$$I = \frac{2}{5} 200(0.3)^2$$

$$I = 7.2 \text{ kg m}^2$$

3.7 **A satellite is orbiting the Earth at an altitude of 200 km. Assuming the Earth's radius is 6400 km, calculate the orbital speed of the satellite.**

Given:

Height of satellite above the surface of Earth = $h = 200$ Km

$= 200 \times 10^3$m

$= 0.2 \times 10^6$ m

Mass of the Earth = 6×10^{24}Kg

Radius of Earth = 6400Km

$= 6.4 \times 10^6$m

To find:

Orbital speed of satellite = v = ?

Calculation:

$$v = \sqrt{\frac{GM}{R+h}}$$

$$v = \sqrt{\frac{6.673 \times 10^{-11} \times 6 \times 10^{24}}{6.4 \times 10^6 + 0.2 \times 10^6}}$$

$$v = \sqrt{\frac{6.673 \times 6 \times 10^{-11+24}}{(6.4+0.2) \times 10^6}}$$

$$v = \sqrt{\frac{39.72 \times 10^{13}}{6.6 \times 10^6}}$$

$$v = \sqrt{6.018 \times 10^{13-6}}$$

$$v = \sqrt{6.018 \times 10^7}$$

$$v = \sqrt{60.18 \times 10^6}$$

$$v = 7.75 \times 10^3 \text{ m s}^{-1}$$

$$v = 7.75 \text{ km s}^{-1}$$

3.8 **A space station has a radius of 20 m and rotates at an angular velocity of 0.5 rad s^{-1}. What is the artificial gravity experienced by the astronauts on the space station?**

Given

Radius of space station = R = 20 m

Angular velocity = ω = 0.5 rad s^{-1}

Artificial gravity = g = ?

Solution

$$\omega = \sqrt{\frac{g}{R}}$$

$$g = \omega^2 R$$

$$g = 0.5^2 \times 20 = 0.25 \times 20 = 5 \text{ m s}^{-2}$$

A bicycle wheel has an angular momentum of 10 kg m² s⁻¹ and angular velocity of 2 rad s⁻¹. Find the value of its moment of inertial.

Given

Angular momentum of wheel = L = 10 kg $m^2 s^{-1}$

Angular Velocity = ω = 2 rad s^{-1}

To determine

Moment of inertia = I = ?

Solution:

$$L = I\omega$$

$$10 = I(2)$$

$$I = 5 \text{ kg m}^2$$

3.10 **A diver comes off a board with arms straight up and legs straight down, giving him a moment of inertia of 18 kg m² about his rotation axis. Then tucks into a small ball, decreasing his moment of inertia to 3.6 kg m². While tucked, he makes two complete rotations in 1.0 second. If he had not tucked at all, how many revolutions would he have made in 1.5 s from board to water?**

Given

Moment of inertia of diver when arm and leg fully stretched = I_1 = 18 kg m^2

Moment of inertia of diver in tucks position = I_2 = 3.6 kg m^2

Angular velocity in tucked position =

$$\omega_2 = \frac{\theta_1}{t_1} = \frac{2}{1} = 2 \text{ rev s}^{-1}$$

Angular velocity with stretched form =

$$\omega_1 = \frac{\theta_1}{t_1} = \frac{\theta_1}{1.5}$$

Solution:

According to law of conservation of angular momentum

$$I_1 \omega_1 = I_2 \omega_2$$

$$18 \times \frac{\theta_1}{1.5} = 3.6 \times 2$$

$$\theta_2 = \frac{3.6 \times 2}{18} \times 1.5 = 0.6 \text{ rev.}$$

CSCRCSCRCSCR

CHAPTER 4 WORK AND ENERGY

STUDENT LEARNING OBJECTIVES

After studying this chapter, the students will be able to:

- Derive the formula for kinetic energy [using the equations of motion]
- Derive an expression for absolute potential energy of a body at a certain position in the gravitational field [including escape velocity]
- Deduce the work done from force-displacement graph.
- Differentiate between conservative and non-conservative forces.
- State and use the work - energy theorem in a resistive medium to solve problems.

4.1 WORK DONE BY CONSTANT FORCE

Q Define the term work? How work is done by constant force? Explain with examples.

Ans

WORK DONE BY CONSTANT FORCE

"The work done on a body by a constant force is defined as the __product__ of the __magnitude of displacement__ and the __component of the force__ in the direction of the displacement." When a force acts on a body and displaces it in its direction then work is said to be done on the body.

$$\text{Work} = F d \cos\theta = \vec{F} \cdot \vec{d}$$

S I unit of work:

S I unit of work is joule which is defined as.

Joule:

The work done is said to be __one joule__ if __one newton__ force acts on a body and the body covers a displacement of __one meter__ in the direction of force.

That is 1 joule = 1N × 1m.

It is represented by J = N m

Note: The unit of work in CGS is erg and is FPS is foot–lb(foot-pound).

Dimension of work:

Work = Force × displacement.

joule = newton × meter.

$$J = \text{kg m/s}^2 \times \text{m}.$$

$$= \text{kg m}^2/\text{s}^2$$

$$[J] = \frac{[M][L^2]}{[T^2]}$$

Dimension of work = $[ML^2T^{-2}]$

Fig. 4.1

Fig. 4.2

Explanation

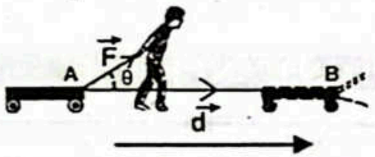
Consider an object which is being pulled by a constant force $\vec{F}$ at an angle θ to the direction of motion. The force moves the object from position A to B through a displacement $\vec{d}$. The work done by the force is defined as the scalar product of $\vec{F}$ and $\vec{d}$

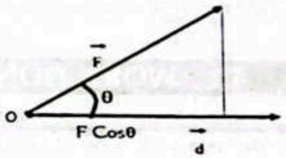
$$W = \vec{F} \cdot \vec{d} = F\,d\,\text{Cos}\theta$$
$$W = (F\,\text{Cos}\theta)d \quad ....(i)$$

Fcosθ = the component of force in the direction of displacement. Equation (i) may be written as

$$\text{Work} = F\,d\,\text{Cos}\theta = \vec{F} \cdot \vec{d}$$

Thus, work is also defined as the ***dot product*** *of* ***force*** *and* ***displacement.***





Examples of Work:

(i) Work Done on the Pail:

If a person holding the pail in his hand walks, no work is done because the angle between force and displacement is 90°.

As $W = Fd\,\text{Cos}\theta = Fd\,\text{Cos}\,90^\circ = 0$

$W = 0$ $(\because \text{Cos}\,90^\circ = 0)$

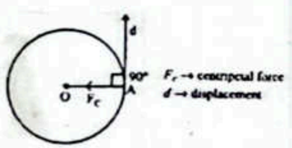
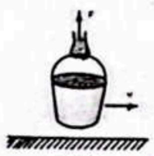
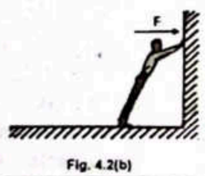
Similarly work done by centripetal force is zero in circular motion.

(ii) Work Done on the Wall:

When a man pushes a wall it does not move. It means displacement of the wall is zero.

$W = Fd\,\text{Cos}\theta = F(0)\ \text{Cos}\theta = 0$ $\because d = 0$

$W = 0$

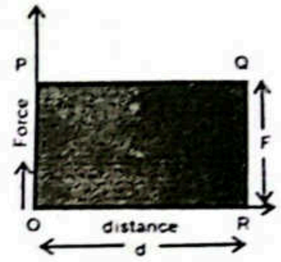
	 Fig. 4.2(a)	 Fig. 4.2(b)
Zero work done	Zero Work done	Zero work done

Work Done By Graphical Method

- When a constant force acts on a body and body covers a distance d then Fd graph can be plotted as shown in figure.
- The distance is taken along x-axis and force is taken along y-axis. In this case the graph will be a horizontal straight line.
- There are two cases

i. If force and displacement are in the same direction, we plot a graph between $\vec{F}$ and $\vec{d}$.

ii. If force makes an angle θ with the displacement, we plot a graph between FCosθ and d.



The work done by a force can be calculated by calculating the area under F d curve as

$$\left[\begin{aligned} &\text{Area} = OP \times OR = F\,\text{Cos}\theta \times d \\ &\text{Area} = OP \times OR = (F\,\text{Cos}\theta) \times d \\ &\text{Area} = \text{Force} \times \text{displacement} \\ &\text{Area} = \text{Work} \end{aligned}\right]$$

Special Cases of work done

There are some special cases that can be derived from the definition of work, these are

(i) Positive Work:

When the angle between the force and displacement is less than 90° i.e., θ< 90° then positive work is said to be done. Also, when the force and displacement are in same direction or when θ= 0°, the maximum or positive work is said to be done.

Mathematically we can write.

When θ = 0°

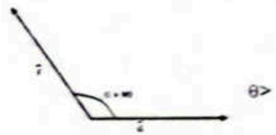
$W = \vec{F} \cdot \vec{d}$
$= F\,d\,\text{Cos}\theta$
$= F\,d\,\text{Cos}0^\circ$ $(\text{Cos}0^\circ = 1)$
$W = Fd$ (Max work)

When θ< 90°

$W = \vec{F} \cdot \vec{d}$
$W = F\,d\,\text{Cos}\theta$ (Positive quantity)

For Examples:

When the car is at rest and we move it by applying a force then work done is said to be positive work.



(ii) Negative Work:

When the angle between force and displacement is greater than 90°i.e. 90° then work done is said to be negative work."

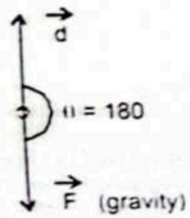
Mathematically

When θ> 90°

$W = \vec{F} \cdot \vec{d}$
$W = F\,d\,\text{Cos}\theta$ (Negative work)

For Examples:

If the body is lifted against the gravity, the angle between the force and displacement is 180°.

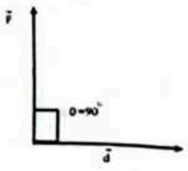


So, $W = \vec{F}.\vec{d}$
$= F\,d\,\text{Cos}\theta$
$= F\,d\,\text{Cos}180^\circ$ $(\cos 180^\circ = -1)$
$W = -F\,d = -mgh.$

(work done against the friction is –ve work)

(iii) Zero work (Minimum work)

When the angle between the force and displacement is 90° i.e., $\theta = 90°$, the force and displacement are right angle to each other and work done is said to be zero.



Mathematically:

When $\theta = 90°$

$$W = \vec{F} \cdot \vec{d}$$
$$W = F\, d \cos 90°$$
$$= F\, d \cos 90°$$
$$W = F\, d\,(0) \qquad (\cos 90° = 0)$$
$$W = 0$$

Characteristics of Work:

From all the above cases we conclude that.

i) Work is a scalar quantity.
ii) If $\theta < 90°$, the work done is said to be positive work.
iii) If $\theta = 90°$, no work is done.
iv) If $\theta > 90°$, the work done is said to be negative work.
v) SI units of work is N m which is known as joule (J).

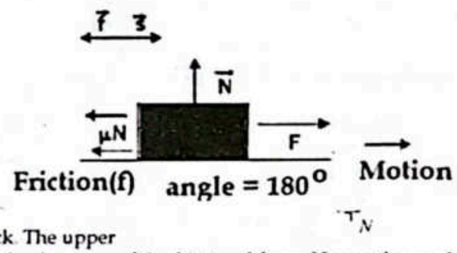
For Your Information Work done by Friction

The work done is the product of the force exerted on the body and the displacement of the body in the direction of that force. Positive work is done by a force when the displacement of the body is in the direction of the force applied whereas negative work is done by the force when the direction of the displacement of the body is opposite to the direction of the force. Zero work is done when the displacement of the body in the direction of the force is zero.

Now, let us analyze the options one by one.

Positive Work By Friction

The work done by frictional force will be positive when the displacement of the body is in the direction of the force. This can be understood by an example. Consider there are two blocks stacked one over the other. If the lower block starts to move slowly in one direction, then there is a frictional force between the two blocks that prevents the sliding of the two blocks. This force acts on the upper block in the direction of motion of the lower block. The upper block also moves along with the lower block, that is, in the direction of the frictional force. Hence, the work done by friction is positive in this case.



Negative Work by Friction

The work done by friction can be negative. For example, when a block is sliding over a rough ground, the frictional force acts in a direction opposite to the direction of motion of the block and tries to bring it to a stop. Since, the direction of motion of the block and the direction of frictional force is opposite to each other, the work done by friction is negative.

Zero Work by Friction

The work done by friction can also be zero. For example, if we try to push a very heavy block on a rough floor, the block does not move. This is because the force of static friction opposes our force. However, since the block does not move, the displacement is zero. Therefore, zero work is done by friction. Therefore, friction can do positive, negative as well as zero work.

SLO BASED SHORT QUESTIONS & ANSWERS

- **Explain why carrying a load horizontally does not involve work done by the gravitational force.**

Ans: Work done is given by $W = Fd\cos\theta$. When carrying a load horizontally, the gravitational force acts vertically downwards, while the displacement is horizontal. The angle (θ) between the gravitational force and the horizontal displacement is 90°. Since $\cos 90° = 0$, the work done by the gravitational force is zero.

- **What is the significance of the angle θ in the formula $W = Fd\cos\theta$?**

Ans: The angle θ in the formula $W = Fd\cos\theta$ represents the angle between the direction of the applied force (F) and the direction of the displacement (d). It determines the component of the force that is effective in causing displacement. If $\theta = 0°$, the force is entirely in the direction of displacement. If $\theta = 90°$, the force is perpendicular to displacement and does no work. If $\theta = 180°$, the force directly opposes displacement, resulting in negative work.

- **Provide an example of a situation where negative work is done, and explain why it is negative.**

Ans: An example of negative work being done is the work done by friction when an object slides across a surface. Friction always acts in a direction opposite to the motion (displacement) of the object. Since the angle between the frictional force and the displacement is 180° ($\cos 180° = -1$), the work done by friction ($W = F_f d \cos 180° = -F_f d$) is always negative. This negative work signifies that energy is being removed from the object's kinetic energy and typically dissipated as heat.

- **What do you understand by work and energy? Give their units. (SGD, FSD 2022 GII)**

Ans: **Energy:**

- Energy is the ability to cause change or to do work. It can exist in many forms, like kinetic (motion), potential (stored), thermal (heat), light, sound, electrical, and more.
- Energy is a **scalar quantity**, meaning it only has a magnitude (amount) and not a direction.
- The unit of energy is the **joule (J)**.

Work:

- Work is done when a force causes an object to move through a distance in the direction of the force. ($W = Fd$)
- Work is a **scalar quantity**, similar to energy.
- The unit of work is also the **joule (J)**.

The Connection:

- The amount of work done is equal to the change in energy of the object. This principle is called the work-energy theorem.

- **Does the work done in raising a box on the platform depend upon how fast it is raised up? (BWP 2022 G-I)**

Ans: No, the work done in raising a box onto a platform does not depend on how fast it is raised.

Reason:

- Work is defined as the product of force and displacement (**$W = Fd$**)
- When we raise a box, the force required to lift it against gravity stays the same regardless of speed.
- The displacement is simply the vertical height the box is lifted.

Therefore, as both the force and displacement are independent of speed, the work done (force x displacement) will also be independent of how fast you raise the box.

- **A car is moving along a circle of radius r. It completes four revolutions and terminates its journey at starting point. How much work is done by the car? Explain. (LHR 2019)**

Ans: The work done by the car is zero.

Reason:

Since Work is defined as $W = F \cdot d$

In circular motion the force (centripetal force) is perpendicular to the displacement (tangential to the circle), the dot product ($F \cdot d$) becomes zero.

As the dot product is zero, the work done (W) by the car is also zero.

Also the displacement is zero because car returns to the starting point so work is zero.

MULTIPLE CHOICE QUESTIONS

- **A body of mass 10 kg, initially at rest, is moved by a horizontal force of 2N on a smooth horizontal surface. Find the work done by the force in 10 s.**

  A. 40 J B. 20 J C. 30 J D. 10 J

Solution:

$$a = \frac{F}{m} = \frac{2}{10} = 0.2\,ms^{-2}$$

Now distance covered in 10s is:

$$S = \frac{1}{2}\times a\times t^2 \Rightarrow S = \frac{1}{2}\times 0.2\times(10)^2 \Rightarrow S = \frac{1}{2}\times 0.2\times 100 \Rightarrow S = \frac{1}{2}\times\frac{2}{10}\times 100 = 10m$$

So work done by the force

$$w = F\times S = 2\times 10 = 20J$$

- **In which scenario is the work done by a constant force on an object zero?**

  (A) A box being pushed across a rough floor.
  (B) A satellite orbiting Earth in a perfectly circular path.
  (C) A person lifting a weight upwards.
  (D) A car accelerating on a straight road.

Answer: B

Explanation: Work done is zero when the force is perpendicular to the displacement (θ=90). In a circular orbit, the gravitational force (towards the center) is always perpendicular to the satellite's instantaneous displacement (tangential to the circle).

- **A force of 20 N acts on an object, causing it to displace 5 m in the direction of the force. What is the work done by the force?**

  (A) 4 J (B) 25 J (C) 100 J (D) 0 J

Answer: C

Explanation: Since the force is in the direction of displacement, θ=0 , so cosθ=1 Work done W=Fd=20 N×5 m=100 J.

- **If the angle between the applied force and the displacement is 120°, the work done by the force is:**

  (A) Positive (B) Negative
  (C) Zero (D) Dependent on the magnitude of force only

Answer: B

Explanation: When θ > 90 (obtuse angle), cosθ is negative. Therefore, W = Fdcosθ will be negative. This typically occurs when a force opposes motion, like friction.

- **A body is moving with uniform speed along a circular track what will be the ratio of its distance to displacement when it has covered $\frac{1}{4}$th part of its time period?**

  (a) $\frac{\pi}{\sqrt{2}}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2\sqrt{2}}$

4.2 WORK DONE BY VARIABLE FORCE

Q.2 What is a variable force? How can you calculate the work done by variable force?

Ans

VARIABLE FORCE

A variable force is that force whose __magnitude or direction__ or both do not __remain same__ is called variable force.

Examples

1. Force of gravity varies inversely by the square of the distance from the center of the earth i.e.

$$F = \frac{GMm}{r^2}$$

2. Force exerted by a spring on a body varies as distance $F = kx$

WORK DONE BY A VARIABLE FORCE

Consider a variable force F moves a particle from 'a' to 'b' in x-y plane as shown in the Fig. Divide the total path into n very short displacements $\Delta\vec{d}_1, \Delta\vec{d}_2, \Delta\vec{d}_3, \ldots\ldots\Delta\vec{d}_n$ and $\vec{F}_1, \vec{F}_2, \vec{F}_3\ldots\ldots\vec{F}_n$ are the forces acting in these displacements respectively. $\theta_1, \theta_2, \theta_3, \ldots\ldots \theta_n$ are the angles between forces and displacements respectively. During each small displacement force is supposed to be constant.

Work done during $\Delta\vec{d}_1 = \Delta W_1 = \vec{F}_1.\Delta\vec{d}_1 = F_1\Delta d_1 \cos\theta_1$

Work done during $\Delta\vec{d}_2 = \Delta W_2 = \vec{F}_2.\Delta\vec{d}_2 = F_2\Delta d_2 \cos\theta_2$

and so on,

Work done during $\Delta\vec{d}_n = \Delta W_n = \vec{F}_n.\Delta\vec{d}_n = F_n\Delta d_n \cos\theta_n$

Total Work The total work done in moving the object from a to b can be calculated by adding all these terms

$$W_{total} = \Delta W_1 + \Delta W_2 + \Delta W_3 + \ldots\ldots + \Delta W_n$$
$$= F_1\Delta d_1\cos\theta_1 + F_2\Delta d_2\cos\theta_2 \ldots\ldots + F_n\Delta d_n\cos\theta_n$$

$$W_{total} = \sum_{i=1}^{n} F_i\Delta d_i\cos\theta_i$$

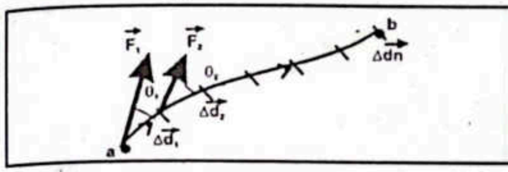


Figure: A particle acted upon by a variable force moves along the path shown from point a to point b

Graphical Method to find Work:

Plot interval $\Delta d_1, \Delta d_2, \ldots\ldots, \Delta d_n$ along x-axis and FCosθ along y-axis. FCosθ is the force at the start of each interval as shown by dotted horizontal lines at the top of each interval.

W_{total} = Sum of Areas of all rectangles

Calculation of More Accurate Work

If we divide the total displacement in "n" very small intervals, then work done become more accurate. If each $\Delta d \rightarrow 0$ for 'n' number of intervals the exact result for work done is obtained such as

$$W_{total} = \lim_{\Delta d\rightarrow 0}\sum_{i=1}^{n} F_i\Delta d_i\cos\theta_i$$

Thus work done by a variable force is equal to the area under FCosθ verses d curve.

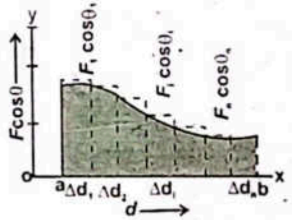


Fig.4.5

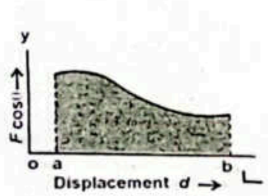


Fig. 4.6

MULTIPLE CHOICE QUESTIONS

- **When calculating work done by a variable force, why is the path divided into small intervals?**
  A) To make the force constant within each interval
  B) To simplify the calculation of total displacement
  C) To ignore the effect of the force
  D) To find the average force over the entire path

Answer: A

Explanation: For a variable force, its magnitude or direction changes along the path. By dividing the path into infinitesimally small intervals, the force can be assumed to be approximately constant within each tiny segment, allowing the calculation of work for that segment using $W=Fd\cos\theta$. The total work is then the sum of these small works.

- **Which of the following is an example of a situation where work is done by a variable force?**
  A) Pushing a heavy box across a smooth, level floor with constant effort.
  B) A car moving at a constant speed on a straight road.
  C) The force exerted by a spring as it is compressed or stretched.
  D) A book resting on a table.

Answer: C

Explanation: The force exerted by a spring (Hooke's Law, $F=-kx$) is directly proportional to its displacement from equilibrium. As the spring is compressed or stretched, the force it exerts changes, making it a variable force.

- **If a force-displacement graph shows a force that first increases linearly and then becomes constant, how would you calculate the total work done?**
  A) By calculating the area of a rectangle.
  B) By calculating the area of a triangle.
  C) By calculating the sum of the areas of a triangle and a rectangle.
  D) By simply multiplying the maximum force by the total displacement.

Answer: C

Explanation: A force that increases linearly forms a triangular area on a force-displacement graph, while a constant force forms a rectangular area. To find the total work, you would sum the areas of these distinct sections.

SLO BASED SHORT QUESTIONS & ANSWERS

- **Describe the concept of a variable force and provide two physical examples.**

Ans: A variable force is a force whose magnitude or direction (or both) changes as the object on which it acts moves through a displacement. This means the force is not constant over the entire path. Two examples include: (i) the force of gravity on an object far from Earth's surface (which varies with distance), and (ii) the force exerted by a spring as it is compressed or stretched (which varies linearly with displacement).

- **How does the graphical representation of work done by a variable force differ from that of a constant force?**

Ans: For a constant force, the force-displacement graph is a horizontal line, and the work done is simply the area of a rectangle. For a variable force, the force-displacement graph is typically a curve, meaning the force value changes with displacement. The work done is still the area under this curve, but calculating it requires methods for finding the area under an irregular shape, such as integration or approximating the area using small rectangles/trapezoids.

- **Consider a spring being stretched. How does the force exerted by the spring vary with its extension, and how does this relate to the work done?**

Ans: According to Hooke's Law, the force exerted by an ideal spring is directly proportional to its extension (or compression) from its equilibrium position, given by $F=kx$, where k is the spring constant and x is the displacement. Since the force changes with x, it is a variable force. The work done in stretching or compressing a spring is not simply F_x, but rather the area under the force-displacement graph, which for a spring is a triangle, resulting in $W=1/2kx^2$

4.3 CONSERVATIVE AND NONCONSERVATIVE FORCES

Q.3 Define conservative force and field also define the gravitational field of the Earth. Show that the work done in gravitational field is independent of the path followed and gravitational field is conservative.

Ans

CONSERVATIVE FIELD

*"A field in which the **work done** is **independent** of **path** followed is called **conservative field**." Or "A field in which **work done** along a **closed path** is **zero** is called **conservative field**."*

CONSERVATIVE FORCE

*The **force** by which work done is **independent of path** followed is called conservative force or work done along closed path is zero is called conservative force.*

Examples of Conservative Forces

(1) Gravitational Force (2) Elastic Spring Force (3) Electric Force

GRAVITATIONAL FIELD:

*The **space** around the Earth in which its **gravitational force** acts on a body is called the gravitational field. The strength of earth's gravitational field decreases with the height from the earth.*

WORK DONE IN A GRAVITATIONAL FIELD:

When an object is moved in the gravitational field the work is done on it by gravitational field. Force acting on it is equal to gravitational force.

The gravitational Force = $F = w = mg$ and

Displacement = $d = h$

Thus the work done by the gravitational Force = $W = mgh$

Vertically downward If displacement is in the direction force (vertically downward) then work done is positive, i.e.,

$W = mgh\cos\theta = mgh\cos 0^\circ = mgh$

Vertically upward If displacement is opposite to the force (vertically upward) then work done is negative, i.e.,

$W = mgh\cos\theta = mgh\cos 180^\circ$

$W = mgh(-1) = -mgh$

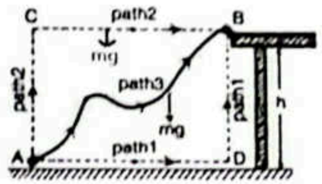
Horizontally If displacement is at right angle to the force (horizontally) then work done is zero, i.e.,

$W = mgd\cos\theta = mgd\cos 90^\circ$

$W = mgd(0) = 0$

Work done along three paths

Consider a body of mass 'm' moving with constant velocity from point A to B on different paths in the Earth's gravitational field. The different paths are shown in fig.



Path 1, from A to D and then D to B

Path 2, from A to C and then C to B

Path 3, from A to B along curved path

Work Done Along Path 1:

To find the work done path 1 is divided into two parts

The work done along AD $= W_{AD} = mgd_1\cos 90^\circ = 0$

The work done along DB $= W_{DB} = mgh\cos 180^\circ = -mgh$

The total work done $= W_{ADB} = W_{AD} + W_{DB}$

$W_{ADB} = 0 + (-mgh) = -mgh$ ------ (A)

Work Done Along Path 2:

To find the work done along path 2 is also divided into two parts

The work done along AC $= W_{AC} = mgh\cos 180^\circ = -mgh$

The work done along CB $= W_{CB} = mgd_2\cos 90^\circ = 0$

The total work done = $W_{ACB} = W_{AC} + W_{CB}$

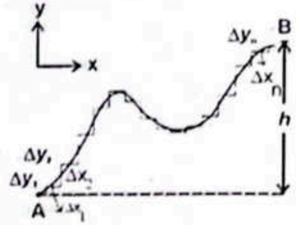
$W_{ACB} = (-mgh) + 0 = -mgh$ ——— (B)

Work Done Along Path 3:

Now consider the curved path between A and B. This path is divided into a large number of horizontal and vertical steps. Let $\Delta x_1, \Delta x_2, \Delta x_3 \ldots \Delta x_n$ be the horizontal distances and $\Delta y_1, \Delta y_2, \Delta y_3, \ldots \Delta y_n$ be the vertical distances.

The Work done along the horizontal steps = $mg\,\Delta x \cos\theta = mg(\Delta x)\cos 90 = 0$

Work done along the vertical steps is



$W_1 = mg.\Delta y_1 = mg\Delta y_1 \cos 180^0 = -mg\Delta y_1$

$W_2 = mg.\Delta y_2 = mg\Delta y_2 \cos 180^0 = -mg\Delta y_2$

. . . . .

. . . . .

. . . . .

$W_n = mg.\Delta y_n = mg\Delta y_n \cos 180^0 = -mg\Delta y_n$

The total work done is given by

$W_{AB} = W_1 + W_2 + \text{————} + W_n$

$W_{AB} = mg\,\Delta y_1 - mg\,\Delta y_2 - \text{————} - mg.\Delta y_n$

$W_{AB} = -mg(\Delta y_1 + \Delta y_2 + \text{————} + \Delta y_n)$

As $\Delta y_1 + \Delta y_2 + \text{————} + \Delta y_n = h$,

So $W_{AB} = -mgh$ ———— (C)

From equations (A), (B) and (C) it is clear that the work done along Path1, Path2 and Path3 remains same. Thus, Work done in the gravitational field is independent of the path followed and is conservative field.

QUESTION: SHOW THAT THE WORK DONE ALONG CLOSED PATH IS ZERO.

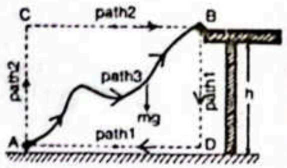
Work Done in a Conservative Field along closed path:

Consider a body of mass m moving with constant velocity in a closed path ABDA, as shown in Fig., in the gravitational field of the earth. We divide the whole path into three parts AB, BD and DA.

Work done along AB = $W_{AB} = -mgh$

Work done along path BD = $W_{BD} = mg.h = mgh\cos 0^o =$ mgh

Work done along path DA = $W_{DA} = mg.h = mgh\cos 90^o = 0$



Thus $W_{ABDA} = W_{AB} + W_{BD} + W_{DA}$

Putting values from the above equations

$W_{ABDA} = -mgh + mgh + 0 = 0$

It means that the work done by gravitational field of the earth along a closed path is zero. Hence the gravitational field is a conservative field.

Q **What is a conservative field and non-conservative field? Show that the work done along closed path is zero.**

Ans

CONSERVATIVE FIELD

*"A field in which the **work done** is **independent** of **path** followed is called **conservative field**." Or "A field in which work done along a **closed path** is **zero** is called **conservative field**."*

CONSERVATIVE FORCE

*The **force** by which work done is **independent of path** followed is called conservative force or work done along closed path is zero is called conservative force*

Examples of Conservative Forces

(1) Gravitational Force (2) Elastic Spring Force (3) Electric Force

NON-CONSERVATIVE FIELD

*"A field in which work is **dependent** on path followed is called **non conservative** force."*

e.g., if an object is moved over a rough surface then work is always done against the frictional force.

Examples of Non-Conservative Forces

1) Frictional Force
2) Air resistance
3) Propulsion Force of a rocket
4) Tension in a string
5) Normal Force
6) Propulsion Force of a motor

MULTIPLE CHOICE QUESTIONS

- **Which of the following is a characteristic of a conservative force?**
  (A) The work done depends on the path taken
  (B) The work done in a closed path is always negative.
  (C) The work done in a closed path is zero
  (D) It always dissipates energy

Answer: C

Explanation: A defining property of a conservative force is that the total work done by it on an object moving along any closed path is zero. This implies that the work done between two points is independent of the path.

- **If an object is moved from point A to point B by a conservative force, and then moved back from B to A by the same conservative force, what is the total work done by the force?**
  (A) Positive (B) Negative
  (C) Zero (D) Dependent on the path taken

Answer: C

Explanation: For a conservative force, the work done along a closed path (A to B and back to A) is always zero. This means the energy expended in one direction is recovered in the other.

- **Which statement best describes why kinetic frictional force is a non-conservative force?**
  (A) It always acts in the direction of motion.
  (B) The work done by friction is independent of the path taken.
  (C) It always does negative work and dissipates energy.
  (D) It can store potential energy.

Answer: C

Explanation: Kinetic friction always opposes motion, meaning it always does negative work on the moving object. This work is converted into heat, and thus mechanical energy is dissipated, making it a non-conservative force. The amount of work done against friction depends on the length of the path.

- **A book is lifted from the floor to a shelf. In this process, work is done:**
  (a) Only by the person lifting the book
  (b) Only by the gravitational field
  (c) By both the person and the gravitational field
  (d) Neither by the person nor by the gravitational field

Explanation: The person applies a force against the gravitational pull to lift the book. Work done is force multiplied by distance moved in the direction of the force. Here, the person's force overcomes the gravitational force, resulting in work done. So work done by person will be positive and by gravitational field is negative in his case.

- **A satellite orbits the Earth. The gravitational field of Earth does work on the satellite.**
  (a) True **(b) False**

Explanation: In a stable orbit, the gravitational force provides the centripetal force that keeps the satellite moving in a circular path. However, work is defined as force acting in the direction of motion. Since the gravitational force is perpendicular to the satellite's motion at any given point, it does not do any work.

- **A ball is dropped from a certain height. The work done on the ball during the fall is done by:**
  (a) The ball's own weight (b) The air resistance
  (c) The Earth's gravitational field (d) All of the above

Explanation: The ball's weight is a force due to gravity, not a force doing work. Air resistance opposes the motion, so it does negative work. The Earth's gravitational field pulls the ball down, causing it to move in the same direction as the force, resulting in positive work done.

- **A spring is stretched on a table. The gravitational field of Earth does positive work on the spring.**
  (a) True **(b) False**

Explanation: The spring is not moving in the direction of the gravitational force. Work is done only when the force acts in the direction of motion.

- **When a rocket is launched from Earth, the work done to overcome Earth's gravitational pull is done by:**
  (a) The rocket's fuel (b) Earth's gravitational field
  (c) Both the rocket's fuel and Earth's gravitational field
  (d) Neither the rocket's fuel nor Earth's gravitational field

Explanation: The rocket's burning fuel creates a force that propels it upwards, overcoming Earth's gravity. The gravitational field itself doesn't do work on the rocket.

- **The work done by a force in moving an object between two points depends only on the starting and ending positions. This force is associated with a:**
  (a) Non-conservative field **(b) Conservative field**
  (c) Both (a) and (b) (d) Neither (a) nor (b)

Explanation: In a conservative field, the work done by the force depends solely on the initial and final positions of the object, regardless of the path taken.

- **A ball rolling on a flat surface experiences a(n):**
  (a) Conservative force only (b) Non-conservative force only
  (c) Combination of conservative and non-conservative forces (d) None of the above
  Explanation: Gravity acting on the ball is a conservative force (work done depends only on initial and final height). However, air resistance acting on the ball is a non-conservative force (work done depends on the path taken).
- **The work done by a force acting along a closed loop (returning to the starting point) is always zero. This force is associated with a:**
  (a) Non-conservative field **(b) Conservative field**
  (c) Both (a) and (b) (d) Neither (a) nor (b)
  Explanation: A key property of a conservative field is that the work done along a closed loop is zero. This is because in a conservative field, the potential energy gained/lost cancels out when returning to the starting point.
- **Friction acting on a block sliding on a rough surface is an example of a:**
  (a) Conservative force **(b) Non-conservative force** (c) Both (a) and (b) (d) Neither (a) nor (b)
  Explanation: Friction opposes the motion of the block, and the work done by friction depends on the path taken (longer path means more work done by friction).
- **When a spring is stretched or compressed, the force exerted by the spring is:**
  (a) Conservative force (b) Non-conservative force
  (c) Depends on the material of the spring (d) None of the above
  Explanation: The elastic force exerted by a spring depends only on the initial and final extension/compression of the spring, making it a conservative force.
- **Which of the following force is the one which depends upon path to work done between two points?**
  A. Frictional force B. Air resistance C. Normal force **D. All of these**
  Solution:
  As all of these are from the family of non-conservative forces so it means they depend upon path followed for work to be done.

SLO BASED SHORT QUESTIONS & ANSWERS

- **Explain why gravitational force is considered a conservative force.**

Ans: Gravitational force is conservative because the work it does on an object depends only on the change in the object's vertical position (initial and final heights), not on the actual path taken. If an object is moved along any path and then returned to its starting point, the net work done by gravity over the entire closed loop is zero.

- **Why does the work done by kinetic frictional force always result in energy dissipation?**

Ans: Kinetic frictional force always opposes the direction of motion, meaning it always does negative work on the moving object. This negative work signifies that mechanical energy is being removed from the system and converted into other forms of energy, primarily heat (due to molecular agitation at the surfaces in contact), which is then dissipated into the environment. This energy is not recoverable as mechanical energy.

- **If an object is pushed around a rough circular track and returns to its starting point, what can be said about the total work done by friction?**

Ans: If an object is pushed around a rough circular track and returns to its starting point, the total work done by friction will be negative and non-zero. Since friction is a non-conservative force, the work done by it depends on the length of the path. As the object moves along the track, friction continuously opposes its motion, doing negative work. This energy is dissipated as heat, and it is not recovered when the object completes the loop, thus the net work done by friction over a closed path is never zero.

4.4 POWER

Q.5 Define Power, Average Power, and Instantaneous Power. Also give the units and dimensions of Power.

Ans

POWER:

"The __rate__ at which the __work__ is being done is called __power__"

$$\text{Power} = \frac{\text{Work}}{\text{time}}$$

$$P = \frac{W}{t}$$

SI Unit of Power: The SI unit of power is watt.

Watt: It can be defined as *"if __onejoule__ of work is done in __one second__ then power is said to be __one watt__."*

Mathematically

$$1\,W = \frac{1J}{1s} = 1Js^{-1}$$

For Your Information	
Approximate Powers	
Device	Power (W)
Jumbo Jet Aircraft	1.3×10^8
Car at 90 km h^{-1}	1.1×10^5
Electric Heater	12×10^3
Colour TV	1.20
Flash Light(two Cells)	1.5
Pocket Calculator	7.5×10^{-4}

Dimensions of power:

As we have

$$1\text{watt} = \frac{1J}{1s.} = \frac{1N \times m}{1s.} = \frac{(kgm^2/s^2}{s.} = \frac{kgm^2}{s^2 \times s}$$

$$\text{watt} = \frac{kgm^2}{s^3}$$

$$\text{Dimension of power} = \frac{[ML^2]}{[T^3]}$$

$$\text{Dimension of power} = [ML^2T^{-3}]$$

AVERAGE POWER:

It is the ratio between total work done and total time taken. "If work ΔW is done in a time interval Δt, then average power P_{av} during the interval Δt is defined as

$$P_{av} = \frac{\Delta W}{\Delta t}$$

INSTANTANEOUS POWER:

It is defined as "The Power at any instant of time *t*." If ΔW is the work done in very short interval of time Δt following the instant *t* then instantaneous power is given by

$$P_{inst} = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t}$$

Question What is the commercial unit of power? Show that 1kWh=3.6 × 10⁶J.

COMMERCIAL UNIT OF ELECTRICAL ENERGY

The commercial unit of electrical energy is Kilowatt-hour (kWh).

Kilowatt-hour

"If a __power__ of __one kilowatt__ is maintained for __one hour__, then work done is __one kilowatt hour__."

$$1\text{watt} = \frac{1J}{1s}$$

$$1\text{watt} \times s. = 1J$$

$$1000\text{watt} \times s. = 1000J$$

$$1\text{Kwatt} \times 3600s. = 1000 \times 3600J$$

$$1\text{Kwatt hour} = 3.6 \times 10^6 J$$

$$1\text{Kwh} = 3.6 \times 10^6 J$$

$$= 3.6\ MJ$$

Do you know?

It takes about 9×10^9 J of energy to make a car and the car then uses about 1×10^{12} J of energy from petrol in its life time.

(Prove that $P = \vec{F}.\vec{v}$)

Relation Between Power Force and Velocity

Let a constant force $\vec{F}$ acting on an object moving at the constant velocity $\vec{v}$. For example, when the propeller of a motor boat causes the water to exert a constant force $\vec{F}$ on the boat it moves with the constant velocity $\vec{v}$. The power delivered by the motor at any instant is given by:

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} \quad \text{.......(1)}$$

For your information

Approximate Energy Values

Source	Energy (J)
Burning 1 ton coal	30×10^9
Burning 1 litre petrol	5×10^7
K.E. of a car at 90 km h⁻¹	1×10^6
Running Person at 10 km h⁻¹	3×10^3
Fission of one atom of uranium	1.8×10^{-11}
K.E. of a molecule of air	6×10^{-21}

We know $\Delta W = \vec{F}.\Delta\vec{d}$ Putting this value of 'Δw' in equation (1)

$$= \lim_{\Delta t \to 0} \frac{\vec{F}\ \Delta\vec{d}}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{\vec{F}.\Delta\vec{d}}{\Delta t}$$

$$= \vec{F}\ \lim_{\Delta t \to 0} \frac{\Delta\vec{d}}{\Delta t}$$ As we know $\lim_{\Delta t \to 0} \frac{\Delta\vec{d}}{\Delta t} = \vec{v}$ So we get,

$$P = \vec{F}.\vec{v} \quad \text{----------- (2)}$$

This is relation between the power and velocity. The equation (2) shows that the power is the scalar or dot product of force and velocity. Hence power is a scalar quantity.

Question: Prove that 1 h p = 746 watt? (Short Question)

1 h p = 550 Foot Pound per second

- 1 h p = 550 foot-pounds per second
- 1 pound = 4.448 N
- 1 foot = 0.3048 m

1 h p = 550 x 4.448 x 0.3048 m

1 h p = 745.69 Nms⁻¹

1 h p = 746 watt

Hence Prove that

1 h p = 746 watt

SLO BASED SHORT QUESTIONS & ANSWERS

- **A person pushes a wall with a force of 500 N for 10 seconds. What is the power output of the person? Explain your answer.**

Ans: The power output of the person is zero. Power is the rate of doing work, and work requires both a force and a displacement in the direction of the force. Although the person applies a force to the wall, the wall does not move (displacement is zero). Since no work is done (W=Fd=500 N×0 m=0 J), the power output (P=W/t=0 J/10 s=0 W) is also zero.

- **How does increasing the speed of an object while applying a constant force affect the power delivered to the object?**

Ans: Increasing the speed of an object while applying a constant force in the direction of motion will increase the power delivered to the object. This is directly evident from the formula P=F·v. If the force F is constant and in the direction of motion, then P∝v. Therefore, a higher velocity v will result in a proportionally higher power P. This means work is being done at a faster rate.

- **Define joule and watt. (MTN, BWP 2018 GI) (SGD, BWP 2018 GII) (BWP 2019 GI)**

Joule (J):

Ans: Joule is the unit of energy in the International System of Units (SI).

It represents the amount of energy required when a force of one newton (N) acts through a distance of one meter (m).

Watt (W):

Watt is the unit of power in the SI system.

One Watt is equal to one Joule per second (W = J/s).

Watts are used to describe how fast an appliance uses energy. Higher wattage indicates the appliance uses energy more quickly.

- **Why is kilowatt-hour (kWh) a unit of energy, not power, despite having "watt" in its name?**

Ans: Kilowatt-hour (kWh) is a unit of energy because it is the product of power (kilowatt, kW) and time (hour, h). Power is the rate of energy transfer, so multiplying a rate by time gives the total amount of energy transferred or consumed. Just as Watt-second is a Joule (unit of energy), Kilowatt-hour is a larger unit of energy, commonly used for billing electricity consumption.

MULTIPLE CHOICE QUESTIONS

- **A 100 kg object is lifted vertically by a crane at a constant speed of 0.5 m/s. What is the power output of the crane? (Assume g=9.8 m/s2)**

(A) 50 W (B) 490 W (C) 980 W (D) 1960 W

Answer: B

Explanation: The force required to lift the object at constant speed is equal to its weight, F = mg = 100 kg ×9.8 m s⁻² = 980 N. Using P = Fv, the power is P = 980 N × 0.5 m s⁻¹ = 490 W.

- A body of mass "m" moves at constant speed "v" for a distance "S" against a constant force F. What is the power required to sustain this motion?

A. Fv B. $\frac{1}{2}FS$ C. $\frac{1}{2}mv^2$ D. FS

Solution: $P_{av} = F.V = FV$ = maximum average power developed

- A 2000 kg car can accelerate from rest to a speed of 25 m s^{-1} in 10s. what average power must the engine of the car produce in order to cause this acceleration (Ignore frictional losses)?

A. 31.25kW B. 62.50kW C. 120.50kW D. 1000kW

Solution: The work done in accelerating the car = increase in K.E = $\frac{1}{2}mv^2$

$$\text{Power} = \frac{\frac{1}{2}mv^2}{t} = \frac{\frac{1}{2}\times 2000\times(25)^2}{10} = 62.50\text{kW}$$

- A pump draws a volume of water V from a slow-moving river up to a vertical height "h" in a time t and discharges it through a nozzle with a speed v. If the density of the water is ρ, what is the effective power developed by pump? GIKI2016

A. $\frac{\rho Vgh}{t}$ B. $\frac{\rho Vv^2}{2t}$ C. $\frac{\rho Vgh}{t}, \frac{\rho Vv^2}{2t}$ D. $\frac{\rho Vgh}{t}, \frac{2\rho Vv^2}{2t}$

Solution:

$$P = \frac{w}{t} = \frac{mgh + \frac{1}{2}mv^2}{t} = \frac{mgh}{t}, \frac{mv^2}{2t}$$

$m = V\rho$

- A body of mass 2 kg, initially at rest, is acted upon simultaneously by two forces, one of 4N and the other of 3N, acting at right angles to each other. The kinetic energy of the body after 20s is:

A. 500J B. 1250J C. 2500J D. 5000J

Solution: Resultant force on the body is;

$F = \sqrt{(4)^2 + (3)^2} = \sqrt{16+9} = \sqrt{25} = 5N$ and $a = \frac{F}{m} = \frac{5}{2}ms^{-2}$ So $v = at = \frac{5}{2}\times 20 = 50ms^{-1}$ Now $K.E = \frac{1}{2}mv^2$

$= \frac{1}{2}\times 2\times(50)^2 = 2500J$

4.5 ENERGY

Q.6 Define Energy. Give the types of Mechanical Energy. And also prove the mathematical form of Kinetic Energy?

Ans

ENERGY:

*The **ability** of a body to **do work** is called energy*

SI Unit of Energy:

The SI unit of energy is "joule" it is same for energy and work.

Dimensions:

$[W] = [ML^2T^{-2}]$

TYPES OF MECHANICAL ENERGY:

There are two main types of mechanical energy

1) Kinetic Energy
2) Potential energy

1. **Kinetic Energy:**

*It is the **energy** possessed by a body due to its **motion** and is given by a formula*

$$K.E = \frac{1}{2}mv^2$$

Where m is the mass of the body and v is the speed of the body.

2. **Potential Energy:**

*It is the **energy** possessed by a body because of its **position** in a **force** field e.g., in a gravitational field*

Examples of Potential Energy:

(i) **Gravitational Potential Energy:**

The Potential Energy due to gravitational field near the surface of the Earth at height h is called gravitational potential energy

$$P.E. = mgh$$

This the gravitational P.E. of a body relative to the surface of the Earth which is taken as reference point in this case.

(ii) **Elastic Potential Energy**

The energy stored in a compressed spring is due to its compressed state and is called elastic potential energy

$$E.P.E = \frac{1}{2}kx^2$$

Approximate Energy Values

Source	Energy(J)
Burning 1 ton coal	30×10^9
Burning 1 litre petrol	5×10^7
K.E of a car at 90 kmh⁻¹	1×10^6
Running person at 10 kmh⁻¹	3×10^3
Fission of one atom of uranium	1.8×10^{-11}
K.E of a molecule of air	6×10^{-21}

Do You Know

- It takes about 9×10^7 J to make a car and car then uses about 1×10^{12} J of energy from petrol in its life time.
- All the food you eat in one day has about the same energy as 1/3 litre of petrol.

Do you know?

Q: How much energy does it take to make a car, and how much does it use in its lifetime?

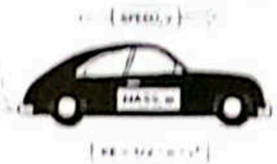
A: It takes about 9×10^7 J of energy to make a car, and the car then uses about 1×10^{12} J of energy from petrol in its lifetime.

Question: Derive the Mathematical form of Kinetic Energy? OR Prove that $K.E = \frac{1}{2}mv^2$

Consider a car running with constant speed 'v' that eventually stops due to friction when the engine is switched off.

Work Done by Car: As the car moves, it does work against the force of friction (f) over a distance (d). This work is equal to the car's kinetic energy: K.E. = fd.

Newton's Second Law: The acceleration 'a' produced by friction is negative (deceleration) because friction opposes motion: f = −ma, or a= −f/m.



Now we can determine the value of (fd) by using the third equation of motion, i.e;

$$2aS = v_f^2 - v_i^2$$

Here,

Initial velocity = $v_i = v$
Final velocity = $v_f = 0$
Distance = $S = d$
Acceleration = $a = -\frac{f}{m}$

Putting values in the above equation of motion, we have

$$2(-\frac{f}{m})d = (0)^2 - (v)^2$$

$$2(-\frac{f}{m})d = -(v)^2$$

$$fd = \frac{1}{2}mv^2$$

As fd is equal to the kinetic energy of the body, therefore

$$\text{Kinetic Energy} = \frac{1}{2} mv^2$$

Since, kinetic energy is equal to work which the body is capable of doing, so the unit of kinetic energy must be that of work, i.e. joule (J).

Question: Define Potential Energy and hence Gravitational Potential Energy?

POTENTIAL ENERGY:

"The energy possessed by a body due to its position is called potential energy."

Example:

- A raised weight.
- Water that is behind a dam.
- A car that is parked at the top of a hill.
- A yoyo before it is released.
- River water at the top of a waterfall.
- A book on a table before it falls.

There are two types of potential energy:

- Gravitational potential energy
- Elastic Potential energy

GRAVITATIONAL POTENTIAL ENERGY:

"The energy possessed by a body due to gravitational force near the surface of the Earth at a height 'h' is called Gravitational force".

Mathematically:

The potential energy due to gravitational field near the surface of the earth at a height h is given by the formula.

$$P.E = mgh$$

This is called **gravitational potential energy**.

The gravitational P.E is always determined relative to some arbitrary position which is assigned the value of zero P.E.

The two points taken as zero reference level of gravitational P.E. are:

i. The surface of Earth

ii. A point at infinity from the earth.

Question: What is elastic potential eneygy?

ELASTIC POTENTIAL ENERGY:

"The energy stored in a compressed spring or the potential energy possessed by the spring due to its compress or stretched state. This form of energy is called the elastic potential energy."

Mathematically:

$$P.E = \frac{1}{2} Kx^2$$

Where:

K = spring constant,

x = extension

MULTIPLE CHOICE QUESTIONS

- **A car of mass 1000 kg is moving with a speed of 20 m/s. What is its kinetic energy?**

(A) 10,000 J (B) 20,000 J (C) 200,000 J (D) 400,000 J

Answer: C

Explanation: Using the formula K.E =1/2mv2:

$K.E = 1/2 \times 1000\ kg \times (20\ m\ s^{-1})^2 = 500\ kg \times 400\ m^2\ s^{-2} = 200,000\ J$.

- **Potential energy is the energy possessed by a body due to its:**

(A) Temperature (B) Motion

(C) Position or constrained state (D) Chemical composition

Answer: C

Explanation: Potential energy is stored energy related to an object's position within a force field (like gravitational potential energy) or its physical configuration (like elastic potential energy in a spring).

- **The absolute gravitational potential energy of an object at a certain position is defined as the work done by the gravitational force in displacing the object from that position to:**

(A) The Earth's surface (B) The center of the Earth (C) Infinity (D) Its initial position

Answer: C

Explanation: Absolute gravitational potential energy is defined with respect to a reference point where gravitational force is considered zero, which is conventionally taken as infinity. At infinity, the potential energy is zero.

- **What will be the velocity of the particle if its momentum and kinetic energy are equal in magnitudes? (DGK 2021 GII)**

Answer: Since P = K.E $\Rightarrow mv = \frac{1}{2}(mv^2)$ $\Rightarrow v = 2\ m\ s^{-1}$

- **When P.E of an object increases then:**

A. Work done by gravity is negative B. Work done against gravity is positive

C. Both "A" & "B" D. None of these

Solution: An increase in P.E means gain in height or to move against gravity so in this case work by gravity will be negative as shown in figure but at the same time if we consider work done against gravity it must have opposite sign so will be taken as positive.

SLO BASED SHORT QUESTIONS & ANSWERS

- **Differentiate between gravitational potential energy and elastic potential energy, providing an example for each.**

Ans: **Gravitational Potential Energy:** This is the energy an object possesses due to its position in a gravitational field. It depends on the object's mass, the acceleration due to gravity, and its vertical height relative to a reference point.

Example: A book held above the ground.

Elastic Potential Energy: This is the energy stored in an elastic object (like a spring or a stretched rubber band) due to its deformation (stretching or compression).

Example: A compressed spring in a toy gun.

- **Why is the absolute gravitational potential energy always negative, and what does it mean for potential energy to increase as an object moves away from Earth?**

Ans: The absolute gravitational potential energy is always negative because the reference point for zero potential energy is taken at infinity, and the gravitational force is always attractive. As an object moves closer to the Earth, the gravitational force does positive work, meaning energy is released, and the potential energy becomes more negative. Conversely, as an object moves away from Earth (increasing 'r'), work must be done on the object against gravity. This means its potential energy increases, becoming less negative (closer to zero at infinity).

- **A car is moving at a certain speed. If its speed is doubled, how does its kinetic energy change?**

Ans: If the speed of the car is doubled, its kinetic energy will become four times its original value. This is because kinetic energy is directly proportional to the square of the velocity ($K.E = (1/2)mv^2$. If v becomes 2v, then v^2 becomes $(2v)^2 = 4v2$. So, the new kinetic energy will be $(\frac{1}{2})m(4v^2) = 4 \times (1/2)(mv^2)$, which is four times the original kinetic energy.

- **Point out the positions where gravitational potential energy is taken as zero. (GRW 2022 GI)**

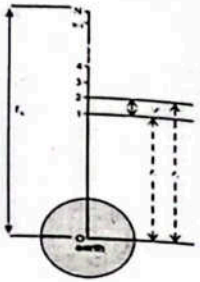
Ans: Infinity. This reference point is often used for problems involving objects moving very far away from the Earth, like rockets escaping Earth's gravity. So the GPE of the object at infinity is considered zero.

Q. 8 What is absolute potential energy? Calculate the value of absolute potential energy in the gravitational field.

Ans

ABSOLUTE POTENTIAL ENERGY

"The absolute gravitational potential energy at a point is the work done by the gravitational force in displacing the object from that point to infinity where the force of gravity becomes zero."



Expression for Absolute P.E.

Consider a body of mass "m" is lifted from a point 1 to another point N in gravitational field. The gravitational force does not remain constant between point 1 and N. So, the distance between point 1 and N is divided into a very large number of small steps each of length Δr at points 1,2,3,.......N-1,N . As Δr is very small so the gravitational force during each step remains constant.

Work between points 1 and 2

To find the work done in moving the body from point 1 to N, first we find the work done in moving body from point 1 to 2

$$W_{1\to2} = \vec{F}\cdot\vec{\Delta r}$$

r_1 = Distance of point 1 from the center of the Earth

r_2 = Distance of point 2 from the center of the Earth

r_3 = Distance of point 3 from the center of the Earth

r_N = Distance of point N from the center of the Earth

$\Delta r = r_2 - r_1 = r_3 - r_2 = r_4 - r_3 = \ldots\ldots\ldots\ldots r_N - r_{N-1}$

$\Rightarrow r_2 = \Delta r + r_1$

And $\quad F = \dfrac{GMm}{r^2}$

r = average distance of the center of the step from the center of the Earth.

$$r = \frac{r_1 + r_2}{2}$$

putting value of r_2 $\quad r_2 = \Delta r + r_1$

$$r = \frac{r_1 + \Delta r + r_1}{2}$$

$$r = \frac{2r_1 + \Delta r}{2}$$

$$r = r_1 + \frac{\Delta r}{2}$$

Squaring both sides

$$r^2 = \left(r_1 + \frac{\Delta r}{2}\right)^2 = r_1^2 + r_1\Delta r + \frac{\Delta r^2}{4}$$

As Δr is very small quantity hence $\frac{\Delta r^2}{4}$ being very very small can be neglected

$$r^2 = r_1^2 + r_1\Delta r$$

$$r^2 = r_1^2 + r_1\left[r_2 - r_1\right]$$

$$r^2 = r_1^2 + r_1r_2 - r_1^2$$

$$r^2 = r_1r_2$$

If M is the mass of the earth, m is the mass of the body and G is the gravitational constant then gravitational force at the center of this step is

$$F = \frac{GMm}{r^2}$$

Then putting the value of 'r', we get

$$F = \frac{GMm}{r_1r_2}$$

As this force is constant during the interval Δr, so the work done in moving the body from point 1 to 2 is

$$W_{1\to2} = \vec{F}\cdot\vec{\Delta r} = F\Delta r\cos 180^\circ$$

$$W_{1\to2} = -F\Delta r = -\frac{GMm}{r_1r_2}(r_2 - r_1)$$

$$W_{1\to2} = -GMm\left(\frac{r_2}{r_1r_2} - \frac{r_1}{r_1r_2}\right)$$

$$W_{1\to2} = -GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Negative sign shows that work is done on the body in moving it from point 1 to 2. Similarly work done in moving the body from point 2 to 3 and is given as

Do you know?

There is more energy reaching Earth in 10 days of sunlight than in all the fossil fuels on the Earth.

$$W_{2\to3} = -GMm\left(\frac{1}{r_2} - \frac{1}{r_3}\right)$$

$$W_{3\to4} = -GMm\left(\frac{1}{r_3} - \frac{1}{r_4}\right)$$

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$$W_{N-1\to N} = -GMm\left(\frac{1}{r_{N-1}} - \frac{1}{r_N}\right)$$

Total Work done

Total work done is calculated by adding all above equations, we get

$$W_{1\to N} = W_{1\to2} + W_{2\to3} + W_{3\to4}\ldots\ldots W_{N-1\to N} + W_{1\to N}$$

$$W_{1\to N} = -GMm\left(\frac{1}{r_1} - \frac{1}{r_N}\right)$$

N at infinity

If the point N is at an infinite distance from the earth so

$$r_N = \infty \qquad so \qquad \frac{1}{r_N} = \frac{1}{\infty} = 0$$

Hence $\quad W_{1\to\infty} = -GMm\left(\dfrac{1}{r_1} - \dfrac{1}{\infty}\right)$

$$W_{1\to\infty} = -\frac{GMm}{r_1}$$

Absolute P.E

Thus the absolute potential energy of a body at a distance r from the center of the Earth is

$$U = -\frac{GMm}{r}$$

Negative sign shows that when 'r' increases, the gravitational force does negative work and 'U' increases i.e., U become less negative. When 'r' decreases, the body falls towards the earth, the work is positive and P.E. decreases.

Negative sign also shows that the Earth's gravitational field for mass 'm' is attractive.

On the surface of the earth:

The absolute P.E. on the surface of the Earth is found by putting r = R in expression, we get

$$U_g = -\frac{GMm}{R}$$

Tidbit

More coal has been used since 1945 than was used in the whole of history before that.

Do you know?

Q: How does the energy from the Sun compare to Earth's fossil fuels?

A: There is more energy reaching Earth in 10 days of sunlight than in all the fossil fuels available on the Earth.

Tidbit:

More coal has been used since 1945 than was used in the whole of history before that.

SLO BASED SHORT QUESTIONS & ANSWERS

- **Define absolute potential energy and write only its formula. (BWP 2017) (FBD 2021 G1)**

Ans: *"The absolute gravitational potential energy at a point is the* ***work done*** *by the* ***gravitational*** *force in displacing the object from that point* ***to infinity*** *where the force of gravity becomes zero."*

On the surface of the earth:

The absolute P.E. on the surface of the Earth is $U_g = -\frac{GMm}{R}$

- **What does negative sign show in the expression: $U_g = -\frac{GMm}{R}$ (LHR 2017 GII) (AJK 2021)**

Ans: The negative sign in the formula for absolute gravitational potential energy indicates:

- A negative sign indicates that the object is in a bound system with another object, like an apple near Earth.
- The negative sign implies that the work done by gravity is negative. When we lift an object against gravity (increasing its distance 'r'), we are doing positive work. Gravity, however, is constantly pulling the object downwards (towards a lower PE), so its work is considered negative.
- The negative sign indicates the attractive nature of gravity and the work needed to overcome it.

4.6 ESCAPE VELOCITY

Q Define Escape Velocity. Derive an expression for escape velocity and calculate escape velocity on the surface of the earth.

Ans.

Escape Velocity

The initial velocity of an object with which it goes out of Earth's gravitational field, is known as escape velocity.

Explanation

It is our daily life experience that an object projected upward comes back to the ground after rising to certain height. This is due to the force of gravity acting downwards. With increased initial velocity the object rises to the more height before coming back to ground. If we go on increasing the initial velocity of the object, a stage comes when the object will not return to the ground it will escape out of the influence of gravity.

Expression for escape velocity

We know that the absolute P.E. of a body of mass m on the surface of earth is

$$U = -\frac{GmM}{R} \quad \text{------ (1)}$$

As the body goes out of gravitational field, it P.E. becomes zero.

$$U = 0 \quad \text{------ (2)}$$

Now initial K.E. is

$$K.E. = \frac{1}{2}mv_{esc}^2 \quad \text{------ (3)}$$

And change in absolute potential energy is

$$\Delta U = U_f - U_i$$

Using equations 1 and 2

$$\Delta U = 0 - (-\frac{GmM}{R})$$

$$\Delta U = \frac{GmM}{R} \quad \text{------ (4)}$$

Comparing equation 3 & 4 we get

$$K.E. = \Delta U$$

$$\frac{1}{2}mv_{esc}^2 = \frac{GmM}{R}$$

$$v_{esc}^2 = \frac{2GM}{R}$$

$$v_{esc} = \sqrt{\frac{2GM}{R}} \quad \text{------ (5)}$$

Force of gravity acting on a body is given by

$$F = \frac{GmM}{R^2}$$

But F = mg

So comparing we get

$$mg = \frac{GmM}{R^2}$$

$$g = \frac{GM}{R^2}$$

$$gR = \frac{GM}{R} \quad \text{------ (6)}$$

using equation 6 in equation 5 we get

$$v_{esc} = \sqrt{2gR} \quad \text{------ (7)}$$

It is the required relation for the escape velocity of an object. It shows that escape velocity does not depend upon mass of the body but it depends on planet's size.

Value of escape velocity on Earth

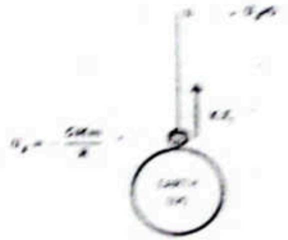
As $g = 9.8 m/sec^2$ and $R = 6.4 \times 10^6 m$

So $v_{esc} = \sqrt{2 \times 9.8 \times 6.4 \times 10^6}$

$= 11.2 \times 10^3 m/sec$

OR $= 11.2 \times 10^3 m/sec$

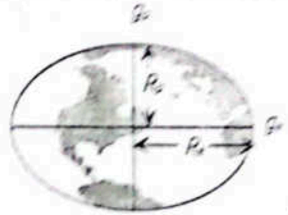
OR $v_{esc} = 11.2 km/sec$



Tidbits

All the food you eat in one day has about the same energy as 1/3 litre of petrol.

Important Point



At Poles: Escape velocity has large value.

At equator: Escape velocity has small value.

$$v_{esc} = \sqrt{\frac{2GM}{R}} \propto \frac{1}{\sqrt{R}}$$

For your information

Some Escape speeds ($km s^{-1}$)

Heavenly body	Escape speed
Moon	2.4
Mercury	4.3
Mars	5.0
Venus	10.4
Earth	11.2
Uranus	22.4
Neptune	25.4
Saturn	37.0
Jupiter	61

MULTIPLE CHOICE QUESTIONS

- **The derivation of escape velocity relies on the principle that the initial kinetic energy of the object must be equal to the:**
  (A) Work done against air resistance.
  (B) Total mechanical energy at the Earth's surface.
  (C) Increase in potential energy required to move it to infinity.
  (D) Gravitational force at infinity

Answer: C

Explanation: For an object to escape, its initial kinetic energy must be sufficient to overcome the gravitational potential energy, meaning it must equal the change in potential energy from the surface to infinity (where potential energy is zero).

- **Which of the following factors does escape velocity NOT depend on?**
  (A) Mass of the celestial body. (B) Radius of the celestial body.
  (C) Mass of the escaping object. (D) Gravitational constant (G).

Answer: C

Explanation: As shown in the derivation v_{esc}=2GM/R, the mass of the escaping object (m) cancels out. Therefore, escape velocity is independent of the mass of the object being launched.

SLO BASED SHORT QUESTIONS & ANSWERS

- **Define escape velocity and explain its physical significance.**

Ans: Escape velocity is the minimum initial velocity that an object must have at the surface of a celestial body (like Earth) to completely escape its gravitational field and never return.
Physical Significance: Its physical significance is that it represents the threshold speed at which an object's kinetic energy is just enough to overcome the gravitational potential energy binding it to the celestial body, allowing it to reach an infinite distance with zero kinetic energy remaining.

- **Explain why the mass of the object being launched does not affect its escape velocity.**

Ans: The mass of the object being launched does not affect its escape velocity because, during the derivation, the mass 'm' of the object appears on both sides of the energy conservation equation (½)mv_{esc}^2=GMm/R). Therefore, 'm' cancels out, leaving the escape velocity dependent only on the gravitational constant (G), the mass of the celestial body (M), and the radius of the celestial body (R). This means a feather and a rocket require the same escape velocity from Earth, assuming no air resistance.

- **If a planet has the same radius as Earth but twice its mass, how would its escape velocity compare to Earth's?**

Ans: If a planet has the same radius as Earth but twice its mass, its escape velocity would be 2 times greater than Earth's escape velocity. This is because the escape velocity formula is v_{esc}=2GM/R. If M is doubled to 2M, then v_{esc} becomes 2G(2M)/R=2(2GM/R)=2×v_{esc},Earth. A more massive planet has a stronger gravitational pull, requiring a higher escape velocity.

- **What happens if an object is launched with a velocity less than its escape velocity?**

Ans: If an object is launched with a velocity less than its escape velocity, it will eventually fall back to the celestial body from which it was launched. Its initial kinetic energy will not be sufficient to overcome the gravitational potential well. It will rise to a certain maximum height where its kinetic energy becomes zero, and then gravity will pull it back down towards the surface.

4.7 WORK-ENERGY THEOREM

Q State and prove Work-Energy Principle.

Ans

WORK-ENERGY PRINCIPLE

"Work done on a body is equal to change in its kinetic energy"

Proof.

Consider a body of mass **m** moving with velocity v_i. A constant force F acting through a distance 'd' increases the velocity to v_f, then from equation of motion

$$2ad = v_f^2 - v_i^2$$

$$d = \frac{1}{2a}\left(v_f^2 - v_i^2\right)$$

From second law of motion F = ma

Multiplying equation by F we get

$$Fd = ma \times \frac{1}{2a}\left(v_f^2 - v_i^2\right)$$

$$Fd = \frac{1}{2}m\left(v_f^2 - v_i^2\right)$$

$$Work = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$Work = (K.E)_f - (K.E)_i$$

$$Work = \Delta(K.E)$$

> Potential energy exists only for conservative forces. It does not exist for nonconservative forces. All the central forces are conservative.
> (a) Elastic potential energy PE = 1/2ky² is always positive
> (b) Electric potential energy PE = $\frac{q_1q_2}{4\pi\epsilon_o r}$ may be positive or negative
> (c) Gravitational potential energy
> PE = $\frac{-Gm_1m_2}{r}$ may be positive or negative
> = mgh (if height is not very large)

i.e. work done on a body is equal to its change in kinetic energy. If a body is raised up from the earth surface then

$$Work = \Delta(P.E)$$

Applicability: The work-energy theorem is applicable for any direction of the force relative to the displacement and even if the force varies from point to point.

SLO BASED SHORT QUESTIONS & ANSWERS

- **Why is the Work-Energy Theorem considered a powerful and general principle in mechanics?**

Ans: The Work-Energy Theorem is powerful and general because it applies to any type of force (constant or variable, conservative or non-conservative) and any direction of force relative to displacement. It provides a direct link between the forces acting on an object and the change in its motion (specifically its speed), often simplifying problems that would be more complex to solve, especially when forces are variable.

- **A car brakes and comes to a stop. Using the Work-Energy Theorem, explain the energy transformation involved.**

Ans: When a car brakes and comes to a stop, its initial kinetic energy is reduced to zero (final kinetic energy is zero). The Work-Energy Theorem states that the change in kinetic energy is equal to the net work done. In this case, the work done by the braking force (which is a non-conservative frictional force) is negative, as it opposes the car's motion. This negative work removes kinetic energy from the car, converting it primarily into heat and sound, demonstrating the dissipation of mechanical energy.

- **If an object's speed doubles, how does the net work done on it change, assuming its mass remains constant?**

Ans. If an object's speed doubles, its kinetic energy increases by a factor of four (since $K.E. \propto v^2$). According to the Work-Energy Theorem ($W_{net} = \Delta K.E.$), if the change in kinetic energy is four times greater, then the net work done on the object would also need to be four times greater to achieve that doubling of speed if initial speed is zero. More precisely, if initial speed is not zero but v_i and final speed is $2v_i$, then $\Delta K.E = \frac{1}{2}m(2v_i)^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}m(4v_i^2 - v_i^2) = 3/2\, mv_i^2$. This is 3 times the initial kinetic energy.

MULTIPLE CHOICE QUESTIONS

- **If the net work done on an object is negative, what can be concluded about its motion?**
  (A) Its speed is increasing (B) Its speed is decreasing
  (C) Its speed remains constant. (D) It is moving upwards.

Answer: B

Explanation: A negative net work done means $\Delta K.E = K.E_f - K.E_i$ is negative. This implies $K.E_f < K.E_i$, meaning the final kinetic energy is less than the initial kinetic energy, and thus the object's speed is decreasing

- **A 5 kg object initially moving at 10 m/s is acted upon by a net force that does 100 J of work on it. What is the final kinetic energy of the object?**
  (A) 150 J (B) 250 J (C) 350 J (D) 450 J

Answer: C

Explanation: Initial K.E. = $\frac{1}{2}mv_i^2 = \frac{1}{2} \times 5\ kg \times (10\ m/s)^2 = \frac{1}{2} \times 5 \times 100 = 250\ J$.

According to the Work-Energy Theorem, $W_{net} = K.E_f - K.E_i$.

So, $100\ J = K.E_f - 250\ J$

$K.E_f = 100\ J + 250\ J = 350\ J$.

- **A ball is thrown upwards. As it moves up, the work done by gravity is negative. According to the Work-Energy Theorem, what does this imply about the ball's kinetic energy?**
  (A) Its kinetic energy increases. (B) Its kinetic energy remains constant.
  (C) Its kinetic energy decreases (D) Its potential energy decreases.

Answer: C

Explanation: As the ball moves upwards, gravity does negative work on it (force downwards, displacement upwards). According to the Work-Energy Theorem, negative work done by the net force (or a component of it) means a decrease in the object's kinetic energy, which is consistent with the ball slowing down as it rises

- **Car X is travelling at half the speed of car Y. Car X has twice the mass of car Y. which statement is correct.**
  A. Car X has half the kinetic energy of car Y.
  B. Car X has one quarter of the kinetic energy of car Y.
  C. Car X has twice the kinetic energy of car Y
  D. The two cars have the same kinetic energy

Solution:

$V_x = \frac{V_y}{2}$ $m_x = 2m_y$ Let $K.E_x = \frac{1}{2}m_x V_x^2 = \frac{1}{2}(2m_y) \times \left(\frac{V_y}{2}\right)^2 = \frac{1}{4}m_y \times V_y^2$

$K.E_x = \frac{1}{2} \times \frac{1}{2}m_y V_y^2 = \frac{1}{2}K.E_y$

- **A piston in a gas supply pump has an area of 400cm². The pump moves the gas against a fixed pressure of 3000 Pa. During part of its stroke, the piston moves a distance of 25 cm in one direction. How much work is done by the piston during this movement?**
  A. 30 J B. 3.0×10^3 J C. 3.0×10^5 J D. 3.0×10^7 J

Solution: $A = 400cm^2 = 400 \times 10^{-4} m^2$

$w = PAd = 3000 \times 400 \times 10 \times 25 \times 10^{-2} = 12 \times 10^5 \times 10^{-4} \times 10^{-2} \times 25 = 12 \times 10^{-1} \times 25 = 1.2 \times 25 = 30J$

- **Energy required to accelerate a car from 10 to 20 m/s compared with that required to accelerate from 0 to 10 m/s in the same interval of time covering the same distance, is:**
  A. Twice **B. Three times** C. Four times D. Same

Solution: As we know that according to work-energy principle,

$W_1 = \Delta K.E_1 = \frac{1}{2}m(V_f^2 - V_i^2) = \frac{1}{2} \times m \times ((20)^2 - (10)^2) = \frac{1}{2} \times m \times (400 - 100) = \frac{1}{2} \times m \times 300 = 150m$

$W_2 = \Delta K.E_2 = \frac{1}{2}m(V_f^2 - V_i^2) = \frac{1}{2}m((10)^2 - (0)^2) = \frac{1}{2}m \times 100 = 50m$

$\therefore W_1 = 3W_2$ so $E_1 = 3E_2$

- **A force F stops a body of mass "m" moving with a velocity "v" in a distance "S". The force required to stop a body of double the mass moving with double the velocity in the same distance is:**
  A. 2F B. 6F C. 4F **D. 8F**

Solution: As we know that;

$W = \Delta K.E = \frac{1}{2}mv^2 = FS \Rightarrow F \propto mv^2$ Now if $m' = 2m$ and $V' = 2v$ then $F' \propto 2m(2v)^2$ and $F' \propto 8mv^2 \propto 8F$

$F' = 8F$

- **A body is lifted by a man to a height of 1 m in 30s. Another man lifts the same mass to the same height in 60 s. The work done by them are in the ratio:**
  A. 1:2 **B. 1:1** C. 2:1 D. 4:1

Solution:

$W = mgh \Rightarrow W \propto h \Rightarrow \therefore h_1 = h_2, m_1 = m_2$ so $w_1 = w_2$

- **A fat person weighing 80 Kg falls on a concrete floor from 2m. If whole of the mechanical energy is converted into heat energy, then heat produced in SI units will be nearly:**
  A. 1600 J B. 1400 J C. 1500 J D. 1600 K cal

Solution:

P.E = mgh = converted into heat

P.E = 80 × 10 × 2 = 1600 J

4.8 INTERCONVERSION OF POTENTIAL AND KINETIC ENERGY

Q.10 **Show that when a body is dropped from some certain height, its Potential Energy is converted into Kinetic Energy.**
Discuss inter-conversion of P.E. and K.E with and without friction. (DGK. 2014)

Ans

In the Absence of Frictional Force:

Consider a body of mass *m* at rest at a height *h* from the surface of the earth as shown in the Fig.

At position A

P.E. at A = mgh

K.E. at A = 0

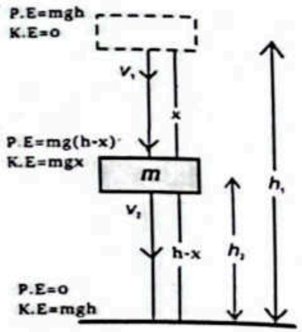
Total Energy at A: $E = K.E. + P.E.$

$E = 0 + mgh = mgh$ ------------(A)

In the absence of air friction when the body is allowed to fall its P.E. is converted into K.E.

At position B

When body is reached at point B, it has fallen down a distance " x " ignoring air friction



$$\text{P.E at B} = mg(h-x)$$

$$\text{K.E at B} = \frac{1}{2}mv_B^2 \text{ -------------- (1)}$$

Calculation of v_B:

The value of v_B can be calculated as follow

$$2gS = v_f^2 - v_i^2$$

$$v_f = v_B \quad v_i = 0 \quad \text{and} \quad S = x$$

$$2gx = v_B^2 - 0$$

$$v_B^2 = 2gx$$

putting this value in eq. (1)

$$\text{K.E at B} = \frac{1}{2}m2gx = mgx$$

Total Energy at B $\quad$ E = K.E. + P.E

$$E = mgx + mg(h-x) = mgh \text{------ (B)}$$

At position C

When body is reached at point C, just before hitting the ground it has fallen down a distance " h " ignoring air friction

$$\text{P.E. at C} = mg(0) = 0 \qquad \text{because } h = 0$$

$$\text{K.E at C} = \frac{1}{2}mv_c^2 \text{ -------------- (2)}$$

Calculation of v_C:

The value of v_C can be calculated as follow

$$2gS = v_f^2 - v_i^2$$

$$v_f = v_C, \ v_i = 0 \qquad \text{and} \qquad S = h$$

$$2gh = v_c^2 - 0$$

$v_c^2 = 2gh$ putting this value in eq. 2

$$\text{K.E at C} = \frac{1}{2}v_c^2 = \frac{1}{2}m(2gh)$$

Total Energy at C $\quad$ E = K.E. + P.E.

$$E = mgh + 0 = mgh \text{------ (C)}$$

Some Escape Speeds (kms⁻¹)	
Heavenly Body	**Escape Speed (kms⁻¹)**
Moon	2.4
Mercury	4.3
Mars	5.0
Venus	10.4
Earth	11.2
Uranus	22.4
Neptune	25.4
Saturn	37.0
Jupiter	61

Conclusion 1: K.E. and P.E. are inter-convertible but total energy remains conserved when body is dropped from height 'h'

When a body falls, its velocity increases. The increase in velocity results in the increase in its K.E. On the other hand as body falls its height decrease and hence its P.E. also decreases. Thus

$$\textit{Loss in P.E. = Gain in K.E.} \text{-------- (D)}$$

Let a body is at height h_1 falling with velocity v_1 at another height h_2 its velocity becomes v_2. In the absence of air friction this result can be represented as

$$mg(h_1 - h_2) = \frac{1}{2}m(v_2^2 - v_1^2) \text{ ------------ (E)}$$

In the presence of frictional force

If a frictional force f is present during the downward motion, then a part of P.E. is used in doing work against friction equal to " fh ". So the remaining $P.E. = mgh - fh$, is converted into K.E. So

$$mgh - fh = \frac{1}{2}mv^2$$

$$mgh = \frac{1}{2}mv^2 + fh$$

$$\textit{Loss in P.E. = Gain in K.E. - Work done against friction} \text{ ------ (F)}$$

SLO BASED SHORT QUESTIONS & ANSWERS

- **Describe the energy transformations of a ball thrown vertically upwards, ignoring air resistance, from the moment it leaves the hand until it reaches its maximum height.**

Ans: When the ball leaves the hand, it has maximum kinetic energy and some initial potential energy. As it travels upwards, its speed decreases, causing its kinetic energy to decrease. Simultaneously, its height increases, causing its gravitational potential energy to increase. This process is an interconversion where kinetic energy is continuously transformed into potential energy. At its maximum height, its velocity momentarily becomes zero, so all its initial kinetic energy (plus its initial potential energy) is converted into maximum gravitational potential energy.

- **How does the presence of frictional force alter the conservation of mechanical energy? Provide the modified energy equation.**

Ans: The presence of frictional force (a non-conservative force) means that mechanical energy is no longer conserved. Friction does negative work, converting a portion of the mechanical energy into other forms, that is heat, which is dissipated from the system. The modified energy equation for a system with friction is:

Loss in P.E. = Gain in K.E. + Work done against friction

Alternatively, it can be stated as:

Initial Total Mechanical Energy = Final Total Mechanical Energy + Energy lost to non-conservative forces (e.g., friction).

- **Consider a car rolling down a hill. If there is significant air resistance and rolling friction, what happens to the car's total mechanical energy as it descends?**

Ans: As the car rolls down the hill, its gravitational potential energy decreases, and its kinetic energy increases. However, due to significant air resistance and rolling friction (both non-conservative forces), a portion of the mechanical energy will be continuously converted into heat and sound. Therefore, the car's total mechanical energy will decrease as it descends. The loss in potential energy will be greater than the gain in kinetic energy, with the difference being the energy dissipated by friction.

- If we consider air friction, write the equation **ball of mass 50kg is dropped from a height of 10m. After rebound, it comes to the height of 7.5m. The percentage loss in P.E is:**

A. 20% **B. 25%** C. 30% D. 50%

Solution:

$\therefore P \propto h$ and $h' = 7.5m$, $h = 10m$, $h - h' = 2.5m$ or so 25% loss in height means 25% loss in P.E

4.8 LAW OF CONSERVATION OF ENERGY

Q. 11 Define and explain law of conservation of energy orState law of conservation of energy. Give some example. (LHR. 2014) or What the law of conservation of energy. (SGD 2019 GI) (DGK 2019 GII) (GRW 2021)

Ans

LAW OF CONSERVATION OF ENERG:

"Energy cannot be destroyed. It can be transformed from one kind into another, but the total amount of energy remains constant."

Conservation of Mechanical Energy:

The kinetic and potential energies are different forms of mechanical energy. The total mechanical energy of a body is sum of P.E. and K.E. For a falling body the P.E. changes into K.E. and vice versa, but the total energy remains constant. Mathematically

Total energy = P.E + K.E

This is special case of law of conservation of energy.The P.E. of a falling body changes into its K.E., but on striking the ground the K.E. changes into heat and sound and disappears.

All forms of energy can be transformed from one form to another form but electrical and chemical energy are more easily transformed.

For Your Information		Energy Sources	
Source of energy	**Original Source**	**Renewable**	**Nonrenewable**
Solar	Sun	Hydroelectric	Coal
Bio mass	Sun	Wind	Natural gas
Fossil Fuels	Sun	Tides	Oil
Wind	Sun	Geothermal	Uranium
Waves	Sun	Biomass	Oil shale
Hydro Electric	Sun	Sunlight	Tar sands
Tides	Moon	Ethanol(individual fields may run off)	
Geothermal	Earth	Methanol(Renewable when made from bio mass)	

TEXT BOOK EXERCISE WITH SOLUTION

MULTIPLE CHOICE QUESTIONS

4.1 A 1kg mass has potential energy of 1 joule relative to the ground when it is at a height of:

(a) 0.102m (b) 1m
(c) 9.8m (d) 32m

- **Explanation:** Potential Energy (P.E.) = mgh

Given P.E. = 1 J, m = 1 kg, g = 9.8 m s⁻²

h = P.E./(mg) = 1 J/(1 kg × 9.8 m s⁻²) = 1/9.8 m = 0.102 m.

4.2 An iron sphere whose mass is 30 kg has the same diameter as an aluminum sphere whose mass is 10.5kg. The spheres are simultaneously dropped from a cliff When they are 10m from the ground, they have identical:

(a) accelerations (b) momentums
(c) potential energies (d) kinetic energies

- **Explanation:** According to the principle of free fall, in the absence of air resistance, all objects accelerate downwards at the same rate due to gravity, regardless of their mass.

4.3 A body at rest may have:

(a) speed (b) velocity
(c) momentum **(d) energy**

- **Explanation:** a body at rest can possess energy, specifically potential energy. For example, a book resting on a shelf has gravitational potential energy relative to the floor.

4.4 The height above the ground of a child on a swing varies from 0.5 m of his lowest point to 1.5 mat his highest point. The maximum speed of the child is approximately:

(a) $1.5ms^{-1}$ **(b) $4.4ms^{-1}$**
(c) $9.8\ m\ s^{-1}$
(d) Depends upon child's mass

Explanation: $PE_{lost} = KE_{gained} \Rightarrow mgh = ½mv^2$. The mass 'm' cancels out from both sides, meaning the maximum speed is independent of the child's mass. $gh = ½v^2 \Rightarrow v^2 = 2gh \Rightarrow v = \sqrt{(2gh)}$ Substituting $g = 9.8\ m\ s^{-2}$ and $h = 1.0\ m$: $v = \sqrt{(2 \times 9.8\ m\ s^{-2} \times 1.0\ m)} = \sqrt{(19.6\ m^2/s^2)} = 4.427\ m/s$. Therefore, the maximum speed is approximately $4.4\ ms^{-1}$

4.5 When a ball is thrown vertically upward and then falls back to the ground, which force can be considered conservative in this scenario?

(a) Air resistance **(b) Gravity**
(c) Friction between ball and air
(d) Contact force with hand

Explanation: A conservative force is one for which the work done in moving an object between two points is independent of the path taken. Gravity is a prime example of a conservative force because the work done by gravity only depends on the initial and final vertical positions of the object, not the specific path it follows.

4.6 According to work-energy principle in linear motion, the work done on body is equal to:

(a) change in K.E. (b) change in P.E.
(c) zero
(d) sum of K.E. and P.E.

Explanation: The work-energy theorem (or principle) states that the net work done by all forces on an object is equal to the change in its kinetic energy.

4.7 Power of a lamp is 6W. How much energy does a lamp give out in 2min ?

(a) 12J (b) 20J
(c) 3J **(d) 720J**

Explanation: Power (P) is the rate at which energy (E) is transferred or consumed, given by the formula P = E/t. To find the energy, we rearrange the formula to E = P × t. Given power P = 6 Watts. Given time t = 2 minutes. First, convert minutes to seconds: 2 min × 60 s/min = 120 seconds. Now, calculate the energy: E = 6 W × 120 s = 720 Joules.

4.8 A dry battery can deliver 3000J of energy to a 2Wsmall electric motor before the battery is exhausted. For how many minutes does the battery run?

(a) 1500 min (b) 100 min
(c) 50 min **(d) 25 min**

Explanation: This question also uses the power-energy-time relationship, E = P × t. Here, we need to find the time (t). So, we rearrange the formula to t = E/P. Given energy E = 3000 Joules. Given power P = 2 Watts. Calculate the time in seconds: t = 3000 J / 2 W = 1500 seconds. To convert seconds to minutes, divide by 60: t = 1500 s / 60 s/min = 25 minutes.

4.9 The kinetic energy acquired by a mass m after travelling a fixed distance from rest under the action of a constant force is directly proportional to:

(a) $\sqrt{m}$ (b) $^1/_{\sqrt{m}}$
(c) m **(d)independent of m**

Explanation: In this equation, F (constant force) and d (fixed distance) are given as constant values. Therefore, the kinetic energy acquired (Fd) is determined solely by the constant force and the fixed distance, **and does not depend on the mass (m)** of the object.

4.10 A body moves a distance of 10m along a straight line under the action of 5N force. If the work done is 25J, the angle which the force makes with the direction of motion of the body is:

(a) 0° (b) 30°
(c) 60° (d) 90°

Explanation: The formula for work done (W) by a constant force (F) over a displacement (d) is $W = Fd\cos\theta$, where θ is the angle between the force and the displacement. We are given: W = 25 J, F = 5 N, d = 10 m Putting these values into the formula: $25\text{ J} = (5\text{ N})(10\text{ m})\cos\theta \Rightarrow 25 = 50\cos\theta \Rightarrow \cos\theta = 25/50 = 0.5$ Finally, to find the angle θ, take the inverse cosine (arccos) of 0.5. $\theta = \arccos(0.5) = 60^\circ$

SHORT ANSWER QUESTIONS

4.1 Why is electrical power required at all when the elevator is descending? Why should there be a limit on the number of passengers in this case?

Ans: **Control & Braking:** In case of descending an Electrical power is required by elevator to control its speed otherwise elevator will fall freely under the action of gravity and attain dangerous speed.

Safety Systems: A limit on the number of passengers is to ensure safe operation of elevator. A heavier load will increase the downward force, which in turn may increase required stopping force and slow it down.

4.2. A body is being raised to a height H from surface of Earth. What is the sign of work done by both? Justify.

Ans: **Work done by the body (or the lifting agent/external force):**

- **Sign: Positive (+ve)**
- **Justification:** When a body is raised, the external force applied (by the body or a lifting agent) is in the upward direction, which is the same direction as the displacement. Since work = Fd cosθ and θ = 0° (cos0° = 1), the work done is positive. This work increases the potential energy of the body.

Work done by the Earth (gravitational force):

- **Sign: Negative (-ve)**
- **Justification:** The Earth's gravitational force (weight) acts downwards, while the displacement of the body is upwards. The angle between the gravitational force and the displacement is 180°. Since work = Fd cosθ and θ = 180° (cos180° = -1), the work done by gravity is negative. This negative work signifies that the gravitational field is doing work *against* the motion, or that energy is being stored in the gravitational field.

4.3 A body falls towards the Earth in air. Will its total mechanical energy be conserved during fall? Justify.

Ans: No, the total mechanical energy of a body falling towards the Earth in air will not be conserved.

Justification:

Mechanical energy (the sum of kinetic energy and potential energy) is conserved only when conservative forces (like gravity) are doing work. However, when a body falls through the air, air resistance (a non-conservative force) acts on it.

Air resistance does negative work on the falling body, converting some of its mechanical energy into non-mechanical forms, thermal energy (heat) and sound. This energy is dissipated into the surroundings. As a result, the sum of the body's kinetic and potential energy continuously decreases throughout the fall.

4.4. Calculate power of a crane in kilowatt which lifts a mass of 1000kg to a height of 100 min 20 second.

Given

Mass lift by crane	= m = 1000 kg
Height	= 100 m
Time	= 20 s

Solution

$$P = \frac{mgh}{t} = \frac{1000 \times 9.8 \times 100}{20} = \frac{980000}{20}$$

$= 49000\text{ W} = 49\text{ kW}$

4.5. A trolley of mass 1500kg carrying sand bags of 500kg is moving uniformly with a speed of 40kmh⁻¹ on a frictionless track. After sometime, sand starts leaking out of whole sand bags on the road at a rate of 0.05kgs⁻¹. What is the speed of the trolley after entire sand bags are empty?

Ans. **Concept:** This problem involves the conservation of linear momentum. Since the track is frictionless, there are no external horizontal forces acting on the trolley-sand system. The leakage of sand is an internal process (the sand simply falls out, it's not "pushed" out relative to the trolley in a way that generates thrust). Therefore, the total momentum of the trolley and the remaining sand (or empty trolley) must remain constant.

As there is no external force acting on the trolley so the momentum of the trolley remains conserved.

Given

Mass of empty trolley = m = 1500 kg
Mass of sand bag = m = 500 kg
Total mass of trolley = M
= m + m = 1500 + 500 = 2000 kg
Speed of trolley with sand = V = 40 km h⁻¹

$$= \frac{40 \times 1000}{3600} = 11.11\text{ m s}^{-1}$$

Flow rate of sand $= \Delta m / \Delta t = 0.05\text{ kg s}^{-1}$

To Find

Speed of empty trolley = v = ?

Solution

According to law of conservation of momentum

$MV = mv$
$2000\,(11.11) = 1500v$
$v = 14.8\text{ m s}^{-1}$

Therefore, the speed of the trolley after the entire sand bags are empty will be approximately 14.81 m/s.

4.6. Give absolute and gravitational units of work on M.K.S & C.G.S systems.

The absolute and gravitational units of work in both M.K.S (Meter-Kilogram-Second) and C.G.S (Centimeter-Gram-Second) systems.

1. **M.K.S. (SI) System:**
   - **Absolute Unit of Work: Joule (J)**
     - **Definition:** 1 Joule = 1 Newton-meter (1 Nm). It is the work done when a force of 1 Newton moves an object through a distance of 1 meter in the direction of the force.
   - **Gravitational Unit of Work: Kilogram-force meter (kgf-m)**
     - Definition: 1 kgf-m is the work done when a force of 1 kilogram-force (which is the weight of a 1 kg mass, or 9.8 N) moves an object through a distance of 1 meter.
     - Conversion: 1 kgf-m = 9.8 J

2. **C.G.S. System:**
   - **Absolute Unit of Work: Erg**
     - **Definition:** 1 Erg = 1 Dyne-centimeter (1 dyne-cm). It is the work done when a force of 1 dyne moves an object through a distance of 1 centimeter in the direction of the force.
     - **Conversion to Joule:** $1\text{ J} = 10^7$ erg (since $1\text{ N} = 10^5$ dynes and $1\text{ m} = 10^2$ cm)
   - **Gravitational Unit of Work: Gram-force centimeter (gf-cm)**
     - **Definition:** 1 gf-cm is the work done when a force of 1 gram-force (which is the weight of a 1 g mass, or 980 dynes) moves an object through a distance of 1 centimeter.
     - **Conversion:** 1 gf-cm = 980 erg

4.7. A body dropped from a height of H reaches the ground with a speed of $1.2\sqrt{gH}$. Calculate work done by air friction.

Given

$v_f = 1.2\sqrt{gH}$

To find

$W_f = ?$

Solution:

$P.E = W_f + K.E$

$$mgH = W_f + \frac{1}{2}mv^2$$

$$W_f = mgH - \frac{1}{2}mv^2$$

$$W_f = mgH - \frac{1}{2}m(1.2\sqrt{gH})^2$$

$$W_f = mgH - \frac{1}{2}m(1.44gH)$$

$W_f = mgH - 0.72mgH = -0.28\text{ mgH}$

(This means 28% of the initial potential energy was dissipated by air friction.)

4.8. A bicycle has a K.E. of 150 J. What K.E. would the bicycle have if it had?

i. Same mass but has speed double?

ii. Three times mass and was moving with one half of the speed?

Given

$$K.E. = \frac{1}{2}mv^2 = 150\text{ J}$$

(i) Kinetic energy when speed is double

K.E.=

$$\frac{1}{2}m(2v)^2 = 4(\frac{1}{2}mv^2) = 4(150) = 600\text{ J}$$

(ii) kinetic energy by increasing mass 3 times and speed become half

K.E =

$$\frac{1}{2}(3m)(\frac{1}{2}v)^2 = \frac{3}{4}(\frac{1}{2}mv^2) = \frac{3}{4}(150) = 112.5\text{ J}$$

4.9. What will be the effect on K.E. of the body having mass m, moving with velocity v When its momentum becomes double? Justify.

$$K.E. = \frac{1}{2}mv^2 = \frac{1}{2m}(mv)^2 = \frac{1}{2m}p^2$$

On doubling the momentum

$$K.E. = \frac{1}{2m}(2p)^2 = 4\frac{1}{2m}p^2 = 4K.E.$$

On doubling the momentum, the K.E. becomes four times.

4.10. Does the international space-station have gravitation P.E. Or and Kinetic energy? Explain.

Ans: Yes, the International Space Station (ISS) has both gravitational potential energy (GPE) and kinetic energy.

1. **Gravitational Potential Energy (P.E.):**
   - The ISS orbits at an altitude of approximately 400 km above the Earth's surface. While this is relatively low compared to the Earth's radius (about 6371 km) but it is still significantly above the surface.
   - The gravitational potential energy is due to the position of ISS with respect of earth.
2. **Kinetic Energy (K.E.):**
   - The ISS is in constant motion, orbiting the Earth at a very high speed of approximately 7.66 km/s (about 27,600 km/h).
   - The kinetic energy is due to constant motion around the earth.

CONSTRUCTED RESPONSE QUESTIONS

4.1 When will you say that a force is conservative? Give two conditions.

Ans: A force is considered as conservative if the work done depends only on the initial and final positions of an object and not the path followed.

Two conditions that define a conservative force:

- A conservative force can be expressed as the gradient of a scalar function.
- Work done by conservative force is independent of the path followed.
- Work done in a closed path by conservative force is zero.

4.2 A light and heavy body have same linear momentum, which one has greater *K.E.*?

Ans: $K.E. = \frac{1}{2m}p^2$ if p = constant then

$$\Rightarrow K.E. \propto \frac{1}{m}$$

- So the lighter mass should have greater K.E. to compensate it mass for equal momentum.

Explanation: Since p is the same for both bodies, and $m_1 < m_2$, the denominator ($2m_1$) for the lighter body's kinetic energy will be smaller than the denominator ($2m_2$) for the heavier body's kinetic energy.

Therefore, $KE_1 > KE_2$.

4.3 A motorcycle is running with constant speed on a horizontal track. Is any work being done on the motorcycle, if no net force is acting on it?

Ans: If a motorcycle is running with constant speed on a horizontal track, and there is no net force acting on it, then no net-work is being done on the motorcycle.

Explanation:

1. **Work-Energy Theorem:** The work-energy theorem states that the net work done on an object is equal to the change in its kinetic energy ($W\_net = \Delta KE$).
2. **Constant Speed:** If the motorcycle is moving at a constant speed, its velocity is constant (assuming constant direction on a straight track). Therefore, its kinetic energy ($\frac{1}{2}mv^2$) is also constant.
3. **No Change in Kinetic Energy:** Since $\Delta KE = 0$ ($KE\_final - KE\_initial = 0$), according to the work-energy theorem, the net work done on the motorcycle must also be zero.
4. **No Net Force:** The statement "no net force is acting on it" directly implies that the acceleration is zero (Newton's Second Law, $F\_net = ma$). Zero acceleration means constant velocity (or rest), which again leads to no change in kinetic energy and thus no net work done.

It's important to distinguish between *net work* and work done by individual forces. If the engine is running to maintain constant speed, it is doing positive work, but this work is precisely balanced by the negative work done by non-conservative forces like air resistance and rolling friction, resulting in zero net-work.

4.4 A force acts on a ball moving with 14 ms^{-1} speed and brings its speed to 6 ms^{-1}. Has the forced one positive or negative work? Explain your answer.

Ans: The force has done negative work.

Explanation:

1. **Work-Energy Theorem:** The work-energy theorem states that the net work done on an object is equal to the change in its kinetic energy ($W_{net} = \Delta KE = KE_{final} - KE_{initial}$).
2. **Change in Speed:**
   - Initial speed ($v_{initial}$) = 14 m s^{-1}
   - Final speed (v_{final}) = 6 m s^{-1}
   - Since the final speed (6 m s^{-1}) is less than the initial speed (14 m s^{-1}), the kinetic energy of the ball has decreased.
3. **Kinetic Energy Calculation:**
   - $\Delta KE = KE\_final - KE_{initial} = \frac{1}{2}m(36) - \frac{1}{2}m(196)$
     $= \frac{1}{2}m(36 - 196) = \frac{1}{2}m(-160) = -80\ m$
4. **Conclusion:** Since the change in kinetic energy (ΔKE) is negative, the net work done on the ball (W_{net}) must also be negative. Negative work indicates that the force acting on the ball opposed its motion, causing it to slow down.

4.5 As low moving truck can have more kinetic energy than a fast moving car. How is this possible?

Ans: This is possible because kinetic energy depends on both mass and the square of velocity ($KE = \frac{1}{2}mv^2$). While the car is "fast moving" and the truck is "slow moving," the truck's significantly larger mass can compensate for its lower speed, resulting in greater kinetic energy.

Explanation:

Let's consider an example:

- **Car:**
  - Mass (m_{car}) = 1000 kg
  - Speed (v_{car}) = 30 m/s (approx. 108 km h^{-1}, quite fast)
  - $KE_{car} = \frac{1}{2}(1000\ kg)(30\ m\ s^{-1})^2 = 500\ kg \times 900\ m^2 s^{-2} = 450{,}000\ J = 450\ kJ$
- **Truck:**
  - Mass (m_{truck}) = 10,000 kg (a typical large truck, 10 times the car's mass)
  - Speed (v_{truck}) = 10 m/s (approx. 36 km h^{-1}, relatively slow)
  - $KE_{truck} = \frac{1}{2}(10{,}000\ kg)(10\ m\ s^{-1})^2 = 5000\ kg \times 100\ m^2 s^{-2} = 500{,}000\ J = 500\ kJ$

In this example, the slow-moving truck (10 m s^{-1}) has more kinetic energy (500 kJ) than the fast-moving car (30 m s^{-1}) because its mass is significantly greater. The kinetic energy's dependence on the square of velocity means that velocity has a stronger impact, but a large enough difference in mass can easily override this.

4.6. Why work done against friction is non-conservative in nature? Explain briefly.

Ans: Work done against friction is non-conservative because:

- **Path Dependent:** The amount of work done against friction varies with the length of the path taken between two points. A longer path means more work.
- **Energy Dissipation:** It converts mechanical energy into unusable forms like heat and sound, which cannot be recovered as mechanical energy. This means that work done against friction over a closed loop is never zero.

4.6 -Does wind contain kinetic energy? Explain.

Ans: Yes, wind absolutely contains kinetic energy.

Explanation:

Wind is essentially moving air. Any object that has mass and is in motion possesses kinetic energy, according to the formula $KE = \frac{1}{2}mv^2$.

Since air has mass (even though it's relatively light per unit volume) and wind is defined by the movement of this air so it carries kinetic energy.

Example: This kinetic energy is used by wind turbines to generate electricity. The blades of the turbine capture the kinetic energy of the moving air, converting it into rotational mechanical energy, which then drives a generator to produce electrical energy.

COMPREHENSIVE QUESTIONS

4.1 Define *K.E.* derive an expression for the same.

4.2 How work is done by a:
(i) constant force
(ii) variable force?

4.3 Define conservative field. Show that gravitational field is conservative in nature.

4.4 What is meant by absolute P.E.? Derive an expression for absolute P.E.

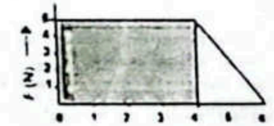
4.5 State and explain work-energy theorem in a resistive medium.

4.6 Define escape velocity. Show that an expression for escape velocity can be expressed $\sqrt{2Rg}$, where R and g denote radius of the Earth and acceleration due to gravity, respectively. Also find its numerical value.

SOLVED EXAMPLES

EXAMPLE 4.1

A force Fading on an object varies with distance das shown in Fig. 4.8. Calculate the work done by the force as the object moves from d = 0 to d = 6 m.



Solution

The work done by the force is equal to the total area under the curve from d = 0to d = 6m. This area is equal to the area of the rectangular section from d = 0 to d = 4 m, plus the area of triangular section from d = 4 m to d= 6m.

Hence

Work done represented by the area of rectangle

$= 4\,m \times 5\,N = 20Nm = 20J$

Work done represented by the area of triangle

$= \frac{1}{2} \times 2\,m \times 5\,N = 5Nm = 5J$

Therefore, the total work done = 20J+5J=25J

EXAMPLE 4.2

A 70 kg man runs up a long flight of stairs in 4.0 s. The vertical height of the stairs is 4.5 m. Calculate his power output in watts.

SOLUTION

Work done = *mgh*

$$Power = \frac{mgh}{t}$$

$$P = \frac{70\,kg \times 9.8\,m\,s^{-2} \times 4.5\,m}{4s}$$

$P = 7.7 \times 10^2\,kg\,m^2\,s^{-3} = 7.7 \times 10^2\,W$

Example 4.3

A car weighing 18620 N is running with a speed of 16 m s^{-1}. Brakes are applied and it is brought to rest in a distance of 80 m. Determine the average force of friction acting on it

Solution

Given that

$v = 16\,m\,s^{-1}$, $d = 80\,m$, $w = 18620\,N$ and $f = ?$

The kinetic energy of the car is equal to the work done by it before stopping, i.e.,

$$\frac{1}{2}mv^2 = fd$$

Here

$$m = \frac{w}{g} = \frac{18620\,N}{9.8\,m\,s^{-2}} = 1900\,kg$$

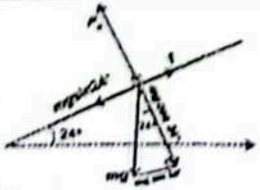
Putting the value in the above equation, we have

$$\frac{1}{2} \times 1900\,kg \times (16\,m\,s^{-1})^2 = 80$$

Or $f = 3040\,N$

EXAMPLE 4.4

A motorcycle rider weighing 60 kg is coasting down a 24° slope. The weight of motorcycle is 30 kg. At the top of the slope, the speed of motorcycle is 3.2 m s^{-1}. If the kinetic frictional force is 100 N, what will be the speed of the motorcycle 72 m downhill?



SOLUTION

The normal force F_N is balanced by the component of weight (mgcos24°) perpendicular to the slope, Let the kinetic frictional force is f, then the net force F is:

$F = mg\sin 24° - f$

Where m = total mass = 60 kg + 30 kg = 90 kg

$F = (90\,kg \times 9.8\,m\,s^{-2} \times 0.4) - 100\,N$

$F = 252.8\,N$

Work done $W = Fd = 252.8\,N \times 72\,m = 18201.6\,J$

As work is positive, so applying work - energy theorem,

$W = (K.E)_f - (K.E)_i$

Form here,

$(K.E)_f = W + (K.E)_i$

Putting the values, we have

$$\frac{1}{2}mv_f^2 = W + \frac{1}{2}mv_i^2$$

$$\frac{1}{2} \times 90\,kg \times v_f^2 = 18201\,J + \frac{1}{2} \times 90\,kg \times (3.2\,m\,s^{-1})^2$$

This gives,

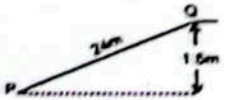
$$v_f^2 = 414.7\,m^2\,s^{-2}$$

or

$$v_f = \sqrt{414.7\,m^2\,s^2} = 20.4\,m\,s^{-1}$$

EXAMPLE 4.5

A car weighing 1100 kg is moving with a velocity of 12 m s^{-1}. When it is at point P, its engine stops. If the frictional force is 120 N, what will be its velocity at point Q? How far beyond Q will it go before coming to rest?



SOLUTION

The kinetic energy possessed by the car at point P will partly be converted into P.E. and partly used up in doing work against friction as it reaches point Q. Therefore, Loss of K.E. = Gain in P.E. + Work against friction

$$\frac{1}{2}m(v_i^2 - v_f^2) = wh + fd$$

$$\frac{1}{2} \times 1100\,kg(144\,m^2\,s^{-2} - v_f^2)$$

$$= (1100\,kg \times 9.8\,m\,s^{-2} \times 1.5\,m) + 120\,N \times 24\,m$$

$$550\,kg(144\,m^2\,s^{-2}) = 16170\,kg\,m^2\,s^{-2} + 2880\,kg\,m^2\,s^{-2}$$

$$(144\,m^2\,s^{-2} - v_f^2) = \frac{16170\,kg\,m^2\,s^{-2} + 2880\,kg\,m^2\,s^{-2}}{550\,kg} = 34.6\,m^2\,s^{-2}$$

Velocity at point Q, $v_f = \sqrt{109\,m^2\,s^{-2}} = 10.5\,m\,s^{-1}$

Now if the car stops at point R, then using the formula

$$\frac{1}{2}mv^2 = fS$$

$$\frac{1}{2} \times 1100\,kg \times 109.4\,m^2\,s^{-2} = 120\,kg\,m\,s^{-2} \times S$$

$S = 501\,m$ approximately

EXAMPLE 4.6

An object of mass 3 kg falls from a height of 15 m. If it strikes the ground with a velocity of 16 m s^{-1}, calculate the average frictional force of the air.

Solution

Loss of P.E=Gain in K.E + Work done against friction

$\because v_i = 0$, $\quad mgh = \frac{1}{2}mv^2 + fh$

$$3kg \times 9.8\,m\,s^{-2} \times 15\,m = \frac{1}{2}3kg(16\,m\,s^{-1})^2 + f \times 15\,m$$

$$441\,kg\,m^2\,s^{-2} = 384\,kg\,m^2\,s^{-2} = 384\,kg\,m^2\,s^{-2} + 15\,m \times f$$

Or

$$f = \frac{441\,kg\,m^2\,s^{-2} - 384\,kg\,m^2\,s^{-2}}{15\,m} = 3.8\,kg\,m\,s^{-2} = 3.8\,N$$

NUMERICAL PROBLEMS

4.1 **A machine gun fires 6 bullets per minute with a velocity of 700ms^{-1}. If each bullet has a mass of 40 g, then find power developed by the gun?**

Given

No. of bullets fire per minutes = n = 6

No. of bullet fire per second = n = 6/60 = 0.1 bullet/s

Velocity of bullet = v = 700 m s^{-1}

Mass of each bullet = m = 40 g = 0.04 kg

To Find

Power develop by gun = p =?

Solution:

Kinetic energy gain by bullet = K.E $= \frac{1}{2}mv^2$

$$= \frac{1}{2}(0.04)(700)^2 = 9800\,J$$

P = n K.E = 0.1(9800) = 980 watt

4.2 **A family uses 10kW of power. Direct solar energy is incident on horizontal surface at an average rate of 300 per square metre. If 75% of this energy can be converted into useful electrical energy, how large area is needed to supply 10kW?**

Given

Power used by family = 10 kW = 10000 W

Energy delivered per unit area = 300 J/m^2

Efficiency = η = 75% = 0.75

To Find

Area = A =?

Solution

Formulas:

$P_{generated} = P_{required}$

$P_{generated}$ = (Incident Solar Power) × η

Incident Solar Power = I × A

Set up the equation for power generated:

$P_{required}$ = (I × A) × η

10,000 W = (300 W/m^2 × A) × 0.75

Simplify the right side:

10,000 W = 225 W/m^2 × A

Solve for Area (A):

A = 10,000 W / (225 W/m^2)

A = 44.44 m^2

4.3 **If mass of the Earth is 6.0×10^{24} kg and mass of Sun is 1.99×10^{30} kg. The sun is 160 million km away from the Earth. Find the value of gravitational P.E of the Earth.**

Given

Mass of Earth = M = 6.0×10^{24} kg

Mass of Sun = m = 1.99×10^{30} kg

Distance of sun from earth = r = 160 million km = 160000000 km = 1.6×10^{11} m

To Find

Gravitational P.E = U = ?

Solution:

$$U = -\frac{GMm}{r}$$

$$U = -\frac{6.673 \times 10^{-11} \times 6 \times 10^{24} \times 1.99 \times 10^{30}}{1.6 \times 10^{11}}$$

$$U = -\frac{6.673 \times 6 \times 1.99 \times 10^{-11+24+30-11}}{1.6}$$

$U = -49.79 \times 10^{32} = -4.97 \times 10^{33}$ J

4.4 **An object weighing 98 N is dropped from a height of 10 m. Its speed just before hitting the ground is 12 m s^{-1}. What is the frictional force acting on it? (Ans: 26 N)**

Given:

Weight (W) = 98 N

Height (h) = 10 m

Initial velocity ($v_{initial}$) = 0 (dropped)

Final velocity (v_{final}) = 12 m s^{-1}