After collision:

Final velocity of first trolley $v_1' = 0$ m s^{-1}

Final velocity of second trolley $v_2' = 1.0$ m s^{-1}

Check for Momentum Conservation:

Initial momentum (P_i) = $m_1v_1 + m_2v_2 = (1.0 \text{ kg})(1.0 \text{ m s}^{-1}) + (1.0 \text{ kg})(0 \text{ m s}^{-1}) = 1.0$ kg m s^{-1}

Final momentum (P_f) = $m_1v_1' + m_2v_2' = (1.0 \text{ kg})(0 \text{ m s}^{-1}) + (1.0 \text{ kg})(1.0 \text{ m s}^{-1}) = 1.0$ kg m s^{-1}

Since $P_i = P_f$, momentum is conserved

Check for Kinetic Energy Conservation:

Initial Kinetic Energy ($K.E_i$) = $(1/2)m_1v_1^2 + (1/2)m_2v_2^2 = (1/2)(1.0)(1.0)^2 + (1/2)(1.0)(0)^2 = 0.5$ J

Final Kinetic Energy ($K.E_f$) = $(1/2)m_1v_1'^2 + (1/2)m_2v_2'^2 = (1/2)(1.0)(0)^2 + (1/2)(1.0)(1.0)^2 = 0.5$ J

Since $K.E_i = K.E_f$, kinetic energy is conserved

Because both momentum and kinetic energy are conserved, this is an example of an elastic collision.

(ii) Inelastic Collision Scenario:

After collision:

The trolleys stick together and move with a common final velocity $v_f = 0.5$ m s^{-1}.

The combined mass $M = m_1 + m_2 = 1.0$ kg $+ 1.0$ kg $= 2.0$ kg

Check for Momentum Conservation:

Initial momentum (P_i) = $m_1v_1 + m_2v_2 = (1.0 \text{ kg})(1.0 \text{ m s}^{-1}) + (1.0 \text{ kg})(0 \text{ m s}^{-1}) = 1.0$ kg m s^{-1}

Final momentum (P_f) = $Mv_f = (2.0 \text{ kg})(0.5 \text{ m s}^{-1}) = 1.0$ kg m s^{-1}

Since $P_i = P_f$, momentum is conserved

Check for Kinetic Energy Conservation:

Initial Kinetic Energy ($K.E_i$) = $(1/2)m_1v_1^2 + (1/2)m_2v_2^2 = (1/2)(1.0)(1.0)^2 + (1/2)(1.0)(0)^2 = 0.5$ J

Final Kinetic Energy ($K.E_f$) = $(1/2)Mv_f^2 = (1/2)(2.0 \text{ kg})(0.5 \text{ m s}^{-1})^2 = (1/2)(2.0)(0.25) = 0.25$ J

Since $K.E_i \neq K.E_f$ (0.5 J ≠ 0.25 J), kinetic energy is not conserved

Because momentum is conserved but kinetic energy is not, this is an example of an inelastic collision.

2. A railway wagon of mass 4×10^4 kg moving with velocity of 3 m s^{-1} collides with another wagon of mass 2×10^4 kg which is at rest. They stick together and move off together. Find their combined velocity. (Ans: 2 m s^{-1})

Solution:

This is an inelastic collision where the wagons stick together.

Given:

Mass of first wagon $m_1 = 4\times10^4$ kg

Initial velocity of first wagon $v_1 = 3$ m s^{-1}

Mass of second wagon $m_2 = 2\times10^4$ kg

Initial velocity of second wagon $v_2 = 0$ m s^{-1} (at rest)

Let v_f be their combined velocity after sticking together.

The total mass after collision $M = m_1 + m_2 = 4\times10^4$ kg $+ 2\times10^4$ kg $= 6\times10^4$ kg

Using the law of conservation of momentum:

Initial momentum = Final momentum

$m_1v_1 + m_2v_2 = (m_1 + m_2)v_f$

$(4\times10^4 \text{ kg})(3 \text{ m s}^{-1}) + (2\times10^4 \text{ kg})(0 \text{ m s}^{-1}) = (6\times10^4 \text{ kg})v_f$

12×10^4 kg m $s^{-1} + 0 = (6\times10^4 \text{ kg})v_f$

$12\times10^4 = (6\times10^4)v_f$

$v_f = (12\times10^4)/(6\times10^4)$ m s^{-1}

$v_f = 2$ m s^{-1}

3. A car with mass 575 kg moving at 15.0 m s^{-1} smashes into the rear end of a car with mass 1575 kg moving at 5 m s^{-1} in the same direction.

(a) What is the final velocity if the wrecked car lock together?

(b) How much kinetic energy is lost in the collision?

(Ans: (a) 7.67 m s^{-1}, (b) 2.11×10^4 J)

Solution:

This is an inelastic collision where the cars lock together.

Given:

Mass of first car $m_1 = 575$ kg

Initial velocity of first car $v_1 = 15.0$ m s^{-1}

Mass of second car $m_2 = 1575$ kg

Initial velocity of second car $v_2 = 5$ m s^{-1} (in the same direction as v_1)

(a) What is the final velocity if the wrecked car lock together?

Let v_f be the combined final velocity

Total mass $M = m_1 + m_2 = 575$ kg $+ 1575$ kg $= 2150$ kg

Using the law of conservation of momentum:

$m_1v_1 + m_2v_2 = (m_1 + m_2)v_f$

$(575 \text{ kg})(15.0 \text{ m s}^{-1}) + (1575 \text{ kg})(5 \text{ m/s}) = (2150 \text{ kg})\, v_f$

8625 kg m $s^{-1} + 7875$ kg m $s^{-1} = 2150\, v_f$

16500 kg m $s^{-1} = 2150\, v_f$

$v_f = 16500/2150$ m s^{-1}

$v_f = 7.674$ m $s^{-1} \approx 7.67$ m s^{-1}

(b) How much kinetic energy is lost in the collision?

Initial Kinetic Energy ($K.E_i$):

$K.E_i = (1/2)m_1v_1^2 + (1/2)m_2v_2^2$

$K.E_i = (1/2)(575)(15.0)^2 + (1/2)(1575)(5)^2$

$K.E_i = (1/2)(575)(225) + (1/2)(1575)(25)$

$K.E_i = 64687.5 \text{ J} + 19687.5$ J

$K.E_i = 84375$ J

Final Kinetic Energy ($K.E_f$):

$K.E_f = (1/2)(m_1 + m_2)v_f^2$

$K.E_f = (1/2)(2150)(7.6744186)^2$ (Using more precise v_f to minimize rounding error for KE loss)

$K.E_f = (1/2)(2150)(58.8957)$ J

$K.E_f = 63307.3$ J

Kinetic energy lost ($\Delta K.E$) = $K.E_i - K.E_f$

$\Delta K.E = 84375 \text{ J} - 63307.3$ J

$\Delta K.E = 21067.7$ J

$\Delta K.E = 2.11\times10^4$ J

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CHAPTER 3

CIRCULAR MOTION

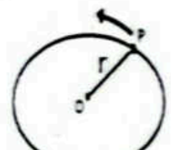
STUDENT LEARNING OBJECTIVES

After studying this chapter, students will be able to:

- Express angles in radians.
- Define and calculate angular displacement, angular velocity and angular acceleration. (This involves use of $S = r\theta$, $v = r\omega$, $a = r\alpha$, $\omega = v/r$, and $\alpha = a/r$, to solve problems)
- Use equations of angular motion to solve problems involving rotational motions.
- Discuss qualitatively motion in a curved path due to a perpendicular force.
- Define and calculate centripetal force. [Use: $F_c = mv^2/r$, $a = v^2/r$]
- Analyze situations involving circular motion in terms of centripetal force (e.g., situations in which centripetal acceleration is caused by a tension force, a frictional force, a gravitational force, or a normal force).
- Define and calculate average orbital speed for a satellite, from the equation $v = 2\pi r/T$ (where r is the average radius of the orbit and T is the orbit period); apply this equation to solve numerical problems).
- Explain why the objects in orbiting satellites appear to be weightless.
- Describe how artificial gravity is created to counter weightlessness.
- Define and calculate moments of inertia of a body and angular momentum.
- Derive and apply the relation between torque, moment of inertia and angular acceleration. Illustrate the applications of conservation of angular momentum in real life. (such as by flywheels to store rotational energy, by gyroscopes in navigation systems, by ice skaters to adjust their angular velocity).
- Describe how a centrifuge is used to separate materials using centripetal force.

Among all possible motions of the material bodies, the circular motion is one that appears to be working in the most of the natural world. Satellites moving in circular orbits around the Earth, orbital and spin motion of the Earth itself, a car turning around a curved road, and a stone whirled around by a string are the familiar examples. When objects move in circular paths, their direction is continuously changing. Since velocity is a vector quantity, this change of direction means that their velocities are not constant.

CIRCULAR MOTION:



"The motion of an object along circumference of a circle is called circular motion."

In circular motion, the direction of motion of body continuously changes.

For Examples:

- a stone whirled around by a string,
- a car turning around a corner
- a satellite in an orbit around the Earth executes the circular motion.

Axis of Rotation:

All the part of body in rotation performs circular motion about fixed centers. The line joining their centers is called axis of rotation.

3.1 ANGULAR MRASUREMENTS

Q **Define angular displacement, determine its direction. Give different units of angular displacement.**

Ans

ANGULAR DISPLACEMENT:

"*Change in **angular position** of rotating or circulating body is called **angular displacement**.*" It is the angle swept by the radius of a rotating body at the center of a circle with the reference axis.

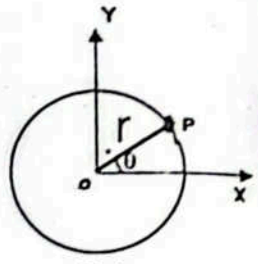


Fig.1

- It is denoted by θ.
- In SI units, angular displacement is expressed in radian (rad).
- Angular displacement is dimensionless quantity.
- The angular displacement Δθ is positive in counter clockwise(anticlockwise) motion
- Angular displacement is negative in clockwise motion
- For very small value of Δθ, the angular displacement is a vector quantity
- The direction of the Δθ is obtained by the right-hand rule
- It is considered vector quantity if $\theta < 10^{o}$

Units of Angular Displacement:

The units of angular displacement which are commonly used are,

i. degree ii. revolution iii. radian

Right Hand Rule:

According to right hand rule, the right-hand fingers are curled in the direction of rotation and the erect thumb is pointed in the direction of angular displacement. If the rotation is in the xy-plane then by the right hand rule, the direction of angular displacement is along the axis of rotation. i.e Z-axis as shown in Figure.

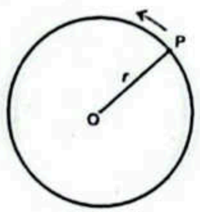


Fig. 3.2(a)

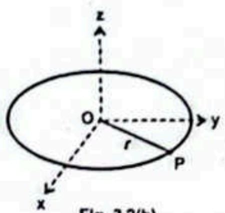


Fig. 3.2(b)

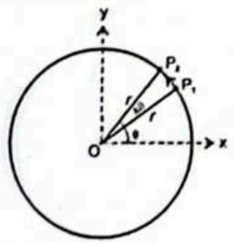


Fig. 3.2(c)

Angle Δθ defines the angular displacement of OP during the time interval Δt. For very small values of Δθ, the angular displacement is a vector quantity.

Q **Define radian, show that 1 rad = 57.3°**
1 revolution = 2π rad = 360°

Ans

DEGREE:

"*The **1/360th** part of **angular displacement** of a body in **complete round trip** along circular path is one degree.*"

RADIAN:

"*One **radian** is the angle subtended at the center of a circle by an **arc length equal to the radius** of the circle.*"

The various conversions are:

$360^\circ = 2\pi$ rad $= 6.28$ rad

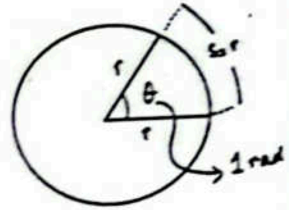
$180^\circ = \pi$ rad $= 3.14$ rad

$90^\circ = \frac{\pi}{2}$ rad $= 1.57$ rad

$60^\circ = \frac{\pi}{3}$ rad $= 1.05$ rad

$45^\circ = \frac{\pi}{4}$ rad $= 0.79$ rad

$1^\circ = \frac{\pi}{180}$ rad $= 0.01745$ rad $= 1.745 \times 10^{-2}$ rad



Answer: Relation between radian and degree:

As, 2π rad $= 360^\circ$

$$1\text{rad} = \frac{360^\circ}{2\pi}$$

$$1\text{rad} = \frac{360^\circ}{2\times 3.14}$$

So, 1 rad = 57.3°

Or $1^\circ = \frac{\pi}{180}$ rad $= 0.01745$ rad $= 1.745 \times 10^{-2}$ rad

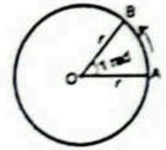


Fig. 3.1

Relation between Linear Displacement and Angular Displacement:

*The **ratio** of **arc length to the radius** of arc is the **angular displacement** in radians.*

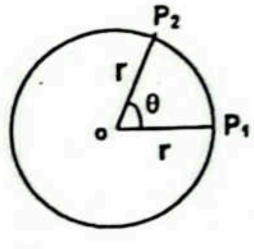
Consider an arc P_1P_2 has radius r and subtends an angle θ. Its arc length is S.

According to the above statement,

$$\frac{\text{Arc.length}}{\text{radius}}(\text{rad}) = \theta$$

Or, $\theta = \frac{S}{r}$ (rad)

$S = r\theta$ ---------- (i)



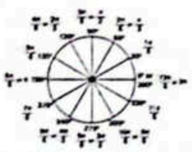
For one complete revolution, arc length is $2\pi r$ (length of circumference), so

$$\frac{S}{r} = \frac{2\pi r}{r} = 2\pi \text{ rad}$$

Thus, **1 revolution = 2π rad = 360°**

Note: Angular displacement is dimensionless quantity.

For Your Information



Radians $= \left(\frac{\pi}{180^\circ}\right) \times$ degrees

Degrees $= \left(\frac{180^\circ}{\pi}\right) \times$ radians

MULTIPLE CHOICE QUESTIONS

- A body is moving with uniform speed along a circular track what will be the ratio of its distance to displacement when it has covered $\frac{1}{4}$th part of its time period?

  (a) $\frac{\pi}{\sqrt{2}}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2\sqrt{2}}$

Explanation: Answer is (d) $\frac{\pi}{2\sqrt{2}}$ Since in quarter circle, distance is $2\pi r/4 = \pi r/2$ and displacement is $\sqrt{r^2+r^2} = \sqrt{2r^2} = \sqrt{2}r$. Hence ratio of distance to displacement in quarter circle is $(\pi r/2)/(\sqrt{2}r) = \pi/2\sqrt{2}$

- **A merry-go-round rotates 45 degrees counter-clockwise. If it continues rotating in the same direction, what is the angular displacement after a further 90-degree rotation?**

  (a) 45 degrees counter-clockwise (b) 135 degrees counter-clockwise
  (c) 45 degrees clockwise (d) 90 degrees clockwise

Explanation: Angular displacement is the total angle traced out by an object in rotation. Since the merry-go-round continues in the same direction (counter-clockwise), we add the angles. 45 degrees + 90 degrees = 135 degrees counter-clockwise

- **A clock hand sweeps across an angle of 60 degrees every hour. What is the angular displacement of the hour hand after 2 hours?**

  (a) 360 degrees (b) 120 degrees (c) 720 degrees (d) 60 degree

  Explanation:
  - The hour hand rotates a full circle (360 degrees) every 12 hours.
  - Therefore, it moves 360 degrees / 12 hours = 30 degrees per hour
  - In 2 hours, it displaces 2 hours x 30 degrees/hour = 60 degrees

- **A fan blade rotates back and forth. If the blade moves 30 degrees counter-clockwise followed by 30 degrees clockwise, what is the total angular displacement?**

  (a) 0 degrees (b) 30 degrees counter-clockwise (c) 30 degrees clockwise (d) 60 degrees

  Explanation: Angular displacement considers the net change in angle. Since the fan blade moves in opposite directions with the same angle, the total displacement is zero (30 degrees counter-clockwise cancels out the 30 degrees clockwise).

- **A wheel rotates 10 times. If in one complete rotation, the wheel traces an angle of 360 degrees, what is the total angular displacement of the wheel?**

  (a) 360 degrees (b) 10 rotations (c) 3600 degrees (d) none of these

Solution: Since the wheel rotates 10 times, and each rotation covers 360 degrees, the total angular displacement is 10 rotations x 360 degrees/rotation = 3600 degrees. It should be noted that 10 rotations are incorrect because - rotation is not a measure of angle.

- **When objects move in circular paths, their velocity is not constant because:**

  (a) Their speed is changing. (b) Their direction is continuously changing
  (c) Their mass is changing. (d) They are at rest.

Answer: (b) Their direction is continuously changing

Explanation: Velocity is a vector, so a change in direction, even at constant speed, means the velocity is not constant.

- **A car turning around a curved road is cited as an example of:**

  (a) Linear motion. (b) Circular motion. (c) Oscillatory motion. (d) Translational motion only.

Answer: (b) Circular motion

Explanation: Turning on a curved road involves motion along an arc of a circle.

- **What key aspect of motion is continuously changing for objects in circular paths?**

  (a) Speed. (b) Magnitude of velocity. (c) Direction of velocity. (d) Kinetic energy.

Answer: (c) Direction of velocity

Explanation: The continuous change in direction is the defining characteristic of circular motion when speed is constant.

- **For angular displacement to be considered a vector quantity, it must be:**

  (a) A large angle. (b) An angle in degrees. (c) A very small angle. (d) Always positive

Answer: (c) A very small angle.

Explanation: Only infinitesimally small angular displacements are considered vector quantities because large angular displacements do not obey vector addition rules (commutativity).

- **Which of these units is NOT used to express angular displacement?**

  (a) Degrees (b) Revolutions (c) Joules. (d) Radians.

Answer: (c) Joules.

Explanation: Joules (J) is a unit of energy, not angular displacement.

- **A particle moves in a circle of radius r. In half the period of revolution, its displacement and distance covered are:**

  (a) $2r, 2\pi r$ (b) $r\sqrt{2}, \pi r$ (c) $2r, \pi r$ (d) $r, \pi r$

Solution: In a period of half of revolution, the particle will cover half of the circumference as shown below. So best option is "C"

- **A particle moves in a circle of radius 25cm at two revolutions per second. The acceleration of the particle in ms^{-2} is:**

  (a) π^2 (b) $8\pi^2$ (c) $4\pi^2$ (d) $2\pi^2$

Solution:
$$r = 25cm = 0.25m$$
$$\omega = 2rev s^{-1} = 2\times 2\pi\, rad\, s^{-1} = 4\pi\, rad\, s^{-1}$$
$$a_c = r\omega^2 = 0.25\times(4\pi)^2 = 0.25\times 16\pi^2$$
$$a_c = 4\pi^2$$

- **A rigid body of moment of inertia I has an angular acceleration α. If the power supplied to the body is P, its instantaneous angular velocity is**

  (a) $P I\alpha$ (b) $P/I\alpha$ (c) PI/α (d) $P\alpha/I$

Solution: $P = Fv \rightarrow$ For translational motion

When transformed in angular form we get
$$P = \tau\omega$$
$$P = I\alpha\omega$$
$$\alpha = \frac{P}{I\omega} \Rightarrow \omega = \frac{P}{I\alpha}$$

SLO BASED SHORT QUESTIONS & ANSWERS

- **Define angular displacement and its unit.**

Ans: The angle subtended by circular moving object at center of circle is called angular displacement. It is a vector quantity for small angle and its direction can be found by using right hand rule

It is unit less physical quantity

- **Show that $S = r\theta$ where S = Arc length, r = radius of the circle, θ= angle in radian.**

Ans: Suppose an object is moving in circle from point A to point B by subtending an angle θ at center of circle. The angle subtended by an arc(s) is found as
$$\theta = \frac{\text{arclength}}{\text{radius}} = \frac{s}{r}$$
$$\theta = \frac{s}{r}$$
$$s = r\theta$$

- **Prove that 2 radian = 114.6°**

Ans: As we know that

1 rad=57.3°

Then

2 rad. = 2× 57.3°

2 rad. =114.6°

- **How many radians are there in 2 degree?**

Ans: As we know that

$57.3^\circ = 1$ rad

Dividing by 57.3° on both sides then we get

$1^\circ = 0.0174$ rad

$2^\circ = 0.0349$ rad

- **How many radians are there in one revolution?**

Ans: There are 2π radians in one revolution.

- **What unit of angular displacement is generally used besides degrees and revolutions?**

Ans: Besides degrees and revolutions, radian is generally used to express angular displacement.

- **If an object completes half a revolution, what is its angular displacement in radians?**

Ans: Its angular displacement is π radians.

ANGULAR VELOCITY

Q **Define angular velocity, average angular velocity and instantaneous velocity. Give unit and dimension.**

Ans

ANGULAR VELOCITY:

"It is the time rate of change of angular position of an object." It is denoted by $\vec{\omega}$ (magnitude by ω). If a circulating particle sweeps angular displacement θ in time interval t, then mathematically it is written as,

$$\vec{\omega} = \frac{\vec{\theta}}{t}$$

- The units of angular velocity in SI are rad s^{-1}.
- The other units of angular velocity as degree s^{-1} and revolution s^{-1} are also used.
- Sometime revolution per minute is used.
- The dimension of Angular velocity is $[T^{-1}]$
- For small angular displacements it is a vector quantity
- Its direction is determined by right hand rule.

Right hand rule

The direction of angular velocity is determined by right hand rule. According to right hand rule, *if we hold the axis with our right hand and rotate the fingers in the direction of motion of the rotating body then erect thumb will point the direction of the angular velocity.*

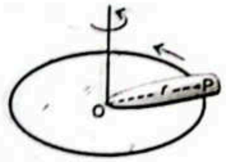


Fig. 3.4

Average Angular Velocity:

"It is the total angular displacement divided by total time taken." If $\Delta\theta$ is the change in angular displacement in the time interval Δt then average angular velocity may be expressed as:

$$\vec{\omega}_{av} = \frac{\Delta\vec{\theta}}{\Delta t} \quad \text{---------- (i)}$$

Instantaneous Angular Velocity:

It is the rate of angular displacement at any particular instant of time interval. It may be expressed in mathematical form as:

$$\vec{\omega} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\theta}}{\Delta t} \quad \text{---------- (ii)}$$

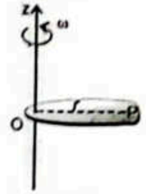


Fig. 3.5(a)

ROTATION OF A RIGID BODY

Till now we have been considering the motion of a particle on a circular path. The point was fixed at the end of a rotating massless rigid rod. Now consider the rotation of a rigid body as shown in Fig. 3.4. Imagine a point on the rigid body. The axis of rotation or reference axis is perpendicular to the plane of rigid body. Take a reference line on the rigid body. As the body rotates, line on the body also rotates with the same angular velocity and angular acceleration. Thus, the rotation of a rigid body can be described by the rotation of the reference line. Henceforth, while dealing with rotation of a rigid body, we take its motion by reference line.

Important Points Rotation of a Rigid Body:

- For a rigid body, consider a point P and a reference line OP.
- As the body rotates, OP rotates with the same angular velocity and angular acceleration.
- Rigid body rotation can be described by the rotation of the reference line OP.
- Terms defined using rotating line OP are valid for rigid body rotational motion.
- Rigid body rotation can be represented by its reference line OP.

MULTIPLE CHOICE QUESTIONS

- **Which equation is used to calculate angular velocity?**

(a) $v = r\alpha$ (b) $v = r\theta$ (c) $v = r\omega$ (d) $v = ra$

Answer: (c) $v = r\omega$

Explanation: Angular velocity (ω) is related to linear velocity (v) and radius (r) by the equation $v = r\omega$.

- **Which of the following is a vector quantity that changes in circular motion?**

(a) Speed (b) Velocity (c) Distance (d) Time

Answer: (b) Velocity

Explanation: Velocity is a vector quantity, and in circular motion, its direction changes continuously, making it non-constant.

- **What is the angular velocity of the hour hand of a clock?**

(a) 7.27×10^{-5} rad s^{-1} (b) 1.75×10^{-3} rad s^{-1} (c) 2.90×10^{-4} rad s^{-1} **(d) 1.45×10^{-4} rad s^{-1}**

Answer: (d) 1.45×10^{-4} rad s^{-1}

Explanation: $\omega = \frac{2\pi}{T} = \frac{2\pi}{12}\times\frac{1}{3600} = 1.45\times10^{-4}\,\text{rad s}^{-1}$

SLO BASED SHORT QUESTIONS & ANSWERS

- **Do all points on a rotating rigid body have the same linear speed and acceleration?**

Ans: No, points at different distances from the axis of rotation have different linear speeds and accelerations. Since $v = r\omega$ and $a = r\alpha$

- **What quantities are the same for all points on a rigid body rotating about a fixed axis?**

Ans: Angular displacement (θ), angular speed (ω), and angular acceleration (α).

- **State the angular motion equations that are analogous to the linear motion equations.**

Ans:

- $\omega = \omega_0 + \alpha t$ (analogous to $v = v_0 + at$)
- $2\alpha\theta = \omega^2 - \omega_0^2$ (analogous to $2aS = v^2 - v_0^2$)
- $\theta = \omega_0 t + 1/2\,\alpha t^2$ (analogous to $S = v_0 t + 1/2\,at^2$)

- **What is the relationship between arc length and the angle subtended at the center of a circle?**

Ans: The arc length (S) is equal to the product of the radius (r) and the angle (θ) in radians: $S = r\theta$.

- **What is the difference between average and instantaneous angular velocity?**

Ans: Average angular velocity is the angular displacement over a time interval ($\Delta\theta/\Delta t$), while instantaneous angular velocity is the limit of this ratio as the time interval approaches zero.

- **Is angular velocity a vector quantity? If so, what is its direction?**

Ans: Yes, angular velocity is a vector quantity. Its direction is along the axis of rotation, given by the right-hand rule.

- A Ferris wheel rotates at a constant speed. If a seat at the outer edge completes one full rotation (2π radians), what's the angular velocity for a seat halfway between the center and the edge?

Ans: The angular velocity is the same for both seats. **Reason** As long as they share the same axis of rotation (the center pole of the Ferris wheel), their angular velocities will be identical, regardless of distance from the center.

- **A car tyre spins at a constant angular velocity. If the tyre completes 10 rotations in 2 seconds, what is the angular velocity of the tyre?**

Ans: The angular velocity is 10π radians per second.

Reason

1. We know the number of rotations (10) and the time (2 seconds).
2. We want to find the angular velocity (ω) in radians per second.
3. Angular velocity = total angle covered (in radians) / time taken (in seconds).
4. Since the tyre completes 10 full rotations, the total angle covered is 10 rotations × 2π radians/rotation = 20π radians.
5. Therefore, the angular velocity (ω) = 20π radians / 2 seconds = 10π radians per second.

- **What are banked track? Explain briefly.**

Ans: Banked tracks are the types of racing tracks which are designed as the turns are raised or "banked" to allow vehicles to maintain higher speed through the turns. This design enables drivers to take turns at faster speeds and reduces the need for braking. The banking allows drivers to generate more speed and maintain control while taking turns.

ANGULAR ACCELERATION

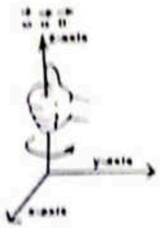
Q Define angular acceleration, average and instantaneous angular acceleration.

Ans

ANGULAR ACCELARATION:

*"It is the time **rate of change** of **angular velocity** and is denoted by $\vec{\alpha}$ (magnitude by α)."*

- The unit of angular acceleration in SI is rad s^{-2}.
- Also rev s^{-2} and deg s^{-2} are used for angular acceleration.
- Angular acceleration is a vector quantity.
- Its direction is found by the right hand rule and is always along the axis of rotation.
- The dimension of angular acceleration is $[T^{-2}]$.



Average Angular Acceleration:

*It is the ratio of **change in angular velocity** to **time interval** for this change.* If a rotating body has initial angular velocity $\vec{\omega}_i$ at t_i and final angular velocity of $\vec{\omega}_f$ at t_f the average angular acceleration is expressed as:

$$\vec{\alpha}_{av} = \frac{\vec{\omega}_f - \vec{\omega}_i}{t_f - t_i} = \frac{\Delta\vec{\omega}}{\Delta t} \quad \text{.............. (i)}$$

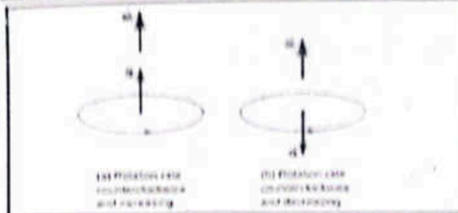
Instantaneous Angular Acceleration:

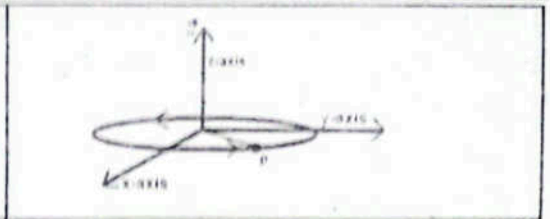
"It is the rate of change of angular velocity at any particular instant of time interval." Mathematically, it is expressed as:

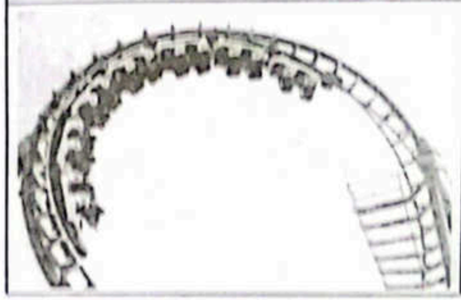
$$\vec{\alpha} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\omega}}{\Delta t} \quad \text{.............. (ii)}$$

Sr. No	U.C.M	Non-U.C.M
i	Circular motion with constant angular speed is known as uniform circular motion	Circular motion with variable angular speed is called as non-uniform circular motion
ii	For U.C.M, $\alpha = 0$	For non U.C.M, $\alpha \neq 0$
iii	In U.C.M work done by tangential force is zero	In non-U.C.M work done by tangential force is not zero
iv	Example: Motion of the earth around the sun	Example: Motion of a body on vertical circle

Note: If angular velocity is increasing then angular acceleration will be parallel to angular velocity, otherwise both will be antiparallel.







Think to Ponder

You may feel scared at the top of roller coaster ride in the amusement parks but you never fall down even when you are upside down. Why?

Answer: This happens due to centripetal force and the speed of the roller coaster. When you are upside down at the top of the loop, gravity pulls you downwards, but the centripetal force required to keep you moving in a circle is provided by the normal force from the track. If the speed is high enough, the required centripetal force is greater than your weight, ensuring the normal force remains positive and you stay pressed against the track, not falling.

MULTIPLE CHOICE QUESTIONS

- The Instantaneous angular acceleration (α) is the limit of the ratio $\Delta\omega/\Delta t$ as Δt approaches:

(a) Infinity (b) One (c) Zero (d) Maximum value

Answer: (c) Zero.

Explanation: Instantaneous angular acceleration is the derivative of angular velocity with respect to time, involving a very small time interval.

SLO BASED SHORT QUESTIONS & ANSWERS

- **What happens to an object's angular velocity if it has angular acceleration?**

Ans: If an object has angular acceleration, its angular velocity will change (increase or decrease).

- **What is the direction of angular acceleration if angular velocity is increasing?**

Ans: If angular velocity is increasing, the angular acceleration vector points in the same direction as the angular velocity.

- **What does it mean if an object has zero angular acceleration?**

Ans: If an object has zero angular acceleration, its angular velocity is constant.

RELATION BETWEEN LINEAR AND ANGULAR VELOCITY

Q Give relation between angular and linear velocities. Or Show that $v = r\omega$

Ans

RELATION BETWEEN ANGULAR AND LINEAR VELOCITIES:

Consider a point 'P' moving in a circle of radius r and Let it is at 'P_1' and after an interval 'Δt' it is at 'P_2'. The arc length between these two points is 'ΔS' and the angle swept by 'OP' for this arc length is '$\Delta\theta$'. The relation between arc length and angular displacement is:

$$S = r\,\theta$$

Or

$$\Delta S = r\,\Delta\theta \quad \text{.............. (i)}$$

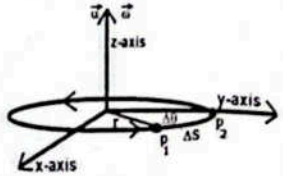
dividing both sides by Δt,

$$\frac{\Delta S}{\Delta t} = r\frac{\Delta \theta}{\Delta t} \quad \text{------------ (ii)}$$

For the time interval approaches to zero, above equation is:

$$\lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t} = \lim_{\Delta t \to 0} r\frac{\Delta \theta}{\Delta t} \quad \text{------------ (iii)}$$

Since $v = \lim_{\Delta t \to 0} \frac{\Delta S}{\Delta t}$ and $\omega = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t}$



Therefore, the eq (iii) is;

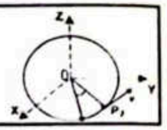
$$v = r\omega$$

In vector form, it may be written as:

$$\vec{v} = \vec{\omega} \times \vec{r} \quad \text{------------ (iv)}$$

The direction of velocity v at any point P is always along tangent to the circular path. So the linear velocity at point P is called tangential velocity.

TANGENTIAL VELOCITY:
The velocity with which object is moving on circumference of circle is called tangential velocity because the direction of motion of body is along tangent to circular path when $\Delta t \to 0$



Q.6 Give relation between angular and linear acceleration. Or Prove that a = r α

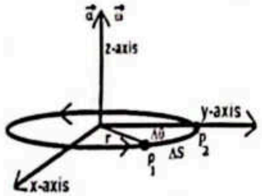
Ans

RELATION BETWEEN ANGULAR AND LINEAR ACCELERATION:

Consider a body of mass 'm' is rotating in a circle of radius 'r' with angular velocity 'ω'. Its linear velocity is 'v'. The relation between angular and linear velocities is;

$$v = r\omega$$

or $\quad \Delta v = r\,\Delta\omega$



Dividing the above eq. By time interval Δt;

$$\frac{\Delta v}{\Delta t} = r\frac{\Delta \omega}{\Delta t}$$

When the time interval is small then,

$$\lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \lim_{\Delta t \to 0} r\frac{\Delta \omega}{\Delta t} \qquad \text{(ii)}$$

Since,

$$\lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \text{linear or tangential acceleration} = a$$

And,

$$\lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \text{Angular acceleration} = \alpha$$

Therefore, eq.(ii), is;

$a = r\alpha$

In vector form, it is;

$$\vec{a} = \vec{\alpha} \times \vec{r}$$

Relations between Linear and Angular variables

$S = r\theta \quad v = r\omega \quad a = r\alpha$

These equations express arc length S, velocity v, and acceleration a, depends on the radius of the circulating points on a body. That is the points on a rotating body at different distances from the axis of rotation do not have the same speed and acceleration but all the points on a rotating body have the same angular displacement, speed and acceleration at any instant.

Q.7 Compare equation of linear and angular motion.

Ans

EQUATIONS OF ANGULAR MOTION:

The rectilinear equations of motions may be transformed into angular equations of motion if linear displacement 'S' is replaced by angular displacement 'θ', linear velocity and acceleration 'a' are replaced by angular velocity 'ω' and acceleration 'α' respectively. These are given below.

Linear Equations Of Motion	Angular Equation Of Motion
$s = vt$	$\theta = \omega t \longrightarrow (i)$
$v_f = v_i + at$	$\omega_f = \omega_i + \alpha t \longrightarrow (ii)$
$S = v_i t + \frac{1}{2}at^2$	$\theta = \omega_i t + \frac{1}{2}\alpha t^2 \longrightarrow (iii)$
$2as = v_f^2 - v_i^2$	$2\alpha\theta = \omega_f^2 - \omega_i^2 \longrightarrow (iv)$
$v_{av} = \frac{v_i + v_f}{2}$	$\omega_{av} = \frac{\omega_i + \omega_f}{2} \longrightarrow (v)$

The angular equations of motion are applicable only when all the angular vectors are in the same direction along the fixed axis of rotation. In this case all the angular vectors are manipulated as scalars.

MULTIPLE CHOICE QUESTIONS

- **If a rigid body is rotating with angular acceleration α, the linear acceleration a of a point on the body is given by:**
  (a) a = rα (b) a = α/r (c) a = rα² (d) a = α²r
  Answer: (a) a = rα
  Explanation: This equation relates linear acceleration to angular acceleration.
- **For a rigid body rotating about a fixed axis, which of the following is the same for all points on the body?**
  (a) Linear speed (b) Linear acceleration (c) Angular speed (d) Tangential velocity
  Answer : (c) Angular speed
  Explanation: All points on a rigid body have the same angular speed and angular acceleration
- **A body rotates with uniform speed in a circle of radius r and takes time T to complete onerevolution. What are the magnitudes of the angular velocity , the linear velocity v andthe acceleration a?**

	Angular velocity,ω	Linear velocity, v	Acceleration, a
(a)	$\frac{1}{T}$	$\frac{4\pi r}{T}$	$\frac{2\pi r}{T^2}$
(b)	$\frac{2\pi}{T}$	$\frac{2\pi r}{T}$	$\frac{2\pi r}{T^2}$
(c)	$\frac{2\pi}{T}$	$\frac{2\pi r}{T}$	$\frac{4\pi^2 r}{T^2}$
(d)	$\frac{2\pi}{T}$	$\frac{4\pi r}{T}$	$\frac{4\pi^2 r}{T^2}$

Solution:

Angular velocity $= \omega = \frac{2\pi}{T}$

Linear velocity $= V = r\omega = r \times \frac{2\pi r}{T}$

Acceleration $a_c = \frac{V_2}{r} = \frac{(r\omega)^2}{r} = \frac{r^2\omega^2}{r} = r\omega^2$

$$a = r \times \left(\frac{2\pi}{T}\right)^2 = r \times \frac{4\pi^2}{T^2}$$

$$a = \frac{4\pi^2 r}{T^2}$$

- **For a rigid body rotating about a fixed axis, all points on the body have the same:**
  (a) Linear speed (b) Linear acceleration (c) Angular speed (d) Tangential velocity

Answer: (c) Angular speed

Explanation: While linear speed varies with distance from the axis, all points on a rigid body rotate through the same angle in the same time, thus having the same angular speed.

- **The linear velocity of a point on a rotating body is also known as:**
  (a) Radial velocity (b) Centripetal velocity (c) Tangential velocity (d) Rotational velocity

Answer: (c) Tangential velocity

Explanation: The direction of linear velocity for a point in circular motion is always tangent to the circular path

- **If the axis of rotation is fixed, the angular velocity can be manipulated as a scalar because:**
  (a) Its magnitude is constant (b) Its direction always remains the same (c) It is a dimensionless quantity (d) It has no effect on linear motion

Answer: (b) Its direction always remains the same

Explanation: When the axis is fixed, the direction of the angular velocity vector does not change, allowing it to be treated as a scalar in calculations involving only its magnitude

- **Which statement is true for points at different distances from the axis on a rotating body?**
  (a) They have the same linear speed (b) They have the same tangential acceleration (c) They have the same angular displacement (d) They have different angular speeds

Answer: (c) They have the same angular displacement

Explanation: All points on a rigid body rotating about a fixed axis undergo the same angular displacement, angular speed, and angular acceleration

- **The angular equations of motion hold true only when:**
  (a) The angular velocity is constant (b) The axis of rotation is fixed (c) There is no angular acceleration (d) Linear and angular motions are independent

Answer: (b) The axis of rotation is fixed

Explanation: A fixed axis ensures that all angular vectors have the same direction, allowing them to be manipulated as scalars

- **If an electric fan rotating at 3 rev s^{-1} comes to rest in 18.0 s with uniform deceleration, its final angular velocity is:**
  (a) 3 rev s^{-1} (b) 0 rev s^{-1} (c) -0.167 rev s^{-2} (d) 27 rev

Answer: (b) 0 rev s^{-1}

Explanation: "Comes to rest" indicates that the final angular velocity is zero

SLO BASED SHORT QUESTIONS & ANSWERS

- **What is the relationship between linear velocity (v) and angular velocity (ω) for a point on a rotating body?**

Ans: The relationship is $v = r\omega$, where r is the perpendicular distance from the axis of rotation.

- **Why is the linear velocity of a point in circular motion called tangential velocity?**

Ans: It is called tangential velocity because its direction is always along the tangent to the circle at that point

For your information

As the wheel turns through an angle θ, it lays out a tangential distance $S = r\theta$

- **Do all points on a rigid body rotating about a fixed axis have the same linear speed? Explain.**

Ans: No, points on a rigid body at different distances from the axis have different linear speeds ($v = r\omega$), with points further away moving faster linearly

- ***How does using angular variables simplify the description of motion for an entire rigid body?***

Ans: By using angular variables, the motion of the entire body can be described in a simple way because all points share the same angular displacement, speed, and acceleration

- **How can linear equations of motion be transformed into angular equations of motion?**

Ans: Linear equations can be transformed into angular equations by replacing linear variables (S, v, a) with their angular counterparts (θ, ω, α, respectively)

- **Why is it important for the axis of rotation to be fixed for the angular equations of motion to hold true?**

Ans: A fixed axis of rotation ensures that all angular vectors (displacement, velocity, acceleration) have the same direction, allowing them to be treated as scalars in the equations

- **For a body undergoing uniform angular deceleration, what would be the sign of its angular acceleration?**

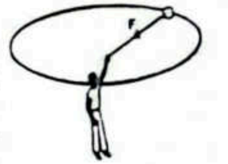
Ans: The sign of its angular acceleration would be negative (opposite to the direction of initial angular velocity)

3.2 CENTRIPETAL FORCE

Q **Define and explain centripetal force. Derive relation for centripetal acceleration and force.**
Define Centripetal force. Find the formula for it. (GRW. 2015)
What is centripetal force? Derive relations for centripetal force and centripetal acceleration. (LHR. 2016)

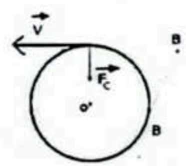
Ans

CENTRIPETAL FORCE:



A force which is needed to move a body along circular path is called centripetal force.
*The **force** needed to **bend** the normally **straight path** of the particle into a **circular path** is called the centripetal force.*

Explanation:



This force is only a cause to change the direction of motion of a rotating body. The force at each point of the circular path is directed towards the center of circle. Newton's second law of motion states that when a force acts on a body, it produces acceleration in the same direction. A force acting on a moving body along the direction of its velocity will change magnitude of the velocity (speed) keeping the direction unchanged. On the other hand, a constant force acting perpendicular to the velocity of a body moving in a circular path will change the direction but magnitude of velocity (speed) will remain the same. Such force makes the body move in a circle by producing a radial (or centripetal) acceleration and is called centripetal force (Centre seeking) force.

Examples of Centripetal force:

(i) When a ball tied at the end of a string is whirled in a horizontal circle, the tension in the string provides the centripetal force.

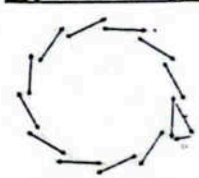
(ii) The planets moving in nearly circular orbits around the Sun has a centripetal force due to gravitational attraction between Sun and planet.

(iii) Motion of nuclear particles in accelerators, motion of flywheels, etc is due to centripetal force.

(iv) Our Earth is revolving around the sun under the action of gravitational force. This gravitational force is a centripetal force

(v) In an atom, coulomb's force between electron and nucleus compel the electron to move in an orbit around the nucleus. Thus, coulomb's force is a centripetal force

Do you know?



Direction of motion changes continuously in circular motion

(vi) For an object placed on a turntable, the friction is the centripetal force

(vii) The gravitational force is the cause of the Earth orbiting around the Sun, Moon and artificial satellites revolving around the Earth

(ix) A normal or perpendicular magnetic force compels a charge particle moving along a straight path into a circular path.

(x) When a vehicle takes turn on a road, it also needs centripetal force which is provided by the friction between the tyres and the road. If the road is slippery, then at high speed, the friction may not be sufficient enough to provide necessary centripetal force. Hence, vehicle will not be able to take turn and may skid or may even be toppled. To overcome this difficulty, the highway road is banked on turns. That is, the outer edge of the track is kept slightly higher than that of the inner edge.

Relation for Centripetal Acceleration:

$$a_c = \frac{v^2}{r} \qquad \text{------------ (i)}$$

Since $v = r\omega$

Or $$a_c = \frac{r^2\omega^2}{r} = r\omega^2$$

This acceleration is the instantaneous centripetal acceleration.

Direction of Acceleration

The direction of acceleration is along the direction of change in velocity $\Delta\vec{v}$ and is perpendicular to instantaneous velocity. Hence the acceleration is also directed towards the center. This normal acceleration towards the center is called centripetal acceleration or radial acceleration. The centripetal acceleration in the vector form is:

$$\vec{a} = \frac{v^2}{r}(-\hat{r}) = -\frac{v^2}{r}\hat{r} = -r\omega^2\hat{r}$$

Centripetal Force:

$$F_c = m a_c \qquad \text{------------ (ii)}$$

Putting equation (i) in (ii)

$$F = \frac{mv^2}{r} \qquad \text{------------ (iii)}$$

Since $v = r\omega$, therefore centripetal expressed in terms of angular velocity as:

$$F = \frac{mr^2\omega^2}{r} = mr\omega^2 \qquad \text{------------ (iv)}$$

Direction of Centripetal Force

The direction of centripetal force is towards the center of circle i.e opposite to $\hat{r}$ and in vector form is:

$$\vec{F} = \frac{-mv^2}{r}\hat{r} = -(mr\omega^2)\hat{r} \qquad \text{------------ (v)}$$

APPLICATIONS OF CENTRIPETAL FORCE

1. **Centrifuge:**
   - **Function:** A highly useful laboratory device that separates denser and lighter particles from a mixture.
   - **Mechanism:** The mixture is rotated at very high speeds in sample tubes. Denser particles, requiring a larger centripetal force to follow the circular path, are "forced" outwards to the bottom of the tube. Lighter particles, requiring less centripetal force, stay closer to the axis of rotation, rising to the top.

Fig 3.9

2. **Washing Machine Dryer:**
   - **Function:** Dries wet clothes quickly.
   - **Mechanism:** The dryer consists of a cylinder with many small holes. Wet clothes are placed inside, and the cylinder rotates rapidly. Water, being denser than the fabric and needing a larger centripetal force, moves outwards towards the cylinder wall and is drained out through the holes.

Fig 3.8(b)

Fig 3.8(a)

3. **Cream Separator**
   - **Function:** Separates cream from milk.
   - **Mechanism:** Milk (a mixture of lighter cream particles and denser skim milk particles) is whirled rapidly. The lighter cream gathers near the axis of rotation, while the denser skim milk particles are "forced" outwards, allowing for easy separation.

Fig 3.10

MULTIPLE CHOICE QUESTIONS

- **When a ball is whirled in a horizontal circle with a string, the centripetal force is provided by:**
  (a) The weight of the ball (b) Air resistance (c) Tension in the string (d) The force of gravity

Answer: (c) Tension in the string.
Explanation: The tension in the string acts radially inward, supplying the necessary centripetal force.

- **For an object placed on a turntable, the centripetal force that keeps it moving in a circle is provided by:**
  (a) Normal force (b) Gravitational force (c) Friction (d) Air pressure

Answer: (c) Friction.
Explanation: The static friction between the object and the turntable surface provides the inward force for circular motion.

- **When a vehicle takes a turn on a road, centripetal force is provided by:**
  (a) The vehicle's engine. (b) Air pressure on the sides.
  (c) Friction between the tires and the road. (d) The steering wheel.

Answer: (c) Friction between the tires and the road.
Explanation: The sideways frictional force exerted by the road on the tires provides the necessary inward force.

- **If friction is insufficient for a vehicle turning at high speed on a slippery road, the vehicle may skid or topple because:**
  (a) Its speed increases. (b) It needs more tangential force.
  (c) It cannot get enough centripetal force. (d) Its mass changes.

Answer: (c) It cannot get enough centripetal force.
Explanation: Without sufficient centripetal force, the vehicle cannot maintain the curved path and tends to move in a straight line.

- **The centrifuge functions on the principle that if the applied force falls short of the required centripetal force, the object will:**
  (a) Move towards the center. (b) Increase its speed.
  (c) Move away from the center of the circle. (d) Remain at rest.

Answer: (c) Move away from the center of the circle.
Explanation: The centrifuge separates particles based on their inertia; less dense particles move inward while denser ones are pushed outward by insufficient centripetal force.

- **In a laboratory centrifuge, when a mixture is rotated at high speed, the denser particles will:**
  (a) Rise to the top of the sample tubes. (b) Gather near the axis of rotation.
  (c) Settle at the bottom of the sample tubes. (d) Remain uniformly distributed.

Answer: (c) Settle at the bottom of the sample tubes.
Explanation: Denser particles have more inertia and experience a greater tendency to move outwards, thus settling at the bottom.

- **The dryer of a washing machine uses the principle of a centrifuge to remove water from clothes by:**
  (a) Heating the clothes. (b) Spinning them slowly.
  (c) Rotating them rapidly, forcing water outwards through holes.
  (d) Using a vacuum.

Answer: (c) Rotating them rapidly, forcing water outwards through holes.
Explanation: Rapid spinning creates an effect similar to artificial gravity, pushing the water out through the perforations.

- **In a cream separator, when milk is whirled rapidly, cream (lighter particles) gathers near the:**
  (a) Outer wall. (b) Bottom of the container. (c) Axis of rotation. (d) Top of the container.

Answer: (c) Axis of rotation.

Explanation: Lighter particles (cream) have less inertia and move inward towards the axis of rotation while denser milk particles move outwards.

- **Which of the following is NOT an application directly functioning on the principle of a centrifuge?**
  (a) Washing machine dryer (b) Cream separator (c) Laboratory centrifuge (d) A simple pendulum

Answer: (d) A simple pendulum

Explanation: A simple pendulum demonstrates oscillatory motion, not the principle of separation via centripetal force.

SLO BASED SHORT QUESTIONS & ANSWERS

- **What is the effect of a force acting perpendicular to the velocity of a body in circular motion?**

Ans: It changes the direction of the body's velocity without changing its speed.

- **What is the primary role of centripetal force?**

Ans: The primary role of centripetal force is to continuously bend the straight path of a particle into a circular path.

- **If a ball whirled in a horizontal circle has its string snapped, what path will it take and why?**

Ans: It will follow a straight line path tangent to the circle at that point due to inertia (Newton's First Law)

- **What is the underlying physical principle that all centrifuge applications rely on?**

Ans: All centrifuge applications rely on the principle that objects with greater inertia (denser particles) have a stronger tendency to move away from the center if the centripetal force is insufficient for their mass/velocity.

For Your InformationMotion on Banked Track

Banked tracks are needed for turns that are taken so quickly that friction alone cannot provide energy for centripetal force.

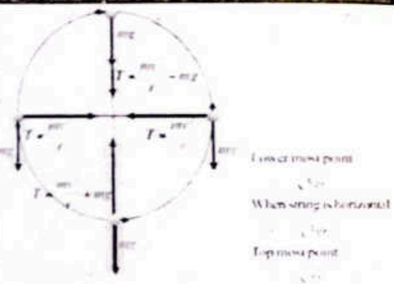
Intresting Information

Curved flight at high speed requires a large centripetal force that makes the stunt dangerous even if the air planes are not so close.

Do you know?

Q: What about the safety of curved flight at high speed for airplanes?

A: Curved flight at high speed requires a large centripetal force; this can make stunts dangerous, even if the airplanes are not physically close to each other, due to the extreme forces involved.



TALENT INFORMATION Uniform vs. Non-Uniform Circular Motion

Feature	Uniform Circular Motion (UCM)	Non-Uniform Circular Motion (NUCM)
Definition	Motion of an object along a circular path at constant speed.	Motion of an object along a circular path with varying speed.
Speed	Constant	Changes (increases or decreases)
Velocity	Changes continuously (due to changing direction)	Changes continuously (due to changing magnitude and direction)
Direction of Motion	Changes continuously (tangent to the circle)	Changes continuously (tangent to the circle)
Angular Speed (ω)	Constant	Varies
Angular Velocity (ω)	Constant (magnitude and direction along axis of rotation)	Varies (magnitude and/or direction along axis of rotation)
Angular Displacement	Equal in equal time intervals	Unequal in equal time intervals
Acceleration	Only centripetal (radial) acceleration (a_c) present.	Both centripetal (radial) acceleration (a_c) and tangential acceleration (at) are present.
Centripetal Accel	Constant in magnitude ($a_c=v^2/r=\omega^2r$)	Varies in magnitude (as v changes, ac changes)
Tangential Accel.	Zero (at=0)	Non-zero (at ≠0), responsible for changing speed
Net Acceleration	Directed towards the center of the circle	Not directed towards the center; has both radial and tangential components.
Centripetal Force	Constant in magnitude (Fc =mv2/r=mω2r)	Varies in magnitude (as v changes, Fc changes)
Tangential Force	Zero (no force along the tangent)	Non-zero, responsible for changing the object's speed
Work Done by Centripetal Force	Zero (force is perpendicular to displacement)	Zero (force is perpendicular to instantaneous displacement)
Work Done by Tangential Force	Zero	Non-zero (force acts along the direction of displacement)
Kinetic Energy (KE)	Constant ($KE=(1/2)mv^2$)	Varies (as speed v changes, KE changes)
Momentum	Changes continuously (due to changing velocity direction)	Changes continuously (due to changing velocity magnitude and direction)
Examples	- Satellite orbiting Earth at constant speed	- Rollercoaster moving through a loop - A ball tied to a string swung in a vertical circle
	- Tip of a clock's second hand - A fan blade spinning at a steady rate	- Car accelerating/decelerating while turning a curve

MULTIPLE CHOICE QUESTIONS

- **If a CD spins at 210 rpm, its angular velocity in rad s^{-1} is:**
  (a) 3.5π (b) 7.0π (c) 14π (d) 210π

Answer: (b) 7.0π
Explanation: The angular velocity is calculated by converting rpm to Hz and then using the formula $\omega = 2\pi f$.

- **In vertical motion, the tension in the string when the ball is at bottom is:**
  (a) $T = mv^2/r$ (b) $T = mg$ (c) $T = mv^2/r - mg$ (d) $T = mv^2/r + mg$

Answer: (d) $T = mv^2/r + mg$
Explanation: The tension in the string must balance both the weight of the ball and provide the centripetal force.

- **In vertical motion, the tension in the string will be zero if:**
  (a) $v^2 = r$ (b) $v = gr$ (c) $v^2 = gr$ (d) $v = g/r$

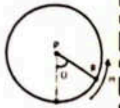
Answer: (c) $v^2 = gr$
Explanation: When $v^2 = gr$, the centripetal force is just equal to the weight, and the tension in the string becomes zero.

- **The direction of centripetal force is always:**
  (a) Along the tangential velocity
  (b) Opposite to the tangential velocity
  (c) Perpendicular to the tangential velocity and towards the center of the circle
  (d) Away from the center of the circle

Answer: (c) Perpendicular to the tangential velocity and towards the center of the circle
Explanation: Centripetal force is always directed towards the center of the circular path.

- **The formula for centripetal force is:**
  (a) $Fc = mv^2/r$ (b) $Fc = mr^2v$ (c) $Fc = mv/r$ (d) $Fc = mr\omega$

Answer: (a) $Fc = mv^2/r$
Explanation: This formula gives the centripetal force required to keep an object of mass m moving with velocity v in a circular path of radius r.

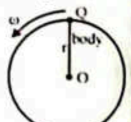


- **The moon revolves around the earth in approximately 27.3 days. If the mean distance between the centres of the earth and the moon is 3.84×10^8 m, estimate the centripetal acceleration experienced by the moon in its revolution around the earth.**
  (a) $1.57ms^{-2}$ (b) $2.75\times10^{-3}ms^{-2}$ (c) $0.24ms^{-2}$ (d) $1.37\times10^{-3}ms^{-2}$

Solution: $a_c = r\omega^2 \Rightarrow a_c = r\times\left(\frac{2\pi}{T}\right)^2 = \frac{3.84\times10^8\times4\pi^2}{(27.3\times24\times60\times60)^2} = 2.75\times10^{-3}ms^{-2}$

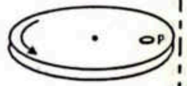
- **A body of mass "m" is whirled in a circle in a vertical plane while tied to a string. The radius of the circle is r. When the body is at the uppermost position i.e point Q, the angular speed is ω. What is the tension in the string, assuming that it is still tout?**
  (a) $mr\omega^2$ (b) mg (c) $mg+mr\omega^2$ (d) $mr\omega^2-mg$

Solution: At the uppermost position at Q, the net force towards "0", the centre of the vertical circle is "mg+T";
Using 2nd law of Newton,
$T + mg = ma_c = mr\omega^2 \Rightarrow T = mr\omega^2 - mg$



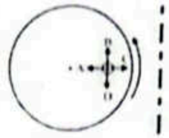
- **The body in previous question passes point R at some later instant of time. PR makes an angle θ with vertical. What is the tension in the string, if at point R, the angular speed is ω_1?**
  (a) $mg\cos\theta + mr\omega_1^2$ (b) $mg\cos\theta - mr\omega_1^2$ (c) $mr\omega_1^2 - mg\cos\theta$ (d) $mr\omega_1^2\cos\theta$

Solution: $\therefore F_r = T - mg\cos\theta$
$mr\omega_1^2 = T - mg\cos\theta$
$T = mg\cos\theta + mr\omega_1^2$



- **A turntable rotates at constant speed. A coin is placed on the turntable at P. The frictional force between the coin and the turntable keeps the coin in the same position on the turntable.**
  In which direction does the friction force act?
  (a) B (b) C (c) A (d) D

Solution: We must remember that in this case centripetal force compels the coin to remain on the rotating turn-table so frictional force must be provided by centripetal force so best option is "A" towards the centre.



- **Three cars A, B and C are moving in three different bridge straight, concave up and concave down respectively. Which passenger of car will experience maximum tension?**
  (a) A (b) B
  (c) C (d) All will feel same tension

Solution: The option "C" is the lower portion of a vertical circle which has maximum speed and tension at the lowest part so it is the best option.

- **A ball tied to a string is swinging along a vertical circle, the tension in the string is minimum at:**
  (a) Top (b) Bottom (c) Mid-way (d) Same at each point

Solution:
In a vertical circle at the highest point these is weightlessness so tension will be the lowest at the top.

- **A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion takes place in a plane. It follows that:**
  (a) Its velocity is constant (b) Its kinetic energy is constant
  (c) It moves in a circular path (d) **Both "B" and "C"**

Solution: Along a circular path F_c & d make $\theta = 90°$

$W = \vec{F}.\vec{d} = Fs\cos90° = zero$
$W = \Delta K.E = Zero \Rightarrow K.E = constant$

Speed remains same only direction of velocity changes

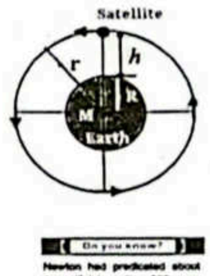
3.3 ARTIFICIAL SATELLITES

Q **What are artificial satellites? Derive relation for orbital speed of satellite.**

Ans

ARTIFICIAL SATELLITES:

"Artificial satellite are __man-made__ objects that __orbit around the Earth__." These are scientific laboratories, which are launched by the rockets at a height where atmospheric friction is negligible.



Uses:
These satellites are communications, weather, spy, war, research satellites and etc.

Orbital Speed of satellite: (Critical value)

If a satellite of mass m is moving around the Earth with a tangential velocity $\vec{v}$ at a distance 'r' from the center of Earth then gravitational pull on the satellite is:

$$F = \frac{GmM}{r^2} \quad \text{-------(i)}$$

Where M is the mass of Earth.
This gravitational force provides the centripetal force to the satellite, which is;

$$F = \frac{mv^2}{r} \quad \text{--------------- (ii)}$$

Equating the (i) and Iii);

$$\frac{mv^2}{r} = \frac{GmM}{r^2}$$

$$v^2 = \frac{GM}{r}$$

Do you know?
Newton had predicated about the artificial satellites 300 years ago. The above figure has been taken from his well-known book "Principia Mathematica". According to this book, if an object is thrown horizontally with a particular speed from a place which is sufficiently high, it will start revolving around the Earth.

$$v = \sqrt{\frac{GM}{r}} \quad \text{.............. (iii)}$$

Or,

If 'R' is the radius of Earth and 'h' is the height of satellite from the surface of Earth then $r = R + h$, and eq (iii) is,

$$v = \sqrt{\frac{GM}{R+h}} \quad \text{.............. (iv)}$$

When a satellite is close to the surface of Earth then,

$$R + h = R$$

and

$$v = \sqrt{\frac{GM}{R}} \quad \text{.............. (v)}$$

> **For Your Information**
> In a circular orbit, the orbital speed is near about 27000km/h and in an elliptical orbit, the orbital speed is nearly 30000km/h. The escape velocity is $11.2 kms^{-1}$ = $40,000 kmh^{-1}$

But

$$G = 6.673 \times 10^{-11} Nm^2 Kg^{-2}$$

$$M = 6 \times 10^{24} kg$$

And

$$R = 6.4 \times 10^6 m$$

So,

$$v = \sqrt{\frac{6.673 \times 10^{-11} \times 6 \times 10^{24}}{6.4 \times 10^6}}$$

$$v = 7.9 \times 10^3 m s^{-1}$$

$$v = 7.9 km s^{-1}$$

This is the minimum velocity to put a satellite into the orbit and called critical velocity.

Time Period of Satellite: (Critical value)

*"The **time** interval required for a satellite to **complete one revolution** is its **time period** or period of motion."*

The distance travelled by satellite with speed 'v' in a complete revolution is $2\pi R$

The time required to complete this distance is time period 'T'. Thus,

$$v = \frac{s}{t} = \frac{2\pi R}{T}$$

$$T = \frac{2\pi R}{v} = \frac{2\pi(6.4 \times 10^6)}{7.9 \times 10^3} = 5060 s$$

$$T = 5060/60 = 84.33 min = 84.33/60 = 1.4 hr$$

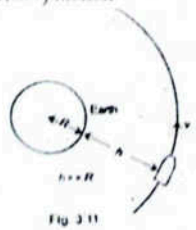


Fig 3.11

Global Positioning System:

Close orbiting satellites orbit the Earth at a height of **400 km**. **Twenty-four** such satellites form the Global positioning system. The moment we switch ON our mobile phone; our location can be tracked immediately by global positioning system (GPS).

For your information - Satellite Orbits (Approximate Speeds):

- Circular orbit: 27,000 km/h
- Elliptical orbit: 30,000 km/h
- Escape velocity: 11 km/s or 40,000 km/h

Do you know?

Q: **Did Newton predict artificial satellites?**

Ans: Yes, Isaac Newton predicted the existence of artificial satellites about 300 years ago. In his book "Principia Mathematica," he described how if an object is thrown horizontally with a particular speed from a sufficiently high place, it would start revolving around the Earth.

Tidbits:

Q: ***How does a mobile phone relate to satellite tracking?***

Ans: The moment you switch on your mobile phone, your location can be tracked immediately by the Global Positioning System (GPS), which relies on a network of Earth-orbiting satellites.

Tidbits:

In 1984, Bruce McCandless became the first human satellite of the Earth when he stepped into space from a space shuttle at a height of 100 km above Hawaii island, moving at a speed of 29,000 km/h.

MULTIPLE CHOICE QUESTIONS

- **Satellites orbit the Earth in nearly circular paths and are held in these orbits by:**
  (a) Air pressure (b) Their own thrust
  (c) The gravitational pull of the Earth (d) Magnetic fields

Answer: (c) The gravitational pull of the Earth

Explanation: Gravity provides the centripetal force necessary to keep satellites in orbit.

- **If there were no gravitational pull, a satellite in orbit would:**
  (a) Spiral inwards (b) Fly off in a straight line tangent to the orbit
  (c) Stop immediately (d) Accelerate towards the sun

Answer: (b) Fly off in a straight line tangent to the orbit

Explanation: Without the centripetal force of gravity, the satellite would follow its inertial path, which is a straight line.

- **The minimum velocity required to put a satellite into orbit, especially near the Earth's surface, is called the:**
  (a) Escape velocity (b) Terminal velocity (c) Critical velocity (d) Orbital velocity

Answer: (c) Critical velocity

Explanation: Critical velocity is the minimum tangential speed needed for an object to maintain a stable circular orbit.

- **According to Newton's prediction, if an object is thrown horizontally with a particular speed from a sufficiently high place, it will:**
  (a) Fall directly down (b) Go into an elliptical orbit
  (c) Start revolving around the Earth (d) Move in a parabolic path

Answer: (c) Start revolving around the Earth.

Explanation: This describes the conceptual basis for achieving orbit by matching the tangential velocity to the rate of fall due to gravity.

SLO BASED SHORT QUESTIONS & ANSWERS

- **What force holds artificial satellites in their orbits around the Earth?**

Ans: The gravitational pull of the Earth holds artificial satellites in their orbits.

- **What is meant by the critical velocity of a satellite?**

Ans: The critical velocity is the minimum velocity necessary to put a satellite into orbit near the Earth's surface.

- **What happens if a satellite's speed is less than its required orbital speed?**

Ans: Any speed less than the required orbital speed will cause the satellite to tumble back to the Earth.

- **How does the orbital velocity of a satellite depend on its mass?**

Ans: The orbital velocity of a satellite is independent of its mass.

- **If a satellite is at a much greater distance 'h' from the Earth's surface, how does the gravitational acceleration change?**

Ans: The gravitational acceleration decreases inversely as the square of the distance from the center of the Earth.

For Your Information

ORBITAL VELOCITY

Q.16 What is orbital velocity? Derive its general formula for satellite orbiting in radius 'r' about Earth.

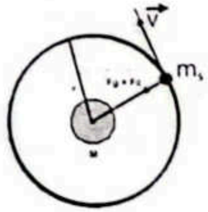
Ans

ORBITAL VELOCITY:

"When a __body__ is __moving in an orbit__ (circular or elliptical), its motion is called __orbital motion__ and its velocity __orbital velocity__."

Orbital Motion:

The Earth and all other planets in our solar system revolve around the Sun in nearly circular paths. The artificial satellites launched by men also adopt nearly circular paths around the Earth. This type of motion is called orbital motion.



Equation of Orbital Velocity:

Consider an artificial satellite is circling around the Earth. Let the mass of satellite is 'm_s' and 'V' is its orbital speed. The mass of Earth is 'M' and 'r' represents the radius of the orbit. A centripetal force $\frac{m_s v^2}{r}$ is required to hold the satellite in orbit. This force is provided by the gravitational force of attraction between the Earth and the satellite i.e.

$$\frac{GMm_s}{r^2}$$

Equating the two force, we get,

$$\frac{m_s v^2}{r} = \frac{GMm_s}{r^2}$$

$$v^2 = \frac{GM}{r}$$

or

$$v = \sqrt{\frac{GM}{r}}$$

TID-BITS

In 1984 at the height of 100 km above Hawaii island with the speed of 29000 kmh^{-1} Bruce McCandless stepped into space from a space shuttle and became the first human satellite of the Earth.

Conclusion:

- The above equation shows that the orbital speed is independent of the mass of a satellite.
- The above equation shows that the orbital speed depends upon altitude of satellite and radius of planet.

MULTIPLE CHOICE QUESTIONS

- **The relation for the orbital speed of an earth satellite is $V=\sqrt{\frac{GM}{r}}$, if we have satellites orbiting around the earth in two different radii of the same period of time then the true relation between the orbital speed and radius must be:**

(a) $v \propto \frac{1}{\sqrt{r}}$ (b) $V^2 \propto \frac{1}{r}$ (c) Both "A" & "B" **(d) $V \propto r$**

Solution: The relation $V = \sqrt{\frac{GM}{r}}$ is not used, here we use

$V = r\omega \Rightarrow \because \omega$ = constant $\Rightarrow v \propto r$

- **An orbital satellite is moving with an orbital speed "V" with a radius "r" and it is making 2 revolutions in 24 hours around the planet whose satellite it is. How will it effect the frequency of rotation of such a satellite if the radius is quadrupled keeping the orbital speed constant?**

(a) Will become $\frac{1}{2}$ revolution in 24 hours (b) Will become 4 revolutions in 24 hours

(c) Will remain same (d) Cannot be predicted

Solution: Again simply using

$V = r\omega \Rightarrow v$ = constant $\Rightarrow \omega \propto \frac{1}{r} \propto \frac{1}{T}$

$r' = 4r$ so $\therefore T' = 4T, \omega' = \frac{\omega}{4} = \frac{2\text{rev per day / 24 hours}}{4}$

$= \frac{1}{2}$ rev per day / 24 hours

- **Which statement about geostationary orbits is false?**

(a) A geostationary orbit must be directly above the equator

(b) All satellites in a geostationary orbit must have the same mass.

(c) The period of a geostationary orbit must be 24 hours

(d) There is only one possible radius for a geostationary orbit

Solution: In a geo-stationary orbit a satellite may have any mass which shows the relation for time periodof a geo-stationary orbit is independent of mass.

- **For an earth satellite longer orbits will correspond to:**

(a) Longer periods and larger velocity **(b) Longer periods and smaller velocity**

(c) Smaller periods and smaller velocity (d) Smaller periods and larger velocity

Solution: When "r" increases a satellite has to cover more distance, hence "v" is less and it takes more time to complete it's period of rotation.

- **A weight is suspended from the roof of a stationary lift by a spring balance. The balance reads 80g. If the cable supporting the lift breaks and the lift starts falling freely under gravity, the reading of the spring balance will be.**

(a) Zero (b) 800 g (c) 80 g (d) 8 g

Solution: Weightlessness (free fall)

- **A fighter aircraft is moving in a vertical circle. The minimum velocity at the highest point is (Given: r = radius of circle)**

(a) $\sqrt{\frac{1}{2}gr}$ (b) $\sqrt{2gr}$ (c) $\sqrt{gr}$ (d) $\sqrt{3gr}$

Solution: We must remember that for a vertical loop the speed at the highest point is minimum;

$v = \sqrt{gR}$

- **Two satellites are to be launched into same orbits from the surface of earth. Satellite I has mass 10 kg and volume 1500 cm^3 while satellite II has mass 5 kg and volume 1000 cm^3. Assume the velocities of satellite I and II are v_1 and v_2. The relation between v_1 and v_2 is: (ECAT-2016)**

(a) $v_1 = v_2$ (b) $v_1 > v_2$

(c) $v_1 < v_2$ (d) Relation depends upon launch

Solution: As their orbits are same so irrespective of whatever is their mass, speeds have to be same. i.e. $v_1 = v_2$

- The ratio of escape to orbital velocity:

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 2 (d) 1

Solution: $V_{esc} = \sqrt{2gR} \to$ (i) and $V_{orb} = \sqrt{gR} \to$ (ii) **Dividing equation (i) by equation (ii) we get;** $\frac{V_{esc}}{V_{orb}} = \frac{\sqrt{2gR}}{\sqrt{gR}} = \sqrt{2}$

SLO BASED SHORT QUESTIONS & ANSWERS

- **What two forces are equated to derive the formula for orbital velocity?**

Ans: The gravitational force of attraction and the required centripetal force are equated.

- **Does the mass of the satellite affect its orbital speed? Explain.**

Ans: No, the mass of the satellite does not affect its orbital speed because its mass cancels out in the orbital velocity derivation.

- **If a satellite is at a higher altitude, how does its required speed compare to a lower altitude satellite?**

Ans: A satellite at a higher altitude will require a slower orbital speed.

WEIGHTLESSNESS IN SATELLITES

Q **Discuss the phenomenon of weightlessness in satellites and gravity free system.**

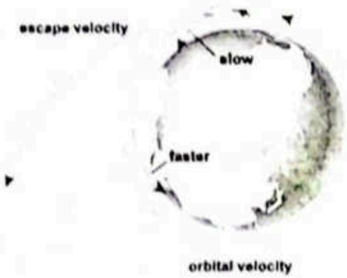
Ans

A satellite is like a freely falling body. Since a free falling body is weightless so a satellite and all objects in it are in the condition of weightlessness.

Effect of tangential velocity on curvature of path

When a satellite is launched at a height, where the air friction is negligible. The launching tangential speed is very important for the geometry of the orbit like the trajectory of projectile. If the projectile is thrown at successively larger speeds then during its free-fall to the Earth, the curvature of the path decreases with increasing horizontal speeds.

Similarly, when the satellite is pushed fast enough parallel to the Earth, the curvature of its path will match the curvature of the Earth. In this case, the satellite will round the Earth in a circular orbit.



Radial Acceleration of Objects in a Satellite:

The gravitational forces of Earth on the satellite compel it to move along a circular path around the Earth. Thus this gravitational force is the centripetal force, which is acting on the satellite, and it is:

$$mg = \frac{GmM}{r^2} = \frac{mv^2}{r}$$

$$mg = \frac{mv^2}{r} \Rightarrow g = \frac{v^2}{r} = \text{Radial acceleration}$$

Force on an Object in Satellite:

Consider a satellite of mass m revolving in its orbit of radius r around the Earth. A body of mass m inside the satellite suspended by a spring balance from the ceiling of the satellite is under the action of two forces.

- Its weight mg acting downward.
- The supporting force, called normal force F, or tension in the spring acting upward.
- Resultant force is equal to the centripetal force required by the mass m which is acting towards the centre of the Earth, and is expressed as

$$mg - F = mv^2/r \quad (1)$$

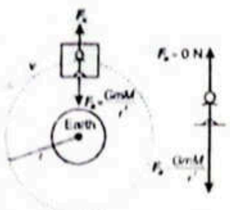


Fig. 3.13

where mv^2/r is the centripetal force. Hence

$$F = mg - mv^2/r \quad (2)$$

It may be noted that the centripetal force responsible for the revolution of the satellite around the Earth is provided by the gravitational force of attraction between the Earth and the satellite.

$$F_c = F_g$$

$$GmM/r^2 = mv^2/r = mg$$

So, $v^2 = GM/r$ and $g = GM/r^2$ putting in eq (2)

Hence, Eq. (2) becomes

$$F = mg - mg$$

$$F = 0$$

This shows that the supporting force which is acting on a body inside the satellite is zero. Therefore, the bodies as well as the astronauts in a satellite find themselves in a state of apparent weightlessness.

Weightlessness and Gravity Free System:

Spaceship is accelerating towards the center of Earth at all times and this radial acceleration is 'g'. Hence, it can be said that spaceship is falling towards the center of Earth at all the times but the curvature of the Earth prevents the spaceship from hitting the ground.

Since the spaceship is in free fall, all the objects within it appear to be weightless. Thus no force is required to hold an object falling in the frame of reference of spaceship (satellite). Such a system is called **gravity free system**.


Astronaut floating inside the cabin of a spaceship

Do you know?

Q: **Does your weight change in an elevator?**

Ans: Your weight slightly changes when the velocity of the elevator changes (i.e., accelerates or decelerates) at the start and end of a ride. However, your weight remains constant during the rest of the ride when the elevator's velocity is constant. This is because apparent weight is related to the normal force, which changes with acceleration.

Do you know
Your weight slightly changes when the velocity of the elevator changes at the start and end of a ride, not during the rest of the ride when that velocity is constant.

Can you tell?

Q: **When a bucket full of water is rapidly whirled in a vertical circular path, water does not fall out even if the bucket is inverted at the maximum height. Why is it so?**

Ans: At the top of the vertical circle, both the tension in the string (or the normal force from the bucket's bottom) and the weight of the water are directed downwards, towards the center of the circle. These forces combine to provide the necessary centripetal force to keep the water moving in a circle. If the speed of rotation is high enough, the required centripetal force is greater than the water's weight. This ensures that the normal force (or the apparent weight pushing the water against the bucket's bottom) remains inward, preventing the water from falling out, even when the bucket is inverted. The water has sufficient tangential velocity that its inertia tends to carry it forward, keeping it "stuck" to the bottom of the bucket as it rounds the top of the loop.

Can you tell
When a bucket full of water is rapidly whirled in a vertical circular path, water does not fall out even if the bucket is inverted at the maximum height. Why is it so?

MULTIPLE CHOICE QUESTIONS

- **Everything inside an orbiting satellite experiences weightlessness because the satellite is:**

(a) Moving too fast for gravity to affect it (b) Out of Earth's gravitational field
(c) Accelerating towards the center of the Earth as a freely falling body
(d) In a vacuum, eliminating all forces

Answer: (c) Accelerating towards the center of the Earth as a freely falling body

Explanation: Weightlessness is apparent, as the satellite and its contents are constantly falling together around the Earth.

- **The supporting force acting on a body inside a freely falling satellite is:**

(a) Equal to its weight (b) Greater than its weight (c) Zero (d) Negative

Answer: (c) Zero

Explanation: Since the body is accelerating with the satellite (in free fall), there is no normal force or tension to provide apparent weight.

- **If an astronaut floats inside a spaceship, it implies a state of:**
  (a) True gravity (b) Apparent weightlessness (c) Strong acceleration (d) Equilibrium

Answer: (b) Apparent weightlessness

Explanation: Floating is a direct observation of apparent weightlessness, where the supporting force is absent.

- **The centripetal force responsible for the revolution of the satellite around the Earth is provided by:**
  (a) The satellite's engines (b) Air pressure
  (c) The gravitational force of attraction (d) The normal force from the Earth.

Answer: (c) The gravitational force of attraction

Explanation: This re-emphasizes that gravity is the primary force for orbital motion.

SLO BASED SHORT QUESTIONS & ANSWERS

- **Why do bodies and astronauts inside a satellite find themselves in a state of apparent weightlessness?**

Ans: They find themselves in apparent weightlessness because the satellite and everything within it are continuously accelerating towards the center of the Earth as a freely falling body.

- **What is the magnitude of the supporting force on a body inside a freely falling satellite?**

Ans: The magnitude of the supporting force on a body inside a freely falling satellite is zero.

- **Does weightlessness in satellites mean they are out of Earth's gravitational pull? Explain.**

Ans: No, weightlessness does not mean they are out of Earth's gravitational pull; it means they are in a constant state of free fall under gravity, so there's no normal force to feel their weight.

- **What effect might extend periods of weightlessness have on astronauts?**

Ans: Extended periods of weightlessness may affect the performance and health of astronauts.

- **What force provides the centripetal force for the satellite itself to revolve around the Earth?**

Ans: The gravitational force of attraction between the Earth and the satellite provides the centripetal force for the satellite's revolution.

ARTIFICIAL GRAVITY

Q **How artificial gravity is created in space ship? Derive expression for frequency of space ship to produce artificial gravity.**

Ans

ARTIFICIAL GRAVITY:

Fig. 3.14

*"The **gravity** produced in a **space ship** due to its **spin motion** is called artificial gravity."*

Explanation

When a spaceship is revolving around the Earth due to gravity, it gains the radial acceleration equal to 'g'. Thus spaceship as well as everything in it seems to be falling freely and create the sense of weightlessness. In the state of weightlessness, the astronauts cannot perform their functions properly and they face many other difficulties. In order to overcome this difficulty, an artificial gravity is produced in the spaceship. To produce the gravity in the spaceship, it is rotated around its own axis. Because of this rotation, the astronaut is pressed towards the outer rim and exerts a force on the floor of the spaceship in much the same way as on the Earth.

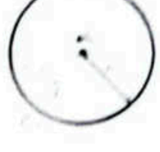
Expression for Frequency:

Consider a spaceship of ring shape having outer radius 'R'. It is rotated around its own axis with angular velocity 'ω'. The radial acceleration produced in it is:

$$a_c = R\omega^2 \quad \text{------------ (i)}$$

If it completes one revolution (θ = 2π) in the time period t = T, then:

$$\omega = \frac{\theta}{t} = \frac{2\pi}{T}$$



Put the value of in eq. (i)

$$a_c = R\left(\frac{2\pi}{T}\right)^2 = \frac{4\pi^2 R}{T^2}$$

Or $$T^2 = \frac{4\pi^2 R}{a_c}$$

Or $$T = 2\pi\sqrt{\frac{R}{a_c}} \quad \text{------------ (ii)}$$

Which gives the period of rotation of space ship.

The reciprocal of the time period is frequency which gives the period of rotation of spaceship i.e.

$$f = \frac{1}{T} = \frac{1}{2\pi\sqrt{\frac{R}{a_c}}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{a_c}{R}}$$

The surface of the rotating space ship pushes on an object with which it is in contact and thereby provides the centripetal force needed to keep the object moving on a circular path.

In order to produce the gravity equal to the gravity of Earth, radial acceleration must be equal to g

i.e. $a_c = g$

and spaceship must be rotated with the frequency;

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{R}}$$

MULTIPLE CHOICE QUESTIONS

- **To overcome the difficulties posed by extended weightlessness, artificial gravity can be created in a spacecraft by:**
  (a) Increasing its speed (b) Decreasing its radius
  (c) Setting the spaceship into rotation around its own axis
  (d) Adding more mass

Answer: (c) Setting the spaceship into rotation around its own axis

Explanation: Rotation simulates gravity by pressing occupants against the outer rim due to inertia.

- **When artificial gravity is created by rotation, the astronauts are pressed towards the:**
  (a) Center of the spaceship (b) Outer rim of the spaceship
  (c) Top of the spaceship (d) Axis of rotation

Answer: (b) Outer rim of the spaceship

Explanation: The inertial tendency of the astronauts is to move in a straight line, but the rotating wall provides a normal force, creating the sensation of weight.

- **The purpose of creating artificial gravity in a spacecraft is to:**
  (a) Increase its speed
  (b) Enable the crew to function in an almost normal manner
  (c) Reduce fuel consumption (d) Protect from cosmic rays

Answer: (b) Enable the crew to function in an almost normal manner

Explanation: Artificial gravity helps astronauts perform tasks and maintain health similar to Earth's conditions.

- **The surface of a rotating spaceship provides artificial gravity by pushing on an object with which it is in contact, thereby providing the:**
  (a) Gravitational force (b) Centrifugal force (c) Centripetal force (d) Tangential force

Answer: (c) Centripetal force

Explanation: From the perspective of an inertial observer, the wall provides the centripetal force needed to keep the object moving in a circular path

SLO BASED SHORT QUESTIONS & ANSWERS

- **Why is artificial gravity sometimes needed in spacecraft for extended missions?**

Ans: Artificial gravity is needed to counter the negative effects of prolonged weightlessness on astronaut performance and health

- **How does the rotation of a spacecraft create artificial gravity?**

Ans: The rotation presses the astronauts towards the outer rim, causing them to exert a force on the floor, similar to experiencing weight on Earth

- **What two components are needed for a rotating spaceship to provide artificial gravity according to the "Do you know?" box?**

Ans: The rotating surface pushes on an object (contact force) to provide the centripetal force needed to keep the object moving on a circular path

3.4 MOMENT OF INERTIA

Q Define and Explain Moment of Inertia.

Ans

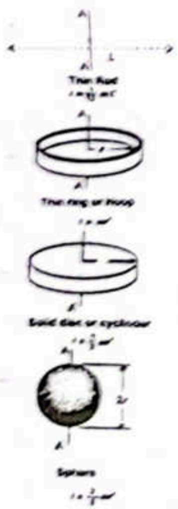
MOMENT OF INERTIA:

By the second law of motion, the mass of a body is the measure of its "inertia" or resistance to linear acceleration. Similarly, *The measure of the resistance of a body to angular acceleration is called "moment of inertia".* It plays the same role in angular motion as played by mass in linear motion. $I = mr^2$

- Its units in SI are $Kg\text{-}m^2$
- Its dimension is $[ML^2]$

For your information

Moments of inertia of various bodies about axis AA



Dependence: $I = mr^2$

Relation shows moment of inertia dependence upon two factors.

1. mass 'm' of rotating body.
2. Square of the distance from axis of rotation, r^2.

Question: Deduce expression for moment of inertia.

Expression Of Moment of Inertia:

Consider a mass m attached to the end of a massless rod. Now apply the force 'F' on the mass 'm' perpendicular to the rod so that it has angular motion in a horizontal plane. From Newton's second law of motion, it will cause a linear acceleration 'a'.

$$F = ma \quad (1)$$

If the force 'F' causes the mass to rotate about 'O', the torque 'τ' tending to rotate the mass about 'O' is given by

$$\tau = rF$$

Put equation (1) in it

$$\tau = r\,(ma)$$
$$\tau = rm\,(r\alpha) \qquad a = r\alpha$$
$$\tau = mr^2\alpha$$

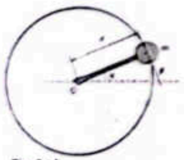


Fig. 3.15
The force F causes a torque about the axis O and gives the mass m an angular acceleration α about the pivot point.

Note: This relation is analogous to F = ma where F is replaced by τ and a is replaced by 'α'. Hence we say that the torque on the system is proportional to angular acceleration. Where constant of proportionality mr^2 is called the moment of inertia of the mas 'm'.

As

$$I = mr^2$$
$$\tau = I\alpha$$

Question: Find expression of moment of inertia of rigid body

Moment of Inertia of a Rigid Body

A rigid body is one whose shape cannot be changed easily by applying force. In most of rigid bodies, the mass distribution is not uniform. The rigid body is made up of small pieces of masses $m_1, m_2, m_3, \ldots m_n$ at distance $r_1, r_2, r_3 \ldots r_n$ from axis of rotation 'O', as shown in the figure (a).

Let the body be rotating with angular acceleration 'α' so magnitude of the torque acting on 'm_1' is

$$\tau_1 = m_1 r_1^2\alpha_1 \qquad (\because \tau = mr^2\alpha)$$

Similarly, the torque on m_2 is

$$\tau_2 = m_2 r_2^2\alpha_2$$

and so on

Now total torque is

$$\tau_{total} = m_1 r_1^2\alpha_1 + m_2 r_2^2\alpha_2 + \ldots + m_n r_n^2\alpha_n$$

Since the body is rigid, so all the masses are rotating with same angular acceleration 'α' so

$$\alpha_1 = \alpha_2 = \alpha_3 \ldots \alpha_n$$

Now total torque will be

$$\tau_{total} = m_1 r_1^2\alpha + m_2 r_2^2\alpha \ldots m_n r_n^2\alpha$$
$$= (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 \ldots m_n r_n^2)\alpha$$

In sigma(summation) form

$$\tau_{total} = \sum_{i=1}^{n} (m_i r_i^2)\alpha$$
$$\tau_{total} = I\alpha$$

Where $I = \sum_{i=1}^{n} m_i r_i^2$ = moment of inertia of rigid body

Moment Of Inertia of rigid body depends upon

- Mass distribution of body
- Shape of body
- Position of axis of rotation inside the body

Question:

Moment Of Inertia of Various Bodies:

The moments of inertia of various bodies are given as

(i) The moments of inertia of a uniform rod around a perpendicular axis passing through its center is $\frac{1}{12} mL^2$.

(ii) The moment of inertia of a thin ring or hoop around a perpendicular axis through its center is mr^2

(iii) The moment of inertia of solid disk or cylinder about a perpendicular axis or about the axis of the cylinder is $\frac{1}{2} mr^2$

(iv) The moment of inertia of a solid sphere about its diameter is $\frac{2}{5} mr^2$.

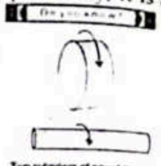
Question: What is the significance of moment of inertia?

Answer: The physical significance of the moment of inertia is **similar to the mass in translational motion**. In translational motion, the mass of a body is used for measuring inertia. As the mass increases, inertia becomes larger. The force required for producing the linear acceleration will also increase. Similarly, same role is performed by moment of inertia in rotational motion. The moment of inertia is a measure of an object's resistance to rotational motion, similar to how mass measures an object's resistance to linear motion. Specifically, it is a property that describes how much torque is required to cause an object to rotate about a given axis.

Do you know?

Q: How does diameter affect rotational inertia for cylinders of equal mass?

A: Two cylinders of equal mass. The one with the larger diameter has the greater rotational inertia (moment of inertia). This is because its mass is distributed further from the axis of rotation.



Two cylinders of equal mass. The one with the larger diameter has the greater rotational inertia.

For your information - Moments of Inertia of various bodies about axis AA:

MULTIPLE CHOICE QUESTIONS

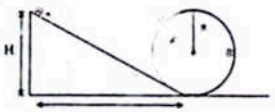
- **A flywheel rotating about a fixed axis has a kinetic energy of 100 J, when it's angular speed was 20 rad s^{-1}. The moment of inertia of the wheel about the axis of rotation is;**

(a) 0.6 kg m^2 (b) 0.15 kg m^2 (c) 0.5 kg m^2 (d) 0.8 kg m2

Solution:

$$E=\frac{1}{2}I\omega^2 \quad \text{so} \quad I=\frac{2E}{\omega^2}=\frac{2\times100}{400}=\frac{1}{2}=0.5\text{kgm}^2$$

- **A body slides on a frictionless inclined plane of height 'H' along track as shown in the following figure and it loops the loop in the vertical circle of radius "R". What should be the minimum value of "H" in terms of "R" so that body is just able to loop the loop?**



(a) H = 2R (b) $H=\frac{5R}{2}$ (a) H = 5R (d) H = 4R

Solution: $v=\sqrt{5gR}\rightarrow(i)$

$v=\sqrt{2gH}\rightarrow(ii)$

Comparing equation (i) & equation (ii) we can write;

$$\sqrt{2gH}=\sqrt{5gR}$$

Taking square on both sides we get;

$$2gH=5gR$$

$$H=\frac{5}{2}\times R$$

- **A particle is projected so as to just move along a vertical circle. The ratio of the tension in the string when the particle is at the lowest and highest point on the vertical circle is;**

(a) Zero (b) 1 (c) Finite but large **(d) Infinite**

Solution: At the highest point there is weightlessness.

- **In rotational motion, moment of inertia (I) plays the same role as which quantity in linear motion?**

(a) Force (b) Velocity. (c) Mass. (d) Acceleration.

Answer: (c) Mass

Explanation: Moment of inertia is the rotational analogue of mass, representing resistance to changes in motion.

- **The moment of inertia of a mass 'm' attached to the end of a massless rod of length 'r' is:**

(a) mr (b) mr^2 (c) m/r (d) m^2r

Answer: (b) mr^2

Explanation: This is the basic formula for the moment of inertia of a point mass.

- **The moment of inertia of a body depends on its mass and also on its:**

(a) Linear velocity (b) Angular velocity

(c) Distribution of mass about the axis of rotation (d) Applied torque

Answer: (c) Distribution of mass about the axis of rotation

Explanation: Moment of inertia is not just about total mass, but how that mass is spread out relative to the pivot

- **Two cylinders of equal mass, where one has a larger diameter, will have:**

(a) The same rotational inertia

(b) The one with larger diameter has greater rotational inertia

(c) The one with smaller diameter has greater rotational inertia

(d) It depends on their material

Answer: (b) The one with larger diameter has greater rotational inertia

Explanation: More mass is distributed further from the axis of rotation for the larger diameter cylinder, increasing its moment of inertia.

SLO BASED SHORT QUESTIONS & ANSWERS

- **What physical quantity is the rotational analogue of mass?**

Ans: Moment of inertia is the rotational analogue of mass.

- **How is moment of inertia denoted, and what is its SI unit?**

Ans: Moment of inertia is denoted by 'I', and its SI unit is kg m^2.

- **Does moment of inertia depend o 'y on mass, or on other factors as well?**

Ans: Moment of inertia depends not only on the mass but also on the distribution of mass about the axis of rotation.

- **What is the rotational analogue of Newton's second law of motion (F = m.a)?**

Ans: The rotational analogue is τ = Iα (Torque = Moment of inertia × Angular acceleration)

3.5 ANGULAR MOMENTUM

Q Define and explain angular momentum. Give its SI unit and dimension.

Ans

ANGULAR MOMENTUM (MOMENT OF LINEAR MOMRNTUM):

Momentum of a body moving in a circle or rotating about an axis is called angular momentum. Angular momentum is the rotational analog of linear momentum. A particle is said to possess an angular momentum about a reference axis if its angular position changes relative to that reference axis.

$$\vec{L}=\vec{r}\times\vec{p}$$

- In SI units, it is expressed in kg-m^2 s^{-1} or J-s.
- Its dimension is $[ML^2T^{-1}]$
- It is a vector quantity
- *The **direction** of angular momentum is alc g the **axis of rotation***
- *Its direction is determined by using **right hand rule**.*

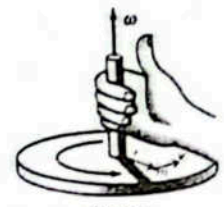


Fig 3.18

Right Hand Rule

It states that the thumb of our right-hand points in the direction of angular momentum when we curl our fingers in the direction of the rotation.

TYPES OF ANGULAR MOMENTUM

Orbital Angular Momentum

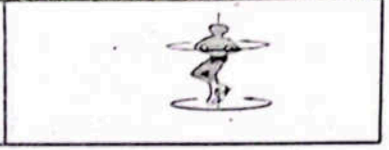
When a body revolves in an orbit, then its angular momentum is called orbital angular momentum L.

Spin Angular Momentum:

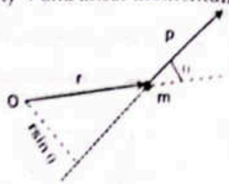
When a body rotates about its own axis then it is a spin motion and its momentum is spin angular momentum L.

Total Angular Momentum

So, the total angular momentum of a body is equal to the sum of its spin and orbital angular momentum. The orbital and spin angular momentum are very useful to explain the atomic structure. $L = L_o + L_s$

Orbital Angular Momentum L_o	Spin Angular Momentum L_s
	

Explanation: (Prove that $L_o = mvr$): Consider a body of mass 'm' is moving with velocity $\vec{v}$ and linear momentum, $\vec{p} = m\vec{v}$. It is changing its position relative to the reference axis passing through, O.



The angular momentum $\vec{L}$ possessed by the body may be expressed as:

$$\vec{L} = \vec{r} \times \vec{p} \qquad \text{------------ (i)}$$

If the angular between $\vec{r}$ and $\vec{p}$ is θ then angular momentum may be written as:

$$\vec{L} = (rp\sin\theta)\hat{n} \qquad \text{------------ (ii)}$$

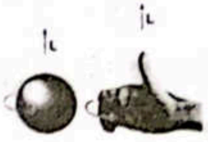
$\hat{n}$ is normal to the plane of $\vec{r}$ and $\vec{P}$ and is determined by the right-hand rule. Thus, the angular momentum is directed along the axis of rotation.

In magnitude $L = rp\sin\theta$

If θ=90° and p = mv then $L = rmv\sin 90° = mvr$

Or $L_o = mvr$

For your information



(a) (b)

The sphere in (a) is rotating in the sense given by the gold arrow. Its angular velocity and angular momentum are taken to be upward along the rotational axis, as shown by the right-hand rule in (b).

Question: Derive relation of angular momentum for a body moving in a circle?

Angular Momentum of Particle moving in circle

The angular momentum is tangent at each point of the curved path, that is θ=90° between $\vec{p}$ and $\vec{r}$. Thus the magnitude of angular momentum is

$L = rp\sin\theta$

$L = r\,mv = mvr\sin 90° = mvr$ ------------ (iii)

The relation between linear velocity 'v' and angular velocity ω is v = rω. Using this value in eq. (iii), we have;

$L = m(r\omega)r = (mr^2)\omega$ ------------ (iv)

Where mr^2 = 'I', the moment of inertia.

So, $L = I\omega$ ------------ (v)

Alternative Unit:

$$Kg\frac{m^2}{s} = \frac{Kgm^2}{s} \times \frac{s}{s}$$

$$Kg\frac{m^2}{s} = \frac{Kgm^2}{s^2} \times s$$

$$Kg\frac{m^2}{s} = J\text{-}s$$

$$(\because Kg\frac{m^2}{s^2} = J)$$

So, alternative unit of angular momentum is J-s

Question: Determine the relation of angular momentum of rigid body?

Angular Momentum of a Rigid Body

Consider a symmetric rigid body made up of large number of small mass pieces, which is rotating with angular velocity ω. A small piece of this body has mass m_i at distance r_i from the axis of rotation. The angular momentum of this piece is

$$L = (m_i r_i^2)\omega$$

Every piece of the rigid body is rotating with same angular velocity ω. The total angular momentum of the rotating rigid body is the sum of angular momentum of all the pieces i.e.

$$L = m_1 r_1^2\omega + m_2 r_2^2\omega + \ldots\ldots + m_n r_n^2\omega$$

$$L = (m_1 r_1^2 + m_2 r_2^2\omega + \ldots\ldots + m_n r_n^2)\omega$$

$$L = \sum_{i=1}^{n}(m_i r_i^2)\omega \qquad \text{------------ (vi)}$$

Where $I = \sum_{i=1}^{n}(m_i r_i^2)$ is the total moment of inertia of the rotating rigid body, so,

$L = I\omega$

What is Difference Between Spin Angular Momentum and Orbital Angular Momentum?

Spin angular momentum	Orbital angular momentum
i. The motion of body about an axis passing through the center of mass	i. The motion of body about an axis which is passing out side of mass.
ii. $L_o = r \times P$	ii. $L_s = I\omega$
iii. The radius of rotation is less than size of object.	iii. The radius of circle is greater than size of object.

Do you know?

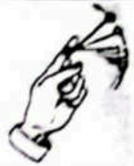
Q: Why does a moving bicycle stay upright and stable, but a bike at rest falls?

Ans: If you try to sit on a bike at rest, it falls because its center of mass is unstable. But if the bike is moving, the angular momentum of its spinning wheels resists any tendency to change its orientation. This gyroscopic effect helps to keep the bike upright and stable, illustrating the conservation of angular momentum.

Do you know?

Q: What happens to the ball's speed as the string wraps around the finger?

Ans: When the string wraps around the finger, the radius 'r' of the circular path decreases. By the conservation of angular momentum ($L = I\omega = mr^2\omega$ = constant), if 'r' decreases, and mass 'm' is constant, the angular velocity 'ω' (and thus linear speed v=rω) must increase. So, the ball's speed increases.



For Your Information:

Q: How do melting ice caps affect Earth's rotation?

A: When ice on the Earth's polar caps melts, the water flows towards the equator (increasing its average distance from the Earth's rotational axis). This causes the moment of inertia of the water, and thus the Earth's total moment of inertia, to increase slightly. By the conservation of angular momentum, the Earth's angular velocity must decrease. Consequently, the duration of a day increases slightly (though negligibly).

Point to Ponder!

Q: Why does the coasting rotating system slow down as water drips into the beaker?

Ans: As water drips from the funnel into the beaker below, the mass of the beaker-water system increases, and the mass is added at a distance from the axis of rotation. This increases the total moment of inertia (I) of the rotating system. By the law of conservation of angular momentum ($L = I\omega$ = constant), if 'I' increases, the angular velocity 'ω' must decrease, causing the system to slow down.

MULTIPLE CHOICE QUESTIONS

- **The orbital speed of the Earth is calculated using the formula:**

(a) v = πr / T (b) v = 2πrT (c) v = 2πr / T (d) v = πr² / T

Answer: (c) v = 2πr / T

Explanation: This formula calculates speed as distance traveled (circumference of the orbit) divided by time taken.

- **The orbital angular momentum of the Earth is given by:**

(a) L = mrT (b) L = mv / r (c) L = mvr (d) L = mr²v

Answer: (c) L = mvr

Explanation: This is the standard formula for angular momentum.

- **If a spinning body alters its moment of inertia, its angular speed:**

(a) Remains constant (b) Increases

(c) Decreases (d) Changes to conserve angular momentum

Answer: (d) Changes to conserve angular momentum

Explanation: The product of moment of inertia and angular speed remains constant

- **A particle performs uniform circular motion with an angular momentum L. If the angular frequency of the particle is doubled and kinetic energy is halved, its angular momentum becomes.**

(a) 4L (b) $\frac{L}{2}$ (c) 2L (d) $\frac{L}{4}$

Solution: (D) $P = \sqrt{2mE}$ so $L = \sqrt{2IE_{rot}}$ and $L^2 = 2IE_{rot} \Rightarrow L = I\omega \Rightarrow I = \frac{L}{\omega} \Rightarrow L^2 = 2 \times \frac{L}{\omega} \times E_{rot}$

$$L = 2\frac{E_{rot}}{\omega} \Rightarrow L' = 2\frac{\frac{E_{rot}}{2}}{2\omega} \Rightarrow L' = 2\frac{E_{rot}}{2 \times 2\omega} = \frac{1}{4}\left(2\frac{E_{rot}}{\omega}\right) \Rightarrow L' = \frac{1}{4} \times L \Rightarrow L' = \frac{L}{4}$$

- **The motion of planets in the solar system is an example of the conservation of:**

(a) **Angular momentum** (b) Energy (c) Linear momentum (d) Mass

Solution: All planets move with a constant angular speed and have a specific moment of inertia so, $I\omega$ =constant or L = constant

- **If net torque acting on a system is zero such that somehow it's angular speed becomes double then:**

(a) Its moment of inertia will reduce to half (b) It's angular momentum remains same

(c) Both "A" & "B" (d) None of these

Solution: Simple apply law of conservation of angular momentum.

- **Two skaters of equal mass, facing each other and holding hands with arms outstretched, are spinning at angular velocity ω about a vertical axis midway between them. If they move together so that they halve their separation, their new angular velocity is:**

(a) $\frac{w}{2}$ (b) ω (c) 2ω (d) 4ω

Solution: Since $I = mr^2$ so $I \propto r^2$ and $I \propto \frac{1}{w}$ so $\omega \propto \frac{1}{r^2}$ and $r' = \frac{r}{2}$ so $\omega' \propto \frac{4}{r^2}$ so $\omega' = 4\omega$

- **How is rotational kinetic energy of a rigid body related to angular momentum?**

(a) $L_f = IE_k$ (b) $L = \frac{E_k}{2}$ (c) $L = E_K$ (d) $L = \sqrt{2IE_K}$

Solution: $P = \sqrt{2mE}$

Converting whole equation into angular form:

$L = \sqrt{2IE_{rot}}$

- **What will be duration of day and night if earth expands to double the present radius?**

(a) 24 hrs (b) 6 hrs (c) 96 hrs (d) 3 hrs

Solution: We must remember that;

$L = I\omega$ = constant So $I \propto \frac{1}{w} \propto T$

But as $I \propto r^2$ so $T \propto r^2 \propto \frac{1}{w}$

$\therefore R' = 2R$ which makes $T' = 4T$

So $T' = 4(24 \text{ hours}) = 96$ hours

- **The angular momentum of a 20g particle moving with velocity $10\hat{i}\,ms^{-1}$ and having position vector $r = (20\hat{i} + 12\hat{j})$ m about the origin is:**

(a) $-0.2\hat{k}Js$ (b) $+0.2\hat{k}Js$ (c) $-2.4\hat{k}Js$ (d) $+2.4\hat{k}Js$

Solution: $\therefore \vec{L} = \vec{r} \times \vec{P} = \vec{r} \times m\vec{V} = m\vec{r} \times \vec{v}$

$$= \frac{20}{1000}(20\hat{i} + 12\hat{j}) \times (10\hat{i}) = \frac{2}{100} \times (120)(-\hat{k}) = \frac{240}{100}\hat{k} = -2.4\hat{k}Js$$

- **A wheel whose moment of inertia is 24 kg m² has an initial angular velocity of 60 rad s⁻¹. A constant rate of change of angular momentum is observed as 12 Nm which acts on wheel. What is the time in which wheel is accelerated to 200 rad s⁻¹?**

(a) 28sec (b) **280sec** (c) 140sec (d) 240sec

Solution: $\tau = I\alpha$

$\tau = I\left(\frac{\omega_f - \omega_i}{t}\right)$

$t = \frac{I}{\tau}(\omega_f - \omega_i) = \frac{24}{12}(200 - 60) = 2(140)$

$t = 280$ sec

- **The SI unit of angular momentum is:**

(a) $kg\ m\ s^{-2}$ (b) N m (c) $kg\ m^2\ s^{-1}$ (d) J

Answer: (c) $kg\ m^2\ s^{-1}$

Explanation: From L = mvr, the units are kg × m/s × m = $kg\ m^2\ s^{-1}$ This is also equivalent to J s

- **For a particle moving in a circle of radius 'r' with uniform angular velocity 'ω', its angular momentum is given by:**

(a) $L = m\omega$ (b) $L = mr\omega^2$ (c) $L = mr^2\omega$ (d) $L = m\omega^2$

Answer: (c) $L = mr^2\omega$ **Explanation:** Substituting $v = r\omega$ into L = mvr gives $L = mr^2\omega$, which is also $L = I\omega$

- **The direction of angular momentum (L) is perpendicular to the plane formed by:**

(a) r and v (b) r and p (c) m and v (d) ω and α

Answer: (b) r and p

Explanation: Since L = r × p, its direction is perpendicular to the plane containing both the position vector (r) and the linear momentum (p)

- **The law of conservation of angular momentum states that if no external torque acts on a system, the total angular momentum of the system:**

(a) Increases (b) Decreases (c) Remains constant (d) Becomes zero

Answer: (c) Remains constant. **Explanation:** This is the definition of the law, indicating that angular momentum is conserved in the absence of external torque

- **If a body's moment of inertia (I_1) spinning with angular speed (ω_1) alters its moment of inertia to (I_2), its new angular speed (ω_2) will satisfy:**

(a) $I_1\omega_1 = I_2\omega_2$ (b) $I_1\omega_2 = I_2\omega_1$ (c) $I_1 + \omega_1 = I_2 + \omega_2$ (d) $I_1 - \omega_1 = I_2 - \omega_2$

Answer: (a) $I_1\omega_1 = I_2\omega_2$

Explanation: This equation represents the conservation of angular momentum ($L = I\omega$ = constant)

- **The angular momentum is a vector quantity with direction along the axis of rotation. According to the conservation law, this direction:**

(a) Constantly changes (b) Remains fixed

(c) Becomes perpendicular to the axis (d) Points towards the center

Answer: (b) Remains fixed. **Explanation:** Not only the magnitude but also the direction of total angular momentum is conserved in the absence of external torque

- **If a spinning isolated body alters its moment of inertia, its angular speed also changes to maintain:**

(a) Constant kinetic energy (b) Constant linear momentum

(c) Constant angular momentum (d) Constant force

Answer: (c) Constant angular momentum. **Explanation:** The change in moment of inertia directly affects angular speed to keep the product $I\omega$ constant

- **The Earth's axis of rotation remains fixed in one direction with reference to the universe around us primarily because:**

(a) It has a very large mass

(b) No other sizeable torque is experienced by the Earth from the Sun

(c) It has a constant speed (d) Its orbital path is circular

Answer: (b) No other sizeable torque is experienced by the Earth from the Sun

Explanation: The absence of significant external torques allows the Earth's large angular momentum to be conserved, keeping its axis orientation fixed

SLO BASED SHORT QUESTIONS & ANSWERS

- **Define angular momentum of a particle.**

Ans: Angular momentum of a particle is a vector quantity defined as the vector product of its position vector (r) and its linear momentum (p), i.e., **L = r × p.**

- **What is the SI unit of angular momentum?**

Ans: The SI unit of angular momentum is kg m² s⁻¹ (or J s).

- **What is the direction of angular momentum, and what rule determines it?**

Ans: The direction of angular momentum is perpendicular to the plane formed by r and p, and its sense is given by the right-hand rule of vector product.

- **For a particle moving in a circle, what is the angle between its position vector (r) and tangential velocity (v)?**

Ans: The angle between its position vector (r) and tangential velocity (v) is 90°.

- **How is the total angular momentum of a rigid body expressed in terms of its moment of inertia (I) and angular velocity (ω)?**

Ans: The total angular momentum (L) of a rigid body is expressed as L = Iω.

3.6 LAW OF CONSERVATION OF ANGULAR MOMENTUM

Q. State and explain law of conservation of angular momentum.

Ans

LAW OF CONSERVATION OF ANGULAR MOMENTUM:

*This law states that **total angular momentum** of a system **remains constant** when **no external torque** acts on the system.*

Mathematically, it may be expressed as:

$$\vec{L} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 + \ldots\ldots\ldots\ldots + \vec{L}_n = \text{Constant}$$

Explanation:

Example of Diver:

In order to illustrate the law of conservation of angular momentum, consider a diver who pushes himself off the board with a small angular velocity about a horizontal axis through his center of gravity. Upon lifting off from the board, the diver's legs and arms are fully extended which means that diver has a large moment of inertia and is considerably reduced to a new value I_2, when the legs and arms are bent into the closed tuck position. As the angular momentum is conserved, so

$$L_1 = L_2$$

$$I_1\omega_1 = I_2\omega_2$$

Hence the diver must spin faster when moment of inertia becomes smaller to conserve angular momentum. This enables the diver to take extra somersaults.

For Your Information:

Melting ice at the polar caps increases the Earth's moment of inertia, causing its angular velocity to decrease the duration of the day to increase.

Conservation of Direction of $\vec{L}$

In this motion direction of angular momentum along the axis of rotation also remain fixed. *"Thus, the axis of rotation of an object will not change its orientation unless an external torque causes it to do so."*

Importance of Conservation of Angular Momentum

This fact is of great importance for the Earth as it moves round the Sun. The Earth experiences no other sizeable torque, because major force acting on it is the pull of the Sun. The Earth's axis of rotation, therefore remains fixed in one direction with reference to the universe around us. The law of conservation of angular momentum is important in many sports particularly in diving, gymnastics and ice-skating.

Applications of Conservation of Angular Momentum (Daily Life Examples):

1. **Diver (Fig. 3.19):**
   - A diver jumping from a springboard curls his body by rolling arms and legs inwards.
   - **Effect:** This action significantly decreases his moment of inertia (I).
   - **Result:** To conserve angular momentum (L=Iω), his angular velocity (ω) increases, allowing him to perform multiple somersaults in the air.
   - When he is about to enter the water, he stretches his limbs out, increasing his moment of inertia and decreasing his angular velocity, allowing for a smooth and controlled entry.

Fig. 3.19
A man diving from a diving board.

2. **Spinning Ice Skater (Fig. 3.20):**
   - An ice skater spinning with outstretched arms and leg has a large moment of inertia.
   - **Effect:** By folding her arms inwards and bringing her stretched leg close to the other leg, she dramatically decreases her moment of inertia.
   - **Result:** Her angular speed increases significantly, conserving angular momentum. When she extends her limbs again, her moment of inertia increases, and her angular speed decreases.

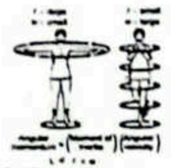


Fig. 3.20
An ice skater using angular momentum

3. **Person on a Turntable with Weights (Fig. 3.21):**
   - A person standing on a turntable with heavy masses (dumbbells) held out to the sides has a large moment of inertia.
   - **Effect:** As the person draws their hands (and weights) inwards towards their body, their moment of inertia decreases.
   - **Result:** Their angular speed on the turntable immediately increases to conserve angular momentum.

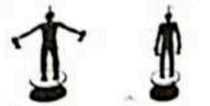


Fig. 3.21
Man with masses in his hands on a turntable. Conservation of angular momentum requires that as the man pulls his arms in, the angular velocity increases.

For your Information:

Q: **How do melting ice caps affect Earth's rotation?**

Ans: When ice on the Earth's polar caps melts, the water flows towards the equator (increasing its average distance from the Earth's rotational axis). This causes the moment of inertia of the water, and thus the Earth's total moment of inertia, to increase slightly. By the conservation of angular momentum, the Earth's angular velocity must decrease. Consequently, the duration of a day increases slightly (though negligibly).

Point to ponder!

Q: **Why does the coasting rotating system slow down as water drips into the beaker?**

Ans: As water drips from the funnel into the beaker below, the mass of the beaker-water system increases, and the mass is added at a distance from the axis of rotation. This increases the total moment of inertia (I) of the rotating system. By the law of conservation of angular momentum (L=Iω=constant), if 'I' increases, the angular velocity 'ω' must decrease, causing the system to slow down.

MULTIPLE CHOICE QUESTIONS

- **The Earth's axis of rotation remains fixed due to:**

  (a) Earth's magnetic field (b) Sun's gravitational pull (c) Earth's atmosphere (d) Earth's rotation speed

Answer: (b) Sun's gravitational pull

Explanation: The Sun's gravity exerts the major force on Earth, resulting in negligible torque.

- **A diver curls his body while diving to:**

  (a) Increase air resistance (b) Decrease air resistance
  (c) Decrease moment of inertia (d) Increase moment of inertia

Answer: (c) Decrease moment of inertia

Explanation: This decrease in moment of inertia increases angular velocity.

- **When an ice skater stretches his hands and a leg outward, his angular velocity:**
  (a) Increases (b) Decreases (c) Remains constant (d) Becomes zero

Answer: (b) Decreases

Explanation: Stretching increases the moment of inertia, which decreases angular velocity.

- **A person standing on a turntable with weights in his hands increases his angular velocity by:**
  (a) Stretching his hands outward (b) Drawing his hands inward
  (c) Lifting his hands upward (d) Lowering his hands downward

Answer: (b) Drawing his hands inward

Explanation: Drawing hands inward decreases the moment of inertia.

- **Melting ice at the polar caps causes the Earth's day to:**
  (a) Become shorter (b) Become longer
  (c) Remain the same (d) Initially shorten, then lengthen

Answer: (b) Become longer

Explanation: Melting ice increases Earth's moment of inertia, slowing its rotation and lengthening the day.

- **The law of conservation of angular momentum states that if no external torque acts on a system, the total angular momentum of the system:**
  (a) Increases. (b) Decreases. (c) Remains constant. (d) Becomes zero.

Answer: (c) Remains constant

Explanation: This is the definition of the law, indicating that angular momentum is conserved in the absence of external torque.

- **If a body's moment of inertia (I_1) spinning with angular speed (ω_1) alters its moment of inertia to (I_2), its new angular speed (ω_2) will satisfy:**
  (a) $I_1\omega_1 = I_2\omega_2$ (b) $I_1\omega_2 = I_2\omega_1$ (c) $I_1 + \omega_1 = I_2 + \omega_2$ (d) $I_1 - \omega_1 = I_2 - \omega_2$

Answer: (a) $I_1\omega_1 = I_2\omega_2$

Explanation: This equation represents the conservation of angular momentum ($L = I\omega$ = constant)

- **The angular momentum is a vector quantity with direction along the axis of rotation. According to the conservation law, this direction:**
  (a) Constantly changes. (b) Remains fixed.
  (c) Becomes perpendicular to the axis. (d) Points towards the center.

Answer: (b) Remains fixed.

Explanation: Not only the magnitude but also the direction of total angular momentum is conserved in the absence of external torque.

- **If a spinning isolated body alters its moment of inertia, its angular speed also changes to maintain:**
  (a) Constant kinetic energy. (b) Constant linear momentum.
  (c) Constant angular momentum. (d) Constant force.

Answer: (c) Constant angular momentum.

Explanation: The change in moment of inertia directly affects angular speed to keep the product $I\omega$ constant.

- **The Earth's axis of rotation remains fixed in one direction with reference to the universe around us primarily because:**
  (a) It has a very large mass.
  (b) No other sizeable torque is experienced by the Earth from the Sun.
  (c) It has a constant speed. (d) Its orbital path is circular.

Answer: (b) No other sizeable torque is experienced by the Earth from the Sun.

Explanation: The absence of significant external torques allows the Earth's large angular momentum to be conserved, keeping its axis orientation fixed.

SLO BASED SHORT QUESTIONS & ANSWERS

- **Why does the Earth's axis of rotation remain fixed?**

Ans: Because the Sun's gravitational pull exerts no significant torque on the Earth.

- **How does a diver use the conservation of angular momentum to perform somersaults?**

Ans: By curling his body to decrease his moment of inertia and increase his angular velocity.

- **What happens to a diver's body just before entering the water and why?**

Ans: He stretches out his arms and legs to increase his moment of inertia and decrease his angular velocity for a smooth dive.

- **How does an ice skater increase their angular velocity?**

Ans: By folding their arms and bringing their stretched leg close to the other leg, which decreases their moment of inertia.

- **What happens to an ice skater's angular velocity when they stretch their arms and a leg outward?**

Ans: The angular velocity decreases because the moment of inertia increases.

- **What happens to a person's angular velocity when they draw their hands inward while standing on a turntable with weights in their hands?**

Ans: Their angular velocity increases because their moment of inertia decreases.

- **How does melting ice at the polar caps affect the Earth's rotation?**

Ans: It increases the Earth's moment of inertia, causing its angular velocity to decrease and the duration of the day to increase.

- **State the law of conservation of angular momentum.**

Ans: The law of conservation of angular momentum states that if no external torque acts on a system, the total angular momentum of the system remains constant.

- **What is the direction of angular momentum for a rotating object, and what happens to this direction under conservation?**

Ans: Angular momentum is a vector quantity whose direction is along the axis of rotation, and under conservation, this direction also remains fixed.

- **If a body's moment of inertia decreases, how does its angular speed change to conserve angular momentum?**

Ans: If a body's moment of inertia decreases, its angular speed increases to conserve angular momentum ($I\omega$ = constant).

- **Why is the law of conservation of angular momentum considered one of the fundamental principles of Physics?**

Ans: It is considered fundamental because it has been verified from cosmological to sub-microscopic levels.

- **What property of the Earth's interaction with the Sun ensures its axis of rotation remains fixed?**

Ans: The fact that the Earth experiences no other sizeable torque from the Sun ensures its axis of rotation remains fixed due to angular momentum conservation.

FLYWHEEL:

- A flywheel is a mechanical device with a heavy wheel and an axle.
- It stores rotational energy, it makes the performance, power, or value that comes out of a system more consistent and less variable.
- It aims to achieve a more stable or desired average performance, and provides stability.
- Used in bicycles, vehicles, industrial machinery, gyroscopes, ships, and spacecraft.
- A spinning flywheel resists changes to its orientation due to its angular momentum.

The flywheel called the balance wheel regulates the time keeping mechanism in mechanical clocks and watches by maintaining controlled oscillations rate.

Fig. 3.22 A flywheel

GYROSCOPE:

- A gyroscope is a device that maintains its orientation relative to Earth.
- It consists of a mounted flywheel pivoted in supporting rings.
- It works on the principle of conservation of angular momentum.
- A gyroscope spinning at high speed has large angular momentum, making it difficult to change its rotational axis orientation.
- Changing the orientation requires a large torque.
- A tilted gyroscope remains levitated without falling.
- Gyroscopes are used to maintain orientation.

Do you know?

Q: What is the role of the balance wheel in mechanical clocks?

Ans: The flywheel, specifically called the balance wheel, regulates the time-keeping mechanism in mechanical clocks and watches by maintaining a controlled oscillation rate, which provides precise stability.

TALENT INFORMATION

Earth as a Natural Gyroscope:

- **Spinning Mass:** The Earth is a massive object that is constantly spinning on its axis (its daily rotation). This rotation gives it significant angular momentum.
- **Conservation of Angular Momentum:** Like any gyroscope, the Earth tends to resist changes in the orientation of its rotational axis. This is due to the principle of conservation of angular momentum. A spinning object will maintain its orientation unless acted upon by an external torque.

Precession of the Earth's Axis:

While the Earth's rotation axis is relatively stable, it's not perfectly fixed in space. It undergoes a slow, continuous wobble, much like a spinning top that is slowing down. This wobble is called precession.

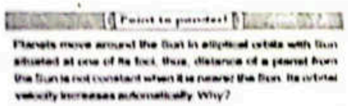
Point to Ponder

"Planets move around the Sun in elliptical orbits with Sun situated at one of its foci, thus, distance of a planet from the Sun is not constant when it is nearer the Sun. Its orbital velocity increases automatically. Why?"

Answer: When a planet gets closer to the Sun in its elliptical orbit, its orbital velocity increases. This happens because of the **conservation of angular momentum**. Imagine the planet spinning around the Sun: as it gets closer, the "lever arm" (distance from the Sun) shortens, so to keep its "spin" (angular momentum) constant, it has to speed up. This is also described by **Kepler's Second Law**, which states that a planet sweeps out equal areas in equal times, a faster speed is needed when closer to the Sun to cover the same area.

TALENT INFORMATION: Kepler's Laws of Planetary Motion describe how planets orbit the Sun

1. **Law of Orbits (First Law):** Planets move in **elliptical orbits** with the Sun located at one of the two foci of the ellipse. This means orbits aren't perfect circles.
2. **Law of Areas (Second Law):** A line connecting a planet to the Sun sweeps out **equal areas in equal intervals of time**. This implies that planets move faster when they are closer to the Sun and slower when they are farther away.


Point to ponder!
Planets move around the Sun in elliptical orbits with Sun situated at one of its foci, thus, distance of a planet from the Sun is not constant when it is nearer the Sun. Its orbital velocity increases automatically. Why?

3. **Law of Periods (Third Law): The square of a planet's orbital period** (the time it takes to complete one orbit) **is directly proportional to the cube of the semi-major axis** of its orbit (the average distance from the Sun). In simpler terms, planets farther from the Sun take disproportionately longer to orbit.

MULTIPLE CHOICE QUESTIONS

- **When a person on a turntable draws their hands inward, their angular speed:**
  (a) Decreases (b) Increases (c) Remains constant (d) Becomes zero

Answer: (b) Increases
Explanation: This is because drawing the hands inward decreases the moment of inertia.

- **A flywheel is used to:**
  (a) Increase friction (b) Store rotational energy (c) Decrease stability (d) Increase output fluctuations

Answer: (b) Store rotational energy
Explanation: Flywheels are designed to store rotational energy.

- **Which of the following devices uses a flywheel to maintain orientation?**
  (a) Electric fan (b) Refrigerator (c) Gyroscope (d) Washing machine

Answer: (c) Gyroscope
Explanation: Gyroscopes use flywheels to maintain their orientation.

- **A gyroscope works on the principle of:**
  (a) Conservation of energy (b) Conservation of linear momentum
  (c) Conservation of angular momentum (d) Newton's first law

Answer: (c) Conservation of angular momentum

Explanation: The gyroscope's stability depends on the conservation of angular momentum.

- **Why is it difficult to change the orientation of a spinning gyroscope?**
  (a) Small moment of inertia (b) Large moment of inertia
  (c) Zero angular momentum (d) Low spinning speed

Answer: (b) Large moment of inertia
Explanation: A large moment of inertia gives it a large angular momentum, resisting changes in orientation.

- **Which of the following is NOT a common application area for flywheels?**
  (a) Bicycles (b) Industrial machinery (c) Navigation systems (d) Chemical reactions

Answer: (d) Chemical reactions
Explanation: Flywheels are mechanical devices for energy storage and stability in mechanical and navigation systems.

- **A flywheel's angular momentum allows it to resist changes to its orientation, thereby maintaining:**
  (a) Its linear velocity (b) Its temperature (c) Stability (d) Its mass

Answer: (c) Stability
Explanation: The large angular momentum of a spinning flywheel makes it resistant to changes in its rotational axis orientation, providing stability.

- **In mechanical clocks and watches, the balance wheel, a type of flywheel, regulates the time-keeping mechanism by maintaining:**
  (a) Constant torque (b) Controlled oscillations rate
  (c) Constant angular momentum (d) High linear speed

Answer: (b) Controlled oscillations rate
Explanation: The balance wheel acts as a resonator, controlling the escapement mechanism to keep time accurately.

- **A flywheel's ability to smooth out output fluctuations makes it useful in devices with:**
  (a) Constant power output (b) Variable power output (c) No moving parts (d) Only linear motion

Answer: (b) Variable power output
Explanation: Flywheels store energy during high power phases and release it during low power phases, smoothing the overall output.

- **A gyroscope consists of a mounted flywheel pivoted in supporting rings. When the flywheel spins at a large angular speed, it gains:**
  (a) Large linear momentum (b) Large kinetic energy (c) Large angular momentum (d) Large torque

Answer: (c) Large angular momentum
Explanation: High angular speed and moment of inertia lead to significant angular momentum.

- **Why is it difficult to change the orientation of a gyroscope's rotational axis?**
  (a) It has a very small mass (b) It experiences strong gravitational force
  (c) It has a large moment of inertia and large angular momentum
  (d) Its pivot points are fixed

Answer: (c) It has a large moment of inertia and large angular momentum
Explanation: A large angular momentum requires a corresponding large external torque to change its direction.

- **Which of the following is NOT a main application of gyroscopes?**
  (a) Guiding system of airplanes (b) Controlling kitchen appliances
  (c) Guiding system of submarines (d) Guiding system of space vehicles

Answer: (b) Controlling kitchen appliances
Explanation: Gyroscopes are used in navigation and stabilization for vehicles that need precise orientation control.

- **Even if a gyroscope is tilted, it still keeps levitated without falling because:**
  (a) It becomes weightless
  (b) Its angular momentum resists the change in orientation
  (c) It generates its own anti-gravity (d) Its pivot points create an upward force

Answer: (b) Its angular momentum resists the change in orientation
Explanation: This is a key demonstration of the gyroscopic effect, where the precessional motion occurs instead of falling.

SLO BASED SHORT QUESTIONS & ANSWERS

- **Why does angular speed increase when a person on a turntable draws their hands inward?**

Ans: Because drawing the hands inward decreases the moment of inertia.

- **What is a flywheel?**

Ans: A mechanical device consisting of a heavy wheel with an axle.

- **What are the uses of a flywheel?**

Ans: To store rotational energy, smooth out output fluctuations, and provide stability.

- **Where are flywheels used?**

Ans: In bicycles, vehicles, industrial machinery, gyroscopes, ships, and spacecraft.

- **Why does a spinning flywheel resist changes to its orientation?**

Ans: Due to its angular momentum.

- **What is a gyroscope?**

Ans: A device used to maintain its orientation relative to Earth.

- **How does a gyroscope work?**

Ans: On the principle of conservation of angular momentum, due to its large moment of inertia.

- **Why is it difficult to change the orientation of a spinning gyroscope's rotational axis?**

Ans: Because it has a large angular momentum.

- **What is required to change the direction of a large angular momentum?**

Ans: A correspondingly large torque.

- **How does a flywheel contribute to the stability of a system?**

Ans: A spinning flywheel resists changes to its orientation due to its angular momentum, thereby maintaining stability.

- **In what specific way does a balance wheel function in mechanical clocks?**

Ans: The balance wheel regulates the time-keeping mechanism by maintaining a controlled oscillations rate.

- **How does a flywheel help in smoothing out output fluctuations?**

Ans: It helps by storing excess rotational energy during periods of high input and releasing it during periods of low input, thus stabilizing the output.

- **Why is it difficult to change the orientation of a spinning gyroscope's rotational axis?**

Ans: It is difficult because a spinning gyroscope gains large angular momentum, and changing the direction of a large angular momentum requires a corresponding large torque.

- **List two main applications of gyroscopes.**

Ans: Two main applications are in the guiding systems of aeroplanes and submarines.

- **What happens if a gyroscope is tilted while spinning, and why?**

Ans: If a gyroscope is tilted, it still keeps levitated without falling because its large angular momentum resists the change in orientation, leading to precession.

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TEXT BOOK EXERCISE WITH SOLUTION

MULTIPLE CHOICE QUESTIONS

Tick (✓) the correct answer

Multiple Choice Questions

3.1 The ratio of angular speed of minute's hand and hour's hand of a watch is:

(a) 1:6 (b) 6:1 (c) 1:12 (d) 12:1

- **Answer:** (d) 12:1
- **Explanation:**
  - Angular speed of minute hand (ω_m): Completes 1 revolution in 60 minutes. $\omega_m = 2\pi/(60 \text{ min})$.
  - Angular speed of hour hand (ω_h): Completes 1 revolution in 12 hours (12 × 60 minutes = 720 minutes). $\omega_h = 2\pi/(720 \text{ min})$.
  - Ratio $\omega_m/\omega_h = (2\pi/60)/(2\pi/720) = 720/60 = 12/1 = 12:1$.

3.2 A body traveling in a circle at constant speed:

(a) has constant velocity

(b) has an inward radial acceleration

(c) is not accelerated

(d) has an outward radial acceleration

- **Answer:** (b) has an inward radial acceleration
- **Explanation:** Even at constant speed, a body in circular motion continuously changes direction, which means its velocity (a vector) is constantly changing. A change in velocity implies acceleration. This acceleration, called centripetal (or radial) acceleration, is always directed towards the center of the circle (inward).

3.3 A stone at the end of long string is whirled in vertical circle at a constant speed. The tension in the string will be minimum when the stone is:

(a) at the top of the circle

(b) half way down

(c) at the bottom of circle

(d) anywhere in the circle

- **Answer:** (a) at the top of the circle
- **Explanation:** At the top of the vertical circle, the stone's weight (mg) acts downwards, in the same direction as the tension (T) needed for centripetal force. The required centripetal force (mv^2/r) is provided by T + mg. So, $T = (mv^2/r) - mg$. To maintain the circle, tension is still required, but it is minimal because gravity is helping to pull the stone towards the center. At the bottom, tension must overcome both gravity and provide the centripetal force, so it is maximum.

3.4 Every point of rotating rigid body has:

(a) same linear velocity

(b) same angular velocity

(c) same linear acceleration

(d) same linear distance

- **Answer:** (b) same angular velocity
- **Explanation:** In a rotating rigid body, all points rotate through the same angle in the same amount of time, meaning they all have the same angular displacement, angular velocity, and angular acceleration. However, points at different radii will have different linear velocities and linear accelerations ($v = r\omega$, $a_t = r\alpha$).

3.5 The minimum velocity necessary to put a satellite into the orbit is called:

(a) terminal velocity (b) critical velocity

(c) artificial velocity (d) angular velocity

- **Answer:** (b) critical velocity
- **Explanation:** The minimum velocity required for a satellite to achieve a stable orbit (usually a low Earth orbit) is known as the critical velocity.

3.6 An astronaut is orbiting around the Earth in a large capsule. Then,

(a) he is freely falling towards the Earth

(b) he will be in a state of weightlessness with respect to capsule

(c) a ball projected at an angle has a straight line path as observed by him

(d) all the above

- **Answer:** (d) all the above
- **Explanation:**
  - (a) An orbiting astronaut and capsule are continuously "falling" towards Earth due to gravity, but their tangential velocity keeps them in orbit.
  - (b) Because they are freely falling together, there is no normal force, and the astronaut experiences apparent weightlessness relative to the capsule.
  - (c) Due to weightlessness (absence of

significant gravitational force relative to the falling frame), any ball projected by the astronaut would follow a straight line path (Newton's first law) within the capsule.

- Therefore, all these statements are true

3.7 **An object in uniform circular motion makes 10 revolutions in 2 seconds. Which of the following statement is true?**

(a) Its period is 2.0 s
(b) Its frequency is 5 Hz
(c) Its period is 20 s
(d) Its frequency is 0.2 Hz

- **Answer:** (b) Its frequency is 5 Hz
- **Explanation:**
  - Frequency (f) = Number of revolutions / Time taken = 10 revolutions / 2 s = 5 rev/s = 5 Hz
  - Period (T) = 1 / f = 1 / 5 Hz = 0.2 s.
  - So, statement (b) is true

3.8 **A man inside the artificial satellite feels weightlessness because the force of attraction due to the Earth is:**

(a) zero at pole
(b) balanced by the force of attraction due to the moon
(c) equal to the centripetal force
(d) non-effective due to some particular design of the satellite

- **Answer:** (c) equal to the centripetal force
- **Explanation:** The reason for apparent weightlessness is that the gravitational force of attraction from the Earth provides exactly the centripetal force needed for the satellite and its contents to orbit. Since this force is entirely used for circular motion, there is no residual force to create a "normal force" sensation, leading to the feeling of weightlessness.

3.9 **A bottle of soda water is grasped from the neck and swung briskly in a vertical circle. Near which portion of the bottle do the bubbles collect?**

(a) Near the bottom
(b) In the middle of bottle
(c) Bubbles remain distributed throughout the volume of the bottle
(d) Near the neck of the bottle

- *Answer: (a) Near the bottom*

Explanation: In a rotating system, the denser components tend to move outwards due to the requirement for a larger centripetal force. Bubbles (gas) are less dense than the soda water. Therefore, the denser water is pushed outwards towards the "bottom" (outermost part of the circle), and the lighter bubbles collect inwards, near the axis of rotation or towards the inner part of the curve, which would be the "bottom" of the bottle when it's swinging in a circle.

3.10 **The moment of inertia of body depends upon:**

(a) mass of the body and its distribution about axis of rotation
(b) volume of the body
(c) kinetic energy of the body
(d) angular momentum of the body

- **Answer:** (a) mass of the body and its distribution about axis of rotation
- **Explanation:** Moment of inertia is fundamentally dependent on both the total mass of the body and how that mass is distributed relative to the axis of rotation. The further the mass is from the axis, the greater the moment of inertia.

SHORT ANSWER QUESTIONS

3.1 **State second law of motion in case of rotation.**

Ans: The sum of the torques on a rotating system or a body about a fixed axis equals moment of inertia time angular acceleration

$$\sum \tau = I\alpha$$

This is the rotational analog to Newton's second law of linear motion.

3.2 **What is the effect of changing the position of a diver while diving in the pool?**

Ans: Changing the position of a diver while diving affects their moment of inertia and, by the law of conservation of angular momentum, their angular speed. When a diver curls their body (tucks limbs inwards), their moment of inertia decreases, causing their angular speed to increase, allowing them to perform multiple somersaults. When they stretch their limbs out, their moment of inertia increases, and their angular speed decreases, allowing for a controlled entry into the water. $L = I\omega$

3.3 **How do we get butter from the milk?**

Ans: We get butter (or cream) from milk using a centrifuge by exploiting the difference in densities between milk components. Milk is

placed in a centrifuge and spun at high speeds. The denser components (skim milk) are forced outwards towards the bottom of the container, while the lighter components (cream/fat) collect closer to the center of rotation, making it easy to separate the cream, which can then be churned into butter.

3.4 **Mass is a measure of inertia in linear motion. What is its analogue in rotational motion? Describe briefly.**

Ans: The analogue of mass in linear motion for rotational motion is moment of inertia ($I = mr^2$).

Description: Just as mass quantifies an object's resistance to changes in its linear state of motion (linear acceleration), moment of inertia quantifies a rigid body's resistance to changes in its rotational state of motion (angular acceleration). It depends not only on the total mass but also on how that mass is distributed relative to the axis of rotation; the further the mass is from the axis, the greater the moment of inertia.

3.5 **Why is it harder for a car to take turn at higher speed than at lower speed?**

Ans: When a car takes a turn require centripetal force and this centripetal force is provided by the friction between road and tyre, which is also called grip of tyre.

The requirement of centripetal force is directly proportional to the square of speed of car ($F = mv^2/r$). A little increase in speed considerably increase the requirement of centripetal force and grip of tyre become less than required centripetal force at high speed and the car may skid out of road.

3.6 **What are the benefits of using rare wheels of heavy vehicles consisted of double tires?**

Ans: The advantages of double rear wheels in heavy vehicles provide better stability with large surface area in contact with road and better grip. This also reduce tyre wear due to distribution of weight over four tyres instead of two tyre.

3.7 **When a moving car turns around a corner to the left, in what direction do the occupants tend to fall? Explain briefly.**

Ans: When a moving car turns around a corner to the left, the occupants tend to fall (or feel pushed) to the right.

Explanation: This is due to inertia. Before the turn, the occupants were moving in a straight line. As the car turns left, their inertia (tendency to resist a change in motion) causes them to continue moving in a straight line relative to their original direction. Since the car itself is changing direction to the left, the occupants effectively feel a push to the right, which is the direction opposite to the center of the turn. This sensation is often referred to as a "fictitious force" or centrifugal force, but it's a consequence of inertia.

3.8 **Why is the acceleration of a body moving uniformly in a circle, directed towards the centre?**

Ans: The acceleration of a body moving uniformly in a circle is directed towards the center because, even though its speed is constant, its velocity is continuously changing direction. Velocity is a vector quantity, and its direction is always tangent to the circular path. For the direction to change in a circle, a force (and thus an acceleration $F \propto a$) must constantly be applied perpendicular to the velocity vector, always pointing towards the center of the circle. This is known as centripetal (center-seeking) acceleration.

3.9 **How does an astronaut feel weightlessness while orbiting from the Earth in a space-ship?**

Ans: When a satellite is launched by a rocket in its desired orbit around the Earth, then it has been observed practically that everything inside the satellite experiences weightlessness because the satellite is accelerating towards the centre of the Earth as a freely falling body.

The net force acting on astronaut

$$F_N = mg - mg = 0$$

This shows that the supporting force which is acting on a body inside the satellite is zero.

Therefore, the bodies as well as the astronauts in a satellite find themselves in a state of apparent weightlessness.

CONSTRUCTED RESPONSE QUESTIONS

3.1 **If angular velocity of different particles of a rigid body is constant, will the linear velocity of these particles also constant?**

Ans: If the angular velocity of particles in a rigid body is constant but the linear velocities will not be constant. The relationship between linear and angular velocity is given by $v = r\omega$, it shows that v is directly related to r, the distance from the axis of rotation. Since the distance r is different for different particles in the rigid body from axis of rotation, even with a constant angular velocity ω, the linear velocities v will be different and not constant.

3.2 A loaf of bread is lying on rotating plate. A crow takes away the loaf of bread and the plates rotation increase. Why?

- **Ans:** The rotation of plate increases according to law of conservation of angular momentum. When the crow takes away the loaf of bread, the mass decreases which also decrease moment of inertia and the angular velocity increases. According to the conservation of angular momentum ($L=I\omega$=constant), if the moment of inertia (I) decreases, the angular velocity (ω) must increase proportionally to keep the product constant. This is why the plate's rotation speeds up.

3.3 Why do we tumble when we take the sharp turn with large speed?

Ans: We tumble or feel like tumbling when taking a sharp turn at high speed due to inertia and the insufficient centripetal force.

Explanation:

- **Inertia:** Our bodies, by inertia, tend to continue moving in a straight line tangential to the curve.
- **Centripetal Force:** To make a turn, a centripetal force is required, pulling us (and the vehicle) towards the center of the curve. This force is usually provided by friction between the tires and the road, or by the normal force on a banked road.
- **Insufficient Force:** At high speeds, the required centripetal force ($F_c=mv^2/r$) becomes very large, especially for a sharp turn (small 'r'). If the available friction or other forces cannot provide this sufficient centripetal force, our bodies (and the vehicle) will tend to continue moving tangentially. This outward tendency, from an inertial frame of reference, makes us feel like we are "thrown outwards" or that we might tumble, as we struggle to follow the sharp curve.

3.4 What will be time period of a simple pendulum in an artificial satellite?

Ans: The time period of a simple pendulum in an artificial satellite is infinite. The effective value of acceleration due to gravity is zero inside the artificial satellite.

The time period of a simple pendulum is

$$T = 2\pi\sqrt{\frac{l}{g}}$$

If $g = 0$ then $T = \infty$

Consequently, the pendulum will not oscillate and will appear to be in a state of free fall.

3.5 Is the motion of a satellite in its orbit, uniform or accelerated?

Ans: The motion of satellite in its orbit is accelerated motion. The speed of satellite in its orbit is uniform but the velocity is not constant due to change in direction. The change in direction is the cause of acceleration and this acceleration is called centripetal acceleration provided by gravitational force toward the centre of the circular orbit.

3.6. What are the advantages that radian has been preferred as SI unit over degree?

Ans: The angle in Radian is considered as more natural than degrees because it is directly related to the arc length on the circle. This makes easier to work with in trigonometric functions and calculus.

Coherence with SI Units: The radian is a dimensionless unit (length/length), making it coherent with other SI units and simplifying dimensional analysis in equations. Degrees are an arbitrary division of a circle.

3.7 In uniform circular motion, what are the average velocity and average acceleration for one revolution? Explain.

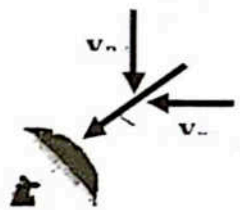
Ans: In uniform circular motion, for one complete revolution:

- **Average Velocity:** The average velocity is zero.
  - **Explanation:** Average velocity is defined as total displacement divided by total time ($v_{avg}=\Delta d/\Delta t$). For one complete revolution, the object starts and ends at the same point. Therefore, the total displacement (Δd) is zero. Since the numerator is zero, the average velocity is zero, regardless of the time taken.
- **Average Acceleration:** The average acceleration is also zero.
  - **Explanation:** Average acceleration is defined as the change in velocity divided by the time interval ($a_{avg}=\Delta v/\Delta t$). In uniform circular motion, the *speed* is constant, but the *velocity vector* is continuously changing direction. However, after one complete revolution, the object returns to its starting point with the exact same velocity vector (same magnitude and same direction). Therefore, the change in velocity ($\Delta v=v_f-v_i$) is zero ($v_f=v_i$). Since the change in velocity is zero, the average acceleration over one full revolution is zero.

3.8 In a rainstorm with a strong wind, what determines the best position to hold an umbrella?

Ans: The best position to hold an umbrella in a rainstorm with a strong wind against the direction of the resultant vector of the raindrops' velocity and the wind's velocity.

For example, if the wind is blowing towards the west from east and the raindrops are falling vertically downward, the resultant vector will be pointing towards the west from the vertical line. So the umbrella must facing toward east from the vertical line. The best position for the umbrella is to orient it against the resultant direction of the wind and rain, minimizing the amount of rain hitting the person. This usually means tilting the umbrella slightly into the wind.



3.9 A ball is just supported by a string without breaking. If it is whirled in a vertical circle, it breaks. Explain why?

Ans: When a ball is just supported by a string, the tension equals its weight (mg). However, when whirled in a vertical circle, the tension constantly changes. At the **bottom of the circle,** the string must not only support the ball's weight but also provide the additional centripetal force needed to keep it moving in a circle ($T=mg+mv^2/r$).

This means the tension at the bottom (T_{bottom}) is significantly greater than the ball's weight (mg). Since the string's breaking strength was only slightly above mg, this increased tension exceeds its limit, causing it to break.

3.10 How the centripetal force supplied in the following cases:

(a) A satellite orbiting around the Earth.
(b) A car taking a turn on a level road.
(c) A stone whirled in a circle by means of a string.

(a) The gravitational force provides the necessary centripetal force to keep the satellite in circular orbit.
(b) The friction between road and tyre or tyre grip provide the necessary centripetal force
(c) The tension in a string provide centripetal force.

COMPREHENSIVE QUESTIONS

3.1 **What is meant by angular momentum? Explain the law of conservation of angular momentum with daily life examples.**

3.2 **Show that orbital angular momentum; L = Iω**

3.3 **Define moment of inertia. Prove that torque acting on rotating rigid body is equal to the product of its moment of inertia and angular momentum.**

3.4 **What are artificial satellites? Calculate the minimum time period necessary to put a satellite into the orbit.**

3.5 **Define orbital velocity. Derive an expression for the same.**

3.6 **Write a note on artificial gravity. Derive an expression for frequency with which the spaceship rotates to provide artificial gravity.**

3.7. **Prove that;**
(i) $v=r\omega$ and (ii) $a=r\alpha$

SOLVED EXAMPLES

EXAMPLE 3.1:

An electric fan rotating at 3 rev s^{-1} is switched OFF. It comes to rest in 18.0 s. Assuming deceleration to be uniform, find its value. How many revolutions did it turn before coming to rest?

Solution: In this problem, we have

$\omega_i = 3.0$ rev s^{-1}, $\omega_f = 0$, t = 18.0 s and α = ?, θ = ?

From equation 3.9, we have

$$\alpha = \frac{w_f - w_i}{t} = \frac{(0-3.0)\,rev\,s^{-1}}{18.0s} = 0.167\,rev\,s^{-1}$$

and, we have

$\theta = \omega_i t + \frac{1}{2}\alpha t^2$

$\theta = 3.0\ rev\ s^{-1} \times 18.0\ s + \frac{1}{2}(-0.167\ rev\ s^{-2})$

$\times (18.0\ s)^2$

$\theta = 54\ rev - 27\ rev = 27\ rev$

EXAMPLE 3.2:

If a CD spins at 210 rpm, what is the radial acceleration of a point on the outer rim of the CD? The CD is 12 cm in diameter.

Solution:

We convert 210 rpm into a frequency in revolutions per second (Hz).

Thus $f = 210\ \frac{rev}{min} \times \frac{1}{60}\frac{min}{s} = 3.5\ \frac{rev}{s} = 3.5\ Hz$

For each revolution, the CD rotates through an angle of 2π radians. The angular velocity is:

$\omega = 2\pi f = 2\pi \times 3.5 = 7.0\pi\ rad\ s^{-1}$

The radial acceleration is:

$a = r\omega^2 = (7.0\pi\ rad\ s^{-1})^2 \times 0.06\ m = 29\ m\ s^{-2}$

EXAMPLE 3.3:

A ball tied to the end of a string, is swung in a vertical circle of radius r under the action of gravity as shown in Fig. 3.7. What will be the tension in the string when the ball is at the point A of the path and its speed is v at this point?

Solution:

For the ball to travel in a circle, the force acting on the ball must provide the required centripetal force. In this case, at point A, two forces act on the ball, the pull of the string and the weight w of the ball. These forces act along the radius at A, and so their vector sum must furnish the required centripetal force. We, therefore, have

$T + w = \frac{mv^2}{r}$ as $w = mg$

$T = \frac{mv^2}{r} - mg = m\left[\frac{v^2}{r} - g\right] mv^2/r + mg$

If $\frac{v^2}{r} = g$, then T will be zero and the centripetal force is just equal to the weight.

EXAMPLE 3.4

An Earth is in circular orbit at distance of 384,000 km from the Earth's surface. What is its period of one revolution in days? Take mass of the Earth M = 6.0 $\times 10^{24}$ kg and its radius R = 6400 km.

Solution:

As $r = R + h = (6400 + 384000)\ km = 390400\ km$

Using $v = \sqrt{\frac{GM}{r}}$

$= \sqrt{\frac{6.67\times10^{-11}\ N\ m^2\ kg^{-2} \times 6\times10^{24}\ kg}{390400\ km}}$

$= 1.025\ km\ s^{-1}$

Also

$T = \frac{2\pi R}{v}$

$= \frac{2 \times 3.14 \times 390400\ km}{1.025\ km\ s^{-1}} \times \frac{1\ day}{60 \times 60 \times 24\ s}$

$= 27.7$ days

EXAMPLE 3.5

The mass of the Earth is 6.0 $\times 10^{24}$ kg. The distance r from Earth to the Sun is 1.50 $\times 10^{11}$ m. As seen from the direction of the North Star, the Earth revolves counter-clockwise around the sun. Determine the orbital angular momentum of the Earth about the sun, assuming that it traverses a circular orbit about the Sun once a year (3.10^7 s).

Solution:

To find the Earth's orbital angular momentum, we must first know its orbital speed from the given data. When the Earth moves around a circle of radius r, it travels a distance $2\pi r$ in one year. Its orbital speed $v_o = \frac{2\pi r}{t}$

Orbital angular momentum of the Earth

$L_o = m\ v_o\ r$

$L_o = \frac{2\pi r^2\ m}{t}$

$L_o = \frac{2\pi(1.50\times10^{11}\ m)^2 \times (6.00\times10^{24}\ kg)}{3.16\times10^7\ s}$

$L_o = 2.67 \times 10^{40}\ kg\ m^2\ s^{-1}$

The sign is positive because the revolution is counter clockwise.

NUMERICAL PROBLEMS

3.1 **A laser beam is directed from the Earth to the moon. The beam spreads over a diameter of 2.50 cm at the moon surface, what is divergence angle of the beam? The distance of moon from the Earth is 3.8 x 10^8m.**

Given:

Diameter of Laser beam at the Moon = S = 2.5 cm

Distance of Moon from the Earth = r = 3.8×10^8m

To fine:

Angle of divergence = θ = ?

Calculation:

As we know that,

$S = r\theta$

$\theta = \frac{S}{r}$

$\theta = \frac{2.5}{3.8\times10^8}$

$\theta = 6.578\times10^{-9}$ rad

$\theta = 6.6\times10^{-9}$ rad Ans.

3.2 **A car is moving with a speed of 108 km h^{-1}. If its wheel has a diameter of 60 cm, find its angular speed in rad s^{-1} and rev s^{-1}.**

Given:

Speed of car = v = 108 kmh^{-1}

$= \frac{108\times1000}{3600} = 30\ ms^{-1}$

Diameter of wheel = D = 60 cm = 0.6 m

Radius of wheel = r = 0.3 m

Time interval, = Δt = 18.0s

To Find:

Angular velocity = ω = ?

Solution:

$\omega = \frac{v}{r}$

$\omega = \frac{30}{0.3}$

$\omega = 100\ rad\ s^{-1}$

or $\omega = 100 \times \frac{1}{2\pi}\ revs^{-1}$

$= 15.91\ rad\ s^{-1} = 16\ rev\ s^{-1}$

3.3 ***An electric motor is running at 1800 rev min^{-1}. On switching OFF it comes to rest in 20 s. If angular retardation is uniform, find the number of revolutions it makes before stopping.***

Given:

Initial angular velocity, = ω_i = 1800 rev min^{-1} = 30 rev s^{-1}

Final angular velocity, = ω_f = 0 rev s^{-1}

Time interval= Δt = 20s

To Find:

Angular deceleration= α= ?

No. of revolution completed=θ=?

Solution:

$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{0-30}{20} = -1.5\ rev\ s^{-2}$

Now $\theta = \omega_i t + \frac{1}{2}\alpha t^2$

$\theta = 30 \times 20 + \frac{1}{2}(-1.5)(20)^2$

$\theta = 600 - 300 = 300$ rev

3.4 **A string 0.5 m long holding a stone can withstand maximum tension of 35.6 N. Find the maximum speedat which a stone of 0.5 kg can be whirled with it in a vertical circle.**

Given

Length of string = l = r = 0.5 m

Maximum tension = T_{max} = 35.6 N

Mass of stone = 0.5 kg

Solution

For vertical circle the maximum tension will be at the bottom of circle i.e.

$T_{max} = \frac{mv^2}{r} + mg$

⇒ $v^2 = r(\frac{T_{max}}{m} - g)$ ---------- (i)

$v^2 = 0.5(\frac{35.6}{0.5} - 9.8) = 30.7 m^2 s^{-2}$

$v = 5.57\ m\ s^{-1}$

3.5 ***The flywheel of an engine is rotating at 2100 rev min^{-1}.when the power source is shut off. What torque is required to stop it in 3 minutes? The moment of inertia of the flywheel is 36 kg m^2.***

Given

Initial angular velocity = ω_i = 2100 rev min^{-1}

$= 2100 \times 2\pi/60 = 219.8\ rad\ s^{-1}$

Final angular velocity = ω_f = 0 rad s^{-1}

Time taken = t = 3 min = 3 × 60 = 180 s

Moment of inertia of flywheel = I = 36 kg m^2

To find

Acceleration of flywheel = ?

Torque on flywheel = τ = ?

Solution:

$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{0-219.8}{180}$

$= -1.22\ rad\ s^{-2}$

Now $\tau = I\alpha$

$\tau = -36 \times 1.22 = -43.96 = -44\ N\ m$

The negative sign indicates that the torque opposes the initial rotation.

3.6 **What is the moment of inertia of a 200 kg sphere *whose diameter is 60 cm.***

Given

Mass of sphere = m = 200 kg

Diameter of sphere = D = 60 cm = 0.6 m

Radius of Sphere = r = 0.3 m

To find:

Moment of inertia of sphere = ?

Solution:

$I = \frac{2}{5}mr^2$