

SPECIAL THEORY OF RELATIVITY

STUDENT LEARNING OBJECTIVES

After studying this chapter, the students will be able to:

- Distinguish between inertial and non-inertial frames of reference.
- Describe the significance of Einstein's assumption of the constancy of the speed of light.
- Describe that if c is constant then space and time become relative.
- State the postulates of Special theory of relativity
- Explain qualitatively and quantitatively the consequences of special relativity Specifically in the case of
 - a. The relativity of simultaneity.
 - b. The equivalence between mass and energy.
 - c. Length contraction.
 - d. Time dilation.
 - e. Mass increase
- State that space time is a mathematical model in relativity that treats time as a fourth dimension of the traditional three dimensions of space
- (It can be thought of as a metaphorical sheet of paper that can bend, and when it bends it can cause effects such as stretching and compression seen when gravitational waves pass through objects.)

INTRODUCTION

This chapter explores Albert Einstein's Special Theory of Relativity, which revolutionized our understanding of space, time, and motion, especially at very high velocities.

Failure of Classical Physics

At the beginning of the 20th century, classical physics (based on Newton's laws) couldn't explain phenomena involving extremely small particles or very high velocities. This led to the development of relativistic mechanics, which fundamentally changed our view of the universe.

Albert Einstein's theory

Albert Einstein's Special Theory of Relativity, introduced in 1905, resolved these issues by proposing that the laws of Physics are the same for all observers and that the speed of light is constant, regardless of the observers motion.

11.1 RELATIVE MOTION

Q What is meant by relative motion and how will you give the concept of rest, motion and direction. Explain with example.

Ans

RELATIVE MOTION:

The change in position of a body with respect to another body is called relative motions.

Concept of rest and motion

Rest and motion cannot be defined absolutely they are relative terms. The state of rest and motion of an object are not same for different observers. If an object is at rest with respect to some observer, it may be in state of motion with respect to some other observer.

For example: the walls of a moving train seem stationary to passengers inside the train but appear to be moving to someone standing on the ground. Thus, we cannot definitively say whether an object is absolutely at rest or in motion; all motions are relative to the observer or to the reference frame being used.

Concept of direction in motion

The direction of motion of an object is not absolute, it is also relative term. Consider if someone is throwing a ball facing you, to your right but for him, the direction appears to his left. This illustrates that direction is a relative concept.

Explanation:

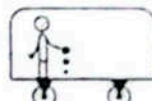
Motion is always relative, meaning it depends on the observer or the frame of reference being used. There is no absolute state of rest or motion.

Example: Dropping a Ball in a Train

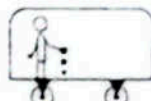
Consider an observer inside a closed train compartment dropping a ball.

Case (a): Stationary Train:

- The observer inside the train sees the ball fall straight down, landing directly below where it was dropped.
- An observer outside the train also sees the ball fall straight down.
- Both observers agree on the ball's motion.



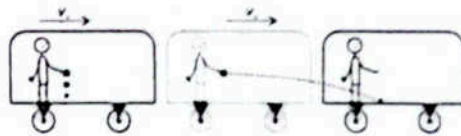
Observer in the train



Observer outside the train

Case (b): Train Moving with Uniform Velocity:

- The observer inside the train still sees the ball fall straight down, landing directly below where it was dropped. This is because both the ball and the observer move horizontally with the same uniform velocity as the train.
- An observer outside the train sees the ball follow a **projectile path** (a curve), as the ball has an initial horizontal velocity from the train's motion while also falling vertically.



Observer in the moving train

Observer outside the moving train

- **Conclusion:** Observers in different frames of reference, moving with uniform velocity relative to each other, will perceive motion differently, but the laws of physics describing the motion remain the same for both.

MULTIPLE CHOICE QUESTIONS

If an observer inside a uniformly moving train drops a ball, they see it fall straight down. An observer outside the train sees the ball follow a parabolic path. This demonstrates that:

- (a) The laws of physics are different for different observers.
- (b) Motion is always relative to the observer's frame of reference.
- (c) The ball is subject to different forces in different frames.
- (d) One observer's view is correct, and the other's is incorrect.

Answer: (b) Motion is always relative to the observer's frame of reference.

Explanation: The ball's path appears different because the observers are in different states of motion relative to each other, highlighting that motion is not absolute but always depends on the viewpoint of the observer.

According to the concept of relative motion, there is no absolute state of:

- (a) position
- (b) velocity
- (c) acceleration
- (d) rest or motion

Answer: (d) rest or motion

Explanation: Relative motion implies that whether something is considered "at rest" or "in motion" depends entirely on the chosen frame of reference; there is no universal, fixed point to determine absolute rest.

Consider a person walking on a moving bus. Relative to a stationary observer on the ground, the person's velocity is the sum of their walking velocity and the bus's velocity. This concept is an example of:

- (a) Constant velocity paradox
- (b) Absolute motion
- (c) Relative velocity
- (d) Accelerated motion



The speed of light c emitted by the flashlight is measured same by two observers, one moving in the car with speed v and other standing on the road.

Answer: (c) Relative velocity

Explanation: Relative velocity describes how the velocity of an object is measured differently by observers in different frames of reference, often by vector addition of velocities.

- If two observers are moving with uniform velocity relative to each other, how do the laws of physics apply to them?
 - (a) The laws of physics are different for each observer.
 - (b) The laws of physics are the same for both observers.
 - (c) Only one observer can apply the laws of physics correctly.
 - (d) The laws of physics apply differently depending on their relative speed.

Answer: (b) The laws of physics are the same for both observers.

Explanation: This is a core idea leading to Einstein's first postulate: the laws of physics hold true regardless of the constant velocity motion of the observer's frame of reference.

- Which statement accurately describes the essence of relative motion?
 - (a) All motion can be precisely measured against a fixed, universal background.
 - (b) The perception of motion depends on the observer's frame of reference, but the underlying physical laws remain consistent.
 - (c) Only observers at rest can accurately measure motion.
 - (d) Motion is an illusion created by varying perspectives.

Answer: (b) The perception of motion depends on the observer's frame of reference, but the underlying physical laws remain consistent.

Explanation: This captures the idea that while perceptions of motion differ, the fundamental laws governing motion do not change across inertial frames.

SLO BASED SHORT QUESTIONS & ANSWERS

What does it mean to say that "motion is always relative"?

Ans: It means that the description of an object's motion (its position, velocity, or acceleration) depends entirely on the chosen frame of reference or the observer from which the motion is being observed. There is no absolute state of rest or motion.

• Give an everyday example of relative motion where the same event appears differently to two observers.

Ans: A person walking towards the front of a moving bus: An observer inside the bus sees the person walking slowly (relative to the bus), while an observer on the ground sees the person moving much faster (relative to the ground), as the sum of their walking speed and the bus's speed.

• Does the concept of relative motion imply that the laws of physics change for different observers?

Ans: No, the concept of relative motion, particularly for observers in uniform motion (inertial frames), implies that the laws of physics remain the same, even though the description of events might appear different to each observer.

• If a ball is thrown upwards inside a smoothly moving car, how does its observed path differ for a passenger inside the car versus someone standing by the road?

Ans: The passenger inside the car sees the ball go straight up and straight down. The person standing by the road sees the ball follow a parabolic trajectory because it retains the car's forward motion while moving vertically.

• Why the Earth is often treated as an inertial frame of reference despite its rotation and revolution?

Ans: The Earth is often treated as an inertial frame for most everyday purposes because its acceleration due to rotation and revolution is relatively small and negligible compared to the accelerations typically observed in terrestrial experiments.

11.2 FRAMES OF REFERENCE

Q What is meant by the "frame of reference"? Distinguish between inertial frame of reference and non-inertial frame of reference by giving examples.

Ans

FRAME OF REFERENCE:

A coordinate system relative to which measurements are taken, is known as frame of reference.

For examples:

- (i) If position of the table in the room is located relative to walls of the room, then room is the frame of reference.
 - (ii) If measurements are taken in a moving train, then the train becomes the frame of reference.
 - (iii) The position of a spaceship can be described relative to the positions of distant stars. A coordinate system based on these stars is then the frame of reference.
- The frame of reference is classified into two classes:
- Inertial frame of reference
 - Non-inertial frame of reference.

INERTIAL FRAME OF REFERENCE:

A frame of reference in which the law of inertia is applicable is known as inertial frame of reference.

OR

A frame of reference, which is at rest or moving with uniform velocity, is known as inertial frame of reference. In inertial frame, the other laws of nature are also valid.

Examples:

- (i) Consider an object lying on earth surface, which remains at rest unless an unbalance force is applied upon it. Thus, earth may be considered as an inertial frame of reference.
- (ii) Now consider an object placed in a car, which is moving with uniform velocity with respect to earth. The object remains at rest in the car unless an external force is applied on it or brakes are applied on the car. Thus, a moving car is also considered as inertial frame of reference.

NON-INERTIAL FRAME OF REFERENCE:

A frame of reference in which the law of inertia is not applicable is known as non-inertial frame of reference.

OR

An accelerated frame is a non-inertial frame of reference.

For example:

- (i) Consider a body placed in a car moving with a uniform velocity. When the moving car is suddenly stopped, the body does not remain at rest. So the accelerated car is a non-inertial frame of reference.
- (ii) As earth is rotating and revolving and hence the Earth is not an inertial frame. Because of very small acceleration, Earth can be treated as an inertial frame.

MULTIPLE CHOICE QUESTIONS

An inertial frame of reference is defined as a frame where:

- (a) Objects are always at rest
- (b) Newton's law of inertia is valid
- (c) Objects always accelerate.
- (d) All forces are fictitious.

Answer: (b) Newton's law of inertia is valid

Explanation: An inertial frame is one where an object at rest stays at rest and an object in uniform motion stays in uniform motion unless acted upon by a net force, without the need for fictitious forces.

• Which of the following scenarios describes a non-inertial frame of reference?

- (a) A spaceship drifting at constant velocity in deep space
- (b) A train moving at a steady 100 km/h on a straight track

For your information
Relativity is the study of the way in which observers from moving frame of reference affect your perception of the world.

- (c) An elevator accelerating upwards.
 (d) A stationary laboratory on Earth (for most experiments).

Answer: (c) An elevator accelerating upwards

Explanation: A non-inertial frame is an accelerating frame. An accelerating elevator would make objects inside appear to have a fictitious force acting on them.

In which type of frame of reference might you observe an object accelerate without any apparent external force acting on it?

- (a) Inertial frame (b) Non-inertial frame (c) Rest frame (d) Stationary frame

Answer: (b) Non-inertial frame

Explanation: In an accelerating (non-inertial) frame, objects appear to accelerate due to fictitious forces that arise from the acceleration of the frame itself, not from real external interactions.

A frame of reference moving with uniform velocity relative to an inertial frame is:

- (a) Always a non-inertial frame (b) Also an inertial frame.
 (c) A special type of non-inertial frame (d) Not a valid frame for physics experiments

Answer: (b) Also an inertial frame.

Explanation: Any frame moving at a constant velocity with respect to an inertial frame is also considered an inertial frame, as Newton's laws still hold.

If you are standing on a merry-go-round that is spinning, you are in a:

- (a) Stationary frame (b) Inertial frame (c) Non-inertial frame (d) Uniform velocity frame

Answer: (c) Non-inertial frame

Explanation: A rotating frame is an accelerating frame (due to constant change in direction of velocity), and thus is non-inertial. You would feel a "centrifugal force" pushing you outwards, which is a fictitious force.

SLO BASED SHORT QUESTIONS & ANSWERS

Q: What is the defining characteristic of an inertial frame of reference?

Ans: The defining characteristic of an inertial frame of reference is that Newton's First Law of Motion (the law of inertia) is valid within it, meaning an object at rest stays at rest, and an object in motion stays in uniform motion unless acted upon by a net force.

Provide an example of a non-inertial frame of reference.

Ans: An example of a non-inertial frame of reference is a car taking a sharp turn or an elevator accelerating upwards or downwards.

Why "fictitious forces" are sometimes introduced in non-inertial frames?

Ans: Fictitious forces are introduced in non-inertial frames to account for the apparent accelerations of objects that are not caused by actual interactions but by the acceleration of the reference frame itself.

If you are in a closed train compartment moving at a constant velocity, can you perform any experiment to determine if the train is moving or at rest? Explain briefly.

Ans: No, you cannot. According to the principle of relativity, all laws of physics are the same in all inertial frames. Any experiment performed inside the closed compartment would yield the same results whether the train is moving at a constant velocity or is at rest.

Is the surface of the Earth a perfect inertial frame? Justify your answer.

Ans: No, the surface of the Earth is not a perfect inertial frame because the Earth rotates and revolves around the Sun, resulting in centripetal accelerations. However, for most everyday physics experiments, these accelerations are small enough that the Earth can be approximated as an inertial frame.

11.3 SPECIAL THEORY OF RELATIVITY

Q: What is special theory of relativity? Write down its postulates. Also summarize the results of special theory of relativity.

Ans:
SPECIAL THEORY OF RELATIVITY:

In 1905, Einstein proposed the theory of relativity, which is concerned, the way in which observers are in a state of relative motion and describe physical phenomena. OR

The theory of relativity deals with how observers in different states of relative motion describe physical phenomena.

The theory of relativity is classified into two classes.

i. **Special Theory of Relativity:** Which deals the problems involving inertial or non-accelerating frames of reference.

ii. **General Theory of Relativity:** Which deals the problems involving non-inertial or accelerating frame of reference.

Einstein's Postulate of Special Theory of Relativity:

The special theory of relativity is based upon two postulates, which can be stated as follows:

Postulate 1: (Principle of relativity) The law of physics (Newton's laws of motion as well as Maxwell's equations) are the same in all inertial frames.

Postulate 2: (principle of constancy of light) The speed of light in free space has the same value for all observers, regardless of their state of motion.

Explanation:

• The first postulate is the generalization of the fact that all physical laws are same in frames of reference at rest or moving with uniform velocity with respect to each other. If the laws of physics differed for observers in relative motion, those observers could determine which was stationary and which was moving. However, such a distinction does not exist, implying that there is no way to detect Absolute uniform motion.

• The second postulate states an experimental fact that speed of light in free space is the universal constant c ($c = 3 \times 10^8 \text{ ms}^{-1}$). For example, if you are sitting in a train moving at the speed of light and you hold up a mirror in front of you at arm's length, you will still see your reflection in the mirror. This is because, according to the principle of relativity, no experiment can detect the constant motion of the train relative to the person inside it.

Q: What is the concept of simultaneity in case of relativity? Explain.

THE RELATIVITY OF SIMULTANEITY

If two events in different locations are observed by one observer to be simultaneous, they will generally **not** be observed as simultaneous by another observer in a different frame of reference that is moving relative to the first observer. Whether two events are seen as simultaneous depends on the observer's frame of reference.

Example: Light-Operated Doors on a Train:

• A traveler in the middle of a train compartment (moving at half the speed of light) turns on a light switch. The light travels to both doors at the same speed. The traveler sees both doors opening simultaneously.

• An observer outside the train will see the back door open before the front door. This is because the back door is moving towards the light waves, while the front door is moving away from them. Light still travels at 'c' relative to all observers.

For your information:

If you are in a frame of reference moving at constant velocity from which you cannot see any other frame of reference, there is no way to know if you are moving or at rest.

Q: Summarize the results of special theory of relativity.

Result Obtained From Special Theory Of Relativity:

Some interesting results of the special theory of relativity can be summarized as follows, without going into their mathematical details.

TIME DILATION

According to the special theory of relativity, time is not an absolute quantity, it depends on the relative motion of the observer and the frame of reference where events occur. Moving clocks appear to run slower.

For your information

If you are in a frame of reference moving at constant velocity from which you cannot see any other frame of reference, there is no way to know if you are moving or at rest.

Proper Time (t_0): This is the time interval between two events measured by an observer who is at rest relative to where the events occur. It is the shortest possible time interval.

Dilated Time (t): If an observer is moving with a relativistic velocity 'v' with respect to the frame of events, the time 't' measured by this moving observer will be greater than the proper time to.

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Explanation:

- The term $\sqrt{1 - \frac{v^2}{c^2}} < 1$, so $t > t_0$ is always less than one (since $v < c$).
- Therefore, 't' is always greater than t_0 . This means time has "dilated" or "stretched" due to relative motion.
- This effect applies to all physical, chemical, and biological processes, including aging.

Example: If an event takes 1 second for a traveler on a plane moving at $0.8c$, it takes 1.7 seconds for a person outside the plane.

LENGTH CONTRACTION

The length of an object or the distance between two points appears shorter when measured by an observer who is in relative motion with respect to the object or points. This contraction occurs **only along the direction of motion**.

Proper Length (ℓ_0): The length of an object or the distance between two points measured by an observer who is at rest relative to them. It is the longest possible length.

Contracted Length (ℓ): If an object and an observer are in relative motion with speed 'v', the contracted length 'l' is given by:

$$\ell = \ell_0 \sqrt{1 - \frac{v^2}{c^2}}$$

As $\sqrt{1 - \frac{v^2}{c^2}} < 1$, so $\ell < \ell_0$

- The term $\sqrt{1 - \frac{v^2}{c^2}} < 1$, so $\ell < \ell_0$ is always less than one (since $v < c$).
- Therefore, 'l' is always smaller than ℓ_0 . This means length has "contracted" or "compressed" due to relative motion.
- This effect applies to all lengths in the direction of motion only.

Example: A train measured to be 100 m long at rest will be observed as only 60 m long by a person standing by the track if the train travels at $0.8c$.

MASS VARIATION (RELATIVISTIC MASS)

The mass of an object is not a constant quantity; it is a variable that depends on the object's speed. As an object's speed approaches the speed of light, its mass increases.

Rest Mass (m_0): The mass of an object measured when it is at rest.

Relativistic Mass (m): When an object is observed to be moving at speed 'v', its mass 'm' is given by:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Explanation:

- This increase in mass indicates an increase in the object's inertia (resistance to changes in motion) at high speeds.
- As 'v' approaches 'c' (the speed of light), the denominator $\sqrt{1 - \frac{v^2}{c^2}} < 1$, approaches zero, so $m > m_0$.
- Consequently, the relativistic mass 'm' approaches infinity ($m \rightarrow \infty$).
- An infinite mass would require an infinite force to accelerate it further. Since infinite forces are not available, it is impossible for any material object to be accelerated to the speed of light 'c' in free space. The

speed of light is an ultimate speed limit in the universe.

Everyday Life vs. Relativistic Effects:

In our everyday lives, speeds are extremely small compared to the speed of light (e.g., Earth's orbital speed is 30 km/s compared to $c = 300,000$ km/s). This is why Newton's laws of motion are valid and accurate for most common situations. However, when dealing with subatomic particles moving at velocities approaching the speed of light (e.g., in particle accelerators), relativistic effects become very prominent, and experimental results cannot be explained without considering Einstein's equations.

MULTIPLE CHOICE QUESTIONS

The first postulate of special relativity states that:

- The speed of light is constant for all observers.
- The laws of physics are the same in all inertial frames of reference.
- All motion is relative.
- Mass and energy are equivalent.

Answer: (b) The laws of physics are the same in all inertial frames of reference.

Explanation: This is known as the Principle of Relativity, which is the first postulate. The constancy of light speed is the second.

According to the second postulate of special relativity, the speed of light in free space (c):

- Varies depending on the observer's speed.
- Is the same for all observers, regardless of their motion.
- Is faster when the source is moving towards the observer.
- Is slower when the source is moving away from the observer.

Answer: (b) Is the same for all observers, regardless of their motion.

Explanation: This is the core of the second postulate: the invariance of the speed of light.

A clock moving at a relativistic speed relative to an observer will appear to the observer to run:

- Faster
- Slower
- At the same rate
- Only in reverse

Answer: (b) Slower

Explanation: This phenomenon is called time dilation, where a moving clock is observed to run slower than a stationary one.

As an object's speed approaches the speed of light, its relativistic mass:

- Decreases
- Remains constant
- Approaches zero
- Approaches infinity

Answer: (d) Approaches infinity

Explanation: According to the relativistic mass formula, as 'v' approaches 'c', the denominator approaches zero, causing the mass to approach infinity.

The length of an object measured by an observer moving relative to it will appear:

- Longer
- Shorter in the direction of motion
- Shorter perpendicular to the direction of motion
- Unchanged

Answer: (b) Shorter in the direction of motion

Explanation: This is length contraction, which only occurs along the dimension of relative motion.

The special theory of relativity is applicable to the object moving with maximum velocity equal to:

- speed of light
- more than speed of light
- less than speed of light
- double the speed of light

(speed of body cannot be equal to or greater than speed of light)

At what speed would the mass of an electron become double of its rest mass? (FSD 2015) (LHR 2016)

- $0.5c$
- $0.707c$
- $0.866c$
- $0.99c$

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \sqrt{1 - \frac{v^2}{c^2}} = \frac{m_0}{m} \Rightarrow \sqrt{1 - \frac{v^2}{c^2}} = \frac{m_0}{2m_0} \Rightarrow 1 - \frac{v^2}{c^2} = \frac{1}{4} \Rightarrow \frac{v^2}{c^2} = \frac{3}{4} \Rightarrow v = \frac{\sqrt{3}}{2} c = 0.866c$$

The mass of object will be double at speed. (SGD 2017 GI)

(a) $2.5 \times 10^8 \frac{m}{s}$ (b) $1.6 \times 10^8 \frac{m}{s}$ (c) $3.6 \times 10^8 \frac{m}{s}$ (d) $0.6 \times 10^8 \frac{m}{s}$

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow \sqrt{1 - \frac{v^2}{c^2}} = \frac{m_0}{m} \Rightarrow \sqrt{1 - \frac{v^2}{c^2}} = \frac{m_0}{2m_0} \Rightarrow 1 - \frac{v^2}{c^2} = \frac{1}{4} \Rightarrow \frac{v^2}{c^2} = \frac{3}{4} \Rightarrow v = \sqrt{\frac{3}{4}} c = 0.866c = 2.5 \times 10^8 \text{ m/s}$$

SLO BASED SHORT QUESTIONS & ANSWERS

- State the two postulates of Einstein's Special Theory of Relativity.
 - Ans: The laws of physics are the same in all inertial frames of reference. 2. The speed of light in free space is the same for all observers, regardless of the motion of the source or the observer.
 - Define "proper time" and "proper length."
 - Ans: Proper time (t_0) is the time interval between two events measured by an observer who is at rest relative to where the events occur. Proper length (l_0) is the length of an object or distance between two points measured by an observer who is at rest relative to them. These are the shortest time and longest length, respectively.
 - Explain the phenomenon of time dilation.
 - Ans: Time dilation is a consequence of special relativity where a time interval measured by an observer moving relative to a clock will appear longer than the time interval measured by an observer at rest relative to the same clock. Moving clocks run slower.
 - Explain why no material object can reach the speed of light.
 - Ans: As a material object (with rest mass) approaches the speed of light, its relativistic mass would approach infinity. Accelerating an infinite mass would require an infinite amount of energy, which is physically impossible to provide.
 - What is meant by the "relativity of simultaneity"?
 - Ans: The "relativity of simultaneity" means that two events that are observed as happening at the same time (simultaneously) by one observer will generally not be observed as simultaneous by another observer who is in relative motion with respect to the first.
 - Does the dilation means that time really passes more slowly in moving system or that it only seems to pass more slowly?
 - Ans: Yes, the time dilation means that time interval of an event is really passes more slowly due to relative motion according to relation $t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$
 - **Explanation** The time dilation (or running of clock slowly) is purely due to relative motion. It really happens when observers are in relative motion. The result is applied to the timing process physical, chemical and biological. Even aging process of the human body is slowed down by motion at very high speed. It should be noted that we cannot detect such changes in a daily life because we are not moving at speed comparable to the speed of light.
 - **For example**, pions created at rest, has life time about $26 \times 10^{-9} \text{ s} = 26 \text{ ns}$. But when pions are moving with velocity $0.6c$, then they have lifetime about $31 \times 10^{-9} \text{ s}$ or 31 ns i.e. their lifetime increases due to relative motion.
 - If you are moving in space ship at a very high speed relative to the Earth, would you notice a difference (a) in your pulse rate (b) in the pulse rate of people on Earth?
 - Our Pulse Rate
 - a. In a fast moving space ship relative to earth, we are at rest with respect to spaceship. Hence we will not observe any change in pulse rate because there is no change in the time interval of pulse.
 - b. **Pulse Rate of People on Earth** Because the spaceship is moving faster relative to the earth, hence we will observe that the pulse rate of people on earth will decrease.
- Since Pulse Rate = $\frac{\text{Number of Pulses}}{\text{Time}}$ and according to time dilation relation,

$$t = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \text{ Time increases so pulse rate decreases}$$

Since mass is a form of energy, can we conclude that a compressed spring has more mass than the same spring when it is not compressed?

Ans: According to special theory of relativity,

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

When spring is compressed, $v = 0$, Then $m = m_0$.

Hence mass of compressed spring will not change. The mass of object increases only if it is moving with a velocity approaching the velocity of light.

The length of a spacecraft might vanishes if it moves with the speed of light. Explain?

Ans: According to length contraction, length of body moving with high speed comparable to speed of light decreases in the direction of motion

$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}} \text{ if } v = c \text{ then } l = l_0 \sqrt{1 - \frac{c^2}{c^2}} = l_0 \sqrt{1 - 1} = 0$$

so the length of a spacecraft might vanishes if it moves with the speed of light

11.4 THE EQUILANCE BETWEEN MASS AND ENERGY:

- Q. Describe the Einstein's mass-energy equation; why cannot we observe its effects in everyday life? What are its significant consequences? Give examples.
Or What do you mean by equilance between mass and energy?

Ans

Mass-Energy Equivalence

According to the special theory of relativity, mass and energy are not separate entities but are interconvertible. Mass can be converted into energy, and energy can be converted into mass.

Einstein's Mass-Energy Equivalence Equation:

The total energy (E) and mass (m) of an object are related by the famous expression:

$$E = mc^2 \text{ (Equation 11.4)}$$

Where 'c' is the speed of light. This equation shows that mass is a form of energy.

Rest Mass Energy (E_0):

The energy equivalent of an object's mass when it is at rest is called its rest mass energy (E_0).

$$E_0 = m_0 c^2$$

Kinetic Energy (K.E.):

When an object is in motion, its total energy ($E = mc^2$) is greater than its rest mass energy ($E_0 = m_0 c^2$). The difference in energy is due to its motion and represents the kinetic energy (K.E.) of the mass.

$$\text{K.E.} = E - E_0 = mc^2 - m_0 c^2$$

$$\text{K.E.} = (m - m_0) c^2$$

Mass Change due to Energy Change:

From $E = mc^2$, a change in energy (ΔE) is related to a change in mass (Δm) by:

$$\Delta m = \Delta E / c^2$$

Why don't we observe mass changes in everyday life?

Because c^2 is a very large quantity ($c^2 = (3 \times 10^8)^2 = 9 \times 10^{16} \text{ m}^2 \text{ s}^{-2}$), this implies that even small changes in mass are equivalent to very large changes in energy. In our everyday world, energy changes (e.g. from burning fuel) are too small to produce measurable mass changes. However, energy and mass changes in nuclear reactions (like fission and fusion) are found to be exactly in accordance with these equations, providing strong evidence for mass-energy equivalence.

SLO BASED SHORT QUESTIONS & ANSWERS

- State Einstein's famous mass-energy equivalence equation and briefly explain what it means.
Ans: Einstein's equation is $E=mc^2$. It means that mass and energy are interconvertible, mass can be converted into energy, and energy can be converted into mass.
- What is the significance of the c^2 term in the equation $E=mc^2$?
Ans: The c^2 term, which is a very large number ($9 \times 10^{16} \text{ m}^2 \text{ s}^{-2}$), signifies that even a tiny amount of mass is equivalent to an enormous amount of energy.
- How is the kinetic energy of a relativistic particle defined in terms of its total energy and rest mass energy?
Ans: The kinetic energy (K.E.) of a relativistic particle is the difference between its total energy (E) and its rest mass energy (E_0). $K.E. = E - E_0$.
- If a hypothetical process completely converts 1 kg of matter into energy, how much energy would be released (in Joules)?
Ans: If 1 kg of matter were completely converted into energy, the energy released would be $E=mc^2 = (1 \text{ kg}) \times (3 \times 10^8 \text{ m s}^{-1})^2 = 9 \times 10^{16} \text{ J}$.
- Why the mass-energy equivalence is often demonstrated using nuclear reactions rather than everyday processes?
Ans: Nuclear reactions involve significantly larger energy changes per unit of mass than everyday processes (like chemical reactions). This leads to measurable changes in mass that can be observed and verified against $E=mc^2$, unlike the negligible mass changes in chemical reactions.

MULTIPLE CHOICE QUESTIONS

- Einstein's famous equation $E=mc^2$ primarily signifies:
 (a) The relationship between kinetic energy and momentum
 (b) The conservation of energy in isolated systems
 (c) The interconvertibility of mass and energy
 (d) The speed limit of the universe
Answer: (c) The interconvertibility of mass and energy. **Explanation:** The equation reveals that mass is a form of energy and can be converted into energy, and vice-versa.
- The rest mass energy (E_0) of an object is given by:
 (a) $m_0 v^2$ (b) $1/2 m_0 c^2$ (c) $m_0 c^2$ (d) $(m - m_0) c^2$
Answer: (c) $m_0 c^2$. **Explanation:** This is the definition of rest mass energy, where m_0 is the rest mass and c is the speed of light.
- If an object is in motion, its total energy (E) is related to its rest mass energy (E_0) and kinetic energy (K.E.) by:
 (a) $E = E_0 - K.E.$ (b) $E = K.E. / E_0$ (c) $E = E_0 + K.E.$ (d) $E = E_0 \times K.E.$
Answer: (c) $E = E_0 + K.E.$ **Explanation:** The total energy of a moving object is the sum of its rest mass energy and the energy it possesses due to its motion (kinetic energy).
- Why are mass changes due to energy transformations not noticeable in everyday chemical reactions?
 (a) Chemical reactions do not involve energy changes
 (b) The speed of light squared (c^2) is a very small number
 (c) The energy changes are too small to result in measurable mass changes
 (d) Mass is always conserved in chemical reactions
Answer: (c) The energy changes are too small to result in measurable mass changes.
Explanation: Because c^2 is an extremely large number, even a large energy change corresponds to an immeasurably tiny change in mass in typical chemical reactions.
- Which type of reaction provides the most compelling direct evidence for the mass-energy equivalence?
 (a) Chemical burning (b) Dissolving salt in water
 (c) Nuclear fission or fusion (d) Boiling water

- Answer:** (c) Nuclear fission or fusion. **Explanation:** Nuclear reactions involve significantly larger energy changes than chemical reactions, resulting in measurable changes in mass that precisely match Einstein's prediction.
- 0.1 kg is equivalent to the energy of: (DGK 2015 GII)
- (a) $5 \times 10^8 \text{ J}$ (b) $6 \times 10^{16} \text{ J}$ (c) $9 \times 10^{16} \text{ J}$ (d) $9 \times 10^{15} \text{ J}$
- Explanation:** ($E=mc^2 = 0.1 \times (3 \times 10^8)^2 = 9 \times 10^{16} \text{ J}$)

11.5 SPACE-TIME IN RELATIVITY

- Q.** State the Einstein's concept about the space-time. Describe the view of gravity according to this concept. or
- What do you mean by the concept of space time and also discuss the view of Newton's and Einstein's on gravity?

Ans.
 SPACE-TIME IN RELATIVITY
 What is Space?

Space is a three-dimensional extent in which all objects and events occur. It provides a framework to define the position and motion of various objects.

What is Time in Relativity?

In the theory of relativity, time is not absolute; it is considered the fourth dimension. Events happen not just at a location (x, y, z) but also at a specific time (t). Time is also relative; it can pass more slowly for an observer moving at extremely high speeds (time dilation).

What is Space-Time?

Space-time is a mathematical model that unifies the traditional three dimensions of space and the dimension of time into a single, four-dimensional continuum. It is a concept used to describe all points of space and time and their relationship to each other.

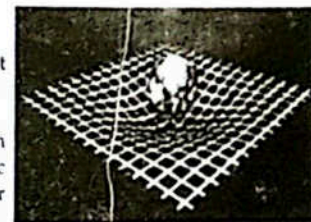
Einstein's View of Gravity (General Relativity):

According to Einstein's theory (specifically General Relativity), space-time is curved, especially near massive bodies and for objects moving at speeds approaching the speed of light.

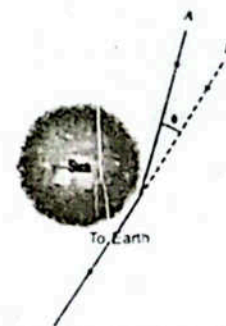
- **Visualization:** Imagine space-time as a stretched fabric sheet. If a heavy ball (representing a massive object like Earth) is placed on this sheet, it causes the fabric to curve or indent.
- **Gravity as Curvature:** Objects like stars and planets cause space-time to curve around themselves. The more massive the object, the deeper the curve.
- **Motion in Curved Space-Time:** Consequently, we don't speak of a "force of gravity" pulling bodies. Instead, bodies and light rays simply move along the "straightest possible paths" (called geodesics) in this curved space-time. A body at rest or moving slowly near a massive object would naturally follow a geodesic towards that object, which we perceive as being "pulled" by gravity.

Advancement over Newton's Theory of Gravity:

- Newton's law of gravity accurately describes the force of gravity but doesn't explain why it works that way.
- Einstein's theory explains gravity as the curvature of space-time caused by mass and energy. It yields the same results as Newton's theory in weak gravitational fields but provides more accurate predictions in strong fields.
- **Verification:**



Interesting information



Bending of starlight by the Sun
 Light from the star A is deflected as it passes close to the Sun on its way to Earth. We see the star in the apparent direction B, shifted by the angle ϕ . Einstein predicted that $\phi = 1.745$ seconds of angle which was found to be the same during the solar eclipse of 1919.

It yields the same results as Newton's theory in weak gravitational fields but provides more accurate predictions in strong fields.

- **Bending of Starlight:** The bending of starlight caused by the Sun's gravity was first measured during a solar eclipse in 1919. The results perfectly matched Einstein's predictions (an angle of $\phi=1.745$ arc seconds), confirming his theory over Newton's.
- **Gravitational Waves:** Einstein's theory also predicted the existence of gravitational waves, ripples in space-time caused by accelerating massive objects (like merging black holes). These waves were first directly detected in 2015 and announced in 2016.

The special theory of relativity explains that space and time are related to each other. It describes how space and time are influenced by gravity and speed, such as the bending of light around massive objects like stars.

Space-time is, in fact, a mathematical model that unifies space-time into a single continuum. It is a concept used to describe all points of space and time and their relation to each other.

NEWTON'S AND EINSTEIN'S VIEWS OF GRAVITATION:

Newton's view about gravitation

1. Gravitation is intrinsic property.
2. Gravitation force obeys inverse square law

$$F \propto \frac{1}{r^2}$$

3. No reason why gravitational force obeys inverse square law.
4. Gravitational force is directly proportional to product of masses.

$$F \propto m_1 m_2$$

5. Newton's idea of about gravitation is based upon the idea that light consist of tiny particles that deflected by gravity.

Einstein's idea about gravitation:

1. Space and time are curved just like rubber sheet.
2. Heavy planets produce geodesics in space and travel on it. Einstein's theory gives a physical picture of how gravity works.
3. It tells why gravitational force obeys inverse square law.
4. Einstein view is much better than Newton's view.
5. Light bends by gravity twice than as predicted by Newton.

For your information

The faster you are moving or close to a strong source of gravity, the slower the time goes for you.

Interesting information

If you are on some spaceship moving extremely fast through space near a black hole like in movie, "Interstellar" then you could miss 7 years on the Earth in every hour.

Bending of Starlight near the Sun:

- When a light from a distant star travels to Earth after first gazing the edge of the Sun, its path is bent as it follows the most direct route through the curved space as shown in the figure.
- The apparent position of the star viewed from the Earth is shifted somewhat from its true position. For stars near the Sun to be visible, the observations must be made during a solar eclipse.
- Another success of Einstein's theory was the detection of gravitational waves, produced by some celestial events causing disturbances (squeezes and stretches) in the curvature of space-time. These waves were detected in 2015 and announced in 2016.

Several measurements have been done, the earliest coming in 1919, just after Einstein proposed his theory. Here again theory and experiment are in excellent agreement.

Hypothetical Example of Space-Time (from text):

If a spaceship travels to a star at half the speed of light, and it takes 8 years from Earth's perspective, the clocks on the spaceship will move slower (time dilation). From the spaceship's occupants' point of view, the length of the journey has contracted, so they cover the distance in less time, perhaps 7 years instead of 8 years.

Interesting information:

If you are on a spaceship moving extremely fast through space near a black hole (like in the movie "Interstellar"), you could miss 7 years on Earth in every hour due to extreme time dilation caused by both high speed and strong gravity.

FOR YOUR INFORMATION

Bending of star light by the sun. Light from the star A is deflected as it passes close to the Sun on its way to Earth. We see the star in the apparent direction B, shifted by the angle ϕ . Einstein predicted that $\phi=$

1.745 seconds of angle which was found to be the same during the solar eclipse of 1919.

Do You Know?

Gravity can bend light. The gravity of a star could be used to focus light from stars.

MULTIPLE CHOICE QUESTIONS

In the context of relativity, time is considered the:

- (a) First dimension (b) Second dimension (c) Third dimension (d) Fourth dimension

Answer: (d) Fourth dimension

Explanation: Space-time is a mathematical model that unifies the traditional three dimensions of space with time as a fourth dimension.

Einstein's General Theory of Relativity explains gravity as:

- (a) A fundamental force transmitted by gravitons.
 (b) The curvature of space-time caused by mass and energy.
 (c) An attractive force acting instantaneously between all masses.
 (d) A quantum field interaction in flat space.

Answer: (b) The curvature of space-time caused by mass and energy.

Explanation: General Relativity describes gravity as the effect of massive objects warping the fabric of space-time.

Objects and light rays in curved space-time move along paths known as:

- (a) Straight lines (b) Parabolic arcs (c) Geodesics (d) Hyperbolas

Answer: (c) Geodesics

Explanation: Geodesics are the "straightest possible paths" in curved space-time that objects follow in the absence of other forces.

Which experimental observation provided early and strong evidence for the curvature of space-time predicted by General Relativity?

- (a) The decay rate of muons. (b) The bending of starlight around the Sun.
 (c) The increase in mass of high-speed particles. (d) The constant speed of light

Answer: (b) The bending of starlight around the Sun.

Explanation: Observations during solar eclipses showed starlight bending by an amount consistent with Einstein's prediction of space-time curvature near a massive object like the Sun.

The concept of space-time implies that:

- (a) Space and time are completely independent.
 (b) All events occur at absolute, fixed points in space and time.
 (c) Measurements of space and time are intertwined and relative to an observer's motion.
 (d) Gravity is a direct force between masses.

Answer: (c) Measurements of space and time are intertwined and relative to an observer's motion.

Explanation: Space-time unifies space and time into a single continuum where measurements of distances and durations are relative, challenging the classical notions of absolute space and time.

What did Einstein's theory of general relativity propose about gravity?

- (a) Gravity is a force that pulls objects together.
 (b) Gravity is caused by the curvature of space-time.
 (c) Gravity is a result of electromagnetic radiation.
 (d) Gravity is a property of matter.

Answer: B

According to general relativity, how does the mass of an object affect gravity?

- (a) The more massive an object, the stronger its gravity.
 (b) The less massive an object, the stronger its gravity.
 (c) The mass of an object does not affect gravity.
 (d) Gravity is not related to mass.

Answer: A

Interesting information

If you are on some spaceship moving extremely fast through space near a black hole like in movie, "Interstellar" then you could miss 7 years on the Earth in every hour.

For your information

The faster you are moving or close to a strong source of gravity, the slower the time goes for you.

TEXT BOOK EXERCISE WITH SOLUTION

MULTIPLE CHOICE QUESTIONS

- Tick the correct answer.
- 11.1 Relativistic mechanics yields results different from classical mechanics for objects moving with:
 (a) low velocity
 (b) velocity equal to that of sound waves
 (c) velocity greater than sound waves
 (d) velocity approaching that of light
- 11.2 If an observer is moving in the same direction as a sound wave, the velocity of the wave seems to be:
 (a) more (b) less
 (c) constant (d) sum of the two velocities
- Explanation:
 (The relative velocity of the sound wave with respect to the observer (v_{r1}) is given by $v_{r1} = v_s - v_o$. Since the observer is moving in the same direction as the sound wave, their velocity subtracts from the sound wave's velocity from the observer's perspective. Therefore, the velocity of the wave seems to be less than its actual velocity.)

- 11.3 If the rest mass of a particle m_0 increases to m due to its high speed, then its kinetic energy is:
 (a) $1/2 m_0 c^2$ (b) $1/2 m v^2$
 (c) $(m - m_0) c^2$ (d) $1/2 (m - m_0) c^2$
- (The kinetic energy (KE) is the difference between the total energy and the rest energy. $KE = E - E_0$, so $KE = mc^2 - m_0 c^2$ or $KE = (m - m_0) c^2$)
- 11.4 The speed of beam light of a car while moving with high speed as compared to its rest position is:
 (a) greater (b) less
 (c) same (d) zero
 (speed of light is a constant as per special theory of relativity)
- 11.5 A photon is a particle of light. What is its mass when it moves with $0.9c$?
 (a) $9.1 \times 10^{-31} \text{ kg}$ (b) $1.67 \times 10^{-27} \text{ kg}$
 (c) $1.67 \times 10^{-27} \text{ kg}$ (d) Zero

SHORT ANSWER QUESTIONS

- 11.2 What is meant by inertial frame of reference and a non-inertial frame of reference?
 Ans: INERTIAL FRAME OF REFERENCE:
 A frame of reference in which the law of inertia is applicable is known as inertial frame of reference. OR

A frame of reference, which is at rest or moving with uniform velocity, is known as inertial frame of reference. In inertial frame, the other laws of nature are also valid.

NON-INERTIAL FRAME OF REFERENCE:
 A frame of reference in which the law of inertia is not applicable is known as non-inertial frame of reference.

OR An accelerated frame is a non-inertial frame of reference.

- 11.2 What are the two postulates of special theory of relativity?

Ans: The special theory of relativity is based upon two postulates, which can be stated as follows:

Postulate 1: (Principle of relativity) The law of physics (Newton's laws of motion as well as Maxwell's equations) are the same in all inertial frames.

Postulate 2: (principle of constancy of light) The speed of light in free space has the same value for all observers, regardless of their state of motion.

- 11.3 Describe why it is impossible for a material particle to move with speed of light.

Ans: The increase in mass indicates the increase in inertia at high speed. As v approach c , then v^2/c^2

approach 1. Therefore $\sqrt{1 - \frac{v^2}{c^2}}$ approach 0. Thus

$m \rightarrow \infty$ and it requires an infinite force to change the speed of object. Because infinite forces are not available, hence, an object cannot be accelerated to the speed of light c in free space.

- 11.4 Does theory of relativity contradict Newton's laws of motion? Explain briefly.

Ans: The theory of relativity doesn't entirely contradict Newton's laws, rather, it supersedes and expands upon them.

Newton's laws are an excellent approximation for everyday conditions (low speeds, weak gravity). However, at very high speeds (near the speed of light) or in strong gravitational fields, relativity provides a more accurate description, accounting for phenomena Newton's laws don't (like time dilation or space-time curvature).

Newtonian physics is essentially a limiting case of relativistic physics when speeds are much less than the speed of light and gravity is weak.

- 11.5 What is meant by proper time, and proper length?

Proper time

How did Einstein explain the phenomenon of gravitational lensing?

- (a) Gravity causes light to bend as it travels through space.
 (b) Gravity causes stars to appear brighter than they actually are.
 (c) Gravity causes galaxies to rotate faster than expected.
 (d) Gravity has no effect on light or other forms of radiation.

Answer: A

According to Newton's theory of gravity, how does the strength of gravity between two objects depend on their distance apart?

- A. The strength of gravity increases with distance.
 B. The strength of gravity decreases with distance.
 C. The strength of gravity is not affected by distance.
 D. The strength of gravity is inversely proportional to distance.

Answer: D

What is the role of mass in Newton's theory of gravity?

- A. Mass determines the strength of gravity between objects.
 B. Mass has no effect on gravity.
 C. Mass affects the direction of gravity.
 D. Mass affects the speed of gravity.

Answer: A

What is the main difference between Newton's theory of gravity and Einstein's theory of general relativity?

- A. Newton's theory of gravity is more accurate than general relativity.
 B. Newton's theory of gravity describes gravity as a force between objects, while general relativity describes gravity as a curvature of space-time.
 C. Newton's theory of gravity only applies to massive objects, while general relativity applies to all objects.
 D. Newton's theory of gravity and general relativity are essentially the same.

Answer: B

Einstein's theory gives a physical picture of how the (Mtn, 18)

- (a) Earth moves (b) Gravity works
 (c) Movement of inertia produced (d) Weight losses creates.

Answer: b

SLO BASED SHORT QUESTIONS & ANSWERS

What is "space-time" in the context of Einstein's theory of relativity?

Ans: Space-time is a mathematical model that unifies the three dimensions of space and the dimension of time into a single, four-dimensional continuum within which all physical events occur.

How does Einstein's General Theory of Relativity explain the phenomenon of gravity, contrasting it with Newton's view?

Ans: Newton described gravity as a force attracting objects with mass. Einstein's General Relativity explains gravity as the curvature or warping of space-time itself, caused by the presence of mass and energy, where objects then follow paths along this curved fabric.

Provide an example of an everyday phenomenon where the relativistic effects of space-time curvature are significant and must be accounted for.

Ans: Global Positioning System (GPS) satellites require continuous adjustments based on both special relativity (due to their speed) and general relativity (due to the weaker gravitational field at their altitude affecting time) to maintain their high accuracy.

If space-time is like a "fabric," how does a massive object like a star affect this fabric?

Ans: A massive object like a star causes the fabric of space-time to curve or warp around it, creating an indentation or distortion that dictates the paths of other objects and light in its vicinity.

What are "geodesics" in curved space-time?

Ans: Geodesics are the "straightest possible paths" that objects (including light) naturally follow through curved space-time in the absence of any other forces. We perceive motion along these geodesics as being influenced by gravity.

Ans: The time interval between to events measure by an observer which is at rest with respect to frame of event is called proper time and it is represented by t_0 .

Proper length

The length or distance between two points measured by an observer who is at rest relative to them is called the proper length l_0 .

11.6 What is meant by relativistic mass, length and time?

Relativistic mass

Ans: The mass of an object moving with very high speed measured by an observer at rest is called relativistic mass. If m_0 is the mass of an object at rest in observer's frame of reference and m is its mass when it is moving with velocity v with respect to observer, then,

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Relativistic length

The length measured by an observer which is in relative motion along the length is called relativistic length.

If an object and an observer are in relative motion with speed v , then length of the rod appears to decrease along the direction of motion and contracted length is given by,

$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}}$$

Relativistic time

The time interval between to events measure by an observer which is in relative motion with respect to frame of event is called relativistic time. If the observer is moving with respect to frame of events with velocity v or if the frame of events is moving with respect to observer with a uniform velocity v , then the time measured by the observer will be,

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

11.7 Why mass of a moving object increases?

Ans: The "increase in mass" (relativistic mass) of a moving object is due to its increased kinetic energy. As an object moves faster, it gains energy, and according to Einstein's $E=mc^2$, this added energy is equivalent to an increase in its mass. This also means it becomes harder to accelerate (its inertia increases) as it approaches the speed of light. However, its fundamental "rest mass" remains constant. The physical mass

of the object does not change.

11.8 All motion are relatives. Does space-time is absolute? Explain briefly.

Ans: No, space-time is not absolute. In relativity, space and time are unified into a single fabric called space-time. This fabric is not fixed, instead, it can be bent or curved by mass and energy. This means its geometry changes depending on what's in it.

11.9 Explain that speed of light is an ultimate limit for any object.

Ans: The speed of light is the ultimate limit because as any object with mass approaches it, its energy and effective mass increase infinitely, requiring an impossible amount of energy to accelerate further, which is not possible. So speed of light is an ultimate limit for any object. If $v=c$ then $m=\infty$, $t=\infty$, $E=\infty$.

11.10 Give examples where the results of special theory of relativity have been verified.

Ans: Examples verifying Special Relativity are:

- Muon Decay:** Muons, unstable particles created in the atmosphere, survive much longer than expected due to time dilation as they travel near the speed of light.
- GPS Systems:** Accurate GPS navigation requires constant corrections for time dilation effects on satellite clocks, proving relativity's practical impact.
- Particle Accelerators:** Particles accelerated close to the speed of light exhibit the predicted increase in energy/mass and are unable to reach the speed of light itself, confirming the universal speed limit.
- Michelson-Morley Experiment:** Showed that the speed of light is constant regardless of the observer's motion, a key postulate of special relativity.

CONSTRUCTED RESPONSE QUESTIONS

11.1 Speed of sound is affected by relative motion between the observer and the source. Does this apply to speed of light as well? Describe briefly.

Ans: No, the principle that affects the speed of sound (Doppler effect due to relative motion) does not apply to the speed of light in the same way. The speed of light is a universal constant according to Einstein's second postulate of special theory of relativity and it is not affected by relative motion between the observer and the source.

11.2 Is it ever possible to see a star moving away from us at a uniform velocity equal to the

velocity of light?

Ans: No, it is never possible to see a star moving away from us at a uniform velocity exactly equal to the velocity of light.

Explanation

Material Objects: Stars are massive objects. As established by special relativity, no object with a non-zero rest mass can reach or exceed the speed of light. Its mass would become infinite, requiring infinite energy.

Observational Limit: Even if such a theoretical scenario, were possible, as the star's speed approached 'c' relative to us, the red shift emitted by star would be of infinite value. That is, it would disappear from the observable sight limit of our eyes due to the extreme relativistic effects.

11.3 If the speed of light is just 50 m s^{-1} , how would every day events appear to?

Ans: If the speed of light were only 50 m s^{-1} , everyday events would appear distorted due to extreme relativistic effects.

- Time Dilation:** Clocks would run noticeably slower for anyone moving at typical speeds (e.g., walking or driving), causing a significant lag compared to stationary observers.
- Length Contraction:** Moving objects would appear dramatically shorter and compressed in their direction of motion.
- Mass Increase:** Even at low speeds, objects would become much heavier and harder to accelerate.
- Light Delay:** Light would travel incredibly slowly, causing noticeable delays in seeing things across a room or street, and making communication very difficult.

11.4 If the speed of light were infinite, what would the equations of special theory of relativity reduce to?

Ans: If the speed of light were infinite, no relativity effect will be observed according to theory of relativity. As we have, $c = \infty$ (infinite) then no relativistic phenomenon will be observed as equations of special theory of relativity reduced to

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{t_0}{\sqrt{1 - \frac{v^2}{\infty^2}}} = \frac{t_0}{\sqrt{1 - 0}} = t_0$$

$t = t_0$ (Time becomes absolute.)

Similarly, $l = l_0$ (Length becomes absolute.)

And $m = m_0$ (Mass becomes constant, independent of speed.)

It means there would be no change in physical

object due to relative motion.

11.5 According to Einstein's equation; $E = mc^2$, is it possible to create a single electron from energy? Explain.

Ans: While $E=mc^2$ conceptually allows energy to become mass, creating a single electron from energy is not possible in isolation due to the strict conservation laws of charge and momentum. Instead, energy can be converted into an electron-positron pair through a process called pair production, which fulfills to all these fundamental laws.

- The creation of a single electron from energy alone would violate the law of charge conservation.
- Electrons have a negative charge.
- To conserve charge, a positron (anti-electron with positive charge) must also be created.
- Therefore, energy can create an electron-positron pair, but not a single electron.

COMPREHENSIVE QUESTIONS

11.1 What is meant by the "frame of reference"? Distinguish between inertial frame of reference and non inertial frame of reference by giving examples.

11.2 Describe the Einstein's mass-energy equation, why cannot we observe its effects in everyday life? What are its significant consequences? Give examples.

11.3 State the Einstein's concept about the space-time. Describe the view of gravity according to this concept.

SOLVED EXAMPLES

11.1 The period of a pendulum is measured to be 3.0 s in the inertial reference frame of the pendulum. What is its period measured by an observer moving at a speed of $0.95c$ with respect to the pendulum?

Solution:

Given:

Proper time (t_0) = 3.0 s (time measured in the pendulum's rest frame)

Speed of observer (v) = $0.95c$

We need to find the dilated time (t).

Using the time dilation formula,

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t = \frac{3}{\sqrt{1 - \frac{(0.95c)^2}{c^2}}} \Rightarrow t = \frac{3}{\sqrt{1 - (0.95)^2}}$$

$$t = \frac{3}{\sqrt{1 - 0.9025}} \Rightarrow t = \frac{3}{\sqrt{0.0975}}$$

$$t = \frac{3}{\sqrt{0.0975}} \Rightarrow t = \frac{3}{0.31224}$$

$$t = 9.608 \text{ s}$$

Answer: The period measured by the moving observer is approximately 9.6 s

11.2 A bar 1.0 m in length and located along the x-axis moves with a speed of 0.75 c with respect to a stationary observer. What is the length of the bar as measured by the stationary observer?

Solution:

Given:

Proper length (l_0) = 1.0 m (length when at rest)

Speed (v) = 0.75 c

We need to find the contracted length (l).

Using the length contraction formula:

$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$l = 1 \times \sqrt{1 - \frac{(0.75c)^2}{c^2}}$$

$$= \sqrt{1 - (0.75)^2}$$

$$= \sqrt{0.4375}$$

$$= 0.66 \text{ m}$$

$$l = 0.66 \text{ m}$$

Answer: The length of the bar as measured by the stationary observer is approximately 0.66 m

11.3 Find the mass m of a moving object with speed 0.8 c.

Solution:

Given:

Speed (v) = 0.8 c

We need to find the relativistic mass (m) in terms of rest mass (m_0).

Using the mass variation formula:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$m = \frac{m_0}{\sqrt{1 - (0.8)^2}}$$

$$m = \frac{m_0}{\sqrt{1 - (0.8c)^2/c^2}}$$

$$= \frac{m_0}{\sqrt{1 - 0.64}} = \frac{m_0}{\sqrt{0.36}}$$

$$= \frac{m_0}{0.6}$$

$$m = 1.67m_0$$

Answer: The mass of the moving object is approximately 1.67 times its rest mass

NUMERICAL PROBLEMS

11.1 An electron is accelerated to a speed of 0.995 c which passes down an evacuated tube 500 m long. How long will the tube appear to the electron?

Solution:

$$l_0 = 500 \text{ m}, \quad v = 0.995c, \quad t = ?$$

$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$l = 500 \times \sqrt{1 - \frac{(0.995c)^2}{c^2}} = 500 \times \sqrt{1 - (0.995)^2}$$

$$l = 49.94 \text{ m} \approx 50 \text{ m}$$

11.2 A neutron, being not a stable particle, disintegrates in 20 minutes on the average. How long will it seem to exist if shoots out from a nucleus with a speed of 0.8 c?

Solution:

$$t_0 = 20 \text{ min}, \quad v = 0.8c$$

$$t = ?$$

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{20 \text{ min}}{\sqrt{1 - \frac{(0.8c)^2}{c^2}}} = \frac{20 \text{ min}}{\sqrt{1 - (0.8)^2}}$$

$$t = \frac{20 \text{ min}}{\sqrt{1 - 0.64}} = \frac{20 \text{ min}}{\sqrt{0.36}}$$

$$t = 33.33 \text{ min} = 33 \text{ min}$$

11.3 A spaceship is measured 100 m long while it is at rest with respect to an observer, if this spaceship now flies by the observer with a speed of 0.99 c, what length will the observer find for the spaceship?

Solution:

$$l_0 = 100 \text{ m}, \quad v = 0.99c, \quad t = ?$$

$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$l = 100 \times \sqrt{1 - \frac{(0.99c)^2}{c^2}} = 100 \times \sqrt{1 - (0.99)^2}$$

$$l = 100 \times \sqrt{1 - 0.9801} = 100 \times \sqrt{0.0199}$$

$$l = 14 \text{ m}$$

11.4 The rest mass of an electron is $9.11 \times 10^{-31} \text{ kg}$. Calculate the corresponding rest mass energy.

Given:

$$m_0 = 9.11 \times 10^{-31} \text{ kg}$$

$$E = ?$$

Solution:

$$E = m_0 c^2$$

$$E = 9.11 \times 10^{-31} \times (3 \times 10^8)^2$$

$$E = 9.11 \times 10^{-31} \times 9 \times 10^{16} \text{ J}$$

$$E = 81.99 \times 10^{-15} \text{ J}$$

$$E = 8.2 \times 10^{-14} \text{ J}$$

$$E = \frac{8.2 \times 10^{-14}}{1.6 \times 10^{-19}} = 5.125 \times 10^5 \text{ eV}$$

$$E = 0.51 \times 10^6 \text{ eV}$$

$$E = 0.51 \text{ MeV}$$

11.5 An electron is accelerated to a speed $v = 0.85 c$. Calculate its total energy and kinetic energy in electron volt.

Given:

$$\text{Speed of electron} = v = 0.85 c$$

$$\text{Rest mass energy of electron} = m_0 c^2 = 0.51 \text{ MeV}$$

$$\text{Relativistic mass of proton} = m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{m_0}{\sqrt{1 - \frac{(0.85c)^2}{c^2}}} = \frac{m_0}{\sqrt{1 - 0.85^2}} = \frac{m_0}{0.53}$$

$$\text{Total energy} = T E = ?$$

$$\text{Kinetic energy} = K E = ?$$

$$T E = m c^2 = \frac{m_0}{0.53} c^2$$

$$= \frac{9.11 \times 10^{-31} (3 \times 10^8)^2}{0.53} = 154.69 \times 10^{-15} \text{ J}$$

$$T E = \frac{154.69 \times 10^{-15}}{1.6 \times 10^{-19}} = 96.69 \times 10^4 \text{ eV} = 0.97 \times 10^6 \text{ eV}$$

$$T E = 0.97 \text{ MeV}$$

$$K E = T E - m_0 c^2$$

$$K E = 0.97 - 0.51 = 0.46 \text{ MeV}$$

11.6 At what speed, would the mass of a proton in a particle accelerator be tripled?

$$\text{Rest mass of proton} = m_0$$

$$\text{Relativistic mass of proton} = m = 3m_0$$

$$\text{Speed of proton} = v = ?$$

Solution:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow 3m_0 = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$3 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Squaring both sides

$$9 = \frac{1}{1 - \frac{v^2}{c^2}} \Rightarrow 1 - \frac{v^2}{c^2} = \frac{1}{9}$$

$$\frac{v^2}{c^2} = 1 - \frac{1}{9} \Rightarrow v^2 = \frac{8}{9} c^2 = 0.89 c^2$$

$$v = 0.943c$$

11.7 The period of pendulum is measured to be 3 s in an inertial frame of reference. What will be the period measured by an observer in a spaceship with a constant speed of 0.95 c with respect to the pendulum?

Solution:

$$t_0 = 3 \text{ s}, \quad v = 0.95c$$

$$t = ?$$

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t = \frac{3 \text{ s}}{\sqrt{1 - \frac{(0.95c)^2}{c^2}}} = \frac{3 \text{ s}}{\sqrt{1 - (0.95)^2}}$$

$$t = \frac{3 \text{ s}}{\sqrt{1 - 0.9025}} = \frac{3 \text{ s}}{\sqrt{0.0975}}$$

$$t = 9.6 \text{ s}$$

11.8 Hypothetically, if a ball of mass 0.5 kg is projected with a velocity of 0.9 c, what will be its mass in flight?

$$\text{Rest mass of ball} = m_0 = 0.5 \text{ kg}$$

$$\text{Velocity of ball} = 0.9 c$$

$$\text{Relativistic mass of ball} = m = ?$$

Solution:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{0.5 \text{ kg}}{\sqrt{1 - \frac{(0.9c)^2}{c^2}}} = \frac{0.5 \text{ kg}}{\sqrt{1 - 0.9^2}} = \frac{0.5 \text{ kg}}{0.436}$$

$$m = 1.15 \text{ kg}$$

