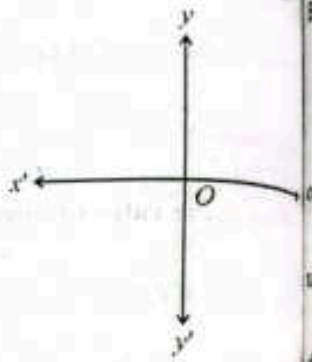
$= \frac{1}{2}\theta r^2$

— ✲✲✲ —

Unit 7 Coordinate Geometry

Coordinate Plane

Draw in a plane two mutually perpendicular number lines x' x and y' y, one horizontal and the other vertical. Let be their point of intersection, origin and the real number 0 of both the lines is represented by O. The two lines are called the **coordinate axes.** The horizontal line x'Ox is called the x-axis and the vertical line y'Oy is called the **y-axis.**



Suppose P is any point in the plne. Then **P** can be located by using an ordered pa of real numbers.

The ordered pair (x, y) gives us enough information to locate the point P. Thus, has coordinates (x, y). The first component of the ordered pair (x, y) is called x-coordina or abscissa and the second component is called y-coordinate or ordinate of P. Th method of pairing off in a one-to-one fashion the points in a plane with ordered pairs o real numbers is called the two dimensional rectangular (or Cartesian) coordinate system.

Quadrants of Plane:

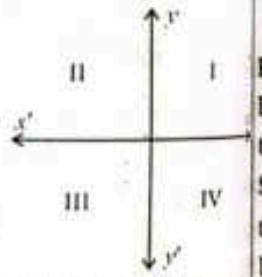
The coordinate axes divide the plane into four equal parts called quadrants. They ar defined as follows:

Quadrant I: All points (x, y) with $x > 0, y > 0$

Quadrant II: All points (x, y) with $x < 0, y > 0$

Quadrant III: All points (x, y) with $x < 0, y < 0$

Quadrant IV: All points (x, y) with $x > 0, y < 0$



The point P in the plane that corresponds to an ordered pair (x, y) is called the graph of (x, y).

Thus, given a set of ordered pairs of real numbers, the graph of the set is th aggregate of all points in the plane that correspond to ordered pairs of the set.

The Distance Formula

Let A (x_1, y_1) and B (x_2, y_2) be two points in the plane.

To find the distance $d = |\overline{AB}|$, we draw a horizontal line from A to a point C lies directly below B, forming a right triangle ABC.

Note that:
$|\overline{AB}|$ stands for $m\overline{AB}$.

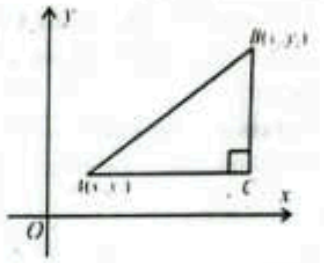
So that $|\overline{AC}| = |x_2 - x_1|$ and $|\overline{BC}| = |y_2 - y_1|$

By using Pythagorean Theorem, we have

$$d^2 = |\overline{AB}|^2 = |\overline{AC}|^2 + |\overline{BC}|^2$$

$$= (x_2 - x_1)^2 + (y_2 - y_1)^2$$

or $$\boxed{d = |AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}} \quad \ldots(i)$$



The distance is always taken to be non-negative. It is not a directed distance from A to B.

If A and B lie on a line parallel to one of the coordinate axes, then by the formula (i), the distance $\overline{AB}$ is absolute value of the directed distance $\overline{AB}$.

The formula (i) shows that any of the two points can be taken as first point.

Example 1: Find the distance between the points:

(i) A(5, 6), B(5, –2) **(ii) C(–4, –2), D(0, 9)**

Solution: By the distance formula, we have

(i) $d = |\overline{AB}| = \sqrt{(5-5)^2 + (-2-6)^2}$

$d = |\overline{AB}| = \sqrt{(0)^2 + (-8)^2}$

$d = |\overline{AB}| = \sqrt{0 + 64} = 8$

(ii) $d = |\overline{CD}| = \sqrt{(0-(-4))^2 + (9-(-2))^2}$

$d = |\overline{CD}| = \sqrt{(0+4)^2 + (9+2)^2}$

$d = |\overline{CD}| = \sqrt{4^2 + 11^2}$

$d = |\overline{CD}| = \sqrt{16 + 121} = \sqrt{137}$

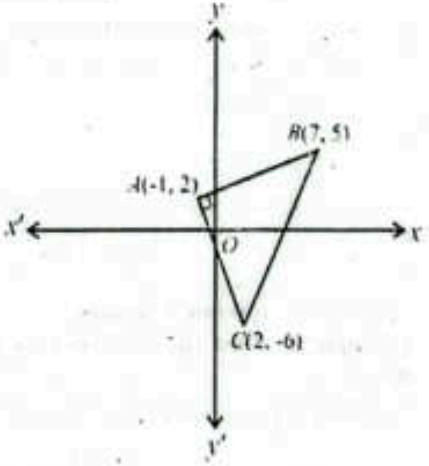
Example 2: Show that the points A(-1, 2), B(7, 5) and C (2, -6) are vertices of a right triangle.

Solution: Let a, b and c denote the lengths of the sides BC, CA and AB respectively. By the distance formula, we have

$$c = |\overline{AB}| = \sqrt{(7-(-1))^2 + (5-2)^2} = \sqrt{73}$$

$$a = |\overline{BC}| = \sqrt{(2-7)^2 + (-6-5)^2} = \sqrt{146}$$

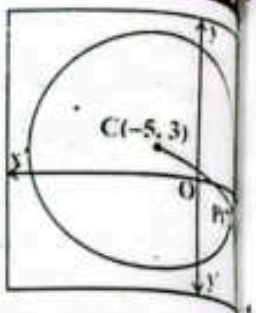
$$b = |\overline{CA}| = \sqrt{(2-(-1))^2 + (-6-2)^2} = \sqrt{73}$$



Clearly: $a^2 = b^2 + c^2$

Therefore, ABC is a right triangle with right angle at A.

Example 3: The point C (–5, 3) is the centre of a circle and P (7, –2) lies on the circle. What is the radius of the circle?

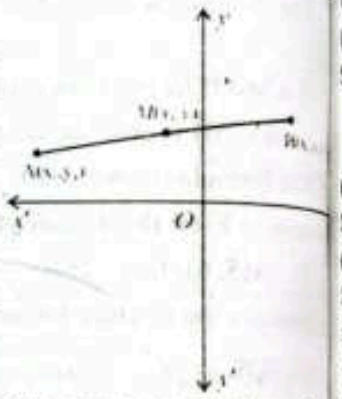


Solution: The radius of the circle is the distance from C to P. By the using distance formula, we have

$$\text{Radius} = CP = \sqrt{(7-(-5))^2+(-2-3)^2}$$
$$= \sqrt{144+25} = \sqrt{169}$$
$$= 13 \text{ units}$$

Mid Point Formula

The midpoint formula is used in geometry to find center point between two given points in a coordinate plane. This formula is particularly useful when you need to divide a line segment into two equal halves or parts.



Derivation of the Midpoint Formula

Consider two points $A(x_1, y_1)$ and $B(x_2, y_2)$ on a two-dimensional plane. The line segment joining these two points has a midpoint M(x, y), where x and y are the coordinates of the midpoint. To derive the formula for M(x, y) we need to find the average of the x-coordinates and y-coordinates of p A and B separately.

1. x-Coordinate of the Midpoint

The x-coordinate of the midpoint is the average of the x-coordinates of points A and B

$$\text{i.e., } x = \frac{x_1 + x_2}{2}$$

2. y-Coordinate of the Midpoint

Similarly, the y-coordinate of the midpoint is the average of the y-coordinate points A and B.

$$\text{i.e., } = \frac{y_1 + y_2}{2}$$

Thus, the coordinates of the midpoint M(x, y) are:

$$M(x, y) = \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$$

This is the **Midpoint Formula.**

Example 4: Find the midpoint of the line segment joining the points A(2,3) B(8,7).

Solution: Using the midpoint formula:

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Substitute $x_1 = 2$, $y_1 = 3$, $x_2 = 8$ and $y_2 = 7$, into the midpoint formula.

$$M(x, y) = \left(\frac{2+8}{2}, \frac{3+7}{2}\right)$$

$$M(x, y) = \left(\frac{10}{2}, \frac{10}{2}\right) = (5, 5)$$

Solved Exercise 7.1

1. Describe the location in the plane of the point P(x, y) for which

(i) x > 0

Sol. x > 0

The point is located on the right side of the y-axis.

(ii) x > 0 and y > 0

Sol. x > 0 and y > 0

The point is located in the I quadrant.

(iii) x = 0

Sol. The point is located on the y-axis.

(iv) y = 0

Sol. The point is located on the x-axis.

(v) x > 0 and y ≤ 0

Sol. The point is located in IV quadrant and on negative y – axis.

(vi) y = 0, x = 0

Sol. y = 0, x = 0

The point is located on origin.

(vii) x = y

Sol. x = y

The point can be located in first and third quadrants.

(viii) x ≥ 3

Sol. x ≥ 3

The point is located on x-axis having abscissa less than or equal to –3 or greater than or equal to 3.

(ix) y > 0

Sol. The set of points laying above x axis.

(x) x and y have opposite signs.

Sol. The point is located in 2nd or 4th quadrants.

2. Find the distance between the points.

(i) A(6, 7), B(0, – 2)

Sol. $d = |\overline{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(0-6)^2 + (-2-7)^2}$

$= \sqrt{(-6)^2 + (-9)^2}$

$= \sqrt{36+81}$

$= \sqrt{117}$

$= \sqrt{13 \times 9}$

$= \sqrt{3 \times 3 \times 13}$

$= \sqrt{3^2 \times 13} = 3\sqrt{13}$

(ii) C (–5, –2), D(3, 2)

Sol. $d = |\overline{CD}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{[3-(-5)]^2 + [2-(-2)]^2}$

$= \sqrt{(3+5)^2 + (2+2)^2}$

$= \sqrt{(8)^2 + (4)^2}$

$= \sqrt{64+16} = \sqrt{80}$

$= \sqrt{2 \times 2 \times 2 \times 2 \times 5}$

$= \sqrt{2^2 \times 2^2 \times 5}$

$= 2 \times 2 \times \sqrt{5} = 4\sqrt{5}$

(iii) L(0, 3), M(−2, −4)

Sol. $|\overline{LM}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(-2-0)^2+(-4-3)^2}$

$= \sqrt{(-2)^2+(7)^2}$

$= \sqrt{4+49}$

$= \sqrt{53}$

(iv) P(−8, −7), Q(0, 0)

Sol. $d = |\overline{PQ}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{[0+(-8)]^2+[0-(-7)]^2}$

$= \sqrt{(0+8)^2+(0+7)^2}$

$= \sqrt{(8)^2+(7)^2}$

$= \sqrt{64+49} = \sqrt{113}$

3. Find in each of the following:

(i) The distance between the two given points.

(ii) Midpoint of the line segment joining the two points:

(a) A(3 , 1) ; B (−2 , −4)

Sol. A(3 , 1) ; B (−2 , −4)

(i) $d = |\overline{AB}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(-2-3)^2+(-4-1)^2}$

$= \sqrt{(-5)^2+(-5)^2}$

$= \sqrt{25+25}$

$= \sqrt{50}$

$= \sqrt{2\times5\times5}$

$= \sqrt{2\times5^2}$

$= 5\sqrt{2}$ units

(ii) $M(x, y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{3+(-2)}{2}, \frac{1+(-4)}{2}\right)$

$= \left(\frac{3-2}{2}, \frac{1-4}{2}\right)$

$= \left(\frac{1}{2}, \frac{-3}{2}\right)$

(b) A (−8 , 3) ; B (2, −1)

Sol. A(−8 , 3) ; B (2, −1)

(i) $d = \sqrt{(2-(-8)^2+(-1-3)^2}$

$= \sqrt{(2+8)^2+(-4)^2}$

$= \sqrt{(10)^2+(-4)^2} = \sqrt{100+16}$

$= \sqrt{116} = \sqrt{2\times2\times29}$

$= \sqrt{2^2\times29} = 2\sqrt{29}$

(ii) $M(x, y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{-8+2}{2}, \frac{3+(-1)}{2}\right)$

$= \left(\frac{-6}{2}, \frac{3-1}{2}\right)$

$= \left(\frac{-6}{2}, \frac{2}{2}\right)$

$= (-3 , 1)$

(c) $A\left(-\sqrt{5}, -\frac{1}{3}\right), B\left(-3\sqrt{5}, 5\right)$

Sol. $d = |\overline{AB}| = \sqrt{\left[-3\sqrt{5}-(-\sqrt{5})\right]^2+\left[5-\left(-\frac{1}{3}\right)\right]^2}$

$= \sqrt{(-3\sqrt{5}+\sqrt{5})^2+\left(5+\frac{1}{3}\right)^2}$

$= \sqrt{(-2\sqrt{5})^2+\left(\frac{15+1}{3}\right)^2}$

$= \sqrt{(4\times5)+\left(\frac{16}{3}\right)}$

$= \sqrt{20+\frac{256}{9}}$

$= \sqrt{\frac{180+256}{9}} = \sqrt{\frac{436}{9}}$

$= \sqrt{\frac{4\times109}{9}} = \frac{2\sqrt{109}}{3}$

$M(x,y) = \left(\frac{x_1+x_2}{2}, \frac{xy+y_2}{2}\right)$

$= \left(\frac{-\sqrt{5}+(-3\sqrt{5})}{2}, \frac{-\frac{1}{3}+5}{2}\right)$

$= \left(\frac{-\sqrt{5}-3\sqrt{5}}{2}, \frac{-\frac{1+15}{3}}{2}\right)$

$= \left(\frac{-\cancel{4}^{2}\sqrt{5}}{\cancel{2}_{1}}, \frac{\cancel{16}^{7}}{3}\times\frac{1}{\cancel{2}_{1}}\right)$

$= \left(-2\sqrt{5}, \frac{7}{3}\right)$

4. Which of the following points are at a distance of 15 units from the origin?

(i) $(\sqrt{176}, 7)$

Sol. $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(\sqrt{176}-0)^2+(7-0)^2}$

$= \sqrt{(\sqrt{176})^2+(7)^2}$

$= \sqrt{176+49} = \sqrt{225} = 15$

Thus, the point $(\sqrt{176}, 7)$ is at a distance of 15 units from the origin.

(ii) (10, −10)

Sol. $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(10-0)^2+(-10-0)^2}$

$= \sqrt{(10)^2+(-10)^2} = \sqrt{100+100} = \sqrt{200}$

$= \sqrt{2\times2\times2\times5\times5} = \sqrt{2^2\times5^2\times2} = 2\times5\sqrt{2} = 10\sqrt{2}$

Thus, the point (10, −10) is not at a distance of 15 units from the origin.

(iii) (1, 15)

Sol. $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(1-0)^2+(15-0)^2}$

$= \sqrt{(1)^2+(15)^2} = \sqrt{1+225} = \sqrt{226}$

Thus, the point (1, 15) is not at distance of 15 units from the origin.

5. Show that:

(i) the points A(0, 2), B($\sqrt{3}$, 1) and C(0, –2) are vertices of a right triangle.

Sol. A(0, 2), B(3, 1), C(0, –2)

$|\overline{AB}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{\left(\sqrt{3}-0\right)^2+(1-2)^2}$

$= \sqrt{\left(\sqrt{3}\right)^2+(-1)^2}$

$= \sqrt{3+1} = \sqrt{4} = 2$

$|\overline{BC}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(0-\sqrt{3})^2+(-2-1)^2}$

$= \sqrt{(\sqrt{3})^2+(-3)^2}$

$= \sqrt{3+(-1)^2} = \sqrt{3+1} = \sqrt{4} = 2$

$= \sqrt{3+9} = \sqrt{(12)} = \sqrt{2\times2\times3}$

$= \sqrt{2^2\times3} = 2\sqrt{3}$

$|\overline{CA}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(0-0)^2+(2-(-2))^2}$

$= \sqrt{(0)^2+(2-2)^2}$

$= \sqrt{0+(4)^2}$

$= \sqrt{0+16} = \sqrt{16} = 4$

By Pythagoras theorem

$\left(\overline{CA}\right)^2 = (AB)^2+(BC)^2$

$4^2 = \left(2\sqrt{3}\right)^2+(2)^2$

$16 = (4\times3)+4$

$16 = 12+4$

$16 = 16$ (true)

Thus, the points A, B and C vertices of a right triangle.

(ii) the points A(3, 1), B(–2, –3) and C(2, 2) are vertices of an isosceles triangle.

Sol. A(3, 1), B(–2, –3), C(2, 2)

$|\overline{AB}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(-2-3)^2+(-3-1)^2}$

$= \sqrt{(-5)^2+(-4)^2}$

$= \sqrt{25+16} = \sqrt{41}$

$|\overline{BC}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(2-(-2))^2+(2-(-3))^2}$

$= \sqrt{(2+2)^2+(2+3)^2}$

$= \sqrt{(4)^2+(5)^2}$

$= \sqrt{16+25} = \sqrt{41}$

$|\overline{CA}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(3-2)^2+(1-2)^2}$

$= \sqrt{(1)^2+(-1)^2}$

$= \sqrt{1+1}$

$= \sqrt{2}$

$|\overline{AB}| = |\overline{BC}|$ = (Length of two s are equal)

So, A B and C are vertices o isosceles triangle.

(iii) the points A(5, 2), B(–2, 3), C(–3, –4) and D(4, –5) are vertices of a parallelogram.

Sol. A(5, 2), B(–2, 3), C(–3, –4), D(4, –5)

$|\overline{AB}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)}$

$= \sqrt{(-2-5)^2+(3-2)^2}$

$= \sqrt{(-7)^2+(1)^2}$

$= \sqrt{49+1}$

$= \sqrt{50} = \sqrt{2\times5\times5} = \sqrt{2\times5^2} = 5\sqrt{2}$

$|\overline{BC}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(-3-(-2))^2+(-4-3)^2}$

$= \sqrt{(-3+2)^2+(-7)^2}$

$= \sqrt{(-1)^2+49}$

$= \sqrt{1+49}$

$= \sqrt{50} = \sqrt{2\times5\times5} = \sqrt{2\times5^2} = 5\sqrt{2}$

$|\overline{CD}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(4-(-3))^2+(-5-(-4))^2}$

$= \sqrt{(4+3)^2+(-5+4)^2}$

$= \sqrt{(7)^2+(-1)^2}$

$= \sqrt{49+1}$

$= \sqrt{50} = \sqrt{2\times5\times5} = \sqrt{2\times5^2} = 5\sqrt{2}$

$|\overline{AD}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(4-5)^2+(-5-2)^2}$

$= \sqrt{(-1)^2+(-7)^2} = \sqrt{1+49}$

$= \sqrt{50} = \sqrt{2\times5\times5} = \sqrt{2\times5^2} = 5\sqrt{2}$

$|\overline{AB}| = |\overline{CD}|$, $|\overline{AD}| = |\overline{BC}|$ (opposite sides lengths)

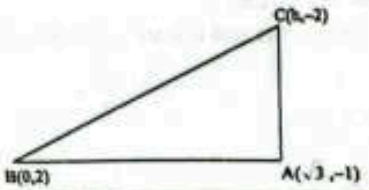
So, ABCD is a parallel logram

Now

6. Find h such that the points A($\sqrt{3}$, –1), B(0, 2) and C(h, –2) are vertices of a right triangle with right angle at the vertex A.

Sol. A($\sqrt{3}$, –1), B(0, 2) and C(h, –2)

$m\angle A = 90°$



By using the distance formula, we have

$$|\overline{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 + y_1)^2}$$
$$= \sqrt{(0 - \sqrt{3})^2 + (2 - (-1))^2}$$
$$= \sqrt{(-\sqrt{3})^2 + (2+1)^2}$$
$$= \sqrt{3 + (3)^2}$$
$$= \sqrt{3+9} = \sqrt{12}$$

$$|\overline{AC}| = \sqrt{(x_2 - x_1)^2 + (y_2 + y_1)^2}$$
$$= \sqrt{(h - \sqrt{3})^2 + (-2 - (-1))^2}$$
$$= \sqrt{h^2 + (\sqrt{3})^2 - 2h(\sqrt{3}) + (-2+1)^2}$$
$$= \sqrt{h^2 + 3 - 2\sqrt{3}h + (-1)^2}$$
$$= \sqrt{h^2 + 3 - 2\sqrt{3}h + 1}$$
$$= \sqrt{h^2 - 2\sqrt{3}h + 4}$$

$$|\overline{BC}| = \sqrt{(x_2 - x_1)^2 + (y_2 + y_1)^2}$$
$$= \sqrt{(h - 0)^2 + (-2 - 2)^2}$$
$$= \sqrt{(h)^2 + (-4)^2}$$
$$= \sqrt{h^2 + 16}$$

By Pythagorean theorem

$$|\overline{BC}|^2 = |\overline{AB}|^2 + |\overline{AC}|^2$$
$$(\sqrt{h^2+16})^2 = (\sqrt{12})^2 + (\sqrt{h^2 - 6h + 4})^2$$
$$h^2 + 16 = 12 + h^2 - 2\sqrt{3}h + 4$$
$$h^2 + 16 = 16 + h^2 - 2\sqrt{3}h$$
$$h^2 - h^2 - 2\sqrt{3}h = 16 - 16$$
$$2\sqrt{3}h = 0$$
$$h = \frac{0}{2\sqrt{3}}$$
$$h = 0$$

7. Find h such that A(–1, h), B(3, 2) and C(7, 3) are collinear.

Sol. Since the points A, B, C are collinear, So

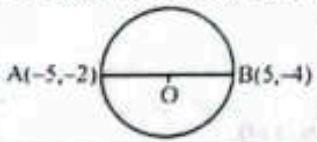
$$\begin{vmatrix} -1 & h & 1 \\ 3 & 2 & 1 \\ 7 & 3 & 1 \end{vmatrix} = 0$$

$$-1\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} - h\begin{vmatrix} 3 & 1 \\ 7 & 1 \end{vmatrix} + 1\begin{vmatrix} 3 & 2 \\ 7 & 3 \end{vmatrix} = 0$$

$$-1(2-3) - h(3-7) + 1(9-14) = 0$$
$$-1(-1) - h(-4) + 1(-5) = 0$$
$$1 + 4h - 5 = 0$$
$$4h - 4 = 0$$
$$4h = 4$$
$$h = \frac{4}{4} \Rightarrow h = 1$$

8. The points A(–5, –2) and B (5, –4) are ends of a diameter of a circle. Find the centre and radius of the circle.

Sol. Let O be the centre of the circle and it is the midpoint of the endpoints of a diameter.



$$(x,y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$= \left(\frac{-5+5}{2}, \frac{-2+(-4)}{2}\right)$$
$$= \left(\frac{0}{2}, \frac{-6}{2}\right) = (0, -3)$$

So centre = O (0, –3)

Radius of circle is the distance $\overline{OA}$ or $\overline{OB}$

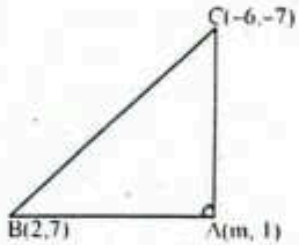
$$\text{Radius} = |\overline{OB}| = \sqrt{(x_2 - x_1)^2 + (y_2 + y_1)^2}$$
$$= \sqrt{(5-0)^2 + (-4-(-3)^2}$$
$$= \sqrt{(5)^2 + (-4+3)^2}$$
$$= \sqrt{25 + (-1)^2}$$
$$= \sqrt{25+1} = \sqrt{26}$$

Radius = $\sqrt{26}$

Find h such that the points A(h , 1), B(2, 7) and C(–6, –7) are vertices of a right triangle with right angle at the vertex A.

Sol. A(h, 1), B(2, 7), C(–6,–7) and m∠A = 90°

By using the distance formaula, we have



$$|\overline{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 + y_1)^2}$$
$$= \sqrt{(2-h)^2 + (7-1)^2}$$
$$= \sqrt{(4 + h^2 - 2(2)(h) + (6)^2}$$
$$= \sqrt{(4 + h^2 - 4h + 36}$$
$$= \sqrt{(h^2 - 4h + 40}$$

$$|\overline{AC}| = \sqrt{(x_1 - x_2)^2 + (y_1 + y_2)^2}$$
$$= \sqrt{(h-(-6))^2 + (1-(-7))^2} = \sqrt{(h+6)^2 + (1+7)^2}$$
$$= \sqrt{h^2 + 36 + 2(h)(6) + (8)^2} = \sqrt{h^2 + 36 + 12h + 64}$$
$$= \sqrt{h^2 + 12h + 100}$$

$$|\overline{BC}| = \sqrt{(x_2 - x_1)^2 + (y_2 + y_1)^2}$$
$$= \sqrt{(-6-2)^2 + (-7-7)^2} = \sqrt{(-8)^2 + (-14)^2}$$
$$= \sqrt{64 + 196} = \sqrt{260}$$

By Pythagores theorem

$|\overline{BC}|^2 = |\overline{AB}|^2 + |\overline{AC}|^2$

$(\sqrt{260})^2 = (\sqrt{h^2 - 4h + 40})^2 + (\sqrt{h^2 + 12h + 100})^2$

$260 = h^2 - 4h + 40 + h^2 + 12h + 100$

$260 = 2h^2 + 8h + 140$

$-2h^2 - 8h - 140 + 260 = 0$

$-2h^2 - 8h + 120 = 0$

$-2(h^2 + 4h - 60) = 0$

$h^2 + 4h - 60 = 0$

$h^2 + 10h - 6h - 60 = 0$

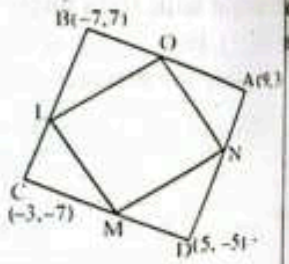
$h(h + 10) - 6(h + 10) = 0$

$(h + 10)(h - 6) = 0$

$h + 10 = 0$	$h - 6 = 0$
$h = -10$	$h = 6$

10. A quadrilateral has the points A(9, 3), B(–7, 7), C(–3, –7) and D(5, –5) as its vertices. Find the midpoints of its sides. Show that the figure formed by joining the midpoints consecutively is a parallelogram.

Sol. Let L, M, N and O be the mid-points of $\overline{BC}$, $\overline{CD}$, $\overline{DA}$, $\overline{AB}$ respectively.



$L = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{-7 + (-3)}{2}, \frac{7 + (-7)}{2}\right)$

$= \left(\frac{-10}{2}, \frac{0}{2}\right)$

$= (-5, 0)$

$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{-3 + 5}{2}, \frac{-7 + (-5)}{2}\right)$

$= \left(\frac{2}{2}, \frac{-12}{2}\right)$

$= (1, -6)$

$N = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{5 + 9}{2}, \frac{-5 + 3}{2}\right) = \left(\frac{14}{2}, \frac{-2}{2}\right)$

$= (7, -1)$

$O = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{9 + (-7)}{2}, \frac{3 + 7}{2}\right)$

$= \left(\frac{2}{2}, \frac{10}{2}\right)$

$= (1, 5)$

No by distance formula

using the distance formula, we have

$|\overline{LM}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(1 - (-5))^2 + (-6 - 0)^2}$

$= \sqrt{(1 + 5)^2 + (-6)^2}$

$= \sqrt{(6)^2 + (-6)^2}$

$= \sqrt{36 + 36}$

$= \sqrt{72}$

$|\overline{ON}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(7 - 1)^2 + (-1 - 5)^2}$

$= \sqrt{(6)^2 + (-6)^2}$

$= \sqrt{36 + 36}$

$= \sqrt{72}$

$|\overline{OL}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-5 - 1)^2 + (0 - 5)^2}$

$= \sqrt{(-6)^2 + (-5)^2}$

$= \sqrt{36 + 25} = \sqrt{61}$

$|\overline{MN}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(7 - 1)^2 + (-1 - (-6))^2}$

$= \sqrt{(6)^2 + (-1 + 6)^2}$

$= \sqrt{36 + 25} = \sqrt{61}$

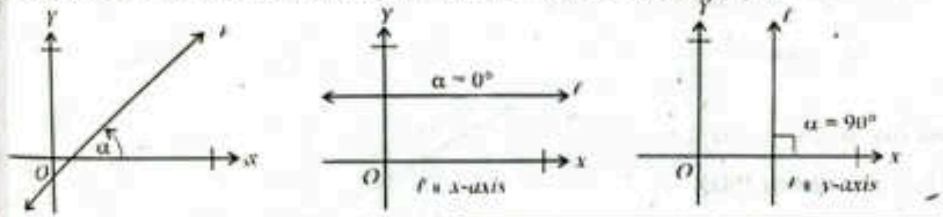
We see that

$|\overline{LM}| = |\overline{ON}| = \sqrt{72}$ and $|\overline{OL}| = |\overline{MN}| = \sqrt{61}$

Since the opposite sides are equal so it is a parallelogram.

uations of Straight Lines

clination of a Line: The angle $\alpha (0° < \alpha < 180°)$ measured counterclockwise from itive x-axis to a non-horizontal straight line l is called the inclination of ℓ.



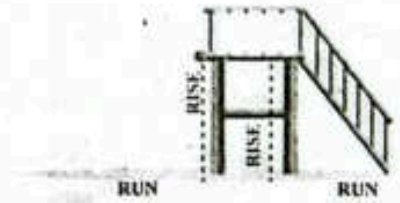
bserve that the angle α in the different sitions of the line ℓ is α, 0° and 90° spectively.

Note:
(i) If ℓ is parallel to x-axis, then $\alpha = 0°$
(ii) If ℓ is parallel to y-axis, then $\alpha = 90°$

lope or Gradient of a Line

When we walk on an inclined plane, we over horizontal distance (**run**) as well as ertical distance (**rise**) at the same time.

It is harder to climb a steeper inclined lane. The measure of steepness (ratio of rise o the run) is termed as slope or gradient of the nclined path and is denoted by m.



$$m = \frac{\text{rise}}{\text{run}} = \frac{y}{x} = \tan\alpha$$

In analytical geometry, **slope or gradient m** of a non-vertical straight line with its inclination is defined by: $m = \tan \alpha$.

If ℓ is horizontal its slope is zero and if ℓ is vertical then its slope is undefined.

If $0° < \alpha < 90°$, m is positive and if $90° < \alpha < 180°$, then m is negative.

Slope or Gradient of a Straight Line Joining Two Points

Theorem 1: If a non-vertical line ℓ with inclination α passes through two points $P(x_1, y_1)$ and $Q(x_2, y_2)$, then the slope or gradient m of ℓ is given by $m = \frac{y_2 - y_1}{x_2 - x_1} = \tan \alpha$

Proof:

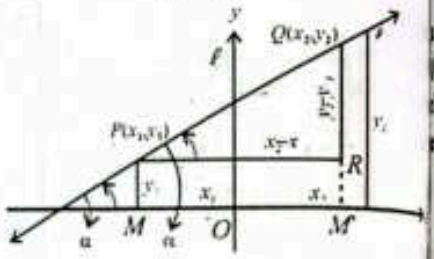
Let m be the slope of the line ℓ Draw perpendiculars PM and QM' on x-axis and a perpendicular PR on QM'.

Then $m\angle RPQ = \alpha$,

$m\overline{PR} = x_2 - x_1$ and $m\overline{QR} = y_2 - y_1$

The slope or gradient of ℓ is defined as:

$$m = \tan \alpha = \frac{y_2 - y_1}{x_2 - x_1}$$



Why are slopes important?
The concept of slope is widely used in engineering, architecture, and even sports like skiing, where understanding the steepness of a hill or ramp is essential.

Case (i). When $0 < \alpha < \frac{\pi}{2}$

In the right triangle PRQ, we have

$$m = \tan \alpha = \frac{y_2 - y_1}{x_2 - x_1}$$

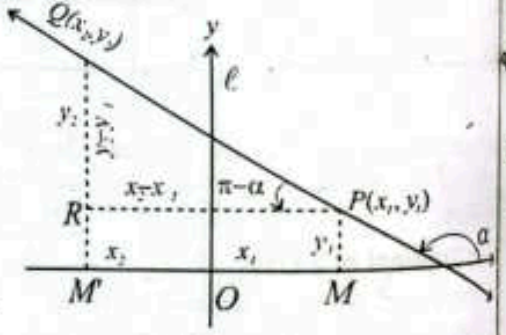
Case (ii). When $\frac{\pi}{2} < \alpha < \pi$

In the right triangle PRQ,

$$\tan (\pi - \alpha) = \frac{y_2 - y_1}{x_1 - x_2}$$

or $$-\tan \alpha = \frac{y_2 - y_1}{x_1 - x_2}$$

or $$\tan \alpha = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad m = \frac{y_2 - y_1}{x_2 - x_1}$$



Thus if $P(x_1, y_2)$ and $Q(x_2, y_2)$ are two points on a line, then slope of PQ is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad m = \frac{y_1 - y_2}{x_1 - x_2}$$

Note: (i) $m \neq \frac{y_2 - y_1}{x_1 - x_2}$ and $m \neq \frac{y_1 - y_2}{x_2 - x_1}$

(ii) ℓ is horizontal, iff m = 0 $(\because \alpha = 0°)$

(iii) ℓ is vertical, iff m is not defined $(\because \alpha = 90°)$

(iv) If slope of AB = slope of BC, then the points A, B and C are collinear.

Theorem 2: The two lines ℓ_1 and ℓ_2 with slopes m_1 and m_2 respectively are

parallel iff $m_1 = m_2$

perpendicular iff $m_1 = \frac{-1}{m_2}$ or $m_1 m_2 = 1$

Rembert that:
The symbol
(i) || stands for "parallel".
(ii) ∦ stands for "not parallel".
(iii) ⊥ stands for "perpendicular"

Example 5: Show that the points A(–3, 6), B(3, 2) and C(6, 0) are collinear.

Notice that:
Slope of AB = slope of AC

Solution: We know that the points A, B and C are collinear if the line AB and BC have same slopes.

Here Slope of $AB = \frac{2-6}{3-(-3)} = \frac{-4}{3+3} = \frac{-4}{6} = \frac{-2}{3}$ and slope of $BC = \frac{0-2}{6-3} = \frac{-2}{3}$

$\therefore$ Slope of AB = Slope of BC

Thus A, B and C are collinear.

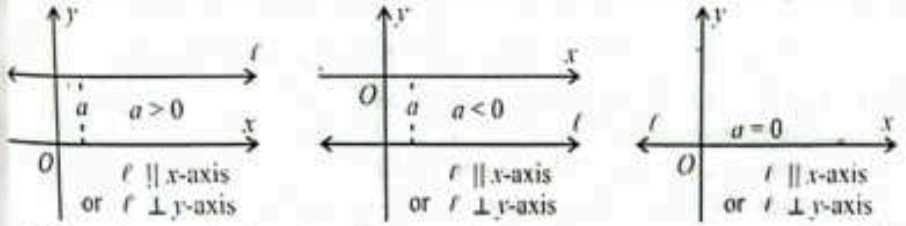
Example 6: Show that the triangle with vertices A(1, 1), B(4, 5) and C(12, –1) is a right triangle.

Solution: Slope of $AB = m_1 \frac{5-1}{4-1} = \frac{4}{3}$ and slope of $BC = m_2 = \frac{-1-5}{12-4} = \frac{-6}{8} = \frac{-3}{4}$

Since $m_1 . m_2 = \left(\frac{4}{3}\right)\left(-\frac{3}{4}\right) = -1$, therefore, AB ⊥ BC

So ΔABC is a right triangle.

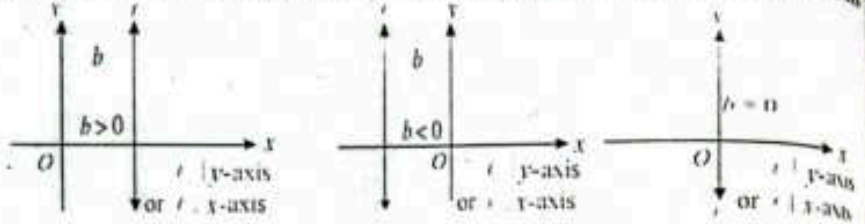
Equation of a Straight Line Parallel to the x-axis (or perpendicular to the y-axis)



All the points on the line ℓ parallel to x-axis remain at a constant distance (say a) from x-axis. Therefore, each point on the line has its distance from x-axis equal to a, which is its y-coordinate (ordinate). So, all the points on this line satisfy the equation: $y = a$

Note: **(i)** If a > 0, then the line ℓ is above the x-axis.
(ii) If a < 0, then the line ℓ is below the x-axis.
(iii) If a = 0, then the line ℓ becomes the x-axis.
Thus the equation of x-axis is y = 0

Equation of a Straight Line Parallel to the y-axis (or perpendicular to the x-axis)

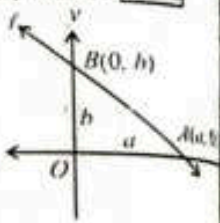


All the points on the line ℓ parallel to y-axis remain at a constant distance (say) from y-axis. Each point on the line has its distance from y-axis equal to b, which is x-coordinate (abscissa). So, all the points on this line satisfy the equation: $x = b$

Standard Forms of Equations of Straight Lines

Intercepts of a line

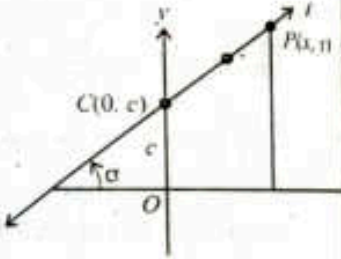
- If a line intersects x-axis at (a, 0), then a is called **x-intercept** of the line.
- If a line intersects y-axis at (0, b), then b is called **y-intercept** of the line.



1. Slope-Intercept form of Equation of a Straight Line

Theorem 3: Equation of a non-vertical straight line with slope m and y-intercept c is given by:

$$\boxed{y = mx + c}$$



Proof: Let P (x, y) be an arbitrary point of the straight line ℓ with slope m and y-intercept c. As C (0, c) and P (x, y) lie on the line, so the slope of the line is:

$$m = \frac{y-c}{x-0} \quad \text{or} \quad y - c = mx \quad \text{or} \quad y = mx + c \text{ is an equation of } \ell.$$

The equation of the line for which c = 0 is y = mx

In this case the line passes through the origin.

Example 7: Find an equation of the straight line if

(a) its slope is 2 and y-intercept is 5

(b) it is perpendicular to a line with slope -6 and its y-intercept is $\frac{4}{3}$

Solution: (a) The slope and y-intercept of the line are respectively:

m = 2 and c = 5

Thus y = 2x + 5 (Slope-intercept form: y = mx + c) is the required equation.

(b) The slope of the given line is

$m_1 = -6$

$\therefore$ The slope of the required line is: $m_2 = -\frac{1}{m_1} = \frac{1}{6}$

$$y = -\frac{1}{6}x + \frac{4}{3}$$

The slope and y-intercept of the required line are respectively:

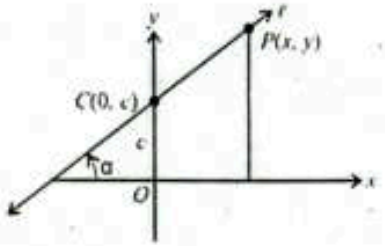
$$m_2 = \frac{1}{6} \text{ and } c = \frac{4}{3}$$

Thus, $y = \frac{1}{6}(x) + \frac{4}{3}$ or $6y = x + 8$ is the required equation.

Point-slope Form of Equation of a Straight Line

Theorem 4: Equation of a non-vertical straight line ℓ with slope m and passing through a point Q (x_1, y_1) is given by:

$$y - y_1 = m(x - x_1)$$



Proof: Let P (x, y) be an arbitrary point of the straight line with slope m and passing through $Q(x_1, y_1)$. As $Q(x_1, y_1)$ and P(x, y) both lie on the line, so the slope of the line is

$$\boxed{m = \frac{y - y_1}{x - x_1} \text{ or } y - y_1 = m(x - x_1)}$$

which is an equation of the straight line passing through (x_1, y_1) with slope m.

Symmetric Form of Equation of a Straight Line

We have $m = \frac{y - y_1}{x - x_1} = \tan\alpha$ where α is the inclination of the line.

$$\boxed{\frac{y - y_1}{x - x_1} = \frac{\sin\alpha}{\cos\alpha} \text{ or } = \frac{x - x_1}{\cos\alpha} = \frac{y - y_1}{\sin\alpha} = r \text{ (say)}}$$

This is called **symmetric** form of equation of the line.

Example 8: Write down an equation of the straight line passing through (5, 1) and parallel to a line passing through the points (0, –1), (7, –15).

Solution: Let m be the slope of the required straight line, then

$$m = \frac{-15 - (-1)}{7 - 0} \quad (\because \text{ Slopes of parallel lines are equal})$$

$$= -2$$

As the point (5, 1) lies on the required line having slope –2 so, by point-slope form of equation of the straight line, we have

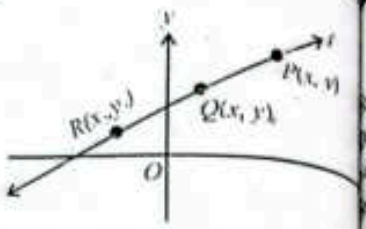
$$y - 1 = -2(x - 5)$$

or $y = -2x + 11$

or $2x + y - 11 = 0$

is an equation of the required line.

4. Two-point Form of Equation of a Straight Line



Theorem 5: Equation of a non-vertical straight line passing through two points $Q(x_1, y_1)$ and $R(x_2, y_2)$ is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \text{ or } y - y_2 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_2)$$

Proof: Let P (x, y) be an arbitrary point of the line passing through $Q(x_1, y_1)$ and $R(x_2, y_2)$. So

$$\frac{y - y_1}{x - x_1} = \frac{y - y_2}{x - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$$ (P, Q and R are collinear points)

We take

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

or $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$, the required equation of the line PQ.

or $(y_2 - y_1)x - (x_2 - x_1)y + (x_1y_2 - x_2y_1) = 0$

We may write this equation in determinant form as: $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

$y - y_2 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_2)$ can be derived similarly.

Example 9: Find an equation of line through the points (–2,1) and (6,–4).

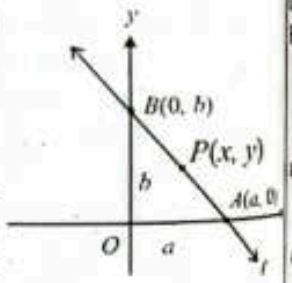
Solution: Using two-points form of the equation of straight line, the required equation

$$y - 1 = \frac{-4 - 1}{6 - 2(-2)}[x - (-2)] \quad \text{or} \quad y - 1 = \frac{-5}{8}(x + 2) \text{ or } 5x + 8y + 2 = 0$$

5. Intercept Form of Equation of a Straight Line

Theorem 6: Equation of a line whose non-zero x and y-intercepts are a and b respectively

$$\boxed{\frac{x}{a} + \frac{y}{b} = 1}$$



Proof: Let P(x, y) be an arbitrary point of the line whose non-zero x and y-intercepts are a and b respectively. Obviously, the points A(a, 0) and B(0, b) lie on the required line. So, by the two-point form of the equation of line, we have

$$y - 0 = \frac{b - 0}{0 - a}(x - a)$$ (P, A and B are collinear)

or $-ay = b(x - a)$ or $bx + ay = ab$

or $\frac{x}{a} + \frac{y}{b} = 1$ (dividing by ab)

ve the result.

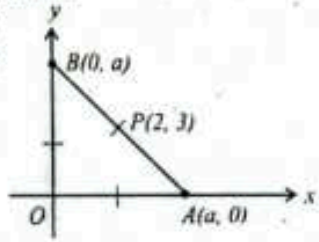
mple 10: Write down an equation of the line which cuts the x-axis at (2, 0) and is at (0, –4).

lution: As 2 and –4 are respectively x and y-intercepts of the required line, so by -intercepts form of equation of a straight line, we have

$$\frac{x}{2} + \frac{y}{-4} = 1 \quad \text{or} \quad 2x - y - 4 = 0$$

hich is the required equation.

mple 11: Find an equation of the line through the point P(2, 3) which forms an sceles triangle with the coordinate axes in the first quadrant.



lution: Let OAB be an isosceles triangle so that the line B passes through A = (a, 0) and B(0, a), where a is some sitive real number.

ope of $AB = \frac{a - 0}{0 - a} = -1$. But AB passes through P (2, 3).

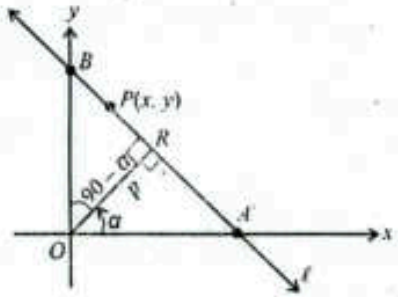
uation of the line through P(2, 3) with slope –1 is

$y - 3 = -1(x - 2)$ or $x + y - 5 = 0$

Normal Form of Equation of a Straight Line

Theorem 7: An equation of a non-vertical straight-line ℓ, such that length of the perpendicular from the origin to ℓ is p and a is the inclination of this perpendicular, is

$$\boxed{x\cos\alpha + y\sin\alpha = p}$$



Proof: Let the line ℓ meet the x-axis and y-axis at the points A and B respectively. Let P (x, y) be an arbitrary point of AB and let $\overline{OR}$ be perpendicular to the line ℓ. Then $|\overline{OR}| = P$

From the right triangles ORA and ORB, we have

$$\cos\alpha = \frac{p}{|OA|} \quad \text{or} \quad |\overline{OA}| = \frac{p}{\cos\alpha}$$

and $\cos(90° - \alpha) = \frac{p}{|OB|}$ or $= |\overline{OB}| = \frac{p}{\sin\alpha}$

$[\because \cos(90° - \alpha) = \sin\alpha]$

As $|\overline{OA}|$ and $|\overline{OB}|$ are the x and y-intercepts of the line AB, so equation of AB is:

$$\frac{x}{p/\cos\alpha} + \frac{x}{p/\sin\alpha} = 1$$ (Two-intercept form)

That is x cos a + y sin a = p is the required equation.

Example 12: The length of perpendicular from the origin to a line is 5 units and the inclination of this perpendicular is 120°. Find the slope and y-intercept of the line.

Solution. Here p = 5, a = 120°.

Equation of the line in normal form is

$$x\cos 120° + y\sin 120° = 5$$

$$\Rightarrow \quad -\frac{1}{2}x + \frac{\sqrt{3}}{2}y = 5$$

$$\Rightarrow \quad x - \sqrt{3}y + 10 = 0$$

To find the slope of the line, we re-write (1) as: $y = \frac{x}{\sqrt{3}} + \frac{10}{\sqrt{3}}$

which is slope-intercept form of the equation.

Here $m = \frac{1}{\sqrt{3}}$ and $c = \frac{10}{\sqrt{3}}$

A Linear Equation in two Variables Represents a Straight Line

Theorem 8: The linear equation ax + by + c = 0 in two variables x and y represe straight line. A linear equation in two variables x and y is:

$$\boxed{ax + by + c = 0} \quad \ldots(i)$$

where a, b and c are constants and a and b are not simultaneously zero.

Remember that:
The equation (i) represe straight line and is called the ge equation of a straight line.

Proof: Here a and b cannot be both zero. So the following cases arise:

Case I: $a \neq 0$, $b = 0$

In this case equation (i) takes the form:

$$ax + c = 0 \text{ or } x = -\frac{c}{a}$$

which is an equation of the straight line parallel to the y-axis at a directed distance from the y-axis.

Case II: $a = 0$, $b \neq 0$

In this case equation (i) takes the form:

$$by + c = \text{ or } y = -\frac{c}{b}.$$

which is an equation of the straight line parallel to x-axis at a directed distance $\frac{-c}{b}$ the x-axis.

Case III: $a \neq 0$, $b \neq 0$

In this case equation (i) takes the form:

$$by = -ax - c \text{ or } y = \frac{-a}{b}x - \frac{c}{b} = mx + c$$

which is the slope-intercept form of the straight line with slope $\frac{-a}{b}$ and y-intercept

Thus the equation ax + by + c = 0, always represents a straight line.

Transform the General Linear Equation to Standard Forms

Let us transform the equation ax + by + c = 0 in the standard form

i. Slope-Intercept Form

We have: $by = -ax - c$ or $y = \frac{-a}{b}x - \frac{c}{b} = mx + c$, ...(i) where $m = \frac{-a}{b}, c = \frac{-c}{b}$

ii. Point - Slope Form

We note from (i) above that slope of the line ax + by + c = 0 is $\frac{-a}{b}$. A point on the line is $\left(\frac{-c}{a}, 0\right)$.

Equation of the line becomes $y - 0 = \frac{-a}{b}\left(x + \frac{c}{a}\right)$

which is in the point-slope form.

iii. Symmetric Form

$$m = \tan\alpha = \frac{-a}{b}, \sin\alpha = \frac{a}{\pm\sqrt{a^2 + b^2}}, \cos\alpha \frac{b}{\pm\sqrt{a^2 + b^2}}$$

A point on ax + by + c = 0 is, $\left(\frac{-c}{a}, 0\right)$

Equation in the symmetric form becomes $\frac{x - \left(-\frac{c}{a}\right)}{b/\pm\sqrt{a^2 + b^2}} = \frac{y - 0}{a/\pm\sqrt{a^2 + b^2}} = r\,(say)$

is the required transformed equation. Sign of the radical to be properly chosen.

iv. Two -Point Form

We choose two arbitrary points on ax+by+c=0. Two such points are $\left(\frac{-c}{a}, 0\right)$ and $\left(0, \frac{-c}{b}\right)$.

Equation of the line through these points is:

$$\frac{y - 0}{0 - \frac{c}{b}} = \frac{x + \frac{c}{a}}{-\frac{c}{a} - 0} \qquad \text{i.e., } y - 0 = \frac{-a}{b}\left(x + \frac{c}{a}\right)$$

v. Intercept Form.

$$ax + by = -c \quad \text{or} \quad \frac{ax}{-c} + \frac{by}{-c} = 1 \quad \text{i.e.,} \quad \frac{x}{-c/a} + \frac{y}{-c/b} = 1$$

which is an equation in two intercepts form.

vi. Normal Form.

The equation, ax + by + c = 0 ...(i)

can be written in the normal form as:

$$\frac{ax + by}{\pm\sqrt{a^2 + b^2}} = \frac{-c}{\pm\sqrt{a^2 + b^2}} \qquad \ldots(ii)$$

The sign of the radical to be such that the right hand side of (ii) is positive.

Proof. We know that an equation of a line in normal form is

$x\cos\alpha + y\sin\alpha = p$ ...(iii)

If (i) and (iii) are identical, we must have

$$\frac{a}{\cos\alpha}=\frac{b}{\sin\alpha}=\frac{-c}{p}$$

i.e., $$\frac{p}{-c}=\frac{\cos\alpha}{a}=\frac{\sin\alpha}{b}=\frac{\sqrt{\cos^2\alpha+\sin^2\alpha}}{\pm\sqrt{a^2+b^2}}=\frac{1}{\pm\sqrt{a^2+b^2}}$$

Hence, $\cos\alpha=\frac{a}{\pm\sqrt{a^2+b^2}}$ and $\sin\alpha=\frac{b}{\pm\sqrt{a^2+b^2}}$ and $P=\frac{-c}{\pm\sqrt{a^2+b^2}}$

Substituting for cos α, sin α and p into (iii), we have

$$\frac{ax+by}{\pm\sqrt{a^2+b^2}}=\frac{-c}{\pm\sqrt{a^2+b^2}}$$

Thus (i) can be reduced to the form (ii) by dividing it by $\pm\sqrt{a^2+b^2}$. The sign of the radical to be chosen so that the right hand side of (ii) is positive.

Example 13: Transform the equation 5x – 12y + 39 = 0 into

(i) Slope intercept form (ii) Two-intercept form (iii) Normal form

(iv) Point-slope form (v) Two-point form (vi) Symmetric form

Solution:

(i) We have $12y=5x+39$ *or* $y=\frac{5}{12}x+\frac{39}{12}, m=\frac{5}{12}, y-\text{intercept } c=\frac{39}{12}$

(ii) $5x-12y=-39$ or $\frac{5x}{-39}+\frac{12y}{39}=1$ *or* $\frac{x}{-39/5}+\frac{y}{39/12}=1$ is the required equation.

(iii) $5x-12y=-39$. Divide both sides by $\pm\sqrt{5^2+12^2}=\pm13$. Since R.H.S is to be positive we have to take negative sign.

Hence $\frac{5x}{-13}+\frac{12y}{13}=3$ is the normal form of the equation.

(iv) A point on the line $\left(\frac{-39}{5},0\right)$ is and its slope is $\frac{5}{12}$.

Equation can be written as: $y-0=\frac{5}{12}\left(x+\frac{39}{5}\right)$

(v) Another point on the line is $\left(0,\frac{39}{12}\right)$. Line through $\left(\frac{-39}{5},0\right)$ and $\left(0,\frac{39}{12}\right)$ is

$$\frac{y-0}{0-\frac{39}{12}}=\frac{x+\frac{39}{5}}{\frac{-39}{5}-0}$$

(vi) We have $\tan\alpha=\frac{5}{12}=m$, so $\sin\alpha=\frac{5}{13}$, $\cos\alpha=\frac{12}{13}$. A point of the line is $\left(\frac{-39}{5},0\right)$.

Equation of the line in symmetric form is

$$\frac{x+39/5}{12/13}=\frac{y-0}{5/13}=r \text{ (say)}$$

Solved Exercise 7.2

1. Find the slope and inclination of the line joining the points:

(i) (-2, 4) ; (5, 11)

Sol. (-2, 4) ; (5, 11)

$$\text{Slope} = m = \frac{y_2-y_1}{x_2-x_1}$$

$$=\frac{11-4}{5-(-2)}=\frac{7}{7}$$

$m = 1$

$m = \tan\alpha = 1$

$\alpha = \tan^{-1}(1)$

$\alpha = 45°$

(ii) (4, 6) ; (4, 8)

Sol. (4, 6) ; (4, 8)

$$\text{Slope} = m = \frac{y_2-y_1}{x_2-x_1}$$

$$=\frac{8-6}{4-4}=\infty$$

$\boxed{m=\infty}$

$m = \tan\alpha = \infty$

$\alpha = \tan^{-1}\infty$

$\boxed{\alpha = 90°}$

(ii) (3, –2) ; (2, 7)

Sol. (3, –2) ; (2, 7)

$$\text{Slope} = m = \frac{y_2-y_1}{x_2-x_1}$$

$$=\frac{7-(-2)}{2-3}=-\frac{9}{1}$$

$m = -9$

$m = \tan\alpha = -9$

$\alpha = \tan^{-1}(-9)$

$= 180° - \tan^{-1}(9)$

$= 180° - 83.66°$

$\boxed{\alpha = 96.34°}$

2. By means of slopes, show that the following points lie on the same line:

(i) A(–1, –3) ; B(1, 5) ; C(2, 9)

Sol. A(–1, –3) ; B(1, 5) ; C(2, 9)

$$\text{Slope of side } \overline{AB} = m_1 = \frac{y_2-y_1}{x_2-x_1}$$

$$=\frac{5-(-3)}{1-(-1)}=\frac{8}{2}=4$$

$$\text{Slope of side } \overline{BC} = m_2 = \frac{y_2-y_1}{x_2-x_1}=\frac{9-5}{2-1}=4$$

As $m_1 = m_2$ so points A(–1, –3), B(1, 5)) and (2, 9) lie on same line

(ii) P(4 , –5) ; Q(7, 5) ; R(10, 15)

Sol. P(4 , –5) , Q(7, 5) , R(10, 15)

$$\text{Slope of } \overline{PQ} = m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5-(-5)}{7-4} = \frac{10}{3}$$

$$\text{Slope of } \overline{QR} = m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{15-5}{10-7} = \frac{10}{3}$$

As $m_1 = m_2$ so points P(4, –5), Q(1, 5) and R(10, 15) lie on the same lines.

(iv) X(a, 2b) ; Y(c, a + b); Z(2c – a, 2a)

Sol. X(a, 2b) : Y(c, a + b): Z(2c – a, 2a)

$$\text{Slope of } \overline{XY} = m_1 = \frac{a+b-2b}{c-a}$$

$$m_1 = \frac{a-b}{c-a}$$

(iii) L(–4, 6) ; M(3, 8) : N(10, 10)

Sol. L(–4, 6) : M(3, 8) : N(10, 10)

$$\text{Slope of } \overline{LM} = m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_1 = \frac{8-6}{3-(-4)} = \frac{2}{7}$$

$$\text{Slope of } \overline{MN} = m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10-8}{10-3} = \frac{2}{7}$$

As $m_1 = m_2$ so points, L (–4, 6), M (3, 8) and N (10, 10) lie on the same line.

$$\text{Slope of } \overline{YZ} = m_2 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2a-(a+b)}{2c-a-c} = \frac{2a-a-b}{c-a} \Rightarrow m_2 = \frac{a-b}{c-a}$$

As $m_1 = m_2$ so points, X (a, 2b), Y(c, a+b) and Z(2c–a, 2a) lie on the same line.

3. Find k so that the line joining A(7, 3) ; B(k, –6) and the line joining C(–4, 5) ; D (–6, 4) are:

(i) parallel

Sol. $\text{Slope of } \overline{AB} = m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-3}{k-7} = \frac{-9}{k-7}$

$$\text{Slope of } \overline{CD} = m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-5}{-6-(-4)} = \frac{-1}{-2} = \frac{1}{2}$$

As lines are parallel so,

$m_1 = m_2$

$$\frac{-9}{k-7} = \frac{1}{2}$$

$$-18 = k - 7$$

$$k = -18 + 7$$

$$\boxed{k = -11}$$

(ii) Perpendicular

Sol. As lines are prependicular so

$$m_1 m_2 = -1$$

$$\left(\frac{-9}{k-7}\right)\left(\frac{1}{2}\right) = -1$$

$$9 = 2k - 14$$

$$2k = 23$$

$$\boxed{k = \frac{23}{2}}$$

4. Using slopes, show that the triangle with its vertices A(6,1),B(2,7)and C(–6, –1) is a right triangle.

Sol. $\text{Slope of AB} = m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-1}{2-6} = \frac{6}{-4} = -\frac{3}{2}$

$$\text{Slope of BC} = m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7-7}{-6-2} = \frac{-14}{-8} = \frac{7}{4}$$

$$\text{Slope of AC } m_3 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7-1}{-6-6} = \frac{-8}{-12} = \frac{2}{3}$$

$$\text{Since } m_1 m_3 = -\frac{3}{2} \times \frac{2}{3} = -1$$

Since the product is –1, sides $\overline{AB}$ and $\overline{AC}$ are perpendicular

So, the triangle is right triangle. The right angle is at vertex A.

Two pairs of points are given. Find whether the two lines determined by these points are:

(i) parallel (ii) perpendicular (iii) none.

(a) (1, –2), (2, 4) and (4, 1), (–8, 2)

Let the points be A(1, –2), B(2, 4) and C(4, 1), D(–8 ,2)

$$\text{Slope of } \overline{AB} = m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-2)}{2-1} = 6$$

$$\text{Slope of } \overline{CD} = m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-1}{-8-4} = \frac{1}{-12}$$

$$m_1 m_2 = 6 \times \frac{1}{-12} = \frac{-1}{2}$$

As $m_1 \neq m_2$ and $m_1 m_2 \neq -1$ so the are niether parallel nor prependicular.

(b) (–3, 4), (6, 2) and (4, 5), (–2, –7)

Sol. Let the points be A(–3, 4), B(6, 2) and C(4, 5), D(–2, –7)

$$\text{Slope of } \overline{AB} = m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-4}{6-(-3)}$$

$$m_1 = \frac{-2}{9}$$

$$\text{Slope of } \overline{CD} = m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7-5}{-2-4} = \frac{-12}{-6} = 2$$

$$m_1 m_2 = -\frac{2}{9} \times 2 = \frac{-4}{9}$$

As $m_1 \neq m_2$ and $m_1 m_2 \neq -1$ so the lines are neither parallel nor perpendicular.

6. Find an equation of:

(a) the horizontal line through (7, –9)

Sol. As line is horizontal so slope = m = 0

(x, y) = (7, –9)

Using point slope form

$y - y_1 = m(x - x_1)$

$y - (-9) = 0\,(x - 7)$

$$\boxed{y + 9 = 0}$$

(b) the vertical line through (–5, 3)

Sol. As vertical line does not intersect y-axis, so the equation is

$x = -5$

$$\boxed{x + 5 = 0}$$

(c) through A(–6, 5) having slope 7

Sol. Here

$x_1 = -6,\ y_1 = 5,\ m = 7$

Using point slope form

$y - y_1 = m(x - x_1)$

$y - 5 = 7(x - (-6))$

$y - 5 = 7(x + 6)$

$y - 5 = 7x + 42$

$7x - y + 42 + 5 = 0$

$\boxed{7x - y + 47 = 0}$

(e) through (–8 , 5) having slope undefined

Sol. As slope is undefined so line is vertical which only inrtersects x-axis, equation is:

$x = -8$

$\boxed{x + 8 = 0}$

(g) y-intercept: –7 and slope: –5

Sol. Here

$c = -7,\ m = -5$

Using slope intercept form

$y = mx + c$

$y = -5x + (-7)$

$y = -5x - 7$

$\boxed{5x + y + 7 = 0}$

(h) x-intercept: –3 and y-intercept: 4

Sol. Here

$a = -3,\ b = 4$

Using two intercepts form

$\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow \frac{x}{-3} + \frac{y}{4} = 1$

$-\frac{x}{3} + \frac{y}{4} = 1$

$\frac{-4x + 3y}{+12} = 1$

$-4x + 3y = 12$

$-4x + 3y - 12 = 0$

$\boxed{-4x + 3y - 12 = 0}$

(d) through (8,– 3) having slope 0

Sol. Here

$x_1 = -8,\ y_1 = 3,\ m = 0$

Using point slope form

$y - y_1 = m(x - x_1)$

$y - (-3) = 0\ (x - 8)$

$\boxed{y + 3 = 0}$

(f) through (–5, –3) and (9, –1)

Sol. Slope $= m = \frac{y_2 - y_1}{x_2 - x_1}$

Slope $= m = \frac{-1 - (-3)}{9 - (-5)}$

Slope $= m = \frac{2}{14} = \frac{1}{7}$

Using point(–5, –3) and slope $= \frac{1}{7}$, required equation is

$y - y_1 = m(x - x_1)$

$y - (-3) = \frac{1}{7}(x - (-5))$

$y + 3 = \frac{1}{7}(x + 5)$

$7(y + 3) = x + 5$

$7y + 21 = x + 5$

$-x + 7y + 21 - 5 = 0$

$-x + 7y + 16 = 0$

$\boxed{x - 7y - 16 = 0}$

(i) x-intercept: –9 and slope: –4

Sol. As x-intercept: is –9, so

$(x_1, y_1) = (-9, 0)$

Also $m = -4$

Using point slope form

$y - y_1 = m(x - x_1)$

$y - 0 = -4(x - (-9))$

$y = -4(x + 9)$

$y = -4x - 36$

$\boxed{4x + y + 36 = 0}$

7. Find an equation of the perpendicular bisector of the segment joining the points A (3 ,5) and B (9, 8).

Sol. Slope of $\overline{AB} = m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 5}{9 - 3}$

$m_1 = \frac{3}{6} = \frac{1}{2}$

Mid-point of segment $\overline{AB} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{3 + 9}{2}, \frac{5 + 8}{2}\right)$

$= \left(6, \frac{13}{2}\right)$

Slope of prependicular bisector

$= m = -\frac{1}{m_1}$

$m = -\frac{1}{(\frac{1}{2})} = -1 \times 2 = -2$

Also $(x_1, y_1) = (6, \frac{13}{2})$

Using point slope form

$y - y_1 = m(x - x_1)$

$y - \frac{13}{2} = -2(x - 6)$

$2(y) - 2 \times \left(\frac{13}{2}\right) = 2 \times -2(x - 6)$

$2y - 13 = -4\ (x - 6)$

$2y - 13 = -4x + 24$

$4x + 2y - 13 - 24 = 0$

$\boxed{4x + 2y - 37 = 0}$

8. Find an equation of the line through (–4, –6) and perpendicular to a line having slope $\frac{-3}{2}$.

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Sol. Slope of given line $= m_1 = -\frac{3}{2}$

Slope of required line $= m = -\frac{1}{m_1} = \frac{2}{3}$

Using point (–4, –6) and slope $= \frac{2}{3}$, required equation is:

$y - y_1 = m(x - x_1)$

$y - (-6) = \frac{2}{3}(x - (-4))$

$y + 6 = \frac{2}{3}(x + 4)$

$3(y + 6) = 2(x + 4)$

$3y + 18 = 2x + 8$

$-2x + 3y + 18 - 8 = 0$

$-2x + 3y + 10 = 0$

$\boxed{2x - 3y - 10 = 0}$

9. Find an equation of the line through (11,–5) and parallel to a line with slope –24

Sol. Slope of given line = $m_1 = -24$

Slope of required line = $m = m_1 = -24$

Using point (11, –5) and slope = –24, required equation is:

$y - y_1 = m(x - x_1)$

$y - (-5) = -24(x - 11)$

$y + 5 = -24x - 264$

$24x + y + 5 - 264 = 0$

$\boxed{24x + y - 259 = 0}$

10. Convert each of the following equations into slope intercept form, two intercept form and normal form:

(a) $2x - 4y + 11 = 0$ (b) $4x + 7y - 2 = 0$ (c) $15y - 8x + 3 = 0$

(a) $2x - 4y + 11 = 0$

Sol. $2x - 4y + 11 = 0$

$4y = 2x + 11$

$y = \frac{2}{4}x + \frac{11}{4} \Rightarrow y = \frac{1}{2}x + \frac{11}{4}$

Hence, $y = \frac{1}{2}x + \frac{11}{4}$ is the slope intercept form of the equation.

Now $2x - 4y + 11 = 0$

$2x - 4y = -11$

$\frac{2x}{-11} - \frac{4y}{-11} = \frac{-11}{-11} \Rightarrow \frac{x}{\frac{-11}{2}} + \frac{y}{\frac{11}{4}} = 1$

Hence, $\frac{x}{\frac{-11}{2}} + \frac{y}{\frac{11}{4}} = 1$ is the slope intercept form of the equation.

And $2x - 4y + 11 = 0$

$-2x + 4y = 11$

$\sqrt{(2)^2 + (-4)^2} = \sqrt{20}$

Dividing both sides by $\sqrt{20}$, we get

$\frac{-2x}{\sqrt{20}} + \frac{4y}{\sqrt{20}} = \frac{11}{\sqrt{20}}$

$\frac{-2x}{2\sqrt{5}} + \frac{4y}{2\sqrt{5}} = \frac{11}{2\sqrt{5}} \quad \because \sqrt{20} = 2\sqrt{5}$

$\frac{-x}{\sqrt{5}} + \frac{2y}{\sqrt{5}} = \frac{11}{2\sqrt{5}}$

Where $\cos\alpha = -\frac{1}{\sqrt{5}}$, $\sin\alpha = \frac{2}{\sqrt{5}}$

$\Rightarrow \tan\alpha = \frac{\frac{2}{\sqrt{5}}}{-\frac{1}{\sqrt{5}}} = -\frac{2}{\sqrt{5}} \times -\frac{\sqrt{5}}{1} = -2$

$\Rightarrow \alpha = \tan^{-1}(-2)$

$\alpha = \tan^{-1}(2) = 63.43°$

$\Rightarrow \alpha = 63.43°$

$\alpha = 180° - 63.43°$

$\alpha = 116.57°$

So the required normal form

$\boxed{x\cos 116.57° + y\sin 116.57° = \frac{11}{2\sqrt{5}}}$

Length of perpendicular from (0.0) to the line $= \frac{11}{2\sqrt{5}}$

(b) $4x + 7y - 2 = 0$

Sol. $4x + 7y - 2 = 0$

$7y = -4x + 2$

$y = -\frac{4}{7}x + \frac{2}{7}$

Hence, $y = -\frac{4}{7}x + \frac{2}{7}$ is the slope intercept form of the equation.

And $4x + 7y - 2 = 0$

$4x + 7y = 2$

$\sqrt{4^2 + 7^2} = \sqrt{65}$

Dividing both side by $\sqrt{65}$

$\frac{4}{\sqrt{65}}x + \frac{7}{\sqrt{65}}y = \frac{2}{\sqrt{65}}$

Where $\cos\alpha = \frac{4}{\sqrt{65}}$, $\sin\alpha = \frac{7}{\sqrt{65}}$

$\Rightarrow \tan\alpha = \frac{7/\sqrt{65}}{4/\sqrt{65}} = \frac{7}{\sqrt{65}} \times \frac{\sqrt{65}}{4} = \frac{7}{4}$

$\Rightarrow \alpha = \tan^{-1}\left(\frac{7}{4}\right)$

$\Rightarrow \alpha = 60.26°$

So the required normal form is $x\cos 60.26° + y\sin 60.26 = \frac{2}{\sqrt{65}}$

Length of perpendicular from origin (0.0) to the line $= \frac{2}{\sqrt{65}}$

Now, $4x + 7y = 2$

$\frac{4x}{2} + \frac{7y}{2} = \frac{2}{2}$

$2x + \frac{7y}{2} = 1 \Rightarrow \frac{x}{\frac{1}{2}} + \frac{y}{\frac{2}{7}} = 1$

Hence, $\frac{x}{\frac{1}{2}} + \frac{y}{\frac{2}{7}} = 1$ is the two intercept form of the equation

(c) $15y - 8x + 3 = 0$

Sol. $15y - 8x + 3 = 0$

$15y = 8x - 3$

$y = \frac{8x}{15} - \frac{3}{15}$

$y = -\frac{8}{15}x - \frac{1}{5}$

Hence, $y = -\frac{8}{15}x - \frac{1}{5}$ is the two intercept form of the equation.

And $15y - 8x + 3 = 0$

$8x - 15y = 3$

$\sqrt{8^2 + (-15)^2} = \sqrt{64 + 225} = \sqrt{289} = 17$

Dividing both sides by 17, we get

$\frac{8}{17}x - \frac{15}{17}y = \frac{3}{17}$

Now, $15y - 8x + 3 = 0$

$8x - 15y = 3$

$\frac{8}{3}x - \frac{15}{3}y = \frac{3}{3}$

$\frac{8}{3}x - 5y = 1 \Rightarrow \frac{x}{\frac{3}{8}} + \frac{y}{\frac{-1}{5}} = 1$

Hence, $\frac{x}{\frac{3}{8}} + \frac{y}{\frac{-1}{5}} = 1$ is the two intercept form of the equation.

Where $\cos\alpha = \frac{8}{17}$, $\sin\alpha = \frac{-15}{17}$

$\Rightarrow \tan\alpha = \frac{-15/17}{8/17} = \frac{-15}{17} \times \frac{17}{8} = \frac{-15}{8}$

$\Rightarrow \alpha = \tan^{-1}\left(\frac{15}{8}\right) = 61.93$

$\Rightarrow \alpha = 360° - \phi$

$= 360° - 61.93°$

$\alpha = 298.07°$

So the required normal form is:

$$x\cos 298.07° + y\sin 298.07° = \frac{3}{17}$$

Length of perpendicular from origin (0,0) to the line $= \frac{3}{17}$

11. In each of the following check whether the two lines are

(i) parallel (ii) perpendicular

(iii) neither parallel nor perpendicular

(a) $2x + y - 3 = 0$; $4x + 2y + 5 = 0$

Sol: $2x + y - 3 = 0$

$m_1 = -\frac{a}{b}$

$= -\frac{2}{1} = -2$

$4x + 2y + 5 = 0$

$m_2 = -\frac{a}{b}$

$= -\frac{4}{2} = -2$

As $m_1 = m_2$ So lines are parallel.

(b) $3y = 2x + 5$; $3x + 2y - 8 = 0$

Sol: $3y = 2x + 5 \Rightarrow 2x - 3y + 5$

$m_1 = -\frac{a}{b} = \frac{-2}{-3} = \frac{2}{3}$

$3x + 2y - 8 = 0$

$m_2 = \frac{a}{b} = \frac{-3}{2}$

Since $m_1 m_2 = \left(\frac{2}{3}\right)\left(\frac{-3}{2}\right) = -1$

As product of slopes is '–1' so the lines are perpendicular.

(c) $4y + 2x - 1 = 0$; $x - 2y - 7 = 0$

Sol: $4y + 2x - 1 = 0$

$m_1 = \frac{-a}{b} = \frac{-2}{4} = \frac{-1}{2}$

$x - 2y - 7 = 0$

$m_2 = \frac{-a}{b} = \frac{-1}{-2} = \frac{1}{2}$

$m_1 m_2 = -\frac{1}{2} \times \frac{1}{2} = -\frac{1}{4}$

$m_1 \neq m_2$ and $m_1 m_2 \neq -1$ so the lines are neither parallel nor perpendicular.

2. Find an equation of the line through (–4,7) and parallel to the line $2x - 7y + 4 = 0$

Sol. Slope of given line $= m_1 = -\frac{a}{b} = \frac{-2}{-7} = \frac{2}{7}$ As slopes of parallel lines are same so

Slope of required line $= m = m_1 = \frac{2}{7}$ Also $(x_1, y_1) = (-4, 7)$

Using point slope form

$y - y_1 = m(x - x_1)$

$y - 7 = \frac{2}{7}(x - (-4))$

$7(y - 7) = 2(x + 4)$

$7y - 49 = 2x + 8$

$-2x + 7y - 49 - 8 = 0$

$-2x + 7y - 57 = 0$

$$2x - 7y + 57 = 0$$

3. Find an equation of the line through (5, -8) and perpendicular to the join of A(-15, -8), B(10, 7).

Sol. Slope of $\overline{AB} = m_1 = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{7 - (-8)}{10 - (-15)}$

$m_1 = \frac{15}{25} = \frac{3}{5}$

Slope of required line $= m = -\frac{1}{m_1} = -\frac{5}{3}$

Also $(x_1, y_1) = (5, -8)$

$y - y_1 = m(x - x_1)$

$y - (-8) = -\frac{5}{3}(x - 5)$

$3(y + 8) = -5(x - 5)$

$3y + 24 = -5x + 25$

$5x + 3y + 24 - 25 = 0$

$\boxed{5x + 3y - 1 = 0}$

Applications of Coordinate Geometry in Real life Situation

Example 14: On a map, Town A is at coordinates (2, 3) and Town B is at (−4, −1). What is the distance between the two towns?

Solution: Use the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute the values:

$$d = \sqrt{(-4-2)^2 + (-1-3)^2}$$
$$= \sqrt{(-6)^2 + (-4)^2}$$
$$= \sqrt{36+16} = \sqrt{52} \approx 7.21 \text{ unit}$$

Thus, the distance between Town A and Town B is approximately 7.21 units.

Example 15: Suppose two cities, City A and City B, are represented by coordinates (3, 4) and (7, 1) on a map. Find the straight-line distance between two cities.

Solution: We apply the distance formula:

$$d = |\overline{AB}| = \sqrt{(7-3)^2 + (1-4)^2}$$
$$d = |\overline{AB}| = \sqrt{(4)^2 + (-3)^2}$$
$$d = |\overline{AB}| = \sqrt{16+9}$$
$$= \sqrt{25} = 5$$

Thus, the straight line distance between City A and City B is 5 units.

Example 16: An Engineer is building a bridge between two points on a river. Suppose the coordinates of the two points where the bridge will start and end are (2,5) and (8, 9). Find the coordinates of the midpoint, which will represent the center of the bridge.

Solution: We apply the midpoint formula:

$$M = \left(\frac{2+8}{2}, \frac{5+9}{2}\right)$$

$$M = \left(\frac{10}{2}, \frac{14}{2}\right) = (5, 7)$$

Thus, the center of the bridge is at the point (5, 7)

Example 17: A landscaper is designing a triangular garden with corners at points A(2, 3), B(5, 7), and C(6, 2). Calculate the lengths of the sides of the triangle.

Solution: Use the distance formula to find the length of each side:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|\overline{AB}| = \sqrt{(5-2)^2 + (7-3)^2}$$
$$|\overline{AB}| = \sqrt{(3)^2 + (4)^2}$$
$$|\overline{AB}| = \sqrt{9+16} = \sqrt{25} = 5 \text{ units}$$
$$|\overline{BC}| = \sqrt{(6-5)^2 + (2-7)^2}$$
$$|\overline{BC}| = \sqrt{(1)^2 + (-5)^2}$$
$$|\overline{BC}| = \sqrt{1+25} = \sqrt{26} = 5.10 \text{ units}$$
$$|\overline{AC}| = \sqrt{(6-2)^2 + (2-3)^2}$$
$$|\overline{AC}| = \sqrt{(4)^2 + (-1)^2}$$
$$|\overline{AC}| = \sqrt{16+1} = \sqrt{17} = 4.12 \text{ units}$$

Thus, the lengths of the sides of the triangular garden are:
$m\overline{AB} = 5$ units, $m\overline{BC} \approx 5.10$ units, $m\overline{AC} \approx 4.12$ units

Example 18: A pilot needs to travel from city A(50, 60) to city B(120, 150). Determine the heading angle the plane should take realtive to the east direction.

Solution: The heading angle can be calculated using the slope:

$$m = \frac{150-60}{120-50} = \frac{90}{70} = \frac{9}{7}$$

Let θ be the required angle, then

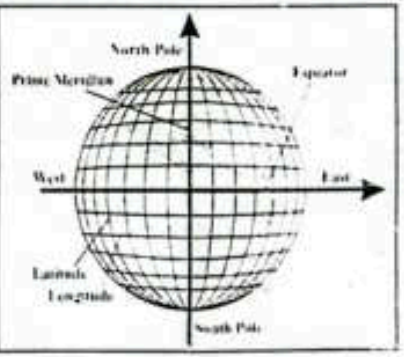
$$\tan\theta = m = \frac{9}{7}$$
$$\theta = \tan^{-1}\left(\frac{9}{7}\right)$$
$$\theta = \tan^{-1}(1.2857)$$
$$\theta = 52.13°$$

Thus, the plane should take a heading angle of 52.13° north of east.

Do you know?
Aviation is the operation and flight of aircraft, including airplanes, helicopters and drones.
Navigation is the process of determining and controlling the route of a vehicle, such as an aircraft, from one place to another.

Latitude measures how far a location is from the equator. It ranges from 0° at the equator to 90° north (at the North Pole) or 90° south (at the South Pole).

Longitude measures how for a location is from the Prime Meridian (which runs through Greenwich, London) It ranges from 0° at the Prime Meridian to 180° east and 180° west.



Example 19: Abdul Hadi is traveling from point A (Latitude 10° N, Longitude 50° to point B (Latitude 20° N, Longitude 60° E). Find the midpoint of his journey terms of latitude and longiude.

Solution: Given that,

Point A (Latitude 10° N, Longitude 50° E)

Point B (Latitude 20° N, Longitude 60° E)

$$\text{Midpoint latitude} = \frac{10^\circ + 20^\circ}{2} = 15^\circ N$$

$$\text{Midpoint longitude} = \frac{50^\circ + 60^\circ}{2} = 55^\circ E$$

Thus, the midpoint of Abdul Hadi's journey would be at Latitude. 15°N, Longitude 55°

Example 20: A landscaper is designing a straight pathway from P(2, 3) to Q(8, What is the length of the pathway?

Solution: The length of the straight pathway can be found using the distance formula:

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (x_2 - y_1)^2} = \sqrt{(8-2)^2 + (9-3)^2}$$

$$= \sqrt{(6)^2 + (6)^2} = \sqrt{36+36} = 6\sqrt{2}$$

So, the length of the pathway is approximately $6\sqrt{2}$ units.

Solved Exercise 7.3

1. If the houses of two friends are represented by coordinates (2, 6) and (9, 12) a grid. Find the straight line distance between their houses if the grid represent kilometres?

Sol. We should use the distance formula to find the straight line distance between houses.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(9-2)^2 + (12-6)^2} = \sqrt{(7)^2 + (6)^2}$$

$$= \sqrt{49+36} = \sqrt{85}$$

$$= 9.22 \text{ km}$$

So, the straight. line distance betwen these two houses is approximately 9.22 kilometr

2. Consider a straight trail (represented by coordinate plane) that starts at p (5, 7) and ends at point (15, 3). What are the coordinates of the midpoint?

Sol. use the midpoint formula for the points (5,7) and (15, 3)

$$M(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{5+15}{2}, \frac{7+3}{2}\right)$$

$$= \left(\frac{20}{2}, \frac{10}{2}\right) = (10, 5)$$

So, the coordinates of the midpoint are (10, 5)

An architect is designing a park with two buildings located at (10, 8) and (4, 3) on the grid. Calculate the straight-line distance between the buildings. Assume the coordinates are in meters.

Sol. Using the distance formula for the points (10, 8) and (4,3)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(4-10)^2 + (3-8)^2} = \sqrt{(-6)^2 + (-5)^2}$$

$$= \sqrt{36+25} = \sqrt{61}$$

$$= 7.81 \text{ m}$$

A delivery driver needs to calculate the distance between two delivery locations. One location is at (7, 2) and the other at (12, 10) on the city grid map, where each unit represents kilometers. What is the distance between the two locations?

Sol. Use distance formula for the points (7.2) and (12, 10) to find the distance between the two locations.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(12-7)^2 + (10-2)^2}$$

$$= \sqrt{(5)^2 + (8)^2} = \sqrt{25+64} = \sqrt{89}$$

$$= 9.43 \text{ km}$$

So, the distance is approximately 9.43 kilometers.

5. The start and end points of a race track are given by coordinates (3,9) and (9,13). What is the midpoint of the track?

Sol. Using the midpoint formula for the points (3,9) and (9,13)

$$M(x, y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{3+9}{2}, \frac{9+13}{2}\right) = \left(\frac{12}{2}, \frac{22}{2}\right) = (6, 11)$$

The midpoint is (6,11).

6. The coordinates of two points on a road are A(3,4) and B(7,10). Find the midpoint of the road.

$$M(x, y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{3+7}{2}, \frac{4+10}{2}\right) = \left(\frac{10}{2}, \frac{14}{2}\right) = (5, 7)$$

7. A ship is navigating from Port A located at (12°N,65°W) to Port B at (20°N,45°W). If the ship travels along the shortest path on the surface of the Earth, describe the process to calculate the straight line distance between the points.

Sol. Port A located at (12° N, 65°W) $\Rightarrow (x_1, y_1) = (65, 12)$

Port B located at (20° N, 45°W) $\Rightarrow (x_2, y_2) = (45, 20)$

Using distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - x_1)}$$

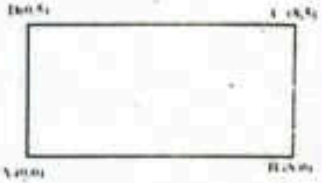
$$= \sqrt{(45-65)^2 + (20-12)^2}$$

$$= \sqrt{(-20)^2 + (8)^2} \quad = \sqrt{400 + 64}$$

$$= \sqrt{464} = 21.54 \text{ units.}$$

8. Farah is fencing building a fence around a rectangular field with corners at (0,0), (0,5), (8,5) and (8,0) How much fencing material will she need to cover the entire perimeter of the field?

Using the distance formula for the points (0, 0), (0, 5), (8, 5) and (8, 0) to find the fencing material she need to conver the entires perimeter of the field.



Sol. $|\overline{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{(8-0)^2 + (0-0)^2}$$

$$= \sqrt{(8)^2 + 0^2}$$

$$= \sqrt{64 + 0} = \sqrt{64} = 8 \text{ units}$$

$$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(8-8)^2 + (5-0)^2} = \sqrt{(0)^2 + (5)^2}$$

$$= \sqrt{0 + 25} = \sqrt{25} = 5 \text{ units}$$

Perimeter of field $= 2(\overline{AB} + \overline{BC})$

$= 2(8 + 5)$

$= 2(13) = 26$ units

So, we need 26 units fencing material to cover the entire perimeter of the field.

9. An airplane is flying from city X at (40° N, 100° W) to city Y at (50° N, 80° W). Use coordinate geometry, calculate the shortest distance between these two cities.

Sol. Convert the points of cities X and Y into the points of xy- plane then use the

distance formula $(x_1, y_1) = (100, 40)$ for city X, $(x_2, y_2) = (80, 50)$ for city y

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - x_1)^2}$$

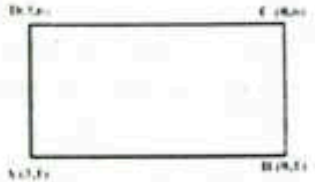
$$= \sqrt{(80-100)^2 + (50-40)^2}$$

$$= \sqrt{(-20)^2 + (10)^2}$$

$$= \sqrt{400 + 100}$$

$$= \sqrt{500} = 22.4 \text{ units}$$

10. A land surveyor is marking out a rectangular plot of land with corners at (3, 1), (3, 6), (8, 6), and (8, 1). Calculate the perimeter?



Sol. Let ther vertices of the rectangle ABCD are

A (3,1), B(8,1), C (8,6) and (3,6)

By using distance formula the length of horizontal side $\overline{AB}$

$$|\overline{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(8-3)^2 + (1-1)^2}$$

$$= \sqrt{(5)^2 + (0)^2} = \sqrt{25 + 0} = \sqrt{25} = 5$$

By distance formula the length of vertical lin $\overline{BC}$.

$$|\overline{BC}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(8-8)^2 + (6-1)^2}$$

$$= \sqrt{0^2 + (5)^2} = \sqrt{0 + 25} = \sqrt{25} = 5$$

Perimeter of plot $= 2(\overline{AB} + \overline{BC})$

$= 2(5 + 5)$

$= 2(10) = 20$ units

11. A landscaper needs to install a fence around a rectangular garden. The garden has its corners at the coordinates: A(0, 0), B(5, 0), C(5, 3), and D(0, 3). How much fencing is required?

Sol. $|\overline{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

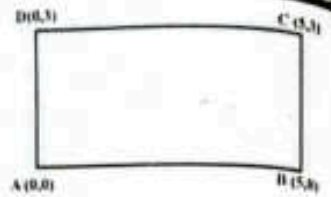
$$= \sqrt{(5-0)^2 + (0-0)^2} = \sqrt{(5)^2 + (0)^2}$$

$= \sqrt{25+0} = \sqrt{25} = 5$ units

$|BC| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(5-5)^2+(3-0)^2} = \sqrt{(0)^2+(3)^2}$

$= \sqrt{0+9} = \sqrt{9} = 3$ units



Perimeter of field $= 2[|\overline{AB}| + |\overline{BC}|]$

$= 2(5+3)$

$= 2(8) = 16$ units

So, 16 units fencing is required.

Review Exercise 7

1. Four options are given against each statement. Encircle the correct option.

(i) The equation of a straight line in the slope - intercept form is written as:

(a) $y = m(x + c)$ (b) $y - y_1 = m(x - x1)$

(c) $y = c + mx$ (d) $ax + by + c = 0$

(ii) The gradients of two parallel lines are:

(a) equal (b) zero

(c) negative reciprocals of each other (d) always undefined

(iii) If the product of the gradients of two lines is – 1, then the lines are:

(a) Parallel (b) perpendicular (c) Collinear (d) coincident

(iv) Distance between two points P(1, 2) and Q(4, 6) is:

(a) 5 (b) 6 (c) $\sqrt{13}$ (d) 4

(v) The midpoint of a line segment with endpoints (–2, 4) and (6, –2)is:

(a) (4, 2) (b) (2, 1) (c) (1, 1) (d) (0, 0)

(vi) A line passing through points (1, 2) and (4, 5) is:

(a) $y = x + 1$ (b) $y = 2x + 3$ (c) $y = 3x - 2$ (d) $y = x + 2$

(vii)The equation of a line in point - slope form is:

(a) $y = m(x + c)$ (b) $y - y_1 = m(x - x_1)$

(c) $y = c + mx$ (d) $ax + by + c = 0$

(viii)$2x + 3y - 6 = 0$ in the slope-intercept form is:

(a) $y = \frac{-2}{3}x + 2$ (b) $y = \frac{2}{3}x - 2$ (c) $y = \frac{2}{3}x + 1$ (d) $y = \frac{-2}{3}x - 2$

(ix) The equation of a line in symmetric form is:

(a) $\frac{x}{a} + \frac{y}{b} = 1$ (b) $\frac{x-x_1}{l} + \frac{y-y_1}{m} = \frac{z-z_1}{l}$

(c) $ax + by + c = 0$ (d) $y - y_1 = m(x - x_1)$

10. The equation of a line in normal form is:

(a) $y = mx + c$ (b) $\frac{x}{a} + \frac{y}{b} = 1$

(c) $\frac{x-x_1}{\cos\alpha} = \frac{y-y_1}{\sin\alpha}$ (d) $x\cos\alpha + y\sin\alpha = p$

Answers:

i. $y = c + mx$ ii. equal iii. perpendicular iv. 5

v. (2, 1) vi. $y = x + 1$ vii. $y - y_1 = m(x - x_1)$ viii. $y = \frac{-2}{3}x + 2$

ix. $ax + by + c = 0$ x. $x\cos\alpha + y\sin\alpha = p$

2. Find the distance between two points A(2, 3) and B(7, 8) on a coordinate plane.

Sol. Using distance formula

$|\overline{AB}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(7-2)^2+(8-3)^2}$

$= \sqrt{(5)^2+(5)^2}$

$= \sqrt{25+25}$

$= \sqrt{50}$

$= \sqrt{5\times5\times2}$

$= \sqrt{5^2\times2}$

$= 5\sqrt{2}$

3. Find the midpoint of the line segment joining the points (4, -2) and (-6, 3).

Sol. $M(x, y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{(4+(-6)}{2}, \frac{-2+3}{2}\right)$

$= \left(\frac{-2}{2}, \frac{1}{2}\right)$

$= \left(-1, \frac{1}{2}\right)$

4. Calculate the gradient (slope) of the line passing through the points (1, 2) and (4, 6).

Sol. Using slope formula

Slope $(m) = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{6-2}{4-1} = \frac{4}{3}$

5. Find the equation of the line in the form $y = mx + c$ that passes through the points (3,7) and (5,11).

Sol. First colculate the slope of points (3,7), (5,11)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{11-7}{5-3}$$

$$= \frac{4}{2} = 2$$

Given point (3, 7)

Using point slope form

$y - y_1 = m(x - x_1)$

$y - 7 = 2(x - 3)$

$y - 7 = 2x - 6$

$y = 2x - 6 + 7$

$y = 2x + 1$

6. If two lines are parallel and one line has a gradient of $\frac{2}{3}$, what is the gradient of the other line?

Sol. Gradient of the given line = $\frac{2}{3}$

The gradient of the parallel line will be same.

So, gradient of the other line = $\frac{2}{3}$

7. An airplane needs to fly from city A at coordinates (12, 5) to city B at coordinates (8,–4). Calculate the straight-line distance between these two cities.

Sol. Using the distance formula.

$$|\overline{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)}$$

$$= |\overline{AB}| = \sqrt{(8-12)^2 + (-4-5)^2}$$

$$= \sqrt{(-4)^2 + (-9)^2}$$

$$= \sqrt{16 + 81}$$

$$= \sqrt{97}$$

$$= 9.8 \text{ units.}$$

So, the distance between two cities is 9.8 units.

In a landscaping project, the path starts at (2, 3) and ends at (10, 7). Find the midpoint.

Sol. Using midpoint formula

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{2+10}{2}, \frac{3+7}{2}\right)$$

$$= \left(\frac{12}{2}, \frac{10}{2}\right)$$

$$= (6, 5)$$

A drone is flying from point (2,3) to point (10,15) on the grid. Calculate the gradient of the line along which the drone is flying and the total distance travelled.

Sol. Using the gradient formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{15-3}{10-2}$$

$$= \frac{12}{8}$$

$$= \frac{3}{2}$$

To calcualte the distance travelled, we use the distance formula.

Using distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(10-2)^2 + (15-3)^2}$$

$$= \sqrt{(8)^2 + (12)^2}$$

$$= \sqrt{64 + 144}$$

$$= \sqrt{208}$$

$$= 14.42 \text{ units}$$

So, the drone is flying along a line with a gradient fo 3/2 and it travels approximately 14.42 units.

10. For a line with a gradient of –3 and a y-intercept of 2, write the equation of the line in:

(a) Slope-intercept form

Sol. $m = -3$

y – intercept $(c) = 2$

Slople intercept form

$y = mx + c$ – (i)

put $m = -3$ and $c = 2$ in (i)

$y = -3x + 2$

(c) Two-point form using the points (1, 2) and (4, -7)

Sol. Given points = (1, 2), (4, – 7)

Using two point form

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 2 = \frac{-7-2}{4-1}(x-1)$$

$$\frac{y-2}{-7-2} = \frac{x-1}{4-1}$$

(e) Symmetric form

Sol. $y = -3x + 2$

$3x + y - 2 = 0$ — (i)

Compare it with

$ax + by + c = 0$

$a = 3, b = 1, c = -2$

$$r = \sqrt{a^2+b^2} = \sqrt{3^2+1^2} = \sqrt{9+1} = \sqrt{10}$$

$$\text{Cos}\,\alpha = \frac{a}{r} = \frac{3}{\sqrt{10}},\ \text{Sin}\,\alpha = \frac{b}{r} = \frac{1}{\sqrt{10}}$$

by put in $x = 1$ in (i)

$3(1) + y - 2 = 0$

$3 + y - 2 = 0$

$y + 1 = 0 \Rightarrow y = -1$

So point = (1, – 1)

Symmetric form:

$$\frac{x - x_1}{\cos\alpha} = \frac{y - y_1}{\text{Sin}\,\alpha} = r$$

$$\frac{x-1}{\frac{3}{\sqrt{10}}} = \frac{y-(-1)}{\frac{3}{\sqrt{10}}} = \sqrt{10}$$

(b) Point-slope form using the point (1, 2)

Sol. $m = -3$

Point = (1,2)

Using the point slope form

$y - y_1 = m(x - x_1)$

$y - 2 = -3(x - 1)$

(d) Intercepts form

Sol. $Y = -3x + 2$

$3x + y = 2$

Dividing both sides by 2, we get

$$\frac{3}{2}x + \frac{y}{2} = \frac{2}{2}$$

$$\frac{3}{2}x + \frac{y}{2} = 1$$

$$\frac{x}{\frac{2}{3}} + \frac{y}{2} = 1$$

(f) Normal form

Sol. $y = -3x + 2$

$3x + y = 2$

$$\sqrt{(3)^2+1^2} = \sqrt{9+1} = \sqrt{10}$$

Dividing both sides by $\sqrt{10}$, we get

$$\frac{3x}{\sqrt{10}} + \frac{y}{\sqrt{10}} = \frac{2}{\sqrt{10}}$$

Objective Type Questions

Additional MCQ's

Multiple Choice Questions prepared in the light of new Examination Techniques (Knowledge, Understanding, Application, Analytical & Conceptual)

7.1 Coordinate System

Encircle the correct option.

1. **In plane two lines x'x and y'y intersect each other at:**
(a) O (b) x (c) y (d) none of above
2. **The horizontal line x'ox called:**
(a) origin (b) x – axis (c) y – axis (d) none of above
3. **The vertical line y'oy is called:**
(a) origin (b) x – axis (c) y – axis (d) none of above
4. **All points on y – axis above x'ox are:**
(a) positive (b) negative (c) neutral (d) none of above
5. **All point on y – axis below x'ox are:**
(a) positive (b) negative (c) neutral (d) none of above
6. **All points on x – axis and lying on right of origin will be:**
(a) positive (b) negative (c) neutral (d) none of above
7. **All points on x - axis and lying on left of origin will be:**
(a) positive (b) negative (c) neutral (d) none of above
8. **The _____ gives us enough information to locate the point P in plane.**
(a) x'ox (b) y'oy (c) order pair (x,y) (d) none of above
9. **If (x, y) are the coordinates of a point p then first member of ordered pair is:**
(a) x – coordinate (b) y – coordinate (c) z – coordinate (d) none of above
10. **In P (x, y) the second member is known as:**
(a) x – coordinate (b) y – coordinate (c) z – coordinate (d) none of above
11. **In p (x, y) x is known as:**
(a) abscissa (b) ordinate (c) coordinate (d) none of above
12. **In p (x, y), y is known as:**
(a) abscissa (b) ordinate (c) coordinate (d) none of above
13. **The coordinate axes divide the plane into parts.**
(a) two (b) three (c) four (d) five
14. **For quadrant I, which of them is true:**
(a) $x > 0, y > 0$ (b) $x < 0, y > 0$ (c) $x < 0, y < 0$ (d) $x >, y < 0$
15. **For quadrant II, which of them is true:**
(a) $x > 0, y > 0$ (b) $x < 0, y > 0$ (c) $x < 0, y < 0$ (d) $x >, y < 0$
16. **For quadrant III which of them is true:**
(a) $x > 0, y > 0$ (b) $x < 0, y > 0$ (c) $x < 0, y < 0$ (d) $x > 0, < 0$

17. For quadrant IV which of them is true:
(a) $x > 0, y > 0$ (b) $x < 0, y > 0$ (c) $x < 0, y < 0$ (d) $x > 0, y < 0$

18. If $A(x_1, y_1)$, $B(x_2, y_2)$ are two points then distance formula is:
(a) $d = |AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)2}$ (b) $d = |AB| = \sqrt{(x_2 - y_2)^2 + (x_1 - y_1)^2}$
(c) $d = |AB| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ (d) $d = |AB| = \sqrt{(y_2 - x_2)^2 + (y_1 - x_1)^2}$

19. If A (– 1, 2), B (7, 5) are two points then the distance between A and B is:
(a) $\sqrt{72}$ (b) $\sqrt{73}$ (c) $\sqrt{74}$ (d) $\sqrt{75}$

20. If we have $A(x_1, y_1)$, $B(x_2, y_2)$ then mid point is:
(a) $\left(\frac{x_1 - x_2}{2}, \frac{y_1 - y_2}{2}\right)$ (b) $\left(\frac{x_1 - x_2}{2}, \frac{y_1 + y_2}{2}\right)$
(c) $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ (d) $\left(\frac{x_1 + y_1}{2}, \frac{x_2 + y_2}{2}\right)$

21. If A (2, 3), B (8, 7) are two points then mid point is:
(a) (2, 2) (b) (3, 3) (c) (4, 4) (d) (5, 5)

7.2 Equations of Straight Lines

22. If ℓ is inclination of line and parallel to x - axis then:
(a) $\alpha = 90°$ (b) $\alpha = 45°$ (c) $\alpha = 30°$ (d) $\alpha = 60°$

23. If ℓ is in inclination of line and parallel to x - axis then:
(a) $\alpha = 90°$ (b) $\alpha = 45°$ (c) $\alpha = 30°$ (d) $\alpha = 60°$

24. Formula to find slop is:
(a) $m = \sin\alpha$ (b) $m = \cos\alpha$ (c) $m = \tan\alpha$ (d) $m = \cot\alpha$

25. When ℓ is horizontal its slope is _____ and if ℓ is vertical its slope is undefined.
(a) zero (b) one (c) two (d) three

26. If m (slope) is positive then:
(a) $0° > \alpha > 90°$ (b) $0° < \alpha < 90°$ (c) $0 < \alpha > 90°$ (d) $0 > \alpha < 90°$

27. If m (slope) is negative then:
(a) $90° < \alpha > 180°$ (b) $90° > \alpha < 180°$ (c) $90° > \alpha > 180°$ (d) $90° < \alpha < 180$

28. The gradient of line ℓ is:
(a) $m = \tan\alpha = \frac{y_2 - y_1}{x_2 - x_1}$ (b) $m = \sin\alpha = \frac{y_2 - y_1}{x_2 - x_1}$
(c) $m = \cos\alpha = \frac{y_2 - y_1}{x_2 - x_1}$ (d) $m = \sec\alpha = \frac{y_2 - y_1}{x_2 - x_1}$

29. Line ℓ is horizontal iff m = 0 when:
(a) $\alpha = 0°$ (b) $\alpha = 30°$ (c) $\alpha = 60°$ (d) $\alpha = 90°$

30. Line ℓ is vertical iff m is:
(a) a defined (b) not defined (c) unlimited (d) none of above

31. If slope of AB is equal to slop of BC then three point A, B, C are:
(a) triangle (b) collinear (c) rectangle (d) circle

32. The two lines ℓ_1, ℓ_2 with slopes m_1, m_2 are parallel iff.
(a) $m1 \neq m_2$ (b) $m_1 < m_2$ (c) $m_1 = m_2$ (d) $m_1 > m_2$

33. The two lines ℓ_1, ℓ_2, with slopes m_1, m_2, are perpendicular iff:
(a) $m_1 = m_2$ (b) $m_1 \neq m_2$ (c) $m_1 = \frac{-1}{m_2}$ (d) none of abvoe

34. If $a > 0$ then the line ℓ is:
(a) right the x–axis (b) left the axis (c) above the x-axis (d) below the axis

35. If $a < 0$ then the line ℓ is:
(a) right the x–axis (b) left the axis (c) above the x-axis (d) below the x-axis

36. If $a = 0$ then the line ℓ be comes the:
(a) x–axis (b) y–axis (c) z–axis (d) none of above

37. If line ℓ intersects x–axis at (a, 0) then a is called _____.
(a) x–intercept (b) y–intercept (c) z-intercept (d) none of above

38. If line ℓ intersect y–axis at (0, b) then b is called _____.
(a) x–intercept (b) y–intercept (c) z-intercept (d) none of above

39. If slope is 2 and y–intercept is 5 then the equation is:
(a) $y = x + 5$ (b) $y = 2x$ (c) $y = 2x + 5$ (d) $y = 5x + 2$

40. Which of them is the equation of point slope form of equation a straight line.
(a) $m = \frac{y}{x}$ (b) $m = \frac{y_1 - y}{x_1 - x}$ (c) $m = \frac{y_1}{x_1}$ (d) $m = \frac{y - y_1}{x - x_1}$

41. Which of them is the equation of two point form of a straight line.
(a) $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ (b) $\frac{y - y_1}{x - x_1}$
(c) $\frac{y_2 - y_1}{x_2 - x_1}$ (d) none of above

42. If $x_1 - x_2$ then the slope becomes undefined, so the line is:
(a) horizontal (b) vertical (c) both (a) and (b) (d) none of above

43. If we have A (–2, 1), B (6, – 4) then the equation is:
(a) $5x - 8y - 2 = 0$ (b) $5x + 8y - 2 = 0$ (c) $5x + 8y + 2 = 0$ (d) $5x - 8y + 2 = 0$

44. Which of them is the intercept form of equation of a straight line:
(a) $\frac{x}{a} + \frac{y}{b} = 1$ (b) $\frac{a}{x} + \frac{b}{y} = 1$ (c) $\frac{a}{b} + \frac{x}{y} = 1$ (d) none of above

45. Which of them is the normal form of equation of a straight line?
(a) $x\cos\alpha + y\sin\alpha = p$ (b) $\cos\alpha + \sin\alpha = p$
(c) $x\cos\alpha - y\sin\alpha = p$ (d) $xy(\cos\alpha - \sin\alpha) = p$

46. $y = -\frac{a}{b}(x + \frac{c}{a})$ is known as:
(a) symmetric form (b) point slope form (c) two point form (d) intercept form

47. $\frac{x-\left(\frac{-c}{a}\right)}{\frac{b}{\pm\sqrt{a^2+b^2}}} = \frac{y-0}{\frac{a}{\pm\sqrt{a^2+b^2}}} = r$ is:

(a) point slope form (b) two point form (c) symmetric form (d) intercept form

48. $y-0=\frac{-a}{b}\left(x+\frac{c}{a}\right)$ is known as:

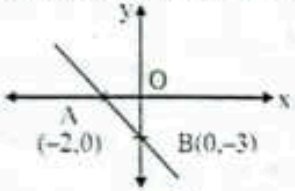
(a) point slope form (b) two point form (c) symmetric form (d) intercept form

49. $\frac{x}{\frac{c}{a}}+\frac{y}{\frac{-c}{b}}=1$ is known as:

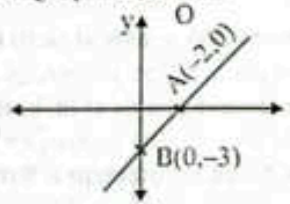
(a) point slope form (b) two point form (c) symmetric form (d) intercept form

50. According to equation $3x + 2y + 6 = 0$ which graph is correct.

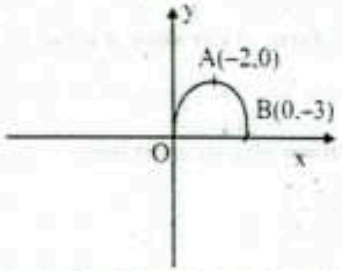
(a)



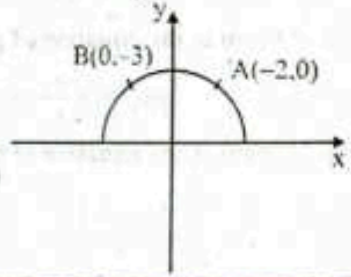
(b)



(c)



(d)



7.3 Application of co-ordinates Geometry in Real life situation

51. If A (2, 3) and B(–4, –1) then distance between two points is:
(a) 4.51 units (b) 5.21 units (c) 6.21 units (d) 7.21 units

52. If A (3, 4), B (7, 1) then the distance between A and B.
(a) 3 units (b) 4 units (c) 5 units (d) 6 units

53. If A (2, 5) and B (8, 9) two points then the co-ordinate the mid point is:
(a) (5, 5) (b) (5, 7) (c) (7, 7) (d) (7, 5)

54. If A (2, 3) and B (5, 7) are two points then distance between A and B is:
(a) 5 units (b) 6 units (c) 7 units (d) 8 units

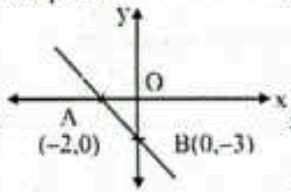
55. If A (5, 7) and B (6, 2) one two points then the distance between A and B is:
(a) 8.10 units (b) 7.10 units (c) 6.10 units (d) 5.10 units

56. If A (2, 3) and B (6, 2) are two points then the distance between A and B is:
(a) 2.12 units (b) 3.12 units (c) 4.12 units (d) 5.12 units

Answers:

1. 0	2. x-axis	3. y-axis	4. positive	5. negative
6. negative	7. Positive	8. order pair (x,y)	9. x-coordinate	10. y-coordinate
11. abscissa	12. ordinate	13. four	14. $x > 0, y > 0$	15. $x < 0, y > 0$
16. $x<0, y<0$	17. $x > 0, y < 0$	18. $d = \lvert AB\rvert = \sqrt{(x_2-x_1)^2+(y_2-y_1)2}$		
19. $\sqrt{73}$	20. $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$		21. (5, 5)	22. $\alpha = 90°$
23. $\alpha = 90°$	24. $m = \tan\alpha$	25. zero	26. $0° < \alpha < 90°$	27. $90° < \alpha < 180$
28. $m = \tan\infty = \frac{y_2-y_1}{x_2-x_1}$		29. $\alpha = 0°$	30. not defined	31. collinear
32. $m_1 = m_2$	33. $m_1 = \frac{-1}{m_2}$	34. above the x-axis		35. below the x-axis
36. x-axis	37. x–intercept	38. y–intercept	39. $y = 2x + 5$	40. $m = \frac{y_1-y}{x_1-x}$
41. $y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$		42. vertical	43. $5x + 8y + 2=0$	44. $\frac{x}{a}+\frac{y}{b}=1$
45. $x\cos\alpha + y\sin\alpha = p$		46. point slope form		47. symmetric form

48. two point form 49. intercept form 50.



51. 7.21 units

52. 5 units 53. (5, 7) 54. 5 units 55. 5.10 units 56. 4.12 units

Conceptual MCQ's

Encircle the correct option.

1. If A, B are two points then AB for:
(a) $m\overline{AB} = |\overline{AB}|$ (b) $m\overline{AB} = \{AB\}$ (c) $m\overline{AB} = |\overline{A}||\overline{B}|$ (d) None of above

2. If $x > 0, y > 0$ this shows which quadrant:
(a) 1st (b) 2nd (c) 3rd (d) 4th

3. If $x < 0, y < 0$ this shows which quadrant:
(a) 1st (b) 2nd (c) 3rd (d) 4th

4. If an angle $\infty = 0°$ this shows line is _____ to x-axis:
(a) parallel (b) Perpendicular (c) passing through origin (d) None of above

5. In determinant form of two point form of equation of a straight line is:

(a) $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 2 \\ x_3 & y_3 & 3 \end{vmatrix} = 0$ (b) $\begin{vmatrix} x & y & z \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{vmatrix} = 0$

(c) $\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix} = 0$ (d) $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

6. The general equation of straight line is:

(a) $ax + bx = 0$ (b) $ax + by + c = 0$ (c) $ay + by = 0$ (d) $ax + by + cg =$

7. If $5x - 12y + 39 = 0$ is a line then point slope form is:

(a) $y = 5(x + 39)$ (b) $y = 2(x + 39)$

(c) $y - 0 = \frac{5}{2}(x + \frac{39}{5})$ (d) $y = \frac{2}{5}(x + \frac{2}{5})$

8. If we have two points A (2, 5), B (8, 9) then the center of points is:

(a) (5, 7) (b) (5, 5) (c) (7, 7) (d) (7, 5)

9. If we have $ax + by + c = 0$ and $(\frac{-c}{a}, 0), (0, \frac{-c}{b})$ then intercept form is:

(a) $\frac{x}{c} + \frac{y}{b} = 1$ (b) $\frac{x}{c/a} + \frac{y}{-c/b} = 1$ (c) $\frac{x}{a} + \frac{y}{b} = 1$ (d) None of above

10. $\frac{x}{a} + \frac{y}{b} = 1$ it is known as:

(a) Intercept form of straight line (b) Normal form

(c) Slope form (d) None of above

Answers:

1. $m\overline{AB} = |\overline{AB}|$ 2. 1st 3. 3rd 4. parallel 5. $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} =$

6. $ax+by+c=0$ 7. $y - 0 = \frac{5}{2}(x + \frac{39}{5})$ 8. (5, 7)

9. $\frac{x}{c/a} + \frac{y}{-c/b} = 1$ 10. Intercept form of straight line

Additional Short Questions

Short Answer Questions prepared in the light of new Examination Techniques (Knowledge, Understanding, Application, Analytical & Conceptual

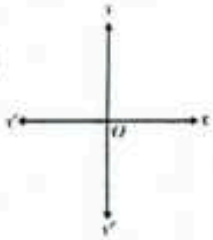
7.1 Coordinate System

1. What do you know about Geometry?

Ans. Geometry is one of the most important branch of mathematics. Fre mathematician Rene Descartes (1596-1650 AD) introduced algebraic metho geometry which gave birth to analytical geometry.

What are coordinate axes?

Ans. Draw in a plane two mutually perpendicular number lines x'x d y'y one horizontal and other vertical both intersect each other at ese two lines are called coordinate axes.



What are x-axis and y – axis?

Ans. The horizontal line x'ox is called x-axis while vertical line y'oy called y-axis.

What do you know about Cartesian coordinate?

Ans. The Cartesian coordinate system was invented by French mathematician Rene Descartes. When he tried to describe the path of a fly crawling along criss-cross beams on the ceiling while he lay on his bed. This system created a link between algebra and geometry.

Where the numbers are positive an number line?

Ans. All points on y-axis above x'ox and all points on right side of O laying on x-axis are associated with positive numbers.

Where the numbers are negative on number line?

Ans. All point on the y-axis below x'ox and all point an the left of O and laying the on x-axis will be negative.

What do you know about ordered pair?

Ans. The ordered pair (x, y) gives us enough information to locate the point P, with every point P in plane, we can associate an ordered pair of real numbers (x, y) and we say that P has coordinates (x, y).

What do you know about x-coordinate and y-coordinate?

Ans. If (x, y) are the coordinate of a point P, then the first element of the ordered pair is called x-coordinate or abscissa of P and second element of ordered pair is called y-coordinate or ordinate of P.

What are quadrants?

Ans. The coordinate axes divide the plane in four equal points called quadrants. They are defined as.

Quadrant I: All points (x, y) with $x > 0, y > 0$

Quadrant II: All points (x, y) with $x < 0, y > 0$

Quadrant III: All points (x, y) with $x < 0, y < 0$

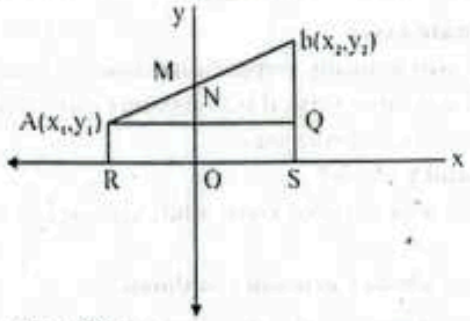
Quadrant IV: All points (x, y) with $x > 0, y < 0$

10. What is graph of (x, y)?

Ans. The point P in the plane that corresponds to an ordered pair (x, y) is called the graph of (x, y).

11. Write distance formula briefly:

Ans. Left A (x_1, y_1) and B (x_2, y_2) be two points in a plane. According to figure by Pythagorean theorem: $d = (AB)^2 = AQ^2 + QB^2$ _____ (1)



$|AQ| = |RS| = |RO + OS|$

$= |-OR + OS| = |x_2 - x_1|$

$|QB| = |SB - SQ| = |OM - ON| = |y_2 - y_1|$

now from (1)

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$d = |AB| = \sqrt{(x_2 - x_1) + (y_2 - y_1)^2}$

12. If we have two points A (-1, 2), B (7, 5) then find the distance between two point A and B.

Ans. A (-1, 2), B (7, 5)

$$d = |AB| = \sqrt{(7-(-1))^2 + (5-2)^2}$$

$$= \sqrt{(7+1)^2 + (3)^2}$$

$$= \sqrt{(8)^2 + (3)^2}$$

$$= \sqrt{64+9}$$

$$= \sqrt{73}$$

13. The point C (-5, 3) is the center of circle and *P* (7, - 2) lies on the circle. What is the radius of circle

Ans. *C* (-5, 3), *P* (7, -2)

using distance formula

$$\text{radius} = |CP| = \sqrt{(7-(-5))^2 + (-2-3)}$$

$$= \sqrt{(7+5)^2 + (-5)^2}$$

$$= (12)^2 + (-5)^2$$

$$= \sqrt{144+25}$$

$$= \sqrt{169}$$

$$= 13$$

14. What do you mean by mid point formula?

Ans. The midpoint formula is a method used in geometry to find the exact center point between two defined points in a coordinate plane.

This formula is useful when we need to divide a line segment into two equal halves.

15. Derive midpoint formula.

Ans. We have A (x_1, y_1) and B (x_2, y_2) then the x-coordinate of the midpoint is the average of the x-coordinates of points A and B.

$$x_m = \frac{x_1 + x_2}{2}$$

Now

The y–coordinate of the midpoint is the average of y–coordinates of points A and B

$$y_m = \frac{y_1 + y_2}{2}$$

Then the coordinates of the mid point $m(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

If we have points a (2, 3), b (8, 7) then find the mid point.

Ans. a (2, 3), b (8, 7)

$$M(x_m, y_m) = \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$$

$$M = \left(\frac{2+8}{2}, \frac{3+7}{2}\right)$$

$$M = \left(\frac{10}{2}, \frac{10}{2}\right)$$

$$M = (5, 5)$$

If A (2, 3), B (10, 7) then find the mid point of A and B.

Ans. A (2, 3), B (10, 7)

$$M = \left(\frac{2+10}{2}, \frac{3+7}{2}\right)$$

$$M = \left(\frac{12}{2}, \frac{10}{2}\right)$$

$$M = (6, 5)$$

7.2 Equations of Straight Lines

18. What is inclination of a line?

Ans. The angle α $(0° < \alpha < 180°)$ measured counter clock wise from positive x-axis to a non-horizontal straight line l is called the inclination of l.

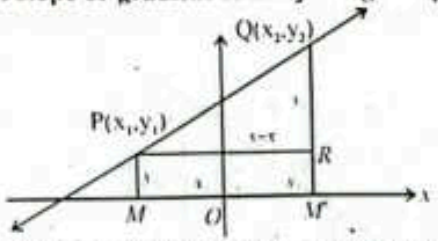
19. Define slope or gradient?

Ans. In analytical geometry slop or gradient m of a non-vertical straight line with a is its inclination is defined by $m = \tan \alpha$

If l is horizontal its slope is zero and if l is vertical then its slope is undefined.

If $0° < \alpha\ 90°$, m is positive and if $90° < \alpha < 180°$. Then m is negative.

20. Find the formula of slope or gradient of line joining two points.



Ans. Let m be the slope of the line l. Draw perpendiculars PM and QM' on x-axis and a perpendicular PR on QM'.

Then $\angle RPQ = \alpha$, $m\overline{P}R = x_2 - x_1$ and $m\overline{Q}R = y_2 - y_1$

Now slope is $m = \tan\alpha = \dfrac{y_2 - y_1}{x_2 - x_1}$

21. When the two lines ℓ_1 and ℓ_2 with respective slopes m_1 and m_2 are parallel?

Ans. ℓ_1 and ℓ_2 are parallel iff $m_1 = m_2$.

22. When two lines ℓ_1, ℓ_2 are perpendicular?

Ans. ℓ_1 and ℓ_2 are two lines with slopes m_1 and m_2 are perpendicular iff $m_1 = \dfrac{-1}{m_2}$

23. Show that A (-3, 6), B (3, 2) and C (6, 0) are collinear points:

Ans. A (-3, 6), B (3, 2) and C (6, 0)

Slope of $AB = \dfrac{2-6}{3-(-3)} = \dfrac{-4}{3+3} = \dfrac{-4}{6} = \dfrac{-2}{3}$

Slope of $BC = \dfrac{0-2}{6-3} = \dfrac{-2}{3}$

Slope of AB = slope of BC. So these are collinear.

24. Show that points A (1, 1), B (4, 5), C (12, -1) are of point of right angle.

Ans. Slope of $AB = m_1 = \dfrac{5-1}{4-1} = \dfrac{4}{3}$

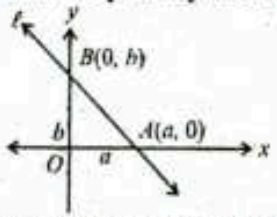
Slope of $\overline{BC} = m_2 \dfrac{-1-5}{12-4} = \dfrac{-6}{8} = \dfrac{-3}{4}$

there $m_1 m_2 = \left(\dfrac{4}{3}\right)\left(\dfrac{-3}{4}\right) = 1$

So $AB \perp BC$

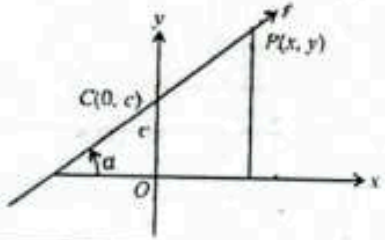
Hence ΔABC is right triangle.

25. According to figure write x-intercept and y-intercept.



Ans. If a line intersects x -axis at (a, 0) then a is called x-intercept of line. While if a line intersects y-axis at (0, b) then b is called y-intercept of the line.

26. Prove $y = mx + c$



Let P (x, y) be an arbitrary point of the straight line ℓ with slope m and y-intercept

As C (0, c) and P (x, y) lie on the line, so the slope of the line is:

$m = \dfrac{y-c}{x-0} = \dfrac{y-c}{x}$

$mx = y - c$

$y = mx + c$

Find the equation of straight line if it is perpendicular to a line with slope –6 and its y-intercept is $\dfrac{4}{3}$

$m = -6$

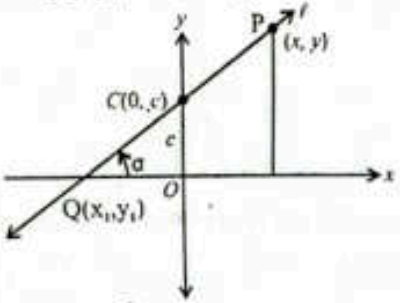
slope of line $= m_2 = -\dfrac{1}{m_1} = \dfrac{1}{6}$

$y = \dfrac{1}{6}x + \dfrac{4}{3}$

$y = \dfrac{x+3}{6}$

$6y = x + 8$ is required equation.

Equation of non-vertical straight line ℓ with slope m and passing through a point Q (x_1, y_1) is given by $y - y_1 = m(x - x_1)$.



Ans. Let P (x, y) be an arbitrary point of the straight line ℓ with slope m and y-intercept c.

As C (0, c) and P (x, y) lie on the line, so the slope of the line is $m = \dfrac{y - y_1}{x - x_1}$

$= (y - y_1) = m(x - x_1)$

$(y - y_1) = m(x - x_1)$

Which is the required equation of straight line passing through (x_1, x_1) with slope m.

29. Write the symmetric form of equation of straight line.

Ans. $\dfrac{x - x_1}{\cos\alpha} = \dfrac{y - y_1}{\sin\alpha} = r$

30. Find the equation of straight line passing through (5, 1) and parallel to a line passing through points (0, -1), (7, -15).

Ans. Suppose that m is slope of line then

$$m = \frac{-15-(-1)}{7-0} = \frac{-15+1}{7} = \frac{4}{7} = 2$$

Now point (5, 1) and m = 2 then equation is

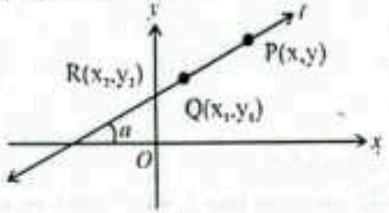
$y - 1 = -2(x - 5)$

$y = -2x + 11$

$2x + y - 11 = 0$

31. Prove that equation of vertical straight line passing through two points $Q(x_1, y_1), R(x_2, y_2)$ is $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

Ans. We have P $(x_1 y)$, Q (x_1, y_1), R (x_2, y_2)



$$\frac{y-y_1}{x-x_1} = \frac{y-y_2}{x-x_2} = \frac{y_2-y_1}{x_2=x_1}$$

P, Q, R are collinear points

Now $= \frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$ or $y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$

or $(y_2-y_1)x - (x_2-x_1)y + (x_1y_2 - x_2y_1) = 0$

32. Find the equation of line through the points (–2, 1), (6, –4).

Ans. $y - 1 = \frac{-4-1}{6-2(-2)}(x-(-2))$

$y - 1 = \frac{-5}{8}(x+2)$

$8y - 8 = -5x - 10$

$5x + 8y + 2 = 0$

33. Find the equation $\frac{x}{a} + \frac{y}{b} = 1$

Ans. Suppose P(x, y) be an arbitrary point of line whole non-zero x and y intercepts are a and b. A(a, 0), B (0, b) les on line 50.

$$y - 0 = \frac{b-0}{0-a}(x-a)$$

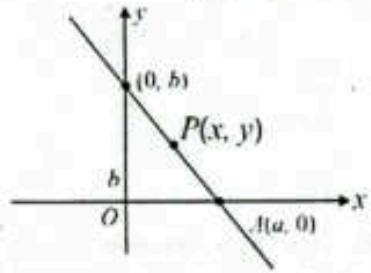
$-ay = b(x - a)$

$-ay = bx - ab$

$bx + ay = ab$

$\frac{bx}{ab} + \frac{ay}{ab} = \frac{ab}{ab}$

$\frac{x}{a} + \frac{y}{b} = 1$



34. Find the equation of line which cuts the x-axis at (2, 0)y and y-axis at (0, –4)

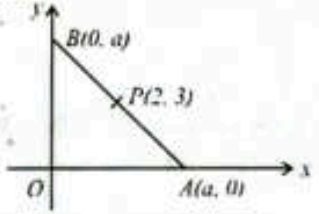
Ans. $\frac{x}{a} + \frac{y}{b} = 1$ As $(a = 2, b = -4)$

then $\frac{x}{2} + \frac{y}{-4} = 1$

$\frac{x}{2} - \frac{y}{4} = 1 \Rightarrow 2x - y + 4 = 0$

35. Find the equation of line through the point P (2, 3) which forms an Isosceles triangle with co-ordinate axes in first quadrant.

Ans. We have P (2, 3), A (a, 0), B (0, a)



m = slope of $AB = \frac{a-0}{0-a} = -1$

Now equation of line is

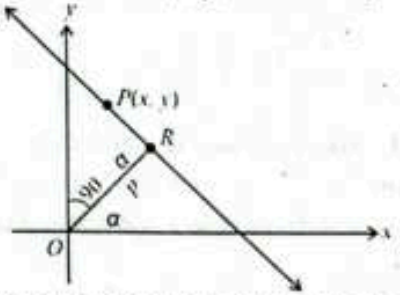
$y - 3 = -1(x - 2)$

$y - 3 = -x + 2$

$x + y - 5 = 0$

36. Find equation $x \cos\alpha + y \sin\alpha = P$

Ans. Suppose line f meet the x-axis and y-axis at points A and B. P(x, y) is an arbitrary point, so $|OR| = P$



$\cos\alpha = \frac{P}{OA} \rightarrow OA = \frac{P}{\cos\alpha}$

and $\cos(90° - \alpha) \Rightarrow OB = \frac{P}{\cos\alpha}$

hence $\frac{x}{\frac{P}{\cos\alpha}} + \frac{x}{\frac{P}{\sin\alpha}} = 1$

or $x\cos\alpha + y\sin\alpha = P$

37. The length of perpendicular from the origin to a line is 5 units and the inclination of this perpendicular is 120°. Find slop and y-intercept of the line.

Ans. $P = 5, \alpha = 120°$

so, $x \cos 120° + y \sin 120° = 5$

$-\frac{1}{2}x + \frac{\sqrt{3}}{2}y = 5$

$$x-\sqrt{3}y+10=0$$

and $y=\frac{4}{\sqrt{3}}+\frac{10}{\sqrt{3}}$

Hence $m=\frac{1}{\sqrt{3}}, C=\frac{10}{\sqrt{3}}$

38. If we have equation 5x - 12y + 39 = 0 find two intercept form.

Ans. $5x-12y+39=0$

$5x-12y=-39$

or $\frac{5x}{-39}+\frac{12}{39}=1$

or $\frac{x}{-39/5}+\frac{y}{39/12}=1$

39. If we have equation 5x - 12y + 39 = 0 find normal form.

Ans. a $5x-12y+39=0$

Divide both sides by $\pm\sqrt{5^2+12^2}=\pm13$

So, $\frac{5x}{-13}+\frac{12y}{13}=3$

40. If we have equation 5x - 12y + 39 = 0 find the symmetric form.

Ans. $5x-12y+39=0$

$\tan\alpha=\frac{5}{12}=m, \sin\alpha\frac{5}{13}, \cos\alpha=\frac{12}{13}$

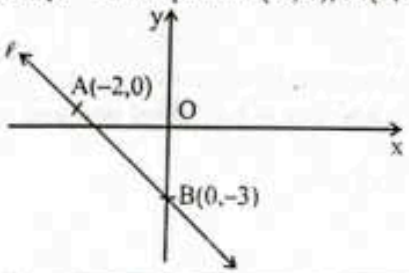
Point $P\left(\frac{-39}{5},0\right)$ then symmetric form is

$$\frac{x+39/5}{12/13}=\frac{y-0}{5/13}=r$$

41. Sketch the line 3x + 2y + 6 = 0.

Ans. $3x+2y+6=0$, here $y=0$, $x=-2$, and if $x=0$, $y=-3$

x-intercept = -2, y intercept = -3 and point A (-2, 0), B (0, -3) then



7.3 Application of co-ordinates Geometry in Real life-situation

1. If we have two points A (2, 3) and (-4, -1) find the distance between two points.

Ans. We have A (2, 3), B (-4, -1)

then $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$d=\sqrt{(-4-2)^2+(-1-3)^2}$

$=\sqrt{36+16}=\sqrt{52}=7.21\text{units}$

2. A (3, 4), B (7, 1) are two points between two places find distance between them.

Ans. A (3, 4), B (7, 1)

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(7-3)^2+(1-4)^2}=\sqrt{16+9}=\sqrt{25}=5\text{units}$

3. If we have two points A (2, 5), B (8, 9) then find the mid point between two points.

Ans. A (2, 5) B (8, 9)

mid point $=M\left(\frac{2+8}{2}, \frac{5+9}{2}\right)$

$=M\left(\frac{10}{2}, \frac{14}{2}\right)$

$=M(5, 7)$

4. Find the distance between A (5, 7) and B (6, 2).

Ans. a

$|AC|=\sqrt{(x_2-x_1)^2(y_2-y_1)^2}$

$=\sqrt{(6-5)^2+(2-7)^2}$

$=\sqrt{(1)^2+(-5)^2}=\sqrt{1+25}=\sqrt{26}=5.10\text{units}$

Conceptual Short Questions

1. Who introduce algebraic method in geometry.

Ans. Rene Descartes (1596-1650 AD)

2. What is abscissa?

Ans. In P(x, y), x is known as abscissa.

3. What is ordinate?

Ans. In P(x, y), y is known as ordinate.

4. All points (x, y) with x < 0, y > 0 locate is which quadrant?

Ans. All points locate in II(2nd) quadrant.

5. All points (x, y) with a > 0, y < 0 locate in which quadrant?

Ans. All points (x, y) locate in IV(4th) quadrant.

6. If two points P(2, 3) and Q(10, 7) then find the mid point?

Ans. P(2, 3), Q(10, 7)

$$\text{mid point } M = \left(\frac{2+10}{2}, \quad \frac{3+7}{2}\right)$$

$$= \left(\frac{12}{2}, \quad \frac{10}{2}\right)$$

$$= (6, 5)$$

is the mid point.

7. If line ℓ is parallel to y-axis then What is the angle?

Ans. If line ℓ is parallel to y-axis then the angle $\alpha = 90°$.

8. What is the angle α measured counter clock wise from positive x-axis to non-horizontal line ℓ.

Ans. The angle is $0° < \alpha < 180°$.

9. What is the equation of line through the points (–2, 1), (6, –4)

Ans. $y - 1 = \dfrac{-4-1}{6-(-2)}(x-(-2))$

$$y - 1 = \frac{-5}{8}(x+2)$$

$$8y - 8 = 5x - 10$$

$$5x + 8y + 2 = 0$$

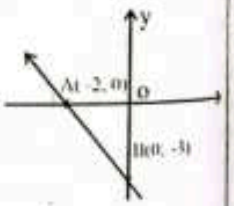
10. Sketch the line 3x + 2y + 6 =0

Ans. $3x + 2y + 6 = 0$

If y = 0, x = – 2 and if x = 0, y = 3 then x = intercept = –2

y – intercept = – 3

So A(–2 – 0), B (0, – 3)



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Unit 8 Logic

Introduction

Logic

Logic is a systematic method of reasoning that enables one to interpret the meanings of statement, examine their truth, and deduce new information from existing facts. Logic plays a key role in problem-solving and decision-making.

Statement

A sentence or Mathematical expression which may be true or false but not both is called a statement.

Here, we discuss some examples of mathematical statements that are all true.

(i) For a non-zero real number x and integers m and n, we have: $x^m \cdot x^n = x^{m+n}$

(ii) The sum of the measures of interior angles of a triangle is 180°

(iii) The circumference of a circle with radius r is $2\pi r$.

(iv) $Q \subseteq R$(Set of rational numbers is a subset of set of real numbers)

(v) $\frac{22}{7} \notin Q'$

(vi) The sum of two odd integers is an even integer

(vii) $x^2 - 5x + 6 = 0$ for x = 2 or x = 3

Further, we discuss some examples of mathematical statements that are all false.

(i) 3 + 4 = 8

(ii) $Z \subseteq W$

(iii) All isosceles triangle are equilateral triangle

(iv) Between any two real numbers there is no real number

(v) $\{1, 2, 3, 4\} \cap \{-1, -2, -3, -4\} = \{1, 2, 3, 4\}$

(vi) If a and b are the length and width of a rectangle then area of a rectangle is $\frac{1}{2}(a \times b)$.

(vii) The sum of interior angle of an n-sided polygon is $(n-1) \times 180°$

(viii) The sum of the interior angles of any quadrilateral is always 180°.

(ix) The set of integers is finite st.

Logical Operators

The letters p, q etc. Will use to donate the statements. A brief list of the symbols which will be used is given below:

Symbols	How to be read	Symbolic expression	How to be read
~	Not	~p	Not p, negation of p
∧	And	p ∧ q	p and q
∨	Or	p ∨ q	p or q

→	If . . . then, implies	$p \rightarrow q$	If p then q, p implies q
↔	Is equivalent to, if and only if	$p \leftrightarrow q$	p if and only if q, p is equivalent to q

Explanation of the use of the Symbols

Negation

If p is any statement its negation is denoted by ~p, read 'not p'. It follows from this definition that if p is true, ~p is false and if p is false, ~p is true. The possible truth values of p and ~p the are given in table: 1, which is called truth table, where the true value is denoted by T and false value is denoted by F.

Table

p	
T	
F	

Conjunction

Conjunction of two statements p and q is symbolically as $p \wedge q$ (p and q). A conjunc is considered to be true only if both statements are true. So, the truth table of $p \wedge q$ is gi in Table: 2.

Table 2

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

Example 1: Whether the following statements are true or false.

(i) Lahore is the capital of the Punjab and Quetta is the capital of Baloachistan.

(ii) $4 < 5 \wedge 8 < 10$ (iii) $2 + 2 = 3 \wedge 6 + 6 = 10$

Solution: Clearly conjunction (i) and (ii) are true whereas (iii) is false.

Disjunction

Disjunction of p and q is symbolically written $p \vee q$ (p or q). The disjunction $p \vee q$ is considered to be true when at least one of the statements is true. It is false when both of them are false. The truth table $p \vee q$ is given in Table: 3.

Table 3

p	q	p∨
T	T	T
T	F	T
F	T	T
F	F	F

Example 2: 10 is a positive integer or 0 is a rational number. Find truth value of this disjunction.

Solution: Since both statements are true, the disjunction is false.

Example 3: Triangle can have two right angles or Lahore is the capital of Sind Find the truth value of this disjunction.

Solution: Both the statements are false, the disjunction is false.

Implication or conditional

A compound statement of the form if p then q ($p \rightarrow q$) also written as **p implies q** called a **conditional** or an **implication.** p is called the **antecedent** or **hypothesis** and q called the **consequent** or the **conclusion.**

A conditional is regarded as false only when the ntecedent is true and consequent is false. In all other ases conditional is considered to be true. So, the truth ble of $p \rightarrow q$ is given in Table: 4.

Table 4

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

iconditional $p \leftrightarrow q$

The statement $p \rightarrow q \wedge q \rightarrow p$ is shortly written as $\leftrightarrow q$ and is called the **biconditional** or **equivalence.** is read **p iff q** (iff stands for "if and only if")

e draw up its truth table.

From the Table 5 it appears that $p \leftrightarrow q$ is true nly when both statements p and q are true or both tatements p and q are false.

Table 5

p	q	p→q	q→p	p↔q
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Conditionals related with a given conditional.

Let p and q be the statements and $p \rightarrow q$ be a given conditional, then

(i) $q \rightarrow p$ is called the converse of $p \rightarrow q$;

(ii) $\sim p \rightarrow \sim q$ is called the inverse of $p \rightarrow q$;

(iii) $\sim q \rightarrow \sim p$ is called the contrapositive of $p \rightarrow q$.

The truth values of these new conditionals are given below in table 6.

Table 6

				Given conditional	Converse	Inverse	Contra positive
p	q	~p	~q	p→q	q→p	~p→~q	~q→~p
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

From the table 6 it appears that

(i) Any conditional and its contrapositive are equivalent therefore any theorem may be proved by proving its contrapositive.

(ii) The converse and inverse are equivalent to each other.

Example 4: Prove that in any universal set, the empty set ϕ is a subset of any set A.

Let U be the universal set. Consider the conditional:

$$\forall x \in U, x \in \Phi \rightarrow x \in A \qquad \text{(i)}$$

The antecedent of this conditional is false because no $x \in U$, is a member of ϕ. Hence the conditional is true.

Example 5: Construct the truth table of $[(p \rightarrow q) \wedge p]$ and $[(p \rightarrow q) \wedge p] \rightarrow q$

Solution: Desired truth table 7 is given below:

Table 7

p	q	$p \to q$	$(p \to q) \wedge p$	$[p \to q \wedge p] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Example 6: Prove the following mathematical statements.

(a) If x is an odd integer, then x^2 is also an odd integers

(b) The sum of two odd numbers is even number

Solution (a): Let x be an odd integer. Then by definition of an odd integer, we express x as:

$x = 2k + 1$ for some $k \in Z$

Now $x^2 = (2k + 1)^2 = 4k^2 + 4k + 1$

$= 2(2k^2 + 2k) + 1$

$= 2m + 1$, where $m = 2k^2 + 2k \in Z$

Thus, $x^2 = 2m + 1$ for some $m \in Z$

Therefore x^2 is an odd integers, by definition of an odd integers

Note:
If x is odd, then x can be expressed in the form:
$x = 2k + 1$ for some $k \in Z$

(b): Let x and y be odd integers. Then by definition of an odd integer, we can express x and y as:

$x = 2k + 1$ and $y = 2n + 1$ for some k and $n \in Z$

Thus, $x + y = (2k + 1) + (2n + 1) = 2k + 2n + 1 + 1$

$= 2(k+n+1) = 2m$, where $k+n+1 = m \in Z$

So, $x + y = 2m$ for some $m \in Z$

Therefore $x + y$ is an even integer, by definition of an even integer

Note:
If x is an even integer, then x can be expressed in the form:
$x = 2k$ for some $k \in Z$

Example 7: Prove that for any two non-empty sets A and B, $(A \cup B)' = A' \cap B'$.

Solution: Let $x \in (A \cup B)'$

$\Rightarrow x \notin (A \cup B)$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \notin A'$ and $x \notin B'$

$\Rightarrow x \notin A' \cap B'$

But $x \in (A \cup B)'$ is an arbitrary element

Therefore $(A \cup B)' \subseteq A' \cap B'$ .....(i)

Now suppose that $y \in A' \cap B'$

$\Rightarrow y \in A'$ and $y \in B'$

$\Rightarrow y \notin A$ and $y \notin B$

$\Rightarrow y \notin (A \cup B)'$

$\Rightarrow y \in (A \cup B)'$

Thus $A' \cap B' \subseteq (A \cup B)'$ .....(ii)

From equations (i) and (ii) we conclude that

$(A \cup B)' = A' \cap B'$

Hence proved

Note:
A set B is a subset of a set A if every element of set B is also an element of set A. Mathematically we write it as $B \subseteq A$ if $\Rightarrow \forall x \in B \Rightarrow x \in A$

Theorem, Conjecture and Axiom

We will now move on to a more advanced concept, known as theorems.

Theorem: A theorem is a mathematical statement that has been proved true based on previously known facts. For example, the following statements are theorem

i) **Theorem:** The sum of the interior angles of a quadrilateral is 360 degrees.

ii) **The Fundamental Theorem of Arithmetic:** Every integer greater than 1 can be uniquely expressed as a product of prime numbers, up to the order of the factors.

iii) **Fermat's Last Theorem:** There are no three positive integers a, b, c, which satisfy the equation $a^n + b^n = c^n$. Where $n \in N$ and $n > 2$.

Conjecture: A **conjecture** is a mathematical statement or hypothesis that is believed to be true based on observations, but has not yet been proved. In mathematics, conjectures often serve as hypotheses, and if a conjecture is proven to be true, it becomes a theorem., Conversely, if evidence is found that disproves it, the conjecture is shown to be false.

Axiom: A mathematical statement that we believe to be true without any evidence or requiring any proof. For example, the following are the statements of axioms.

Axiom: Through a given point, infinitely many lines can pass.

Euclid Axioms: A straight line can be drawn between any two points.

Peano Axioms: Every natural number has a successor, which is also a natural number.

Axiom of Extensionality: Two sets are equal if they have the same elements

Axiom of Power Set: Any set, has a set of all its subsets.

Postulate: An idea that is suggested or accepted as a basic principle before a further idea is formed or developed from it.

Example 8: prove that $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$ where a,b,c and d are non-zero real numbers.

Solution:

$\frac{a}{b} + \frac{c}{d} = \frac{a}{b} \times 1 + \frac{c}{d} \times 1$ (Multiplicative identity) ($\because$ X)

$= \frac{a}{b} \times \left(d \times \frac{1}{d}\right) + \frac{c}{d} \times \left(b \times \frac{1}{b}\right)$ ($\because$ Multiplicative inverse)

$= \frac{a}{b} \times \frac{d}{d} + \frac{c}{d} \times \frac{b}{b}$ ($\because a \times \frac{1}{b} = \frac{a}{b}$)

$= \frac{ad}{bd} + \frac{cb}{db}$ ($\because$ Rule of production of fraction $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$)

$= \frac{ad}{bd} + \frac{bc}{bd}$ ($\because$ Commutative law of multiplication ab=ba)

$= ad \times \frac{1}{bd} + bc \times \frac{1}{bd}$ ($\because a \times \frac{1}{b} = \frac{a}{b}$)

$= (ad + bc) \cdot \frac{1}{bd}$ [Distributive property]

$$= \frac{(ad+bc)}{bd} = \text{R.H.S}$$

Thus, $\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$

Hence proved.

Deductive Proof

As discussed earlier, deductive reasoning is a way of drawing conclusions fr premises believed to be true.

If the premises are true, then the conclusion must also be true. For example: a human beings need to breathe to live. Ahmad is a human. Therefore, Ahmad is a breathing to live.

Similarly in mathematics, deductive proof in algebraic expression is a technique show the validity of a mathematical statement through logical reasoning based on kno rules, theorem 5, axioms, or previously proven statements. Deductive reasoning broadly used in algebra to validate identities and solve equations.

Example 9: Prove that: $(x+1)^2+7=x^2+2x+8$

Deductive Proof: L.H.S $= (x+1)^2+7$

$= (x+1)(x+1)+7$ ($\because x^m . x^n = x^{m+n}$)

$= x(x+1)+1(x+2)+7$ ($\because$ Right distuributive law)

$= x . x + x . 1 + 1 . x + 1 . 1 + 7$ ($\because$ Right distuributive law)

$= x^2 + 1 . x + 1 . x + 1 + 7$ ($\because$ Commutative law $x^m . x^n = x^{m+n}$)

$= x^2 + (1+1)x + 8$ ($\because$ Left distributive law)

$= x^2 + 2x + 8 = \text{R.H.S}$

$\Rightarrow$ L.H.S = R.H.S

Thus, $(x+1)^2+7 = x^2+2x+8$. Hence proved

Example 10: Prove that $\frac{45x+15}{15} = 3x+1$ by justifying each step.

Deductive Proof: L.H.S $= \frac{45x+15}{15}$

$= \frac{1}{15} \times (45x+15)$ $\left(\because \frac{a}{b} = \frac{1}{b} \times a\right)$

$= \frac{1}{15} \times (15 \times 3x + 15 \times 1)$ ($\because$ Multiplicative identity)

$= \frac{1}{15} \times 15(3x+1)$ ($\because$ Distributive law)

$= \left(\frac{1}{15} \times 15\right).(3x+1)$ ($\because$ Associative law)

$= 1.(3x+1)$ ($\because$ Multiplicative Inverse)

$= 3x + 1 = \text{R.H.S}$ ($\because$ Multiplicative identity)

Thus, $\frac{45x+15}{15} = 3x+1$ hence proved.

Solved Exercise 8

Four options are given against each statement. Encircle the correct option.

i. Which of the following expressions is often related to inductive reasoning?

(a) based on repeated experiments (b) if and only if statements

(c) statement is proven by a theorem (d) based on general principles

ii. Which of the following sentences describe deductive reasoning?

(a) general conclusions from a limited number of observations

(b) based on repeated experiments

(c) based on unit of information that are accurate

(d) draw conclusion from well-known facts

iii. Which one of the following statements is true?

(a) The set of integers is finite

(b) The sum of the interior angles of any quadrilateral is always 180°

(c) $\frac{22}{7} \notin Q'$ (d) All isosceles triangles are equilateral triangles

iv. Which of the following statements is the best to represent the negation of the statement "The stove is burning"?

(a) the stove is not burning. (b) the stove is dim

(c) the stove is turned to low heat (d) it is both burning and not burning.

v. The conjunction of two statements p and q is true when:

(a) both p and q are false (b) both p and q are true

(c) only q is true. (d) only p is true

vi. A conditional is regarded as false only when:

(a) antecedent is true and consequent is false.

(b) consequent is true and antecedent is false.

(c) antecedent is true only. (d) consequent is false only.

vii. Contrapositive of $q \rightarrow p$ is:

(a) $q \rightarrow \sim p$ (b) $\sim q \rightarrow p$ (c) $\sim p \rightarrow \sim q$ (d) $\sim q \rightarrow \sim p$

viii. The statement "Every integer greater than 2 is a sum of two prime numbers" is:

(a) theorem (b) conjecture (c) axiom (d) postulates

ix. The statement "A straight line can be drawn between any two points" is:

(a) theorem (b) conjecture (c) axiom (d) logic

x. The statement "The sum of the interior angle of a triangle is 180°" is:

(a) converse (b) theorem (c) axiom (d) conditional

Answers:

i. based on repeated experiments ii. draw conclusion from well-known facts

iii. $\frac{22}{7} \notin Q'$ iv. the stove is not burning. v. both p and q are true

vi. antecedent is true and consequent is false. vii. $\sim p \rightarrow \sim q$

viii. conjecture ix. axiom x. theorem

2. Write the converse, inverse and contrapositive of the following conditionals:

(i) $\sim p \rightarrow q$

Sol. Converse: $q \rightarrow \sim p$

Inverse: $p \rightarrow \sim q$

Contrapositive: $\sim p \rightarrow q$

(ii) $q \rightarrow p$

Sol. Converse: $q \rightarrow p$

Inverse: $\sim p \rightarrow \sim q$

Contrapositive: $\sim q \rightarrow \sim p$

(iii) $\sim p \rightarrow \sim q$

Sol. Converse: $\sim q \rightarrow \sim p$

Inverse: $p \rightarrow q$

Contrapositive: $q \rightarrow p$

(iv) $\sim q \rightarrow \sim p$

Sol. Converse: $\sim p \rightarrow \sim q$

Inverse: $q \rightarrow p$

Contrapositive: $p \rightarrow q$

3. Write the truth table of the following.

(i) $\sim(p \vee q) \vee (\sim q)$

Sol.

p	q	$\sim p$	$p \vee q$	$\sim(p \vee q)$	$\sim(p \vee q) \vee (\sim q)$
T	T	F	T	F	F
T	F	T	T	F	T
F	T	F	T	F	F
F	F	T	F	T	T

(ii) $\sim(\sim q \vee \sim p)$

Sol.

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$\sim(\sim q \vee \sim p)$
T	T	F	F	F	T
T	F	F	T	T	F
F	T	T	F	T	F
F	F	T	T	T	F

(iii) $(p \vee q) \leftrightarrow (p \wedge q)$

Sol.

p	q	$(p \vee q)$	$p \wedge q$	$(p \vee q) \leftrightarrow (p \wedge q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

4. Differentiate Between a Mathematical Statement and its Proof. Give two examples.

Sol. A mathematical statement is an expression which may be true or false but not both, while a mathematical proof is a logic argument that demonstrate the truth of a mathematical statement.

Examples:

(i) The sum of the measures of interior angles of a triangle is 180°. (True)

(ii) $3 + 4 = 8$ (False)

What is the difference between an axiom and a theorem? Give examples of each.

Axiom: A mathematical statement which we believe to be true without any evidence or requiring any proof.

Examples:

(i) Through a given point, there pass infinitely many lines.

(ii) Every natural number has a successor, which is also a natural number.

Theorem: A theorem is a mathematical statement that has been proved true based on previously known facts.

Examples: The sum of the interior angles of a quadrilateral is 360 degrees.

6. What is the importance of logical reasoning in mathematical proofs? Give an example to illustrate your point.

Sol. Logical reasoning is the foundation of mathematical proofs as it ensures that conclusions are derived in a systematic, and valid manner from given assumption or axioms. It eliminates ambiguity and subjectivity, allowing mathematical results to be universally accepted and verified.

Example: To prove that the sum of two odd numbers is even:

Let the two odd numbers be $2m + 1$ and $2n + 1$ (where $m, n \in z$)

Their sun is $(2m + 1) + (2n + 1) = 2m + 2n + 2 = 2(m + n + 1)$

Since $2(m + n + 1)$ is divisible by 2, the sum is even. This logical reasoning ensures the result hold for all odd numbers, not just specific cases.

7. Indicate whether it is an axiom, conjecture, or theorem, and explain your reasoning.

(i) There is exactly one straight line through any two points.

Sol. It is an axiom.

(ii) Every even number greater than 2 can be written as the sum of two prime numbers.

Sol. It is a conjuecture.

(iii) The sum of the angles in a triangle is 180°.

Sol. It is a theorem.

8. Formulate Simple Deductive Proofs For each of the following algebraic expressions, prove that the LHS is equal to the RHS:

(i) Prove that $(x - 4)^2 + 9 = x^2 - 8x + 25$

Sol. $(x - 4)^2 + 9$

L.H.S $= (x - 4)^2 + 9$

$= (x)^2 - 2(x)(4) + (4)^2 + 9$
$= x^2 - 8x + 16 + 9$
$= x^2 - 8x + 25 = R.H.S$

So, it is proved that $(x-4)^2 + 9 = x^2 - 8x + 25$

(ii) Prove that $(x+1)^2 - (x-1)^2 = 4x$

Sol. $(x+1)^2 - (x-1)^2 = 4x$

L.H.S $= (x+1)^2 - (x-1)^2$
$= [(x)^2 + 2(x)(1) + (1)^2] - [(x)^2 - 2(x)(1) + (1)^2]$
$= (x^2 + 2x + 1) - (x^2 - 2x + 1)$
$= x^2 + 2x + 1 - x^2 + 2x - 1$
$= \cancel{x^2} - \cancel{x^2} + 2x + 2x \cancel{+1-1}$
$= 4x = R.H.S$

So, it is proved that $(x+1)^2 - (x-1)^2 = 4x$

(iii) Prove that $(x+5)^2 - (x-5)^2 = 20x$

Sol. $(x+5)^2 - (x-5)^2 = 20x$

L.H.S $= (x+5)^2 - (x-5)^2$
$= [(x)^2 + 2(x)(5) + (5)^2] - [(x)^2 - 2(x)(5) + (5)^2]$
$= (x^2 + 10x + 25) - (x^2 - 10x + 25)$
$= x^2 + 10x + 25 - x^2 + 10x - 25$
$= \cancel{x^2} - \cancel{x^2} + 10x + 10x \cancel{+25-25}$
$= 20x = R.H.S$

So, it is proved that $(x+5)^2 - (x-5)^2 = 20x$

9. Prove that following by justifying each step:

(i) $\frac{4+16x}{4} = 1 + 4x$

sol. $\frac{4+16x}{4} = 1 + 4x$

L.H.S $= \frac{4+16x}{4}$

$= \frac{1}{4} \times (4 + 16x)$ ∵ $\frac{a}{b} = \frac{1}{b} \times a$

$= \frac{1}{4} \times (4 \times 1 + 4 \times 4x)$ ∵ (Multiplicative identity)

$= \frac{1}{4} \times 4(1 + 4x)$ ∵ (Distributive law)

$= (\frac{1}{4} \times 4) . (1 + 4x)$ ∵ (Associative law)

$= 1.(1 + 4x)$ ∵ (Multiplicative inverse)

$= 1 + 4x$ ∵ (Multiplicative identity)
$= R.H.S$

Thus, $\frac{4+16x}{4} = 1 + 4x$ proved.

(ii) $\frac{6x^2+18x}{3x^2-27} = \frac{2x}{x-3}$

Sol. $\frac{6x^2+18x}{3x^2-9} = \frac{2x}{x-1}$

L.H.S $= \frac{6x \times x + 6x \times 3}{3 \times x^2 - 3 \times 9}$ factorize the terms of numerator and denominator

$= \frac{6x(x+3)}{3(x^2-9)}$ Disributive law in numerator and denominator

$= \frac{3 \times 2x(x+3)}{3[(x)^2 - (3)^2]}$ factorize the numerator and use $a.a = a^2$ in denominator

$= \frac{3(x+3) \times 2x}{3(x+3)(x-3)}$ use associative law of multiply in numerator and use $a^2 - b^2 = (a+b)(a-b)$ in denominator

$= 1 \times \frac{2x}{(x-3)}$ use $\frac{a}{a} = 1$ Division property

$= \frac{2x}{x-3}$ Multiplicative identity
$= R.H.S$

Thus, $\frac{6x^2+18x}{3x^2-27} = \frac{2x}{x-3}$ proved.

Note (the statement in the book is not correct)

(iii) $\frac{x^2+7x+10}{x^2-3x-10} = \frac{x+5}{x-5}$

Sol. $\frac{x^2+7x+10}{x^2-3x-10} = \frac{x+5}{x-5}$

L.H.S $= \frac{x^2+7x+10}{x^2-3x-10}$

$= \frac{x^2+5x+2x+10}{x^2-5x+2x-10}$ break the mid terms

$= \frac{x(x+5)+2(x+5)}{x(x-5)+2(x-5)}$ use distributive law

$= \frac{(x+5)(x+2)}{(x-5)(x+2)}$ use again the distributive law

$= \frac{(x+5)}{(x-5)} \times 1$ use $\frac{a}{a} = 1$ division properly

$= \frac{x+5}{x-5}$ Multiplicative identity

= R.H.S

Hence, $\frac{x^2+7x+10}{x^2-3x-10} = \frac{x+5}{x-5}$ is proved.

The question in the book is wrong.

10. Suppose x is an integer. Then x is odd then if 9x + 4 is odd.

Sol. $9x + 4$ _____(i)

Let $x = 2k + 1$ as x is odd

put $x = 2k + 1$ in (i)

$9(2k + 1) + 4$

$= 18k + 9 + 4$

$= 18k + 13$

$= 18k + 12 + 1$

$= 2(9k + 6) + 1$

Let $m = 9k + 6$

$= 2m + 1$ is an odd number.

11. Suppose x is an integer. If x is odd then 7x + 5 is even.

Sol. $7x + 5$ _____(i)

Let $x = 2k + 1$ as x is odd

put $x = 2k + 1$ in (i)

$7(2k + 1) + 5$

$= 14k + 7 + 5$

$= 14k + 12$

$= 2(7k + 6)$

Let $m = 7k + 6$

$= 2m$ is an even number

12. Prove the following statement

(a) If x is an odd integer, then show that it $x^3 - 4x + 6$ is odd.

Sol. (a) $x^3 - 4x + 6$ _____(i)

Let $x = 2k + 1$ as x is odd

put $x = 2k + 1$ in (i)

$(2k + 1)^3 - 4(2k + 1) + 6$

$= [(2k)^3 + 2(2k)(1) + (1)^3] - 8k - 4 + 6$

$= 4k^3 + 4k + 1 - 8k - 4 + 6$

$= 4k^3 + 4k - 8k + 1 - 4 + 6$

$= 4k^3 - 4k + 3 = 4k^3 - 4k + 2 + 1$

Let $m = 4k^3 - 4k + 2$

$= m + 1$ is odd

(b) If x is an even integer then show that $x^3 + 2x + 4$ is even.

$x^3 + 2x + 4$ _____(i)

Let $x = 2k$ as x is even

put $x = 2k$ in (i)

$(2k)^3 + 2(2k) + 4$

$= 4k^3 + 4k + 4 = 2(2k^3 + 2k + 2)$

Let $m = 2k^3 + 2k + 2$

$= 2m$ is even

13. Prove that for any two non-empty set A and B, $(A \cap B)' = A' \cup B'$.

Sol. $(A \cap B)' = A' \cup B'$

Let $x \in (A \cap B)'$

$\Rightarrow x \notin A \cap B$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in A'$ and $x \in B'$

$\Rightarrow x \in A' \cup B'$ _____(1)

But x is an arbitrary member of $(A \cap B)'$

Therefore it means that $(A \cap B)' \subseteq A' \cup B'$ _____(2)

Now suppose that

$y \in A' \cup B'$

$y \in A'$ and $y \in B'$

$y \notin A$ and $y \in B$

$y \notin (A \cap B)$

$y \notin (A \cap B)'$ _____(3)

From (1), (2) and (3) we know that $(A \cap B)' = A' \cup B'$

If x and y are positive real numbers and $x^2 < y^2$ then $x < y$.

$x^2 < y^2$ _____(i)

since $x, y > 0$

take the square root of both sides of (i)

$\sqrt{x^2} < \sqrt{y^2}$

$\Rightarrow x < y$

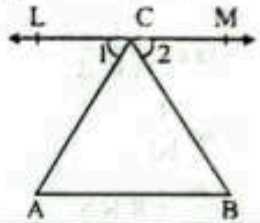
So, it is proved that if x, y are positive real numbers and $x^2 < y^2$ then $x < y$.

The sum of the interior angle of a triangle is 180°.

Given = ΔABC

Prove = $m\angle A + m\angle B + m\angle C = 180$

Construction = Draw a line $\overleftrightarrow{LM}$ through C parallel to $\overline{AB}$. Label the angles 1 and 2.



Statement	Reasons
$m\angle A = m\angle 1$ _____(i)	alternate angles
$m\angle B = m\angle 2$ _____(ii)	alternate angles
$m\angle 1 + m\angle BCA + m\angle 2 = 180°$	angles of a straight line
$m\angle A + m\angle C + m\angle B = 180°$	From (i) and (ii)
So, it is proved that sum of the interior angles of a triangle is 180°.	

16. If a,b and c are non-zero real numbers, prove that:

(a) $\frac{a}{b} = \frac{c}{d} \Leftrightarrow ad = bc$

Sol. $\frac{a}{b} = \frac{c}{d}$

$\Rightarrow \frac{a}{b}(bd) = \frac{c}{d}(bd)$ Multiplicative proprty of equality

$\Rightarrow \frac{a.1}{b}(bd) = \frac{c.1}{d}(bd)$ Multiplicative identity

$a.\left(\frac{1}{b}.b\right).d = c.\left(\frac{1}{d}.bd\right)$ Associative property w.r.t ×

$a.1.d = c.\left(\frac{1}{d}.d.b\right)$ multiplicative inverse

$a.d = c.(1.b)$

$ad = c.b$ Multiplicative identity

$ad = bc$ Commutative property of multiplicative

(b) $\frac{a}{b}.\frac{c}{d} = \frac{ac}{bd}$

Sol. $\frac{a}{b}.\frac{c}{d} = \frac{ac}{bd}$

L.H.S $= \frac{a}{b}.\frac{c}{d}$

$= a.\frac{1}{b}.c\frac{1}{d}$ rule for product of fraction

$= (a.c).\left(\frac{1}{b}.\frac{1}{d}\right)$ associative property w.r.t ×

$= ac.\frac{1}{bd}$ rule for product of fraction

$= \frac{ac}{bc} = R.H.S$

So, it is proved that $\frac{a}{b}.\frac{c}{d} = \frac{ac}{bd}$

(c) $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$

Sol. $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$

L.H.S $= \frac{a}{b} + \frac{c}{b}$

$= a.\frac{1}{b} + b.\frac{1}{b}$ product rule of fraction

$= (a + c)\frac{1}{b}$ left distributive property

$= \frac{a+c}{b} = R.H.S$

So, it is proved that $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$

Objective Type Questions

Additional MCQ's

Multiple Choice Questions prepared in the light of new Examination Techniques (Knowledge, Understanding, Application, Analytical & Conceptual)

8.1 Statement

Encircle the correct option.

1. A sentence or mathematical expression which may be true or false but not both is:
(a) statement (b) argument (c) proof (d) check
2. The sum of two odd integers is an even integer, it is statement.
(a) false (b) true (c) may be true (d) may be wrong
3. All isosceles triangle are equilateral triangle, it is statement
(a) false (b) true (c) may be true (d) may be false
4. "~"this symbol read as:
(a) fales (b) true (c) not (d) and
5. "∧" this symbol read as:
(a) yes (b) not (c) or (d) and
6. "∨" this symbol read as:
(a) or (b) and (c) not (d) none of above
7. If P is any statement its negation is as:
(a) ∧P (b) ∨P (c) ~P (d) ≈P
8.

P	~ P
T	
F	

Tick the correct answer.

(a)

P	~ P
T	T
F	F

(b)

P	~ P
T	F
F	T

(c)

P	~ P
T	F
F	F

(d)

P	~ P
T	T
F	T

9. Conjunction of two statements p and q is written as:
(a) P ~ q (b) P ∨ q (c) P ∧ q (d) ~ P ~ q
10. True example of conjunction is:
(a) $4 < 5 \wedge 8 < 10$ (b) $5 < 5 \wedge 10 < 10$ (c) $2 > 5 \wedge 3 > 6$ (d) none of above
11. Disjunction of P and q is written as:
(a) P ~ q (b) P ∧ q (c) P ∨ q (d) P → q
12. Biconditional of P and q is written as:
(a) P ∨ q (b) P ∧ q (c) P → q (d) P ↔ q

13. If P and q be the statement and $P \rightarrow q$ be given conditional then $q \rightarrow P$ called:
(a) converse (b) inverse (c) contra positive (d) none of above

14. Inverse of $P \rightarrow q$ is written as:
(a) $P \rightarrow q$ (b) $\sim P \rightarrow \sim q$ (c) $\sim q \rightarrow \sim P$ (d) $\sim P \rightarrow \sim q$

15. Contra positive of $P \rightarrow q$ is written as:
(a) $\sim q \rightarrow P$ (b) $\sim P \rightarrow q$ (c) $\sim q \rightarrow \sim P$ (d) $\sim P \rightarrow \sim q$

16. A statement which is true for all possible values of the variables involved in it known as:
(a) tautology (b) absurdity (c) contingency (d) none of above

17. A statement is already false is known as:
(a) tautology (b) absurdity (c) contingency (d) none of above

18. A statement which can be true or false depending upon the truth values of the variables involved in it is known as:
(a) tautology (b) absurdity (c) contingency (d) none of above

19. How many types of quantities:
(a) two (b) three (c) four (d) five

20. Symbol of universal quantifiers is:
(a) $\wedge$ (b) $\forall$ (c) $\vee$ (d) $\sim$

21. Symbol is used for existential quantities:
(a) $\forall$ (b) $\exists$ (c) $\sim$ (d) $\wedge$

22. If x is an odd integer then _____ is also an odd integer:
(a) x^2 (b) 2 (c) both (a) and (b) (d) none of above

23. The sum of two odd numbers is _____ number.
(a) odd (b) even (c) neutral (d) none of above

24. $\frac{a}{b}+\frac{c}{d}=$ _____.
(a) $\frac{ac}{bd}$ (b) $\frac{a+c}{b+d}$ (c) $\frac{ad+bc}{bd}$ (d) $\frac{ab+cd}{bd}$

25. The sum of interior angles of a quadrilateral is _____.
(a) 90° (b) 180° (c) 270° (d) 360°

26. Every integer _____ can be uniquely expressed as a product of prime numbers up to the order of the factors.
(a) greater than 1 (b) less than 1 (c) equal to 1 (d) zero

27. There are no three positive integers a, b, c, $n \in N$, whose $n > 2$ that satisfy the equation.
(a) $a^n + b^n = c^n$ (b) $a^n b^n = c^n$ (c) $a^n - b^n = c^n$ (d) $\frac{a^n}{b^n} = c^n$

28. Who claims that he prove the fermat's last theorem?
(a) Dr. Abdusalam (b) Andrew wiles (c) Al-beruni (d) Newton

29. Every even integer _____ is a sum of two prime numbers.
(a) equal to 2 (b) less then 2 (c) greater than 2 (d) none of above

30. $(x-1)^2 + (x+1)^2 =$ _____.
(a) $5(x^2+1)$ (b) $4(x^2+1)$ (c) $3(x^2+1)$ (d) $2(x^2+1)$

Answers:

statement	2. true	3. false	4. not	5. and
or	7. $\sim P$	8. P | $\sim$P; T | F; F | T	9. $P \wedge q$	10. $4<5 \wedge 8<10$
$P \vee q$	12. $P \leftrightarrow q$	13. converse	14. $\sim P \rightarrow \sim q$	15. $\sim q \rightarrow \sim P$
tautology	17. absurdity	18. contingency	19. two	20. $\forall$
3	22. x^2	23. even	24. $\frac{ad+bc}{bd}$	25. 360°
greater than 1	27. $a^n + b^n = c^n$	28. Andrew wiles	29. greater than 2	30. $2(x^2+1)$

Conceptual MCQ's

1) Encircle the correct option.

$\frac{22}{7}$ = _____ Q'
(a) $\notin$ (b) $\in$ (c) $\neq$ (d) $\not\leq$

2. Between any two real numbers there is no real number:
(a) It is true statement (b) It is false statement
(c) may or may not false (d) None of above

3. Which of them table is correct?
(a) P | ~P; T | F; F | T (b) P | ~P; T | T; F | T (c) P | ~P; T | F; F | T (d) P | ~P; T | T; F | F

4. Which symbol stands for such that:
(a) $\approx$ (b) $<$ (c) $\in$ (d) $\ni$

5. If x is odd then x can be expressed in the form:
(a) $x = 2k+1, k \in z$ (b) $x = k+1, k \in z$ (c) $x = 2k-1, k \in z$ (d) $d = x = k-1, k \in z$

6. If x is even integer, then x can be expresses in the form:
(a) $x = k, k \in z$ (b) $x = 2k, k \in z$ (c) $x = 3k, k \in z$ (d) $x = 4k, k \in z$

7. If set B is a subset of a set A if every element of set B is also an element of a set A. It is written as:
(a) $A \subseteq B, \forall x \in B, \Rightarrow x \in A$ (b) $B \subseteq B, \forall x \in B, \Rightarrow x \in A$
(c) $B \subseteq A, \forall x \in B, \Rightarrow x \in A$ (d) None of above

8. $(A \cup B)^c =$ _____
(a) $A^c \cap B^c$ (b) $A^c \cup B^c$ (c) $B^c \cap A^c$ (d) $A^c \cup B^c$

9. $(A \cap B)^c =$ _____
(a) $A^c \cap B^c$ (b) $A^c \cup B^c$ (c) $B^c \cap A^c$ (d) $A^c \cup B^c$

10. $2\pi r$ is a:
(a) circumference of circle (b) radius of circle
(c) diameter of circle (d) sector of circle

Answers:

1. ∉ 2. It is false statement 3. | P | P | / | T | F | / | F | T | 4. ›
5. $x = 2k + 1, k \in z$ 6. $x = 2k, k \in z$ 7. $B \subset A, \forall x \in B, \Rightarrow x \in A$
8. $A^c \cap B^c$ 9. $A^c \cup B^c$ 10. circumference of circle

Additional Short Questions

Short Answer Questions prepared in the light of new Examination Techniques (Knowledge, Understanding, Application, Analytical & Conceptual)

8.1 Statement

1. Define logic.

Ans. Logic is a systematic method of reasoning that enables you to interpret the meaning of statement, examine the truth of statement and deduce new information from existing facts.

2. How logic plays a key role?

Ans. Logic plays a key role in problem solving and decision making.

3. Define induction with an example.

Ans. A person gets penicillin injection once or twice and experiences reaction soon afterwards. He generalises that he is allergic to penicillin. This way of drawing conclusions is called induction.

4. Define statement.

Ans. A sentence or mathematical expression which may be true or false but not both is called a statement.

5. The circumference of a circle with radius r is $2\pi r$ is it true statement or fals?

Ans. It is true statement.

6. If a and b are the length and width of a rectangle then area of reactangle is $\frac{1}{2}(a \times b)$. Is it true or false statement?

Ans. It is false statement.

7. What is meant by –P?

Ans. –P means "Not P" or negation of P.

8. What is meant by "P ∧ q".

Ans. P ∧ q means "P and q".

9. What is the symbol for "or"?

Ans. Symbol for or is "∨".

10. What is meant by symbol "→"?

Ans. Symbol "→" means if ——— then, implies. p → q (if P then q, p implies q)

11. What is meant by "↔"?

Ans. "↔" is equivalent to if and only if p ↔ q. p if and only if q, p is equivalent to q.

What do you mean by negation?

If P is any statement its negation is denoted by "~P" read not P. If P is true, ~P false and if P is false ~P is true.

P	~P
T	F
F	T

What do you mean by conjunction?

Conjunction of two statements P and q is denoted symbolically as P ∧ q (P and q). A Conjunction is considered to be true only if both statements are true.

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

What is disjunction?

Disjunction of P and q is symbolically written P ∨ q (P or q). The disjunction p ∨ q is considered to be true when at least one of the statements is true. It is false when both of them are false.

p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

15. What do you mean by Implication or conditional?

Ans. A compound statement of the form if P then q also written P implies q is called conditional or an implication. Here P is called antecedent or hypothesis and q is called consequent or the conclusion. A conditional is regarded as false any when the antecedent is true and consequent is false. In all other cases conditional is considered to be true.

16. What do you know about Biconditional (P ↔ q)?

Ans. The statement P → q ∧ q → P is shortly written P ↔ q and is called the bi conditional or equivalence. It read P iff q.

17. Write the converse of P → q.

Ans. The inverse of P → q is q → p.

18. Write the inverse of P → q.

Ans. The inverse of P → q is ~ P → ~ q

19. What is the contra positive of p → q

Ans. The contra positive of P → q is ~ q → ~P

20. Prove that in any universe the empty set ϕ is a subset of any set A.

Ans. Let U be the universal set consider the conditional ∀ x ∈ U, x ∈ ϕ → x ∈ A.
The antecedent of this conditional is false because no x ∈ U is a member of P. Hence the condition is true.

21. Define tautologies.

Ans. A statement which is true for all possible values of the variables involved in it

called tautology e.g; $(P \to q) \leftrightarrow (\sim q \to \sim P)$

22. What is absurdity?

Ans. A statement which is already false is called absurdity or contradiction e.g; $P \to \sim P$

23. What is contingency?

Ans. A statement which can be true or false depending upon values of the variables involved in it is called contingency e.g; $(P \to q) \wedge (P \vee q)$

24. What do you mean by quantifiers?

Ans. The word or symbol which convey the idea of quantity or number are called quantifiers.

25. How many types of quantifiers? Name them,

Ans. There are two types of quantifiers:

(i) Universal quantifier meaning (for all $\forall$)

(ii) Existential quantifier: There exist (some or few, atleast one) $\exists$

26. Prove that if x is an odd integer, then x^2 is also an odd integer.

Ans. Let x is an odd integer

$x = 2k + 1, K \in z$

$x^2 = (2k + 1)^2 = 4k^2 + 4k + 1$

$x^2 = 2(2k^2 + 2k) + 1$

$x^2 = 2m + 1, m = 2k^2 + 2k \in z$

So, x^2 is also an odd integer

27. Show that the sum of two odd numbers is an even number.

Ans. Let x, y are odd integers so,

$x = 2k + 1, \quad y = 2n + 1$

$x + y = 2k + 1 + 2n + 1 = 2k + 2n + 2 = 2(k + n + 1)$

$\therefore \quad k + n + 1 = m$

So, $x + y = m$

Hence x + y is an even integer.

28. Define theorem.

Ans. A theorem is a mathematical statement that has proved true based on previously known facts its e,g;.

The sum of interior angles of a quadrilateral is 360°.

29. What do you mean by conjecture?

Ans. A conjecture is a mathematical statement or hypothesis that is believed to be true based on observations, but has not yet been proved.

30. What axioms?

Ans. A mathematical statement which we believe to be true without evidence or requiring any proof.

31. Show that $(x - 1)^2 + (x + 1)^2 = 2(x^2 + 1)$

Ans. L.H.S $= (x - 1)^2 + (x + 1)^2 = x^2 + 1^2 - 2x + x^2 + 1^2 + 2x$

$= 2x^2 + 2 = 2(x^2 + 1) = R.H.S$

Show that $(x + 2)^2 - (x - 2)^2 = 8x$

L.H.S $= (x + 2)^2 - (x - 2)^2$

$= (x^2 + 4 + 4x) - (x^2 + 4 - 4x)$

$= x^2 + 4 + 4x - x^2 - 4 + 4x$

$= 4x + 4x = 8x = R.H.S$

What is deductive proof?

If premises are true then conclusion must also be true e,g; All humans need to breathe to live. Ahmad is human, So Ahmad is also breathe to live.

Conceptual Short Questions

Is sum of measures of interior angles of a triangle is 360°.

No, the sum of measrue of interior angle of triangle is 180°.

Find intersection of A = {1, 2, 3, 4}, B = {–1, –2, –3, –4}

$A \cap B = \{1, 2, 3, 4\} \cap \{-1, -2, -3, -4\} = \{\ \} = \phi$

Construct the Table $p \wedge q$.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Construct the table $p \to q$.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Construct the table $\sim p \to \sim q$.

p	q	$\sim p \to \sim q$
T	T	T
T	F	T
F	T	F
F	F	T

What do you mean by postulate?

An idea that is suggested or accepted as a basic principle before a further idea is formed or developed from it.

What is peano axioms?

Every natural number has a successor which is also natural number, it is peano axioms.

What is meant by Euclid axioms?

A straight line can be draw between any two points, it is Euclid axioms.

4. Construct the Table $p \vee q$.

Ans.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

6. Construct the table $q \to p$.

Ans.

p	q	$q \to p$
T	T	T
T	F	T
F	T	F
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Unit 9 Similar Figure

Similarity of Polygons

Two polygons are similar if their corresponding angles are equal and the corresponding sides are proportional (i.e., the ratios of the lengths of corresponding sides are equal).

Remember! Three or more than three-sided closed figure is called polygon.

Identification of Similar Triangles

(i) If two angles in one triangle are congruent to two corresponding angles in another triangle, the third angle in each triangle must be congruent. Since the angle are the same, the triangles are similar. Simililarity symbol is '~'.

(ii) If the ratio of two corresponding sides and their included angle are equal, then the triangles are similar.

(iii) If the ratio of all the corresponding sides are equal, then the triangles are similar.

Example 1:

If one pair of corresponding sides are parallel to each other, then the triangles so formed as shown in the figure are similar. i.e.,

Need to Know! Proportional sides means one side is k times of its corresponding side.

In the figure $\overline{AB}$ is parallel to $\overline{CD}$ and

$m\angle AOB = m\angle DOC$ (Vertically opposite angles)

$m\angle A = m\angle D$ (Alternate angles of parallel lines)

$m\angle B = m\angle C$ (Alternate angles of parallel lines)

Since all three corresponding angles are equal,

so $\Delta OAB \sim \Delta ODC$

The ratio of corresponding sides are equal i.e.,

$$\frac{m\overline{OA}}{m\overline{OD}} = \frac{m\overline{AB}}{m\overline{DC}} = \frac{m\overline{OB}}{m\overline{OC}}$$

$$\frac{2}{2.8} = \frac{3}{4.2} = \frac{2.5}{3.5}$$

$$\frac{5}{7} = \frac{5}{7} = \frac{5}{7}$$

So, the triangles OAB and ODC are similar.

Example 2:

In the triangles XBC and XDE, find the value of x and y.

Solution: Since $\overline{BC}$ is parallel to $\overline{ED}$, so the triangles XBC and XDE are similar, so the ratio of the corresponding sides are:

$$\frac{m\overline{XB}}{m\overline{XD}} = \frac{m\overline{BC}}{m\overline{DE}} = \frac{m\overline{XC}}{m\overline{XE}}$$

$$\frac{2}{y} = \frac{x}{4} = \frac{1.8}{2.7}$$

$$\frac{x}{4} = \frac{1.8}{0.4} \Rightarrow x = \frac{1.8}{2.7} \times 4 = 2.67\text{cm}$$

$$\frac{2}{y} = \frac{1.8}{2.7} \Rightarrow x = \frac{2.7}{1.8} \times 2 = 3\text{cm}$$

Similarity of Quadrilaterals

Example 3: The Quadrilateral ABCD has side lengths $m\overline{AB}$ = 5cm, $m\overline{BC}$ = 8cm, $m\overline{CD}$ = 10cm, $m\overline{AD}$ = 12cm, and its angles are $m\angle A = 90°$, $m\angle B = 120°$, and $m\angle C = 90°$. Quadrilateral EFGH has side lengths $m\overline{EF}$=16cm, $m\overline{GH}$=20cm, $m\overline{EH}$=24cm and its angles $m\angle E = 90°$, $m\angle F = 120°$ and $m\angle H = 60°$. Prove that quadrilateral ABCD is similar to the quadrilateral EFGH. Diagrams are not drawn to scale.

Solution: We see that in quadrilateral ABCD $m\angle D = 360° - (90° + 120° + 90°) = 60°$.

In the quadrilateral EFGH, $m\angle G = 360° - (90° + 120° + 60° = 90°$.

Now, check if the correponsing angles of the quadrilaterals are congruent:

$m\angle A = m\angle E = 90°$, $m\angle B = m\angle F = 120°$, $m\angle C = m\angle G = 90°$ and $m\angle D = m\angle H = 60°$.

Next, check the ratio of the corresponding sides:

Ratio of $\overline{AB}$ to $\overline{EF}$: $\frac{m\overline{AB}}{m\overline{EF}} = \frac{5}{10} = \frac{1}{2}$, Ratio of $\overline{BC}$ to $\overline{FG}$: $\frac{m\overline{BC}}{m\overline{FG}} = \frac{8}{16} = \frac{1}{2}$

Ratio of $\overline{CD}$ to $\overline{GH}$: $\frac{m\overline{CD}}{m\overline{GH}} = \frac{10}{20} = \frac{1}{2}$, Ratio of $\overline{AD}$ to $\overline{EH}$: $\frac{m\overline{AD}}{m\overline{EH}} = \frac{12}{24} = \frac{1}{2}$

Since the corresponding angles are congruent and the corresponding sides are proportional (with a ratio of $\frac{1}{2}$), so the quadrilateral ABCD is similar to the quadrilateral EFGH.

Example 4: Find whether the parallelograms are similar given that one of the angle between sides is 45° in both the parallelograms.

Solution:

Since opposite angles in a parallelogram are equal and adjacent angles are supplementary, so the corresponding angles in both parallelograms (45°, 135°, 45°, and 135°) are equal. So, the parallelograms are similar.

Measure of the base of smaller parallelogram, b_1 = 2units

Measure of the base of larger parallelogram, $b_2 = 6$ units.
Measure of the height of smaller parallelogram, $h_1 = 1$ unit.
Measure of the height of larger parallelogram, $h_2 = 3$ units.

Ratio of corresponding lengths are equal, i.e., $\frac{b_1}{b_2} = \frac{2}{6} = \frac{1}{3}$ and $\frac{h_1}{h_2} = \frac{1}{3}$

Therefore, $\frac{b_1}{b_2} = \frac{h_1}{h_2}$

Example 5: The perimeter of a regular octagon is 48 cm. Another octagon has all that are 1.2 times the sides of the first octagon. What is the length of side of second octagon?

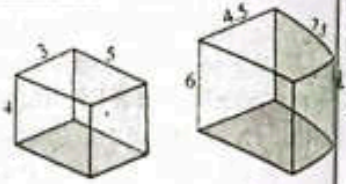
Solution: Perimeter of first regular octagon = 48 cm

Side length of first regular $= \frac{48}{8} = 6$cm.

Side length of second regular octagon $= 6 \times 1.2 = 7.2$ cm.

Solved Exercise 9.1

1. Find whether the solids are similar. All lengths are in cm.



Sol. The ratio of corresponding sides of the solids.

$$\frac{4}{6} = \frac{3}{4.5} = \frac{5}{7.5}$$

$$\frac{2}{3} = \frac{3}{\frac{45}{10}} = \frac{5}{\frac{75}{10}}$$

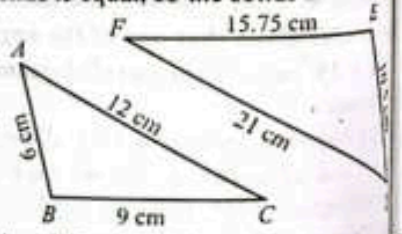
$$\frac{2}{3} = 3 \times \frac{10}{45} = 5 \times \frac{10}{75}$$

$$\frac{2}{3} = \frac{30}{45} = \frac{50}{75} \quad \Rightarrow \quad \frac{2}{3} = \frac{\cancel{30}^{2}}{\cancel{45}_{3}} = \frac{\cancel{50}^{2}}{\cancel{75}_{3}}$$

$$\frac{2}{3} = \frac{2}{3} = \frac{2}{3}$$

Since the ratio of corresponding sides of the slids is equal, so the solids are similar.

2. In triangle ABC, the sides are given as $m\overline{AB} = 6$cm, $m\overline{BC} = 9$cm and $m\overline{CA} = 12$cm. In triangle DEF, the sides are given as $m\overline{DE} = 10.5$cm, $m\overline{EF} = 15.75$cm and $m\overline{FD} = 21$cm. Prove that the triangles are similar.



Sol: The ratio of corresponding sides of the triangles ABC and FDE.

$$\frac{m\overline{BC}}{m\overline{EF}} = \frac{m\overline{AB}}{m\overline{ED}} = \frac{m\overline{AC}}{m\overline{FD}}$$

$$\frac{4}{0.75} = \frac{6}{10.5} = \frac{12}{21}$$

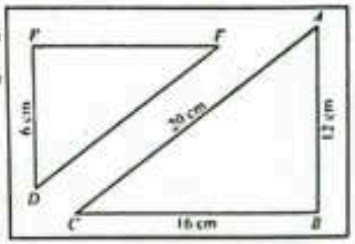
$$\frac{4}{\frac{1575}{100}} = \frac{6}{\frac{105}{10}} = \frac{12}{21}$$

$$\frac{\cancel{100}^{4}}{\cancel{1575}_{63}} \times \cancel{6}^{2} = \frac{\cancel{10}^{2}}{\cancel{105}_{21}} = \frac{\cancel{12}^{4}}{\cancel{21}_{7}}$$

$$\frac{4}{7} = \frac{4}{7} = \frac{4}{7}$$

Since the ratio of corresponding sides are equal, so the triangles ABC and FDE are similar.

In the given figure, $\Delta ABC \sim \Delta DEF$, $m\overline{AB} = 12$cm, $m\overline{AC} = 20$ cm and $m\overline{BC} = 16$cm. In ΔDEF, $m\overline{DE} = 6$cm. Find $m\overline{DF}$ and $m\overline{EF}$.



$\Delta ABC \sim \Delta DEF$ (Given)

So, the ratio of corresponding sides will be equal.

$$\frac{m\overline{DE}}{m\overline{AB}} = \frac{m\overline{FD}}{m\overline{AC}} = \frac{m\overline{EF}}{m\overline{CB}}$$

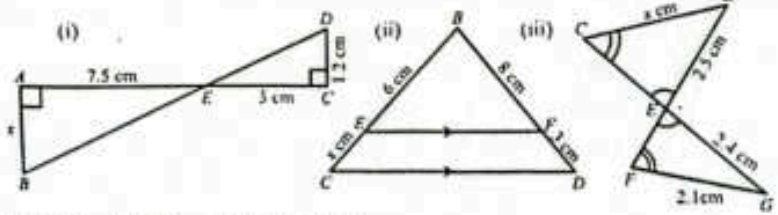
$$\frac{6}{12} = \frac{m\overline{FD}}{20} = \frac{m\overline{EF}}{16}$$

$$\frac{m\overline{FD}}{20} = \frac{6}{12} \qquad \frac{m\overline{EF}}{16} = \frac{6}{12}$$

$$m\overline{FD} = \frac{\cancel{6}^{1}}{\cancel{12}_{2}} \times \cancel{20}^{10} \qquad m\overline{EF} = \frac{\cancel{6}^{1}}{\cancel{12}_{2}} \times \cancel{16}^{8}$$

$m\overline{FD} = 10$cm $\qquad$ $m\overline{EF}$ 8cm

Find the value of x in each of the following:



Sol. (i) $\Delta ABE \sim \Delta CDE$ $\because$ AA ≅ AA

So, the ratio of corresponding sides will be equal.

$$\frac{m\overline{AB}}{m\overline{DC}} = \frac{m\overline{AE}}{m\overline{EC}}$$

$$\frac{x}{1.2} = \frac{7.5}{3}$$

$$x = \frac{\cancel{75}^{25}}{\cancel{10}_{2} \times \cancel{3}_{1}} \times \frac{\cancel{12}^{4}}{\cancel{10}_{2}}$$

$$x = \frac{12}{4} = 3\text{cm}$$

(ii) In triangle BCD, $\overline{EF} \parallel \overline{CD}$. so, $\overline{EF}$ divides $\overline{BC}$ and $\overline{BC}$ into same ratio

$$\overline{BE} : \overline{EC} = \overline{BF} : \overline{FD}$$

$$\frac{\overline{BE}}{\overline{EC}} = \frac{\overline{BF}}{\overline{FD}}$$

$$\frac{6}{x} = \frac{8}{3}$$

$$\frac{3}{\cancel{8}_{4}} \times \cancel{6}^{3} = x \Rightarrow \frac{9}{4} = x$$

$$x = 2.25\text{cm}$$

(iii) $\Delta CED \sim \Delta FEG$ (AA ≅ AA)

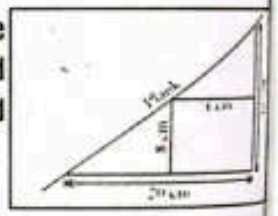
$$\frac{m\overline{CD}}{m\overline{FG}} = \frac{m\overline{ED}}{m\overline{EG}}$$

$$\frac{x}{2.1} = \frac{2.5}{2.4}$$

$$x = \frac{2.5}{2.4} \times 2.1 = \frac{5.25}{2.4}$$

$$x = 2.19\text{cm}$$

5. A plank is placed straight upstairs that 20cm wide and 16cm deep. A rectangular box of height 8cm and width x cm is placed on a stair under the plank. Find the value of x.



Sol.

Base = 20cm

Hight = 16cm

Hight of box = 8cm

Width of box = xcm

By the principal of similar triangles,

$$\frac{\text{Hight of the box}}{\text{Total hight of plank}} = \frac{\text{Width of the box}}{\text{Total width of plank}}$$

$$\frac{8}{16} = \frac{x}{20}$$

$$\cancel{20}^{5} \times \frac{\cancel{8}^{1}}{\cancel{16}_{2}} = x$$

$$10 = x$$

$$x = 10\text{cm}$$

A man who is 1.8m tall casts a shadow of a 0.76m in length. If at the same time a telephone pole costs a 3m shadow, find the height of the pole.

Sol. Hight of the man = 1.8m

Shadow of the man = 0.76m

Hight of the pole = ? = xcm

Shadow of the pole = 3m

Using the principal of similar triangles,

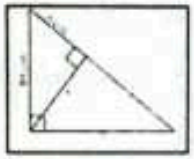
$$\frac{\text{Hight of the man}}{\text{Shadow of the man}} = \frac{\text{Hight of the pole}}{\text{Shadow of the pole}}$$

$$\frac{1.8}{0.76} = \frac{x}{3}$$

$$\frac{3 \times 1.8}{0.76} = x \Rightarrow x = \frac{5.4}{0.76}$$

$$x \cong 7.11\text{m}$$

7. Find the values of x, y and z in the given figure.



Sol. In small triangle by Pythagoras theorem

$$10^2 = y^2 + 6^2$$

$$100 = +y^2 + 36$$

$$100 - 36 = y^2$$

$$64 = y^2$$

$$\sqrt{64} = \sqrt{y^2}$$

$$8 = y$$

$$y = 8\text{cm}$$

Since, the small and big triangles are similar ($\because$ AA ≅ AA)

The ratio of corresponding sides are

$$\frac{6}{y} = \frac{y}{x} = \frac{10}{z}$$

$$\frac{6}{y} = \frac{y}{x} \qquad \left| \quad \frac{6}{y} = \frac{10}{z} \right.$$

$$\frac{6}{8} = \frac{8}{x} \qquad \left| \quad \frac{6}{8} = \frac{10}{z} \right.$$

$$x = \cancel{8}^{4} \times \frac{8}{\cancel{6}_{3}} \qquad \left| \quad z = 10 \times \frac{\cancel{8}^{4}}{\cancel{6}_{3}} \right.$$

$$x = \frac{32}{3} = 10\frac{2}{3}\text{ cm} \qquad z = \frac{40}{3} = 13\frac{1}{3}\text{ cm}$$

8. Draw an isosceles trapezoid ABCD where $\overline{AB} \parallel \overline{CD}$ and $m\overline{AB} > m\overline{CD}$. Draw diagonals $\overline{AC}$ and $\overline{BD}$, intersecting at E. Prove that ΔABE is similar to ΔCDE. If $m\overline{AB} = 8\text{cm}$, $m\overline{CD} = 4\text{cm}$, and $m\overline{AE} = 3\text{cm}$, find the length of $\overline{CE}$.

Sol. Given = an isosceles trapezoid ABCD.

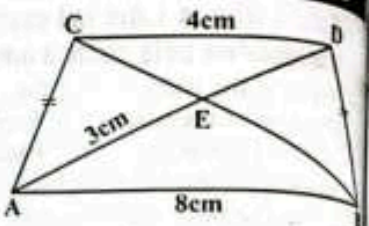
$\Delta ABE \sim \Delta CDE$.

using similarity ratios

$$\frac{m\overline{AB}}{m\overline{CD}} = \frac{m\overline{AE}}{m\overline{CE}}$$

$$\frac{8}{4} = \frac{3}{m\overline{CE}}$$

$$m\overline{CE} = 3 \times \frac{4^1}{8_2} \Rightarrow m\overline{CE} = \frac{3}{2} = 1.5\text{cm}$$



9. A regular dodecagon has its side lengths decreased by a factor of $\frac{1}{\sqrt{2}}$. If the perimeter of the original dodecagon is 72cm. What is the side length of scaled dodecagon?

Sol. Perimeter of the orignal dodecagon $= 72\text{cm}$

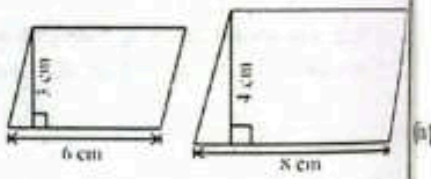
Side length of the orignal dodecagon $= \frac{72\text{cm}}{12} = 6\text{cm}$

Scaling side length of scaled dodecagon $= 6 \times \frac{1}{\sqrt{2}} = \frac{6 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$

$$= \frac{6 \times \sqrt{2}}{(\sqrt{2})^2} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}\text{cm}$$

Area of Similar Figures

There are two parallelograms with corresponding bases 6 cm and 8 cm and corresponding altitudes 3 cm and 4 cm respectively. The ratio between their lengths is 3:4 written as:



$$\frac{\ell_1}{\ell_2} = \frac{3}{4}$$

The area of smaller parallelogram is: A_1 = base x altitude

$= 6 \times 3 = 18\ \text{cm}^2$

The area of larger parallelogram is: A_2 = base x altitude

$= 8 \times 4 = 32\ \text{cm}^2$

The ratio of their areas is: $\frac{A_1}{A_2} = \frac{9}{16} = \left(\frac{3}{4}\right)^2$

$$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

Where A_1 and A_2 are areas and ℓ_2 and ℓ_2 are any two corresponding lengths of similar figures.

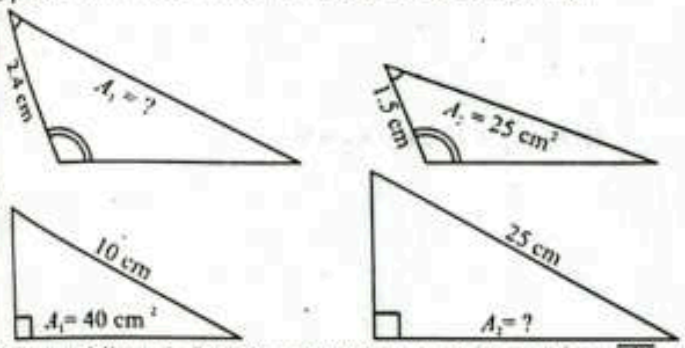
Hence the ratio of the areas of any two similar figures is equal to the square of the of any two corresponding lengths of the figures.

$$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

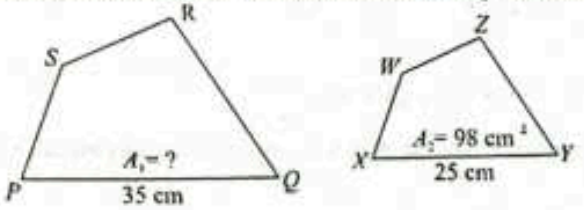
Since each length is k times of the other, we take $\frac{\ell_1}{\ell_2} = k^2$, then $\frac{A_1}{A_2} = k^2$ .i.e. Area is k^2 times the area A_2 k is called scale factor.

ample 6: Find the unknown value in the following:

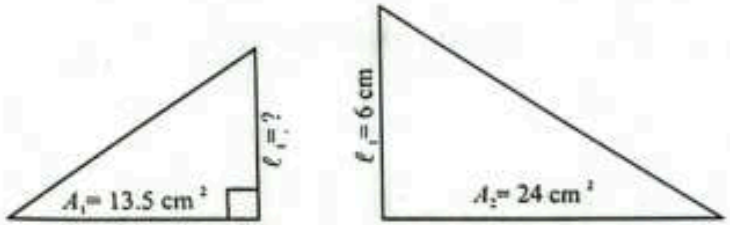
(i)



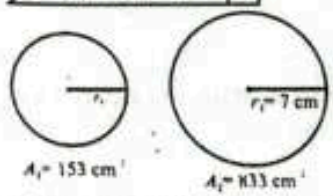
(ii) The quadrilaterals PQRS and XYZW are similar where $m\overline{PQ}$=35 cm and $m\overline{XY}$=25 cm.



(iii)



(iv)



Solution: (i) Since two pairs of corresponding angles are equal i.e., triangles are similar. We use the formula for ratio of areas of similar figures.

$$\frac{A_1}{A_2}=\left(\frac{\ell_1}{\ell_2}\right)^2$$

Here $\ell_1 = 2.4$ cm, $\ell_2 = 1.5$cm, $A_2 = 25$ cm^2, $A_1 = ?$

$$\frac{A_1}{25}=\left(\frac{2.4}{1.5}\right)^2 \Rightarrow \frac{A_1}{25}=\left(\frac{8}{5}\right)^2$$

$$A_1=\frac{64}{25}\times 25 = 64\text{cm}^2$$

(ii) Apply formula:

$$\frac{A_1}{A_2}=\left(\frac{\ell_1}{\ell_2}\right)^2$$

Here $\ell_1 = 10$ cm, $\ell_2 = 25$cm, $A_1 = 40$ cm^2, $A_2 = ?$

$$\frac{40}{A_2}=\left(\frac{10}{25}\right)^2$$

$$\frac{40}{A_2}=\left(\frac{2}{5}\right)^2$$

$$\frac{40}{A_2}=\frac{4}{25}$$

$$A_2 = 40\times\frac{25}{4} = 250\text{cm}^2$$

(iii) It is given that the quadrilateral PQRS is similar to quadrilateral XYZW.

$$\frac{A_1}{A_2}=\left(\frac{\ell_1}{\ell_2}\right)^2$$

Here $\ell_1 = 35$ cm, $\ell_2 = 25$cm, $A_1 = ?$, $A_2 = 98$ cm^2

$$\frac{A_1}{98}=\left(\frac{35}{25}\right)^2$$

$$\frac{A_1}{98}=\left(\frac{7}{5}\right)^2$$

$$A_1=\frac{49}{25}\times 98 = 192.08\text{cm}^2$$

(iv) Since two pairs of corresponding angles in both triangles are equal, so triangles are similar.

$$\therefore \quad \frac{A_1}{A_2}=\left(\frac{\ell_1}{\ell_2}\right)^2$$

Here $\ell_1 = ?$, $\ell_2 = 3$cm, $A_1 = 13.5$ cm^2, $A_2 = 24$ cm^2

$$\frac{13.5}{24}=\left(\frac{\ell_1}{3}\right)^2$$

$$\frac{135}{240}=\left(\frac{\ell_1}{3}\right)^2$$

$$\frac{9}{16}=\left(\frac{\ell_1}{3}\right)^2$$

$$\sqrt{\left(\frac{\ell_1}{3}\right)^2}=\sqrt{\frac{9}{16}} \qquad \text{(Taking square root)}$$

$$\frac{\ell_1}{3}=\frac{3}{4}$$

$$\ell_1=\frac{9}{4}$$

$$= 2.25 \text{ cm}$$

(v) For similar spheres

$$\frac{A_1}{A_2}=\left(\frac{\ell_1}{\ell_2}\right)^2$$

Here $r_1 = ?$, $r_2 = 7$ cm^2, $A_1 = 153$ cm^2, $A_2 = 833$ cm^2

$$\frac{153}{833}=\left(\frac{r_1}{7}\right)^2$$

$$\frac{9}{49}=\left(\frac{r_1}{7}\right)^2$$

$$\sqrt{\left(\frac{r_1}{7}\right)^2}=\sqrt{\frac{9}{49}} \qquad \text{(Taking square root)}$$

$$\frac{r_1}{7}=\frac{3}{7} \quad \Rightarrow \quad r_1 = 3\text{cm}$$

Example 7: Two polygons are similar with a ratio of corresponding sides being $\frac{3}{5}$. If the area of the smaller polygon is 54 cm^2, find the area of the larger polygon.

Solution: The ratio of the areas of two similar polygons is the square of the ratio of corresponding sides. So, $\frac{\text{Area of larger polygon}}{\text{Area of smaller polygon}}=\left(\frac{5}{3}\right)^2=\frac{25}{9}$

Therefore, Area of larger polygon $= \frac{25}{9}\times 54$

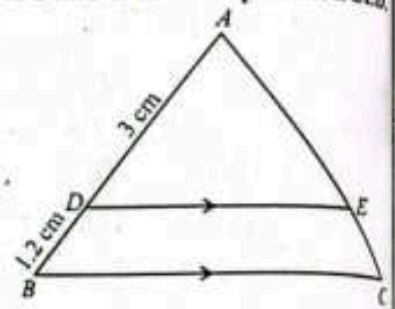
$= 150$ cm^2

Example 8: Given that $\overline{BC} \| \overline{DE}$, prove that the triangles ABC and ADE are similar.

(i) If $m\overline{AB} = 3$ cm and $m\overline{BD} = 1.2$ cm, find the ratio of area of ΔABC to the area of ΔADE.

(ii) If area of ΔADE is 125 cm², find the area of ΔABC and area of trapezium BCED.

Solution: Since $m\angle A = m\angle A$ (common), $m\angle B = m\angle D$ and $m\angle C = m\angle E$ (Corresponding angles of parallel lines $\overline{BC}$ and $\overline{DE}$). Hence ΔABC is similar to ΔADE.



(i) Ratio of sides $\frac{m\overline{AB}}{m\overline{AD}} = \frac{3+1.2}{3} = \frac{4.2}{3} = \frac{7}{5}$

$\therefore \quad \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta ADE} = \left(\frac{\ell_1}{\ell_2}\right)^2 \left(\frac{7}{5}\right)^2 = \frac{49}{25}$

(ii) Area of ΔADE = 125 cm²

$$\frac{\text{Area of } \Delta ABC}{125} = \frac{49}{25}$$

$$\Rightarrow \text{Area of } \Delta ABC = \frac{49}{\cancel{25}} \times \cancel{25}^{5} = 245 \text{ cm}^2$$

Area of trapezium BCED = Area of ΔABC – Area of ΔADE

= 245 – 125 = 120 cm²

Solved Exercise 9.2

1. Find the ratio of the areas of similar figures if the ratio of their corresponding lengths are: (i) 1:3 (ii) 3:4 (iii) 2:7 (iv) 8:9 (v) 6:5

(i) 1:3

Sol.

Ratio of the corresponding lengths = 1:3

Ratio of the areas of similar figures = ?

Use the formula for ratio of areas of similar figures

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$

$$\frac{A_1}{A_2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

$A_1 : A_2 = 1 : 9$

(ii) 3 : 4

Sol.

Ratio of the corresponding lengths = 3:4

Use the formula for ratio of areas of similar figures

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$

$$\frac{A_1}{A_2} = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

$A_1 : A_2 = 9 : 16$

(iii) 2 : 7

Sol.

Ratio of the corresponding lengths = 2:7

Use the formula for ratio of areas of similar figures

(iv) 8 : 9

Sol.

Ratio of the corresponding lengths = 8:9

Use the formula for ratio of areas of similar figures

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$

$$\frac{A_1}{A_2} = \left(\frac{2}{7}\right)^2 = \frac{4}{49}$$

$A_1 : A_2 = 4 : 49$

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$

$$\frac{A_1}{A_2} = \left(\frac{8}{9}\right)^2 = \frac{64}{81}$$

$A_1 : A_2 = 64 : 81$

(v) 6:5

Sol. Ratio of the corresponding lengths = 6:5

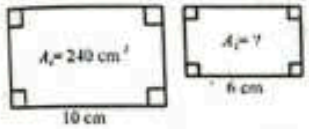
Use the formula for ratio of areas of similar figures

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 \Rightarrow \frac{A_1}{A_2} = \left(\frac{6}{5}\right)^2 = \frac{36}{25}$$

$A_1 : A_2 = 36 : 25$

2. Find the unknowns in the following figures.

(i)



Sol. Use the formula for ratio of areas of similar figures because two pairs of corresponding angles are equal, triangles are similar

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$

Here $l_1 = 10$cm, $l_2 = 6$cm,

$A_1 = 240$cm², $A_2 = ?$

$$\frac{240}{A_2} = \left(\frac{10}{6}\right)^2 \Rightarrow \frac{240}{A_2} = \frac{100}{36}$$

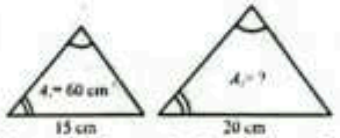
$$\frac{36}{\cancel{10}\,\cancel{0}} \times \cancel{24}\,\cancel{0} = A_2$$

$$\frac{432}{5} = A_2$$

$86.4 = A_2$

$A_2 = 86.4$cm²

(ii)



Sol. Use the formula for ratio of areas of similar figures because two pairs of corresponding angles are equal, triangles are similar figures.

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$

Here $l_1 = 15$cm, $l_2 = 20$cm,

$A_1 = 60$cm², $A_2 = ?$

$$\frac{60}{A_2} = \left(\frac{15}{20}\right)^2 \Rightarrow \frac{60}{A_2} = \frac{225}{400}$$

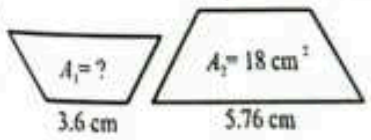
$$\frac{\overset{80}{\cancel{400}}}{\underset{45}{\cancel{225}}} \times \overset{4}{\cancel{60}} = A_2$$

$$\frac{320}{3} = A_2$$

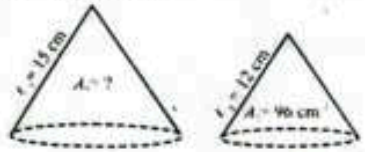
$106.67 = A_2$

$A_2 = 106.67$cm²

(iii)



(iv)



Sol. Use the formula for ratio of areas of similar figures.

$$\frac{A_1}{A_2}=\left(\frac{l_1}{l_2}\right)^2$$

Here $l_1=3.6\text{cm}, l_2=5.76\text{cm}, A_2=18\text{cm}^2, A_1=?$

$$\frac{A_1}{18}=\left(\frac{3.6}{5.76}\right)^2$$

$$\frac{A_1}{18}=\frac{12.96}{33.18}$$

$$A_1=\frac{12.96}{33.18}\times 18$$

$$A_1=\frac{233.28}{33.18}$$

$$A_1=7.031\text{cm}^2$$

Sol. Use the formula for the ratio of areas of similar figures.

$$\frac{A_1}{A_2}=\left(\frac{l_1}{l_2}\right)^2$$

Here $l_1=15\text{cm}$, $l_2=12\text{cm}$, $A_2=96\text{cm}^2$, $A_1=?$

$$\frac{A_1}{96}=\left(\frac{15}{12}\right)^2$$

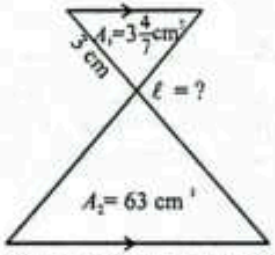
$$\frac{A_1}{96}=\frac{225}{144}$$

$$A_1=\frac{225}{144}\times 96$$

$$A_1=\frac{21600}{144}$$

$$A_1=150\text{cm}^2$$

(v)



Sol. Use the formula for the ratio of areas of similar figures.

$$\frac{A_1}{A_2}=\left(\frac{l_1}{l_2}\right)^2$$

Here $A_1=1\frac{4}{7}\text{cm}^2$, $A_2=63\text{cm}^2$, $l_1=3\text{cm}$, $l_1=?$

$$3\frac{4}{7}\Big/63=\left(\frac{3}{l_2}\right)^2 \Rightarrow \frac{25}{7}\Big/63=\frac{9}{l_2^2}$$

$$\frac{25}{7}\times\frac{1}{63}=\frac{9}{l_2^2}$$

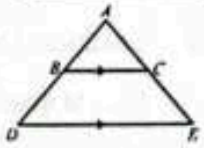
$$l_2^2=9\times\frac{7}{25}\times 63$$

$$l_2^2=158.76$$

$$\sqrt{l_2^2}=\sqrt{158.76}$$

$$l_2=12.6\text{cm}$$

Given that area of $\Delta ABC=36\text{cm}^2$ and $m\overline{AB}=6\text{cm}$, $m\overline{BD}=4\text{cm}$.



Find

(a) the area of ΔADE (b) the area of trapezium BCED

$m\angle A = m\angle A$ (common)

$m\angle B = m\angle D$, $m\angle C = m\angle E$ (corresponding angles of parallel lines $\overline{BC}$ and $\overline{DE}$)

Hence ΔABC is similar ΔADE

use the formula for ratio of areas of similar figures.

$$\frac{A_1}{A_2}=\left(\frac{l_1}{l_2}\right)^2$$

$m\overline{AB}=l_1=6\text{cm}$

$m\overline{AD}=l_2=m\overline{AB}+m\overline{BD}=6\text{cm}+4\text{cm}=10\text{ cm}$

Area of ΔABC (A_1) $=36\text{cm}^2$

Area of ΔADE (A_2) $=?$

$$\frac{36}{A_2}=\left(\frac{6}{10}\right)^2$$

$$\frac{36}{A_2}=\frac{36}{100}$$

$$\frac{100}{36}\times 36=A_2$$

$$100=A_2$$

$$A_2=100\text{cm}^2$$

Area of trapezium BCED = area of ΔADE – Area of ΔABC

$= 100\text{cms}^2 - 36\text{cm}^2$

$= 64\text{cm}^2$

4. Given that ΔABC and ΔDEF are similar, with a scale factor of $k = 3$. If the area of ΔABC is 50cm^2, find the area of triangle ΔDEF?

Sol. $A_1=50\text{cm}^2$, $k=3$, $A_2=?$

Use formula

$$\frac{A_1}{A_2}=K^2$$

$$\frac{50}{A_2}=(3)^2 \Rightarrow \frac{50}{A_2}=9$$

$$\frac{50}{9}=A_2 \Rightarrow 5\frac{5}{9}=A_2$$

So, area of $\Delta DEF=5\frac{5}{9}\text{cm}^2$

5. Quadrilaterals ABCD and EFGH are similar, with a scale factor of $k = \frac{1}{4}$. If the area of quadrilateral ABCD is 64cm², find the area of quadrilateral EFGH.

Sol. Quadrilaterals ABCD and EFGH are similar

Scale factor $k = \frac{1}{4}$

A_1 = Area of ABCD = 64cm², A_2 = Area of EFGH = ?

The areas of similar figures is the square of the scale factor.

$$\frac{A_1}{A_2} = K^2$$

$$\frac{64}{A_2} = \left(\frac{1}{4}\right)^2$$

$$\frac{64}{A_2} = \frac{1}{16}$$

$$A_2 = 16 \times 64$$

$$A_2 = 1024\text{cm}^2$$

So, the area of quadrilateral EFGH = 1024cm²

6. The areas of two similar triangles are 16cm² and 25cm². What is the ratio of pair of corresponding sides?

Sol. Area of a triangle (A_1) = 16cm²

Area of other triangle (A_2) = 25cm²

Use the formula for ratio of areas of similar figure.

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$

$$\frac{16}{25} = \left(\frac{l_1}{l_2}\right)^2 \Rightarrow \sqrt{\frac{16}{25}} = \sqrt{\left(\frac{l_1}{l_2}\right)^2}$$

$$\frac{4}{5} = \frac{l_1}{l_2} \Rightarrow \frac{l_1}{l_2} = \frac{4}{5}$$

$l_1 : l_2 = 4 : 5$

7. The areas of two similar triangles are 144cm² and 81cm². If the base of the large triangle is 30cm, find the corresponding base of the smaller triangle.

Sol. Area of large triangle (A_1) = 144cm²

Area of smaller triangle (A_2) = 81cm²

Base of large triangle (l_1) = 30cm

Base of smaller triangle (l_2) = ?

Use the formula for ratio of area of similar figure.

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 \Rightarrow \frac{144}{81} = \left(\frac{30}{l_2}\right)^2$$

$$\frac{900}{l_2^2} \Rightarrow l_2^2 = 900 \times \frac{9}{16} = \frac{8100}{16}$$

$= 505.61$

$= \sqrt{505.61}$

$= 22.5\text{cm}$

A regular heptagon is inscribed in a larger regular heptagon and each side of the larger heptagon is 1.7 times the side of the smaller heptagon. If the area of the smaller heptagon is known to be 100 cm², find the area of the larger heptagon.

The two regular heptagon are similar

Area of smaller heptagon = 100cm²

Scale factor = 1.7

Area of large heptagon = ?

$$\frac{\text{Area of large heptagon}}{\text{Area of smaller heptagon}} = K^2$$

$$\frac{\text{Area of large heptagon}}{100\text{cm}^2} = (1.7)^2$$

Are of large heptagon = 100 (1.7)²

= 100 (2.89)

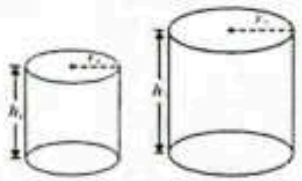
= 289cm²

lume of Similar Solids

Two solids are said to be similar if they have same shape but possibly different sizes. o solids are similar if lengths of the corresponding sides are proportional i.e., the ratio the corresponding lengths are equal, e.g.,

e two cylinders are similar if $\frac{r_1}{r_2} = \frac{h_1}{h_2}$

$r_1 = 4$ cm, $r_2 = 5$ cm, $h_1 = 8$ cm and $h_2 = 10$ cm, then e note that:



$\frac{4}{5}$ and $\frac{8}{10} = \frac{4}{5}$

$\Rightarrow \frac{r_1}{r_2} = \frac{h_1}{h_2}$

Volume of smaller cylinder.

$V_1 = \pi r_1^2 h_1$

$= \pi \times 4^2 \times 8$

$= 128\pi\ \text{cm}^2$

Volume of larger cylinder.

$V_1 = \pi r_2^2 h_2$

$= \pi \times 52 \times 10$

$= 250\pi\ \text{cm}^2$

Ratio of volumes: $\frac{V_1}{V_2} = \frac{128\pi}{250\pi} = \frac{64}{125} = \left(\frac{4}{5}\right)^2$

So, $\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3$ or $\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$

Hence the ratio of the volume of any two similar solids is equal to the cube of the ratio of any two corresponding lengths of the solids.

$$\frac{V_1}{V_2} = \left(\frac{\ell_1}{\ell_2}\right)^3$$

Since each length is k times of the other, we take $\frac{\ell_1}{\ell_2} = k$, then $\frac{V_1}{V_2} = k^3$ .i.e., Volume is k^3 times the volume V_2. k is called scale factor.

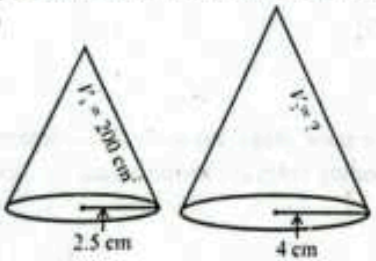
Since mass of a substance is proportional to its volume, the ratio of the masses of similar solids is equal of to the ratio of their volumes. If the masses of two similar solids are w_1 and w_2 and volumes are V_1 and V_2, then

$$\frac{V_1}{V_2} = \frac{w_1}{w_2}$$

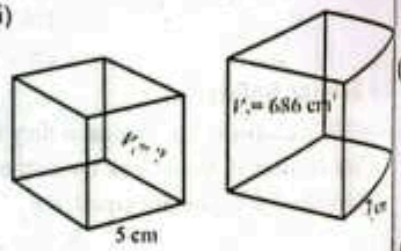
Therefore, $\frac{w_1}{w_2} = \left(\frac{\ell_1}{\ell_2}\right)^3$

Example 9: Find the unknown volume in the following similar solids:

(i)



(ii)



$$\frac{V_1}{V_2} = \left(\frac{\ell_1}{\ell_2}\right)^3$$

$$\frac{200}{V_2} = \left(\frac{2.5}{4}\right)^3$$

$$\frac{200}{V_2} = \left(\frac{5}{8}\right)^3$$

$$V_2 = 200 \times \frac{512}{125}$$

$$V_2 = 819.2 \times \text{cm}^3$$

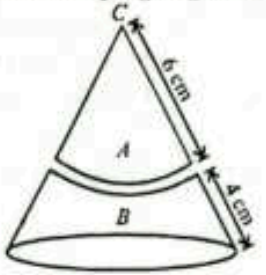
Using formula $\frac{V_1}{V_2} = \left(\frac{\ell_1}{\ell_2}\right)^3$

$$\frac{V_1}{686} = \left(\frac{5}{7}\right)^3 \qquad \left[\ell_2 = 5\text{ cm}, \ell_1 = 7\text{ cm}, V_1 = ?, V_2 = 686\text{ cm}^3\right]$$

$$\frac{V_1}{686} = \frac{125}{343}$$

$$V_1 = \frac{125}{343} \times 686 = 250\text{ cm}^2$$

Example 10: A solid cone C is cut into two pieces A and B with sloping edges 6 cm and 4 cm.



(i) the diameters of the bases of the cones A and C.
(ii) the area of the bases of the cones A and C.
(iii) the vlumes of the two cones A and C.
(iv) If volume of cone A is 72 cm³, find the volume of solid B.

Solution: Let diameter of cone A = d_1

Diameter of cone C = d_2

(i) The ratio of the corresponding lengths are equal because of similarity of the cones.

$$\therefore \frac{d_1}{d_2} = \frac{\ell_1}{\ell_2} = \frac{6}{10}$$

$$= \frac{3}{5} \qquad \text{i.e., } \frac{\ell_1}{\ell_2} = \frac{3}{5}$$

(ii) $\frac{\text{Area of cone A}}{\text{Area of cone C}} = \left(\frac{\ell_1}{\ell_2}\right)^2$

$$= \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

(ii) $\frac{\text{Area of cone A}}{\text{Area of cone C}} = \left(\frac{\ell_1}{\ell_2}\right)^3$

$$= \left(\frac{3}{5}\right)^2 = \frac{27}{125}$$

(iv) V_1 = Volume of cone A = 72 cm³

V_2 = Volume of cone C = ?

$$\therefore \frac{V_1}{V_2} = \left(\frac{\ell_1}{\ell_2}\right)^3$$

$$\frac{72}{V_2}=\frac{27}{125}$$

$$V_2=\frac{72\times125}{27}=333\frac{1}{3}\ cm^3$$

Volume of solid B = Volume of cone C – Volume of cone A

$$=333\frac{1}{3}-72=261\frac{1}{3}\ cm^3$$

Example 11: The mass of sack of rice is 50 kg and height 4 m. Find the mass of similar sack of rice with height of 6m.

Solution: Mass of the smaller sack of rice $w_1 = 50kg$
Height of smaller sack of rice $h_1 = 4m$
Mass of larger sack of rice $w_2 = ?$
Height of smaller sack of rice $h_2 = 6$ m

Using formula $\frac{w_1}{w_2}=\left(\frac{h_1}{h_2}\right)^3$

$$\frac{50}{w_2}=\left(\frac{4}{6}\right)^3$$

$$\frac{50}{w_2}=\frac{8}{27}$$

$$w_2=\frac{27\times50}{8}=168.75\ kg$$

Example 12: The ratio of the corresponding lengths of two similar cylindrical cans is 3 : 2.

(i) The larger cylindrical can has surface area of 67.5 square metres. Find the surface area of the smaller cylindrical can.

(ii) The smaller cylindrical can has a volume of 132 cubic metres. Find the volume of larger tin can.

Solution: (i) Surface area of larger can = $A_1 = 67.5\ m^2$
Surface area of smaller can = $A_2 = ?$

Ratio of corresponding lengths is $\frac{\ell_1}{\ell_2}=\frac{3}{2}$

Using formula for areas of the similar figures:

$$\frac{A_1}{A_2}=\left(\frac{\ell_1}{\ell_2}\right)^2$$

$$\frac{67.5}{A_2}=\left(\frac{3}{2}\right)^2 \quad\Rightarrow\quad A_2=67.5\times\frac{4}{9}=30m^2$$

Volume of smaller can = $V_2 = 132\ m^3$
Volume of larger can = $V_1 = ?$

Using formula for volume of similar figures : $\frac{V_1}{V_2}=\left(\frac{\ell_1}{\ell_2}\right)^3$

$$\frac{V_1}{132}=\left(\frac{3}{2}\right)^3 \quad\Rightarrow\quad V_1=132\times\frac{27}{8}=445.5\ m^3$$

Solved Exercise 9.3

1. The radii of two spheres are in the ratio 3:4. What is the ratio of their volumes?

Sol. The ratio of the radii of two spheres = 3:4
Let the volum of small sphere $= V_1$
Let the volum of big sphere $= V_2$
Using formula for volume of similar figures

$$\frac{V_1}{V_2}=\left(\frac{r_1}{r_2}\right)^3$$

$$\frac{V_1}{V_2}=\left(\frac{3}{4}\right)^3=\frac{27}{64}$$

$V_1 : V_2 = 27 : 64$

2. Two regular tetrahedrons have volume in the ratio 8:27. What is the ratio of their sides?

Sol. The ratio of the volumes of two regular tetrahedron = 8:7
Using formula for volume of similar figures

$$\frac{V_1}{V_2}=\left(\frac{l_1}{l_2}\right)^3 \quad\Rightarrow\quad \frac{8}{27}=\left(\frac{l_1}{l_2}\right)^3$$

$$\frac{2^3}{3^3}=\left(\frac{l_1}{l_2}\right)^3$$

$$\left(\frac{2}{3}\right)^3=\left(\frac{l_1}{l_2}\right)^3$$

$$\sqrt[3]{\left(\frac{2}{3}\right)^3}=\sqrt[3]{\left(\frac{l_1}{l_2}\right)^3}$$

$$\frac{2}{3}=\frac{l_1}{l_2}$$

$$\frac{l_1}{l_2}=\frac{2}{3} \quad\Rightarrow\quad l_1 : l_2 = 2 : 3$$

3. Two right cones have volumes in the ratio 64 : 125. What is the ratio of:

(a) their heights (b) their base areas?

Sol. (a) The ratio of the volumes of two right cones = 16:125

Using formula for volume of similar figures

$$\frac{V_1}{V_2}=\left(\frac{h_1}{h_2}\right)^3$$

$$\frac{64}{125}=\left(\frac{h_1}{h_2}\right)^3$$

$$\frac{4^3}{5^3}=\left(\frac{h_1}{h_2}\right)^3$$

$$\left(\frac{4}{5}\right)^3=\left(\frac{h_1}{h_2}\right)^3$$

$$\sqrt[3]{\left(\frac{4}{5}\right)^3}=\sqrt[3]{\left(\frac{h_1}{h_2}\right)^3}$$

$$\frac{4}{5}=\frac{h_1}{h_2}$$

$$\frac{h_1}{h_2}=\frac{4}{5}$$

$h_1 : h_2 = 5$

(b) Raduis is the cube root of the volume ratio

$$\frac{V_1}{V_2}=\left(\frac{r_1}{r_2}\right)^3$$

$$\frac{64}{125}=\left(\frac{r_1}{r_2}\right)^3$$

$$\frac{4^3}{5^3}=\left(\frac{r_1}{r_2}\right)^3 \Rightarrow \left(\frac{4}{5}\right)^3=\left(\frac{r_1}{r_2}\right)^3$$

Taking cube root

$$\sqrt[3]{\left(\frac{4}{5}\right)^3}=\sqrt[3]{\left(\frac{r_1}{r_2}\right)^3}$$

$$\frac{4}{5}=\frac{r_1}{r_2}$$

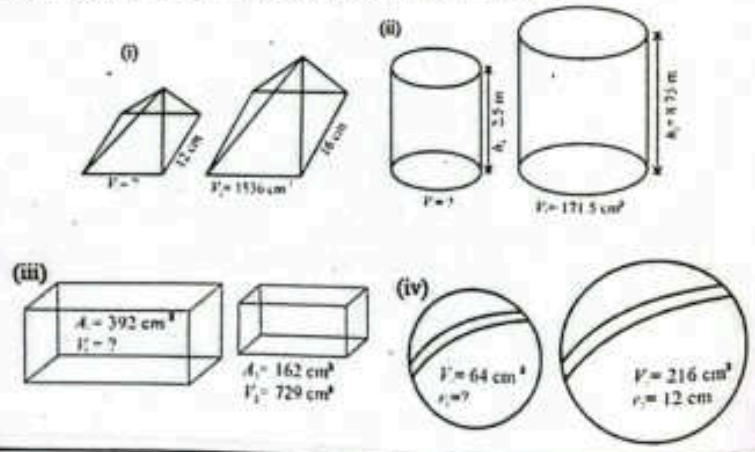
use the formula for ratio of areas of similar figures

$$\frac{A_1}{A_2}=\left(\frac{r_1}{r_2}\right)^2$$

$$\frac{A_1}{A_2}=\left(\frac{4}{5}\right)^2=\frac{16}{25}$$

Thus, the ratio of base areas of cones = 16:25

4. Find the missing value in the following similar solids.



(i) V_1 = ?
h_1 = 12cm
V_2 = 1536cm³
h_2 = 16cm

the formula for valum of similar

$$\frac{V_1}{V_2}=\left(\frac{h_1}{h_2}\right)^3$$

$$\frac{V_1}{1536}=\left(\frac{12}{16}\right)^3$$

$$\frac{V_1}{1536}=\frac{1728}{4096}$$

$$V_1=\frac{1728}{4096}\times 1536$$

$$V_1=\frac{2654208}{4096}=648\text{cm}^3$$

A_1 = 392 cm²
A_2 = 162 cm²
V_2 = 729 cm³
V_1 = ?

First, we use the formula for the ratio similar figures

$$\frac{A_1}{A_2}=\left(\frac{l_1}{l_2}\right)^2 \Rightarrow \frac{392}{162}=\left(\frac{l_1}{l_2}\right)^2$$

$$\frac{196}{81}=\left(\frac{l_1}{l_2}\right)^2 \Rightarrow \frac{14^2}{9^2}=\left(\frac{l_1}{l_2}\right)^2$$

$$\left(\frac{14}{9}\right)^2=\left(\frac{l_1}{l_2}\right)^2$$

by taking square root

$$\sqrt{\left(\frac{14}{9}\right)^2}=\sqrt{\left(\frac{l_1}{l_2}\right)^2}$$

$$\frac{14}{9}=\frac{l_1}{l_2} \Rightarrow \frac{l_1}{l_2}=\frac{14}{9}$$

using formula for the valume of similar figures

(ii) h_1 = 2.5m
h_2 = 8.75m
V_2 = 171.5cm²
V_1 = ?

Using the formula for valum of similar figures

$$\frac{V_1}{V_2}=\left(\frac{h_1}{h_2}\right)^3$$

$$\frac{V_1}{171.5}=\left(\frac{2.5}{8.75}\right)^3$$

$$\frac{V_1}{171.5}=\frac{15.625}{699.92}$$

$V_1 = 0.0223\times 171.5$

$V_1 = 4\text{ cm}^3$

(iv) V_1 = 64 cm²
V_2 = 216
r_2 = 12 cm
r_1 = ?

useing formula for volumes of similar figures

$$\frac{V_1}{V_2}=\left(\frac{r_1}{r_2}\right)^3$$

$$\frac{64}{216}=\left(\frac{r_1}{12}\right)^3$$

$$\frac{64}{216}=\frac{r_1^3}{1728}$$

$$1728\times\frac{64}{216}=r_1^3$$

$$512=r_1^3$$

$$r_1^3=512$$

By taking cube root

$$\sqrt[3]{r_1^3}=\sqrt[3]{512}$$

$r = 8\text{cm}$

$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$

$$\frac{V_1}{729} = \left(\frac{14}{9}\right)^3 = \frac{2744}{729}$$

$$V_1 = \frac{2744}{729} \times 729 = 2744\text{cm}^3$$

5. The ratio of the corresponding lengths of two similar canonical cans is 3:2

(i) The larger canonical can have surface area of 96m². Find the surface area of the smaller canonical can.

(ii) The smaller canonical can have a volume of 240m³. Find the volume of larger canonical can.

Sol. (i) $l_1 : l_2 = 3 : 2$ (Given)

$$\frac{l_1}{l_2} = \frac{3}{2}$$

$A_1 = 96\text{m}^2$

$A_2 = ?$

By using the formula for areas of similar figures

$$\frac{V_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$

$$\frac{96}{A_2} = \left(\frac{3}{2}\right)^2$$

$$\frac{96}{A_2} = \frac{9}{4}$$

$$\frac{4}{9} \times 96 = A_2$$

$$42.67 = A_2$$

$A_2 = 42.67\ \text{m}^2$

(ii) $\frac{l_1}{l_2} = \frac{3}{2}$ (Given)

$V_2 = 240\text{m}^3$

$V_1 = ?$

$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$

$$\frac{V_1}{240} = \left(\frac{3}{2}\right)^3$$

$$\frac{V_1}{240} = \frac{27}{8}$$

$$V_1 = \frac{27}{8} \times 240$$

$$= 810\text{m}^3$$

6. The ratio of the heights of two similar cylindrical water tanks is 5 : 3.

(i) If the surface area of the larger tank is 250 square metres, find the surface area of the smaller tank.

(ii) If the volume of the smaller tank is 270 cubic metres, find the volume of the larger tank.

Sol. (i) $l_1 : l_2 = 5 : 3$ (Given)

$$\frac{l_1}{l_2} = \frac{5}{3}$$

Area of large tanker $(A_1) = 250\text{m}^2$

Area of smaller tank $(A_2) = ?$

Using the formula for the ratio of areas of similar figures

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$

$$\frac{250}{A_2} = \left(\frac{5}{3}\right)^2$$

$$\frac{250}{A_2} = \frac{25}{9}$$

$$\frac{9}{25} \times 250 = A_2$$

$A_2 = 90\text{m}^2$

(ii) $h_1 : h_2 = 5 : 3$ (Given)

$$\frac{h_1}{h_2} = \frac{5}{3}$$

Volume of smaller tank $(V_2) = 270\text{m}^3$

Volume of large tank $(V_1) = ?$

Using the formula for the volume of the similar figures

$$\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$$

$$\frac{V_1}{270} = \left(\frac{5}{3}\right)^3 = \frac{125}{27}$$

$$V_1 = \frac{125}{27} \times 270 = 1250\text{cm}^3$$

Geometrical Properties of Polygons and their Application

Geometrical Properties of Regular Polygons

A regular polygon has all sides and all angles equal. Some of the common regular polygons are equilateral triangles, squares, regular pentagons, regular hexagons, etc.

Sum of Interior Angles: The formula for sum of interior angles of n-sided polygon is $(n-2) \times 180°$.

Interior Angle: For a regular n-sided polygon:

$$\text{Size of each Interior Angle} = \frac{(n-2)\times 180°}{n}$$

Exterior Angle: The sum of all exterior angles of any polygon is always 360° regardless of the number of sides. The exterior angle of each side of a regular n-sided polygon is:

$$\text{Exterior Angle} = \frac{360°}{n}$$

The interior and exterior angles are supplementary at a vertex i.e.,
Interior + exterior angle = 180°

Diagonals: The total number of diagonals in a regular polygon with n sides is $\frac{n(n-3)}{2}$

Symmetry: A regular "-sided polygon has rotational symmetry and reflexive symmetry both of order n.

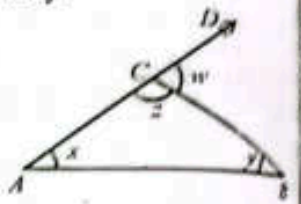
Geometrical Properties of Triangles

Angle sum: The sum of the interior angles in any triangle is always 180°. In equilateral triangle, all sides are equal, and each angle is 60°. It has three lines of symmetry and rotational symmetry of order 3. In isosceles triangle, two sides are equal, and the angles opposite to the equal sides are also equal. It has one line of symmetry.

Exterior angle of a triangle: The measure of an exterior angle in a triangle is equal to sum of the measures of two opposite interior angles i.e.,

In ΔABC, m∠A + m∠B = m∠BCD i.e.,

i.e., x + y = w



Geometrical Properties of Parallelograms

A parallelogram is a quadrilateral whose opposite sides are parallel and equal in length and opposite angles are equal. Its adjacent angles are supplementary. The diagonals of a parallelogram bisect each other (they cross each other at the midpoint). They are not equal in length.

Recall:
Rectangle: All angles are 90° and diagonals are equal.
Rhombus: All sides are equal, and diagonals bisect each other at right angles.
Square: All sides are equal, all angles are 90° and diagonals are equal and bisect each other at right angles.

Example 13: Find the measure of each interior angle of a regular pentagon.

Solution: Interior angle $= \frac{(n-2)\times 180^\circ}{n}$

$= \frac{(5-2)\times 180^\circ}{5} = \frac{540^\circ}{5} = 108^\circ$

Each exterior angle is: $\frac{360^\circ}{5} = 72^\circ$

Application of Polygons

Architects use polygons in building designs, while engineers rely on them to make strong structures like bridges. In art and design, polygons help create beautiful patterns and 3-D models. On maps, polygons show areas like cities or land boundaries. Polygons are also used in video games and animations to build characters and scenes. In science, they appear in molecular shapes, natural patterns like honeycombs and even in the design of telescope mirrors. Their simple, versatile shapes make polygons essential in many fields.

Tessellation

A tessellation is a pattern of shapes that fit together perfectly, without any gaps or overlaps, covering a plane. These shapes can be repeated infinitely to create a repeating pattern. Tessellations can be created using a single shape or a combination of shapes. They can be regular or irregular and they can exhibit various symmetries and patterns.

Equilateral triangles can tessellate perfectly because the internal angle of each equilateral triangle is 60°, and six of these triangles meet at a point to form a 360° angle, allowing them to fill space seamlessly. Squares can tessellate perfectly because each square has an internal angle of 90° and four squares meet at a point to form a 360° angle.

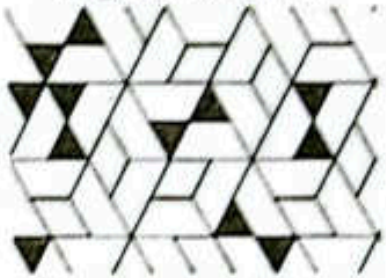
Only three regular polygons can tessellate the plane on their own: equilateral triangles, squares, and regular hexagons. They have symmetries. Hexagons (interior angle 120°) can tessellate perfectly because three hexagons meet at each vertex to form a 360° angle with no space creating a natural look inspired by honeycombs.

Regular pentagons and other polygons with angles that don't add up to 360° at each vertex cannot form gap-free patterns, i.e., Tessellation is not possible.

Regular Tessellation



Irregular Tessellation



Example 14: A tessellation is created using a combination of regular pentagons and decagons. Find the sum of the angles at a vertex where a pentagon and a decagon meet.

Solution: Interior angle of regular decagon $= \frac{(n-2)\times 180^\circ}{n}$

$= \frac{(10-2)\times 180^\circ}{10} = \frac{1440^\circ}{10} = 144^\circ$

$= 144^\circ$

Interior angle of regular pentagon = 108°

Sum of angles = 144°+108°=252°. Since, angle sum ≠ 360°. Tessellation cannot be done.

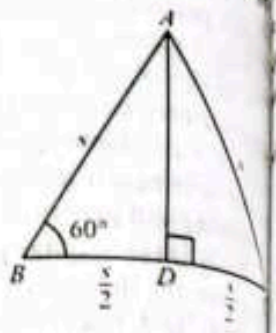
Example 15: A parallelogram-shaped room has a base of 10 meters and a height of 8m. Babar wants to carpet the room using rolls that cover 20 m² each. How many rolls of carpet do he need?

Solution: The area of the parallelogram = A = base × height = 10 × 8 = 80 m²

Number of rolls needed: $\frac{80}{20} = 4$ rolls

Example 16: Find the area of the equilateral triangle ABC of side length s metres.

Solution: Draw perpendicular from A to side BC at point D. In the right angled triangle ABD:



Using trigonometric ratios: $\sin 60° = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

$$\frac{\sqrt{3}}{2} = \frac{m\overline{AD}}{s} \Rightarrow m\overline{AD} = \frac{\sqrt{3}}{2}s$$

Area of triangle ABC $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times s \times \frac{\sqrt{3}}{2} s$

Area of triangle ABC $= \frac{\sqrt{3}}{4} s^2$

Example 17: Ali wants to create a floor design that uses regular hexagons (each with a side length of 1 metre) and equilateral triangles (each with a side length of 1 metre) to cover a rectangular area measuring 10 m by 5 m. Find how many hexagons and triangles Ali will need to complete the tessellation.

Solution: To find the area of an equilateral triangle with side length s, we can use the formula:

$$\text{Area of a triangle} = \frac{\sqrt{3}}{4} . s^2$$

Multiply by 6 (since there are 6 triangles)

$$\text{Area of a hexagon} = \frac{6\sqrt{3}}{4} . s^2 = \frac{3\sqrt{3}}{4} . s^2$$

$$\text{Area of a hexagon} = \frac{3\sqrt{3}}{4} \times s^2 \approx \frac{3\sqrt{3}}{4} \times (1\text{ m})^2 \approx 2.598\text{ m}^2$$

$$\text{Area of an equilateral triangle} = \frac{\sqrt{3}}{4} \times s^2 \approx \frac{\sqrt{3}}{4} \times (1\text{ m})^2 \approx 0.433\text{ m}^2$$

Area of the rectangular floor $= 10\text{m} \times 5\text{m} = 50\text{ m}^2$

Determine the arrangement: Assume a pattern where one hexagon is surrounded by 6 triangles. The area covered by one hexagon and the 6 surrounding triangles:

Total area covered by 1 hexagon and 6 triangles

$= 2.598\text{ m}^2 + 6 \times 0.433\text{ m}^2 \approx 2.598\text{ m}^2 = 5.196\text{ m}^2$

Calculate the total number of hexagons and triangles needed:

$$\text{Number of sets} = \frac{50\text{ m}^2}{5.196\text{ m}^2} \approx 9.62 \text{ sets}$$

Rounding up, you can fit 10 sets of the pattern. Therefore, we need:

- Hexagons: 10
- Triangles: $10 \times 6 = 60$

Example 18: Falak plans to tile a square patio with an area of 100 square metres. She decides to use both square tiles and triangular tiles, each with an area of 0.25 square metres. If 60% of the tiles will be square and 40% will be triangular, how many tiles of each shape are needed?

Solution: Total number of tiles $= \frac{\text{Patio Area}}{\text{Tile Area}} = \frac{100}{0.25}$

$= 400$ tiles

Number of square tiles $= 400 \times 0.6\ 240$

Number of triangular tiles $= 400 \times 0.4 = 160$

Solved Exercise 9.4

(i) What is the sum of the interior angles of a decagon (10-sided polygon)?

(ii) Calculate the measure of each interior angle of a regular hexagon.

(iii) What is each exterior angle of a regular pentagon?

(iv) If the sum of the interior angles of a polygon is 1260°, how many sides does the polygon have?

Sol. (i)

For a decagon number of sides = 10

Sum of interior angles of a decagon

$= (n - 2) \times 180°$

$= (10–2) \times 180°$

$= 8 \times 180$

$= 1440°$

(ii) Number of sides of a hexagon = 6

Each interior angle of hexgon $\frac{(n-2)\times 180°}{n}$

$= \frac{(6-2)\times 180°}{6}$

$= \frac{4\times 180°}{6} = \frac{72°}{6} = 120°$

(iii)

Number of sides of a regular pentagone $= 5$

Each exteroir angle $= \frac{360°}{n}$

$= \frac{360°}{5} = 72°$

(iv) Sum of the interior angles of the polygon = 1440°

Number of sides of the polygon = ?

Sum of interior angles = (n–2)×180°

$1260 = (n–2) \times 180$

$\frac{1260}{180} = n - 2$

$7 = n - 2$

$7 + 2 = n$

$9 = n$

$n = 9$ sides

2. In a parallelogram ABCD, $m\overline{AB} = 10$cm, $m\overline{AD} = 6$cm and $m\angle BAD = 45°$. Calculate the area of ABCD.

Sol. Area of parallelogram $= \overline{AB} \times \overline{AD} \times \sin 45°$

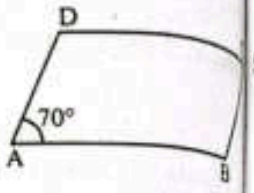
$= 10 \times 6 \times \frac{1}{\sqrt{2}}$

$= 60 \times 0.707$

$= 42.42\ cm^2$

3. In a parallelogram ABCD if m∠DAB=70°, find the measures of all other angles in the parallelogram.

Sol. In a parallelogram ABCD



$m\angle BAD = 70°$

$m\angle BCD = 70°$ (opposite angles of parallelogram are equal)

$m\angle ABC\ 180° - 70° = 110°$

(adjacent angels of parallelogram are supplementary)

$m\angle CDA = 110°$ (opposite angles of parallelogram)

so, $m\angle BCD = 70°, m\angle ABC = 110°, m\angle CDA = 110°$

4. A shape is created by cutting a square in half diagonally and then attaching a right-angled triangle to the hypotenuse of each half. Explain why this shape can tessellate and calculate the interior angle of the new shape.

Sol. (i) Take a square

(ii) Cut the square diagonally into two right angled triangles.

(iii) Attach another right angled triangle along the hypotenuse of each half. This will form a larger shape resembling a kite or parallelogram.

(iv) This shape can tessellate be cause the sides of the shape aline exactly due to congruence of the triangles.

(v) Both the original triangles and attached triangles are right angled triangles, ensuring angles sum up to 360°.

5. A tessellation is created by repeatedly reflecting a basic shape. The basic shape is a right-angled triangle with sides of length 3, 4 and 5 units. Find the minimum number of reflections needed to create a tessellation that covers a square with an area of 3600 square units.

Sol. Lengths of sides of right-angled triangle $= 3, 4$ and 5 units

Length of hypotenuse $= 5$ units (the longest side)

Length of base and hight $= 3, 4$ units

$$\text{Area of the triangle} = \frac{1}{2} \times \text{base} \times \text{hight}$$

$$= \frac{1}{2} \times 3 \times 4 = 6 \text{ units}$$

Total area of tessellation $= 3600$ units

$$\text{Number of reflection triangles} = \frac{\text{total area}}{\text{area of one triangle}}$$

$$= \frac{3600}{6} = 600$$

6. A tessellation is created using regular hexagons. Each hexagon has a side length of 5cm. Find the total area of the tessellation if it consists of 25 hexagons and total perimeter of the outer edge of the tessellation, assuming it's a perfect hexagon.

Sol. Side length of the hexagon $= 5cm$

$$\text{Area of one hexagon} = \frac{3\sqrt{3}}{2} \times 5^2 = \frac{3\sqrt{3}}{2} \times (5)^2$$

$$= \frac{3\sqrt{3} \times 25}{2} = \frac{75\sqrt{3}}{2} = \frac{75(1.732)}{2}$$

$$= \frac{129.9}{2} = 64.95 cm^2$$

Total are of 25 hexagons $= 25 \times 64.95 = 1623.75 cm^2$

A hexangonal cluster of 25 hexagons a hexago with 3 layer.

The sides of outer lair $= 6 \times 3 = 18$ sides

Perimeter $= 18 \times 5cm = 90$ cm

7. A rectangular floor is 12 m by 15 m. How many square tiles, each 1m by 1m, are needed to cover the floor?

Sol. Area of rectangular floor $= 12m \times 15m = 180m^2$

Area of saquare tile $= 1m \times 1m = 1m^2$

$$\text{Number of tiles to cover the floor} = \frac{\text{area of the floor}}{\text{area of a tile}}$$

$$= \frac{180m^2}{1m^2} = 180 \text{ tiles}$$

8. A rectangular wall is 10 m tall and 120 m wide. How many gallons of paint are needed to cover the wall, if one gallon covers 35 m^2?

Sol. Height of the wall $= 10m$

Width of the all $= 120m$

Area of the wall $= 10m \times 120m = 1200m^2$

Area covered by one gallons of point $= 350m^2$

$$\text{Number of gallons needed} = \frac{\text{Area}}{\text{coverage per gallon}}$$

$$= \frac{1200m^2}{35m^2} = 34.29 \cong 35 \text{ gallons}$$

9. A rectangular wall has a length of 10 m and a width of 4 meters. If 1 litre of paint covers 7 m^2, how many liters of paint are needed to cover the wall?

Sol. Length of the wall $= 10m$

Width of the wall $= 4m$

Area of the wall $= L \times W$
$= 10m \times 4m = 40m^2$

Area of wall covered by one litre point $= 7m^2$

$$\text{Amount of point needed} = \frac{\text{Area of wall}}{\text{Area covered by 1 litre point}}$$

$$= \frac{40m^2}{7m^2} = 5.71 \cong 6 \text{ litres}$$

10. A window has a trapezoidal shape with parallel sidews of 3 m and 1.5m and a height of 2m. Find the area of the window.

Sol. Length of parallel sides of window $= 3m, 1.5m$
Hight of window $= 2m$

Area of trapezodil shape window $= \frac{1}{2} \times$ sum of parallel sides $\times$ hight

$= \frac{1}{2} \times (3 + 1.5) \times 2$

$= \frac{1}{2} \times 4.5 \times 2 = 4.5m^2$

Review Exercise 9

1. Four option are given against each statement. Encircle the correct option.

1. Four options are given against each statement. Encircle the correct one.

(i) If two polygons are similar, then:
(a) their corresponding angles are equal. (b) their areas are equal.
(c) their volumes are equal. (d) their corresponding sides are equal.

(ii) The ratio of the areas of two similar polygons is:
(a) equal to the ratio of their perimeters.
(b) equal to the square of the ratio of their corresponding sides.
(c) equal to the cube of the ratio of their corresponding sides.
(d) equal to the sum of their corresponding sides.

(iii) If the volume of two similar solids is 125 cm³ and 27 cm³, the ratio of their corresponding heights is _____.
(a) 3:5 (b) 5:3 (c) 25:9 (d) 9:25

(iv) The exterior angle of regular pentagon is:
(a) 40° (b) 45° (c) 60° (d) 72°

(v) A parallelogram has an area of 64 cm² and a similar parallelogram has an area of 144 cm². If a side of the smaller parallelogram is 8 cm, the corresponding side of the larger parallelogram is:
(a) 10 cm (b) 12 cm (c) 18 cm (d) 16 cm

The total number of diagonals in a polygon with 9 sides is:
(a) 18 (b) 21 (c) 25 (d) 27

Two spheres are similar, and their radii are in the ratio 4:5. If the surface area of the larger sphere is 500π cm², what is the surface area of the smaller sphere?
(a) 256π cm² (b) 320π cm² (c) 400π cm² (d) 40π cm²

A regular polygon has an exterior angle of 30°. How many diagonals does the polygon have?
(a) 54 (b) 90 (c) 72 (d) 108

In a regular hexagon, the ratio of the length of a diagonal to the side length is:
(a) $\sqrt{3}:1$ (b) 2:1 (c) 3:2 (d) 2:3

A regular polygon has an interior angle of 165°. How many sides does it have?
(a) 15 (b) 16 (c) 20 (d) 24

s:
their corresponding angles are equal
equal to the square of the ratio of their corresponding sides.

5:3	iv. 72°	v. 12 cm	vi. 27	vii. 320π cm²	
54	ix 2:1	x. 24			

If the sum of the interior angles of a polygon is 1080°, how many sides does the polygon has?

Sol. Sum of the interior angles of polygon $= 1080°$
Number of sides of polygon $= ?$
Sum of the angles of polygon $= (n-2)\,180°$

$1080° = (n-2)\,180$

$\frac{1080}{180} = n - 2$

$6 = n - 2$

$6 + 2 = n$

$8 = n$

so, number of sides of polygon = 8 sides

Two similar bottles are such that one is twice as high as the other. What is the ratio of their surface areas and their capacities?

Sol. Hight of an bottle $= 1$
Hight of other smimilar bottle $= 2$
Ratio of the hights of similar bottles $= 1:2$
Use the formula for areas of similar figures to find ratio of their areas.

$$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

$$\frac{A_1}{A_2} = \left(\frac{1}{2}\right)^2$$

$$\frac{A_1}{A_2} = \frac{1}{4}$$

$$A_1 : A_2 = 1 : 4$$

To find the ratio of capacities use the formula for the volume of the similar figures

$$\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$$

$$\frac{V_1}{V_2} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$$V_1 : V_2 = 1 : 8$$

4. Each dimension of a model car is $\frac{1}{10}$ of the corresponding car dimension. Find the ratio of:

(a) the areas of their windscreens (b) the capacities of their boots

(c) the widths of the cars (d) the number of wheels they have.

Sol. (a) Ratio of the dimension of two modal car = $\frac{1}{10}$

Use the formula for ratio of areas of similar figures.

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$

$$\frac{A_1}{A_2} = \left(\frac{1}{10}\right)^2 = \frac{1}{100} = 1:100$$

(b) Ratio of the dimension of two car = $\frac{1}{10}$

Use the formula for ratio of volumes areas of similar figures.

$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$

$$\frac{V_1}{V_2} = \left(\frac{1}{10}\right)^3 = \frac{1}{1000} = 1:1000$$

(c) The unidths are proportional to the dimensions.

Width ratio = $\frac{1}{10}$ = 1:10

(d) The number of wheels is same both cars:

Wheel ratio = 1:1

5. Three similar jugs have heights 8cm, 12cm and 16cm. If the smallest jug holds $\frac{1}{2}$ litre, find the capacities of the other two.

Sol. The ratio of heights of three similar jugs = 8 : 12 : 16

The capacities are proportional to cube of their heights

$(8 : 12 : 16)^3 = 512 : 1728 : 4096$

$= 512 : 1728 : 4096$ (Divide by 512)

$= 1 : 3.75 : 8$

$$= \frac{1}{2} : \frac{3.75}{2} : \frac{8}{2} \text{ (Divide by 2)}$$

$$= \frac{1}{2} : 1\frac{7}{8} : 4$$

so, the capacities of other two jugs are $1\frac{7}{8}$ litres and 4 litres.

6. Three similar drinking glasses have heights 7.5 cm, 9 cm and 10.5 cm. If the tallest glass holds 343 millilitres, find the capacities of the other two.

Sol. The height of 1st glass (h_1) = 7.5cm

The height of 2nd glass (h_2) = 9cm

The height of 3rd glass (h_3) = 10.5cm

Let the capacity of the 1st glass = V_1

Let the capacity of the 2nd glass = V_2

Let the capacity of the 3rd glass = V_3

The capacities are proportional to cube of their respective heights.

$$\frac{V_1}{V_3} = \left(\frac{h_1}{h_2}\right)^3$$

$$\frac{V_1}{343} = \left(\frac{7.5}{10.5}\right)^3 = \frac{421.875}{1157.625}$$

$$V_1 = \frac{421.875}{1157.625} \times 343 = 125cm^3 = 125ml$$

$$\frac{V_2}{V_3} = \left(\frac{h_2}{h_3}\right)^3$$

$$\frac{V_2}{V_3} = \left(\frac{9}{10.5}\right)^3 = \frac{729}{1157.625}$$

$$V_2 = \frac{729}{1157.625} \times 343 = 216cm^3 = 216ml$$

so, the capacities of other glasses are 125 ml and 216 ml.

7. A toy manufacturer produces model cars which are similar in every way to the actual cars. If the ratio of the door area of the model to the door area of the car is 1 cm to 2500 cm, find:

(a) the ratio of their lengths

(b) the ratio of the capacities of their petrol tanks

(c) the width of the model, if the actual car is 150 cm wide

(d) the area of the rear window of the actual car if the area of the rear window of the model is 3 cm².

Sol. (a) Ratio of the door area of model car to the door area of actual = 1 : 2500

use the formula for ratio of areas of similar figures.

$$\frac{V_1}{V_2}=\left(\frac{l_1}{l_2}\right)^3 \Rightarrow \frac{1}{2500}=\left(\frac{l_1}{l_2}\right)^2$$

By taking squar root

$$\sqrt{\frac{1}{2500}}=\sqrt{\left(\frac{l_1}{l_2}\right)^2}$$

$$\frac{1}{50}=\frac{l_1}{l_2} \Rightarrow \frac{l_1}{l_2}=\frac{1}{50}$$

$l_1 : l_2 = 1 : 50$

(b) Use the formula for ratio of the volumes of similar figures.

$$\frac{V_1}{V_2}=\left(\frac{l_1}{l_2}\right)^3$$

$$\frac{V_1}{V_2}=\left(\frac{1}{50}\right)^3=\frac{1}{125000}$$

$V_1 : V_2 = 1 : 125000$

(c) Ratio of the lengths $= \frac{1}{50}$

The width of model car is proportional to the length ratio

$$\frac{\text{Width of modal car}}{\text{Width of actual car}}=\frac{l_1}{l_2}$$

$$\frac{\text{Width of modal car}}{150}=\frac{1}{50}$$

$$\text{Width of modal car} = \frac{1}{50}\times 150 = 3\text{cm}$$

(d) Ratio of the lengths $= \frac{1}{50}$

The area of the actual car's window is proportional to the square of length window.

$$\frac{\text{Area of the window of modal car}}{\text{Area of window of actul car}}=\left(\frac{l_1}{l_2}\right)^2$$

$$\frac{3}{\text{Area of window of actul car}}=\left(\frac{1}{50}\right)^2=\frac{1}{2500}$$

$7500 = 3\times 2500 =$ Area of window of actual car

Area of the window of actual car 7500 cm²

8. The ratio of the areas of two similar labels on two similar jars of coffee is 144:169. Find the ratio of

(a) the heights of the two jars (b) their capacities.

Sol. (a) The ratio of areas of two similar Jars = 144:169

We use the formula for ratio of areas of similar figures.

$$\frac{A_1}{A_2}=\left(\frac{l_1}{l_2}\right)^2$$

$$\frac{144}{169}=\left(\frac{l_1}{l_2}\right)^2$$

$$\left(\frac{12}{13}\right)^2=\left(\frac{l_1}{l_2}\right)^2$$

By taking square root

$$\sqrt{\left(\frac{12}{13}\right)^2}=\sqrt{\left(\frac{l_1}{l_2}\right)^2}$$

$$\frac{12}{13}=\frac{l_1}{l_2}$$

$$\frac{l_1}{l_2}=\frac{12}{13}$$

$l_1 : l_2 = 12 : 13$

(b) The ratio of hights of two jars = 12 : 13

so $\frac{h_1}{h_2}=\frac{12}{13}$

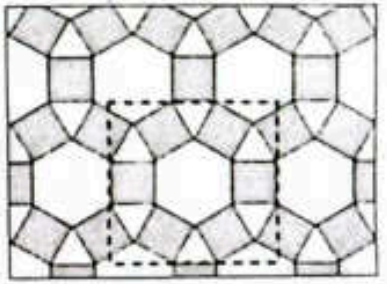
We use the formula for ratio of volume of similar figures.

$$\frac{V_1}{V_2}=\left(\frac{h_1}{h_2}\right)^3 = \frac{V_1}{V_2}=\left(\frac{12}{13}\right)^3$$

$$= \frac{V_1}{V_2}=\frac{1728}{2197}$$

$= 1728:2197$

9. A tessellation of tiles on a floor has been made using a repeating pattern of a regular hexagon, six squares and six equilateral triangles. Find the total area of a single pattern with side length $\frac{1}{2}$ metre of each polygon.



Sol. Lenght of a side of each polygon $= \frac{1}{2}$m

Area of a hexagon $= \frac{3\sqrt{3}a^2}{2}$

$= \frac{3\sqrt{3}}{2}\left(\frac{1}{2}\right)^2$

$= \frac{3\sqrt{3}}{2}\left(\frac{1}{4}\right) = \frac{3\sqrt{3}}{8}m^2$

Area of a square $= a^2$

$= \left(\frac{1}{2}\right)^2 = \frac{1}{4}m^2$

Area of six squares $= \frac{1}{4} \times 6 = \frac{3}{2}m^2$

Area of an equilateral triangle $= \frac{\sqrt{3}}{4}a^2$

$= \frac{\sqrt{3}}{4}\left(\frac{1}{2}\right)^2$

$= \frac{\sqrt{3}}{4} \times \frac{1}{4}$

$= \frac{\sqrt{3}}{16}m^2$

Area of the six triangles $= 6 \times \frac{\sqrt{3}}{16} = \frac{3\sqrt{3}}{8}m^2$

Total area of pattern = Area of hexagon + Area of Squares + Areas of triangles

$= \frac{3\sqrt{3}}{8} + \frac{3}{2} + \frac{3\sqrt{3}}{8}$

$= \frac{3\sqrt{3} + 12 + 3\sqrt{3}}{8}$

$= \frac{6\sqrt{3} + 12}{8}$

$= \frac{6(1.73) + 12}{8}$

$= \frac{10.38 + 12}{8}$

$= \frac{22.38}{8}$

$= 2.80\ m^2$

Objective Type Questions

Additional MCQ's

Multiple Choice Questions prepared in the light of new Examination Techniques
Knowledge, Understanding, Application, Analytical & Conceptual)

9.1 Similarity of polygons

Encircle the correct option.

Similar figures have same shapes but not necessary of _____.
(a) same size (b) same weight (c) same style (d) none of above

If two polygons are _____ one is a scaled version of the other.
(a) different (b) similar (c) corresponding (d) none of above

All equilateral triangles are similar to each other because they have the same angles and the measure of the _____ are proportional.
(a) angles (b) width (c) sides (d) lengths

If two angles in one triangle are congruent to two corresponding angles in another triangle the third angle in each triangle must be:
(a) congruent (b) small (c) greater (d) equal

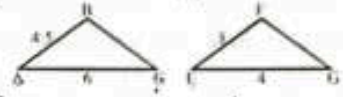
In corresponding of triangles ABC and DEF which of them is true?
(a) $m\angle C = m\angle D$ (b) $m\angle A = m\angle B$ (c) $m\angle B = m\angle F$ (d) $m\angle C = m\angle F$

Symbol of similarity is:
(a) ~ (b) ≅ (c) ≈ (d) ↔

If the ratio of two corresponding sides and their included angles are _____ then triangles are similar:
(a) less than (b) greater than (c) equal (d) none of above

If the ratio of all corresponding sides are _____ than the triangles are similar:
(a) equal (b) less than (c) greater than (d) none of above

According to figure which is true $\frac{m\overline{AC}}{m\overline{EG}}$ =



(a) $\frac{1}{2}$ (b) $\frac{2}{2}$ (c) $\frac{3}{2}$ (d) $\frac{4}{2}$

9.2 Area of similar figures

If base = 6 and altitude = 3 then area of parallelogram is _____:
(a) 12 cm² (b) 14 cm² (c) 16 cm² (d) 18 cm²

According to figure the value of A_1 = _____:



(a) $A_1 = 64\ cm^2$ (b) $A_1 = 62\ cm^2$ (c) $A_1 = 60\ cm^2$ (d) $A_1 = 58\ cm^2$

12. If $r_2 = 7cm$, $A_1 = 153\ cm^2$, $A_2 = 833\ cm^2$ then value of r_1 is _____:
(a) $r_1 = 0$ cm (b) $r_1 = 1$ cm (c) $r_1 = 2$ cm (d) $r_1 = 3$ cm

13. If $\ell_1 = 10$ cm, $\ell_2 = 25$ cm, $A_1 = 40$ cm, A_2 = _____:
(a) 150 cm² (b) 250 cm² (c) 350 cm² (d) 450 cm²

9.3 Volume of similar solids

14. Two solids are said to be similar if they have ____ shape but possibly different sizes.
(a) same (b) different (c) opposite (d) none of above

15. Two solids are similar if _____ of the corresponding sides are proportional.
(a) width (b) length (c) thickness (d) none of above

16. If $r_1 = 4$ cm, $r_2 = 5$ cm, $h_1 = 8$ cm, $h_2 = 10$cm then volume of V_1 = _____:
(a) 126π cm² (b) 127π cm² (c) 128π cm² (d) 129π cm²

17. If $r_1 = 4$ cm, $r_2 = 5$ cm, $h_1 = 8$ cm, $h_2 = 10$cm then volume of cylinder V_2 =
(a) 220π cm² (b) 230π cm² (c) 240π cm² (d) 250π cm²

18. According to figures what is value of V_2 = _____:



(a) 819.2 cm³ (b) 719.2 cm³ (c) 619.2 cm³ (d) 519.2 cm³

19. The given figure is known as _____:



(a) cylinder (b) cube (c) cone (d) rectangle

20. The given figure is known as:



(a) cylinder (b) cube (c) cone (d) triangle

9.4 Geometrical Properties of Polygons and their Application

21. A regular polygon has all sides and all angles are = _____:
(a) different (b) equal (c) bigger (d) smaller

22. Sum of interior angles of n – sided polygon is:
(a) $(n-1)\times 180°$ (b) $(n-2)\times 180°$ (c) $(n-3)\times 180°$ (d) $(n-4)\times 180°$

23. Which of them is interior angle of regular hexagon:
(a) 105° (b) 120° (c) 130° (d) 150°

24. The interior and exterior angles of regular polygon are _____:
(a) supplementary (b) complementary (c) both a & b (d) none of above

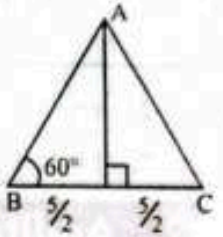
25. The total number of digonal in a regular polygon with n sides is:
(a) $\frac{n(n-1)}{n}$ (b) $\frac{n(n-2)}{n}$ (c) $\frac{n(n-3)}{n}$ (d) $\frac{n(n-4)}{n}$

26. A regular n-sided polygon can be rotated by _____:
(a) $\frac{90°}{n}$ (b) $\frac{180°}{n}$ (c) $\frac{270°}{n}$ (d) $\frac{360°}{n}$

Interior angle of ragular pentagon is _____:
(a) 70° (b) 71° (c) 72° (d) 73°

How many regular polygons can be tessellation?
(a) 2 (b) 3 (c) 4 (d) 5

According to figure what is the area of triangle ABC.



(a) $\sqrt{35}$ (b) $\sqrt{\frac{35}{2}}$ (c) $\sqrt{\frac{35}{3}}$ (d) $\frac{25\sqrt{3}}{4}$

What is the interior angle of regular decagon?
(a) 144° (b) 150° (c) 155° (d) 160°

Answers:

same size	2. similar	3. sides	4. congruent	5. $m\angle C = m\angle F$
-	7. equal	8. equal	9. $\frac{3}{2}$	10. 18 cm²
11. $A_1 = 64$ cm²	12. $r_1 = 3$ cm	13. 250 cm²	14. same	15. length
16. 128π cm²	17. 250π cm²	18. 819.2 cm³	19. cylinder	20. cone
21. equal	22. $(n-2)\times 180°$	23. 120°	24. supplementary	25. $\frac{n(n-3)}{n}$
26. $\frac{360°}{n}$	27. 72°	28. 3	29. $\frac{25\sqrt{3}}{4}$	30. 144°

Conceptual MCQ's

Encircle the correct option.

1. Three or more than three sided closed figure is called:
(a) polygon (b) point (c) line (d) ray

2. Proportionality of sides means _____ side is k times of it, corresponding side:
(a) one (b) two (c) three (d) four

3. For a regular n-sided polygon the size of each interior angle is:
(a) $\frac{(n-1)\times 180°}{n}$ (b) $\frac{(n-2)\times 180°}{n}$ (c) $\frac{(n-3)\times 180°}{n}$ (d) $\frac{(n-4)\times 180°}{n}$

4. The interior angle of regular hexagon is:
(a) 90° (b) 100° (c) 110° (d) 120°

5. The sum of interior angles in any triangle is:
(a) 30° (b) 60° (c) 120° (d) 180°

6. The interior angle of regular pentagon is:
(a) 70° (b) 75° (c) 72° (d) 73°

7. **The interior angle of regular decagon is:**
(a) 140° (b) 144° (c) 150° (d) 160°

8. **The interior angle of regular pentagon is:**
(a) 108° (b) 110° (c) 120° (d) 150°

Answers:

1.	polygon	2.	one	3.	$\frac{(n-2)\times 180^\circ}{n}$	4.	120°	5.	180°
6.	72°	7.	144°	8.	108°				

Additional Short Questions

Short Answer Questions prepared in the light of new Examination Techniques (Knowledge, Understanding, Application, Analytical & Conceptual)

1. **What do you know about similar figures?**
Ans. Similar figures have same shape but not necessarily of same size.

2. **How two polygons are similar?**
Ans. Two polygons are similar if their corresponding angles are equal and the corresponding sides are proportional.

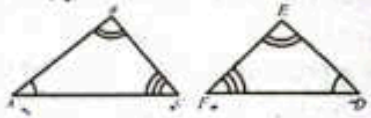
3. **Define scaled version.**
Ans. If two polygons are similar one is a scaled version of the other.

4. **Which type of triangles are similar?**
Ans. All equilateral triangles are similar to each other because they have the same angles and measure of the sides are proportional.

5. **If two angles in one triangle are congruent, what will be happend?**
Ans. If two angles in one triangle are congruent to two corresponding angles in another triangle, the third angle in each triangle must be congruent.

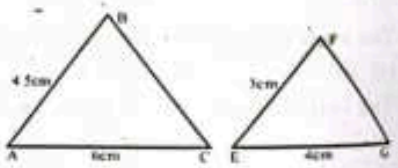
6. **What is the symbol of similarity?**
Ans. The symbol of similarity is "~".

7. **In the corresponding of triangles ABC and DEF, prove ΔABC ~DEF.**
Ans. $m\angle A = m\angle D$
$m\angle B = m\angle E$
$m\angle C = m\angle F$
So, $\Delta ABC \sim \Delta DEF$.



8. **How triangles are similar if the ratio of two corresponding sides are?**
Ans. If the ratio corresponding sides and thier included angle are equal, then the triangles are similar.

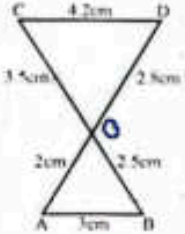
9. **According to figure prove ΔABC and ΔEFG are similar.**
Ans. $\frac{m\overline{AB}}{m\overline{EF}} = \frac{4.5}{3} = \frac{3}{2}$



$\frac{m\overline{AC}}{m\overline{EG}} = \frac{6}{4} = \frac{3}{2}$

So ΔABC and ΔEFG are similar.

According to figure find the ratio of corresponding sides.



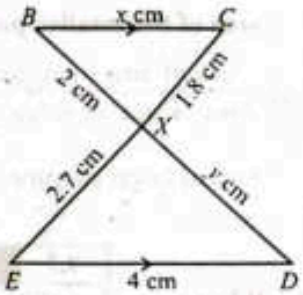
$\frac{m\overline{OA}}{m\overline{OB}} = \frac{m\overline{AB}}{m\overline{BC}} = \frac{m\overline{OB}}{m\overline{OC}}$

$\frac{2}{2.8} = \frac{3}{4.2} = \frac{2.5}{3.5}$

$\frac{5}{7} = \frac{5}{7} = \frac{5}{7}$

So, ΔOAB and ΔODC are similar.

In the ΔXBC and ΔXDE, find values of x and y.



Sol. $\frac{m\overline{XB}}{m\overline{XD}} = \frac{m\overline{BC}}{m\overline{DE}} = \frac{m\overline{XC}}{m\overline{XE}}$

$\frac{2}{y} = \frac{x}{4} = \frac{1.8}{2.7}$

$\frac{x}{4} = \frac{1.8}{2.7}$

$x = \frac{1.8}{2.7} \times 4 = 2.67$ cm

Now

$\frac{2}{y} = \frac{1.3}{2.7} \Rightarrow y = \frac{2.7}{1.8} \times 2 = 3$ cm

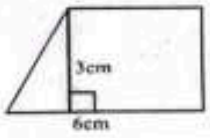
12. **The perimeter of a regular octagon is 48 cm. Another octagon has sides that are 1.2 times of the sides of the first octagon. What is the length of the side of the second octagon?**
Ans. perimeter of first regular octagon = 48 cm.

side length of first regular octagon = $\frac{48}{8} = 6$cm

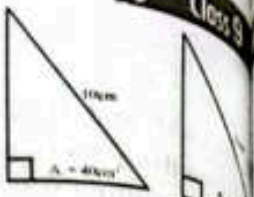
side length of second regular octagon = $6 \times 1.2 = 7.2$ cm.

9.2 Area of Similar Figures

13. **According to figure find area of parallelogram.**
Ans. Area of parallelogram = base × altitude
$= 6 \times 3$
$= 18$ cm²



14. Look at the figure. Find the value of A_2.



Ans. As

$$\frac{A_1}{A_2}=\left(\frac{\ell_1}{\ell_2}\right)^2$$

$$\frac{40}{A_2}=\left(\frac{10}{25}\right)^2 \Rightarrow \frac{40}{A_2}=\left(\frac{2}{5}\right)^2$$

$$\frac{40}{A_2}=\frac{4}{25}$$

$$A_2=40\times\frac{25}{4}=250\ cm^2$$

15. Two polygons are similar with a ratio of corresponding sides being $\frac{3}{5}$. If area of the smaller polygon is 54 cm². Find the area of the larger polygon.

Ans. $\frac{\text{Area of larger polygon}}{\text{Area of smaller polygon}}=\left(\frac{5}{3}\right)^2=\frac{25}{9}$

$$\text{Area of larger polygon}=\frac{25}{9}\times 54$$

$$=150\ cm^2$$

9.3 Volume of similar solids

16. When two sides will be similar?

Ans. Two sides are said to be similar if they have same shape but possibly different size. Two solids are similar of lengths of the corresponding sides are proportional i.e. ratio of the corresponding lengths are equal. e.g. two cylinders are similar.

$$\frac{r_1}{r_2}=\frac{h_1}{h_2}$$

17. If $r_1 = 4cm$, $r_2 = 5cm$, $h_1 = 8cm$, $h_2 = 10$ cm then find the volume of V_1.

Ans. $V_1=\pi r_1^2 h_1$

$=\pi\times 4^2\times 8=128\pi\ cm^2$

18. If $r_1 = 4cm$, $r_2 = 5cm$, $h_1 = 8cm$, $h_2 = 10$ cm then find the volume of V_2.

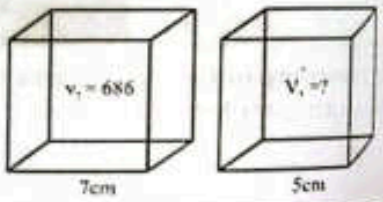
Ans. $V_2=\pi r_2^2 h_2$

$=(\pi)(5)^2(10)$

$=(\pi)(25)(10)$

$=250\pi\ cm^2$

19. Find the unknown volume of the figure.



Ans. $\frac{V_1}{V_2}=\left(\frac{\ell_1}{\ell_2}\right)^3 \Rightarrow \frac{V_1}{686}=\left(\frac{5}{7}\right)^3$

$$\frac{V_1}{686}=\frac{125}{343}$$

$$V_1=\frac{(125)(686)}{343}=250\ cm^3$$

A solid cone C is cut into two pieces A and B with sloping edges 6cm, 4cm. Find the ratio of diameters of the bases of cones A and C.

Diameter of cone A = d_1

Diameter of cone C = d_2

Now

$$\frac{d_1}{d_2}=\frac{\ell_1}{\ell 2}=\frac{6}{10}=\frac{3}{5}$$

The mass of sack of rice is 50 kg and height 4m. Find the mass of the similar sack of rice with height of 6m.

Mass of the smaller sack of rice = $w_1 = 50$ kg

Height of smaller sack of rice = $h_1 = 4m$

Mass of the larger sack of rice = $w_2 = ?$

Height of larger sack of rice = $h_1 = 6m$

Now

$$\frac{w_1}{w_2}=\left(\frac{h_1}{h_2}\right)^3 \Rightarrow \frac{50}{w_2}=\left(\frac{4}{6}\right)^3$$

$$\frac{50}{w_2}=\frac{8}{27} \Rightarrow w_2=\frac{27\times 50}{8}=168.75\ kg$$

The ratio of the corresponding lengths of two similar cylinderical can is 3:2. The larger cylinderical can hase surface area of 67.5 m². Find the surface area of smaller cylinderical can.

Ans. Surface area of larger can = $A_1 = 67.5\ cm^2$

Surface area of smaller can = $A_2 = ?$

Now $\frac{\ell_1}{\ell_2}=\frac{3}{2}$

and $\frac{A_1}{A_2}=\left(\frac{\ell_1}{\ell_2}\right)^2$

$$\frac{67.5}{A_2}=\left(\frac{3}{2}\right)^2 \Rightarrow A_2=67.5\times\frac{4}{5}=30\ m^2$$

9.4 Geometrical Properties of Polygons and their Application

23. Define regular polygon.

Ans. A regular polygon has all sides and all angles equal. For example, equilateral triangles, squares, regular pentagon and hexagon.

24. What is the formula for sum of interior angles of n-sided polygon?

Ans. $(n-2)\times 180°$

25. What is the formula for interior angle of n-sided polygon?

Ans. $\frac{(n-2)\times 180^\circ}{n}$

26. What is sum of all exterior angles of polygon?

Ans. The sum of all exterior angles of any polygon is always 360°.

27. Where interior and exterior angles are supplementary?

Ans. The interior and exterior angles are supplementary at a vertex.
Interior angle + exterior angle = 180°

28. What is the formula to find the total number of diagonals in a regular polygon with n sides.

Ans. $\frac{n(n-3)}{2}$

29. In which angle a regular n-sided polygon can be rotated?

Ans. A regular n-sided polygon can be rotated $\frac{360^\circ}{n}$ and will look the same.

30. What is the sum of interior angles of triangle?

Ans. The sum of interior angles of triangle is 180°.

31. In which triangle all sides are equal?

Ans. In Equilateral triangle all sides are equal in length.

32. What is the interior angle of equilateral triangles?

Ans. 60°.

33. Find the measure of each interior angle of a regular pentagon.

Ans. Interior angle $= \frac{(n-2)\times 180^\circ}{n}$

$= \frac{(5-2)\times 180^\circ}{5} = \frac{540^\circ}{5} = 108^\circ$

34. What is Tessellation?

Ans. A tessellation is a pattern of shapes that fit together perfectly without any gaps or overlaps, covering a plane.

35. How tessellations can be created?

Ans. Tessellations can be created using a single or a combination of shapes.

36. How many polygons can tessellation?

Ans. There are only three regular polygon can be tessallate e,g. Equilateral triangles, squares, regular hexagones.

37. A parallelogram-shaped room has a base of 10 meters, and a height of 8 meters. Sadia wants to carpet the room using rolls that cover 20 m² each. How many rolls of carpet do she need?

Ans. Area of parrallelogram = A = base × height
$= 10 \times 8 = 80\ cm^2$

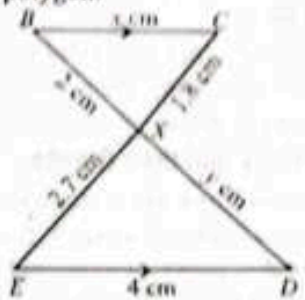
Number of rolls $= \frac{80}{20} = 4$ rolls.

Conceptual Short Questions

Define polygon.

Three or more than three sides closed figure is called polygon.

According to figure find the values of x, y.



$\frac{m\overline{XB}}{m\overline{XD}} = \frac{m\overline{BC}}{m\overline{DE}} = \frac{m\overline{XC}}{m\overline{XE}}$

$\frac{2}{y} = \frac{x}{4} = \frac{1.2}{2.7}$

$\frac{x}{4} = \frac{1.8}{2.7} \Rightarrow x = \frac{1.8}{2.7} \times 4 = 2.67$

$\frac{2}{y} = \frac{1.8}{2.7} \Rightarrow y = \frac{2.7}{1.8} \times 2 = 3$

3. Two polygons are similar with a ratio of corresponding sides being $\frac{3}{5}$. If the area of smaller polygon is 54 cm² find the area of larger polygon.

Ans. $\frac{\text{Area of larger polygon}}{\text{Area of smaller polygon}} = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$

Area of larger polygon $= \frac{25}{9} \times 54 = 150 cm^2$

4. If $r_1 = 4cm$, $r_2 = 5cm$, $h_1 = 8cm$, $h_2 = 10$ cm then find the volume of V_1.

Ans. $V_1 = \pi\ r_1^2\ h_1 = \pi \times 4^2 \times 8 = \pi \times 16 \times 8 = 128\ \pi\ cm^3$

5. If $r_1 = 4cm$, $r_2 = 5cm$, $h_1 = 8cm$, $h_2 = 10$ cm then find the volume of V_2.

Ans. $V_2 = \pi\ r_2^2\ h_2 = (\pi)\ (5)^2\ (10) = (\pi)\ (25)\ (10) = 250\ \pi\ cm^3$

6. Where the interior and exterior angles are supplement?

Ans. The interior and exterior angles are supplement at vertex i.e Interior+Exterior=180°.

7. Deine Rhombus.

Ans. In rhombus all sides are equal and diagonals bisect each other at a right angles.

8. What is the interior and exterior angle of pentagon.

Ans. Interior angle $= \frac{(n-2)\times 180^\circ}{n} = \frac{(5-2)\times 180^\circ}{5} = 108^\circ$

Exterior angle $= \frac{360^\circ}{5} = 72^\circ$

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Unit 10

Graphs of Functions

Graphs

Graphs are powerful tools for visualizing and analyzing relationships betw variables, making them essential in understanding various mathematical functions a their applications.

Functions and their Graphs

Functions are essential tools for representing real-world phenomena us mathematical concepts. A function can be expressed in various forms, including equation, a graph, or numerical table or a verbal description.

Graph of Linear Functions

A linear function is a mathematical expression that represents a straight-li relationship between two variables. Its general form is f(x) = mx + c, where "m" is t slope or gradient of the line, indicating how steep it is and "c" is the y-intercept (the po where the line crosses the y-axis). It can also be written as y = mx + c.

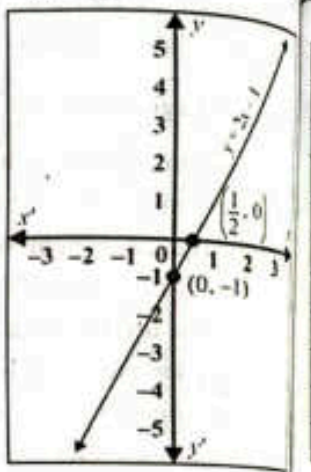
Example 1: Sketch the graph of y = 2x – 1.

Solution: To sketch the graph of linear function, we can find its x and y intercepts.

Put x = 0, we get y = –1. So (0, –1) is the y-intercept.

Put y = 0, we get $x = \frac{1}{2}$. So $\left(\frac{1}{2}, 0\right)$ is the x-intercept.

The graph is a straight line that rises to the right because slope is positive.



Graph of Quadratic Functions

A quadratic function is a type of polynomial function that involves x^2 term. general form is:

$$y = ax^2 + bx + c$$

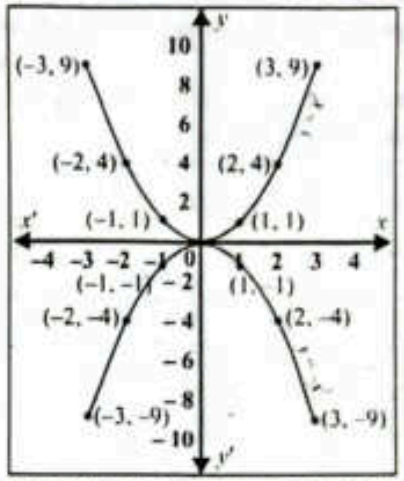
Where a, b, c are constants and $a \neq 0$.

Keep in mind!
The graph of a quadratic function is always a parabo
- If a > 0, then the parabola opens upward like "∪".
- If a < 0, then the parabola opens downward like "∩".

Example 2: Plot the graphs of $y = x^2$ and $y = -x^2$ on the same diagram.

Solution: The following table shows several values of x and the given functions evaluated at these values:

x	$y = x^2$	$y = -x^2$
–3	$(-3)^2 = 9$	–9
–2	$(-2)^2 = 4$	–4
–1	$(-1)^2 = 1$	–1
0	$(0)^2 = 0$	0
1	$(1)^2 = 1$	–1
2	$(2)^2 = 4$	–4
3	$(3)^2 = 9$	–9

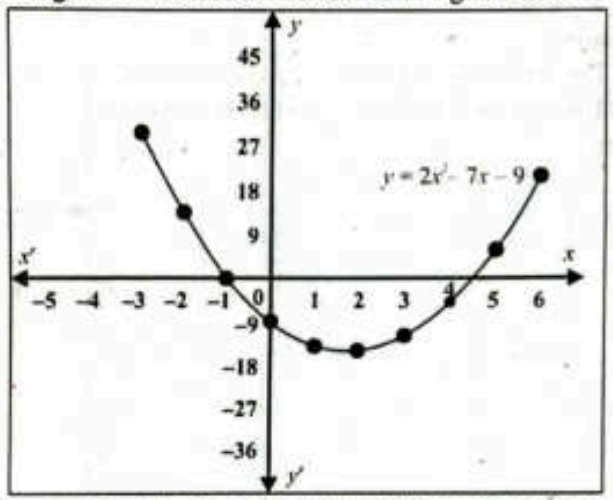


Graph of $y = x^2$ represents parabola, passing through origin and opens upward.
Graph of $y = -x^2$ represents parabola, passing through origin and opens downward.

ample 3: Sketch the graph of $y = 2x^2 - 7x - 9$ for $-3 \leq x \leq 6$.

ution: The values of x and y are given in the table and sketched in figure below:

x	y
–3	30
–2	13
–1	0
0	–9
1	–14
2	–15
3	–12
4	–5
5	6
6	21



Graph of $y = 2x^2 - 7x - 9$ represents parabola, opens upward. It intersects the y-axis (0, –9) and x-axis at (–1, 0) and (4.5, 0).

Graph of Cubic Functions

A cubic function is a type of polynomial function of degree 3. Its standard form is:

$$y = ax^3 + bx^2 + cx + d$$

Where a, b, c, d are constants and $a \neq 0$.

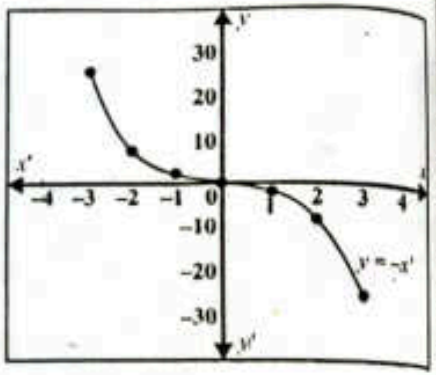
Remember!
- The graph of a cubic function is a curve that can have at most two turning points.
- It has a general "S-shaped" appearance and depending on the coefficients, the shape may vary.
- Such functions are much more complicated and show more varied behaviour than linear and quadratic ones.

Example 4: Plot the graph of the following cubic function for $-3 \leq x \leq 3$:

$$y = -x^3$$

Solution: The following table shows several values of x and the given function evaluated at these values:

x	$y = -x^3$
−3	27
−2	8
−1	1
0	0
1	−1
2	−8
3	−27



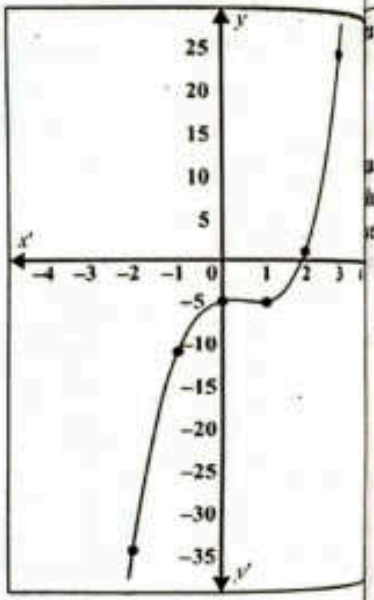
The curve passes through the origin.

Example 5: Plot the graph of $y = 2x^3 - 3x^2 + x - 5$ for $-2 \leq x \leq 3$.

Solution:

The following table shows several values of x and the given function is evaluated at these values:

x	y
−2	−35
−1	−11
0	−5
1	−5
2	1
3	25



The graph tells us that when x = 0, the function's value is −5.

Graph of Reciprocal Functions

A reciprocal function is a function of the form: $y = \frac{a}{x}$

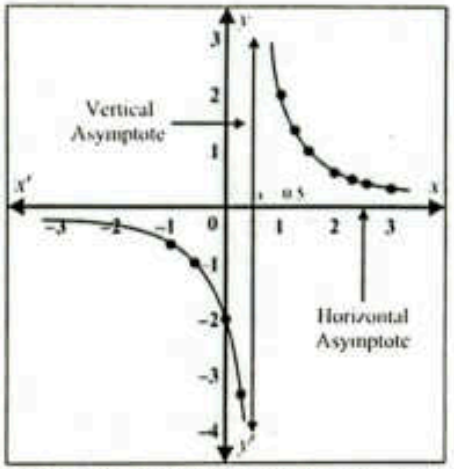
Where a is any real number and $x \neq 0$.

Example 6: Sketch the graph of the following reciprocal function:

$$y = \frac{1}{x - 0.5}, \; x \neq 0.5$$

Solution: The following table shows several values of x and the given function is evaluated at these values:

x	y
−1	−0.67
−0.5	−1
−0.2	−1.43
0	−2
0.2	−3.3
0.5	undefined
1	2
1.2	1.43
1.5	1
2	0.67
2.2	0.59
2.5	0.5
3	0.4



Remember!
An asymptote is a line that a graph approaches but never touches.

Graph of Exponential Functions ($y = ka^x$ where x is real number, $a > 1$)

An exponential function is a mathematical function of the form:

$$y = ka^x$$

Where a, k are constants, x is variable and $a > 1$.

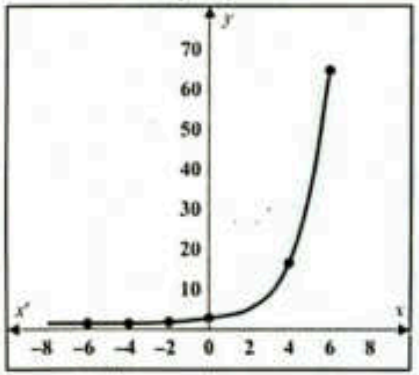
Example 7: Plot the graph of the exponential function $y = 2^x$ for $-6 \leq x \leq 6$.

Solution: The function $y = 2^x$ has base 2 and variable exponent x. Values of (x, y) are given in the table below:

x	−6	−4	−2	0	2	4	6
$y = 2^x$	0.02	0.06	0.25	1	4	16	64

Then graph these points as in figure below.

Graph of $y = 2^x$



The graph of $y = 2^x$ represents the growth curve.

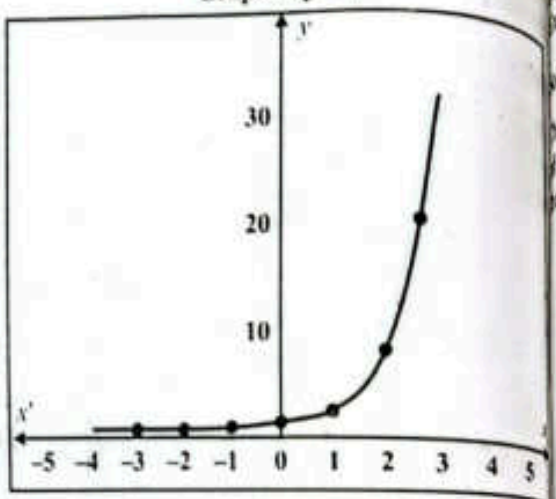
Example 8: Plot the graph of the exponential function, $y = e^x$.

Solution: The function $y = e^x$ has base e and variable power x. We know e = 2.7182818,

correct to two decimal places e = 2.72. Table of x and y values is given below:

x	$y = e^x$
-3	0.05
-2	0.14
-1	0.37
0	1
1	2.72
2	7.40
3	20.09

Graph of $y = e^x$



Graphs of $y = ax^n$ (where n is +ve integer, -ve integer or rational number for $x > 0$ and a is any real number)

The graph of the function $y = ax^n$, where n is a positive integer, negative integer or rational number for $x > 0$ and a is any real number, exhibits distinct behavior depending on the value of n. Following are the examples of these cases:

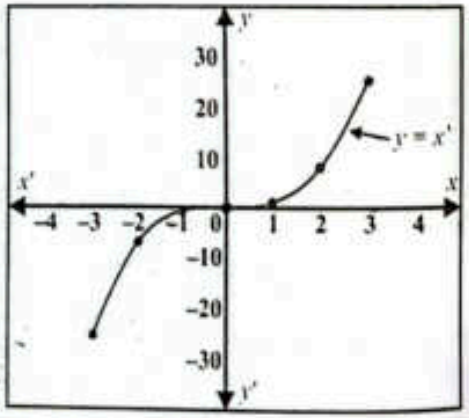
(i) When n is positive integer (n = 3)

Example 9: Plot the graph of $y = x^3$ for $-3 \le x \le 3$.

Solution: The table shows several values of x and the given function is evaluated at these values:

The curve passes through the origin.

x	$y = x^3$
−3	−27
−2	−8
−1	−1
0	0
1	1
2	8
3	27



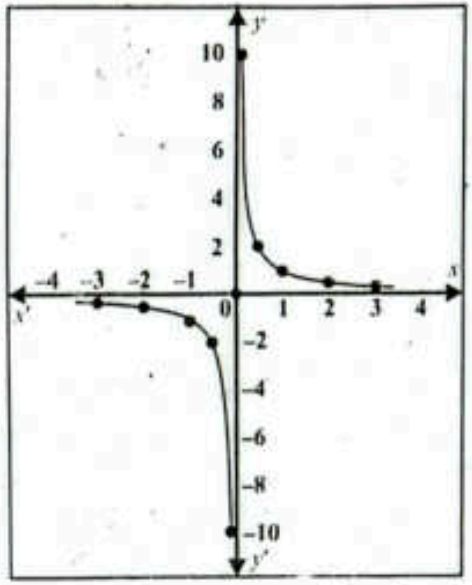
When n is negative integer (n = −1)

Example 10: Plot the graph of $y = x^{-1}$

Solution: $y = x^{-1} = \frac{1}{x}$

The following table shows several values of x and the given function is evaluated at these values:

x	$y = \frac{1}{x}$
−3	−0.3
−2	−0.5
−1	−1
−0.5	−2
−0.1	−10
0.1	10
0.5	2
1	1
2	0.5
3	0.3



The above graph consists of two branches, one in the first quadrant and the other in the third quadrant. Both branches approach but never touch the x-axis or the y-axis.

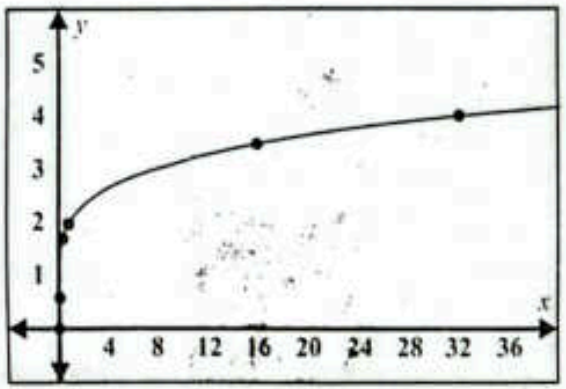
(iii) When n is rational number $\left(n = \frac{1}{5}\right)$

Example 11: Plot the graph of $y = 2x^{\frac{1}{5}}$.

Solution: $y = 2x^{\frac{1}{5}}$

The following table shows several values of x and the given function is evaluated at these values.

x	y
0	0
0.01	0.80
0.5	1.74
1	2
16	3.48
32	4



Solved Exercise 10.1

1. Sketch the graph of the following linear functions:

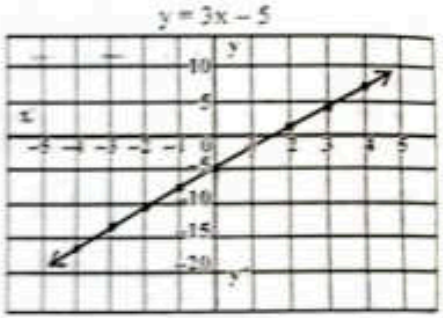
(i) $y = 3x - 5$

Sol. $y = 3x - 5$

$y = 3(1) - 5 = 3 - 5 = -2$
$y = 3(2) - 5 = 6 - 5 = 1$
$y = 3(3) - 5 = 9 - 5 = 4$
$y = 3(4) - 5 = 12 - 5 = 7$
$y = 3(0) - 5 = 0 - 5 = -5$
$y = 3(-1) - 5 = -3 - 5 = -8$
$y = 3(-2) - 5 = -6 - 5 = -11$
$y = 3(-3) - 5 = -9 - 5 = -14$
$y = 3(-4) - 5 = -12 - 5 = -17$

The following table shows several values of x and the given function is evaluated at these values.

x	y	(x, y)
1	−2	(1, −2)
2	1	(2, 1)
3	4	(3, 4)
4	7	(4, 7)
0	−5	(0, −5)
−1	−8	(−1, −8)
−2	−11	(−2, −11)
−3	−14	(−3, −14)
−4	−17	(−4, −17)



(ii) $y = -2x + 8$

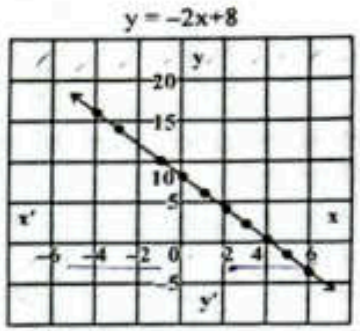
Sol. $y = -2x + 8$

$y = -2(0) + 8 = 0 + 8 = 8$
$y = -2(1) + 8 = -2 + 8 = 6$
$y = -2(2) + 8 = -4 + 8 = 4$
$y = -2(3) + 8 = -6 + 2 = 2$
$y = -2(4) + 8 = -8 + 8 = 0$
$y = -2(5) + 8 = -10 + 8 = -2$
$y = -2(6) + 8 = -12 + 8 = -4$
$y = -2(-1) + 8 = 2 + 8 = 10$
$y = -2(-2) + 8 = -6 + 8 = 14$
$y = -2(-4) + 8 = 8 + 8 = 16$

The following table shows the several values of x and the given function is evaluated

at these values.

x	y	(x, y)
0	8	(0, 8)
1	6	(1, 6)
2	4	(2, 4)
3	2	(3, 2)
4	0	(4, 0)
5	−2	(5, −2)
6	−4	(6, −4)
−1	10	(−1, 10)
−3	14	(−3, 14)
−4	16	(−4, 16)



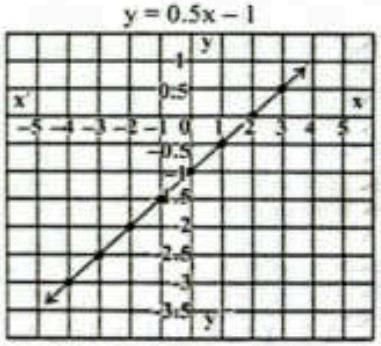
(iii) $y = 0.5x - 1$

Sol. $y = 0.5x - 1$

$y = 0.5(0) - 1 = 0 - 1 = -1$
$y = 0.5(1) - 1 = 0.5 - 1 = -0.5$
$y = 0.5(2) - 1 = 1 - 1 = 0$
$y = 0.5(3) - 1 = 1.5 - 1 = 0.5$
$y = 0.5(-1) - 1 = -0.5 - 1 = -1.5$
$y = 0.5(-2) - 1 = -1 - 1 = -2$
$y = 0.5(-3) - 1 = -1.5 - = -2.5$
$y = 0.5(-4) - 1 = -2 - 1 = -3$

The following table shows the several values of x and the given function is evaluated at these values.

x	y	(x, y)
0	−1	(0, −1)
1	−0.5	(1, −0.5)
2	0	(2, 0)
3	0.5	(3, 0.5)
−1	−1.5	(−1, −1.5)
−2	−2	(−2, −2)
−3	−2.5	(−2, −2.5)
−4	−3	(−4, −3)



2. Plot the graph of the following quadratic and cubic functions:

(i) $y = x^3 + 2x^2 - 5x - 6; -3.5 \le x \le 2.5$

Sol. $y = x^3 + 2x^2 - 5x - 6$

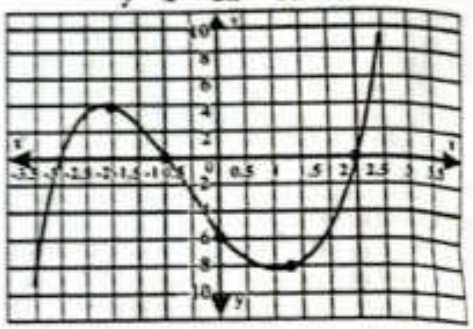
$y = 0^3 + 2(0)^2 - 5(0) - 6 = 0 + 0 - 0 - 6 = -6$

$y = 1^3 + 2(1)^2 - 5(1) - 6 = 1 + 2 - 5 - 6 = -8$
$y = 2^3 + 2(2)^2 - 5(2) - 6 = 8 + 8 - 10 - 6 = 0$
$y = (-1)^3 + 2(-1)^2 - 5(-1) - 6 = -1 + 2 + 5 - 6 = 0$
$y = (-2)^3 + 2(-2)^2 - 5(-2) - 6 = -8 + 8 + 10 - 6 = 4$
$y = (-3)^3 + 2(-3)^2 - 5(-3) - 6 = -27 + 18 + 15 - 6 = 0$

The following table shows the several values of x and the given function is evaluated at these values.

x	y	(x, y)
0	–6	(0, –6)
1	–8	(1, –8)
2	0	(2, 0)
–1	0	(–1, 0)
–2	4	(–2, 4)
–3	0	(–3, 0)

$y = x^3 + 2x^2 - 5x - 6$



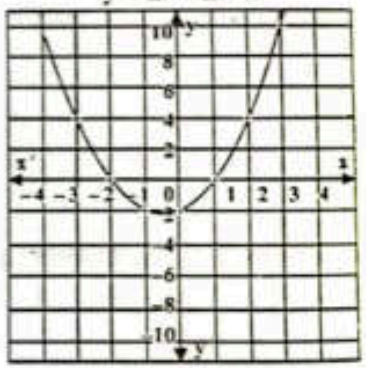
(ii) $y = x^2 + x - 2$

Sol. $y = x^2 + x - 2$
$y = (-3)^2 + (-3) - 2 = 9 - 3 - 2 = 4$
$y = (-2)^2 + (-2) - 2 = 4 - 2 - 2 = 0$
$y = (-1)^2 + (-1) - 2 = 1 - 1 - 2 = -2$
$y = 0^2 + 0 - 2 = 0 + 0 - 2 = -2$
$y = 1^2 + 1 - 2 = 1 + 1 - 2 = 0$
$y = 2^2 + 2 - 2 = 4 + 2 - 2 = 4$
$y = 3^2 + 3 - 2 = 9 + 3 - 2 = 10$

The following table shows the several values of x and the given function is evaluated at these values.

x	y	(x, y)
–3	4	(–3, 4)
–2	0	(–2, 0)
–1	–2	(–1, –2)
0	–2	(0, –2)
1	0	(1, 0)
2	4	(2, 4)
3	10	(3, 10)

$y = x^2 + x - 2$



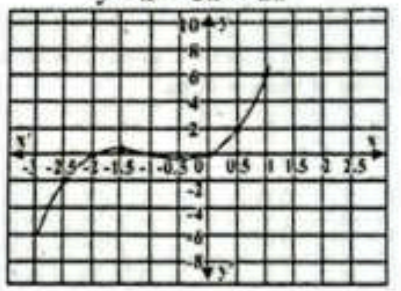
(iii) $y = x^3 + 3x^2 + 2x$; $-2.5 \le x \le 0.5$

Sol. $y = x^3 + 3x^2 + 2x$
$y = (-2.5)^3 + 3(-2.5)^2 + 2(-2.5) = -15.625 + 18.75 - 5 = -1.9$
$y = (-2)^3 + 3(-2)^2 + 2(-2) = -8 + 12 - 4 = 0$
$y = (-1.5)^3 + 3(-1.5)^2 + 2(-1.5) = -3.375 + 6.75 - 3 = 0.4$
$y = (-1)^3 + 3(-1)^2 + 2(-1) = -1 + 3 - 2 = 0$
$y = 0^3 + 3(0)^2 + 2(0) = 0 + 0 + 0 = 0$
$y = (0.5)^3 + 3(0.5)^2 + 2(0.5) = 0.125 + 0.75 + 2 = 2.9$

The following table shows the several values of x and the given function is evaluated at these values.

x	y	(x, y)
–2.5	–1.9	(–2.5, –1.9)
–2	0	(–2, 0)
–1.5	0.4	(–1.5, 0.4)
–1	0	(–1, 0)
0	0	(0, 0)
0.5	2.9	(0.5, 2.9)

$y = x^3 + 3x^2 + 2x$



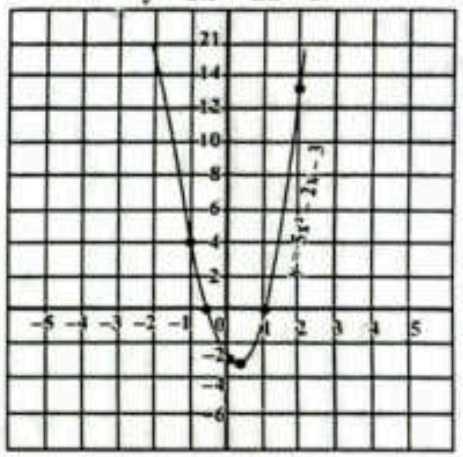
(iv) $y = 5x^2 - 2x - 3$

Sol. $y = 5x^2 - 2x - 3$
$y = 5(-2)^2 - 2(-2) - 3 = 20 + 4 - 3 = 21$
$y = 5(-1)^2 - 2(-1) - 3 = 5 + 2 - 3 = 4$
$y = 5(0)^2 - 2(0) - 3 = 0 - 0 - 3 = -3$
$y = 5(1)^2 - 2(1) - 3 = 5 - 2 - 3 = 0$
$y = 5(2)^2 - 2(2) - 3 = 20 - 4 - 3 = 13$

The following table shows the several values of x and the given function is evaluated at these values.

x	y	(x, y)
–2	21	(–2, 21)
–1	4	(–1, 4)
0	–3	(0, –3)
1	0	(1, 0)
2	13	(2, 13)

$y = 5x^2 - 2x - 3$



3. Plot the graph of the following functions:

(i) $y = 4^x$

Sol. $y = (4)^{-3} = \frac{1}{(4)^3} = \frac{1}{64} = 0.02$

$y = (4)^{-2} = \frac{1}{(4)^2} = \frac{1}{16} = 0.06$

$y = (4)^{-1} = \frac{1}{4} = 0.25$

$y = (4)^0 = 1$

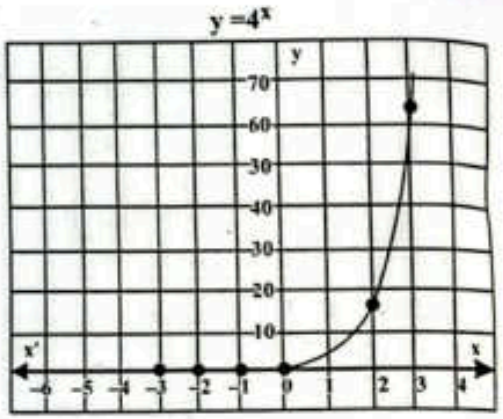
$y = (4)^1 = 4$

$y = (4)^2 = 16$

$y = (4)^3 = 64$

The following table shows the several values of x and the given function is evaluated at these values.

x	y	(x, y)
–3	0.02	(–3, 0.02)
–2	0.06	(–2, 0.06)
–1	0.25	(–1, 0.25)
0	1	(0, 1)
1	4	(1, 4)
2	16	(2, 16)
3	64	(3, 64)



(ii) $y = 5^{-x}$

Sol. $y = 5^{-x}$

$y = 5^{-(-2)} = 5^2 = 25$

$y = 5^{-(-1)} = 5^1 = 5$

$y = 5^{-(0)} = 5^0 = 1$

$y = 5^{-(1)} = 5^{-1} = \frac{1}{5} = 0.2$

$y = 5^{-(2)} = 5^{-2} = \frac{1}{5^2} = \frac{1}{25} = 0.04$

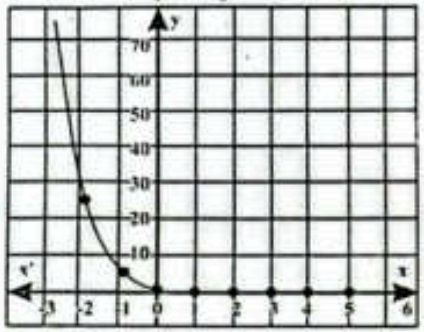
$y = 5^{-3} = 5^{-3} = \frac{1}{5^3} = \frac{1}{125} = 0.008$

$y = 5^{-4} = 5^{-4} = \frac{1}{5^4} = \frac{1}{625} = 0.0001$

The following table shows the several values of x and the given function is evaluated at these values.

x	y	(x, y)
–2	25	(–2, 25)
–1	5	(–1, 25)
0	1	(0, 1)
1	0.2	(1, 0.2)
2	0.04	(2, 0.04)
3	0.008	(3, 0.008)
4	0.0001	(4,0.0001)

Graph of $y = 5^{-x}$



(iii) $y = \frac{1}{x-3}, x \neq 3$

Sol. $y = \frac{1}{x-3}$ $x \neq 3$

$y = \frac{1}{-8-3} = \frac{1}{-11} = -0.09$

$y = \frac{1}{-6-3} = \frac{1}{-9} = -0.11$

$y = \frac{1}{-4-3} = \frac{1}{-7} = -0.14$

$y = \frac{1}{-2-3} = \frac{1}{-5} = -0.2$

$y = \frac{1}{-0-3} = \frac{1}{-3} = -0.33$

$y = \frac{1}{2-3} = \frac{1}{-1} = -1$

$y = \frac{1}{4-3} = \frac{1}{1} = 1$

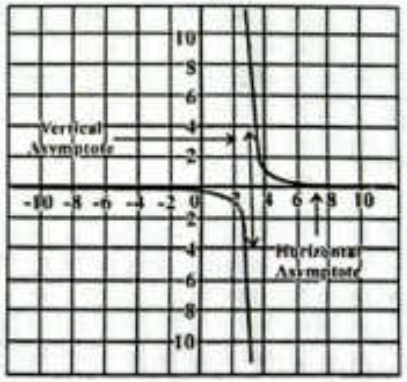
$y = \frac{1}{6-3} = \frac{1}{3} = 0.33$

$y = \frac{1}{8-3} = \frac{1}{5} = 0.2$

The following table shows the several values of x and the given function is evaluated at these values.

$y = \frac{1}{x-3}, x \neq 3$

x	y	(x, y)
–8	–0.09	(–8, –0.09)
–6	–0.11	(–6, –0.11)
–4	–0.14	(–4, –0.14)
–2	–0.2	(–2, –0.2)
0	–0.33	(0, –0.33)
2	–1	(2, –1)
4	1	(4, 1)
6	0.33	(6, 0.33)
8	0.2	(8, 0.2)



The following table shows the several values of x and the given function is evaluated at these values.

(iv) $y = \frac{2}{x} + 3, x \neq 0$

Sol. $y = \frac{2}{x} + 3, x \neq 0$

$y = \frac{2}{-10} + 3 = -0.2 + 3 = 2.8$

$y = \frac{2}{-8} + 3 = -0.25 + 3 = 2.75$

$y = \frac{2}{-6} + 3 = -0.33 + 3 = 2.67$

$y = \frac{2}{-4} + 3 = -0.5 + 3 = 2.50$

$y = \frac{2}{-2} + 3 = -1 + 3 = 2$

$y = \frac{2}{2} + 3 = 1 + 3 = 4$

$y = \frac{2}{4} + 3 = 0.5 + 3 = 3.5$

$y = \frac{2}{6} + 3 = 0.33 + 3 = 3.33$

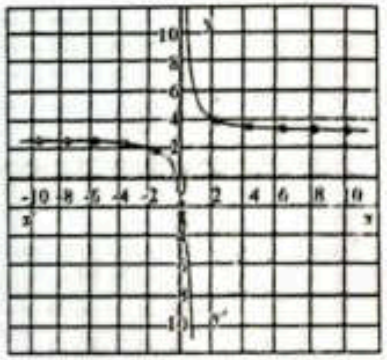
$y = \frac{2}{8} + 3 = 0.25 + 3 = 3.25$

$y = \frac{2}{10} + 3 = 0.2 + 3 = 3.20$

The following table shows the several values of x and the given function is evaluated at these values.

x	y	(x, y)
–10	2.8	(–10, 2.8)
–8	2.75	(–8, 2.75)
–6	2.67	(–6, 2.67)
–4	2.50	(–4, 2.50)
–2	2	(–2, 2)
2	4	(2, 4)
4	3.5	(4, 3.5)
6	3.33	(6, 3.33)
8	3.25	(8, 3.25)
10	3.2	(10, 3.2)

$y = \frac{2}{x} + 3, x \neq 0$



(v) $y = x^{\frac{1}{2}}$

Sol. $y = x^{\frac{1}{2}}$

$y = (1)^{\frac{1}{2}} = 1$

$y = (2)^{\frac{1}{2}} = 1.4$

$y = (3)^{\frac{1}{2}} = 1.7$

$y = (4)^{\frac{1}{2}} = 2$

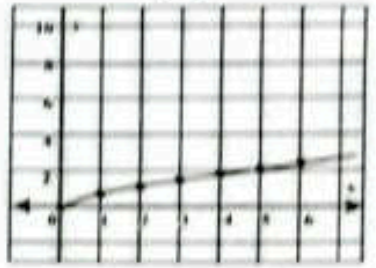
$y = (5)^{\frac{1}{2}} = 2.2$

$y = (6)^{\frac{1}{2}} = 2.4$

The following table shows the several values of x and the given function is evaluated at these values.

$y = x^{\frac{1}{2}}$

x	y	(x, y)
1	1	(1, 1)
2	1.4	(2, 1.4)
3	1.7	(3, 1.7)
4	2	(4, 2)
5	2.2	(5, 2.2)
6	2.4	(6, 2.4)



(vi) $y = 3x^{\frac{1}{2}}$

Sol. $y = 3x^{\frac{1}{2}}$

$y = 3(-3)^{\frac{1}{2}} = 3(-1.44) = -4.3$

$y = 3(-2)^{\frac{1}{2}} = 3(-1.26) = -3.8$

$y = 3(-1)^{\frac{1}{2}} = 3(-1) = -3$

$y = 3(0)^{\frac{1}{2}} = 3(0) = 0$

$y = 3(1)^{\frac{1}{2}} = 3(1) = 3$

$y = 3(2)^{\frac{1}{2}} = 3(1.26) = 3.8$

$y = 3(3)^{\frac{1}{2}} = 3(1.44) = 4.3$

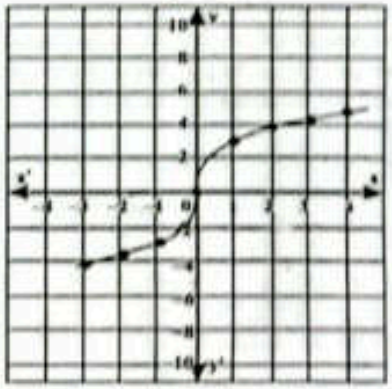
$y = 3(4)^{\frac{1}{2}} = 3(1.59) = 4.8$

$y = 3x^{\frac{1}{2}}$

The following table shows the several values of x and the given function is evaluated at these values.

$y = 3x^{\frac{1}{2}}$

x	y	(x, y)
–3	–4.3	(–3, –4.3)
–2	–3.8	(–2, –3.8)
–1	–3	(–1, –3)
0	0	(0, 0)
1	3	(1, 3)
2	3.8	(2, 3.8)
3	4.3	(3, 4.3)
4	4.8	(4, 4.8)



(vii) $y = 2x^{-2}$

Sol. $y = 2x^{-2}$

$y = 2(-3)^{-2} = 2(0.11) = 0.2 \Rightarrow y = 2(-2)^{-2} = 2(0.25) = 0.5$

$y = 2(-1)^{-2} = 2(1) = 2 \Rightarrow y = 2(1)^{-2} = 2(1) = 2$

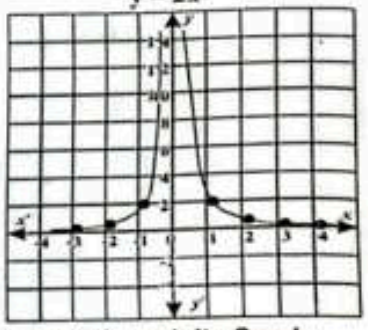
$y = 2(2)^{-2} = 2(0.25) = 0.5 \Rightarrow y = 2(3)^{-2} = 2(0.11) = 0.2$

$y = 2(4)^{-2} = 2(0.06) = 0.12$

The following table shows the several values of x and the given function is evaluated at these values.

x	y	(x, y)
–3	0.2	(–3, 0.2)
–2	0.5	(–2, 0.5)
–1	2	(–1, 2)
1	2	(1, 2)
2	0.5	(2, 0.5)
3	0.2	(3, 0.2)
4	0.12	(4, 0.12)

$y = 2x^{-2}$



Exponential Growth/ Decay of a Practical Phenomenon through its Graph

Exponential growth and decay are widely observed in real-world phenomenon and their graphical representations offer critical insights into these processes. In exponential growth, such as population expansion, compound interest in finance or the spread of infectious diseases, the graph starts slowly but accelerates rapidly as time progresses. The curve increases steeply, showcasing how growth becomes more pronounced with time due to constant proportional changes. Conversely, in exponential decay, observed in cooling of objects or depreciation of assets, the graph starts high and decreases sharply before levelling off, indicating a gradual reduction over time. These graphs are essential for interpreting trends, making predictions and informing decision-making in diverse fields.

Example 12: The population of a village was 753 in 2010. If the population grows according to the equation $p = 753e^{0.03t}$, where p is the number of persons in the population at time t,

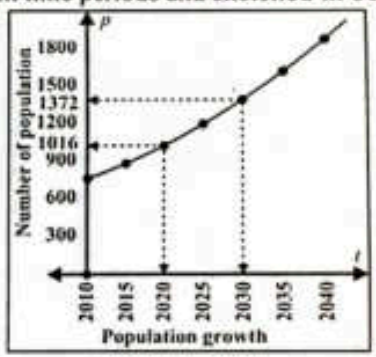
(a) Graph the population equation for t = 0 (in 2010) to t = 30 (in 2040).

(b) From the graph, estimate the population (i) in 2020 and (ii) in 2030.

Solution: (a) The general shape of the exponential is known; however, since the graph is being used for estimations, an accurate graph over the required interval, t = 0 to t = 30, is required.

Calculate a table of values for different time periods and sketched in below figure:

t	p
0	753
5	874.9
10	1016.4
15	1180.9
20	1372.1
25	1594.1
30	1852.1



(b) From graph,

(i) In 2020 (t = 10) the population is 1016 persons.

(ii) In 2030 (t = 20) the population is 1372 persons.

Gradients of Curves by Drawing Tangents

The gradient or slope of a graph at any point is equal to the gradient of the tangent to the curve at that point. Remember that a tangent is a line that just touches a curve only at one point (and doesn't cross it).

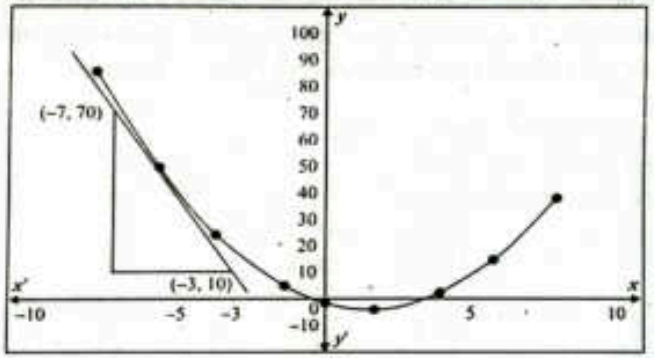
The gradient between two points is defined as:

$$\text{Gradient} = \frac{\text{Change in } y}{\text{Change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 13: Sketch the graph of $y = x^2 - 3x - 2$ for values of x from –8 to 8, draw a tangent line at x = –6 and determine the gradient.

Solution: Calculate the y-values for given values of x. The results are given in the table and sketched in below figure:

x	–8	–6	–4	–2	0	2	4	6	8
y	86	52	26	8	–2	–4	2	16	38



Consider two points (–3, 10) and (–7, 70) on the tangent line.

So, gradient $= \frac{70-10}{-7+3} = -15$. Since the gradient is negative, this indicates that the height of the graph decreases as the value of x increases.

Applications of Graph in Real-Life

Applying concepts of sketching and interpreting graphs to real-life problems enables individuals to visualize and analyse complex relationships, make informed decisions and optimize solutions. In cost and profit analysis, graphing enables businesses to visualize cost-profit relationships, determine break-even points, and optimize production levels.

Example 14: Majid's salary S(x) in rupees is based on the following formula:

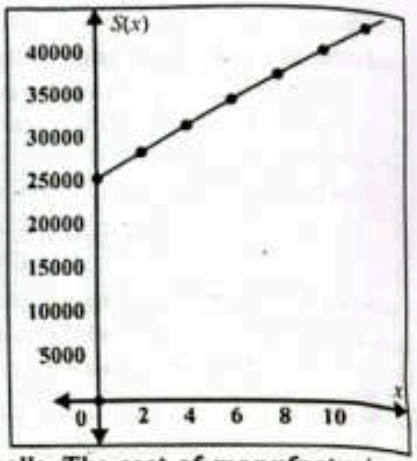
$$S(x) = 25000 + 1500x,$$

where x is the number of years he worked. Sketch and interpret the graph of salary

function for $0 \leq x \leq 10$.

Solution: Table values and graph are given below:

x	$S(x)$
0	25000
2	28000
4	31000
6	34000
8	37000
10	40000



Majid's salary increases linearly with years of service and rises by Rs. 1500 for every year.

Example 15: A company manufactures footballs. The cost of manufacturing x footballs is C(x)=90,000+600x. The revenue from selling x footballs is R(x) = 1,800x. Find the break-even point and determine the profit or loss when 200 footballs are sold. Draw the graphs of both the functions and identify the break-even point.

Solution: Given that

Cost function: $C(x) = 90,000 + 600x$

Revenue function: $R(x) = 1,800x$

The break-even point occurs when $R(x) = C(x)$

$$1800x = 90000 + 600x$$
$$1200x = 90000$$
$$\Rightarrow x = \frac{90000}{1200} = 75$$
$$x = 75$$

So, at the break-even point, 75 footballs are produced or sold.

Next, we find the profit for 150 footballs

When $x = 150$, revenue:

$R(150) = 1,800(150)$

$= \text{Rs. } 270,000$

and $C(150) = 90,000 + 600(150)$

$= \text{Rs. } 180,000$

Now profit: $P(x) = R(x) - C(x)$

Substitute $x = 150$

$P(150) = R(150) - C(150)$

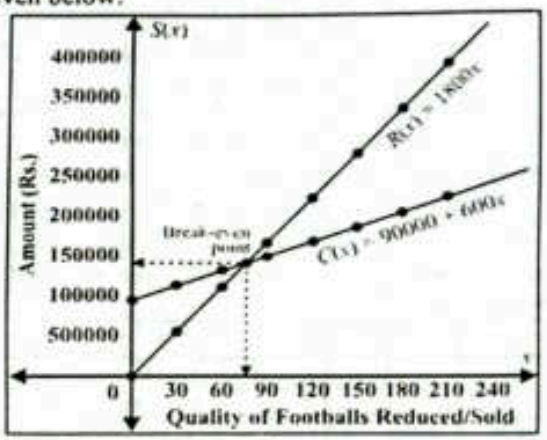
$= \text{Rs. } 270,000 - \text{Rs. } 180,000$

$= \text{Rs. } 90,000$

Thus, a company earns a profit of Rs. 90,000 when selling 150 footballs.

Table values of x, C(x) and R(x) are given below:

x	$C(x)$	$R(x)$
0	90000	0
30	108000	54000
60	126000	108000
90	144000	162000
120	162000	216000
150	180000	27000
180	198000	324000
210	216000	378000



Solved Exercise 10.2

1. Plot the graph of $y = 2x^2 - 4x + 3$ from –1 to 3. Draw tangent at (2, 3) and find the gradient.

Sol. $y = 2x^2 - 4x + 3$

$y = 2(-1)^2 - 4(-1) + 3 = 2(1) + 4 + 3 = 2 + 4 + 3 = 9$

$y = 2(0)^2 - 4(0) + 3 = 2(0) - 0 + 3 = 0 + 0 + 3 = 3$

$y = 2(1)^2 - 4(1) + 3 = 2(1) - 4 + 3 = 2 - 4 + 3 = 1$

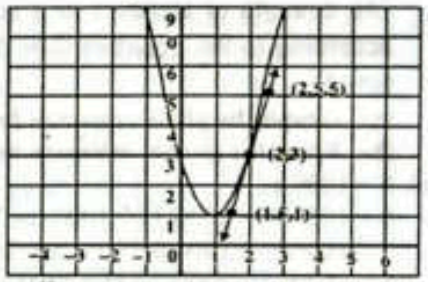
$y = 2(2)^2 - 4(2) + 3 = 2(4) - 8 + 3 = 8 - 8 + 3 = 3$

$y = 2(3)^2 - 4(3) + 3 = 2(9) - 12 + 3 = 18 - 12 + 3 = 9$

The following table shows several values of x and the given function is evaluated at these value.

x	y	(x,y)
–1	9	(–1,9)
0	3	(0,3)
1	1	(1,1)
2	3	(2,3)
3	9	(3,9)

$y = 2x^2 - 4x + 3$



Consider two points (1.5,1) and (2.5,5) on tangent line.

So, gradient $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-1}{2.5-1.5} = \frac{4}{1} = 4$

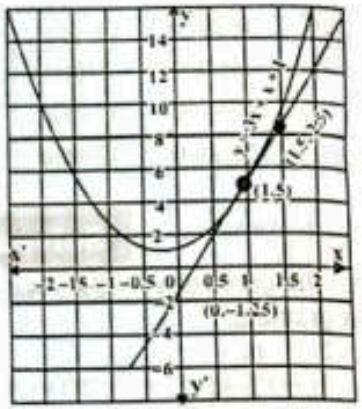
2. Plot the graph of $y = 3x^2 + x + 1$ and draw tangent at (1, 5). Also find gradient of the tangent line at this point.

Sol. $y = 3x^2 + x + 1$

$y=3(-2)^2+(-2)+1 = 3(4)-2+1 = 12-2+1 = 11$
$y=3(-1.5)^2+(-1.5)+1 = 3(2.25)-1.5+1 = 6.75-1.5+1 = 6.25$
$y=3(-1)^2+(-1)+1 = 3(1)-1+1 = 3-1+1 = 3$
$y=3(-0.5)^2+(-0.5)+1 = 3(0.25)-0.5+1 = 0.75-0.5+1 = 1.25$
$y=3(0)^2+0+1 = 3(0)+0+1 = 0+0+1 = 1$
$y=3(0.5)^2+0.5+1 = 3(0.25)+0.5+1 = 0.75+0.5+1 = 2.25$
$y=3(1)^2+1+1 = 3(1)+1+1 = 3+1+1 = 5$
$y=3(1.5)^2+1.5+1 = 3(2.25)+1.5+1 = 6.75+1.5+1 = 9.25$
$y=3(2)^2+2+1 = 3(4)+2+1 = 12+2+1 = 15$

The following table shows the several values of x and the given function is evaluated at these values.

x	y	(x,y)
–2	11	(–2,11)
–1.5	6.25	(–1.5,6.25)
–1	3	(–1,3)
–0.5	1.25	(–0.5,1.25)
0	1	(0,1)
0.5	2.25	(0.5,2.25)
1	5	(1.5)
1.5	9.25	(1.5,9.25)
2	15	(2.15)



Consider two points (0,–1.25) and (1.5,9.25) on tangent line.

So, gradient $= \frac{y_2-y_1}{x_2-x_1} = \frac{9.25-(-1.25)}{1.5-0} = \frac{9.25+1.25}{1.5} = \frac{10.5}{1.5} = 7$

3. The strength of students in a school was 1000 in 2016. If the strength decay according to the equation $S = 1000\,e^{-t}$, where S is the number of students at time t.

(a) Graph the given equation for t = 0 (in 2016) to t = 9 (in 2025).

(b) From the graph, estimate the student's strength in 2019 and in 2023.

Sol. $S = 1000\,e^{-t}$

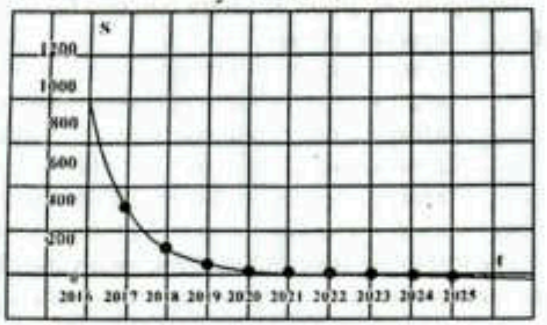
$S = 1000e^{-0} = 1000(1) = 1000$ (in 2016)
$S = 1000e^{-1} = 1000(0.37) = 370$ (in 2017)
$S = 1000e^{-2} = 1000(0.14) = 140$ (in 2018)
$S = 1000e^{-3} = 1000(0.05) = 50$ (in 2019)
$S = 1000e^{-4} = 1000(0.02) = 20$ (in 2020)
$S = 1000e^{-5} = 1000(0.007) = 7$ (in 2021)
$S = 1000e^{-6} = 1000(0.0025) = 2.5$ (in 2022)
$S = 1000e^{-7} = 1000(0.0009) = 0.9$ (in 2023)
$S = 1000e^{-8} = 1000(0.0003) = 0.3$ (in 2024)

$S = 1000e^{-9} = 1000(0.0001) = 0.1$ (in 2025)

The following table show the several values of x and the given function is evaluated at these values.

t	s	(t,s)
0	1000	(0,1000)
1	370	(1,370)
2	140	(2,140)
3	50	(3,50)
4	20	(4,20)
5	7	(5,7)
6	2.5	(6,2.5)
7	0.9	(7,0.9)
8	0.3	(8,0.3)
9	0.1	(9,0.1)

$y = 100e^{-t}$



From the graph students strength in 2019 is approximitly 50, and in 2023 approximitly 1.

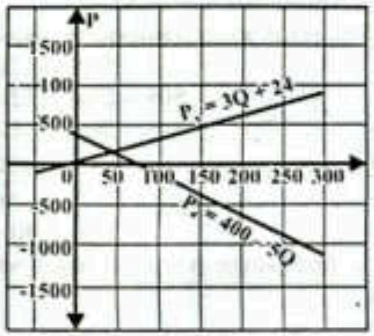
4. The demand and supply functions for a product are given by the equations $P_d = 400 - 5Q$, $P_s = 3Q + 24$:

Plot the graph of each function over the interval Q = 0 to Q = 300.

$P_d = 400 - 5Q$	$P_s = 3Q + 24$
$P_d = 400 - 5(0) = 400 - 0 = 400$	$P_s = 3(0) + 24 = 0 + 24 = 24$
$P_d = 400 - 5(50) = 400 - 250 = 150$	$P_s = 3(50) + 24 = 150 + 24 = 174$
$P_d = 400 - 5(100) = 400 - 500 = -100$	$P_s = 3(100) + 24 = 300 + 24 = 324$
$P_d = 400 - 5(150) = 400 - 750 = -350$	$P_s = 3(150) + 24 = 450 + 24 = 474$
$P_d = 400 - 5(200) = 400 - 1000 = -600$	$P_s = 3(200) + 24 = 600 + 24 = 624$
$P_d = 400 - 5(250) = 400 - 1250 = -850$	$P_s = 3(250) + 24 = 750 + 24 = 774$
$P_d = 400 - 5(300) = 400 - 1500 = -1100$	$P_s = 3(300) + 24 = 900 + 24 = 924$

The following table shows the several values of x and the given functions are given in evaluated at these values.

x	Pd	Ps
0	400	24
5	150	174
100	–100	324
150	–350	474
200	–600	624
250	–850	774
300	–1100	924

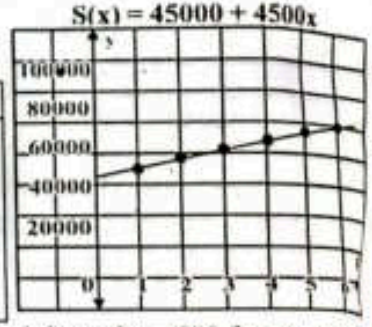


5. Shahid's salary S(x) in rupees is based on the following formula:
S(x) = 45000 + 4500x,
where x is the number of years he has been with the company. Sketch and interpret the graph of salary function for $0 \le x \le 5$.

Sol. $S_{(x)} = 45000 + 4500x$
$S_{(x)} = 4500 + 4500(0) = 45000 + 0 = 45000$
$S_{(x)} = 4500 + 4500(1) = 45000 + 4500 = 49500$
$S_{(x)} = 4500 + 4500(2) = 45000 + 9000 = 54000$
$S_{(x)} = 4500 + 4500(3) = 45000 + 13500 = 58500$
$S_{(x)} = 4500 + 4500(2) = 45000 + 18000 = 63000$
$S_{(x)} = 4500 + 4500(4) = 45000 + 22500 = 67500$

The following table shows the several values of x and the given function is evaluated at these values.

x	s	(x,s)
0	45000	(0,45000)
1	49500	(1,49500)
2	54000	(2,54000)
3	58500	(3,58500)
4	63000	(4,63000)
5	67500	(5,67500)



Shahid's Salary increases linearly with years of service and rises y Rs. 4500 for every yea

6. A company manufactures school bags. The cost function of producing x bags is C(x) = 1200 + 20x and the revenue from selling x bags is R(x)=50x.
(a) Find the break-even point.
(b) Determine the profit or loss when 250 bags are sold.
(c) Plot the graphs of both the functions and identify the break-even point.

Sol. $C(x) = 1200 + 20x$, $R(x) = 50x$
Cost function: $C(x) = 1200 + 20x$
Revenue function $R(x) = 50x$
The break-even point occurs when $R(x) = C(x)$

$$50x = 1200 + 20x$$
$$50x - 20x = 1200$$
$$30x = 1200$$
$$x = \frac{1200}{30}$$
$$x = 40$$

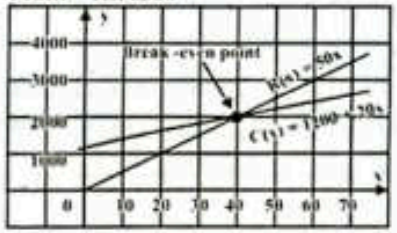
So break-even point, 40 bages sold.
when
$x = 250$
$R(x) = 50(250)$

$= Rs. 12500$
$C(x) = 1200 + 20(250) = 1200 + 5000 = Rs. 6200$
$Profit = R(x) - C(x)$
$= 12500 - 6200 = Rs. 6300$

$C(x) = 1200 + 20x$ $\quad R(x) = 50x$
$C(x) = 1200 + 20(0) = 1200 + 0 = 1200$ $\quad R(x) = 50(0) = 0$
$C(x) = 1200 + 20(10) = 1200 + 200 = 1400$ $\quad R(x) = 50(10) = 500$
$C(x) = 1200 + 20(20) = 1200 + 400 = 1600$ $\quad R(x) = 50(20) = 1000$
$C(x) = 1200 + 20(30) = 1200 + 600 = 1800$ $\quad R(x) = 50(30) = 1500$
$C(x) = 1200 + 20(40) = 1200 + 800 = 2000$ $\quad R(x) = 50(40) = 2000$
$C(x) = 1200 + 20(50) = 1200 + 1000 = 2200$ $\quad R(x) = 50(50) = 2500$
$C(x) = 1200 + 20(60) = 1200 + 1200 = 2400$ $\quad R(x) = 50(60) = 3000$
$C(x) = 1200 + 20(70) = 1200 + 1400 = 2600$ $\quad R(x) = 50(70) = 3500$

x	C(x)	R(x)
0	1200	0
10	1400	500
20	1600	1000
30	1800	1500
40	2000	2000
50	2200	2500
60	2400	3000
70	2600	3500

(a) x = 40 bags
(b) Profit = Rs. 6300
(c)

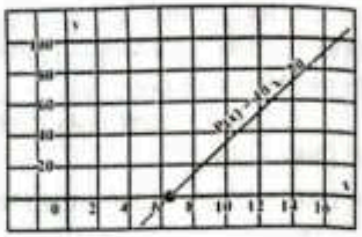


7. A newspaper agency fixed cost of Rs. 70 per edition and marginal printing and distribution costs of Rs. 40 per copy. Profit function is p(x) = 10 x – 70, where x is the number of newspapers., Plot the graph and find profit for 500 newspapers.

Sol. $P(x) = 10x - 70$
$P(x) = 10(7) - 70 = 70 - 70 = 0$
$P(x) = 10(8) - 70 = 80 - 70 = 10$
$P(x) = 10(10) - 70 = 100 - 70 = 30$
$P(x) = 10(12) - 70 = 120 - 70 = 50$
$P(x) = 10(14) - 70 = 140 - 70 = 70$
$P(x) = 10(16) - 70 = 160 - 70 = 90$

The following table shows the several values of x and the given function is evaluated at these values.

x	y = p(x)	(x,y)
7	0	(7,0)
8	10	(8,10)
10	30	(10,30)
12	50	(12,50)
14	70	(14,70)
16	90	(16,90)



Now, profit for 500 for 500 newspaper

$P(x) = 10(500) - 70$

$= 5000 - 70 = Rs.\ 4930$

8. Ali manufactures expensive shirts for sale to a school. Its cost (in rupees) for x shirts is $C(x) = 1500 + 10x + 0.2x^2$, $0 \le x \le 150$. Plot the graph and find the cost of 200 shirts.

Sol. $C(x) = 1500 + 10x + 0.2x^2$

$C(x) = 1500 + 10(0) + 0.2(0)^2 = 1500 + 0 + 0 = 1500$

$C(x) = 1500 + 10(50) + 0.2(50)^2 = 1500+500+0.2\,(2500)=2000+500=2500$

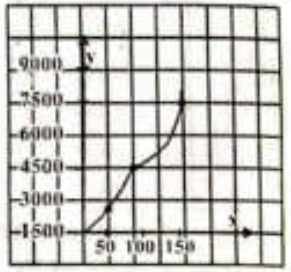
$C(x) = 1500 + 10(100) + 0.2(100)^2 = 1500+1000+0.2\,(10000)=2500+2000=4500$

$C(x) = 1500 + 10(150) + 0.2(150)^2 = 1500+1500+0.2\,(22500)=3000+4500=7500$

$C(x) = 1500 + 10x + 0.2x^2$

The following table shows several values of x and the given function is evaluated at these values.

x	y = cm	(x,y)
0	1500	(0,1500)
50	2500	(50,2500)
100	4500	(100,4500)
150	7500	(150,7500)



Now, for the cost of 200 shirts

$Cx = 1500 + 10(200) + 0.2\,(200)^2$

$= 1500 + 2000 + 0.2\,(40000)$

$= 1500 + 2000 + 8000 = Rs.\ 11500$

Review Exerciese 10

1. Four options are given against each statement. Encircle the correct option.

(i) x = 5 represents:

(a) x-axis (b) y-axis (c) line ∥ to x-axis (d) line ∥ to y-axis

(ii) Slope of the line y = 5x + 3 is:

(a) 3 (b) –3 (c) 5 (d) –5

(iii) The y- intercepts of y = –2x –1 is:

(a) –2 (b) 2 (c) –1 (d) 1

(iv) The graph of $y = x^3$, cuts the x-axis at:

(a) x = 0 (b) x = 1 (c) x = –1 (d) x = 2

(v) The graph of 3^x represents:

(a) growth (b) decay (c) both(a)and(b) (d)a line

(vi) The graph of $y = -x^2 + 5$ opens:

(a) upward (b) downward (c) left side (d) right side

(vii) The graph of $y = x^2 - 9$ opens:

(a) upward (b) downward (c) left side (d) right side

(viii) $y = 5^x$ is _____ function.

(a) linear (b) quadratic (c) cubic (d) exponential

(ix) Reciprocal function is:

(a) $y = 7^x$ (b) $y = \frac{2}{x}$ (c) $y = 2x^2$ (d) $y = 5x^3$

(x) $y = -3x^3 + 7$ is _____ function.

(a) exponential (b) cubic (c) linear (d) reciprocal

Answers:

i. line ∥ to y-axis **ii.** 5 **iii.** –1 **iv.** x = 0

v. growth **vi.** downward **vii.** upward **viii.** exponential

ix. $y = \frac{2}{x}$ **x.** cubic

2. Plot the graph of the following functions:

(i) $y = 3^{-x}$ for x from –2 to 4

(i) Sol.

$y = 3^{-x}$

$y = 3^{-(-2)} = 3^2 = 9$

$y = 3^{-(-1)} = 3^1 = 3$

$y = 3^{-0} = 3^0 = 1$

$y = 3^{-1} = \frac{1}{3} = 0.3$

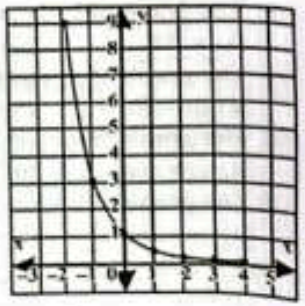
$y = 3^{-2} = \frac{1}{3^2} = \frac{1}{9} = 0.1$

$y = 3^{-3} = \frac{1}{3^3} = \frac{1}{27} = 0.04$

$y = 3^{-4} = \frac{1}{3^4} = \frac{1}{81} = 0.01$

The following tables shows several values of x and the given function is evaluated at these values.

x	y	(x,y)
–2	9	(–2,9)
–1	3	(–1,3)
0	1	(0,1)
1	0.3	(1,0.3)
2	0.1	(2,0.1)
3	0.04	(3,0.04)
4	0.01	(4,0.01)



(ii) $y = \frac{2}{x}, x \neq 0$

Sol. $y = \frac{2}{x}$ _____(i)

Put x = –8 in equation (i)

$y = \frac{2}{-8} = -0.25$

Put x = –6 in equation (i)

$y = \frac{2}{-6} = -0.33$

Put x = –4 in equation (i)

$y = \frac{2}{-4} = -0.5$

Put x = –0.7 in equation (i)

$y = \frac{2}{-0.7} = -2.9$

Put x = –0.5 in equation (i)

$y = \frac{2}{-0.5} = -4$

Put x = –0.4 in equation (i)

$y = \frac{2}{-0.4} = -5$

Put x = –1 in equation (i)

$y = \frac{2}{-1} = -2$

Put x = 0.2 in equation (i)

$y = \frac{2}{0.2} = 10$

Put x = 0.3 in equation (i)

$y = \frac{2}{0.3} = 6.67$

Put x = 0.4 in equation (i)

$y = \frac{2}{0.4} = 5$

Put x = 0.5 in equation (i)

$y = \frac{2}{0.5} = 4$

Put x = 0.7 in equation (i)

$y = \frac{2}{0.7} = 2.9$

Put x = 1 in equation (i)

$y = \frac{2}{1} = 2$

Put x = 4 in equation (i)

$y = \frac{2}{4} = 0.5$

Put x = 6 in equation (i)

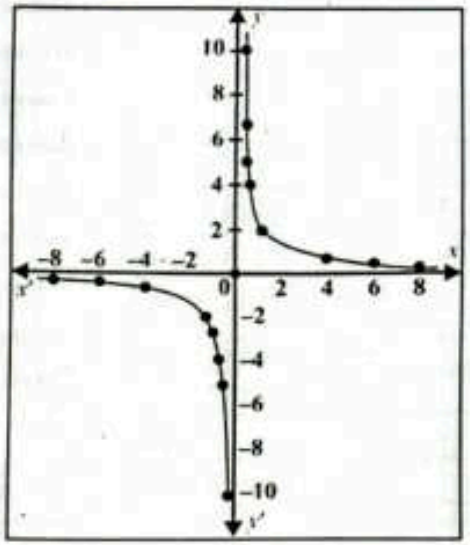
$y = \frac{2}{6} = 0.33$

Put x = 8 in equation (i)

$y = \frac{2}{8} = 0.25$

The following table shows several values of x and the given function is evaluated at these values:

x	y
–8	–0.25
–6	–0.33
–4	–0.5
–0.7	–2.9
–0.5	–4
–0.4	–5
–0.2	–10
–1	–2
0.2	10
0.4	5
0.5	4
0.7	2.9
1	2
4	0.5
6	0.33
8	0.25



1 **Sales for a new magazine are expected to grow according to the equation:**

$S = 200000 (1 - e^{-0.05t})$, where t is given in weeks.

(a) Plot graph of sales for the first 50 weeks.

(b) Calculate the number of magazines sold, when t = 5 and t = 35.

Sol. $S = 200000 (1-e^{-0.05t})$

$S = 200000 (1-e^{-0.05(5)}) = 20000 (1-0.7788) = 200000 (0.2212) = 44240$

$S = 200000 (1-e^{-0.05(10)}) = 20000 (1-0.6) = 200000 (0.4) = 80000$

$S = 200000 (1-e^{-0.05(15)}) = 20000 (1-0.4724) = 200000 (0.5276) = 105550$

$S = 200000 (1-e^{-0.05(20)}) = 20000 (1-0.3679) = 200000 (0.6321) = 126420$

$S = 200000 (1-e^{-0.05(25)}) = 20000 (1-0.2865) = 200000 (0.7135) = 142700$

$S = 200000 (1-e^{-0.05(30)}) = 20000 (1-0.2231) = 200000 (0.7769) = 155380$

$S = 200000 (1-e^{-0.05(35)}) = 20000 (1-0.1738) = 200000 (0.8262) = 165240$

$S = 200000 (1-e^{-0.05(40)}) = 20000 (1-0.1353) = 200000 (0.8647) = 172940$

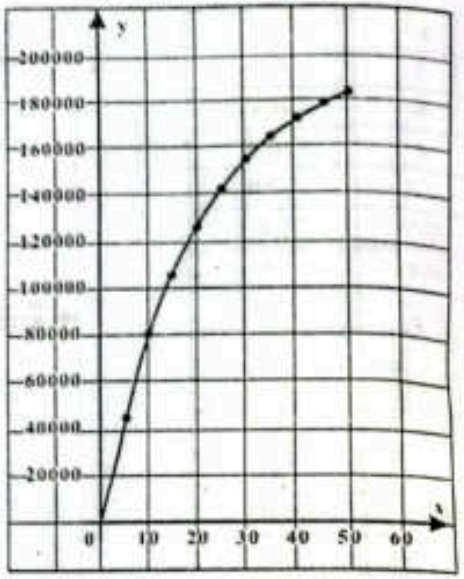
$S = 200000 (1-e^{-0.05(45)}) = 20000 (1-0.1054) = 200000 (0.8946) = 178920$

$S = 200000 (1-e^{-0.05(50)}) = 20000 (1-0.0821) = 200000 (0.9179) = 183580$

The following table shows several values of x and the given function is evaluated at these values.

x = t	y(5)	(x,y) = (t,s)
5	44220	(5,44220)
10	80000	(10,80000)
15	105550	(15,105550)
20	126420	(20,126420)
25	142700	(25,142700)
30	155380	(30,155380)
35	165240	(35,165240)
40	172940	(40,172940)
45	88820	(45,178820)
50	183580	(50,183580)

$S = 200000\,(1 - e^{-0.05t})$



From the above table

For t = 5 , S = 44220

For t = 35 , S = 165240

4. Plot the graph of following for x from -5 to 5:

(i) $y = x^2 - 3$ (ii) $y = 15 - x^2$

Sol. (i) $y = x^2 - 3$

$y = (-5)^2 - 3 = 25 - 3 = 22$

$y = (-4)^2 - 3 = 16 - 3 = 13$

$y = (-3)^2 - 3 = 9 - 3 = 6$

$y = (-2)^2 - 3 = 4 - 3 = 1$

$y = (-1)^2 - 3 = 1 - 3 = -2$

$y = (0)^2 - 3 = 0 - 3 = -3$

$y = (1)^2 - 3 = 1 - 3 = -2$

$y = (2)^2 - 3 = 4 - 3 = 1$

$y = (3)^2 - 3 = 9 - 3 = 6$

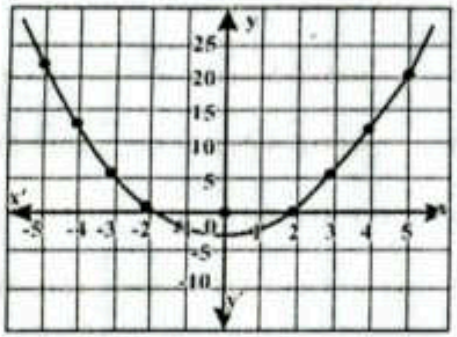
$y = (4)^2 - 3 = 16 - 3 = 13$

$y = (5)^2 - 3 = 25 - 3 = 22$

The following table shows several values of x and the given function is evaluated at these points.

x	y	(x,y)
–5	22	(–5,22)
–4	13	(–4,13)
–3	6	(–3,6)
–2	1	(–2,1)
–1	–2	(–1,–2)
0	–3	(0,–3)
1	–2	(1,–2)
2	1	(2,1)
3	6	(3,6)
4	13	(4,13)
5	22	(5,22)

$y = x^2 - 3$



Sol. (ii) $y = 15 - x^2$

$y = 15 - (-5)^2 = 15 - 25 = -10$

$y = 15 - (-4)^2 = 15 - 16 = -1$

$y = 15 - (-3)^2 = 15 - 9 = 6$

$y = 15 - (-2)^2 = 15 - 4 = 11$

$y = 15 - (-1)^2 = 15 - 1 = 14$

$y = 15 - (0)^2 = 15 - 0 = 15$

$y = 15 - (1)^2 = 15 - 1 = 14$

$y = 15 - (2)^2 = 15 - 4 = 11$

$y = 15 - (3)^2 = 15 - 9 = 6$

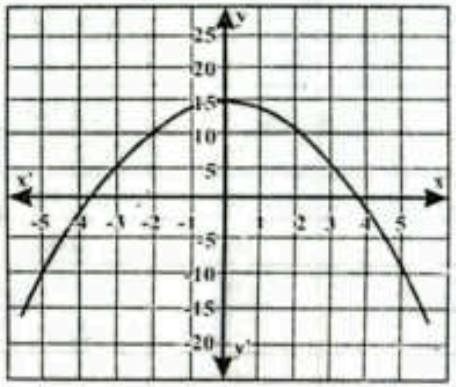
$y = 15 - (4)^2 = 15 - 16 = -1$

$y = 15 - (5)^2 = 15 - 25 = -10$

The following table shows several values of x and the given function is evaluated at these values.

x	y	(x,y)
–5	–10	(–5,–10)
–4	–1	(–4,–1)
–3	6	(–3,6)
–2	11	(–2,11)
–1	14	(–1,14)
0	15	(0,15)
1	14	(1,14)
2	11	(2,11)
3	6	(3,6)
4	–1	(4,–1)
5	–10	(5,–10)

$y = 15 - x^2$



5. Plot the graph of $y = \frac{1}{2}(x+4)(x-1)(x-3)$ from –5 to 4.

Sol. $y = \frac{1}{2}(x+4)(x-1)(x-3)$

$y = \frac{1}{2}(-5+4)(-5-1)(-5-3) = \frac{1}{2}(-1)(-6)(-8) = -\frac{48}{2} = -24$

$y = \frac{1}{2}(-4+4)(-4-1)(-4-3) = \frac{1}{2}(0)(-5)(-7) = 0$

$y = \frac{1}{2}(-3+4)(-3-1)(-3-3) = \frac{1}{2}(1)(-4)(-6) = \frac{24}{2} = 12$

$y = \frac{1}{2}(-2+4)(-2-1)(-2-3) = \frac{1}{2}(2)(-3)(-5) = \frac{30}{2} = 15$

$y = \frac{1}{2}(-1+4)(-1-1)(-1-3) = \frac{1}{2}(3)(-2)(-4) = \frac{24}{2} = 12$

$y = \frac{1}{2}(1+4)(1-1)(1-3) = (5)(0)(-2) = 0$

$y = \frac{1}{2}(2+4)(2-1)(2-3) = \frac{1}{2}(6)(1)(-1) = \frac{-6}{2} = -3$

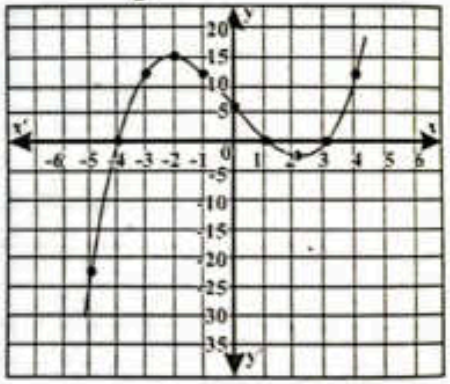
$y = \frac{1}{2}(3+4)(3-1)(3-3) = \frac{1}{2}(7)(2)(0) = 0$

$y = \frac{1}{2}(4+4)(4-1)(4-3) = \frac{1}{2}(8)(3)(1) = \frac{24}{2} = 12$

The following table shows saveral values of x and the given function is evaluated at these values.

x	y	(x,y)
–5	–24	(–5,–24)
–4	0	(–4,0)
–3	12	(–3,12)
–2	15	(–2,15)
–1	12	(–1,12)
0	6	(0,6)
1	0	(1,0)
2	–3	(2,–3)
3	0	(3,0)
4	12	(4,12)

$y = \frac{1}{2}(x+4)(x-1)(x-3)$



6. The supply and demand functions for a particular market are given by the equations:

$P_s = Q^2 + 5$ and $P_d = Q^2 - 10Q$, where P represents price and Q represents quantity.

Sketch the graph of each function over the interval Q = –20 to Q = 20.

Sol. $P_s = Q^2 + 5$, $P_d = Q^2 - 10Q$

$P_s = Q^2 + 5$

$P_s = (-20)^2 + 5 = 400 + 5 = 405$

$P_s = (-15)^2 + 5 = 225 + 5 = 230$

$P_s = (-10)^2 + 5 = 100 + 5 = 105$

$P_s = (-5)^2 + 5 = 25 + 5 = 30$

$P_s = (0)^2 + 5 = 0 + 5 = 5$

$P_s = (5)^2 + 5 = 25 + 5 = 30$

$P_s = (10)^2 + 5 = 100 + 5 = 105$

$P_s = (15)^2 + 5 = 225 + 5 = 230$

$P_s = (20)^2 + 5 = 400 + 5 = 405$

$P_d = Q^2 - 10Q$

$P_d = (-20)^2 - 10(-20) = 400 + 200 = 600$

$P_d = (-15)^2 - 10(-15) = 225 + 150 = 375$

$P_d = (-10)^2 - 10(-10) = 100 + 100 = 200$

$P_d = (-5)^2 - 10(-5) = 25 + 50 = 75$

$P_d = (0)^2 - 10(0) = 0 - 0 = 0$

$P_d = (5)^2 - 10(5) = 25 - 50 = -25$

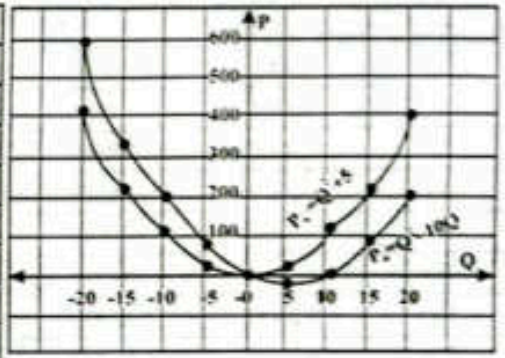
$P_d = (10)^2 - 10(10) = 100 - 100 = 0$

$P_d = (15)^2 - 10(15) = 225 - 150 = 75$

$P_d = (20)^2 - 10(20) = 400 - 200 = 200$

The following table shows the several values of x and the given functions are evaluated at there value.

x	P_s	P_d	(x,P_s)	(x,P_d)
–20	405	600	(–20,405)	(–20,600)
–15	230	375	(–15,230)	(–15,375)
–10	105	200	(–10,105)	(–10,200)
–5	30	75	(–5,30)	(–5,75)
0	5	0	(0,5)	(0,0)
5	30	–25	(5,30)	(5,–25)
10	105	0	(10,105)	(10,0)
15	230	75	(15,230)	(15,75)
20	405	200	(20,405)	(20,200)



7. A television manufacturer company make 40 inches LEDs. The cost of manufacturing x LEDs is C(x) = 60,000 + 250x and the revenue from selling x LEDs is R(x) = 1200x. Find the break-even point and find the profit or loss when 100 LEDs are sold. Identify the break-even point graphically.

Sol. Cost Function C (x) = 60000 + 250x

Revenue Function R(x) = 1200x

The break-even point when R(x) = C(x)

$1200x = 60000 + 250x$

$1200x - 250x = 6000$

$$950x = 60000$$

$$x = \frac{60000}{950}$$

$$x = \frac{1200}{19} = 63.16 \approx 63$$

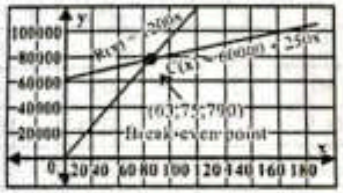
So, break-even point is a appromanily 63 LEDs Sold.

Now,

$C(x) = 60000 + 250x$	$R(x) = 1200(x)$
$C(x) = 60000 + 250(0) = 60000 + 0 = 60000$	$R(x) = 1200(0) = 0$
$C(x) = 60000 + 250(20) = 60000 + 5000 = 65000$	$R(x) = 1200(20) = 24000$
$C(x) = 60000 + 250(40) = 60000 + 10000 = 70000$	$R(x) = 1200(40) = 48000$
$C(x) = 60000 + 250(60) = 60000 + 15000 = 75000$	$R(x) = 1200(60) = 72000$
$C(x) = 60000 + 250(80) = 60000 + 20000 = 80000$	$R(x) = 1200(80) = 96000$
$C(x) = 60000 + 250(100) = 60000 + 25000 = 85000$	$R(x) = 1200(100) = 120000$
$C(x) = 60000 + 250(120) = 60000 + 30000 = 90000$	$R(x) = 1200(120) = 144000$
$C(x) = 60000 + 250(140) = 60000 + 35000 = 95000$	$R(x) = 1200(140) = 168000$
$C(x) = 60000 + 250(160) = 60000 + 40000 = 100000$	$R(x) = 1200(160) = 192000$
$C(x) = 60000 + 250(180) = 60000 + 45000 = 105000$	$R(x) = 1200(180) = 216000$

The following table shows the several values of x and the given function are evaluated at these values.

x	C(x)	R(x)
0	60000	0
20	65000	24000
40	70000	48000
60	75000	72000
80	80000	96000
100	85000	120000
120	90000	144000
140	95000	168000
160	100000	192000
180	105000	216000



Profit Formula $P(x) = R(x) - C(x)$

Profit of 100 LEDs

$$P(100) = 1200x - 60000 + 250(x)$$

$$1200(100) - 60000 + 250(100)$$

$$= 120000 - 60000 + 25000$$

$$= 120000 - 85000$$

$$= \text{Rs. } 35000$$

Objective Type Questions

Additional MCQ's

Multiple Choice Questions prepared in the light of new Examination Techniques (Knowledge, Understanding, Application, Analytical & Conceptual)

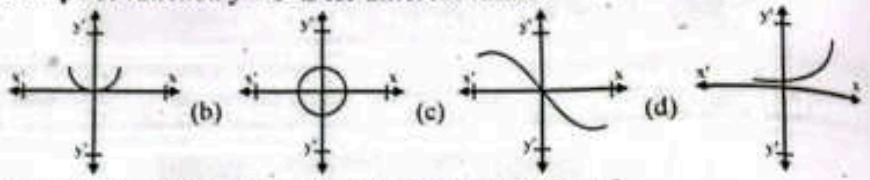
10.1 Functions and Graphs

Encircle the correct option.

1. **In function one variable y depends another variable x. This relationship is expressed as:**
   (a) $y = f(x)$ (b) $f = xy$ (c) $f = \frac{x}{y}$ (d) $f = x + y$
2. **In $y = f(x)$, x is variable:**
   (a) dependent (b) Independent
   (c) It is not a variable (d) None of above
3. **General form of linear equation is:**
   (a) $f(c) = x + m$ (b) $f(m) + x + c$ (c) $f(x) = mx + c$ (d) None of above
4. **The graph of $y = 2x - 1$ is:**
   (a) straight line (b) curve (c) zig Zag (d) None of above
5. **In $y = 2x - 1$, the slope is:**
   (a) negative (b) positive (c) neutral (d) None of above
6. **Equation of quadratic form is:**
   (a) $y^2 = ax + bx + c$ (b) $ax + by + c = 0$ (c) $y = ax + by + c$ (d) $y = ax^2 + bx + c$
7. **The graph of $y = x^2$ represent:**
   (a) parabola (b) hyperbola (c) circle (d) strength line
8. **The graph of $y = -x^2$ shows.**
   (a) Hyperbala (b) Parabola (c) circle (d) straright line
9. **The standard form of Polynomial function of degree 3 is:**
   (a) $y = ax + bx + cx + d$ (b) $y = ax^2 + bx + d$
   (c) $y = ax^3 \cdot bx^2 + cx + d$ (d) None of above
10. **In $y = -x^3$ the shape of graph is:**
    (a) circle (b) curve (c) straight line (d) none of above
11. **If $y = -x^3$, $-3 \le x \le 3$, The curve passes through:**
    (a) origin (b) x-axis (c) y-axis (d) no graph exist.
12. **The reciprocal function is e function of the form.**
    (a) $y = a x + b$ (b) $y = ax^2 + bx + c$ (c) $y = \frac{a}{x}, x \neq 0$ (d) $y = \frac{ax}{by}$
13. **An exponential function is in form:**
    (a) $y = ax^2 + bx + c$ (b) $y = \frac{a}{x}$ (c) $y = ax + b$ (d) $y = Ka^x$

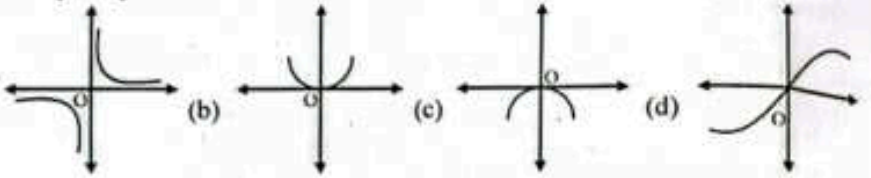
14. The graph $y = 2^x$ represents:
(a) curve (b) circle (c) straight line (d) Hyperbola

15. The shape of function $y = e^x$ is for different values:

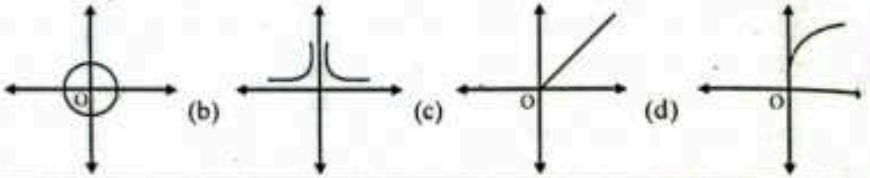


16. In $y = ax^n$, the shope of graph depends upon the value of:
(a) y (b) a (c) n (d) x

17. The shape of $y = x^{-1}$ is for different values is:



18. The graph for $y = 2x^{\frac{1}{3}}$ is:



10.2 Exponential growth / Decay of a practical phenomenon through its graph

19. The gradient of a graph at any point is _____ to the gradient of tangent to the curve at that point.
(a) equal (b) greater than (c) less than (d) zero

20. Gradient = $\frac{\text{Change in y}}{\text{Change in x}}$ = _____.
(a) $\frac{x_2 - x_1}{y_2 - y_1}$ (b) $\frac{y_2 - y_1}{x_2 - x_1}$ (c) $\frac{x_1 - x_2}{y_1 - y_2}$ (d) $\frac{y_1 - y_2}{x_1 - 4_2}$

21. What is the value of $y = x^2 - 3x - 2$ at $x = -6$:
(a) 50 (b) 51 (c) 52 (d) 53

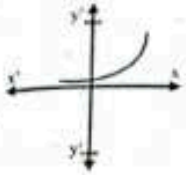
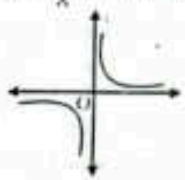
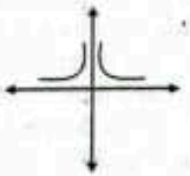
22. What is the value of $S(x) = 25000 + 1500\,x$ at $x = 10$.
(a) 10000 (b) 20000 (c) 30000 (d) 40000

23. What is the value of $p = 753\,e^{0.03t}$ at $t = 5$:
(a) 874.9 (b) 774.9 (c) 674.9 (d) 574.9

24. What is the value of $y = x^2 - 3x - 2$ at $x = 6$.
(a) 14 (b) 16 (c) 18 (d) 20

25. If $S(x) = 25000 + 1500\,x$, then the value at $x = 6$ is:
(a) 32000 (b) 33000 (c) 34000 (d) 35000

Answers:

1. $y = f(x)$ 2. Independent 3. $f(x) = mx + c$ 4. straight line 5. positive
6. $y = ax^2 + bx + c$ 7. parabola 8. Parabola 9. $y = ax^3 + bx^2 + cx + d$
10. curve 11. origin 12. $y = \frac{a}{x}$, $x \neq 0$ 13. $y = Ka^x$ 14. curve
15.  16. n 17.  18. 
19. equal 20. $\frac{y_2 - y_1}{x_2 - x_1}$ 21. 52 22. 40000 23. 874.9
24. 16 25. 34000

Conceptual MCQ's

Encircle the correct option.

1. For visualizing and analyzing relationship between variables, making them essential in understanding various mathematical functions and their application.
(a) graphs (b) lines (c) points (d) None of above

2. One variable y depends on another variable x. This relationship is as:
(a) $f = xy$ (b) $y = f(x)$ (c) $f = \frac{x}{y}$ (d) $f = x + y$

3. The graph of a cubic function is a:
(a) zig zag (b) straight line (c) curve (d) No one

4. An asymptote is a line that a graph approaches but touches:
(a) at one point (b) at two points (c) at three points (d) never

5. The graph of a quadratic function is always :
(a) Parabola (b) Hyperbola (c) Straight line (d) None of above

6. If _____ then the parabola opens upward:
(a) $a = 0$ (b) $a < 0$ (c) $a > 0$ (d) No one

7. If _____ then the parabola opens down word.
(a) $a = 0$ (b) $a < 0$ (c) $a > 0$ (d) No one

8. Gradient = _____
(a) $\Delta x + \Delta y$ (b) $\Delta x - \Delta y$ (c) $\frac{\Delta x}{\Delta y}$ (d) $\frac{\Delta y}{\Delta x}$

Answers:

1. graphs 2. $y = f(x)$ 3. curve 4. never 5. Parabola
6. $a > 0$ 7. $a < 0$ 8. $\frac{\Delta y}{\Delta x}$

Additional Short Questions

Short Answer Questions prepared in the light of new Examination Techniques (Knowledge, Understanding, Application, Analytical & Conceptual)

10.1 Functions and Graphs

1. Why graphs are important?

Ans. Graphs are powerful tools for visualizing and analyzing relationship between variables, making them essential in understanding various mathematical functions and their applications.

2. How functions are essential tool?

Ans. Functions are essential tools for representing real world phenomena using mathematical concepts. A function can be expressed in various forms, including an equation a graph, numerical table or a verbal description. For example the area of a circle depends on it, radius.

3. How we expressed the equation of function?

Ans. One variable y depends on another variable x.

$$y = f(x)$$

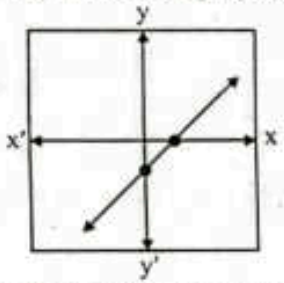
f is function, x is independent variable, y is depended variable.

4. What is linear function?

Ans. A linear function is a mathematical expression that represents a straight li relationship between two variables. Its general form is $f(x) = mx + c$ is slope an is intercept point.

5. If $y = 2x - 1$ then the graph shape is at different points is?

Sol.



6. What is the shape of quadratic function.

Ans. The graph of quadratic function is always a parabola.

7. What do you know about graph $y = x^2$?

Ans. Graph of $y = x^2$ represents parabola passing through origin and opens upward.

8. What do you know about graph of $y = -x^2$?

Ans. Graph of $y = -x^2$ represents parabola parring through origin and opens downward

9. If $y = x^2$ then value of y at $x = 2$ is?

Sol. $y = x^2 \Rightarrow y = (2)^2 = 4$

10. If $y = -x^2$ then find the value of y a t $x = 3$

Sol. $y = -x^2$

$y = -(3)^2 = -9$

11. If $y = 2x^2 - 7x - 9$ at $x = -3$ than find the value of y.

Sol. $y = 2x^2 - 7x - 9$

$y = 2(-3)^2 - 7(-3) - 9$

$y = 18 + 21 - 9$

$y = 39 - 9$

$y = 30$

12. What is the shape of graph of $y = 2x^2 - 7x - a$ at $-3 \le x \le 6$

Ans. The shape of graph is parabola.

13. Define cubic function.

Ans. A cubic function is a type of polynomial function of degree 3. Its standard form is $y = ax^3 + bx^2 + cx + d$.

Here a, b, c are constant, $a \neq 0$

14. What is the value of $y = -x^3$ at $x = 3$

Sol. $y = -x^3$

$y = -(3)^3 = -27$

15. What is the value of $y = 2x^3 - 3x^2 + x - 5$ at $x = 0$

Sol. $y = 2x^3 - 3x^2 + x - 5$

$y = 2(0)^3 - 3(0)^2 + 0 - 5$

$y = 0 - 0 + 0 - 5$

$y = -5$

16. What is reciprocal function?

Ans. A reciprocal function is a function of the form $y = \frac{a}{x}$, a is real number, $x \neq 0$

17. Find the value of $y = \frac{1}{x - 0.5} = \frac{1}{-0.2 - 0.5}$ at $x = -0.2$

Sol. $y = -1.43$

18. What is the value of $y = \frac{1}{x - 0.5}$ at $x = 0.5$ if $x \neq 0.5$

Sol. $y = \frac{1}{x - 0.5} = \frac{1}{0.5 - 0.5}$

$y =$ Undefined.

19. Define exponential function:

Ans. An exponential function is a mathematical function of the form $y = Ka^x$

Here a, K are constant, x is variable and $a > 1$.

20. If $y = 2^x$ at $x = -6$ find the value of y:

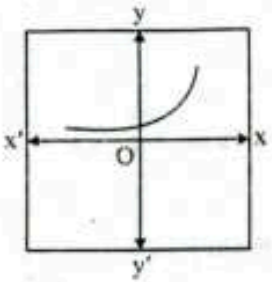
Sol. $y = 2^x$

$$y = 2^{-6} = \frac{1}{2^6} = \frac{1}{2\times2\times2\times2\times2\times2}$$

$$y = \frac{1}{64} = 0.02$$

21. What is the shape of graph of $y = 2^x$ for $-6 \le x \le 6$

Sol.



22. What is the value of $y=e^x$ at x=0

Sol. $y = e^x$, $x = 0$

$y = e^0$

$y = 1$

23. Find the value of $y = x^3$ for $x = 2$

Sol. $y = x^3$ for $x = 2$

$y = 2^3 = 2 \times 2 \times 2 = 8$

24. Find the value of $y = x^{-1}$ for $x = 2$

Sol. $y = x^{-1}$ for $x = 2$

$$y = 2^{-1} = \frac{1}{2} = 0.5$$

25. Find the value of $y = 2x^{\frac{1}{5}}$ for $x = 32$.

Sol. $y = 2x^{\frac{1}{5}}$

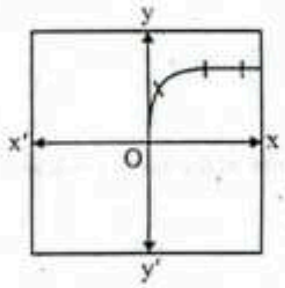
$y = 2(32)^{\frac{1}{5}} \Rightarrow y = 2(2^5)^{\frac{1}{5}}$

$y = 2(2)$

$y = 4$

26. What is the shape of graph of $y = 2x^{\frac{1}{5}}$ at different values.

Sol.



10.2 Exponential growth / decay of a practical phenomenon through its graph.

27. Where exponential trial growth used?

Ans. In exponential growth such as population expansion, compound interest in finance or the spread of infections diseases the graph starts slowly but accelerates rapidly as time progresses.

28. If $P = 753e^{0.03t}$ then at $t = 0$ find the value of p.

Sol. $P = 753\ e^{0.03t}$

$P = 753\ e^{0.03(0)}$

$P = 753\ (e^0)\ \because e^0 = 1$

$P = 753$

29. What do you know about gradient of graph at any point?

Sol. The gradient of graph at any point is equal to the gradient of the tangent to the curve at that point.

30. Define the gradient between two points?

Ans. The gradient between two points is defined as

$$\text{Gradient} = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

31. If $y = x^2 - 3x - 2$, find the value of y for $x = -2$

Sol. $y = x^2 - 3x - 2$

$y = (-2)^2 - 3(-2) - 2$

$y = 4 + 6 - 2$

$y = 8$

32. If $s(x) = 25000 + 1500\ x$ then find the value of s(x) for x=4

Sol. $s(x) = 25000 + 1500\ x$

$s(x) = 25000 + (1500)\ (4)$

$= 25000 + 6000 = 31000$

33. If $c(x) = 90,000 + 600\ x$, then find the value of c(x) for $x = 5$.

Sol. $c(x) = 90,000 + 600\ x$

$= 90,000 + 600\ (5)$

$= 90,000 + 3000 = 93000$

34. What is the shape of graph for $S(x) = 25000\ x$ for different values of $x\ (0 \le x \le 10)$

Ans. The graph is a straight line.

Conceptual Short Questions

1. Which of the essential tools representing real world phenomena using mathematical concepts.

Ans. Functions are essential tools for representing real-world phenomena using

mathematical concepts.

2. From which function parabola made?

Ans. The graph of a quadratic function is always parabola.

3. When the shape of parabola upward?

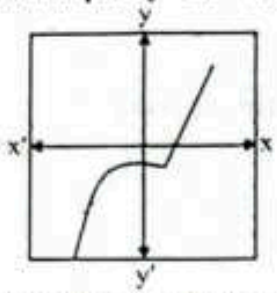
Ans. If $a > 0$ then the parabola opens downward like "$\cup$".

4. When the shape of parabola is downward.

Ans. If $a < 0$ then the parabola opens downward like "$\cap$".

5. What is the shape of $y = 2x^3 - 3x^2 + x - 5$ for $-2 \le x \le 3$.

Sol.

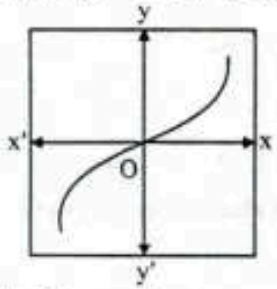


6. What is the form of reciprocal function?

Ans. A reciprocal function is a function of the form $y = \frac{a}{x}$, a is real number and $x \neq 0$

7. What is the graph of $y = x^3$ for $-3 \le x \le 3$.

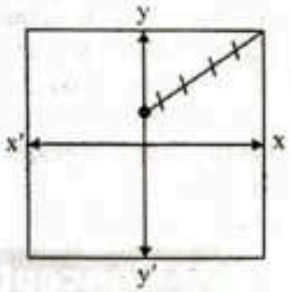
Sol.



8. Draw a graph of the readings:

x	5(x)
0	25000
2	20000
4	31000
6	34000
8	37000
10	4000

Sol.



— ✲✲✲ —

Unit 11

Loci and Construction

Locus

A locus plural loci is a set of points that follow a given rule.

Remember!
There are three types of triangles w.r.t. sides.
Scalene triangle: All sides are of different length.
Isosceles triangles: Two sides are of equal length.
Equilateral triangle: All sides of equal length.
There are three types of triangles w.r.t. angles.
Acute angled triangle: All angles are of measure less than 90°.
Obtuse angled triangle: One angle is of measure greater than 90°. **Right angled triangle:** One angle is of measure equal to 90°.

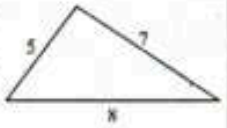
Construction of Triangles

A triangle is a closed figure having three sides and three angles. We construct triangle in the following cases:

(a) When measure of all three sides are given.
(b) When measure of two sides and their included angle are given.
(c) When measure of one side and measure of two angles are given.
(d) When measure of two sides and an angle opposite to one of them is given.

Triangle Inequality Theorem

The sum of the measure of any two sides of a triangle is always greater than the measure of the third side. For example, we can see in the figure adding any two lengths then this will be greater than the third side i.e., $5 + 7 > 8$, $5 + 8 > 7$ and $7 + 8 > 5$



Key fact!
- An equilateral triangle is acute angled triangle.
- A right angled triangle cannot be equilateral.

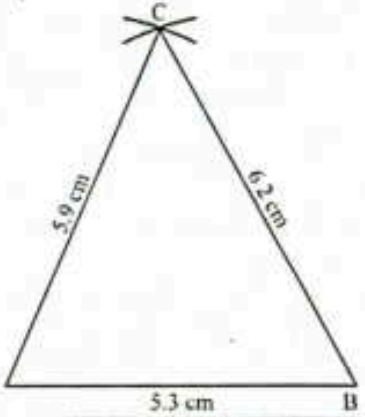
(a) Construction of a triangle when measure of three sides is given

Example 1: Construct a triangle of sides 5.3 cm, 5.9 cm and 6.2 cm.

Solution: Steps of construction:

(i) Draw a line segment AB of length 5.3 cm long.
(ii) Using a pair of compasses, draw two arcs with centres at points A and B of radii 5.9 cm and 6.2 cm respectively.
(iii) These two arcs intersect each other at point C.
(iv) Join A and B with C.

Hence, ΔABC is the required triangle.

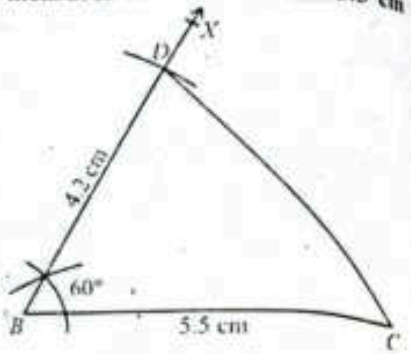


NOTE: The angles 30°, 45°, 60°, 75°, 90°, 105°, 120°, 135° and 150° are constructed with the help a pair of compasses. Other angles are drawn using protractor.

Do you know?
When three sides are given, we can draw any length first.

(b) Construction of a triangle when the measure of two sides and their included angle are given

Example 2: Construct a triangle BCD in which measures of two sides are 5.5 cm and 4.2 cm and measure of their included angle is 60°.



Solution: Steps of construction:

(i) Draw a line segment BC of length 5.5cm.

(ii) Draw an angle 60° at point B using a pair of compasses and draw a ray $\overrightarrow{BX}$ through this angle.

(iii) Draw an arc of radius 4.2 cm with centre at point B intersecting $\overrightarrow{BX}$ at point D.

(iv) Join C and D.

Hence, ΔBCD is the required triangle.

(c) Construction of a triangle when measure of one side and two angles are given

Example 3: Draw a triangle CDE when $m\overline{DE}$=4.3 cm, m∠D = 30° and m∠E = 120°.

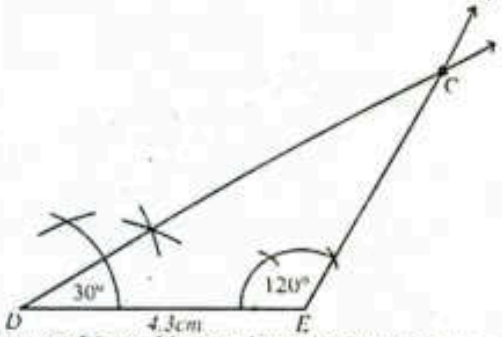
Solution: Steps of construction:

(i) Draw $m\overline{DE}$ = 4.3 cm.

(ii) Draw angles 30° and 120° at points D and E respective using a pair of compasses and draw two rays through the: angles from D and E.

(iii) These two rays intersect each other at point C.

Hence, ΔCDE is the required triangle.



(d) Construction of a triangle when measure of two sides and angle opposite to one of them is given

Consider the given two cases:

(i) If the measure of one angle is greater than or equal to 90°.

(ii) If the measure of angle is less than 90°.

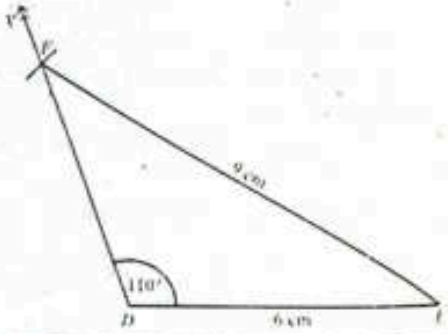
Example 4: Construct a triangle DEF when $m\overline{DE}$=6cm, m∠D=110° and m∠F=9cm.

Solution: Steps of construction:

(i) Draw m $\overline{DE}$ = 6 cm.

(ii) Construct m∠D = 110° using protractor and draw $\overrightarrow{DX}$ through this angle.

(iii) Draw an arc of radius 9 cm with centre at point E intersecting $\overrightarrow{DX}$ at point F.



(iv) Join E and F.

Hence, ΔDEF is the required triangle

If the given angle opposite to the given side is obtuse, only one triangle is possible.

Example 5: Construct triangles DEF and DEF' when $m\overline{DE}$ = 6 cm, m∠D=30° and $m\overline{EF}$ = 3.6cm.

Solution: Steps of construction:

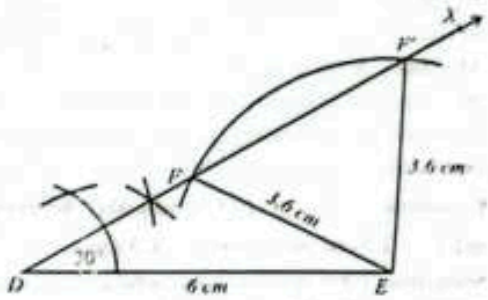
(i) Draw $m\overline{DE}$ = 6 cm.

(ii) Construct an angle 30° at point D using a pair of compasses and draw $\overrightarrow{DX}$ through this angle.

(iii) Draw an arc of radius 3.6 cm with centre at point E.

(iv) This arc intersects $\overrightarrow{DX}$ at two points F and F'.

(v) Join F and F' with E.



We get two triangles DEF and DEF'. This is known as anbiguous case.

Do you know?
The Ambiguous Case (SSA) occurs when we are given two sides and the angle opposite one of these is less than 90°.

Example 6: In the above example if we take:

(a) $m\overline{EF}$ =3 cm (b) $m\overline{EF}$ = 2.5 cm,

we see that in case (a), only one triangle is possible and in case (b), no triangle is possible.

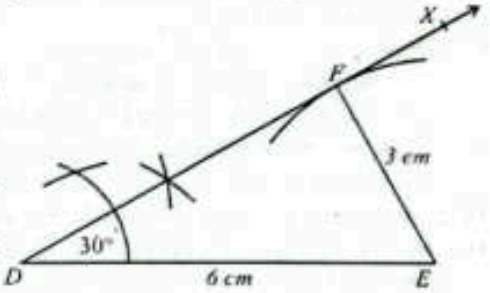
Solution: Steps of construction:

Follow the same steps (i) and (ii) as in Example 5.

Case (a)

(i) Draw an arc of radius 3 cm with centre at point E which touches $\overrightarrow{DX}$ at point F.

(ii) Join E with F. Here, $\overline{EF}$ will be perpendicular to $\overrightarrow{DX}$.



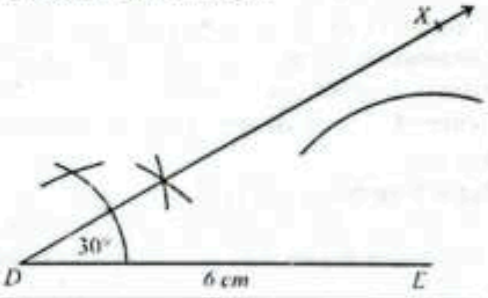
Hence, ΔDEF is the required triangle, which is a right angled triangle.

Case (b)

(i) If we take $m\overline{EF}$ = 2.5cm less than 3cm and draw an arc of radius 2.5cm with centre at E.

(ii) This arc does not intersect $\overrightarrow{DX}$.

So, in this case, no triangle can be formed.



We considered three cases when acute angle is given.
- If $m\overline{EF} > 3$ cm, two triangles are possible.
- If $m\overline{EF} = 3$ cm, only one triangle is possible.
- If $m\overline{EF} < 3$ cm, no triangle is possible.

Perpendicular Bisectors and Medians of a Triangle Perpendicular

Perpendicular Bisector: A perpendicular bisector is a line that intersects a line segment at right angle and dividing it into two equal parts.

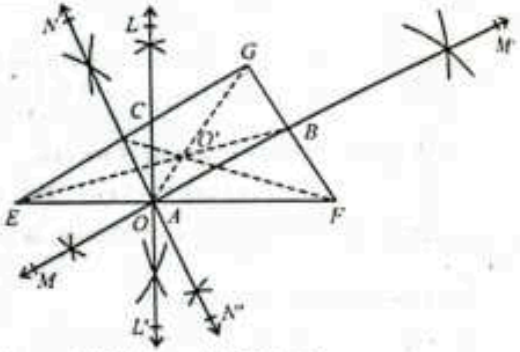
Median: A median of a triangle is a line segment that joins a vertex to the midpoint of the side that is opposite to that vertex.

Point of concurrency: A point of concurrency is the single point where three or more lines, rays or line segments intersect or meet in a geometric figure.

Example 7: Draw perpendicular bisector of the triangle EFG with $m\overline{EF}$ = 5 cm, $m\overline{FG}$ = 2.5cm and $m\overline{EG}$ = 4.3 cm.

Solution: First we draw perpendicular bisectors and then medians.

Steps of construction:



i. Draw ΔGEF as explained in the previous examples.
ii. Draw two arcs above and below $\overline{EF}$ with more than half of $m\overline{EF}$ with centre at E.
iii. Draw two arcs above and below $\overline{EF}$ with radius more than half of $m\overline{EF}$ with centre at F.
iv. Draw a line through the points of intersection of the arcs in steps (ii) and (iii), we get the perpendicular bisector LL' of the side EF at A.
v. Draw two more perpendicular bisectors MM' and NN' of the sides $\overline{FG}$ and $\overline{EG}$ at B and C respectively.
vi. Join the point G with opposite midpoint A so $\overline{GA}$ is the median.
vii. Join the point F with opposite midpoint C, we get median $\overline{FC}$ and join point E with opposite midpoint B, we get median EB.

Hence, we see that the perpendicular bisector $\overleftrightarrow{LL'}$, $\overleftrightarrow{MM'}$ and $\overleftrightarrow{NN'}$ are concurrent at at point O or A and the medians $\overline{GA}$, $\overline{EB}$ and $\overline{FC}$ are concurrent at point O

Circumcentre: The point of concurrency of perpendicular bisector of the sides of a triangle is called circumcentre.

Centroid: The point of concurrency of the medians of a triangle is called centroid of the triangle.

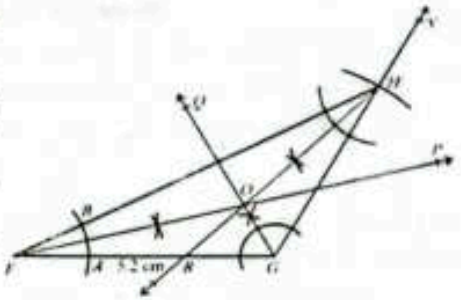
Angle Bisector of a Triangle

An angle bisector is a line or ray that divides an angle into two equal parts, creating two smaller angles that are congruent (each having half the measure of the original angle).

Example 8: Draw angle bisector of a triangle FGH if:
$m\overline{FG}$ = 5.2, $m\overline{GH}$ =4.1 and $m\angle FGH = 120°$

Solution: We first construct triangle FGH, then draw its angle bisector.

Steps of construction:



(i) Construct ΔFGH with given lengths and angle.
(ii) Draw an arc of suitable radius with centre at point F intersecting sides $\overline{FG}$ and $\overline{FH}$ at points A and B.
(iii) Draw two arcs with centres at points A and B with suitable radius.
(iv) Draw a ray from F passing through the point of intersection of the arcs in step (iii).

Which is the required angle bisector $\overrightarrow{FP}$ of the angle F.

(v) Draw two more angle bisectors $\overrightarrow{GQ}$ and $\overrightarrow{HR}$ of the angles G and H. respectively.

We see that all the angle bisectors $\overrightarrow{FP}$, $\overrightarrow{GQ}$ and $\overrightarrow{HR}$ intersect at one point O. i.e, the angle bisectors of the triangle are concurrent.

Incentre: The point of concurrency of the angle bisectors of a triangle is called incentre of the triangle.

Altitudes of Triangle

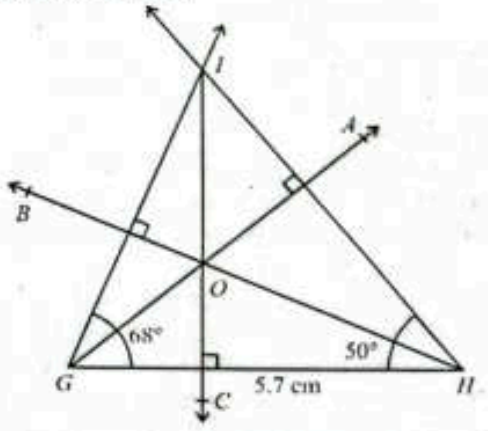
Altitude is a ray drawn perpendicular from a vertex to the opposite side of the triangle. There are three altitudes of the triangle which meet at a single point i.e. the altitudes of a triangle are concurrent.

Orthocentre: The point of concurrency of the altitudes of the triangle is called orthocentre of the triangle.

Example 9: Construct a triangle GHI in which $m\overline{GH}$=5.7, $m\angle G$=68° and $m\angle H$=50°. Prove that altitudes of the ΔGHI are concurrent.

Solution: First, we construct ΔGHI using the given measurements and then draw altitudes of the triangle.

Steps of construction.



(i) Construct ΔGHI using the given measurements.
(ii) Draw perpendicular $\overrightarrow{GA}$ from G to the opposite side $\overline{HI}$.
(iii) Draw two more perpendiculars $\overrightarrow{HB}$ and $\overrightarrow{IC}$. The first is from point H to the opposite side $\overline{GI}$ and the other is from point I to the opposite side $\overline{GH}$.

So, $\overleftrightarrow{GA}$, $\overleftrightarrow{HB}$ and $\overleftrightarrow{IC}$ are the altitudes of ΔGHI and they intersect at one point O. i.e., the altitudes of ΔGHI are concurrent.

Solved Exercise 11.1

1. Construct AABC with the given measurements and verify that the perpendicular bisectors of the triangle are concurrent.

(i) $m\overline{AB}$ = 5 cm, $m\overline{BC}$ = 6 cm and $m\overline{AC}$ = 7 cm

Sol. (i) Given

$m\overline{AB}$ = 5cm, $m\overline{BC}$ = 6cm, $m\overline{AC}$ = 7cm

Required: (i) To construct the ΔABC

(ii) To draw perpendicular bisectors of its sides and to verify that they are concurrent.

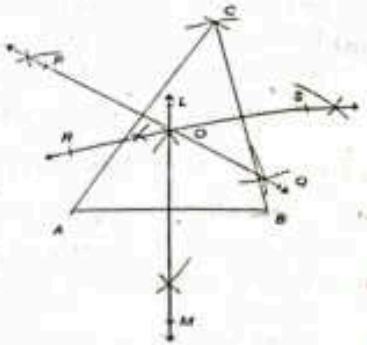
Construction:

(i) Take $m\overline{AB}$ = 5cm

(ii) With B as centre and radius $m\overline{BC}$ = 6cm draw an arc.

(iii) With A as centre and radius $m\overline{AC}$ = 7cm draw another are that intersects the first arc at C.

(iv) Join C to A and B to complete the ΔABC



(v) Draw $\overleftrightarrow{PQ}$ the perpendicular bisector of $\overline{AC}$ and $\overleftrightarrow{LM}$ the perpendicular bisector of $\overline{AB}$ meeting each other at the point O.

(vi) No draw the $\overleftrightarrow{RS}$ perpendicular bisector of the third side $\overline{BC}$. We observe that it also passes through O, the point of intersection of first two perpendicular bisectors.

(vii) Hence, three perpendicular bisectors of ΔABC are concurrent at point O.

(ii) $m\overline{AB}$ = 7.1 cm, $m\angle B$ = 135° and $m\overline{BC}$ = 6.5 cm

Sol. (ii) Given

$m\overline{AB}$ =7.1cm, $m\angle B$ = 135° and $m\overline{BC}$ =6.5cm

Required: (i) To construct the ΔABC

(ii) To draw perpendicular bisectors of its sides and to verify that they are concurrent.

Construction:

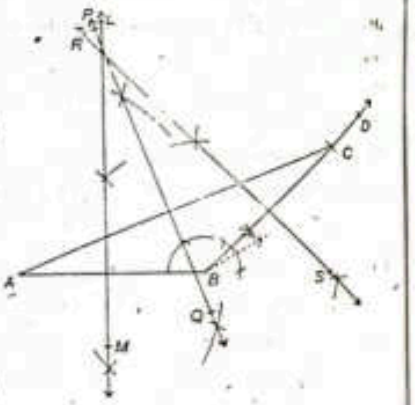
(i) Take $m\overline{AB}$ = 7.1cm

(ii) At point B make an angle $m\angle ABD$ =135°

(iii) With cantre at B, draw an arc of radius 6.5cm that cuts the $\overleftrightarrow{AD}$ at point C.

(iv) Join A to C to complete the ΔABC.

(v) Draw $\overleftrightarrow{PQ}$ the perpendicular bisector of $\overline{AC}$ and $\overleftrightarrow{LM}$ the perpendicular bisector of $\overline{AB}$ meeting each other at the point O.



(vi) Now draw the $\overleftrightarrow{RS}$ perpendicular bisector of the third side $\overline{BC}$. We observe that it also passes through O, the point of intersection of first two perpendicular bisectors.

(vii) Hence, three perpendicular bisectors of ΔABC are concurrent at point O.

2. Construct ΔALMN of the following measurements and verify that the medians of the triangle are concurrent.

(i) $m\overline{LM}$ = 4.9 cm, $m\angle L$ = 51° and $m\angle M$ = 38°

Sol. (i) Given

$m\overline{LM}$ = 4.9cm, $m\angle L$ = 51° and $m\angle M$ = 38°

Required: (i) To construct the ΔLMN

(ii) Draw its medians and verify their concurrency.

Construction:

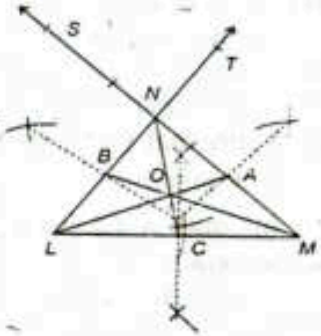
(i) Draw a line segment $m\overline{LM}$ = 4.9cm.

(ii) At point L make $m\angle MLT$ = 51°.

(iii) At point M make $m\angle LMS$ = 38°. The terminal sides of these angles meet at N. Thus LMN is required triangle.

(iv) Find A the mid-point of $\overline{MN}$ and join to A to find median $\overline{LA}$.

(v) Find B, the mid-point of $\overline{LN}$ and join M to B to get the median $\overline{BM}$.



(vi) The medians $\overline{LA}$ and $\overline{MB}$ meet at point O.

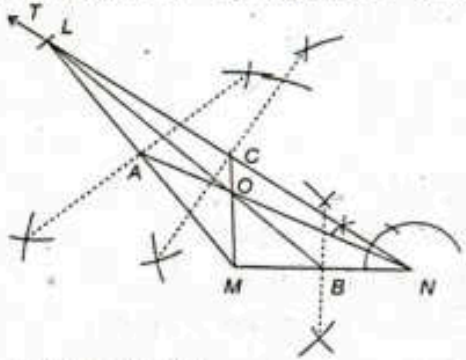
(vii) Find c, the mid-point of $\overline{LM}$ and join C to N to get the thrid median $\overline{CN}$.

(viii) We observe that the third median also passes through the point of intersection O of the first two medians.

(ix) Hence, the three medians of the ΔLMN pass through the same point O. That is their point of concurrency.

(ii) $m\overline{MN}$ = 4.8 cm, $m\angle N$ = 30° and $m\overline{LM}$ = 8.1 cm

Sol. (ii) Given $m\overline{MN}$ = 4.8cm, $m\angle N$ = 30° and $m\overline{LM}$ = 8.1cm



Required: (i) To construct the ΔLMN

(ii) Draw its medians and verify their concurrency.

Construction:

(i) Draw a line segment $m\overline{MN} = 4.8cm$.

(ii) A point N make an $m\angle MNT = 30°$.

(iii) What M as cantre and radius $m\overline{LM} = 8.1cm$ draw an arc that intersect the $\overline{NT}$ at point L.

(iv) Join L to M to complete the ΔLMN.

(v) Find C the mid-point of $\overline{LN}$ and join M to C to find median $\overline{MC}$.

(vi) Find A the mid-point of $\overline{LM}$ and join N to A to find the second median $\overline{NA}$.

(vii) The medians $\overline{MC}$ and $\overline{NA}$ meet at point O.

(viii) Find B, the mid-point of $\overline{MN}$ and join B to L to get the third median $\overline{LB}$.

(ix) We observe that the third median also passes through the point of intersection of the first two medians.

(x) Hence, the three medians of the ΔLMN pass through the same point O. That is their point of concurrency.

3. Verify that the angle bisectors of A ABC are concurrent with the following measurement:

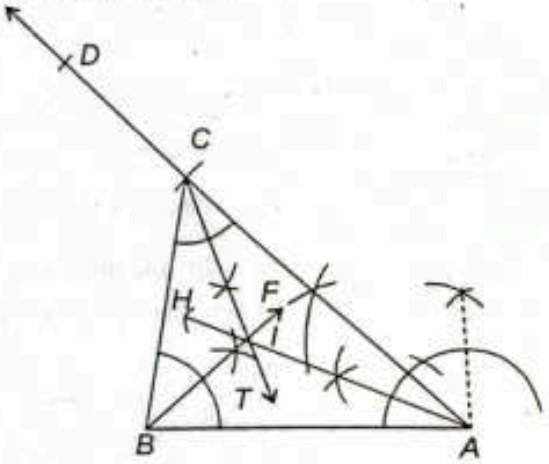
(i) $m\overline{AB} = 4.5$ cm, $m\angle A = 45°$ and $m\overline{AC} = 5.3$ cm

Sol. (i) Given

$m\overline{AB} = 4.5cm$, $m\angle A = 45°$ and $m\overline{AC} = 5.3cm$

Required: (i) To construct the ΔABC

(ii) Draw its angles bisectors ΔABC.



Construction:

(i) Draw a line segment $m\overline{AB} = 4.5cm$.

(ii) At point A make an angle $BAD = 45°$.

(iii) At point A draw an arc of radius 5.3cm that cuts the $\overline{AD}$. at point C.

(iv) Join C to B to complete ΔABC.

(v) At point A draw $\overline{AH}$ the bisector of angle A.

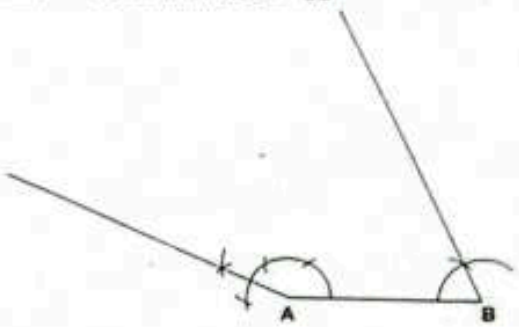
(vi) At point B draw $\overline{BF}$ the bisector of $\overline{LB}$.

(vii) Bisectors of $\overline{LA}$ and $\overline{LB}$ intersect each other at a point I.

(viii) At point C draw $\overline{CT}$ the bisector of $\angle C$.

(ix) We observe that the third angle bisector also passes through the point I.

(ii) $m\overline{AB} = 6$ cm, $m\angle A = 150°$ and $m\angle B = 60°$



Sol. Given

$m\overline{AB} = 6cm$, $m\angle A = 150°$ and $m\angle B = 60°$

Required: (i) To construct the ΔABC (ii) Angles bisectors of this ΔABC.

Construction:

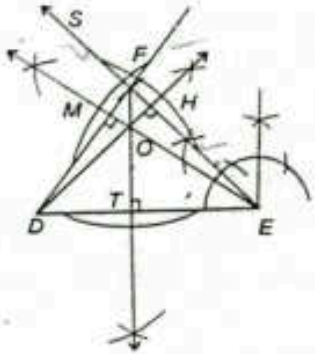
This triangle can not be constructed because the sum of the measures of two given angles $\angle A$ and $\angle B$ is 210°. This is impossible.

4. Given the measurements of ΔDEF: $m\overline{DE} = 4.8$ cm, $m\overline{EF} = 4$cm and $m\angle E = 45°$, draw altitudes of ΔDEF and find orthocentre.

Sol. Given $m\overline{DE} = 4.8cm$, $m\overline{EF} = 4cm$ and $m\angle E = 45$

Required: (i) To construct ΔDEF

(ii) Draw altitudes of ΔDEF and to find orthocentre.



Construction:

(i) Draw $m\overline{DE} = 4.8cm$.

(ii) At point E make an angle $\angle DES = 45°$.

(iii) At B, draw an arc of radius 4cm that cuts the $\overline{ES}$ at point F.

(iv) Join F to D to complete the ΔDEF.

(v) Draw perpendicular $\overline{DH}$ from point D to the opposite side $\overline{EF}$.

(vi) Draw perpendicular $\overline{EM}$ from point E to the opposite side $\overline{DF}$. These two altitudes meet at point O inside the ΔDEF.

(vii) Draw the third perpendicular $\overline{FT}$ from point F to opposite side $\overline{DE}$.

(viii) We observe that this third altitude also passes through the point of intersection O of the first two altitudes.

Hense, O is the orthocenter.

5. Construct the following triangles and find whether there exists any ambiguous case.

(i) ΔBCD ; $m\overline{BC} = 5$ cm, $m\angle B = 62°$ and $m\overline{CD} = 4.7$ cm

Sol. Given

$m\overline{BC}$ = 5cm, $m\angle B = 60°$, $m\overline{CD}$ = 4.7cm

Required: To construct ΔBCD

Construction:

(i) Draw $m\overline{BC}$ = 6cm.

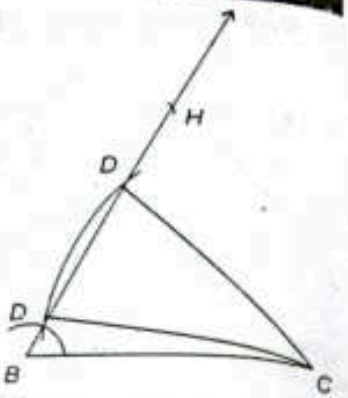
(ii) At point B make an angle $\angle CBH = 60°$.

(iii) Draw an arc of radius 4.7cm with centre at point C.

(iv) This arc intersects $\overline{BH}$ at two points D and D'.

(v) Join C to D and D'.

We get two triangles ΔBCD and ΔBCD'. This is known as ambiguous case.



(ii) ΔKLM; $m\overline{LM}$ = 6 cm, $m\angle M = 42°$ and $m\overline{LN}$ = 5 cm

Sol. Given

$m\overline{LM}$ = 6cm, $m\angle M = 42°$ and $m\overline{LN}$ = 5cm

Required: To construct ΔLMN

Construction:

(i) Draw $m\overline{LM}$ = 6cm.

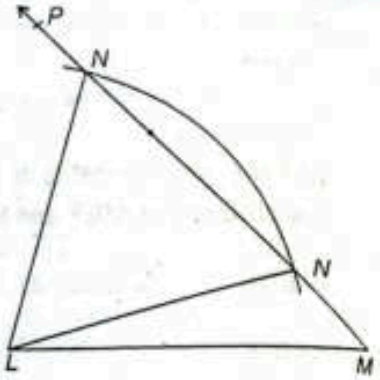
(ii) At point M make and angle $\angle LMP = 42°$.

(iii) Draw an arc of radius 5cm with centre at point L.

(iv) This arc intersects $\overline{MP}$ at two points N and N'.

(v) Join L to N and N'.

We get two triangles ΔLMN and ΔLMN'. This is known as ambiguous.



Locus

A locus (plural loci) is a set of points that follow a given rule. In geometry, loci are often used to define the geometric figures.

Do you know?
word locus is defined by the English term, location.

Remember!
Equidistant: Let A be a fixed point and B be a set of points. If A is at equal distance from all points of B, then A is said to be equidistant from B.

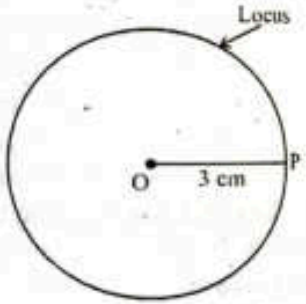
Loci in Two Dimension

We study the loci, circle, parallel lines, perpendicular bisector and angle bisector in two dimensions and apply them to real life situations.

Circle

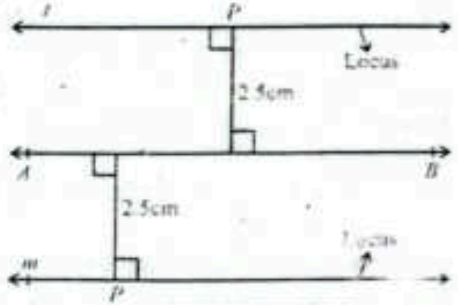
The locus of a point whose distance is constant from a fixed point is called a circle.

For example, the locus of a point P whose distance is 3cm from a fixed-point O is a circle of radius 3 cm and centre at point O.

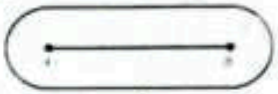


Parallel Lines

The locus of a point whose distance from a fixed line is constant are parallel lines, ℓ and m e.g. the locus of a point P whose distance is 2.5 cm from a fixed line AB are parallel lines at a distance of 2.5 cm from AB.

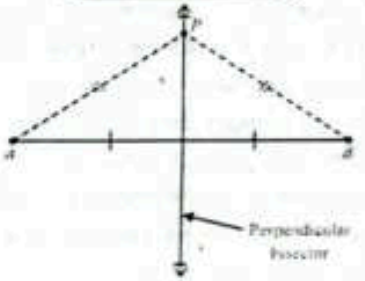


For example, a locus of points equidistant from a line segment creates a sausage shape. We can think of this type of locus as a track surrounding a line segment.



Perpendicular Bisector

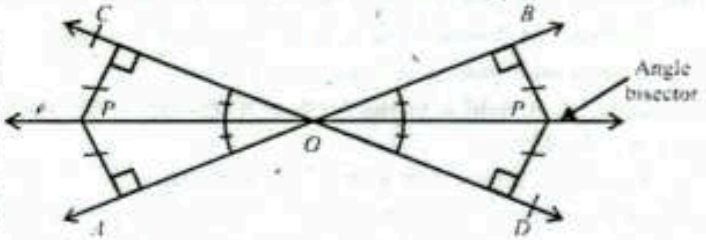
The locus of a point whose distance from two fixed points is constant is called a perpendicular bisector. For example, the locus of a point P whose distance from P fixed points A and B is constant is the perpendicular bisector of the line segment AB.



Angle Bisector

The locus of a point whose distance is constant from two intersecting lines is called an angle bisector.

For example, the locus of a point P whose distance is constant from two lines AB and CD intersecting at O is the angle bisector (ℓ) of $\angle AOC$ and $\angle BOD$.



Remember!

- Locus of points equidistant from a fixed point is a circle and equidistant from two fixed points is a perpendicular bisector.
- Locus of points equidistant from a fixed line are two parallel lines and equidistant from two fixed intersecting lines is angle bisector.

Intersection of Loci

If two or more loci intersect at a point P, then P satisfies all given conditions of the loci. This will be explained in the following examples:

Example 10: Construct a rectangle ABCD with mAB = 5 cm and mBC = 3.2 cm. Draw the locus of all points which are:

(i) at a distance of 3.1 cm from point A. (ii) equidistant from A and B.

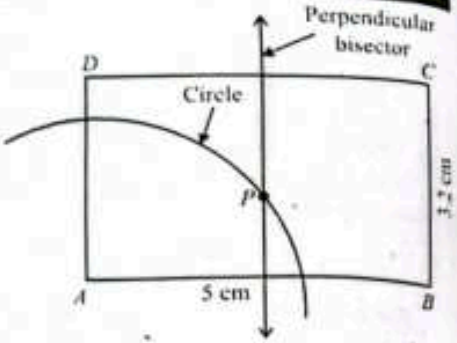
Label the point P inside the rectangle which is 3.1 cm from point A and equidistant from A and B.

Solution: Construct rectangle ABCD with given lengths.

(i) Draw a circle of radius 3.1 cm with centre at A.

(ii) Draw perpendicular bisector of AB.

The two loci intersect at P inside the rectangle which is 3.1 cm from point A and equidistant from A and B.



Example 11: Construct an isosceles triangle DEF with vertical angle 80° at E and $m\overline{EF} = m\overline{DE} = 4.8$ cm. Draw the locus of all points which are:

(i) at a distance of 2.8 cm from point E,

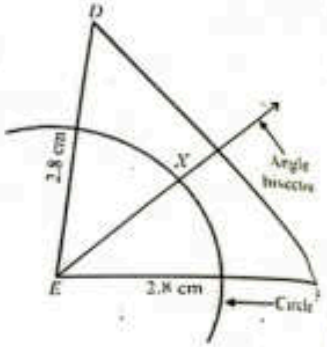
(ii) equidistant from $\overline{DE}$ and $\overline{EF}$.

Label the point X inside the triangle which is 2.8 cm from point E and equidistant from $\overline{ED}$ and $\overline{EF}$.

Solution: Construct triangle DEF with given measurements.

(i) Draw a circle of radius 2.8 cm with centre at E.

(ii) Draw angle bisector of angle DEF. The two loci intersect at Xinside the triangle which is 2.8 cm from point E and equidistant from $\overline{ED}$ and $\overline{EF}$.



Example 12: A field is in the form of a triangle LMN with $m\overline{LM}$=69 m, $m\angle L = 60°$ and $m\angle M = 45°$.

(i) Construct ΔLMN with given measurements. [Scale: 10m = 1cm]

(ii) Draw the locus of all points which are equidistant from L and M, equidistant from $\overline{LM}$ and $\overline{LN}$ and at a distance of 13 m from $\overline{LM}$ inside the triangular field.

(iii) Two trees are to be planted at points P and Q inside the field.

(a) Mark the position of point P which is equidistant from L and M and equidistant from $\overline{LM}$ and $\overline{LN}$.

(b) Mark the position of point Q which is equidistant from $\overline{LM}$ and $\overline{LN}$ and 13 m from $\overline{LM}$.

(c) Find the distance $m\overline{PQ}$.

Solution:

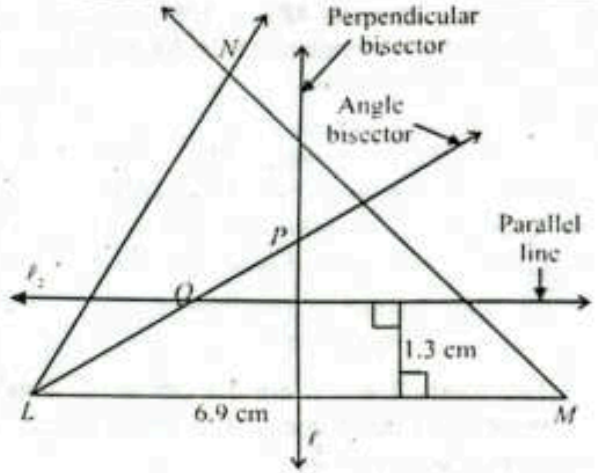
(i) Construct triangle LMN with given measurements using a scale of 10 m to represent 1 cm.

(ii) Draw perpendicular bisector ℓ_1 of $\overline{LM}$. Draw angle bisector of angle MLN. Draw a parallel line ℓ_2 inside the triangle LMN, 1.3 cm from $\overline{LM}$.

(iii) (a) Label the point P which is equidistant from L and M and equidistant from $\overline{LM}$ and $\overline{LN}$. Mark the point P inside the triangle which is equidistant from L and M.

(b) Label the point Q which is equidistant from $\overline{LM}$ and $\overline{LN}$ and 1.3 cm from $\overline{LM}$.

(c) $m\overline{PQ} = 1.2 \times 10 = 12$m



Solved Exercise 11.2

1. Two points A and B are 8.2 cm apart. Construct the locus of points 5 cm from point A.

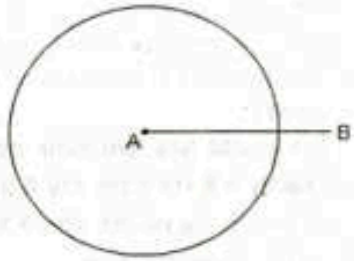
Sol. Steps of construction

(i) Draw a straight line segment $\overline{AB}$ of 8.2cm

(ii) With a compass, take A as the centre and set the radius to 5cm.

(iii) Draw a circle around point A with a radius of 5cm.

(iv) The circle represents the locus of all points that are 5cm from point A.



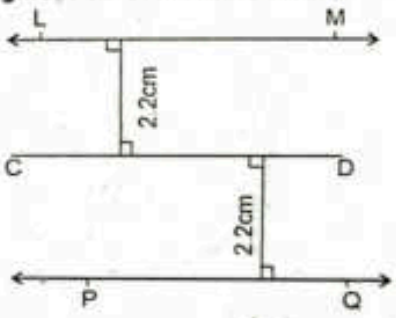
2. Construct a locus of point 2.2 cm from line segment CD of measure 5.7cm.

Sol. Steps of construction:

(i) Draw a line segment $\overline{CD}$ of length 5.7cm

(ii) Draw two parrallel lines $\overleftrightarrow{LM}$ and $\overleftrightarrow{PQ}$ on either sides of the $\overline{CD}$ using a rular and a set square at a distanc of 2.2cm from $\overline{CD}$.

(iii) These two parrallel lines $\overleftrightarrow{LM}$ and $\overleftrightarrow{PQ}$ represent the locus of all points that are 2.2cm away from the line segment $\overline{CD}$.



3. Construct an angle ABC = 105°. Construct a locus of a point P which moves such that it is equidistant from $\overline{BA}$ and $\overline{BC}$.

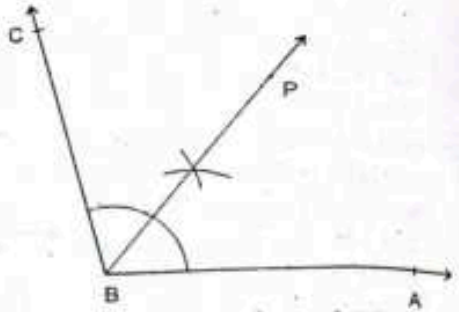
Sol. Steps of construction

(i) Draw a ray $\overrightarrow{BA}$

(ii) At point B draw an angle ∠ABC = 105°.

(iii) Draw $\overrightarrow{BP}$ as an angle bisector of ∠ABC

(iv) Thus $\overrightarrow{BP}$ the bisector of ∠ABC will represent the locus of point P because the angle bisector is equidistance from the two arms $\overrightarrow{BA}$ an $\overrightarrow{BC}$



4. Two points E and F are 5.4 cm apart. Construct a locus of a point P which moves such that it is equidistant from E and F.

Sol. Steps of construction

(i) Draw a line segment $\overline{EF}$ of 5.4cm

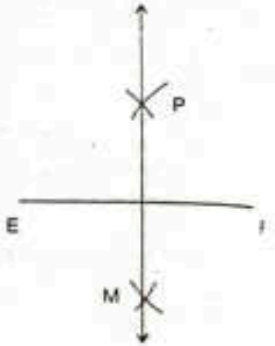
(ii) Take a compass and set the radius slightly greater than the half of $\overline{EF}$.

(iii) Draw ares from E and F on both sides of $\overline{EF}$.

(iv) The intersection points of the arcs determine the perpendicular bisector.

(v) Draw the perpendicular bisector $\overrightarrow{PM}$.

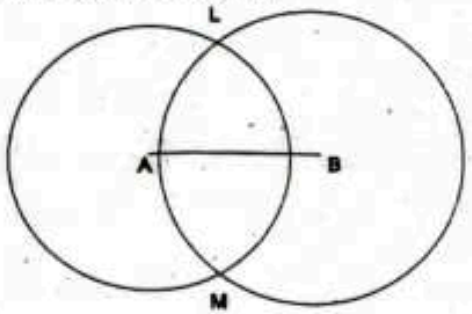
(vi) This line $\overrightarrow{PM}$ is the locus of all points equidistance from E and F.



5. The island has two main cities A and B 8 km apart. Kashif lives on the island exactly 6.8 km from city A and exactly 7.3 km from city B. Mark with a cross the points on the island where Kashif could live.

Sol. Steps of construction

(i) Draw a line segment $\overline{AB}$ of 4cm



Scale
2km = 1cm
8km = 4cm
6.8km = 3.4cm
7.3km = 3.65cm

(ii) With canter point A draw a circle of radius 3.4cm.

(iii) With canter point B draw another circle of radius 3.65cm.

(iv) These two circle intersect at L and M.

(v) Points L and M are the possible locations where Kashif could live.

6. Construct a triangle CDE with m$\overline{CD}$ = 7.6 cm, m∠D = 45° and mDE = 5.9 cm. Draw the locus of all points which are:

(a) equidistant from C and D

(b) equidistant from $\overline{CD}$ and $\overline{CE}$ Mark the point X where the two loci intersect.

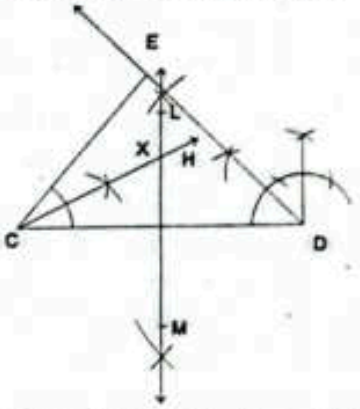
Sol. Steps of construction

(i) Construct ΔCDE with given measurements m$\overline{CD}$=7.6cm, m∠D=45° and m$\overline{DE}$=5.9cm

(ii) Draw the perpendicular bisector $\overrightarrow{LM}$ of the $\overline{CD}$. $\overrightarrow{LM}$ is the locus of all points equidistant from points C and D.

(iii) From point C draw the bisector $\overrightarrow{CH}$ of ∠DCE. $\overrightarrow{CH}$ is the locus of all points which are equidistant from $\overline{CD}$ and $\overline{CE}$.

(iv) X is the point where two loci intersect.

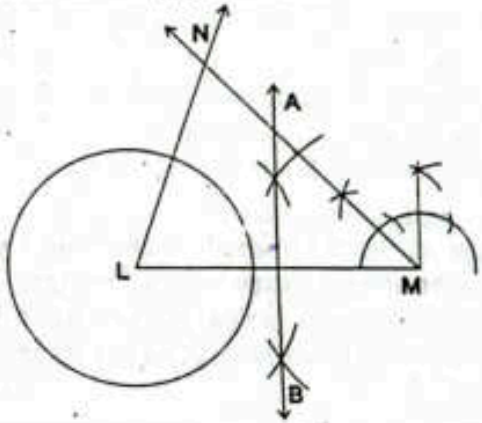


7. Construct a triangle LMN with m$\overline{LM}$ = 7 cm, m∠L = 70° and m∠M = 45°. Find a point within the triangle LMN which is equidistant from L and M and 3cm from L.

Sol. Steps of construction

(i) Construct a triangle ΔLMN with given measurements m$\overline{LM}$ = 7cm, m∠L = 70° and m∠M = 45°

(ii) Draw the perpendicular bisector $\overrightarrow{AB}$ of the $\overline{LM}$.



(iii) Draw a circle of radius 3m at point L.

(iv) The intersection of the circle and bisector $\overleftrightarrow{AB}$ is the required point.

(v) There is no such point which is equidistint from L and M and 3cm from L because two conditions have no valid over lapping region.

8. Construct a right angled triangle RST with $m\overline{RS}$ = 6.8 cm, $m\angle S = 90°$ and $m\overline{ST}$ = 7.5 cm. Find a point within the triangle RST which is equidistant from $\overline{RS}$ and $\overline{RT}$ and 4.5 cm from R.

Sol. Steps of construction

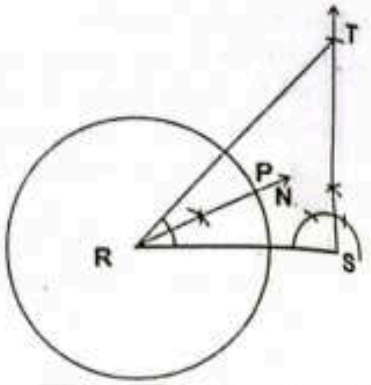
(i) Construct a right triangle RST with given measures
$m\overline{RS}$ = 6.8cm, $m\angle S = 90°$ and $m\overline{ST}$ = 7.5cm

(ii) Taking point R as cantre draw $\overrightarrow{AN}$, an angle bisector of $\angle SRT$.

(iii) Again taking point R as cantre draw a circle of radius = 4.5cm

(iv) $\overrightarrow{AN}$ and circle at point R are two loci.

(v) These two loci intersect at point P.



(vi) Thus, P is the required point which is equidistant from $\overline{RS}$ and $\overline{RT}$ and 4.5cm from R.

9. Construct a rectangle UVWX with $m\overline{UV}$ = 12 cm and $m\overline{VW}$ = 5.6 cm. Draw the locus of points at a distance of 2 cm from $\overline{UV}$ and 3.5 cm from W.

Sol. Steps of construction

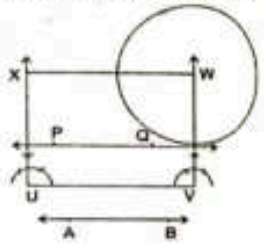
(i) Construct a rectangle with given measurements
$m\overline{UV}$ = 7.2cm, and $m\overline{VM}$ = 5.6cm

(ii) Draw two parallel line $\overleftrightarrow{PQ}$ and $\overleftrightarrow{AB}$ at a distance of 2cm from $\overline{UV}$ on either sides of $\overline{UV}$ using a ruler and a set square.

(iii) Taking point W as cantre draw an arc of a circle of radius 3.5cm. This is a locus.

(iv) Circle of radius 3.5cm line $\overleftrightarrow{PQ}$ are two required loci.

(iv) There is no point where the two loci intersect.
so, the conditions have no valid overlapping vegion



Note: The question in the book is not correct.

10. Imagine two cell towers located at points A and B on a coordinate plane. The GPS-enabled device, positioned somewhere on the plane, receives signals from both towers. To ensure accurate navigation, the device is placed equidistant from both towers to estimate its position. Draw this locus of navigation.

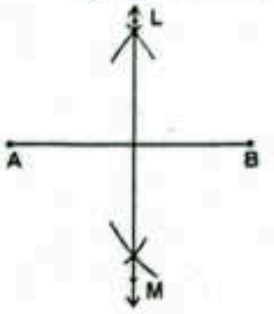
Sol. Steps of construction

(i) Take two points A and B.

(ii) Join A with B to get the line segment $\overline{AB}$.

(iii) Construct $\overleftrightarrow{LM}$ the perpendicular bisector of $\overline{AB}$.

(iv) The perpendicular bisector $\overleftrightarrow{LM}$ represents the locus of all points equidistant from both towers.



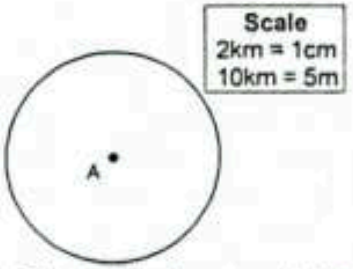
11. Epidemiologists use loci to determine infection zones, especially for contagious diseases, to predict the spread and take containment measures. In the case of a disease outbreak, authorities might determine a quarantine zone within 10 km of the infection source. Draw the locus of all points 10 km from the source defining the quarantine area to monitor and control the disease's spread.

Sol. Steps of construction

(i) Let A be the point of source

(ii) Taking A as cantre draw a circle of the radius 5cm (5cm = 10km)

(iii) Thus, this circle defines the quarantine area to monitor and control the disease's spread with the zone of 10 km.



12. There is a treasure buried somewhere on the island. The treasure is 24 kilometres from A and equidistant from B and C. Using a scale of 1cm to represent 10 km, find where the treasure could be buried.

Sol. Steps of construction

(i) Take three points A, B and C.

(ii) Join B with C to get the line segment $\overline{BC}$.

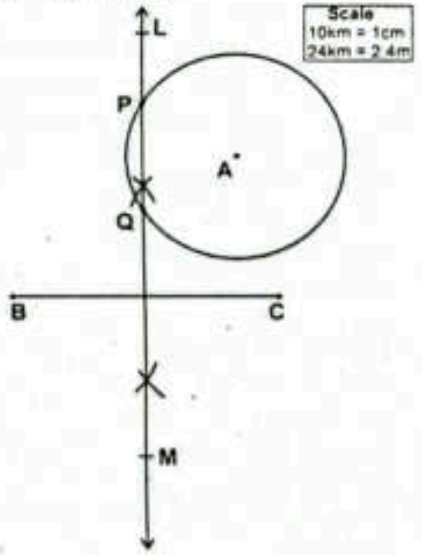
(iii) Draw $\overleftrightarrow{LM}$ the perpendicular bisector of $\overline{BC}$.

(iv) $\overleftrightarrow{LM}$ is the locus of all points equidistant from B and C.

(v) Taking point A as cantre draw a circle of radius of 2.4cm.

(vi) This circle intersects the locus $\overleftrightarrow{LM}$ at point P and Q.

(vii) Thus, the treasure could be buried at point P or Q because these points are 24km from A and equidistant from B and C.



13. There is an apple tree at a distance of 90 metres from banana tree in the garden of Sara's house. Sara wants to plant a mango tree M which is 64 metres from apple tree and between 54 and 82 metres from the banana tree. Using a scale of 1cm to represent 10m, Find the points where the mango tree should be planted.

Sol. Steps of construction

(i) Let A denotes the apple tree and B denotes the banana tree.

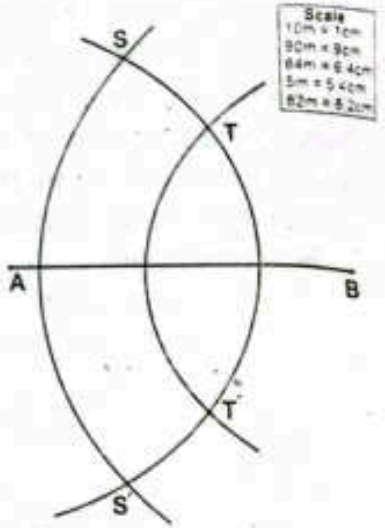
(ii) Draw a line segment $\overline{AB}$ = 9cm (9cm = 90m)

(iii) Taking A as cantre draw an arc of a circle of radius 6.4cm.

(iv) Taking B as cantre draw two arcs of two circles of radius 8.2cm and radius of 5.4cm.

(v) The arcs of these two circles interset the previous arc at the points S, T, T' and S'.

(vi) So, Sara should be planted Mango tree M between the arc $\overset{\frown}{ST}$ or between the arc $\overset{\frown}{S'T'}$ because these arcs represent the area 64m from apple tree and between 54 and 82 metres from the banana tree.



Review Exercise 11

1. Choose the correct option.

I. Four options are given against each statement. Encircle the correct option.

(i) A triangle can be constructed if the sum of the measure of any two sides is _____ the measure of the third side.

(a) less than (b) greater than (c) equal to (d) greater than and equal to

(ii) An equilateral triangle _____.

(a) can be isosceles (b) can be right angled

(c) can be obtuse angled (d) has each angle equal to 50°.

(iii) If the sum of the measures of two angles is less than 90°, then the triangle is_

(a) equilateral (b) acute angled (c) obtuse angled (d) right angled

(iv) The line segment joining the midpoint of a side to its opposite vertex in a triangle is called _____.

(a) median (b) perpendicular bisector (c) angle bisector (d) circle

(v) The angle bisectors of a triangle intersect at _____.

(a) one point (b) two points (c) three points (d) four points

(vi) Locus of all points equidistant from a fixed point is _____.

(a) circle (b) perpendicular bisector (c) angle bisector (d) parallel lines

(vii) Locus of points equidistant from two fixed points is _____

(a) circle (b) perpendicular bisector (c) angle bisector (d) parallel lines

(viii) Locus of points equidistant from a fixed line is/are _____.

(a) circle (b) perpendicular bisector (c) angle bisector (d) parallel lines

(ix) Locus of points equidistant from two intersecting lines is _____.

(a) circle (b) perpendicular bisector (c) angle bisector (d) parallel lines

(x) The set of all points which is farther than 2 km from a fixed point B is a region outside a circle of radius _____ and centre at B.

(a) 1km (b) 1.9 km (c) 2 km (d) 2.1km

Answers:

i. greater than ii. can be obtuse angled iii. obtuse angled iv. median
v. one point vi. circle vii. perpendicular bisector
viii. parallel lines ix. angle bisector x. 2 km

2. Construct a right angled triangle with measures of sides 6 cm, 8 cm and 10 cm.

Sol. Given: Measures of the sides 6cm, 8cm and 10cm.

Required: Construct a right triangle

Steps of Constructions:

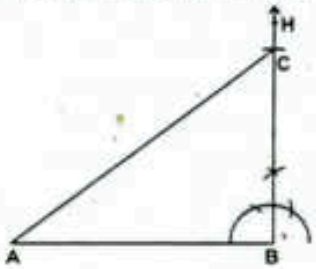
(i) Take m $\overline{AB}$ = 8cm

(ii) With B as cantre draw an angle ∠ABH = 90°

(iii) With B as cantre dra and arc of measure 6cm, which cuts the $\overleftrightarrow{BH}$ at point C

(iv) Join C to A to complete the required triangle.

(v) Thus, ABC is required right triangle.



3. Construct a triangle ABC with m$\overline{AB}$ = 5.3cm, m∠A = 30° and m∠B = 120°. Draw the locus of all points which are equidistant from A and B.

Sol. Given: m$\overline{AB}$=5.3cm, m∠A=30° and m∠B=120°

Required: (i) To Construct ΔABC

(ii) To draw the locus of all points which are equidistant from A and B

Steps of Constructions:

(i) Draw m$\overline{AB}$ = 5.3cm

(ii) Taking A as cantre make an angle ∠BAH = 30°

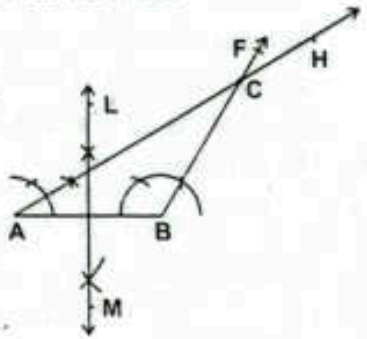
(iii) Taking B as cantre make another angle ∠ABF=120°.

(iv) $\overleftrightarrow{AH}$ and $\overleftrightarrow{BF}$ intersect each other at point C.

(v) So, ABC is the required triangle.

(vi) Draw $\overleftrightarrow{LM}$ the perpendicular bisector of $\overline{AB}$.

(vii) Thus, $\overleftrightarrow{LM}$ is required locus. All the points of $\overleftrightarrow{LM}$ are equidistant from A and B.



4. Construct a triangle with m$\overline{DE}$ = 7.3 cm, m∠D = 42° and m$\overline{EF}$ = 5.4 cm.

Sol. Given: m$\overline{DE}$ = 7.3cm, m∠D = 42° and m$\overline{EF}$ = 5.4cm

Required: To Construct ΔDEF

Steps of Constructions:

(i) Draw $m\overline{DE} = 7.3cm$

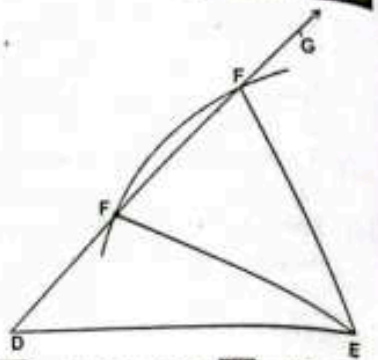
(ii) At point D draw an angle $\angle EDG = 42°$

(iii) From point E draw an arc of radius of 5.4cm.

(iv) This arc intersects $\overline{DG}$ at two points F and F'.

(v) Join F and F' with E.

(vi) Thus, we get two triangles ΔDEF and ΔDEF'. This is known as ambiguous.



5. Construct a triangle XYZ with $m\overline{YX} = 8$ cm, $m\overline{YZ} = 7$ cm and $m\overline{XZ} = 6.5$ cm. Draw the locus of all points which are equidistant from $\overline{XY}$ and $\overline{XZ}$.

Sol. Given: $m\overline{YX} = 8cm$, $m\overline{YZ} = 7cm$ and $\overline{XZ} = 6.5cm$

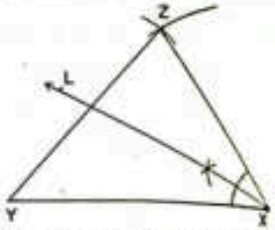
Required: (i) To Construct ΔXYZ

(ii) The locus of all points which are equidistant from $\overline{XY}$ and $\overline{XZ}$.

Steps of Constructions:

(i) Take $m\overline{YX} = 8cm$

(ii) With Y as cantre and radius = 7cm draw an arc.



(iii) With X as cantre and radius = 6.5cm draw another arc that intersect the lost arc at point Z.

(iv) Join Z to Y and X to complete ΔXYZ.

(v) With point X draw $\overleftrightarrow{XL}$ the bisector of $\angle YXZ$.

(vi) $\overleftrightarrow{XL}$ is the required locus of all points on it are equidistant from $\overline{XY}$ and $\overline{XZ}$.

6. Construct a triangle FGH such that $m\overline{FG} = m\overline{GH} = 6.4$ cm, $m\angle G = 122°$. Draw the locus of all points which are:

(a) equidistant from F and G, (b) equidistant from $\overline{FG}$ and $\overline{GH}$.

(c) Mark the point where the two loci intersect.

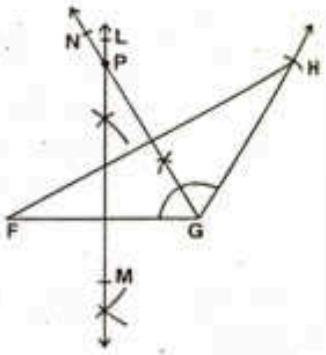
Sol. Steps of Constructions:

(i) Construct a triangle FGH with given measurements. $m = \overline{FG} = m\overline{GH} = 6.4cm$, $m\angle G = 1220°$

(ii) Draw the perpendicular bisector $\overleftrightarrow{LM}$ of $\overline{FG}$.

(iii) $\overleftrightarrow{LM}$ is the locus of all points equidance from the points F and G.

(iv) From point G draw the bisector $\overleftrightarrow{GN}$ of $\angle FGH$. $\overleftrightarrow{GN}$ is the locus of all points which are equidistant from $\overline{FG}$ and $\overline{GH}$.



(v) P is the point where two loci intersect each other.

7. Two houses Q and R are 73 metres apart. Using a scale of 1 cm to represent 10 m, construct the locus of a point P which moves such that it is:

(i) at a distance of 32 metres from Q

(ii) at a distance of 48 metres from the line joining Q and R.

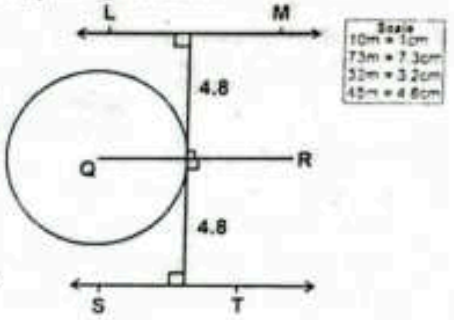
Sol. Steps of construction

(i) Let Q and R denote the two houses.

(ii) Draw $\overline{QR} = 7.3cm$ (7.3cm = 73m)

(iii) Draw two parallel lious $\overleftrightarrow{LM}$ and $\overleftrightarrow{ST}$ on either sides of $\overline{QR}$ using a rulaer and a ST wquare at a distance of 4.8cm from $\overline{QR}$.

(iv) These two parallel lines $\overleftrightarrow{LM}$ and $\overleftrightarrow{ST}$ represnt the locus of all points that are 4.8cm from the $\overline{QR}$.



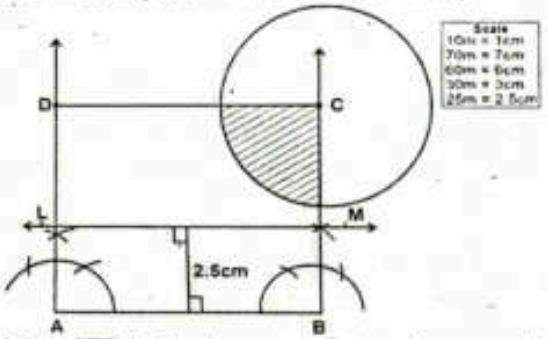
(v) Takign Q as cantre draw a circle of radius 3.2cm. This circle is also a locus of all points at a distance of 3.2cm from Q.

(vi) There is no point where the two loci intersect. so, the conditions have no valid overlapping region.

8. The field is in the form of a rectangle ABCD with $m\overline{AB} = 70m$ and $m\overline{BC} = 60$ m. Construct the rectangle ABCD using a scale of 1cm to represent 10 m. Show the region inside the field which is less than 30 m from C and farther than 25 m from $\overline{AB}$.

Sol. Steps of construction

(i) Construct a rectangle ABCD with given measurements. $m\overline{AB} = 70m$, $m\overline{BC} = 60m$ (70m = 7cm and 60m = 6cm)



(ii) Draw a parrallel line $\overleftrightarrow{LM}$ inside the rectangle at a distance of 2.5cm from $\overline{AB}$ with a ruler and a set square.

(iii) Taking C as cantre draw a circle of radius 3cm. This circle is locus.

(iv) Thus, the shaded area of the circle is the required region inside the field which is less than 30m from C and farther than 25m from $\overline{AB}$.

Objective Type Questions

Additional MCQ's

Multiple Choice Questions prepared in the light of new Examination Techniques (Knowledge, Understanding, Application, Analytical & Conceptual)

11.1 Construction of Triangles

O Encircle the correct option.

1. **A triangle is a closed figure having:**
   (a) three sides (b) two sides (c) four sides (d) five sides
2. **A triangle is a closed figure having:**
   (a) two angles (b) three angles (c) four angles (d) five angles
3. **How many types of triangles w.r.t sides?**
   (a) two (b) three (c) four (d) five
4. **How many types of tirangles w.r.t angles?**
   (a) two (b) three (c) four (d) five
5. **In which triangle all sides are of different length.**
   (a) scalene triangle (b) isosceles triangle
   (c) equilateral triangle (d) none of above
6. **In which triangle two sides are of equal lengths?**
   (a) scalene triangle (b) isosceles triangle
   (c) equilateral triangle (d) none of above
7. **In which triangle all sides are of equal length?**
   (a) scalene triangle (b) isosceles triangle
   (c) equilateral triangle (d) none of above
8. **In which triangle all angles are of measure less than 90°.**
   (a) acute angled triangle (b) obtuse angled triangle
   (c) right angled triangle (d) none of above
9. **In which triangle one angle is of measure greater than 90°.**
   (a) acute angled triangle (b) obtuse angled triangle
   (c) right angled triangle (d) none of above
10. **In which triangle one angle is of measure equal to 90°.**
    (a) acute angled triangle (b) obtuse angled triangle
    (c) right angled triangle (d) none of above

11.2 Perpendicular Bisector and Medium of a Triangle

11. **A perpendicular bisector is a line that intersects a line segment at _____ angle and divide it in to _____ equal parts.**
    (a) 90°, 2 (b) 180°, 2 (c) 90°, 3 (d) 180°, 3

12. **A _____ of a triangle is a line segment that joins a vertex to the mid point of the side that is opposite to that vertex.**
    (a) median (b) mode
    (c) perpendicular point (d) least point
13. **A point of concurrency is the point where _____ lines, rays, line segment intersect or meet in geometric figure called point of concurrency.**
    (a) one or more (b) two or more (c) three or more (d) none of above
14. **The point of concurrency of perpendicular bisector of the sides of a triangle called:**
    (a) circumference (b) circum center (c) centroid (d) none of above
15. **The point of concurrency of the medians of a triangle is:**
    (a) circumference (b) circum center (c) centroid (d) none of above

11.3 Angle Bisector of a Triangle

16. **An angle bisector is a line on ray that divides an angle into equal parts.**
    (a) two (b) three (c) four (d) five
17. **The angle bisectors of the triangle are:**
    (a) congruent (b) concurrent (c) combination (d) none of above
18. **The point of concurrency of the angle bisectors of a triangle is:**
    (a) in centre (b) out centre (c) ortho centre (d) none of above

11.4 Altitudes of Triangles

19. **Altitude is a ray drawn _____ from a vertex to the opposite side of the triangle.**
    (a) parallel (b) perpendicular (c) linear (d) none of above
20. **How many altitudes of a triangle:**
    (a) two (b) three (c) four (d) five
21. **The altitudes of a triangle are _____.**
    (a) concurrent (b) congruent (c) combination (d) none of above
22. **The point of concurrency of the altitudes of the triangle is:**
    (a) in center (b) out center (c) ortho center (d) none of above

11.5 Loci and Construction

23. **Which of them is used to define the position of points relative to one another or to other geometric figures:**
    (a) algebra (b) loci (c) sets (d) matrices
24. **Which of them is singular of Loci?**
    (a) locus (b) lotus (c) lumerous (d) none of above
25. **The locus of a point whose distance is constant from a fixed point is:**
    (a) triangle (b) rectangle (c) square (d) circle
26. **The locus of a point whose distance from a fixed line is constant called:**
    (a) parallel line (b) perpendicular line
    (c) circle (d) none of above

27. **The locus of a point whose distance from two fixed points is constant is called:**
(a) parallel line (b) perpendicular line
(c) circle (d) none of above

28. **The locus of a point whose distance is constant from two intersecting lines is:**
(a) parallel line (b) perpendicular line
(c) angle bisector (d) none of above

29. **If two or more loci intersect at a point P then P satisfies _____ given conditions of the loci.**
(a) one (b) two (c) three (d) all

11.6 Real Life Application of Loci

30. **The concept of _____ has many applications across fields where spatial relationship, distances or specific constraints are important.**
(a) loci (b) algebra (c) sets (d) quadratic equation

31. **A part has two water sources at x_1 and x_2. A fire hydrant needs to be placed so, it is equally accessible to both sources. It is an example of:**
(a) algebra (b) sets (c) matrices (d) loci

32. **A robotic arm in a factory is programmed to work within a specific area without crossing into area where it could interfere with other equipment. It is an example of:**
(a) algebra (b) loci (c) sets (d) none of above

Answers:

1. three sides	2. three angles	3. three	4. three	5. scalene triangle
6. isosceles triangle		7. equilateral triangle		8. acute angled triangle
9. obtuse angled triangle		10. right angled triangle		11. 90°, 2
12. median	13. three or more	14. circum center		15. centroid
16. two				
17. concurrent	18. in centre	19. perpendicular	20. three	21. concurrent
22. ortho center	23. loci	24. locus	25. circle	26. parallel line
27. perpendicular line		28. angle bisector	29. all	30. loci
31. loci	32. loci			

Conceptual MCQ's

Encircle the correct option.

1. **How many types of triangles w.r.t sides.**
(a) two (b) three (c) four (d) five

2. **In scalene triangles all sides are of _____.**
(a) different length (b) same length (c) 90° (d) 180°

3. **In isosceles triangles _____ sides are equal length.**
(a) two (b) three (c) four (d) five

4. **In equilateral triangle all sides are of _____ length.**
(a) different (b) equal (c) 90° (d) 180°

5. **There are types of triangles w.r.t angles.**
(a) two (b) three (c) four (d) five

6. **In acute angled triangle, all angles are of measure less than _____.**
(a) 30° (b) 45° (c) 60° (d) 90°

7. **In obtuse angled triangle one angle is of greater than.**
(a) 30° (b) 60° (c) 90° (d) 120°

8. **In right angled triangle one angle is of:**
(a) 90° (b) 180° (c) 270° (d) 360°

9. **A right triangle be equilateral:**
(a) can (b) can not (c) both (a) & (b) (d) none of above

10. **The ambiguous case occurs when we are given two sides and the angle opposite one of these is less than _____.**
(a) 90° (b) 60° (c) 45° (d) 30°

Answers:

1. three	2. different length	3. two	4. equal	5. three
6. 90°	7. 90°	8. 90°	9. can not	10. 90°

Additional Short Questions

Short Answer Questions prepared in the light of new Examination Techniques (Knowledge, Understanding, Application, Analytical & Conceptual)

11.1 Construction of Triangles

Short Answers Questions:

1. **What is the plural of locus?**
Ans. Loci is the plural of locus.

2. **Where loci are useful?**
Ans. Loci are useful for understanding and predicting patterns.

3. **Define triangle.**
Ans. A triangle is a closed figure having three sides and three angles.

4. **How many types of triangles w.r.t sides?**
Ans. There are three types of triangles w.r.t sides.

5. **How many types of triangle w.r.t angles?**
Ans. There are three types of triangle w.r.t angles.

6. **What is scalene triangle?**
Ans. In scalene triangle all sides of triangle are of different length.

7. **What do you mean isosceles triangle?**
Ans. In isosceles triangle two sides are of equal length.

8. **What is equilateral triangle?**
Ans. In equilateral triangle all sides of triangle are of equal length.

9. **What is acute angled triangle?**
Ans. All angles are of measure less than 90° in acute angled triangle.

10. Define obtuse angled triangle.

Ans. In obtuse angled triangle one angle is of measure greater than 90°.

11. What is right angled triangle?

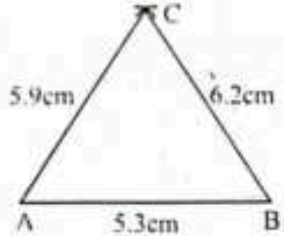
Ans. In right angled triangle one angle is of measure equal to 90°.

12. What is triangle inequality theorem?

Ans. The sum of measure of any two sides of a triangle is always greater than the measure of the third side.

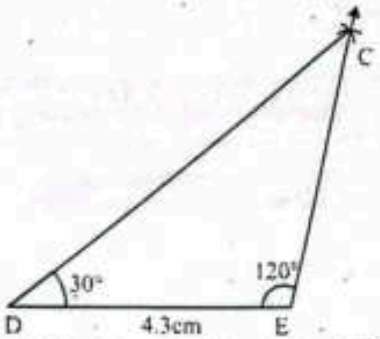
13. If $\overline{AC}$ = 5.9cm $\overline{AB}$ = 5.3cm, $\overline{BC}$ = 6.2cm then construct ΔABC.

Ans.



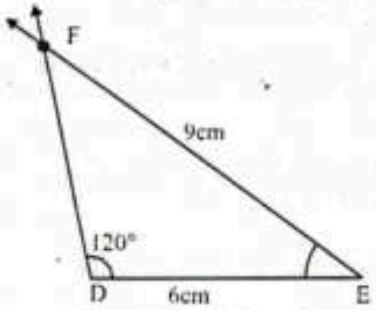
14. Draw a ΔCDE, mDE = 4.3cm m∠D = 30°, m∠DEC = 120°.

Ans.



15. Draw ΔDEF, m$\overline{DE}$ = 6cm, m∠D = 110°, m$\overline{EF}$ = 9cm.

Ans.



16. What do you mean by perpendicular bisector?

Ans. A perpendicular bisector is a line that intersects a line segment at right angle and dividing it into two equal parts.

17. Define median of triangle.

Ans. A median of a triangle is a line segment that joins a vertex to the mid point of the side that is opposite to that vertex.

18. What do you mean by point of concurrency?

Ans. A point of concurrency is the single point where three or more lines, rays or line segments intersects or meet in a geometric figure.

19. What is circum centre?

Ans. The point of concurrency of perpendicular bisector of the sides of a triangle is known as circum centre.

20. What is centroid?

Ans. The point of concurrency of the medians of a triangle is called centroid.

11.3 Angle Bisector of a Triangle

21. What do you mean by angle bisector?

Ans. An angle bisector is a linear line or ray which divides an angle into two equal parts creating two smaller angles which are congruent.

22. When angles of triangle are concurrent?

Ans. The angle bisectors of the triangle are concurrent.

23. What is incentre?

Ans. The point of concurrency of the angle bisectors of a triangle is called in centre of the triangle.

11.4 Altitudes of Triangles

24. Define altitude.

Ans. Altitude is a ray drawn perpendicular from a vertex to the opposite side of the triangle.

25. How many altitudes of a triangle?

Ans. There are three altitudes of a triangle, which meet at a single point, the altitude of a triangle are concurrent.

26. What is orthocenter?

Ans. The point of concurrency of the altitudes of the triangle is called orthocenter of the triangle.

11.5 Loci and Construction

27. What is the plural of locus?

Ans. The plural of locus is loci.

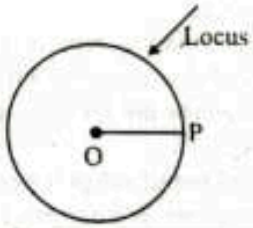
28. How loci are used in geometry?

Ans. In geometry, loci are used to define the positions of points relative to one another or to other geometric figures.

29. Define circle.

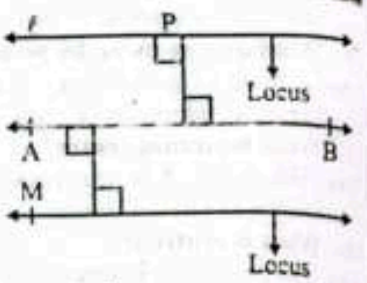
Ans. A locus of a point whose distance is constant from a fixed point is called circle.

O is fixed point and O is center, the locus of a point P whose distance $\overline{OP}$.



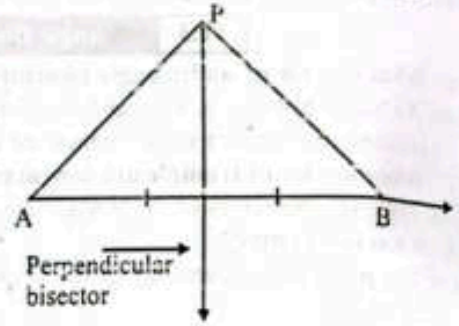
30. What are parallel lines?

Ans. The locus of a point whose distance from a fixed line is constant are parallel lines, ℓ and m.



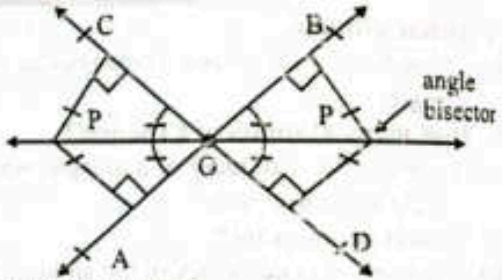
31. What is perpendicular bisector?

Ans. The locus of a point whose distance from two fixed points is constant is called perpendicular bisector.



32. What is angle bisector?

Ans. The locus of a point whose distance is constant from two intersecting lines is called an angle bisector.



33. What happen if two or more loci intersect at a point?

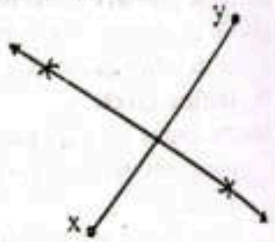
Ans. If two or more loci intersect at a point p, then p satisfies all given conditions of the loci.

11.6 Real Life Application of Loci

34. How park has two water sources fulfill as real life application of loci.

Ans. A park has two water sources at x_1 and x_2. A fire hydrant needs to be placed so it is equally accessible to both sources.

Suppose x_1 and x_2 represent the two water sources in the park. Draw the perpendicular bisector of x_1 and x_2 and represents the locus of all points equidistant from x_1 and x_2.



35. How working of robot is best example of loci?

Ans. A robotic arm in a factory is programmed to work within a specific area without

crossing into areas where it could interfere with other equipment. The loci of the robot's possible positions would be a defined space, such as a circle or rectangular region, ensuring it operates safely within its designated zone.

Conceptual Short Questions

Short Answers Questions:

1. When ambiguous case occurs?

Ans. The ambiguous case occurs when we are given two sides and the angle opposite one of these less than 90°.

2. In Latin what is the meaning of locus in English?

Ans. In Latin the word locus is defined by the English term location.

3. Define equidistant.

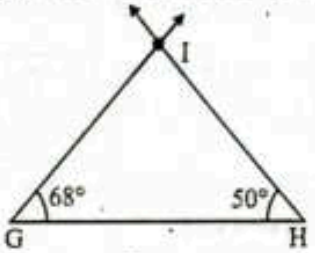
Ans. If A be a fixed point and B be a set of points. If A is at equal distance from all points of B, then A is said to be equidistant from B.

4. What is circle?

Ans. Locus of points equidistant from a fixed point is a circle and equidistant from two fixed points is a perpendicular bisector.

5. Draw a triangle GH I when $m\overline{GH} = 5.7cm$ $m\angle G = 68°$ and $m\angle H = 50°$.

Ans.



6. How many altitudes of a triangle?

Ans. There are three altitudes of a triangle.

7. What is the difference between acute and obtuse angled triangle.

Ans. In acute angled triangle all angles are of measure less than 90° while one angle is of measure greater than 90° in obtuse angled triangle.

8. Name the triangle in which two sides are same in length.

Ans. Isosceles triangle

9. Name the triangle in which all sides are different in length?

Ans. Scalene triangle.

10. In which triangle one angle is 90°?

Ans. In right angled triangle one angle is 90°.

— ✲✲✲ —

Unit 12 Information Handling

Information Handling

Information handling is the process of collecting, organizing, summarizing, analyzing and interpreting numerical data.

Classification of Data

Data is further classified into two categories.

(i) **Discrete data:** It can take only some specific values, whole numbers are used to write discrete data, e.g., number of books sold by a shopkeeper, number of patients visited a hospital in a week etc. This data is only obtained by counting.

(ii) **Continuous data:** It can take every possible value in a given interval. Decimal numbers are used to write continuous data. The data is only obtained by measuring e.g., the mass of students in class i.e., 28.5 kg, 26.5 kg, 27.5 kg etc.

Ungrouped and Grouped Data

Data which is not arranged in any systematic order (groups or classes) is called ungrouped data. For example, the number of toys sold by a shopkeeper in a month is given below:

10, 5, 8, 12, 15, 20, 25, 30, 23, 15, 23, 21, 18, 15 17, 23, 22,15, 20, 21, 24,18,16, 21, 23,21,17,19,21,23. This data is called ungrouped data.

Do you know?
Ungrouped data is also known as raw data.

If we arrange the above given data in groups or classes, then it is called grouped data.

Classes	Tally marks	No. of toys sold
5-9	||	2
10-14	||	2
15-19	𝍸 ||	10
20-24	𝍸 𝍸 ||||	14
25-29	|	1
30-34	|	1

Teachers' note!
By using more examples, clear the concept of grouped data and ungrouped data to the students.

In above grouped data, 5,10,15, 20,25 and 30 are lower class limits and 9,14, 19,24, 29 and 34 are upper class limits.

Frequency Distribution

A distribution or table that represents classes or groups along with their respective class frequencies is called frequency distribution.

Think!
If the size of class limits is 6. The greatest value is 80 and the smallest value is 25. Can you find the number of class limits for the data?

Formation of Frequency distribution

In this method, the raw data or the ungrouped data is presented into a grouped data.

Choice is yours to select the number of classes.

Generally, the size of class limits is determined on the basis of the greatest value, smallest value and the desired number of groups or classes.

Following are the major steps to construct frequency distribution:

(i) Find the range of the data. Range is the difference between the greatest value and the smallest value i.e., Range $= X_{max} - X_{min}$

(ii) Find the size of the class by dividing the range by the number of classes or groups you wish to make.

Keep in mind!
The number of times a value occurs in a data is called the frequency of that value. It is denoted by "f".

(iii) Prepare four columns.
(a) Class limits (b) Tally marks (c) Frequencies (d) Class Boundaries

(iv) Make classes having size of 11. Start from the smallest value.

(v) Look for the class in which each element of ungrouped data falls. Draw a small tally mark (|) against that class and also tick the element concerned with a sign (✓). In this way you can remember that you have counted for the element. Continue this way with the next element that upto the last element of the data set. If 5 or more tallies appear in any class, mark every 5th tally diagonally 𝍸.

(vi) Class boundaries usually are found by the following method:

- Chose the upper class limit of the 1st class and lower class limit of the 2nd class.
- Find the difference between these two limits.
- The difference is divided by 2 and subtract it from the lower class limit and add it to the upper class limit.

Do you know?
Class boundaries may also be obtained from the midpoints (x) as $\left[x \pm \frac{h}{2}\right]$, where h is the difference between any two consecutive values of x.

Example 1: Following are the number of telephone calls made in a week to 30 teachers of a high school.

5	8	11	25	13	16	20	17	15	16	30	21	14	18	19
6	22	26	15	19	35	29	31	23	25	20	10	9	7	26

Construct a frequency distribution with number of classes 7.

Solution: (i) Find range

Greatest value (maximum value) =35, Smallest value (minimum value) = 5

$$\text{Range} = X_{max} - X_{min} = 35 - 5 = 30$$

(ii) Size of class limits $= \dfrac{\text{Range}}{\text{Number of classes}} = \dfrac{30}{7} = 4.28 \approx 5$

(iii) Make class limits having size 5. For example, 5 – 9,10 – 14,15 – 19 and so on. (see 1st column of table: 1).

(iv) Tally marks are used to count the values, fall in the given class limits. (See 2nd column of table: 1).

(v) Now, count the number of tally marks and write the number as frequency in the third column (see 3rd column of table: 1).

(vi) Class boundaries The difference between lower class limit of the second class and upper class limit of the first class is 1. i.e., 10 – 9 = 1. Now, divide

Activity
Collect data of height of 50 students in your class, and convert the data into grouped data.

the difference of the limits by 2 i.e. $\frac{1}{2} = 0.5$.

Lower class boundaries are obtained by "subtracting 0.5" from the lower class limits.
Upper class boundaries are obtained by "adding 0.5" to the upper class limits.

Lower class boundaries	Upper class boundaries
5 – 0.5	9 + 0.5
4.5 and so on.	9.5 and so on.

(see 4th column of the table: 1)

Table 1

Class limits	Tally marks	Frequency (f)	Class Boundaries (C.B)
5 – 9	𝍸	5	4.5 – 9.5
10 – 14	𝍸	4	9.5 – 14.5
15 – 19	𝍸 III	8	14.5 – 19.5
20 – 24	𝍸	5	19.5 – 24.5
25 – 29	𝍸	5	24.5 – 29.5
30 – 34	II	2	29.5 – 34.5
35 – 39	I	1	34.5 – 39.5

Graph of Frequency Distribution

The following are the types of graphs which can be used to represent a frequency distribution on a graph.

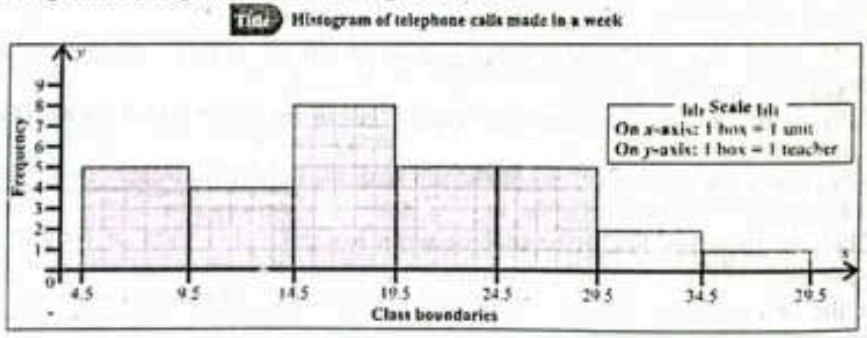
(a) Histogram (b) Frequency polygon

(a) Histogram (with equal class limits)

This is a graph of adjacent rectangles constructed on xy plane. A histogram is similar to bar graph but it is constructed for a frequency distribution. The procedure for making histogram:

Do you know?
Continuous data is mostly represented by using histogram and frequency polygon.

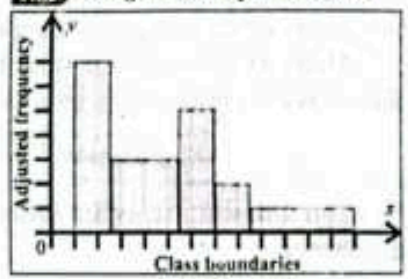
(i) Draw lines as x - axis and as y – axis on a graph paper perpendicular to each other.
(ii) Class boundaries are marked on x – axis and a rectangle is made against each group with its width proportional to the size of class limits and height proportional to the class frequencies.
(iii) Setting a scale, draw frequencies on y – axis. The resulting figure is called histogram. Histogram of table: 1 is given below:



12.1.3 Histogram (with unequal class limits)

The procedure for making histogram is explained below:

i. Draw lines as x – axis and y – axis on a graph paper perpendicular to each other.
ii. Class boundaries are marked on x–axis and a rectangle is made against each group with its width proportional to the size of class limits and height proportional to the class frequencies.
iii. This can be achieved by adjusting the heights of rectangle. The height of each rectangle is obtained by dividing each class frequency on its class limit size.



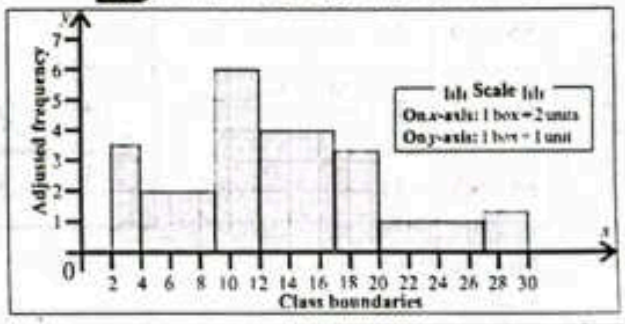
Example 2: The frequency distribution of ages (in years) of 76 members of a locality is available. Draw a histogram for this data.

Class limits	2-4	4-9	9-12	12-17	17-20	20-27	27-30
Frequency (f)	7	10	18	20	10	7	4

Solution: Look at the table, indicates that the width of the class limits is not equal as first class has width 2, second has 5, the third has 3, the fourth has 5, the fifth has 3, sixth class has 7, seventh class has width 3. So, there is need to adjust the heights of the rectangles i e., for the first class we have 2 as width of class and 7 as a frequency, so the height of the first class is $\frac{7}{2} = 3.5$, similarly for the other $\frac{10}{5} = 2$, $\frac{18}{3} = 6$, $\frac{20}{4} = 4$, $\frac{10}{3} = 3.3$, $\frac{7}{1} = 1$, $\frac{4}{3} = 1.3$.

Class limits	Frequency (f)	Width of Class	Height of rectangle (Adjusted frequency)
2 – 4	7	4 – 2 = 2	$\frac{7}{2} = 3.5$
4 – 9	10	9 – 4 = 5	$\frac{10}{5} = 2$
9 – 12	18	12 – 9 = 3	$\frac{18}{3} = 6$
12 – 17	20	17 – 12 = 5	$\frac{20}{5} = 4$
17 – 20	10	20 – 17 = 3	$\frac{10}{3} = 3.3$
20 – 27	7	27 – 20 = 7	$\frac{7}{7} = 1$
27 – 30	4	30 – 27 = 3	$\frac{4}{3} = 1.3$

These proportional heights are also calld adjusted frequencies. Taking class boundaries along x-axis and corresponding adjusted frequencies along y-axis, rectangles are drawn and the histogram is given below:.



Frequency Polygon

A frequency polygon is a closed geometrical figure used to display a frequency distribution graphically. A line graph of a frequency distribution is known as frequency polygon in which frequencies are plotted against their midpoints.

Midpoint is the average value of the lower and upper class limits. Midpoint is also known as class mark. Midpoint is calculated by the given formula:

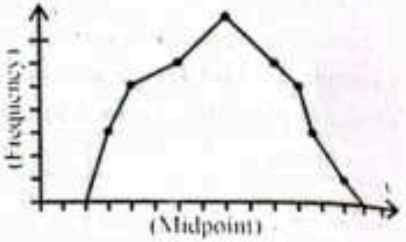
$$\text{Mid point} = \frac{\text{Lower class limit} + \text{Upper class limit}}{2}$$

The following steps are followed to draw a frequency polygon for a frequency distribution:

(i) Draw lines as x-axis and y-axis perpendicular to each other.

(ii) Take midpoints on x-axis and class frequencies on y-axis.

(iii) Put a dot mark against each midpoint corresponding to its class frequency. Join all the dotted marks by straight lines to get the required frequency polygon.

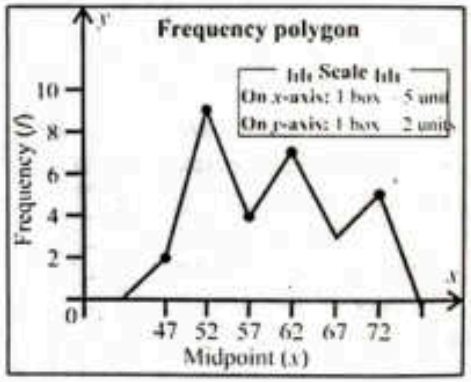


(iv) The lines at both ends are joined together with the next midpoints to touch the bases of x-axis.

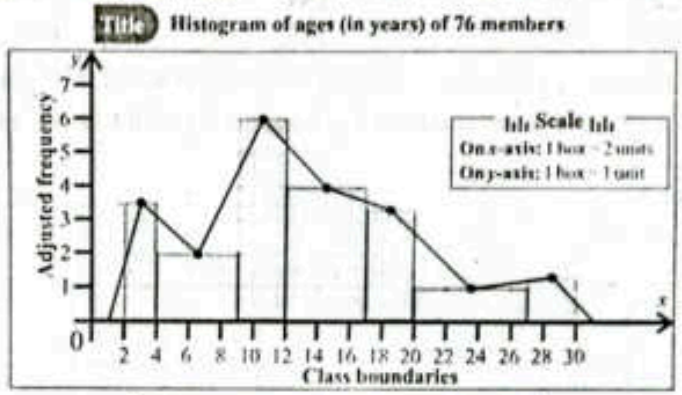
Example 3: The following are the marks obtained by 30 students out of 100 in the subject of Mathematics at their final examination. Construct frequency polygon for the following frequency table.

Marks	45–49	50–54	55–59	60–64	65–69	70–74
Frequency	2	9	4	7	3	5

Marks	f	Midpoints
45 – 49	2	$\frac{45+49}{2} = 47$
50 – 54	9	$\frac{50+54}{2} = 52$
55 – 59	4	$\frac{55+59}{2} = 57$
60 – 64	7	$\frac{60+64}{2} = 62$
65 – 69	3	$\frac{65+69}{2} = 67$
70 – 74	5	$\frac{70+74}{2} = 72$



Remember!
Frequency polygon on histogram: In histogram, we mark the midpoints on the top of rectangles and join all the points. To touch the base of x-axis, we extend the line at both ends to the next midpoints. The resulting graph is a frequency polygon.



Solved Exercise 12.1

1. The following distribution represents the scores achieved by a group of chemistry students in the chemistry laboratory.

Scores	24-28	29-33	34-38	39-43	44-48	49-53	Total
No. of students	3	6	12	23	15	6	65

Answer the following questions.

(i) What is the upper limit of the last class?

Sol: The upper limit of the last class is 53.

(ii) What is the lower limit of the class 39-43?

Sol: The lower limit of the class 39-43 is 39.

(iii) What is the midpoint of the class (34 - 38)?

Sol: The midpoint of class (34-38) is 36.

(iv) What are the class frequencies of the classes 29 - 33 and 44 - 48?

Sol: The class frequency of the class 29-33 is 6 and the frequency of the class 44-48 is 15.

(v) What is the size of the class limits in the above frequency distribution?

Sol: The size of the class limits in the above frequency distribution is 5.

(vi) In which class or group does minimum number of students fall?

Sol: In class (24-28) minimum number of students fall.

(vii) What is the lower limit of the class having 15 as its class frequency?

Sol: The lower limit of the class having 15 as its frequency is 44.

(viii) What is the number of students having scores between 24 and 43?

Sol: The number of students having scores between 24 and 43 is 44.

2. For a school staff, the following expenditures (rupees in hundred) are required for the repair of chairs.

145, 152, 153, 156, 158, 160, 146, 152, 155, 159,
161, 163, 165, 147, 148, 151, 154, 156, 158, 160,
144, 167, 151, 150, 152, 149, 145, 153, 152, 155

Prepare a frequency distribution by tally bar method using 3 as the size of class limits and also write down what are the frequencies of the last three classes?

Sol. Minimum value = 144
Maximum value = 167
size of the class limits = 3 (Given)

Frequency Distribution Table

Class limits	Tally marks	f
144 – 146	𝍸	4
147 – 149	𝍸	3
150 – 152	𝍸 ||	7
153 – 155	𝍸	5
156 – 158	𝍸	4
159 – 161	𝍸	4
162 – 164	|	1
165 – 167	||	2
Total		$\Sigma f = 30$

The frequencies of the last three classes are 4, 1 and 2 respectively.

3. Given below are the weights in kg of 30 students of a high school.

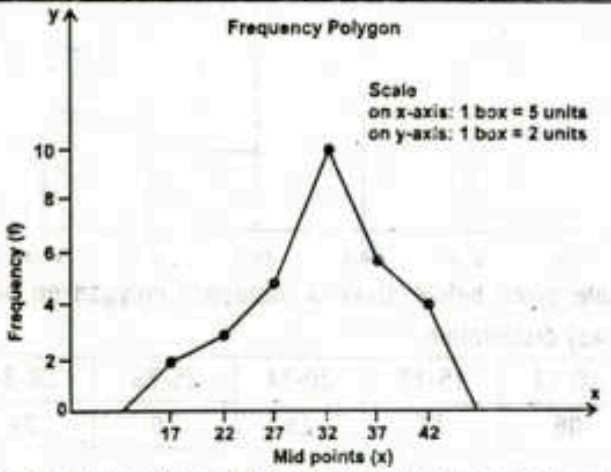
30, 33, 24, 21, 15, 39, 37, 44, 42, 33,
33, 28, 29, 32, 31, 28, 26, 32, 34, 35
38, 36, 41, 30, 35, 41, 23, 26, 18, 34

Taking 5 as the size of the class limit, prepare a frequency table and construct a frequency polygon.

Sol. Minimum value = 15
Maximum value = 44
size of the class limits = 5 (Given)

Class limits	Tally marks	f	Midpoints (x)
15 – 19	||	2	$\frac{15+19}{2} = 17$
20 – 24	𝍸	3	$\frac{20+24}{2} = 22$

25 – 29	𝍸	5	$\frac{25+29}{2} = 27$
30 – 34	𝍸 𝍸	10	$\frac{30+34}{2} = 32$
35 – 39	𝍸 |	6	$\frac{35+39}{2} = 37$
40 – 44	𝍸	4	$\frac{40+44}{2} = 42$
Total		$\Sigma f = 30$	



4. A group of Grade -10 students obtained the following marks out of 100 marks in English test.

58, 59, 58, 33, 40, 58, 45, 46, 43, 45, 45,
50, 52, 49, 50, 57, 52, 55, 49, 50, 62, 49,
48, 44, 42, 47, 46, 47, 46, 53, 40, 44

Classify the data into a frequency distribution by (direct method) taking 6 as the size of class limit. Also find the class limit with least class frequency and construct histogram for the data.

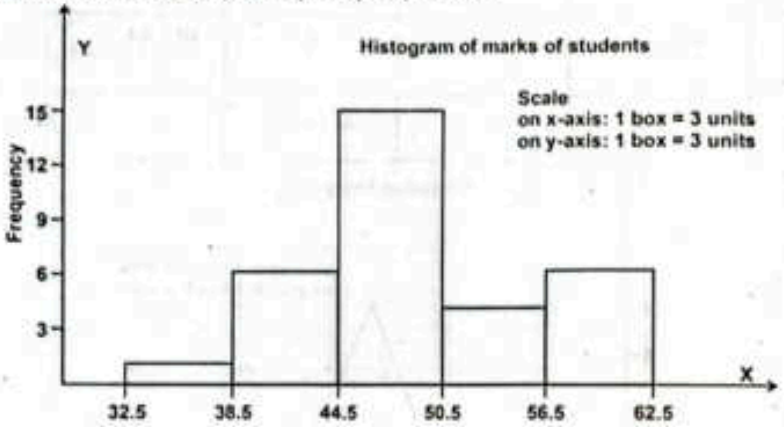
Sol. Minimum value = 33
Maximum value = 62
size of the class limits = 6 (Given)

Frequency Distribution Table

Class limits	Tally marks	f	Class Boundries
33 – 38	|	1	32.5—38.5

39 – 44	𝍸 I	6	38.5—44.5
45 – 50	𝍸 𝍸 𝍸	15	44.5—50.5
51 – 56	IIII	4	50.5—56.5
57 – 62	𝍸 I	6	56.5—62.5
Total		$\Sigma f = 32$	

Class limits with least class frequency is (33–38).



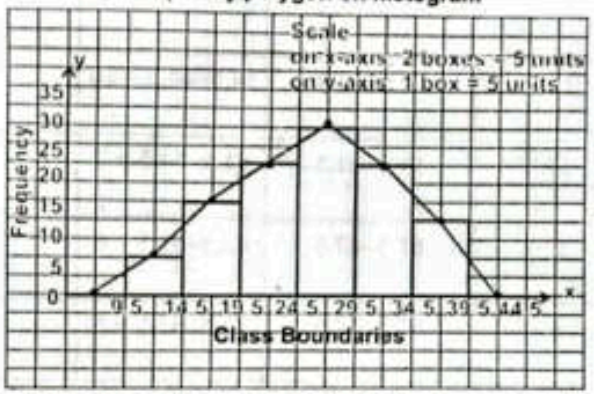
5. From the table given below. Draw a frequency polygon on histogram for the given frequency distribution.

Weight (kg)	10-14	15-19	20-24	25-29	30-34	35-39
Frequency (f)	06	17	23	30	22	13

Sol.

Weight (kg)	Frequency (f)	Midpoints	Class Boundries
10–14	06	$\frac{10+14}{2} = 12$	9.5 — 14.5
15–19	17	$\frac{15+19}{2} = 17$	14.5 — 19.5
20–24	23	$\frac{20+24}{2} = 22$	19.5 — 24.5
25–29	30	$\frac{25+29}{2} = 27$	24.5 — 29.5
30–34	22	$\frac{30+34}{2} = 32$	29.5 — 34.5
35–39	131	$\frac{35+39}{2} = 37$	34.5 — 39.5

Frequency polygon on histogram



The following data shows the number of heads in an experiment of 50 sets of tossing a coin 5 times. Make a discrete frequency distribution from the information.

3, 3, 4, 0, 5, 4, 3, 3, 1, 2, 4, 5, 0, 3, 2, 4, 4, 0, 0, 0, 5, 5, 3, 2, 1
4, 3, 2, 5, 3, 2, 1, 3, 5, 4, 3, 2, 1, 3, 2, 1, 3, 1, 3, 1, 4, 3, 2, 2, 4

Sol: Discrete Frequency Distribution Table

No. of heads	Tally marks	F
0	𝍸	5
1	𝍸 II	7
2	𝍸 IIII	9
3	𝍸 𝍸 IIII	14
4	𝍸 IIII	9
5	𝍸 I	6
Total		$\Sigma f = 50$

7. The marks obtained by the students of Grade - 10 in mathematics test were grouped into the following frequency distribution.

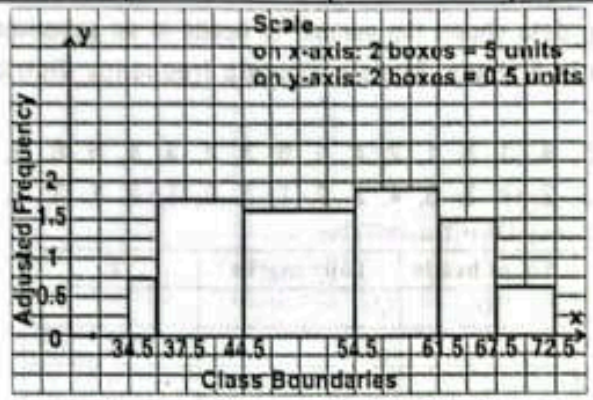
Marks	35–37	38–44	45–54	55–61	62–67	68–72
Frequency (f)	2	12	16	13	9	3

Draw a histogram for the above distribution.

Sol:

Class limits	Frequency (f)	class-Boundries	width of class	Height of rectangles Adjusted Frequency
35–37	2	34.5–37.5	37.5–34.5 = 3	$\frac{2}{3} = 0.7$

38–44	12	37.5–44.5	44.5–37.5 = 7	$\frac{12}{7}$ =1.7
45–54	16	44.5–54.5	54.5–44.5 = 10	$\frac{16}{10}$ =1.6
55–61	13	54.5–61.5	61.5–54.5 = 7	$\frac{13}{7}$ =1.9
62–67	9	61.5–67.5	67.5–61.5 = 6	$\frac{9}{6}$ =1.5
68–72	3	67.5–72.5	72.5–67.5 = 5	$\frac{3}{5}$ =0.6



8. Make a frequency polygon on histogram for the following grouped data:

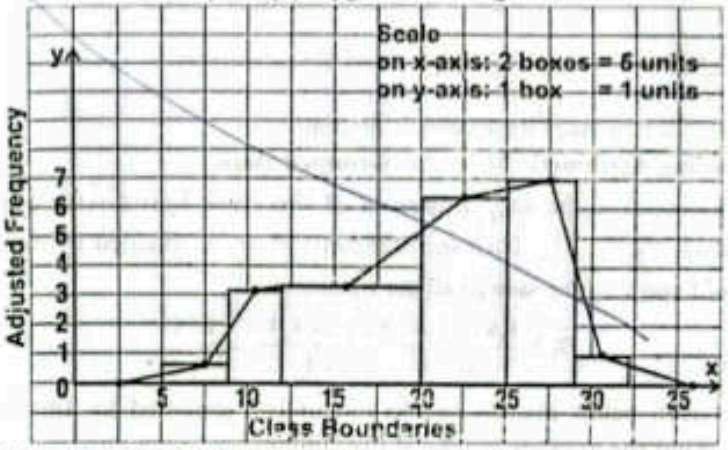
Class limits	5–8	8–12	12–20	20–25	25–27	27–32
Frequency (*f*)	2	12	25	32	14	5

Sol:

Class limits	Frequency (*f*)	Width of class	Height of rectangle (Adjusted frequency)
5 – 8	2	8 – 5 = 3	$\frac{2}{3}$ =0.7
8 – 12	12	12 – 8 = 4	$\frac{12}{4}$ =3
12 – 20	25	20 – 12 = 8	$\frac{25}{8}$ =3.1
20 – 25	32	25 – 20 = 5	$\frac{32}{5}$ =6.4
25 – 27	14	27 – 25 = 2	$\frac{14}{2}$ =7

27 – 32	5	32 – 27 = 5	$\frac{5}{5}$ =1

Frequency polygon on histogram



Measures of Location (Central Tendency)

The measure that gives the centre of the data is called measure of central tendency.

The following are some measure of central tendency will be discussed in this section:

(i) Arithmetic Mean (A.M.) **(ii)** Median **(iii)** Mode **(iv)** Weighted mean

Arithmetic Mean (A.M.)

It is defined as a value of variable which is obtained by dividing the sum of all the values (observations) by their number of observations. Thus, the arithmetic mean of a set of values $x_1, x_2, x_3 \ldots x_n$ is denoted by $\overline{X}$ (read as X-bar) and is calculated as:

$$\overline{X} = \frac{x_1 + x_2 + x_3 + \ldots x_n}{n} = \frac{\sum x}{n} \text{ (Direct method)}$$

where, the sign $\sum$ stands for the sum and n is the number of observations.

Example 4: The marks of a student in five examinations were 64, 75, 81, 87, 90. Find the arithmetic mean of the marks.

Solution: $A.M. = \overline{X} = \frac{\sum x}{n}$

$$= \frac{64+75+81+87+90}{5}$$

$$\text{or } \overline{X} = \frac{397}{5} = 79.4$$

Try Yourself!
The mean of 10, 30, 40, x, 67 and 81 is 50. Find the value of the x.

Example 5: A government allocates funds of Rs.200,000 to five sectors of a school i.e.,

(i) School Library: Rs. 35,000
(ii) Sports facilities: Rs. 25,000
(iii) Parking area: Rs. 40,000
(iv) Room renovation: Rs. 45,000
(v) Furniture: Rs. 55,000

Try Yourself!
The mean of 15 values was 50. It was found on rechecking that the value 25 was wrongly copied as 52. Find the correct mean.

Find the average of fund allocation in each sector of a school.

Solution: To find out the average of each sector, we will find the mean of the given data.

$$\overline{X} = \frac{35,000 + 25,000 + 40,000 + 45,000 + 55,000}{5}$$

$$\overline{X} = \frac{200,000}{5} \Rightarrow \overline{X} = \text{Rs. } 40,000$$

On average, each sector takes Rs.40.000 in funding.

Method of finding Arithmetic Mean for Grouped Data

Let $x_1, x_2, x_3 \ldots x_n$ be the midpoints of the class limits with corresponding frequencies say $f_1, f_2, f_3 \ldots f_n$. Then the arithmetic mean is obtained by dividing sum of the products of f and x by the sum of all the frequencies.

$$\overline{X} = \frac{f_1x_1 + f_2x_2 + \ldots\ldots f_nx_n}{f_1 + f_2 + \ldots\ldots + f_n} = \frac{\Sigma fx}{f}$$

Example 6: Given below are the marks out of 100 obtained by 100 students in a examination. Find the average marks of the students.

Marks	30 – 35	35 – 40	40 – 45	45 – 50	50 – 55	55 – 60
No. of students	14	16	18	23	18	11

Solution:

Marks	Midpoint (x)	Frequency (f)	fx
30-35	32.5	14	455.0
35-40	37.5	16	600.0
40-45	42.5	18	765.0
45-50	47.5	23	1092.5
50-55	52.5	18	945.0
55-60	57.5	11	632.5
Total	------	$\Sigma f = 100$	$\Sigma fx = 4490$

$$\overline{X} = \frac{\Sigma fx}{\Sigma f} = \frac{4490}{100}$$

or $\overline{X} = 44.9$ marks

Hence, the average marks is 44.9 of the surdents.

Short Formula for Computing Arithmetic Mean

Let A be considered as assumed or provisional mean (may be any value from the values of x or any number) and D denotes the deviations of x from A i.e., $D = x - A$. For $x = D + A$, the formula of arithmetic mean becomes;

$$\overline{X} = A + \frac{\Sigma D}{\Sigma n} \quad \text{(for ungrouped data)} \quad ...(i)$$

$$\overline{X} = A + \frac{\Sigma fD}{\Sigma f} \quad \text{(for grouped data)} \quad ...(ii)$$

Example 7: Find the arithmetic mean using short formula for the runs made by a batsman.

Runs: 40, 45, 50, 52, 50, 60, 56, 70.

Solution: Taking deviations from A = 52 (assumed mean)

x	40	45	50	52	50	60	56	70
D=x–A	–12	–7	–2	0	–2	8	4	18

Now: $\Sigma D = -23 + 30 = 7$

$$\because \quad \overline{X} = A + \frac{\Sigma D}{n} \Rightarrow \overline{X} = 52 + \frac{7}{8}$$

$= 52 + 0.875 = 52.88$ or 53 runs.

Example 8: Deviations from 12.5 of ten different values are 6, –2, 3.5, 9, 8.7, –5.5, 14, 11.3, –6.8, –4.2, find the arithmetic mean.

Solution: Deviations from 12.5 are:

6, –2, 3.5, 9, 8.7, –5.5, 14, 11.3, –6.8, –4.2 Now, $\Sigma D = 34.80$

Now, $\Sigma D = 34$. Also, A = 12.5, using the formula we have.

$$\overline{X} = A + \frac{\Sigma D}{n} = 12.5 + \frac{34}{10}$$

or $\overline{X} = 12.5 + 3.4 = 15.9$

Example 9: The heights (in inches) of 200 students are recorded in the following frequency distribution. Find the mean height of the student by short formula.

Height (x) (in inches)	51	52	53	54	55	56	57	58	59	60
Frequency (f)	2	5	8	24	55	45	38	16	6	1

Solution:

Heights (x) (in inches)	Frequency (f)	A = 55 D = x–A	fD
51	2	–4	–8
52	5	–3	–15
53	8	–2	–16
54	24	–1	–24
A ← 55	55	0	0
56	45	1	45
57	38	2	76
58	16	3	48
59	6	4	24
60	1	5	5
Total	$\Sigma f = 200$	$\Sigma fD = 135$	

Now, using the formula (ii), we get

$$\bar{X} = A + \frac{\Sigma fD}{f}$$

$$\bar{X} = 55 + \frac{135}{200}$$

or $\bar{X} = 55 + 0.675$

$\therefore$ $\bar{X} = 55.68$ inches approx.

Hence, the mean height of the students is 55.68 inches.

Example 10: Ten students each from Grade-V section A and B of a well reputed school were taken randomly. Their weights were measured in kg. and recorded as given below:

Weights (kg) Section A	30	28	32	29.5	35	34	31	33	40	37.5
Weights (kg) Section B	35	31.5	34.5	35	32.8	38	29.5	36	36.5	34

(i) Compute the mean weight for section A and B.

(ii) Conclude which section is better on Average?

Solution: (i) We find arithmetic mean for both the sections by direct method. (Any method can be applied).

$X_{(A)}$	$X_{(B)}$
30	35
28	31.5
32	34.5
29.5	35
35	32.8
34	38
31	29.5
33	36
40	36.5
37.5	34
$\Sigma X_{(A)} = 330$	$\Sigma X_{(B)} = 342.8$

As number of observations n = 10

and $\bar{X}_{(A)} = \frac{\Sigma X_{(A)}}{n}$

$\therefore$ $\bar{X}_{(A)} = \frac{330}{10} = 33\text{kg}$

and $\bar{X}_{(B)} = \frac{\Sigma X_{(B)}}{n}$

$\therefore$ $\bar{X}_{(B)} = \frac{342.8}{10} = 34.28\text{ kg}$

(ii) We have seen from the results that $\bar{X}_{(B)}$ is greater than $\bar{X}_{(A)}$. Therefore, we conclude that section B is better on the average.

Median

Median is the middle most value in an arranged (ascending or descending order) data set. Median is the value which divides the data into two equal parts i.e., 50% data is before the median and 50% data after it. Median is denoted by $\tilde{X}$.

Median for ungrouped data

The median of n observations $x_1, x_2, \ldots, x_n$ is obtained as:

$$\text{Median } (\tilde{X}) = \left(\frac{n+1}{2}\right)^{th} \text{ observation } \begin{pmatrix}\text{when n is}\\ \text{odd number}\end{pmatrix}$$

$$\text{Median } (\tilde{X}) = \frac{1}{2}\left(\left(\frac{n}{2}\right)^{th} \text{observation} + \left(\frac{n+2}{2}\right)^{th} \text{observation}\right) \begin{pmatrix}\text{when n is}\\ \text{even number}\end{pmatrix}$$

Example 11: The following are the scores made by a batsman. Find the median of the data. 8,12,18,13,16, 5,20.

Solution: Writing the scores in an ascending order, we have

5, 8, 12, 13, 16, 18, 20

Since, number of observations is odd i.e., n = 7

$$\text{Median } (\tilde{X}) = \left(\frac{n+1}{2}\right)^{th} \text{ observation}$$

$$= \left(\frac{7+1}{2}\right)^{th} \text{ observation} = 4^{th} \text{ observation} = 13$$

Hence, 13 is the median of the given data.

Example 12: Following are the marks out of 100 obtained by 10 students in English. 13, 15, 35, 48, 41, 5, 8, 9, 11, 51. Find the median of the data.

Solution: Arranging the data in an ascending order.

5, 8, 9, 11, 15, 23, 35, 41, 48, 51

Since, number of observation is even, i.e., n = 10

$$\text{Median } (\tilde{X}) = \frac{1}{2}\left(\left(\frac{n}{2}\right)^{th} \text{observation} + \left(\frac{n+1}{2}\right)^{th} \text{observation}\right)$$

As, $\frac{n}{2} = \frac{10}{2} = 5$ and $\frac{n+2}{2} = \frac{12}{2} = 6$

$$\text{Median} = \frac{1}{2}\,[5^{th} \text{ observation} + 6^{th} \text{ observation}]$$

$$\text{Median} = \frac{1}{2}\,[15 + 23] = \frac{38}{2} = 19$$

Hence, 19 is the median of the data.

Median for Grouped Data

The median for grouped data is obtained by the following formula:

$$\text{Median } (\tilde{X}) = \ell + \frac{h}{f}\left(\frac{n}{2} - c\right)$$

Where, ℓ = Lower class boundary of median class,

h = The size of class limits of median class,

f = Frequency of the median class,

n = Total frequency i.e., Σf,

and c = Cumulative frequency preceding the median class.

Remember the following points:

i) The groups of classes must be in a continuous form i.e., we need class boundaries.

ii) Make the column of cumulative frequencies (c.f.) from the column of frequencies.

(iii) Locate median class i.e., $\left(\frac{n}{2}\right)^{th}$ see value in c.f. column wherever it lies.

(iv) Underline the median class, then take the values of f and h of the median class thus obtained

Example 13: The heights of 100 athletes, measured to the nearest (inches) are given in the following table. Find the median.

Heights (in inches)	62.5-63.5	63.5-64.5	64.5-65.5	65.5-66.5	66.5-67.5	67.5-68.5	68.5-69.5	69.5-70.5	70.5-71.5
No. of Students	4	6	10	20	30	13	12	3	2

Solution: In the above data, class boundaries have already been given:

Heights (inches)	Frequency (f)	c.f.	
62.5 – 63.5	4	4	
63.5 – 64.5	6	6 + 4 = 10	
64.5 – 65.5	10	10 + 10 = 20	
65.5 – 66.5	20	20 + 20 = 40 → c	
66.5 – 67.5	30	30 + 40 = 70 →	Median class
67.5 – 68.5	13	13 + 70 = 83	
68.5 – 69.5	12	12 + 83 = 95	
69.5 – 70.5	3	3 + 95 = 98	
70.5 – 71.5	2	2 + 98 = 100 → n	
Total	**$\Sigma f = 100$**	---	

Here, $n = 100$

so, $\frac{n}{2} = \frac{100}{2} = 50$

50^{th} item lies in the class boundaries 66.5 – 67.5

$\ell = 66.5$, $h = 1$, $f = 30$, $c = 40$

$\because$ Median $= \ell + \frac{h}{f}\left(\frac{n}{2} - c\right)$

$= 66.5 + \frac{1}{30}(50 - 40)$ (Putting the values)

$= 66.5 + \frac{10}{30}$

$= 66.5 + 0.33$

$\therefore$ Median $= 66.83$ inches

Example 14: Following are the weights (in kg) of 50 men. Find the median weight.

Weights (kg)	110-114	115-119	120-124	125-129	130-134
No. of men (f)	5	12	23	6	4

Solution: As class boundaries are not given so, first of all we make class boundaries by the usual procedure.

Weight (kg)	Frequency(f)	Class Boundaries	c.f	
110–114	5	109.5–114.5	5	
115–119	12	114.5–119.5	17 → c	
120–124	23	119.5–124.5	40 →	Median class
125–129	6	124.5–129.5	46	
130–134	4	129.5–134.5	50 → n	
Total	**$\Sigma f = 50$**	---	---	

Here $n = 50$ so, $\frac{n}{2} = \frac{50}{2} = 25.25^{th}$ item lies in 119.5 – 124.5.

$\ell = 119.5$, $h = 5$, $f = 23$, $c = 17$

Median $= \ell + \frac{h}{f}\left(\frac{n}{2} - c\right)$

$= 119.5 + \frac{5}{23}(25 - 17)$ (Putting the values)

$= 119.5 + \frac{40}{23} = 119.5 + 1.74$

$\therefore$ Median $= 121.24$ kg

Mode

In a data the values (observation) which appears or occurs most often is called mode of the data. It is the most common value. Mode is denoted by $\hat{x}$.

Mode for Ungrouped Data

Example 15: The marks in mathematics of Jamal in eight monthly tests were 75, 76, 80, 80, 82, 82, 82, 85. Find the mode of the marks.

Solution: As 82 is repeated more than any other number so, clearly mode is 82.

Example 16: Ten students were asked about the number of questions they have solved out of 20 questions last week. Records were 13,14,15,11,16,10,19,20,18,17. Find the mode of the data.

Solution: It is obvious that the given data contains no mode. It is ill-defined. Sometimes data contains several modes. If the data is: 10, 15, 15, 15, 20, 20, 20, 25, 32, then data contains two modes i.e., 15 and 20.

Example 17: A survey was conducted from the 15 students of a school and asked the students about their favourite colour.

The responses are: purple, yellow, purple, yellow, yellow, red, blue, green, yellow, yellow, red, blue, yellow, purple, green. Find mode of the data.

Solution: Mode is the most frequent colour.

Mode = yellow

So, the colour "yellow" is the mode of the given data.

> **Remember!**
> A data can has more than one mode.
> A data may or may not have a mode.

Mode for Grouped Data

Mode can be calculated by the following formula:

$$\text{Mode} = \ell + \frac{(f_m - f_1)}{(f_m - f_1)(f_m - f_2)} \times h$$

Where, ℓ = Lower class boundary of the modal class.
f = Frequency of the modal class.
f_2 = Frequency preceding the modal class.
f_3 = Frequency following the modal class and
h = Size of the modal class.

> **Note:**
> Mode cannot be easily calculated from the data presented in a frequency distribution. As it has no individual values, so we do not know which value appears most frequently. We only assume the class with the highest frequency as a modal class.

Example 18: Following are the heights in (inches) of 40 students in Grade - 8.

Heights (inches)	48-50	50-52	52-54	54-56	56-58	58-60
No. of students (f)	5	7	10	9	6	3

Find mode of the above data.

Solution

Heights (inches)	Frequency (f)
48 – 50	5
50 – 52	$7 \rightarrow f_1$
52 – 54	$10 \rightarrow f_m$,
54 – 56	$9 \rightarrow f_2$
56 – 58	6
58 – 60	3
Total	$\Sigma f = 40$

> **Teachers' note!**
> Collect data of weights of 50 students. Make a frequency distribution and find mean, median and mode of the data.

Solution: In the above data, class boundaries have already been given. Using the formula for grouped data we find mode as:

$\ell = 52, h = 2, f_m = 10, f_1 = 7, f_2 = 9$

$$\text{Mode} = \ell + \frac{(f_m - f_1) \times h}{(f_m - f_1)(f_m - f_2)}$$

or $$\text{Mode} = 52 + \frac{(10-7) \times 2}{(10-7)+(10-9)}$$

or $$\text{Mode} = 52 + \frac{3 \times 2}{3+1} = 52\frac{6}{4}$$

or Mode = 52 + 1.5 = 53.5 (inches)

Weighted Mean

Arithmetic Mean is used when all the observations are given equal importance / weight

but there are certain situations in which the different observations get different weights. In this situation, weighted mean denoted by $\overline{X}_w$ is preferred. The weighted mean of X_1, X_2, X_3, ... X_n with corresponding weights W_1, W_2, W_3, ..., W_n is calculated as:

$$\overline{X}_w = \frac{W_1X_1 + W_2X_2 + W_3X_3 + ... + W_nX_n}{W_1 + W_2 + W_3 + ... W_n} = \frac{\sum_{i=1}^{n} W_iX_i}{\sum_{i=1}^{n} W_i} = \frac{\Sigma WX}{\Sigma W}$$

Example 19: The following data describes the marks of a student in different subjects and weights assigned to these subjects are also given:

Mark (x)	74	78	74	90
Weights (w)	4	3	5	6

Find its weighted mean.

Solution: Weighted mean $(\overline{X}_w) = \frac{\Sigma WX}{\Sigma W}$

$$\overline{X}_w = \frac{4(74) + 3(78) + 5(74) + 6(90)}{4+3+5+6}$$

$$\overline{X}_w = \frac{296 + 234 + 370 + 540}{18} = \frac{1440}{18}$$

$$\overline{X}_w = 80$$

Example 20: A medicine company started marketing of a sample of medicine in seven different areas of a city. The company distributed the packets of medicine in each area of the city and the weight of each area based on the demand of the medicine. Find the mean and weighted mean of the given data.

Area	Number of packets	Weights (kg)
A	15	5
B	25	4
C	18	3
D	23	4
E	15	2
F	10	1
G	8	2

Solution: Mean $= \frac{\Sigma x}{n}$

$$= \frac{15+25+18+23+15+10+8}{7}$$

$$= \frac{114}{7} = 16.29 \approx 16 \text{ packets}$$

So, the average number of packets of the medicine distributed by the company per area is 16.

$$\text{Weighted mean} = \frac{\Sigma WX}{\Sigma W}$$

$$= \frac{15(5)+25(4)+18(3)+23(4)+15(2)+10(1)+8(2)}{5+4+3+4+2+1+2}$$

$$= \frac{377}{21} = 17.95 \approx 18$$

Real Life Situations Involving Mean, Weighted Mean, Median and Mode

Sales and Marketing

Example 21: A toy factory sold toys in a month. Consider the following data.

Class limits	10-20	20-30	30-40	40-50	50-60
f	15	28	45	29	20

(i) Calculate mean, median and mode of the number of toys sold by the factory.

(ii) Also tell the modal class of the distribution.

Solution: (i) For mean

Class limits	f	X	fX	c.f.	
10-20	15	15	225	15	
20-30	28	25	700	28 + 15 = 43	
30-40	45	35	1575	45 + 43 = 48	Modal class
40-50	29	45	1305	29 + 48 = 77	Median class
50-60	20	55	1100	20 + 77 = 97	
Total	**Σ = 137**		**4905**		

$$\text{Mean } (\overline{X}) = \frac{\Sigma fx}{\Sigma f} = \frac{4905}{137} = 35.8 \approx 36$$

Average sale of the toys is 36.

For median: Here, n = 137, so, $\frac{137}{2} = 68.5$; 68.5 lies in 40 – 50.

$\ell = 40$, h = 10, f = 29, n = 137, c = 48

$$\text{Mean } (\tilde{X}) = \ell + \frac{h}{f}\left(\frac{n}{2} - c\right)$$

$$= 40 + \frac{10}{29}\left(\frac{137}{2} - 48\right)$$

$$= 40 + \frac{10}{29}(68.5 - 48)$$

$$= 40 + \frac{10}{29}(20.5)$$

$$= 40 + 7.07$$

$$\text{Median} = 47.07 \approx 47$$

Thus, median of the sold toys by the factory is 47.07.

For mode: $\ell = 30$, h = 10, $f_m = 45$, $f_1 = 28$, $f_2 = 29$

$$\text{Mode } (\hat{X}) = \ell + \frac{(f_m - f_1)}{(f_m - f_1)(f_m - f_2)} \times h$$

$$= 30 + \frac{(45-28)}{(45-28)+(45-29} \times 10$$

$$= 30 + \frac{7}{17+16} \times 10$$

$$= 30 + \frac{17}{33} \times 10$$

$$= 30 + 5.15$$

$$\text{Mode } (\hat{X}) = 35.15 \approx 35$$

Thus, mode of the sold toys by the factory is 35.

(ii) The modal class of sold toys by the factory is (30 - 40).

Solved Exercise 12.2

1. Find the arithmetic mean in each of the following:

(i) 4, 6, 10, 12, 15, 20, 25, 28, 30.

Sol. A.M. $= \overline{X} = \frac{\Sigma x}{n}$

$$\overline{X} = \frac{4+6+10+12+15+20+25+28+30}{9}$$

$$\overline{X} = \frac{150}{9}$$

$$\overline{X} = 16.67$$

(ii) 12, 18, 19, 0, –19, –18, –12

Sol. A.M. $= \overline{X} = \frac{\Sigma x}{n}$

$$\overline{X} = \frac{12+18+19+0+(-19)+(-18)+(-12)}{7}$$

$$\overline{X} = \frac{12+18+19+0-19-18-12}{7}$$

$$\overline{X} = \frac{0}{7}$$

$$\overline{X} = 0$$

(iii) 6.5, 11, 12.3, 9, 8.1, 16, 18, 20.5, 25

Sol. A.M. $= \overline{X} = \frac{\Sigma x}{n}$

$$\overline{X} = \frac{6.5+11+12.3+9+8.1+16+18+20.5+25}{9}$$

$$\overline{X} = 126.4$$

$$\overline{X} = 14.04$$

(iv) 8, 10, 12, 14, 16, 20, 22

Sol. A.M. $= \overline{X} = \frac{\Sigma x}{n}$

$$\overline{X} = \frac{8+10+12+14+16+20+22}{7}$$

$$\overline{X} = \frac{102}{7}$$

$$\overline{X} = 14.57$$

2. Following are the heights in (inches) of 12 students. Find the median height.
55, 53, 54, 58, 60, 61, 62, 56, 57, 52, 51, 63.

Sol. Arrange the heights of students in an ascending order, we have

51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63

since, number of obervation is even, x = 12

$$\text{Median}(\widetilde{X}) = \frac{1}{2}\left[\left(\frac{x}{2}\right)^{th} \text{observation} + \left(\frac{x+2}{2}\right)^{th} \text{observation}\right]$$

$$\widetilde{X} = \frac{1}{2}\left[\left(\frac{12}{2}\right)^{th} \text{observation} + \left(\frac{12+2}{2}\right)^{th} \text{observation}\right]$$

$$\widetilde{X} = \frac{1}{2}\left[6^{th}\text{observation} + 7^{th}\text{observation}\right]$$

$$\widetilde{X} = \frac{1}{2}(56+57)$$

$$\widetilde{X} = \frac{1}{2}(113)$$

$$\widetilde{X} = 56.5$$

Hence, 56.5 inches is the median of the data.

3. Following are the earnings (in Rs.) of ten workers:
88, 70, 72,125,115, 95, 81, 90, 95, 90. Calculate
(i) Arithmetic Mean (ii) Median (iii) Mode

Sol. (i) Arithmetic Mean

A.M. $= \overline{X} = \frac{\Sigma x}{n}$

$$\overline{X} = \frac{88+70+72+125+115+95+81+90+95+90}{10}$$

$$\overline{X} = \frac{921}{10}$$

$$\overline{X} = 92.1$$

(ii) Median

Sol. Arranging the earnings of workers in an ascending order, we have

70, 72, 81, 88, 90, 90, 95, 95, 115, 125

since, number of observation is even, x = 10

$$\text{Median}(\widetilde{X}) = \frac{1}{2}\left[\left(\frac{x}{2}\right)^{th} \text{observation} + \left(\frac{x+2}{2}\right)^{th} \text{observation}\right]$$

$$\widetilde{X} = \frac{1}{2}\left[\left(\frac{10}{2}\right)^{th} \text{observation} + \left(\frac{10+2}{2}\right)^{th} \text{observation}\right]$$

$$\widetilde{X} = \frac{1}{2}\left[5^{th}\text{ observation} + 6^{th}\text{ observation}\right]$$

$$\widetilde{X} = \frac{1}{2}(90+90)$$

$$\widetilde{X} = \frac{1}{2}(180)$$

$$\widetilde{X} = 90$$

Hence, 90 is the median of the data.

(iii) Mode

Sol. 88, 70, 72, 125, 115, 95, 81, 90, 95, 90

As 90 and 95 are repeated more than any other observation so, the data contain two mode, 90 and 95.

4. The Marks obtained by the students in the subject of English are given below.

Marks obtained	15–19	20–24	25–29	30–34	35–39
Frequency	9	18	35	17	5

Find:(i) Arithmetic mean of their marks by direct and short formula.
(ii) Median of their marks.

Sol. (i) Arithmatic mean by direct formula:

Marks obtained	Mid-pointd (x)	Frequency (f)	fx
15–19	$\frac{15+19}{2} = 17$	9	153

20–24	$\frac{20+24}{2}=22$	18	396
25–29	$\frac{25+29}{2}=27$	35	945
30–34	$\frac{30+34}{2}=32$	17	544
35–39	$\frac{35+39}{2}=37$	5	185
		$\Sigma f = 84$	$\Sigma fx = 2223$

$$\overline{X} = \frac{\Sigma fx}{\Sigma f} \Rightarrow \overline{X} = \frac{2223}{84}$$

$$\overline{X} = 26.46$$

Arithmatic mean by short formula:

Marks obtained	Mid-points (x)	Frequency (f)	A = 27, D = x – A	fD
15–19	$\frac{15+19}{2}=17$	9	17–27 = –10	–90
20–24	$\frac{20+29}{2}=22$	18	22–27 = –5	–90
25–29	$\frac{25+29}{2}=27 \rightarrow A$	35	27–27 = 0	0
30–34	$\frac{30+34}{2}=32$	17	32–27 = 5	85
35–39	$\frac{35+39}{2}=37$	5	37–27 = 10	50
		$\Sigma f = 84$		$\Sigma fD = -45$

$$\overline{X} = A + \frac{\Sigma fD}{\Sigma f}$$

$$\overline{X} = 27 + \left(\frac{-45}{84}\right)$$

$$\overline{X} = 27 - 0.54$$

$$\overline{X} = 26.46$$

(ii) Median of their marks.

Sol.

Marks obtained	Frequency (f)	Class boundries	C.f
15-19	9	14.5 – 19.5	9
20-24	18	19.5 – 24.5	9 + 18 = 27

25-29	35	24.5 – 29.5	27 + 35 = 62 Median Class
30-34	17	29.5 – 34.5	62 + 17 = 79
35-39	5	34.5 – 39.5	79 + 5 = 84
	$\Sigma f = 84$		

Here $x = 84$ So, $\frac{x}{2} = \frac{84}{2} = 42$

42^{th} term lies in (24.5 – 29.5)

$\ell = 24.5$, $h = 5$, $f = 35$, $c = 27$

$$\text{Median} = \ell + \frac{h}{f}\left(\frac{x}{2} - c\right)$$

$$= 24.5 + \frac{5}{35}(42 - 27)$$

$$= 24;5 + \frac{5}{35}(15)$$

$$= 24.5 + \frac{75}{35}$$

$$= 24.5 + 2.14$$

$$= 26.24$$

5. Given below is a frequency distribution.

Class interval	5–9	10–14	15–19	20–24	25–29
Frequency	1	8	18	11	2

Find the mode of the frequency distribution.

Sol.

Class Interval	Frequency (f)	Class – Boundries
5–9	1	4.5 – 9.5
10–14	$8 \rightarrow f_1$	9.5 – 14.5
15–19	$18 \rightarrow f_m$	14.5 – 19.5
20–24	$11 \rightarrow f_2$	19.5 – 24.5
25–29	2	24.5 – 29.5
	$\Sigma f = 40$	

Here

$\ell = 14.5$, $h = 5$, $f_m = 18$, $f_1 = 8$, $f_2 = 11$

$$\text{Mode} = \ell + \frac{(f_m - f_1) \times h}{(f_m - f_1) + (f_m - f_2)}$$

$$= 14.5 + \frac{(18-8) \times 5}{(18-8) + (18-11)}$$

$$= 14.5 + \frac{10 \times 5}{10 + 7}$$

$= 14.5 + \frac{50}{17}$

$= 14.5 + 2.94$

$= 17.44$

6. Ten boys work on a petrol pump station. They get weekly wages as follows: Wages (in Rs.) 4250,4350, 4400, 4250, 4350, 4410,4500, 4300, 4500, 4390. Find the arithmetic mean by short formula, median and mode of their wages.

Sol. Arithmatic mean by sort formula:

Taking deviation from A = 4350 (assumed-mean)

x	4250	4350	4400	4250	4350	4410	4500	4300	4500	4390
D=x–A	–100	0	50	–100	0	60	150	–50	150	40

Now, $\Sigma D = -250 + 450 = 200$, $x = 10$

$$\overline{X} = A + \frac{\Sigma D}{x}$$

$$\overline{X} = 4350 + \frac{200}{10}$$

$$\overline{X} = 4350 + 20$$

$$\overline{X} = 4370$$

Median

Arranging data in an ascending order

4250, 4250, 4300, 4350, 4350, 4390, 4400, 4410, 4500, 4500

Since = number of observations is even ; x = 10

$$\text{Median}(\tilde{X}) = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} \text{observation} + \left(\frac{n+2}{2}\right)^{th} \text{observation}\right]$$

$$\tilde{X} = \frac{1}{2}\left[\left(\frac{10}{2}\right)^{th} \text{observation} + \left(\frac{10+2}{2}\right)^{th} \text{observation}\right]$$

$$\tilde{X} = \frac{1}{2}\left[5^{th} \text{ observation} + 6^{th} \text{ observation}\right]$$

$$\tilde{X} = \frac{1}{2}(4350 + 4390)$$

$$\tilde{X} = \frac{1}{2}(8740)$$

$$\tilde{X} = 4370$$

Hence, 4370 is the median of the data.

Mode

Mode = the given data contains three modes 4250, 4350 and 4500 because these number repeated more than any other numbers.

7. The arithmetic mean of 45 numbers is 80. Find their sum.

Sol. $x = 45$

$\overline{X} = 80$

$\Sigma x = ?$

By using formula

$$\overline{X} = \frac{\Sigma x}{x}$$

$$80 = \frac{\Sigma x}{45}$$

$80 \times 45 = \Sigma x$

$3600 = \Sigma x$

So, the sum of given numbers is 3600.

8. Five numbers are 1, 4, 0, 7, 9. Find their mean, median and mode.

Sol. Mean

$$A.M = \overline{X} = \frac{\Sigma x}{x}$$

$$= \frac{1+4+0+7+9}{5}$$

$$= \frac{21}{5} = 4.2$$

Median

Arranging data in an ascending order:

0, 1, 4, 7, 9

Since, number of observations is add, n = 5

Median

$$\text{Median}(\tilde{X}) = \left(\frac{n+1}{2}\right)^{th} \text{observation}$$

$$\tilde{X} = \left(\frac{5+1}{2}\right)^{th} \text{observation}$$

$$\tilde{X} = 3^{rd} \text{observation}$$

$$\tilde{X} = 4$$

Hence, 4 is median of data.

Mode

Data 1, 4, 0, 7, 9

No observation is repeated so, there is no mode in the given data.

9. A set of data contains the values as 148, 145, 160. 157, 156, 160. Show that Mode > Median > Mean

Sol. Mean

$$\overline{X} = \frac{\Sigma x}{x}$$

$$\overline{X} = \frac{148+145+160+157+156+160}{6}$$

$$\overline{X} = \frac{926}{6}$$

$$\overline{X} = 154.3$$

Median

Arranging data in an ascending order

145, 148, 156, 157, 160, 160

Since, the number of observation is even, x = 6

$$\text{Median}(\tilde{X}) = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} \text{observation} + \left(\frac{n+2}{2}\right)^{th} \text{observation}\right]$$

$$\tilde{X} = \frac{1}{2}\left[\left(\frac{6}{2}\right)^{th} \text{observation} + \left(\frac{6+2}{2}\right)^{th} \text{observation}\right]$$

$$\tilde{X} = \frac{1}{2}\left[3^{rd} \text{ observation} + 4^{th} \text{ observation}\right]$$

$$\tilde{X} = \frac{1}{2}(156 + 157)$$

$$\tilde{X} = \frac{1}{2}(313)$$

$$\tilde{X} = 156.5$$

Hence, 156.5 is the median of the data.

Mode

148, 145, 160, 157, 156, 160

160 is repeated more than any other number. So, mode is 160

Mode = 160

Madian = 156.5

Mean = 154.3

There fore

Mode > Median > Mean

160 > 156.5 > 154.3

10. The monthly attendance of 10 students for their lunch in the hostel is recorded as: 21, 15, 16, 18, 14, 17, 15, 12, 13, 11.

Find the median and mode of the attendance. Also find the mean if D = A - 20.

Sol. Median

Arrange data in an ascending order

11, 12, 13, 14, 15, 15, 16, 17, 18, 21

Since, the number of observations is even, n = 10

$$\text{Median}(\tilde{X}) = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} \text{observation} + \left(\frac{n+2}{2}\right)^{th} \text{observation}\right]$$

$$\tilde{X} = \frac{1}{2}\left[\left(\frac{10}{2}\right)^{th} \text{observation} + \left(\frac{10+2}{2}\right)^{th} \text{observation}\right]$$

$$\tilde{X} = \frac{1}{2}\left[5^{th} \text{ observation} + 6^{th} \text{ observation}\right]$$

$$\tilde{X} = \frac{1}{2}(15 + 15) \Rightarrow \tilde{X} = \frac{1}{2}(30)$$

$$\tilde{X} = 15$$

Mode

21, 15, 16, 18, 14, 17, 15, 12, 13, 11

15 is repeated more than any other number. So, mode is 15.

Mean by snort formula

x	21	15	16	18	14	17	15	12	13	11
D = x – 20	1	–5	–4	–2	–6	–3	–5	–8	–7	–9

Now

$\Sigma D = 48$, $n = 10$, $A = 20$

$$\overline{X} = A + \frac{\Sigma D}{n}$$

$$= 20 + \left(\frac{-48}{10}\right)$$

$$= 20 - 4.8 = 15.2$$

11. On a prize distribution day, 50 students brought pocket money as under:

Rupees	5–10	10–15	15–20	20–25	25–30
Frequency	12	9	18	7	4

(i) Find the median and mode of the above data.

(ii) Find the arithmetic mean of the data given above using coding method.

(i) Sol. Median

Rupees	Frequency (f)	C.f	
5 – 10	12	12	
10 – 15	9	12 + 9 = 21 → c	
15 – 20	18	21 + 18 = 39 →	Median Class
20 – 25	7	39 + 7 = 46	
25 – 30	4	46 + 4 = 50 – x	
	$\Sigma f = 50$		

Here $x = 50$. So, $\frac{n}{2} = \frac{50}{2} = 25$.

25th term lies in (15 – 20)

$\ell = 15, h = 5, f = 18, c = 21$

$$\tilde{X} = \ell + \frac{h}{f}\left(\frac{x}{2} - c\right)$$

$$\tilde{X} = 15 + \frac{5}{18}(25 - 21)$$

$$\tilde{X} = 15 + \frac{5}{18} \times 4 \Rightarrow \tilde{X} = 15 + \frac{20}{18}$$

$$\tilde{X} = 15 + 1.11$$

$$\tilde{X} = 16.11$$

Mode

Rupees	Frequency (f)	
5 – 10	12	
10 – 15	9	f_1
15 – 20	18	f_m
20 – 25	7	f_2
25 – 30	4	

$\ell = 15, h = 5, f_m = 18, f_1 = 9, f_2 = 7$

$$\text{Mode } \hat{X} = \ell + \frac{(f_m - f_1)}{(f_m - f_1) + (f_m - f_2)} \times h$$

$$= 15 + \frac{18 - 9}{(18 - 9) + (18 - 7)} \times 5$$

$$= 15 + \frac{9}{9 + 11} \times 5$$

$$= 15 + \frac{9}{20} \times 5$$

$$= 15 + \frac{45}{20}$$

$$= 15 + 2.25$$

$$= 17.25$$

(ii) **Sol.** Mean by coding method.

Rupees	Frequency (f)	Mid-point (x)	$U = \frac{x - A}{h}$, $A = 17.5$, $h = 5$	f.U
5 – 10	12	$\frac{5+10}{2} = 7.5$	$\frac{7.5 - 17.5}{5} = -2$	–24

10 – 15	9	$\frac{10+15}{2} = 12.5$	$\frac{12.5 - 17.5}{5} = -1$	–9
15 – 20	18	$\frac{15+20}{2}$ 17.5→A	$\frac{17.5 - 17.5}{5} = 0$	0
20 – 25	7	$\frac{20+25}{2} = 22.5$	$\frac{22.5 - 17.5}{5} = 1$	7
25 – 30	4	$\frac{20+35}{2} = 27.5$	$\frac{27.5 - 17.5}{5} = 2$	8
	$\Sigma f = 50$			$\Sigma fu = -18$

Here $\Sigma fu = -18, \Sigma f = 50, h = 5, A = 17.5$

$$\overline{X} = A + \frac{\Sigma fu}{\Sigma f} \times h$$

$$= 17.5 + \frac{(-18)}{50} \times 5$$

$$= 17.5 - \frac{90}{50} = 17.5 - 1.8 = 15.7$$

12. The arithmetic mean of the ages of 20 boys is 13 years, 4 months and 5 days. Find the sum of their ages. If one of the boys is of age exactly 15 years. What is the average age of the remaining boys?

Sol. Number of boys (n) = 20

Mean of 20 Boys = 13 years, 4 months, 5 days

$$= \left(13 + \frac{4}{12} + \frac{5}{365}\right) \text{ years}$$

$$= (13 + 0.33 + 0.014) \text{ years}$$

$$= 13.347 \text{ years}$$

By using formula

$$\overline{X} = \frac{\Sigma x}{n}$$

$$13.347 = \frac{\Sigma x}{20} \Rightarrow 13.347 \times 20 = \Sigma x$$

$$266.94 = \Sigma x$$

So, the sum of the ages of 20 boys = 266.94 years

Age of one boy = 15 years

Age of remaining 19 boys = 266.94 – 15 = 251.94 years

Mean of 19 boys

$$\overline{X} = \frac{\Sigma x}{n} \Rightarrow \overline{X} = \frac{251.94}{19}$$

$$\overline{X} = 13.26 \text{ years}$$

So, the average age of remaining 19 boy = 13.26 years
= 13 years, (.26 × 12)
= 13 years, 3.12 months
= 13 years, 3 months, .12 × 30 days
= 13 years, 3 months, 4 days

13. Calculate the arithmetic mean from the following information:

(i) If $D = X - 140$, $\Sigma D = 500$ and $n = 10$

(ii) If $U = \frac{x-130}{6}$, $\Sigma U = -150$ and $n = 15$

(iii) If $D = x - 25$, $\Sigma fD = 300$ and $\Sigma f = 20$

(iv) If $U = \frac{x-120}{6}$, $\Sigma fU = 60$ and $\Sigma f = 100$

Sol. (i) $D = X - 140$

$\Sigma D = 500$

$n = 10$

$A = 140$

$\overline{X} = A + \frac{\Sigma x}{n}$

$\overline{X} = 140 + \frac{500}{10}$

$\overline{X} = 140 + 50$

$\overline{X} = 190$

Sol. (ii) $U = \frac{x-130}{6}$

$\Sigma U = -150$

$n = 15$

$A = 130$

$h = 6$

$\overline{X} = A + \frac{\Sigma U}{n} \times h$

$\overline{X} = 130 + \frac{(-150)}{15} \times 6 = 130 - \frac{900}{15}$

$\overline{X} = 130 - 60 = 70$

Sol. (iii) $D = x - 25$

$\Sigma fD = 300$

$\Sigma f = 20$

$A = 25$

$\overline{X} = A + \frac{\Sigma fD}{\Sigma f}$

$\overline{X} = 25 + \frac{300}{20}$

$\overline{X} = 25 + 15$

$\overline{X} = 40$

Sol. (iv) $U = \frac{x-120}{5}$

$\Sigma fU = 60$

$\Sigma f = 100$

$A = 120$

$h = 5$

$\overline{X} = A + \frac{\Sigma fU}{\Sigma f} \times h$

$= 120 + \frac{60}{100} \times 5 = 120 + \frac{300}{100}$

$= 120 + 3 = 123$

14. The three children Haris, Maham and Minal made the following scores in a game conducted by a group of teachers in the school.

Haris scores	50	55	70	85	90
Maham scores	75	60	60	45	53
Minal scores	80	77	66	42	48

It is decided that the candidate who gets the highest average score will be awarded rupees 1000. Who will get the awarded amount?

Sol. Average score of Haris $\overline{X}_1 = \frac{\Sigma x}{n}$

$= \frac{50+55+70+85+90}{5} = \frac{350}{5} = 70$

Average score of Maham $\overline{X}_2 = \frac{\Sigma x}{n}$

$= \frac{75+60+60+45+53}{5} = \frac{293}{5} = 58.6$

Average score of Minal $\overline{X}_1 = \frac{\Sigma x}{n}$

$= \frac{80+77+66+42+48}{5} = \frac{313}{5} = 62.6$

So, Haris got highest average score 70. Thus, Haris will get award of Rs. 1000.

15. Given below is a frequency distribution derived by making a substitution as $D = X - 20$. Calculate the arithmetic mean.

D	−6	−4	−2	0	2	4	6
f	1	3	6	20	26	12	2

Sol.

D = −20	f	fD
−6	1	−6
−4	3	−12
−2	6	−12
0	20	0
2	26	52
4	12	48
6	2	12
	$\Sigma f = 60$	$\Sigma fD = 82$

$\overline{X} = A + \frac{\Sigma fD}{\Sigma f}$

$= 20 + \frac{82}{60}$

$= 20 + 1.37$

$= 21.37$

16. Being partners Hafsa and Fatima took part in a quiz programme. They made the following number of points 45, 51, 58, 61, 74, 48, 46 and 50. Compute the average number of points using deviation D = x – 58.

Sol.

X	45	51	58	61	74	48	46	50
D=X–A A = 58	–13	–7	0	3	16	–10	–12	–8

$\Sigma D = -50 + 19 = -31,\ A = 58,\ x = 8$

$$\overline{X} = A + \frac{\Sigma D}{x}$$

$$\overline{X} = 58 + \frac{(-31)}{8}$$

$$\overline{X} = 58 - 3.875$$

$$\overline{X} = 54.125$$

17. A person purchased the following food items:

Food item	Quantity (in kg)	Cost per kg (in Rs.)
Rice	10	96
Flour	12	48
Ghee	4	190
Sugar	3	49
Mutton	2	650

What is the weighted mean of cost of food items per kg?

Sol.

Food Item	Quantity (in kg) W	Cost per kg (in Rs.)	Wx
Rice	10	96	960
Flour	12	48	576
Ghee	4	190	760
Sugar	3	49	147
Mutton	2	650	1300
	$\Sigma w = 31$		$\Sigma wx = 3743$

$$\overline{X}_w = \frac{\Sigma wx}{\Sigma w}$$

$$\overline{X}_w = \frac{3743}{31} = 120.74$$

$$\overline{X}_w = 120.74\ ^{Rs}/_{Kg}$$

18. For the following data, find the weighted mean.

Item	Quantity	Cost of item (in thousands)
Washing Machine	5	35
Heater	3	5
Stove	2	13
Dispenser	6	18

Sol.

Item	Quantity W	Cost of item X	Wx
Washing Machine	5	35	175
Heater	3	5	15
Stove	2	13	26
Dispenser	6	18	108
	$\Sigma w = 16$		$\Sigma wx = 324$

$$\overline{X}_w = \frac{\Sigma wx}{\Sigma w}$$

$$\overline{X}_w = \frac{324}{16}$$

$$\overline{X}_w = 20.25$$

So, Weighted Mean = 20.25 thousand

19. A company is planning its next year marketing budget across five years: yearly budgets (in million) are: 5, 7, 8, 6, 7. Find the average budget for the next year.

Sol. Amount of budget (in million): 5, 7, 8, 6, 7

$n = 5$

$$\overline{X} = \frac{\Sigma x}{n}$$

$$\overline{X} = \frac{5+7+8+6+7}{5}$$

$$\overline{X} = \frac{33}{5}$$

$$\overline{X} = 6.6$$

So, average budget = 6.6 million

20. Ahmad obtained the following marks in a certain examination. Find the weighted mean if weights 5,4,2,3,2,4 respectively are allotted to the subjects.

Urdu	English	Science	Math	Islamiyat	Computer
78	65	80	90	85	72

Sol.

Subjects	Weights W	Marks X	Wx
Urdu	5	78	390
English	4	65	260
Science	2	80	160
Math	3	90	270
Islamiyat	2	85	170
Computer	4	72	288
	$\Sigma w = 20$		$\Sigma w x = 1538$

$$\overline{X}_w = \frac{\Sigma wx}{\Sigma w}$$

$$\overline{X}_w = \frac{1538}{20}$$

$\overline{X}_w = 76.9$ marks

Review Exercise 12

1. Four options are given against each statement. Encircle the correct option.

(i) Which data takes only some specific values?

(a) continuous data (b) discrete data
(c) grouped data (d) ungrouped data

(ii) The number of times a value occurs in a data is called:

(a) frequency (b) relative frequency
(b) class limit (d) class boundaries

(iii) Midpoint is also known as:

(a) mean (b) median (c) class limit (d) class mark

(iv) Frequency polygon is also drawn / constructed by using:

(a) histogram (b) bar graph (c) class boundaries (d) class limit

(v) The difference between the greatest value and the smallest value is called:

(a) class limits (b) midpoint (c) relative frequency (d) range

(vi) Measure of central tendency is used to find out the _____ of a data set:

(a) class boundaries (b) cumulative frequency
(c) middle or centre value (d) frequency

(vii) If the mean of 5, 7, 8, 9 and x is 7.5, what will be the value of x?

(a) 10 (b) 8 (c) 8.5 (d) 5.8

(viii) Find the mode of the given data: 2, 5, 8, 9, 0, 1, 3, 7 and 10:

(a) 5 (b) 7 (c) 0 (d) no mode

(ix) In a data the values (observations) which appears or occurs most often is called:

(a) mean (b) mode (c) median (d) weighted mean

(x) Find the median of the given data: 110, 125, 122, 130, 124, 127 and 120:

(a) 124 (b) 120 (c) 125 (d) 127

Answers:

i. discrete data ii. frequency iii. class mark iv. histogram
v. range vi. middle or centre value vii. 8.5 viii. no mode
ix. mode x. 124

2. Define the following:

(i) Frequency distribution:

Sol. A distribution or table that represents classes or groups along with their respective class frequencies is called frequency distribution.

(ii) Histogram (unequal class limits):

Sol. A histogram with unequal class intervals is a graph that displays data with rectangles of different widths. This occurs when the classes, or categories, in data are not equally sized.

(iii) Mean:

Sol. It is defined as a value of variable which is obtained by dividing the sum of all the values (observations) by their numbers of observations.

(iv) Median:

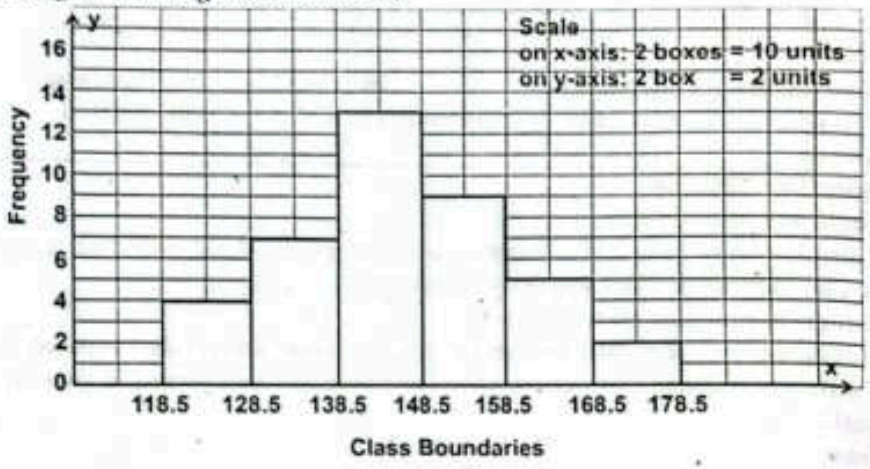
Sol. Median is the middle most value in an arranged (sending or descending order) data set. Median is the value which divides the data into two equal parts i.e, 50% data is before the median and 50% data after it. Median is denoted by $\tilde{x}$.

3. Following are the weights of 40 students recorded to the nearest (lbs). 138,164,150,132,144,125,149,157,146,158,140,147,136,148,152,144, 168,126,138,176,163,119,154,165,146,173,142,147,135,153,140,135 161, 145, 135, 142, 150, 156, 145, 128. (a) Make a frequency table taking size of class limits as 10. (b) Draw histogram. (c) Draw frequency polygon of the given data.

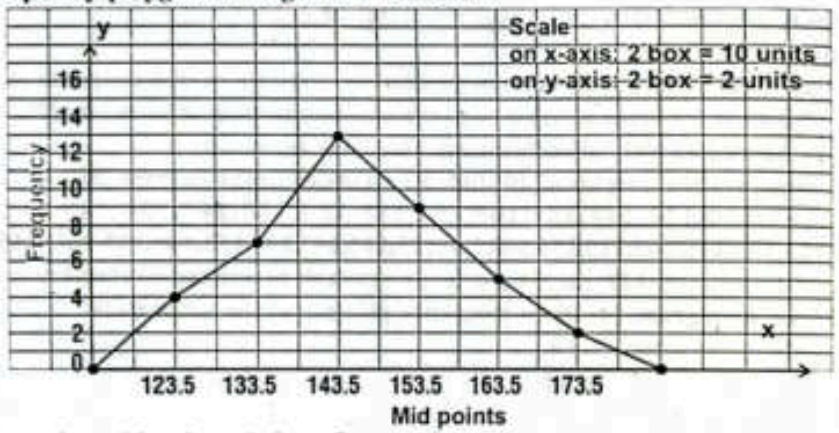
Sol. Minimum value = 19
Maximum value = 176
size of the class limits = 10
Frequency Distribution Table
Frequency table taking size of class limit as 10.

Class limits	Tally marks	Frequency f	Class Boundries	midpoint
119 – 128	||||	4	118.5 – 128.5	123.5
129 – 138	𝍸 ||	7	128.5 – 138.5	133.5
139 – 148	𝍸 𝍸 |||	13	138.5 – 148.5	143.5
149 – 158	𝍸 |||	9	148.5 – 158.5	153.5
159 – 168	𝍸	5	158.5 – 168.5	163.5
169 – 178	||	2	168.5 – 178.5	173.5
Total		$\Sigma f = 40$		

(b) Histogram of weight of 40 students.



(c) Frequency polygon of weight of 40 students.



4. From the table given below. Draw a frequency polygon on histogram for the given frequency distribution.

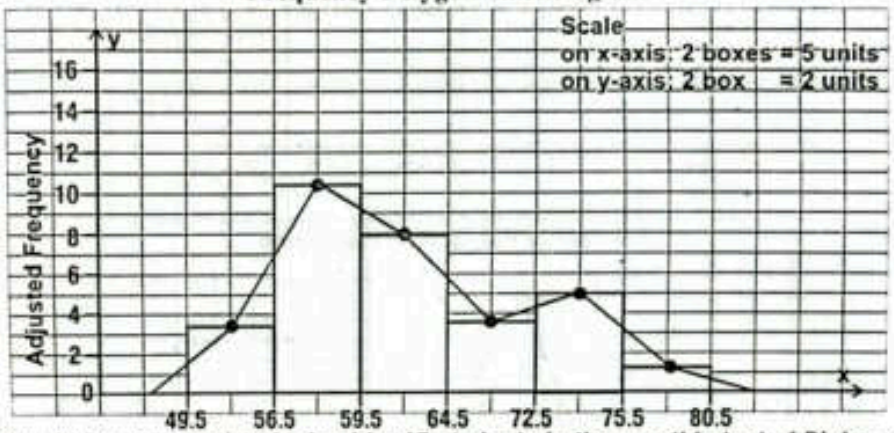
Weight (kg)	50-56	57-59	60-64	65-72	73-75	76-80
Frequency (f)	25	32	40	30	15	8

Sol.

Marks	f	Clxass-Boundries	Width of Class	Hight of rectangles Adjusted Frequency
50 – 56	25	49.5 – 56.5	56.5 – 49.5 = 7	$\frac{25}{7} = 3.6$
57 – 59	32	56.5 – 59.5	59.5 – 56.5 = 3	$\frac{32}{3} = 10.7$

60 – 64	40	59.5 – 64.5	64.5 – 59.5 = 5	$\frac{40}{5} = 8$
65 – 72	30	64.5 – 72.5	72.5 – 64.5 = 8	$\frac{30}{8} = 3.8$
73 – 75	15	72.5 – 75.5	75.5 – 72.5 = 3	$\frac{15}{3} = 5$
76 – 80	8	75.5 – 80.5	80.5 – 75.5 = 5	$\frac{8}{5} = 1.6$

Frequency Polygon on Histogram



5. Given below are marks obtained by 45 students in the monthly test of Biology:

Marks	20-24	25-29	30-34	35-39	40-44	45-49
No. of students	05	08	12	15	03	02

With reference to the above table find the following:

(i) upper class boundary of the 5th class.

Sol. upper class boundary of the 5th class is 44.5.

(ii) lower class boundaries of all the classes.

Sol. lower class boundaries of all the classes are 19.5, 24.5, 29.5, 34.5, 39.5 and 44.5.

(iii) midpoint of all the classes.

Sol. midpoint of all the classes are 22, 27, 32, 37, 42 and 47.

(iv) the class interval with the least frequency.

Sol. the class interval with the least frequency is (45–49).

6. Given below is frequency distribution.

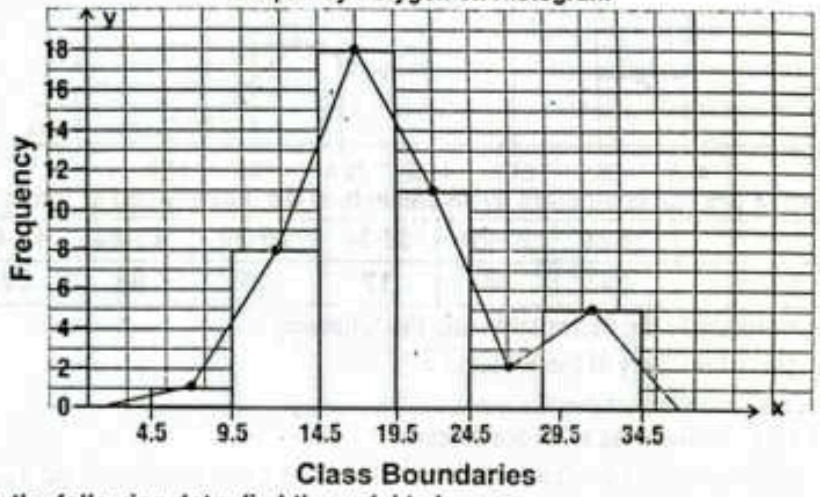
Draw frequency polygon and histogram for the distribution.

Class limits	5-9	10-14	15-19	20-24	25-29	30-34
Frequency	1	8	18	11	2	5

Sol.

Class limits	Frequency f	Class Boundries	Midpoints
5 – 9	1	4.5 — 9.5	$\frac{5+9}{2}=7$
10 – 14	8	9.5 — 14.5	$\frac{10+14}{2}=12$
15 – 19	18	14.5 — 19.5	$\frac{15+19}{2}=17$
20 – 24	11	19.5 — 24.5	$\frac{20+24}{2}=22$
25 – 29	2	24.5 — 29.5	$\frac{25+29}{2}=27$
30 – 34	5	29.5 — 34.5	$\frac{30+34}{2}=32$

Frequency Polygon on Histogram



7. For the following data, find the weighted mean.

Item	Quantity	Cost of item (Rs.)
Chair	20	500
Table	20	400
Black board	10	750
Tube light	25	230
Cupboard	09	950

Sol.

Item	Quantity W	Cost of item (Rs.) X	WX
Chair	20	500	10000
Table	20	400	8000
Black board	10	750	7500
Tube light	25	230	5750
Cupboard	09	950	8550
	$\Sigma W = 84$		$\Sigma W = 39800$

$$\bar{X}_w = \frac{\Sigma WX}{\Sigma W} \Rightarrow \bar{X}_w = \frac{39800}{84}$$

$\bar{X}w = 473.81$

So, weighted mean is Rs. 473.81

8. A principal of a school allocates funds of Rs.50, 000 to five different sectors:
(i) chairs: Rs. 15000 (ii) tables: Rs. 12,000
(iii) black boards: Rs.6,000 (iv) room renovation: Rs. 10,000
(v) gardening: Rs. 7,000
Find the average of funds allocation in each sector of the school.

Sol. Average allocation $= \frac{\text{Total funds}}{\text{Number of sectors}}$

$$\bar{X} = \frac{\Sigma x}{n} \Rightarrow \bar{X} = \frac{15000 + 12000 + 6000 + 10000 + 7000}{5}$$

$$\bar{X} = \frac{50000}{5} \Rightarrow \bar{X} = 10000$$

Thus, average funds allocation is Rs 10,000 Per sector

9. The marks of a student Saad in six tests were 84, 91, 72, 68, 87, 78. Find the arithmetic mean of his marks.

Sol. The marks are : 84, 91, 72, 68, 87, 78

$$\bar{X} = \frac{\Sigma x}{n}$$

$$\bar{X} = \frac{84+91+72+68+87+78}{6}$$

$$\bar{X} = \frac{480}{6} = 80$$

Thus, arithmetic mean of the marks is 80.

10. Adjoining distribution showed maximum load (in kg) supported by certain ropes. Find the mean load using short method.

Max-Load kg	93-97	98 -102	103-107	108-112	113-117	118-122
No. of ropes	2	5	8	12	6	2

Sol:

Load (k.g)	Frequency F	Midpoint x	D = X–A, A = 110	f D
93 — 97	2	$\frac{93+97}{2}=95$	95 – 110 = –15	–30
98 — 102	5	$\frac{98+102}{2}=100$	100 – 110 = –10	–50
103 — 107	8	$\frac{103+107}{2}=105$	105 – 110 = –5	–40
108 — 112	12	$\frac{108+112}{2}=110 \rightarrow A$	110 – 110 = 0	0
113 — 117	6	$\frac{113+117}{2}=115$	115 – 110 = 5	30
118 — 122	2	$\frac{118+122}{2}=120$	120 – 110 = 10	20
	$\Sigma f = 35$			$\Sigma fD = -70$

$$\bar{X} = A + \frac{\Sigma fD}{\Sigma F} \Rightarrow = 110 + \left(\frac{-70}{35}\right)$$

$$= 110 - 2 = 108$$

Thus the mean load is 108 k.g

11. Usman rolled a fair dice eight times. Each time their sum was recorded as 8, 5, 6, 6, 9, 4, 3, 11. Find the median and mode of the sum.

Sol. Arranging the data is an ascending order 3, 4, 5, 6, 6, 8, 9, 11

Since, number of observation is even; n = 8

$$\text{Medin } (\tilde{x}) = \frac{1}{2}\left(\frac{n^{th}}{2} \text{ observation} + \left(\frac{n+2}{2}\right)^{th} \text{ observation}\right)$$

$$= \frac{1}{2}\left(\frac{8^{th}}{2} \text{ observation} + \left(\frac{8+2}{2}\right)^{th} \text{ observation}\right)$$

$$= \frac{1}{2}(4^{th} \text{ observation} + 5^{th} \text{ observation})$$

$$= \frac{1}{2}(6+6) = \frac{1}{2}(12) = 6$$

Hence, 6 is the median of date Mode

Data: 8, 5, 6, 6, 9, 4, 3, 11 6 is repeated more than any of other number. so, mode is 6.

12. Two partners Mr. Aslam and Mrs. Kalsoom run a company. In the following data the weekly wages (in Rs.) of employees who work in the company are given:

Wages (Rs.)	600-700	700 - 800	800 - 900	900-1000	1000-1100
Employees	3	5	7	21	11

Sol.

Wages	Frequency f	Midpoints (x)	f x	c.f	
600–700	3	$\frac{600+700}{2}=650.$	1950	3	
700–800	5	$\frac{700+800}{2}=750$	3750	5 + 3 = 8	
800–900	$7 \rightarrow f_1$	$\frac{800+900}{2}=850$	5950	8+7+15→c	
900–1000	$21 \rightarrow f_m$	$\frac{900+1000}{2}=950$	19950	15+21=36→	Mediam class
1000–1100	$11 \rightarrow f_2$	$\frac{1000+11000}{2}=1050$	11550	36 +11=47	
	$\Sigma f = 47$		$\Sigma f n = 43150$		

(i) Mean

$\Sigma fx = 43150$, $\Sigma f = 47$

$$\bar{X} = \frac{\Sigma fx}{\Sigma F} \Rightarrow \bar{X} = \frac{43150}{47} = 918.9$$

(ii) Mediam

Here, $n = 47$, $\frac{n}{2} = \frac{47}{2} = 23.5$. 25^{th} term lies in class (900 – 1000)

$\ell = 900$, $h = 100$, $f = 21$, $c = 15$

$$\text{median} = \ell + \frac{n}{f}\left(\frac{n}{2} - c\right)$$

$$= 900 + \frac{100}{21}(23.5 - 15) \Rightarrow = 900 + \frac{100}{21}(8.5)$$

$$= 900 + \frac{850}{21} = 900 + 40.48 = 940.48$$

(iii) Mode

Here, $\ell = 900$, $h = 100$, $f_m = 21$, $f_1 = 7$, $f_2 = 11$

$$\text{Mode} = \ell + \frac{(f_m - f_1) \times h}{(f_m - f_1) + (f_m - f_1)}$$

$$= 900 + \frac{(21-7) \times 100}{(21-7)+(21-11)} \Rightarrow = 900 + \frac{14 \times 100}{14+10}$$

$$= 900 + \frac{1400}{24}$$

$$= 900 + 58.3 = 958.3$$

This man = 918.9, Median = 940.48, Mode = 958.3

Objective Type Questions

Additional MCQ's

Multiple Choice Questions prepared in the light of new Examination Techniques (Knowledge, Understanding, Application, Analytical & Conceptual)

Introduction

1. **To know about something is called:**
   (a) information (b) data (c) handling (d) none of above
2. **To represent the information in a manageable way so that useful conclusions can be drawn is:**
   (a) data (b) information handling
   (c) knowledge (d) none of above
3. **The collection of meaningful information in the form of facts and numerical figures is:**
   (a) infromation (b) handling (c) data (d) none of above
4. **Data can be obtained from _____.**
   (a) existing sources (b) non existing sources
   (c) both (a) and (b) (d) none of above
5. **Which of them is the process of collecting organizing, summarizing, analyzing and interpreting numerical data?**
   (a) information handling (b) data
   (c) both (a) and (b) (d) none of above
6. **How many categories of data?**
   (a) only one (b) two (c) three (d) four
7. **It can take only some specific values.**
   (a) data (b) discrete data (c) continuous data (d) none of above
8. **It can take every possible value in a given interval.**
   (a) data (b) discrete data (c) continuous data (d) none of above

12.1 Ungrouped and Grouped data

9. **Data is not arranged in any systematic order.**
   (a) ungrouped data (b) grouped data (c) both (a) and (b) (d) none of above
10. **A table which represents classes or groups along with their respective class frequencies is:**
    (a) frequency distribution (b) grouped data
    (c) ungrouped data (d) none of above
11. **Range = _____.**
    (a) $X_{max} + X_{min}$ (b) $X_{min} - X_{max}$ (c) $X_{max} - X_{min}$ (d) $(X_{min})(X_{max})$

12. $\frac{\text{Range}}{\text{Number of classes}} =$ _____
    (a) size of class (b) width of class (c) length of class (d) none of above
13. **Tally mark is represented as:**
    (a) ∈ (b) — (c) – (d) /
14. **Mark every 5th tally mark as:**
    (a) 𝍸 (b) |||| (c) |||| (d) |V|
15. **The _____ lower class limit of the second class and upper class limit of the first class is 1.**
    (a) addition of (b) difference between
    (c) multiplication of (d) subtraction of
16. **How many types of graphs which can be used to represent a frequency distribution a graph.**
    (a) two (b) three (c) four (d) five
17. **Which of them is a graph of adjacent rectangles constructed on xy - plane.**
    (a) pie graph (b) line graph (c) histogram (d) frequency polygon
18. **_____ of equal width are used to represent individual classes of frequency table.**
    (a) bars (b) paints (c) lines (d) none of above
19. **Which of them is a closed geometrical figure used to display a frequency distribution graphically.**
    (a) frequency polygon (b) frequency distribution
    (c) histogram (d) none of above
20. **A line graph of a frequency distribution is known as _____ is which frequencies are plotted against their midpoints.**
    (a) frequency polygon (b) histogram
    (c) line graph (d) none of above
21. $\frac{\text{Lower class limit} + \text{upper class limit}}{2} =$ _____
    (a) mean point (b) extreme point (c) mid point (d) none of above

12.2 Measure of Location (Central Tendency)

22. **The measure which gives the center of the data called:**
    (a) mean point (b) mid point (c) central tendency (d) upper point
23. **How many measure of central tendency.**
    (a) two (b) three (c) four (d) five
24. **The value of variable which is obtained by dividing the sum of all the values by their number of observations.**
    (a) A.M (b) median (c) mode (d) weighted mean
25. **Arithmetic mean is denoted by:**
    (a) |X| (b) $\bar{X}$ (c) $\underline{X}$ (d) [X]

26. If 64, 75, 81, 87, 90 then A. M is:
(a) 76.4 (b) 77.4 (c) 78.4 (d) 79.4

27. What is the formula to find the method of finding A.M for grouped data.
(a) $\overline{X} = \Sigma fx$ (b) $\overline{X} = \frac{\Sigma fx}{\Sigma f}$ (c) $\overline{X} = \Sigma f$ (d) $\overline{X} = \frac{\Sigma f}{\Sigma fx}$

28. What is the short formula for computing arithmetic mean for ungrouped data?
(a) $\overline{X} = A + \frac{\Sigma D}{\Sigma n}$ (b) $\overline{X} = A - \frac{\Sigma D}{\Sigma n}$ (c) $\overline{X} = A + \frac{\Sigma n}{\Sigma D}$ (d) none of above

29. What is the short formula for computing arithmetic mean for grouped data?
(a) $\overline{X} = A + \frac{\Sigma fD}{\Sigma f}$ (b) $\overline{X} = A - \frac{\Sigma fD}{\Sigma f}$ (c) $\overline{X} = A + \frac{\Sigma f}{\Sigma fD}$ (d) $\overline{X} = A - \frac{\Sigma f}{\Sigma fD}$

30. The A.M of 40, 45, 50, 52, 50, 60, 56, 70 is:
(a) 51.88 (b) 52.88 (c) 53.88 (d) 54.88

31. The _____ is the middle most value in an arranged data set.
(a) mean (b) median (c) mode (d) none of above

32. Median is denoted by:
(a) X (b) |X| (c) $\tilde{X}$ (d) none of above

33. When n is odd then median ($\tilde{X}$) is:
(a) $\left(\frac{n+1}{5}\right)^{th}$ (b) $\left(\frac{n+1}{4}\right)^{th}$ (c) $\left(\frac{n+1}{3}\right)^{th}$ (d) $\left(\frac{n+1}{2}\right)^{th}$

34. When n is even number then median ($\tilde{X}$) is:
(a) $\frac{1}{2}\left(\left(\frac{n}{2}\right)^{th} \text{ observations} + \left(\frac{n+2}{2}\right)^{th} \text{ observations}\right)$
(b) $\frac{1}{2}\left(n^{th} \text{ observations} + (n+1)^{th} \text{ observations}\right)$
(c) $\frac{1}{2}\left((n+1)^{th} \text{ observation} + (n+2)^{th} \text{ observations}\right)$ (d) none of above

35. The median of 8, 12, 18, 13, 16, 5, 20 is:
(a) 12 (b) 13 (c) 14 (d) 15

36. The formula of median for grouped data is:
(a) $\ell - \frac{h}{f}\left(\frac{n}{2} - c\right)$ (b) $\ell + \frac{h}{f}\left(\frac{n}{2} + c\right)$ (c) $\ell + \frac{h}{f}\left(\frac{n}{2} - c\right)$ (d) $\ell - \frac{h}{f}(n + c)$

37. In data to the values which appears or occurs most often is:
(a) mean (b) median (c) mode (d) tendency

38. Mode is denoted by:
(a) X (b) $\overline{X}$ (c) $\tilde{X}$ (d) $\hat{X}$

39. In 75, 76, 80, 80, 82, 82, 82, 85 the mode is:
(a) 82 (b) 80 (c) 75 (d) 85

40. Mode for grouped data is:
(a) $\ell - \frac{(f_m + f_1)}{(f_m - f_1)(f_m - f_2)} \times h$ (b) $\ell + \frac{(f_m - f_1)}{(f_m - f_1)(f_m - f_2)} \times h$
(c) $\ell + \frac{(f_1 + f_2)}{(f_m - f_1)(f_m - f_2)} \times h$ (d) $\ell + \frac{(f_m - f_1)}{(f_m + f_1)(f_m + f_2)} \times h$

41. Symbol of weighted mean is:
(a) $\overline{X}_w$ (b) $|X_w|$ (c) $\hat{X}_w$ (d) $\tilde{X}_w$

42. $\overline{X}_W =$ _____
(a) ΣW (b) ΣX (c) $\frac{\Sigma Wx}{\Sigma W}$ (d) $\frac{\Sigma W}{\Sigma Wx}$

Answers:

1. information	2. information handling	3. data	4. existing sources	
5. information handling	6. two	7. discrete data	8. continuous data	
9. ungrouped data	10. frequency distribution	11. $X_{max} - X_{min}$		
12. size of class	13. /	14. $\mathbb{N}$	15. difference between	
16. two	17. histogram	18. bars	19. frequency polygon	
20. frequency polygon	21. mid point	22. central tendency	23. four	
24. A.M	25. $\overline{X}$	26. 79.4	27. $\overline{X} = \frac{\Sigma fx}{\Sigma f}$	28. $\overline{X} = A + \frac{\Sigma D}{\Sigma n}$
29. $\overline{X} = A + \frac{\Sigma fD}{\Sigma f}$	30. 52.88	31. median	32. $\tilde{X}$	33. $\left(\frac{n+1}{2}\right)^{th}$
34. $\frac{1}{2}\left(\left(\frac{n}{2}\right)^{th} \text{ observations} + \left(\frac{n+2}{2}\right)^{th} \text{ observations}\right)$	35. 13	36. $\ell + \frac{h}{f}\left(\frac{n}{2} - c\right)$		
37. mode	38. $\hat{X}$	39. 82	40. $\ell + \frac{(f_m - f_1)}{(f_m - f_1)(f_m - f_2)} \times h$	
41. $\overline{X}_w$	42. $\frac{\Sigma Wx}{\Sigma W}$			

Conceptual MCQ's

Encircle the correct option.

1. Who used the word data handling first time?
(a) Sir Ronald Fisher (b) Newton
(c) Al-Beruni (d) none of above

2. Ungrouped data also known as _____.
(a) raw data (b) grouped data (c) computer data (d) none of above

3. The number of times a value occurs in a data is:
(a) event (b) frequency (c) sample space (d) experiment

4. **Class boundaries may also be obtained from the mid points (x) as:**

(a) $\left[x \pm h\right]$ (b) $\left[x \pm \frac{h}{3}\right]$ (c) $\left[x \pm \frac{h}{2}\right]$ (d) $\left[x \pm \frac{h}{4}\right]$

5. **_____ is mostly represented by using histogram and frequency polygon.**

(a) continuous data (b) grouped data (c) ungrouped data (d) none of data

6. **If $\bar{X} = \frac{\Sigma X}{n}$ is known as:**

(a) direct method (b) indirect method (c) no method (d) none of above

7. **If we have 64, 75, 81, 87, 90 then A.M is:**

(a) 76.4 (b) 77.4 (c) 78.4 (d) 79.4

8. $\frac{\text{Lower class limit + upper class limit}}{2} =$ _____

(a) mean point (b) extreme point (c) mid point (d) none of above

9. **Histogram and frequency polygon are the types of:**

(a) frequency distribution (b) mode
(c) median (d) mean

10. $X_{max} - X_{min} =$ _____

(a) mode (b) range (c) mean (d) median

Answers:

1.	Sir Ronald Fisher	2.	raw data	3.	frequency	4.	$\left[x \pm \frac{h}{2}\right]$
5.	continuous data	6.	direct method	7.	79.4	8.	mid point
9.	frequency distribution	10.	range				

Additional Short Questions

Short Answer Questions prepared in the light of new Examination Techniques (Knowledge, Understanding, Application, Analytical & Conceptual)

1. Define information.

Ans. To know about something is called information.

2. What is information handling?

Ans. To represent information in a manageable way so that useful conclusions can be drawn is called information handling.

3. What do you mean data?

Ans. The collection of meaningful information in the form of acts and numerical figures is known as data.

4. Where from data obtained?

Ans. Data can be obtained from existing sources e.g. office records.

5. Define information handling.

Ans. Information handling is the process of collecting, organizing, summarizing, analyzing and interpreting numerical data.

6. How many categories of data?

Ans. There are two categories of data.

7. Name the categories of data.

Ans. **(i)** Discrete data **(ii)** Continuous data

8. What is discrete data?

Ans. It can take only specific values. Whole number are used to write discrete data. e.g: No. of books sold by a shopkeeper.

9. What is continuous data?

Ans. It can take every possible value is a given interval. Decimal numbers are used to write continuous data.

10. Define ungrouped data.

Ans. Data which is not arranged in any systematic order (group) is called ungrouped data.

11. What is frequency distribution?

Ans. A distribution or table that represents classes or group along with their respective class frequencies is called frequency distribution.

12. Write steps to construct frequency distribution.

Ans.(i) Find the range (Range = $X_{max} - X_{min}$)

(ii) Find the size of class by dividing the range by the number of classes you wish to make.

(iii) Prepare four columns **(iv)** Make classes

(v) Draw tally marks **(vi)** Make class boundaries

13. Define Range

Ans. Range is the difference between the greatest value and smallest value.

Range = $X_{max} - X_{min}$.

14. What are class boundaries?

Ans. The difference between lower class limit of the second class and upper class limit of the first class.

15. How many types of graphs to represent the frequency distribution?

Ans. There are two graphs. (a) Histogram (b) Frequency polygon

16. What do you know about histogram?

Ans. This is graph of adjacent rectangles. It is limiter to bar graph, but it is construct for frequency distribution. The values of data are represented along horizontal axis and frequencies are shown by bars perpendicular to the horizontal axis. Bars are equal in width.

17. What are the steps to make histogram with equal class limits?

Ans. (i) Draw lines as x and y-axis which are perpendicular to each other.

(ii) Class boundaries are mark on x-axis and rectangle is made against each group with its width proportional to the class size limit and height proportional to class frequencies.

(iii) Setting a scale, draw frequencies on y-axis. The resulting figure is histogram.

18. What are the steps to make histogram with unequal class limits?

Ans. (i) Draw x-axis and y-axis which are perpendicular each other.
(ii) Class boundaries are mark on x-axis and a rectangle is made against each group with its width proportional to the size of class limits and heights proportional to class frequency.
(iii) We obtained adjacent heights of rectangle.

19. Define frequency polygon.

Ans. It is a closed geometrical figure used to display a frequency distribution graphically. A line graph of a frequency distribution is known as frequency polygon in which frequencies are plotted against their midpoints.

20. What is meant by midpoint?

Ans. Midpoint is the average value of lower and upper class limits. It is also known as class work.

$$\text{Mid point} = \frac{\text{Lower class limit} + \text{upper class limit}}{2}$$

21. Write steps to draw a frequency polygon for frequency distribution.

Ans. (i) Draw x and y axis which are perpendicular to each other.
(ii) Take mid points on x-axis and class frequency on y-axis.
(iii) Put a dot mark against each midpoint, corresponding to its class frequency join these dots to get frequency polygon.
(iv) The lines at both ends are joined together with next mid points to touch the bases of x-axis.

12.2 Measure of Location (Central Tendency)

22. What is measure of central tendency?

Ans. The measure that gives the centre of the data is called measure of central tendency.

23. How many types of central tendency?

Ans. There are four types of central tendency.

24. Name the types of central tendency.

Ans. (i) Arithmetic mean (ii) Median (iii) Mode (iv) Weighted mean

25. Defined arithmetic mean.

Ans. A value of variable which is obtained by dividing the sum of all the values by their number of observations.

It is denoted by $\overline{X}$. $\overline{X} = \frac{x_1 + x_2 + x_3 + \ldots\ldots x_n}{n} = \frac{\Sigma X}{n}$

26. If we have 64, 75, 81, 87, 90 then find A.M.

Ans. $A.M = \overline{X} = \frac{\Sigma X}{n} = \frac{64+75+81+81+90}{5}$

$\overline{X} = \frac{397}{5} = 79.4$

27. What is the method of finding arithmetic mean for grouped data?

Ans. Let $x_1, x_2, \ldots x_n$ be the midpoints of class limits with corresponding frequencies say $f_1, f_2 \ldots f_n$ then

$$\overline{X} = \frac{f_1x_1 + f_2x_2 + \ldots + f_nx_n}{f_1 + f_2 + \ldots + f_n} = \frac{\Sigma fx}{\Sigma f}$$

28. What is the short formula to find the A.M for ungrouped data?

Ans. $\overline{X} = A + \frac{\Sigma D}{\Sigma n}$

29. What is the short formula to find the A.M for grouped data?

Ans. $\overline{X} = A + \frac{\Sigma fD}{\Sigma f}$

30. Using short formula find A.M for data 40, 45, 50, 52, 50, 60, 56, 70

Ans. x = 40, 45, 50, 52, 50, 60, 56, 70

Hence A = 52 (assume) D = x - A = -12, -7, -2, 0, -2, 8, 4, 18

$\Sigma\Delta D = -23 + 30 = 7$

$$\overline{X} = A + \frac{\Sigma D}{n} = 52 + \frac{7}{8} = 52 + 0.875 = 52.88 = 53$$

31. Define median.

Ans. It is middle most value in an arranged data set. It is value which divides the data into two equal parts. 50% data a before and 50% after it. It is denoted by $\tilde{X}$.

32. What is the formula of median for when n is odd number?

Ans. $\text{median}(\tilde{X}) = \left(\frac{n+1}{2}\right)^{th}$ observations.

33. What is the formula of median for when n is even number?

Ans. $\tilde{X} = \frac{1}{2}\left(\left(\frac{n}{2}\right)^{th} + \left(\frac{n+2}{2}\right)^{th}\right)$ observations.

34. If we have 8, 12, 18, 13, 16, 5, 20 then find median.

Ans. 8, 12, 18, 13, 16, 5, 20, n = 7

$$\tilde{X} = \left(\frac{n+1}{2}\right)^{th} = \left(\frac{7+1}{2}\right)^{th} = \left(\frac{8}{2}\right)^{th} = 4\text{th} = 13$$

35. If we have 5, 8, 9, 11, 15, 23, 35, 41, 48, 51 then find median.

Ans. 5, 8, 9, 11, 15, 23, 35, 41, 48, 51 n = 10

So, $\tilde{X} = \frac{1}{2}\left(\left(\frac{n}{2}\right)^{th} + \left(\frac{n+2}{2}\right)^{th}\right)$

$= \frac{1}{2}\left(\left(\frac{10}{2}\right)^{th} + \left(\frac{10+2}{2}\right)^{th}\right)$

$= \frac{1}{2}(5+6)$

$= \frac{1}{2}(15+23) = \frac{1}{2}(38) = 19$

36. What is the formula of median for grouped data?

Ans. $\hat{X} = \ell + \frac{h}{f}\left(\frac{n}{2} - c\right)$

37. Define mode.

Ans. In the data the values which appears most often is called mode. It is denoted by $\hat{X}$.

38. In 75, 76, 80, 80, 82, 82, 82, 85 find mode.

Ans. 75, 76, 80, 80, 82, 82, 82.

$\hat{X} = 82$

39. In 13, 14, 15, 11, 16, 10, 19, 20, 18, 17 find the mode.

Ans. 10, 15, 15, 15, 20, 20, 20, 25, 32

$\hat{X}$ = 15 and 20

40. What is the formula to find mode for grouped data?

Ans. $\text{Mode} = \ell + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$

41. If $\ell = 52$, $h = 2$, $f_m = 10$, $f_1 = 7$, $f_2 = 9$ find mode ($\hat{X}$)

Ans. $\text{mode} = \ell + \frac{(f_m - f_1)}{(f_m - f_1) + (f_m - f_2)} \times h$

$= 52 + \frac{(10-7)}{(10-7)+(10-9)} \times 2$

$= 52 + \frac{3}{(3)+(1)} \times 2$

$= 52 + \frac{6}{4} = 52 + 1.5 = 53.5$

42. Write the formula for weighted mean.

Ans. $\bar{X}_w = \frac{W_1X_1 + W_2X_2 + \ldots + W_nX_n}{W_1 + W_2 + \ldots W_n}$

$= \frac{\sum_{i=1}^{n} W_iX_i}{\sum_{i=1}^{n} W_i} = \frac{\Sigma WX}{\Sigma W}$

43. If

Marks x	74	78	74	90
W	4	3	5	6

Find $\bar{X}_w$

Ans. $\bar{X}_w = \frac{\Sigma WX}{\Sigma W} = \frac{4(74) + 3(78) + 5(74) + 6(90)}{4 + 3 + 5 + 6}$

$= \frac{296 + 234 + 370 + 540}{18}$

$= \frac{1440}{18} = 80$

Conceptual Short Questions

1. Whole numbers is the example of which data?

Ans. Discrete data.

2. Decimal numbers is the example of which data?

Ans. Continuous data.

3. Define frequency.

Ans. The number of times a value occurs in data is called frequency of that value. It is denoted by f.

4. If the greatest value is 136, and smallest value is 30 and number of classes is 10 then find size of class.

Ans. $\frac{\text{size of class}}{\text{No. of classes}} = \frac{\text{greatest value} - \text{smallest value}}{\text{No. of classes}}$

$= \frac{136 - 30}{10} = \frac{106}{10} = 10.6 \approx 11$

5. How we obtained class boundaries from mid point?

Ans. Here h is the difference between any two consecutive value of x.

6. If greatest value is 40 and smallest value is 10 then find the range?

Ans. Range = Greatest value - smallest value

$= 40 - 10$

$= 30$

7. If the greatest value is 40 and smallest value is 10 and number of class is 10 then find the size of class limit.

Ans. Size of class limit $= \frac{\text{Range}}{\text{No. of classes}}$

$= \frac{40 - 10}{10} = \frac{30}{10} = 3$

8. Why we used measure of central tendency?

Ans. The measure of central tendency is used to find out the middle or central value of a data set.

9. This formula $\frac{\Sigma WX}{\Sigma W}$ is used to find which type of mean.

Ans. $\frac{\Sigma WX}{\Sigma W}$ is used for weighted mean

— ✲✲✲ —

Unit 13

Probability

Probability

Probability is the chance of occurence of a particular event.

Probability Formula

$$\text{Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

It is written as: $P(A) = \frac{n(A)}{n(s)}$

P(A) = Probability of an event A

n(A) = Number of favourable outcomes

n(S) = Total number of possible outcomes

Basic Concepts of Probability

Experiment: The process which generates results e.g., tossing a coin, rolling a dice, etc. is called an experiment.

Outcomes: The results of an experiment are called outcomes e.g., the possible outcomes of tossing a coin are head or tail, the possible outcomes of rolling a dice are 1, 2, 3, 4, 5, or 6.

Favourable Outcome: An outcome which represents how many times we expect the things to be happened e.g., while tossing a coin, there is 1 favourable outcome of getting head or tail. While rolling a dice, there are 3 favourable outcomes of getting multiples of 2 i.e. {2, 4, 6}

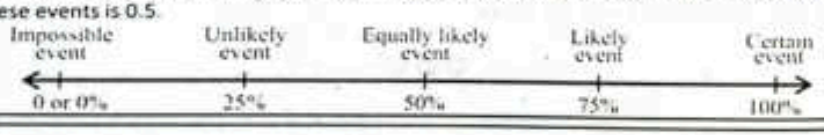
Remember!
Each element of the sample space is called sample point.

Sample Space: The set of all possible outcomes of an experiment is called sample space. It is denoted by 'S' e.g., while tossing a coin, the sample space will be S = {H, T}. While rolling a dice, the sample space will be S = {1, 2, 3,4, 5, 6}.

Event: The set of results of an experiment is called an event e.g., while rolling a dice getting even number is an event i.e., A = {2,4, 6}; n(A) = 3.

Recall! Types of Events:

- **Certain event:** An event which is sure to occur. The probability of sure event is 1.
- **Impossible event:** An event cannot occur in any trial. The probability of this event is 0.
- **Likely event:** An event which will probably occur. It has greater chance to occur.
- **Unlikely event:** An event which will not probably occur. It has less chance to occur.
- **Equally likely events:** The events which have equal chance of occurrence. The probability of these events is 0.5.



Probability of Single Event

Example 1: Abdul Raheem rolls a fair dice, what is the probability of getting the number divisible by 3?

Solution: When a dice is rolled, the sample space will be:

S= {1,2, 3, 4, 5, 6} ; n(S) = 6

Let "A" be the event of getting the number divisible by 3.

A = { 3, 6}; n(A) = 2

$$PA = \frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

The probability of getting the number divisible by 3 is $\frac{1}{3}$.

Keep in mind
The range of probability for an event is $0 \leq P(A) \leq 1$

Teacher's note:
Clear the concept of all the types of events by using different colours of balls or pencils etc.

Example 2: If Zeeshan rolled two fair dice, find the probability of getting:

(i) Even numbers on both dice.

(ii) Multiples of 3 on both dice.

(iii) Even number on the first dice and the number 3 on the second dice.

(iv) At least the number 3 on the first dice and number 4 on the second dice.

Solution: When a pair of fair dice is rolled, the sample space will be:

2nd \ 1st	1	2	3	4	5	6
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

Try Yourself!
Can you find out the sample space when 3 dice are rolled.

(i) Even numbers on both dice.

Let "A" be the event of getting even numbers on both dice.

A = {(2, 2), (2, 4), (2, 6), (4, 2) (4, 4), (4, 6), (6, 2), (6, 4), (6, 6)}

n(A) = 9; n(S) = 36

$$P(A) = \frac{n(A)}{n(S)} \quad \frac{9}{36} = \frac{1}{4}$$

Thus, the probability of getting even numbers on both dice is $\frac{1}{4}$.

(ii) Multiple of 3 on both dice.

Let "B" be the event of getting multiples of 3 on both dice.

B = {(3, 3), (3, 6), (6, 3), (6, 6)}

n(B) = 4; n(S) = 36

$$P(B) = \frac{n(A)}{n(S)} \quad \frac{4}{36} = \frac{1}{9}$$

Thus, the probability of getting multiples of 3 on both dice is $\frac{1}{9}$

(iii) Even number on the first dice and the number 3 on the second dice.

Let "C" be the event of getting even numbers on the first dice and the number 3 on the second dice.

$C = \{(2,3), (4,3), (6,3)\}$

$n(C) = 3; n(S) = 36$

$$P(C) = \frac{n(A)}{n(S)} \quad \frac{3}{36} = \frac{1}{12}$$

Thus, the probability of getting an even number on the first dice and the number 3 on the second dice is $\frac{1}{12}$.

(iv) At least the number 3 on the first dice and number 4 on the second dice.

Let "D" be the event of getting at least the number 3 on the first dice and number 4 on the second dice.

$D = \{(3,4), (4, 4), (5, 4), (6, 4)\}$

$n(D) = 4; n(S) = 36$

$$P(D) = \frac{n(D)}{n(S)} \quad \frac{4}{36} = \frac{1}{9}$$

Thus, the probability of getting at least the number 3 on the first dice and number 4 on the 2nd dice is $\frac{1}{9}$.

Probability of an Event Not Occurring

Sometimes, we are interested in the probability that the head will not occur while tossing a coin.

Let "A" be the event of getting head while tossing a coin, then the event A' be the event of not getting head while tossing a coin.

The probability of not getting head while tossing a coin is known as the complement of that event. It is written as P(A') or $P(A^c)$.

The complement of an event "A" is calculated by the given formula:

$$P(A') = 1 - P(A)$$

For example, while tossing a coin, the probability of getting a head is:

$$P(A) = \frac{1}{2}$$

and the probability of not getting a head is:

$$P(A') = 1 - P(A)$$

Teachers' note:
Give more examples to explain complement of events e.g., if the desired outcome is head on a flipping coin, the complement is tail. The compliment rule states that the sum of the probability of an event and its complement must be equal to 1.

$$= 1 - \frac{1}{2} = \frac{1}{2}$$

Thus, the complement of the event of getting a head is $\frac{1}{2}$.

Example 3: Zubair rolls a dice, what is the probability of not getting the number 6?

Solution: Let "A" be the event of getting the number 6.

The sample space while rolling a dice is: $S = \{1, 2, 3, 4, 5, 6\}$

$n(S) = 6$

$A = \{6\}; n(A) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{6}$$

Remember!
The sum of the probability of an event "A" and the probability of an event not occurring "A" is always "1"
P(A) + 1 – P(A') = 1

To find out probability of not getting the number 6, we have

$$P(A') = 1 - P(A)$$

$$= 1 - \frac{1}{6} = \frac{6-1}{6} = \frac{5}{6}$$

Thus, the probability of not getting the number 6 is $\frac{5}{6}$.

Example 4: If two fair dice are rolled. What is the probability of getting:
(i) not a double six (ii) not the sum of both dice is 8

Solution: Sample space of two fair dice is given by:

$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

$n(S) = 36$

(i) not a double six.

Let "A" be the event that a double six occurs.

$A = \{(6, 6)\}; n(A) = 1$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{36}$$

Let " A' " be the event that not a double six occurs As we know that

$$P(A') = 1 - P(A)$$

$$= 1 - \frac{1}{36} = \frac{36-1}{36} = \frac{35}{36}$$

Thus, the probability of not getting the double six is $\frac{35}{36}$.

(ii) not the sum of both dice is 8.

Let "B" be the event that the sum of both dice is 8.

$B = \{(2, 6), (3, 5), (4, 4), (5, 3), (6,2)\}$

$n(B) = 5$

$$P(B) = \frac{n(A)}{n(S)} = \frac{5}{36}$$

Let " B' " be the event not sum of both dice is 8.

$P(B') = 1 - P(B)$

$= 1 - \frac{5}{36} = \frac{36-5}{36} = \frac{31}{36}$

Thus, the probability of not the sum of both dice be 8 is $\frac{31}{36}$.

Real Life Problems Involving Probability

Example 5: Let A, B and C are three missiles and they are fired at a target. If the probabilities of hitting the target are $P(A) = \frac{1}{4}$, $P(B) = \frac{3}{7}$, $P(C) = \frac{5}{9}$, respectively.

Find the probabilities of

(i) missile A does not hit the target. (ii) missile B does not hit the target.

(iii) missile C does not hit the target.

Solution: (i) missile A does not hit the target.

Since, $P(A) = \frac{1}{4}$

Let 'A' ' be the event that missile A does not hit the target

$P(A') = 1 - P(A)$

$= 1 - \frac{1}{4} = \frac{4-1}{4} = \frac{3}{4}$

Thus, the probability of missile 'A' does not hit the target is $\frac{3}{4}$

(ii) missile 'B' does not hit the target.

Since, $P(B) = \frac{3}{7}$

Let ' B' ' be the event missile B does not hit the target

$P(B') = 1 - P(B)$

$= 1 - \frac{3}{7} = \frac{7-3}{7} = \frac{4}{7}$

Thus, the probability of missile 'B' does not hit the target is $\frac{4}{7}$.

(iii) missile 'C' does not hit the target.

Since, $P(C) = \frac{5}{9}$

Let ' C' ' be the event missile C of not hitting the target

$P(C') = 1 - P(C)$

$= 1 - \frac{5}{9} = \frac{9-5}{9} = \frac{4}{9}$

Thus, the probability of missile 'C' does not hit the target is $\frac{4}{7}$.

Example 6: A bag contains 5 blue balls and 8 green balls. Find the probability of selecting at random:

(i) a blue ball (ii) a green ball. (iii) not a green

Solution: (i) a blue ball

Let 'A' be the event that the ball is blue

Blue balls = $n(A) = 5$

Total balls = $n(S) = 5 + 8 = 13$

$P(A) = \frac{n(A)}{n(S)} = \frac{5}{13}$

Thus, the probability of selecting a blue ball is $\frac{5}{13}$.

Think!
The probability that a person A will be alive 0.75. Can you find out the complement of that event?

(ii) a green ball

Let 'B' be the event that ball is green

Green balls = $n(B) = 8$

Total balls = $n(S) = 5 + 8 = 13$

$P(A) = \frac{n(A)}{n(S)} = \frac{8}{13}$

Thus, the probability of selecting green ball is $\frac{8}{13}$.

(iii) not a green ball

Let ' B' ' be the event that the ball is not green.

$P(B') = 1 - P(B)$

$= 1 - \frac{8}{13} = \frac{13-8}{13} = \frac{5}{13}$

Thus, the probability of not selecting a green ball is $\frac{5}{13}$.

Example 7: A card is drawn at random, from a pack of 52 playing cards. What is the probability of getting:

(i) a card of heart (ii) neigther spade nor heart

Solution: (i) a card of heart

Total number of cards = 52 ; $n(S) = 52$

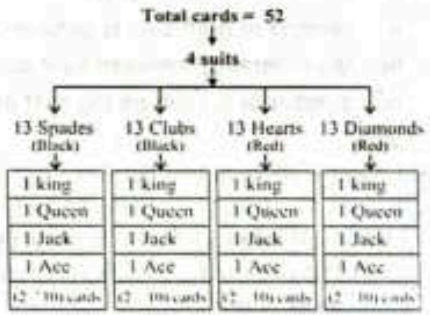
Let 'A' be the event of selecting a card of heart.

Number of heart cards = 13 ; $n(A) = 13$

$P(A) = \frac{n(A)}{n(S)}$

$= \frac{13}{52} = \frac{1}{4}$

Thus, the probability of getting a card of is $\frac{1}{4}$.



(ii) neither spade nor heart

Let 'B' be the event of selecting a card of spade or heart
Number of spade and heart cards = 26 : n(B) = 26

$$P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{26}{52} = \frac{1}{2}$$

Let 'B' ' be the event of selecting neither spade nor heart card.

$$P(B') = 1 - P(B)$$

$$= 1 - \frac{1}{2} = \frac{1}{2}$$

Thus, the probability of getting neither spade nor heart cards is $\frac{1}{2}$.

Solved Exercise 13.1

1. Arshad rolls a dice, with sides labelled L, M, N, O, P, U. What is the probability that the dice lands on consonant?

Sol: Sample space S = {L,M,N,O,P,U}; n(S) = 6
Let "A" be the event of getting a constant

$$A = \{L,M,N,P\}$$

$$n(A) = 4$$

$$p(A) = \frac{n(A)}{n(S)}$$

$$= \frac{4}{6} = \frac{2}{3}$$

So, the probability that dice lands on consonant is $\frac{2}{3}$.

2. Shazia throws a pair of fair dice. What will be the probability of getting:

i. sum of dots is at least 4.

ii. product of both dots is between 5 to 10.

iii. the difference between both the dots is equal to 4.

iv. number at least 5 on the first dice and the number at least 4 on the second dice.

Sol: i. Sample space of two fair dice

S = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

$$n(S) = 36$$

Let "A" be the event that sum of dots in at least 4.

A = {(1,3), (1,4), (1,5), (1,6), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

$$n(A) = 33$$

$$p(A) = \frac{n(A)}{n(S)}$$

$$= \frac{33}{36} = \frac{11}{12}$$

Thus, the probability of getting sum of dots at last 4 is $\frac{11}{12}$.

Let "B" be the event that product of both dots is between 5 to 10.

B = {(1,5), (1,6), (2,3), (2,4), (2,5), (3,2), (3,3), (4,2), (5,1) (5,2), (6,1)}

$$n(B) = 11$$

$$p(B) = \frac{n(B)}{n(S)}$$

$$= \frac{11}{36}$$

Thus, the probability of getting product of both dots between 5 to 10 is $\frac{11}{36}$.

ii. Let "E" be the event difference between both the dots is equal to 4.

E = {(1,5), (2,6), (5,1), (6,2)}

$$n(E) = 4$$

$$p(E) = \frac{n(E)}{n(S)}$$

$$= \frac{4}{36} = \frac{1}{9}$$

Thus, the required probability is $\frac{1}{9}$.

iv. Let "F" be the event that number at least 5 on the first dice and the number at least 4 on the sceond dice.

F = {(5,4), (5,5), (5,6), (6,4), (6,5), (6,6)}

$$n(F) = 6$$

$$p(F) = \frac{n(F)}{n(S)}$$

$$= \frac{6}{36} = \frac{1}{6}$$

Thus, the required probability is $\frac{1}{6}$.

3. One alphabet is selected at random from the word "MATHEMATICS". Find the probability of getting:

i. vowel ii. consonant iii. an E iv. an A

v. not M vi. not T

Sol: Total leters in the word "MATHEMATICS" = 11

i. Let "A" be the event of selecting the alphabet that is vowel. There are A, E, A and I four vowels.

$$n(A) = 4$$

$$p(A) = \frac{n(A)}{n(S)}$$

$$= \frac{4}{11}$$ Thus, the required probability is $\frac{4}{11}$.

ii. Let "B" be the event of selecting the alphabet that is vowel. There are M, T, H, M, T, C, S seven consonants.

$$n(B) = 7$$

$$p(B) = \frac{n(B)}{n(S)}$$

$$= \frac{7}{11}$$ Thus, the probability of getting consonant is $\frac{7}{11}$.

iii. Let "C" be the event of selecting one alphabet that is an E. There is one E.

$$n(E) = 1$$

$$p(E) = \frac{n(E)}{n(S)}$$

$$= \frac{1}{11}$$ Thus, the probability of getting consonant is $\frac{1}{11}$.

iv. Let "E" be the event of selecting one alphabet that is an A. There are A, A two A.

$$n(E) = 2$$

$$p(E) = \frac{n(E)}{n(S)}$$

$$= \frac{2}{11}$$ Thus, the probability of getting an A is $\frac{2}{11}$.

v. Let "M" be the event of selecting one alphabet that is an M. There are M, M two M.

$$n(M) = 2$$

$$p(M) = \frac{n(M)}{n(S)} = \frac{2}{11}$$

Let "M" be the event of selecting one alphabit that is not M.

$$p(M') = 1 - p(M)$$

$$= 1 - \frac{2}{11}$$

$$= \frac{11-2}{11} = \frac{9}{11}$$

Thus, the probability of getting not M is $\frac{9}{11}$.

ii. Let "H" be the event of selecting one alphabet that is an T. There are T, T two T.

$$n(H) = 2$$

$$p(H) = \frac{n(H)}{n(S)} = \frac{2}{11}$$

Let "(H')" be the event of selecting the alphabit that is not T.

$$p(H') = 1 - p(H)$$

$$= 1 - \frac{2}{11}$$

$$= \frac{11-2}{11} = \frac{9}{11}$$

Thus, the probability of getting not T is $\frac{9}{11}$.

4. Aslam rolled a dice. What is the probability of getting the numbers 3 or 4? Also find the probability of not getting the numbers 3 or 4.

Sol: Sample space S = {1,2,3,4,5,6}

Let "A" be the event of getting the numbers 3 or 4.

$$A = \{3,4\}$$

$$n(A) = \{2\}$$

$$p(A) = \frac{n(A)}{n(S)}$$

$$= \frac{2}{6} = \frac{1}{3}$$

Let " A' " be the event of not getting the numbers 3 or 4.

$$A' = 1 - p(A)$$

$$= 1 - \frac{1}{3}$$

$$= \frac{3-1}{3} = \frac{2}{3}$$

Thus, the probability of getting the number 3 or 4 is $\frac{1}{3}$ and the probability of getting the number not 3 or 4 is $\frac{2}{3}$.

5. Abdul Hadi labelled cards from 1 to 30 and put them in a box. He selects a card at random. What is the probability that selected card containing:

i. the number 25 **ii. number between 17 to 22**

iii. number at least 20 **iv. number not 27 and 29**

v. number not between 12-15

Sol: i. Total outcomes = 30 cards

$n(s) = 30$

Let "A" be the event of selecting a card that is the number 25. There is only one card having the number 25.

$A = \{25\}$

$n(A) = 1$

$$p(A) = \frac{n(A)}{n(S)} = \frac{1}{30}$$

Thus, the probability of selecting he 25 is $\frac{1}{30}$

ii. Let "B" be the event of selecting a card that is of the number between 17 to 22. There are 17,18,19,20,21,22 six cards.

$B = \{17,18,19,20,21,22\}$

$n(B) = 6$

$$p(B) = \frac{n(B)}{n(s)} = \frac{6}{30} = \frac{1}{5}$$

Thus, the probability of a card that is of number between 17 and 25 is $\frac{1}{5}$.

iii. Let "C" be the event of selecting a card that has a number at least 20. There are 11 such card that has the number at least 20. (From 20 to 30)

$= \{20,21,22,23,24,25,26,27,28,29,30\}$

$n(C) = 11$

$$p(C) = \frac{n(C)}{n(S)} = \frac{11}{30}$$

Thus, the probability of selecting a case that has a number at 20 is $\frac{11}{30}$.

iv. Let "B" be the event of selecting a card that is of number 27 and 29. There are card having number 27 and 29.

$B = \{27,29\}$

$n(B) = 2$

$$p(B) = \frac{n(B)}{n(S)} = \frac{2}{30} = \frac{1}{15}$$

Let "B" be the event of selecting a card that is number not 27 and 29.

$$p(B') = 1 - p(B)$$

$$= 1 - \frac{1}{15}$$

$$= \frac{15-1}{15} = \frac{14}{15}$$

Thus, the probability of selecting a card that is number not 27 and 29 is $\frac{14}{15}$.

v. Let "C" be the event of selecting a card that is number between 12–15. There are 4 number 12,13,14,15 between 12–15.

$C = \{12,13,14,15\}$

$n(C) = 4$

$$p(C) = \frac{n(C)}{n(S)} = \frac{4}{30} = \frac{2}{15}$$

Let "C" be the event of selecting a card not between 12–15.

$$p(C') = 1 - p(C)$$

$$= 1 - \frac{2}{15}$$

$$= \frac{15-2}{15} = \frac{13}{15}$$

Thus, the probability of selecting a card not between 12–15 and $\frac{13}{15}$.

6. The probability that Ayesha will pass the examination is 0.85. What will be the probability that Ayesha will not pass the examination?

Sol: Let "B" be event that Ayesha will pas the examination an "B" be event that Ayesha will not pas the examination.

$p(B) = 0.85$

$p(B') = 1 - p(B)$

$= 1 - 0.85 = 0.15$

Thus, the probability that Ayesha will not pass the examination is 0.15.

7. Taabish tossed a fair coin and rolled a fair dice once. Find the probability of the following events:

i. tail on coin and at least 4 on dice.

ii. head on coin and the number 2,3 on dice.

iii. head and tail on coin and the number 6 on dice.

iv. not tail on coin and the number 5 on dice.

v. not head on coin and the number 5 and 2 on dice.

Sol: Sample space of tossing a coin and rolling a dice.

$S = \{(H,1), (H,2), (H,3), (H,4), (H,5), (H,6), (T,1), (T,2), (T,3), (T,4), (T,5), (T,6)\}$

i. Let "A" be the event that tail on coin and at least 4 on dice.

$A = \{(T,4), (T,5), (T,6)\}$

$n(A) = 3$

$$p(A) = \frac{n(A)}{n(S)}$$

$$= \frac{3}{12} = \frac{1}{4}$$

Thus, the probability of tail on coin and at least 4 on diece is $\frac{1}{4}$.

ii. Let "B" be the event that head on coin and the number 2, 3 on dice.

$B = \{(H,2), (H,3)\}$

$n(B) = 2$

$$p(B) = \frac{n(B)}{n(S)}$$

$$= \frac{2}{12} = \frac{1}{6}$$

Thus, the probability of head on coin and number 2, 3 on dice is $\frac{1}{6}$.

iii. Let "H" be the event that head and tail on coin and the number 6 on dice.

$H = \{(H,6), (T,6)\}$

$n(H) = 2$

$$p(H) = \frac{n(H)}{n(S)}$$

$$= \frac{2}{12} = \frac{1}{6}$$

Thus, the probability of head and tail on coin and number 6 on dice is $\frac{1}{6}$.

iv. Let "E" be the event tail on coin and the number 5 on dice.

$E = \{(T,5)\}$

$n(E) = 1$

$$p(E) = \frac{n(E)}{n(S)}$$

$$= \frac{1}{12}$$

Let "E" be the event not tail on coin and the number 5 or dice.

$P(E') = 1 - P(E)$

$$n(E) = 1 - \frac{1}{12}$$

$$p(E) = \frac{12-1}{12} = \frac{11}{12}$$

Thus, the probability of not tail on coin and the number 5 on dice is $\frac{11}{12}$.

Let "B" be the event that head on coin and the number 5 and 2 on dice.

$B = \{(H,5), (H,2)\}$

$$p(B) = \frac{n(B)}{n(S)}$$

$$= \frac{2}{12} = \frac{1}{6}$$

Let "B" be the event that not head on coin and the number 5 and 2 on dice.

$p(B') = 1 - p(B)$

$$n(E) = 1 - \frac{1}{6}$$

$$p(E) = \frac{6-1}{6} = \frac{5}{6}$$

Thus, the probability of not head on coin and the number 5 and 2 on dice is $\frac{5}{6}$.

8. A card is selected at random from a well shuffled pack of 52 plying cards. What will be the probability of selecting:

i. a queen **ii. neither a queen nor a jack**

Sol: i. Total number of cards = 52

$n(S) = 52$

Let "A" be the event of selecting a card that a queen. The number of queen cards = 4.

$n(A) = 4$

$$p(A) = \frac{n(A)}{n(S)}$$

$$= \frac{4}{52} = \frac{1}{13}$$

Thus, probability of selecting a card that a queen is $\frac{1}{13}$.

ii. Total number of cards = 52

$n(S) = 52$

Let "B" be the event of selecting that is a queen and jak.

There are 4 queen cards and 4 jack cards in 52 playing cards.

$n(B) = 8$

$$p(B) = \frac{n(B)}{n(S)}$$

$= \frac{8}{52} = \frac{2}{13}$

Let "B" the event of selecting a card that neirther a green nor a jack

$p(B') = 1 - \frac{2}{13}$

$p(B) = \frac{13-2}{13} = \frac{11}{13}$

Thus, the probability of selecting a card that heither a queen nor a jack is $\frac{11}{13}$.

9. A card is chosen at random from a pack of 52 playing cards. Find the probability of getting:

i. a jack **ii. no diamond**

Sol: i. Total number of cards = 52

$n(S) = 52$

Let "A" be the event of selecting a card that is a jack.

There are 4 jack in 52 playing card.

$n(A) = 4$

$p(A) = \frac{n(A)}{n(S)}$

$= \frac{4}{52} = \frac{1}{13}$

Thus, probability of selecting a card that a jack is $\frac{1}{13}$.

ii. Total number of cards = 52

$n(S) = 52$

Let "B" be the event of selecting a card that is diamond.

There are 13 diaminds cards in 52 playing cards.

$n(B) = 13$

$p(B) = \frac{n(B)}{n(S)}$

$= \frac{13}{52} = \frac{1}{4}$

Let "B" be the event of selecting a card that is no diamond.

$p(B') = 1 - p(B)$

$p(B') = 1 - \frac{1}{4}$

$p(B) = \frac{4-1}{4} = \frac{3}{4}$

Thus, the probability of selecting a card that is no diamond is $\frac{3}{4}$.

Relative Frequency as an Estimate of Probability

Relative frequency tells us how often a specific event occurs relative to the total number of frequency event or trials. It is calculated by using the following method:

$$\text{Relative frequency} = \frac{\text{Frequency of specific event}}{\text{Total frequency}} = \frac{x}{N}, \text{ where } N = \Sigma f$$

Example 8: Find the relative frequency of the given date.

X	2	3	4	5	6	7	8
f	3	5	6	9	10	8	2

Solution:

X	f	Relative frequency
2	3	$\frac{3}{43} = 0.07$
3	5	$\frac{5}{43} = 0.12$
4	6	$\frac{6}{43} = 0.14$
5	9	$\frac{9}{43} = 0.21$
6	10	$\frac{10}{43} = 0.23$
7	8	$\frac{8}{43} = 0.19$
8	2	$\frac{2}{43} = 0.04$
Total	$\Sigma f = 43$	

Real Life Application of Relative Frequency

Example 9: A survey was conducted on 80 students of Grade – IX and asked about their favourite colour. The responses are:

> **Keep in mind**
> The sum of all the relative frequencies is always equal to or approximately equal to 1.

(i) Red colour = 23 students (ii) Green colour =15 students

(iii) Pink colour = 25 students (iv) Blue colour =10 students

(v) White colour = 7 students.

Find the relative frquency for each colour.

Solution: Total number of students = 80

(i) Relative frequency for red colour $= \frac{23}{80} = 0.29$

It means that 29% students prefer red colour.

> **Remember!**
> Relative frequency is an estimated probability of an event occurring when an experiment is repeated a fixed number of times.

(ii) Relative frequency for green colour $= \frac{15}{80} = 0.19$

It means that 19% students prefer green colour.

(iii) Relative frequency for pink colour $= \frac{25}{80} = 0.31$

It means that 31% students prefer pink colour.

(iv) Relative frequency for blue colour $\frac{10}{80} = 0.12$

It means that 12% students prefer blue colour.

(v) Relative frequency for white colour $= \frac{7}{80} = 0.09$

It means that 9% students prefer white colour.

Try Yourself!
Out of 200 students in a school, 80 play cricket, 50 play football, 25 play volleyball and 45 do not play any game. Can you find out the probability of the students who do not play any game and relative frequency of the students who play cricket?

Example 10: Abdul Rehman obtained different marks in different subjects out of 100 marks. The detail is as under:

Subject	Urdu	English	Islamiyat	Mathematics	Science	Computer Science
Marks Obtained	75	80	72	95	81	85

Find the relative frequency of above given data.

Solution:

Subject	Marks obtained	Relative frequency
Urdu	75	$\frac{75}{488} = 0.15$
English	80	$\frac{80}{488} = 0.16$
Islamiyat	72	$\frac{72}{488} = 0.15$
Mathematics	95	$\frac{95}{488} = 0.19$
Science	81	$\frac{81}{488} = 0.17$
Computer Science	85	$\frac{85}{488} = 0.17$
Total	$\Sigma f = 488$	

Expected Frequency

Expected frequency is a measure that estimate how often an event should be occurred depended on probability. Expected frequency is found by using the following method:

Teacher's note:
Clear the concept to the students that relative frequency as an estimate of probability by using different real life problems.

Expected frequency = Total number of trials × Probability of the event.

$= N \times P(A)$

Example 11: Six fair dice are rolled 50 times. The probability of occurrence of different number of sixes are given below. Find the expected frequency of the following data:

x	0	1	2	3	4	5	6
P(x)	0.09	0.10	0.12	0.24	0.10	0.20	0.15

Find the expected frequency of occurrence of each six.

Solution:

No. of Sixes (x)	P(x)	Expected frequency = N × P(x) = 50 × P(JC)
0	0.09	50 × 0.09 = 4.5
1	0.10	50 × 0.10 = 5
2	0.12	50 × 0.12 = 6
3	0.24	50 × 0.24 = 12
4	0.10	50 × 0.10 = 5
5	0.20	50 × 0.20 = 10
6	0.15	50 × 0.15 = 7.5

Real Life Application on Expected Frequency

Example 12: Find the average number of times getting 1 or 6, when a fair dice is rolled 300 times.

Solution: Let "S" be the sample space when a fair dice is rolled:

$S = \{1,2,3,4,5,6\} ; n(S) = 6$

Let "B" be the event that 1 or 6 comes up.

$B = \{1,6\} ; n(B) = 2$

So, $P(B) = \frac{n(B)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

Therefore, $E(B) = N \times P(B)$

$= 300 \times \frac{1}{3} = 100$

Thus, the average number of times 1 or 6 comes up is 100.

Remember!
Sum of all expected frequencies is always equal to or approximately equal to a fixed number of trials.

Example 13: If the probability of a defective bolt is 0.3. Find the number of non-defective bolts in a total to 800.

Solution: The probability of defective bolt is = 0.3

Probability of non-defective bolt = 1 − 0.3 = 0.7

Number of non-defective bolts = 0.7 × 800 = 560

Thus, the non-defective bolts will be 560.

Solved Exercise 13.2

1. A researcher collected data on number of deaths from Horse-Ricks in Russian Army crops over to years. The table is as follows:

No. of death	0	1	2	3	4	5	6
Frequency	60	50	87	40	32	15	10

Find the relative frequency of te given data.

Sol: Total Frequency = 60+50+87+40+32+15+10 = 294

No. of Death	frequency (f)	Relative frequency $\left(\frac{f}{294}\right)$
0	60	$\frac{60}{294} = 0.204$
1	50	$\frac{50}{294} = 0.170$
2	87	$\frac{87}{294} = 0.296$
3	40	$\frac{40}{294} = 0.136$
4	32	$\frac{32}{294} = 0.109$
5	15	$\frac{15}{294} = 0.051$
6	10	$\frac{10}{294} = 0.034$
	$\Sigma f = 294$	

2. The frequency of defective products in 750 samples are shown in the following table. Find the relative frequency for the given table.

No. of defectives per sample	0	1	2	3	4	5	6	7	8
No. of sample	120	140	94	85	105	50	40	66	50

Sol: Total Frequency = 120+140+94+85+105+50+40+66+50
= 750

No. of Death	frequency (f)	Relative frequency $\left(\frac{f}{294}\right)$
0	120	$\frac{120}{750} = 0.160$

1	140	$\frac{140}{750} = 0.187$
2	94	$\frac{94}{750} = 0.125$
3	85	$\frac{85}{750} = 0.113$
4	105	$\frac{105}{750} = 0.140$
5	50	$\frac{50}{750} = 0.067$
6	40	$\frac{40}{750} = 0.053$
7	66	$\frac{66}{750} = 0.088$
8	50	$\frac{50}{750} = 0.067$
	$\Sigma f = 750$	

3. A quiz competition on general knowledge is conducted. The number of corrected answers out of 5 questions for 100 sets of questions is given below.

X	0	1	2	3	4	5
F	10	23	15	25	18	9

Find the relative frequencies for the given data.

Sol: Total Frequency = 10+23+15+25+18+9
= 100

No. of Death	frequency (f)	Relative frequency $\frac{f}{100}$
0	10	$\frac{10}{100} = 0.10$
1	23	$\frac{23}{100} = 0.23$
2	15	$\frac{15}{100} = 0.15$
3	25	$\frac{25}{100} = 0.25$
4	18	$\frac{18}{100} = 0.18$

5	09	$\frac{9}{100} = 0.09$
	$\Sigma f = 100$	

4. A survey was conducted from the 50 students of a class and asked about their favourite food. The responses are as under:

Name of food item	Biryani	Fresh Juice	Chicken	Bar. B.Q	Sweets
No. of students	40	7	21	15	25

i. how many percentages of students like biryani?
ii. how many percentages of students like chicken?
iii. which food is the least like by the students?
iv. which food is the most prefer by the students?

Sol: i. Total Students = 40+7+21+15+25
= 108

Number of studetns liking Biryani = 40

Percentage of students liking Biryani = $\frac{\text{Number of studets liking Biryani}}{\text{Total number of students}} \times 100\%$

$= \frac{40}{108} \times 100\%$

$= 37\%$

ii. Total number of students = 108

Number of studetns liking Chicken = 21

Percentage of students liking Chicken = $\frac{\text{Number of studets liking Checken}}{\text{Total number of students}} \times 100\%$

$= \frac{21}{108} \times 100\%$

$= 19.4\%$

iii. Fresh juice is least by the students only 7 students like fresh Juice.
iv. Biryani is the most prefer by the students 40 students like Biryani.

5. In 500 trials of a thrown of two dice, what is expected frequency that the sum will be greater than 8?

Sol: Sample space whent two dice are thrown

$n(S) = 36$

Let "A" be the event that sum of two dice will be greater than 8

$A = \{(3,6), (4,5), (4,6), (5,4), (5,5), (5:6), (6.3), (6,4), (6.5), (6.6)\}$

$n(A) = 10$

$p(A) = \frac{n(A)}{n(S)} \Rightarrow = \frac{10}{36}$

$= \frac{5}{18}$

Expected frequency = Total number of traisl × Probability of event

$E(A) = N \times p(A)$

$= 500 \times \frac{5}{18}$

$= \frac{2500}{18} \Rightarrow = 139$

6. What is the expectation of a person who is to get Rs. 120 if he obtains at least 2 heads in single toss of three coins?

Sol: Sample space of tossing 3 coins

$S = \{(HHH), (HHT), (HTH), (THH), (HTT), (THT), (TTH), (TTT)\}$

$n(S) = 8$

Let "A" be event that at last 2 heads obtained in single toss of three coins.

$A = \{(HHH), (HHT), (HTH), (THH)\}$

$n(A) = 4$

$p(A) = \frac{n(A)}{n(S)}$

$= \frac{4}{8} \Rightarrow = \frac{1}{2} = 0.5$

Expectaion = p(Event) × Amount earned

$= (0.5) \times 120$

= Rs. 60

7. Find the expected frequencies of the given data if the experiment is repeated 200 times.

x	0	1	2	3	4	5	6
P(x)	0.11	0.21	0.17	0.18	0.09	0.17	0.07

Sol:

x	p(x)	Expected frequency $N \times P = 200 \times p(x)$
0	0.11	$200 \times 0.11 = 22$
1	0.21	$200 \times 0.21 = 42$
2	0.17	$200 \times 0.17 = 34$
3	0.18	$200 \times 0.18 = 36$
4	0.09	$200 \times 0.09 = 18$
5	0.17	$200 \times 0.17 = 34$
6	0.07	$200 \times 0.07 = 14$

8. The probability of getting 5 sixes while tossing six dice is $\frac{2}{5}$, the dice is rolled 200 times. How many times would you expect it to show 5 sixes?

Sol: Probability of getting 5 six = $\frac{2}{5}$

total number ot trail = 200 dice tosses

Expectd number = Probability × Total number of trails

$$= \frac{2}{5} \times 200 = \frac{400}{5} = 80$$

Thus, the probability shows approximately 80 times the result willdisplay 5 sixes.

Review Exercise 13

1. Four options are given agains each statement. Encircle the correction option.

i. Each element of the sample space is called:
(a) event (b) experiment (c) sample point (d) outcomes

ii. An outcome which represents how many times we expect the things to be happened is called:
(a) outcomes (b) favourable outcome (c) sample space (d) sample point

iii. Which one tells us how often a specific event occurs relative to the total number of frequency event or trials?
(a) expected frequency (b) sum of relative frequency
(c) relative frequency (d) frequency

iv. Estimated probability of an event occurring is also known as:
(a) relative frequency (b) expected frequency
(c) class boundaries (d) sum of expected frequency

v. The sum of all expected frequencies is equal to the fixed number of:
(a) trials (b) relative frequencies (c) outcomes (d) events

vi. The chance of occurrence of a particular event is called:
(a) sample space (b) estimated probability
(c) probability (d) expected frequency

vii. An event which will probably occur. It has greater chance to occur is called:
(a) equally likely event (b) likely event (c) unlikely event (d) certain event

viii. Find out the total number of possible sample space when 4 dice are rolled:
(a) 6^2 (b) 6^3 (c) 6^4 (d) 6^6

ix. While rolling a pair of dice, what will be the probability of double 2?
(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{5}{6}$ (d) $\frac{1}{36}$

x. A card is chosen from a pack of 52 playing cards, find the probability of getting no jack and king:
(a) $\frac{2}{13}$ (b) $\frac{11}{13}$ (c) $\frac{2}{52}$ (d) $\frac{11}{52}$

Answers:

i. sample point ii. favourable outcome iii. relative frequency
iv. relative frequency v. trials vi. probability vii. likely event
viii. 6^4 ix. $\frac{1}{36}$ x. $\frac{11}{13}$

2. i. Define the following:

i. relative frequency ii. expected frequency

Ans: Relative frequency:

Relative frequency tells us how often a specific event occurs relative to the total number of frequency event or trails. It is calculated by using the following method:

$$\text{Relative frequency} = \frac{\text{Frequency of specific event}}{\text{Total frequency}} = \frac{x}{N}$$

ii. Expected frequency:

Expected frequency is a measure that estimate how often an event should be occurred depended on probability. Expected frequency is found by using the following method:

Expected Frequency = Total number × Probability of the event of trials
$= N \times p(A)$

3. An urn contains 10 red balls, 5 green balls and 8 blue balls. Find the probability of selecting at random.

i. a green ball ii. a red ball iii. a blue ball
iv. not a red ball v. not a green ball

Sol. Number of rad balls = 10
Number of green balls = 5
Number of blue balls = 8
Number of total balls in an urn = 10+5+8
= 23
n(S) = 23

i. Let "G" be the event of selecting a green ball
Number of green balls = 5
n(G) = 5

$$p(G) = \frac{n(G)}{n(S)} = \frac{5}{23}$$

Thus, the probability of selecting a green ball is $\frac{5}{23}$.

ii. Let "R" be the event of selecting a red ball
Number of red balls = 10
n(R) = 10

$$p(R) = \frac{n(R)}{n(S)} \Rightarrow = \frac{10}{23}$$

Thus, the probability of selecting a red ball is $\frac{10}{23}$.

iii. Let "B" be the event of selecting a blue ball
Number of blue balls = 8

$$n(B) = 8$$

$$p(B) = \frac{n(B)}{n(S)} \Rightarrow = \frac{8}{23}$$

Thus, the probability of selecting a blue ball is $\frac{8}{23}$.

iv. Let "R" be the event of selecting a red ball
Number of red balls = 10

$$n(R) = 10, \quad n(S) = 23$$

$$p(R) = \frac{n(R)}{n(S)} \Rightarrow = \frac{10}{23}$$

Thus, the probability of selecting not a red ball is $\frac{10}{23}$.

$$p(R') = 1 - p(R)$$
$$= 1 - \frac{10}{23}$$
$$= \frac{23-10}{23} \Rightarrow = \frac{13}{23}$$

Thus, the probability of selecting not a red ball is $\frac{13}{23}$.

v. Let "G" be the event of selecting a green ball
Number of green balls = 5

$$n(G) = 5, \quad n(S) = 23$$

$$p(G) = \frac{n(G)}{n(S)} \Rightarrow = \frac{5}{23}$$

Let "G" be the equal of selecting not a green ball.

$$p(G') = 1 - p(G)$$
$$= 1 - \frac{5}{23}$$
$$= \frac{23-5}{23} \Rightarrow = \frac{18}{23}$$

Thus, the probability of selecting not a green ball is $\frac{18}{23}$.

4. Three coins are tossed together, what is the probability of getting:
i. exactly three heads **ii. at least two tails**
iii. not at least two heads **iv. not exactly two heads**

Sol. When three coin are tossed together, the sample space is:

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
n(S) = 8

i. Let "A" be the equal of getting exactly three heads.
A = {HHH}
n(A) = 1

$$p(A) = \frac{n(A)}{n(S)} \Rightarrow = \frac{1}{8}$$

Thus, the probability of getting exactly three heads is $\frac{1}{8}$.

ii. Let "B" be the equal of getting at least two tails.
B = {HTT, THT, TTH, TTT}
n(B) = 4 ⇒ n(s) = 8

$$p(B) = \frac{n(B)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

Thus, the probability of getting least two tails is $\frac{1}{2}$.

iii. Let "N" be the event of getting at least two heads.
N = {HHH, HHT, HTH, THH}
n(N) = 4 ⇒ n(s) = 8

$$p(N) = \frac{n(N)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

Let "N" be the event of getting at least two heads.

$$p(N') = 1 - p(N)$$
$$= 1 - \frac{1}{2} = \frac{2-1}{2} = \frac{1}{2}$$

Thus, the probability of getting not at least two heads is $\frac{1}{2}$.

iv. Let "D" be the event of getting exactly two heads.
D = {HHT, HTH, THH}
n(D) = 3 ⇒ n(s) = 8

$$p(D) = \frac{n(D)}{n(S)} = \frac{3}{8}$$

Let "D'" be event to getting not exactly two heads.

$$p(D') = 1 - p(D)$$
$$= 1 - \frac{3}{8} = \frac{8-3}{8} = \frac{5}{8}$$

Thus, the probability of getting not 2 of heads is $\frac{5}{8}$.

5. A card is drawn from a well shuffled pack of 52 playing cards. What will be the pobability of getting.

(i) King or jack of red colour

Sol. Total numbers of cards = 52

$n(S) = 52$

Let "A" be the event of getting king or jack of red colour.

There are 2 red kings (heart, diamond) and 2 red Jack (heart, diamond)

Total = 2 + 2 = 4

$n(A) = 4$

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{25} = \frac{1}{13}$$

Thus, the probability of getting king or jack of red colour is $\frac{1}{13}$.

(ii) not "2" of club and spade

Sol. Let "B" be the event of getting 2 of club and spade

$n(B) = 2, n(S) = 52$

$$P(A) = \frac{n(B)}{n(S)}$$

$$= \frac{2}{25} = \frac{1}{26}$$

Let "B" be the event to getting not 2 of club and spade:

There are 2 red kings (heart, diamond) and 2 red Jack (heart, diamond)

$$P(B') = 1 - P(B)$$

$$= 1 - \frac{1}{26}$$

$$= \frac{26-1}{26}$$

$$= \frac{25}{26}$$

Thus, the probability of getting not 2 of club and spade is $\frac{25}{26}$.

6. Six coins are tossed 600 times. The number of occurrence of tails are recorded and shown in the table given below:

No. of tails	0	1	2	3	4	5	6
Frequency	110	90	105	80	76	123	16

Find the relative frequency of given table.

Sol. Total frequency = 600 (Given)

x	p(x)	Expected frequency $\frac{f}{600}$
0	110	$\frac{110}{600} = 0.183$
1	90	$\frac{90}{600} = 0.15$
2	105	$\frac{105}{600} = 0.175$
3	80	$\frac{80}{600} = 0.133$
4	76	$\frac{76}{600} = 0.127$
5	123	$\frac{123}{600} = 0.205$
6	16	$\frac{16}{600} = 0.027$
	$\Sigma f = 600$	

7. From a lot containing 25 items, 8 items are defective. Find the relative frequency of non-defective items, also find the expected frequency of non-defective items.

Sol. Total item in a lot = 25

n(25)

Let "D" be the equal of defective items

$n(D) = 8$

$$p(D) = \frac{8}{25}$$

Let "D" be the event of non-deective items.

$$p(D') = 1 - \frac{8}{25}$$

$$n(B) = \frac{25-8}{25}$$

$$= \frac{17}{25}$$

$$= 0.68$$

Exprectd frequency of non-defective items = 25 – 8

= 17

Objective Type Questions

Additional MCQ's

Multiple Choice Questions prepared in the light of new Examination Techniques (Knowledge, Understanding, Application, Analytical & Conceptual)

Introduction

○ Encircle the correct option.

1. Which of them is the chance of occurrence of a particular event:
(a) probability (b) events (c) sample (d) none of these

2. $\frac{\text{Number of favourable out comes}}{\text{Total number of possible out comes}}$ = _____.
(a) sample (b) probability (c) outcomes (d) events

3. $\frac{n(A)}{n(S)}$ = _____.
(a) p(A) (b) p(B) (c) p(C) (d) p(D)

4. The process which generates results e.g; rolling a dice called:
(a) sample (b) probability (c) experiment (d) event

5. The results of an experiment called:
(a) probability (b) sample (c) experiment (d) outcomes

6. A _____ which represent show many times we expect the things to be happed:
(a) favorable out comes (b) sample space
(c) probability (d) event

7. The set of all possible out comes of an experiment is called:
(a) favorable out comes (b) sample space
(c) probability (d) event

8. The set of results of an experiment is called:
(a) favorable out comes (b) sample space
(c) probability (d) event

13.1 Probability of Single Event

9. Umer rolls a fair dice, what is the probability of getting a number divisible by 3:
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

10. If Shoaib rolled two fair dice what is the probability of getting even numbers on both sides:
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

11. If Abdullah rolled to fair dice what is the probability of getting multiple of 3 on both sides:
(a) $\frac{1}{6}$ (b) $\frac{1}{7}$ (c) $\frac{1}{8}$ (d) $\frac{1}{9}$

12. If Mariyam rolled two fair dice, What is the probability of getting even number on the first dice and 3 on the second dice:
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{12}$ (d) $\frac{1}{6}$

13. If Fatima rolled two fair dice what is the probability of getting at least the number 3 on the first dice and number 4 on the second dice:
(a) $\frac{1}{6}$ (b) $\frac{1}{9}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

13.2 Probability of an Event Not Occurring

14. The probability of not getting head while tossing a coin is called:
(a) Complement of event (b) Supplement of event
(c) Sample of event (d) Probability of an event

15. "Complement of an event is written as:
(a) P(A') (b) P(A) (c) $P\frac{(A)}{(C)}$ (d) none of these

16. The'complement of an event A is calculated as:
(a) P(A') = P(A) (b) P(A') = 1 – P(A) (c) P(A') = 2 – P(A) (d) P(A') = 3 – P(A)

17. If $P(A) = \frac{1}{2}$ then P(A') is:
(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

18. If S = {1,2,3,4,5,6} then P(A) = _____ and A = {6}, n(A) = 1:
(a) $\frac{1}{4}$ (b) $\frac{1}{4}$ (c) $\frac{1}{6}$ (d) $\frac{1}{8}$

19. If $P(A) = \frac{1}{6}$ then P(A') = _____
(a) $\frac{2}{6}$ (b) $\frac{3}{6}$ (c) $\frac{4}{6}$ (d) $\frac{5}{6}$

20. If two fair dice are rolled the probability of getting not a double six:
(a) $\frac{35}{36}$ (b) $\frac{34}{36}$ (c) $\frac{33}{36}$ (d) $\frac{32}{36}$

21. If two fair dice are rolled, then the probability of getting not the sum of both dice is 8:
(a) $\frac{30}{36}$ (b) $\frac{31}{36}$ (c) $\frac{32}{36}$ (d) $\frac{33}{36}$

13.3 Real Life Problems Involving Probability

22. If $P(A) = \frac{1}{4}, P(B) = \frac{3}{7}, P(C) = \frac{5}{9}$ then probability that A does not hit the target:
(a) $P(A') = \frac{3}{4}$ (b) $P(A') = \frac{2}{4}$ (c) $P(A') = \frac{1}{4}$ (d) 0

23. If $P(A) = \frac{1}{4}, P(B) = \frac{3}{7}, P(C) = \frac{5}{9}$ then probability of B does not hit the target:
(a) $\frac{3}{7}$ (b) $\frac{4}{7}$ (c) $\frac{5}{7}$ (d) $\frac{6}{7}$

24. If $P(A) = \frac{1}{4}, P(B) = \frac{3}{7}, P(C) = \frac{5}{9}$ then probability C does not hit the target:
(a) $P(C') = \frac{2}{9}$ (b) $P(C') = \frac{3}{9}$ (c) $P(C') = \frac{4}{9}$ (d) $P(C') = \frac{5}{9}$

25. A box contains 5 blue and 8 green beads. The probability of selecting a blue bead:
(a) $\frac{5}{13}$ (b) $\frac{4}{13}$ (c) $\frac{3}{13}$ (d) $\frac{2}{13}$

26. A box contains 5 blue and 8 green beads. The probability of selecting a green bead:
(a) $\frac{5}{13}$ (b) $\frac{6}{13}$ (c) $\frac{7}{13}$ (d) $\frac{8}{13}$

27. A box contains 5 blue 8 green beads. The probability of selecting not a green bead:
(a) $\frac{5}{13}$ (b) $\frac{6}{13}$ (c) $\frac{7}{13}$ (d) $\frac{8}{13}$

28. A card is drawn at random from a pack of 52 cards. The probability of getting a card of heart:
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

29. A card is drawn at random, from a pack of 52 cards. The probability of getting neither spade not heart:
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

13.4 Relative Frequency as an Estimate of Probability

30. Which of them tells us how often a specific event occurs relative to the total number of frequency events or trails?
(a) Frequency distribution (b) Relative frequency
(c) Positive frequency (d) Negative frequency

31. Relative frequency = _____.
(a) $\frac{\text{Total frequency}}{\text{Frequency of event}}$ (b) $\frac{\text{Relative frequency}}{\text{Specific event}}$
(c) $\frac{\text{Frequency of Specific event}}{\text{Total frequency}}$ (d) $\frac{\text{Frequency of event}}{\text{Relative frequency}}$

32. If

X	2	3	4	5	6	7	8
f	3	5	6	9	10	8	2

then what is the relative frequency?
(a) 0.07,0.12,0.14,0.21,0.23,0.19,0.04 (b) 0.04,0.19,0.23,0.21,0.14,0.12,0.07
(c) 0.21,0.23,0.19,0.04,0.07,0.12,0.14 (d) 0.21,0.14,0.12,0.07,0.23,0.19,0.04

13.5 + 13.6 + 13.7 Real Life Application of Relative Frequency + Expected Frequency + Real Life Application on Expected Frequency

33. If total number of students = 80, favourite red colour = 23 students then relative frequency is:
(a) 0.26 (b) 0.27 (c) 0.28 (d) 0.29

34. Total students = 80, Farouvite colour green = 15 students then relative frequency is:
(a) 0.17 (b) 0.18 (c) 0.19 (d) 0.20

35. If total students = 80, Favourite colour pink = 25 students then relative frequency = ?
(a) 0.30 (b) 0.31 (c) 0.32 (d) 0.33

36. If total students = 80, Favourite colour blue = 10 then relative frequency is:
(a) 0.12 (b) 0.13 (c) 0.14 (d) 0.15

37. If total students = 80 and favourite colour white = 7 students then relative frequency is:
(a) 0.08 (b) 0.09 (c) 0.10 (d) 0.11

38. Which of them is a measure that estimate how often an event should be occurred depend on probability?
(a) Relative frequency (b) Expected frequency
(c) Initial frequency (d) Final frequency

39. Formula of expected frequency is:
(a) Total no. of trials×probability of event (b) Total no. of trials+probability of event
(c) Total no. of trails–probability of event (d) None of these

40. What is the average number of times getting 1 or 6, when a fair dice is rolled 300 times:
(a) 10 (b) 50 (c) 100 (d) 200

41. If the probability of a defective bolt is 0.3 then the number of non defective bolts in a total to 800:
(a) 260 (b) 360 (c) 460 (d) 560

Answers:

1. probability	2. probability	3. p(A)	4. experiment	5. outcomes
6. favorable out comes	7. sample space	8. event	9. $\frac{1}{3}$	
10. $\frac{1}{4}$	11. $\frac{1}{9}$	12. $\frac{1}{12}$	13. $\frac{1}{9}$	14. Complement of event
15. P(A')	16. P(A')=1− P(A)	17. $\frac{1}{2}$	18. $\frac{1}{6}$	19. $\frac{5}{6}$
20. $\frac{35}{36}$	21. $\frac{31}{36}$	22. P(A') = $\frac{3}{4}$	23. $\frac{4}{7}$	24. P(C') = $\frac{4}{9}$
25. $\frac{5}{13}$	26. $\frac{8}{13}$	27. $\frac{5}{13}$	28. $\frac{1}{4}$	29. $\frac{1}{2}$
30. Relative frequency		31. $\frac{\text{Frequency of Specific event}}{\text{Total frequency}}$		
32. 0.07,0.12,0.14,0.21,0.23,0.19,0.04		33. 0.29	34. 0.19	35. 0.31
36. 0.12	37. 0.09	38. Expected frequency		
39. Total no. of trials×probability of event		40. 100	41. 560	

Conceptual MCQ's

Encircle the correct option.

1. **Each element of the sample space is:**
   (a) Sample point (b) Outcomes (c) Event (d) Experiment
2. **The range of probability for an events is:**
   (a) $0 \le p(A) \le 0$ (b) $0 \le p(A) \le 1$ (c) $1 \le p(A) \le 1$ (d) $2 \le p(A) \le 3$
3. **p(A) + p(A') = _____**
   (a) 0 (b) 1 (c) 2 (d) 3
4. $\frac{\text{Frequency of specific event}}{\text{Total frequency}}$ = _____.
   (a) Event (b) Experiment (c) Sample space (d) Relative frequency
5. **The sum of all the relative frequencies is always equal to or approximately equal to:**
   (a) 0 (b) 1 (c) 2 (d) 3
6. **Sum of all expected frequencies is always equal to or approximately _____ a fixed number of trials.**
   (a) equal to (b) less than (c) grater than (d) none of these
7. **The formula N×P(A) is for:**
   (a) expected frequency (b) estimate frequency
   (c) frequency (d) none of these

8. **If p(A) is $\frac{5}{6}$ then p(A') = ?**
   (a) $\frac{4}{6}$ (b) $\frac{3}{6}$ (c) $\frac{2}{6}$ (d) $\frac{1}{6}$
9. **If p(A') = $\frac{1}{2}$ then p(A) = ?**
   (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

Answers:

1. Sample point	2. $0 \le p(A) \le 1$	3. 1	4. Relative frequency	
5. 1	6. equal to	7. expected frequency	8. $\frac{1}{6}$	9. $\frac{1}{2}$

Additional Short Questions

Short Answer Questions prepared in the light of new Examination Techniques (Knowledge, Understanding, Application, Analytical & Conceptual)

Introduction

1. What do you know about probability?

Ans. Probability is chance of occurrence of particular event.

$$\text{Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$p(A) = \frac{n(A)}{n(S)}$$

2. Define experiment.

Ans. The process which generates results. **For example:** tossing a coin, rolling a dice.

3. Define outcomes.

Ans. The results of an experiment are called outcomes. e.g.: the possible outcomes of rolling a dice.

4. What is favourable outcomes?

Ans. A outcomes which represents how many times we aspect the things to be happened. e.g. while rolling a die, there are 3 favourite outcomes of getting multiples of 2 i.e. {2,4,6}.

5. What is sample space?

Ans. The set of all possible outcomes of an experiment is called samples space. It is denoted by S. e.g. while tossing a coin the sample space will be {H, T}.

6. What do you know about event?

Ans. The set of results of an experiment is called an event. e.g. while a dice getting even number is a event. i.e. A = {2,4,6}, n(A) = 3

13.1 Probability of Single Event

7. Umar rolls of fair a dice, what is the probability of getting the number divisible by 3.

Ans. When dice is roll. We have S = {1,2,3,4,5,6}, n(S) = 6

Now A is event, A = {3,6}, n(A) = 2

and $p(A) = \frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$, Hence probability is $\frac{1}{3}$.

8. If Abdullah rolled two dice, than find the probability of getting even number on the first dice and number 3 on the second dice.

Ans. According to questions:

C = {(2,3), (4,3), (6,3)}, n(C) = 3

$$n(S) = 36 \quad \Rightarrow \quad p(C) = \frac{n(C)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

9. If Shoaib rolled two dice, then find the probability of getting at least the number 3 on the first dice and number 4 on the second dice.

Ans. According to questions:

D = {(3,4), (4,4), (5,4), (6,4)}

$$n(D) = 4 \quad \Rightarrow \quad n(S) = 36 \quad \Rightarrow \quad P(D) = \frac{n(D)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

10. What is the complement of an event?

Ans. The probability of not getting while tossing a coin is known as the complement or $P(A^C)$. So, P(A') = 1 – P(A)

11. Fatima rolls a dice what is the probability of not getting the number 6?

Ans. S = {1,2,3,4,5,6}, n(S) = 6, n(A) = 1

$$p(A) = \frac{n(A)}{n(S)} = \frac{1}{6}$$

$$p(A') = 1 - p(A) = 1 - \frac{1}{6} = \frac{6-1}{6} = \frac{5}{6}$$

12. If two dice are rolled. What is the probability of getting not the sum of both dice is 8.

Ans. According to statement:

A = {(2,6), (3,5), (4,4), (5,3), (6,2)}

n(A) = 5, n(S) = 36

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{36}$$

$$P(A') = 1 - P(A) = 1 - \frac{5}{36} = \frac{36-5}{36} = \frac{31}{36}$$

13.3 Real Life Problems Involving Probability

13. If $P(A) = \frac{1}{4}$, find A does not hit the target.

Sol. $P(A) = \frac{1}{4} \quad \Rightarrow \quad P(A') = 1 - P(A) = 1 - \frac{1}{4} = \frac{4-1}{4} = \frac{3}{4}$

14. If $P(B) = \frac{3}{7}$, then find B does not hit the target.

Sol. $P(B) = \frac{3}{7} \quad \Rightarrow \quad P(B') = 1 - P(B) = 1 - \frac{3}{7} = \frac{7-3}{7} = \frac{4}{7}$

15. If $P(C) = \frac{5}{9}$, then find the C does not hit the target.

Sol. $P(C) = \frac{5}{9} \quad \Rightarrow \quad P(C') = 1 - P(C) = 1 - \frac{5}{9} = \frac{9-5}{9} = \frac{4}{9}$

16. A bag contains 5 blue and 8 green balls. Find the probability of getting at random a blue ball.

Ans. Blue balls = n(A) = 5, green balls = n(B) = 8

Total balls = n(S) = 5 + 8 = 13

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{13}$$

17. A bag contains 5 blue balls and 8 green balls. Find the probability of selecting at random a green ball.

Sol. Green balls = n(B) = 8, blue balls = n(A) = 5

Total balls = n(S) = 5 + 8 = 13

$$p(B) = \frac{n(B)}{n(S)} = \frac{8}{13}$$

18. A bag contains 5 blue balls and 8 green balls. Find the probability of selecting a random not a green balls.

Sol. $P(B') = 1 - P(B) = 1 - \frac{8}{13} = \frac{13-8}{13} = \frac{5}{13}$

19. A card is drown at random, from a pack of 52 playing cards. What is the probability of getting a card of heart?

Sol. Total = n(S) = 52, n(A) = 13

$$P(A) = \frac{n(A)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

20. A card is drawn at random from a pack of 52 paying class. What is the probability of getting neither spade non heart?

Sol. n(S) = 52, n(B) = 26

$$p(B) = \frac{n(B)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

Now,

$$P(B') = 1 - P(B) = 1 - \frac{1}{2} = \frac{2-1}{2} = \frac{1}{2}$$

13.4 Relative Frequency as an Estimate of Probability

21. Define relative frequency.

Ans. The relative frequency tells us how often a specific event occur relative to the total number of frequency event or trails.

$$\text{Relative freqeuncy} = \frac{\text{frequency of specific event}}{\text{Total Frequency}}$$

$$= \frac{x}{N}, \quad N = \Sigma f$$

22. Find the relative frequency of given data:

X	2	3	4	5	6	7	8
f	3	5	6	9	10	8	2

Sol. $\Sigma f = 43$

X	2	3	4	5	6	7	8
f	3	5	6	9	10	8	2
R.F	$\frac{3}{43}$=0.07	$\frac{5}{43}$=0.12	$\frac{6}{43}$=0.14	$\frac{9}{43}$=0.21	$\frac{10}{43}$=0.23	$\frac{8}{43}$=0.19	$\frac{2}{43}$=0.04

13.5 Real Life Application of Relative Frequency

23. A survey was conducted on 80 students and asked about their favourite colour. The responses are:

i. Red colour = 23 students
ii. Green colour = 15 students
iii. Pink colour = 25 students
iv. Blue colour = 10 students
v. White colour = 7 students

Ans. i. Total = 80 students

$$\text{Relative frequency} = \frac{23}{80} = 0.29$$

29% student prefer red colour.

ii. Total = 80 students

$$\text{Relative frequency} = \frac{15}{80} = 0.19$$

19% student prefer green colour.

iii. Total = 80 students

$$\text{Relative frequency} = \frac{25}{80} = 0.31$$

31% student prefer pink colour.

iv. Total = 80 students

$$\text{Relative frequency} = \frac{10}{80} = 0.12$$

12% student prefer blue colour.

v. Total = 80 students

$$\text{Relative frequency} = \frac{7}{80} = 0.09$$

9% student prefer white colour.

24. According to table find relative frequency:

Subject	Urdu	English	Islamiyat	Math	Science	Computer Science
Marks obtains	75	80	72	95	81	85

Sol. Total = 488

Subject	Urdu	English	Islamiyat	Maths	Science	Computer Science
Marsk Obtain	75	80	72	95	81	85
Relative Frequency	$\frac{75}{488}$ = 0.15	$\frac{80}{488}$ = 0.16	$\frac{72}{488}$ = 0.15	$\frac{95}{488}$ = 0.19	$\frac{81}{488}$ = 0.17	$\frac{85}{488}$ = 0.17

13.6 Expected Frequency

25. Define expected frequency.

Ans. It is a measure that estimate how to often an event should be occurred depended on probability. It has following formula.

Expected frequency = Total number of trails × Probability of an event

$= N \times p(A)$

26. According to table find expected frequency.

X	0	1	2	3	4	5	6
p(x)	0.09	0.10	0.12	0.24	0.10	0.20	0.15

Sol.

x	p(x)	Expected frequency
0	0.09	50 × 0.09 = 4.5
1	0.10	50 × 0.10 = 5
2	0.12	50 × 0.12 = 6
3	0.24	50 × 0.24 = 12
4	0.10	50 × 0.10 = 5
5	0.20	50 × 0.20 = 10
6	0.15	50 × 0.15 = 7.5

13.7 Real Life Application on Expected Frequency

27. Find the average number of times getting 1 to 6, when a fair dice is rolled 300 times.

Sol. $S = \{1,2,3,4,5,6\}$, $n(S) = 6$

$B = \{1,6\}$, $n(B) = 2$

$$p(B) = \frac{n(B)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

Now, $E(B) = N \times p(B) = 300 \times \frac{1}{3} = 100$

28. If the probability of a defective computer is 0.3. Find the number of non defective computers in a total to 800.

Ans. The probability of defective computers $= 0.3$
Probability of non-defective computers $= 1 - 0.3 = 7$
Number of non-defective computers $= 0.7 \times 800$
$= 560$

Conceptual Short Questions

1. Tossing a coin or rolling a dice are the examples of which?

Ans. Tossing a coin or rolling a dice are the examples of experiment.

2. How many types of events?

Ans. There are five types of events.

3. Name the types of events.

Ans. i. Certain events **ii.** Impossible event
iii. Likely events **iv.** Unlikely events
v. Equally likely event

4. Define certain event.

Ans. An event which is sure to occur. The probability of sure event is.

5. Define impossible event.

Ans. An event cannot occur in any trial. The probability of this event is 0.

6. Define likely event.

Ans. An event which will probably occur. It has greater chance to occur.

7. What is unlikely event?

Ans. An event which will not probably occur. It has less chance to occur.

8. What is equally likely events?

Ans. The events which have equal chance of occurrence. The probability of these events is 0.5.

9. Define sample point.

Ans. Each element of the sample space is called sample point.

10. How we can define $p(A) + p(A') = 1$?

Ans. The sum of the probability of an event A and the probability of an event not occurring A is always 1.

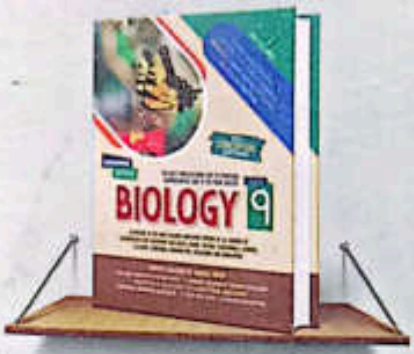
11. What is the range of probability for an event?

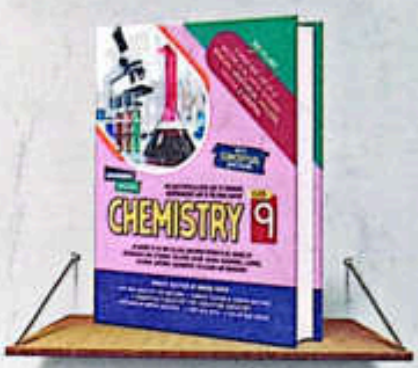
Ans. $0 \leq P(A) \leq 1$

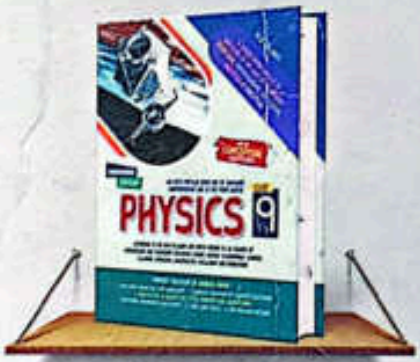
12. Define sample point.

Ans. Each element of sample space is called sample point.

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