2[nd] Way: By multiplying, Danish's father can find the total number of table choices to buy a table with one kind of material and shape.

Total number of table choices = Total types of material × Total types of shape
$= 4 \times 3 = 12$ choices

➤ These examples show that when making a choice involving multiple stages or categories, we can find the total number of outcomes by multiplying the number of options at each stage.

Statement:

Suppose A and B are two events, the event A occurs in m different ways, and the event B occurs in n different ways then the total number of ways that the two events one after another can occur in $m \times n$ ways.

Total number of ways $= mn$

Proof: Let $A = \{a_1, a_2, a_3, \cdots, a_m\}$ and $B = \{b_1, b_2, b_3, \cdots, b_n\}$. Let P denotes the event that both events A and B occur together then $P = \{(a_i, b_j): a_i \in A, b_j \in B, 1 \le i \le m,\ 1 \le j \le n\} = A \times B$. Hence the number of ways in which both events A and B can occur is the number of elements in $A \times B$ which is mn.

➤ This principle can be extended to three or more events. For instance, if event A can occur in m ways, event B can occur in n ways and event C can occur in k ways, the number of ways that three events can occur all together is the product $m \cdot n \cdot k$.

> **Try yourself!**
> If three dice are rolled together, how many total numbers of ways occur?

Factorial (!)

Suppose there are four chairs to be occupied by four students and we are interested in counting all the possible ways the students can be seated.

To occupy the first chair there are 4 options. For the second chair, only 3 students remain, so there are 3 options. Similarly, for the third and fourth chairs, there are 2 and 1 options respectively.

In this way, we have to perform four independent events with 4, 3, 2, and 1 options respectively.

> **History:**
> The factorial notation (!) was introduced by Christian Kramp (1760-1826) in 1808.
> This notation is frequently used to solve permutation and combination.

By the Fundamental Principle of Counting:

Total number of ways to occupy all the chairs = 4.3.2.1 = 24

Such problems frequently occur in daily life, where we have to multiply the first n natural numbers: $1, 2, 3, \cdots, n$. We call this product the factorial of n and denote it by $n!$ or $\lfloor n$.

Definition: For a natural number n, the factorial of n is denoted by $n!$ or $\lfloor n$ and defined as:

$$n! \text{ or } \lfloor n = n(n-1)(n-2)\ldots 3\cdot 2\cdot 1$$

➤ For some reason we also define $0! = 1$.

In general, if n is a non-negative integer, then its factorial is denoted and is defined as

$$n! = \lfloor n = \begin{cases} 1 & \text{if } n = 0 \\ n(n-1)(n-2)\ldots 3\cdot 2\cdot 1 & \text{if } n \ge 1 \end{cases}$$

For example,

$1! = 1$
$2! = 2 . 1 = 2$
$3! = 3 . 2 . 1 = 6$
$4! = 4 . 3 . 2 . 1 = 24$
$5! = 5 . 4 . 3 . 2 . 1 = 120$
$6! = 6 . 5 . 4 . 3 . 2 . 1 = 720$

> **Challenge:**
> Can you find out $\frac{8!}{3!}$?

It can be easily observed that

$$\boxed{n! = n(n-1)! \text{ for } n \ge 1}$$

Example 1: Evaluate $\frac{8!}{6!}$

Solution: $\frac{8!}{6!} = \frac{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1} = 8\cdot 7 = 56$

Example 2: Write 8.7.6.5 in the factorial form.

Solution: $8\cdot 7\cdot 6\cdot 5 = \frac{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{4\cdot 3\cdot 2\cdot 1}$

$= \frac{8!}{4!}$

Example 3: Evaluate $\frac{9!}{6!3!}$

Solution: $\frac{9!}{6!3!} = \frac{(9\cdot 8\cdot 7)6!}{6!(3\cdot 2\cdot 1)} = 84$

Exercise 7.1

1. Evaluate each of the following:

(i) $\frac{10!}{0!8!}$

Solution:

$\frac{10!}{0!8!} = \frac{10\cdot 9\cdot 8!}{1\cdot 8!} = 10\cdot 9 = 90$

(ii) $\frac{12!}{3!(12-3)!}$

Solution:

$\frac{12!}{3!(12-3)!} = \frac{12\cdot 11\cdot 10\cdot 9!}{(3\cdot 2\cdot 1)\cdot 9!}$

$= 2\cdot 11\cdot 10 = 220$

(iii) $\frac{1440}{3!4!} + \frac{2400}{5!2!}$

Solution:

$\frac{1440}{3!\,4!} + \frac{2400}{5!\,2!}$

$= \frac{1440}{(3.2.1)(4.3.2.1)} + \frac{2400}{(5.4.3.2.1)\cdot(2.1)}$

$= \frac{1440}{6.24} + \frac{2400}{120.2}$

$= \frac{1440}{144} + \frac{2400}{240}$

$= 10 + 10 = 20$

(iv) $\frac{(n+2)!}{(n+1)!}$

Solution:

$\frac{(n+2)!}{(n+1)!} = \frac{(n+2)(n+1)!}{(n+1)!}$

$= n + 2$

2. Write each of the following in the factorial form:

(i) $n^3 - n$

Solution:

$n^3 - n = n(n^2 - 1)$

$= n(n+1)(n-1)$

$= \frac{(n+1)(n)(n-1)\cdot(n-2)!}{(n-2)!}$

$= \frac{(n+1)!}{(n-2)!}$

(ii) $n(n-1)(n-2)\ldots(n-r+1)$

Solution:

$n(n-1)(n-2)\cdots(n-r+1)$

$= \frac{n(n-1)(n-2)\cdots(n-r+1)(n-r)!}{(n-r)!}$

$= \frac{n!}{(n-r)!}$

3. Find n, if $(n+4)! = 3024 \cdot n!$.

Solution:

$(n+4)! = 3024\cdot n!$

$\frac{(n+4)!}{n!} = 3024$

$\frac{(n+4)(n+3)(n+2)(n+1)n!}{n!} = 9\cdot 8\cdot 7\cdot 6$

$(n+4)(n+3)(n+2)(n+1) = (5+4)(5+3)(5+2)(5+1)$

Comparing both bides, we have

$n = 5$

4. If $\frac{1}{7!} + \frac{1}{8!} = \frac{x}{9!}$, find x.

Solution:

$\frac{1}{7!} + \frac{1}{8!} = \frac{x}{9!}$

$\frac{1}{7!} + \frac{1}{8\cdot 7!} = \frac{x}{9\cdot 8\cdot 7!}$

Multiply each term by (9.8.7!)

$9\cdot 8 + 9 = x$

$x = 72 + 9$

$x = 81$

5. Prove that:

$\frac{(2n+1)!}{n!} = [1.3.5\ldots(2n-1)(2n+1)]2^n$

Solution:

$\text{L.H.S} = \frac{(2n+1)!}{n!}$

$= \frac{(2n+1)(2n)(2n-1)(2n-2)(2n-3)(2n-4)(2n-5)\cdot 654321}{n!}$

$$= \frac{[2n(2n-2)(2n-4)\cdots 6.4\cdot 2][(2n+1)(2n-1)\cdots 5\cdot 3.1]}{n!}$$

$$= \frac{2^n[n(n-1)(n-2)\cdots 3.2\cdot 1][(2n+1)(2n-1)\cdots 5\cdot 3.1]}{n!}$$

$$= \frac{2^n\cdot n![1.3\cdot 5\cdots(2n-1)(2n+1)]}{n!}$$

$= [1.3\cdot 5\cdots(2n-1)(2n+1)]2^n$

= R.H.S (Proved)

6. Express as a single fraction: $\frac{(n+2)!}{(r+2)!}+\frac{(n+1)!}{(r+1)!}$

Solution:

$$\frac{(n+2)!}{(r+2)!}+\frac{(n+1)!}{(r+1)!}$$

$$=\frac{(n+2)(n+1)!}{(r+2)(r+1)!}+\frac{(n+1)!}{(r+1)!}$$

$$=\frac{(n+1)!}{(r+1)!}\left\{\frac{n+2}{r+2}+1\right\}$$

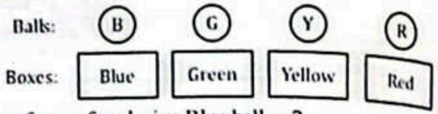
$$=\frac{(n+1)!}{(r+1)!}\left\{\frac{n+2+r+2}{r+2}\right\}$$

$$=\frac{(n+1)!(n+r+4)}{(r+2)(r+1)!}$$

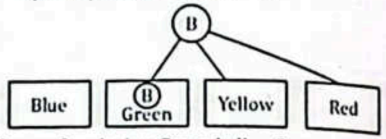
$$=\frac{(n+1)!(n+r+4)}{(r+2)!}$$

7. There are four distinct colored balls and four boxes of same colors as those of the balls. Determine the number of possible ways the balls, one each in a box, can be placed such that a ball does not go to a box of its own colour?

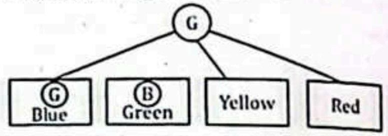
Solution:



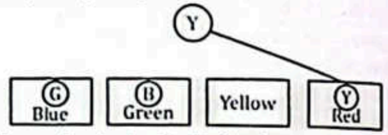
No. of ways for placing Blue ball = 3



No. of ways for placing Green ball = 3



No of ways for placing Yellow ball = 1



No. of ways for placing Red ball = 1

Total no. of ways = 3 × 3 × 1 × 1 = 9 ways

Permutations:

One important application of the fundamental principle of counting is to determine the number of ways that objects can be arranged in order.

Definition: An arrangement of all or part of set of objects in a specific order is called a permutation. Number of permutations of $r(\leq n)$ objects taken from a set of n objects is written as ^nP_r or $P(n,r)$.

$$^nP_r=\frac{n!}{(n-r)!} \quad \text{when } r\leq n$$

According to fundamental principle of counting:

(i) Three books of mathematics for grades 1, 2 and 3 can be arranged in a row taken all at a time (if books are distinct)

$^nP_r = {}^3P_3 \qquad \because n=r=3$

$= \frac{3!}{(3-3)!}=\frac{3!}{0!} \qquad \because 0!=1$

$= 3!=3\cdot2\cdot1=6$ ways

(ii) Number of ways of writing the letters of the WORD taken all at a time

$^nP_r = {}^4P_4 \qquad \because n=r=4$

$= \frac{4!}{(4-4)!}=\frac{4!}{0!} \qquad \because 0!=1$

$= 4!=4\cdot3\cdot2\cdot1=24$ ways

n = Total number of things/objects
r = The number of selected things/objects

Challenge:
Can you make total number of permutations for the "WORD" pictorially?

Do you know:
In 1974, "Erno Rubik" invented a popular puzzle, each turn of the puzzle shows a permutation of the different colours. The name of this puzzle is "Rubik's Cube".

Theorem: Prove that: $^nP_r=n(n-1)(n-2)\dots(n-r+1)=\frac{n!}{(n-r)!}$

Proof: As there are n different objects to fill up r places. So,

The first place can be filled in n ways.

Since repetitions are not allowed, so after placing one object we are left with $(n-1)$ objects, thus

The second place can be filled in $(n-1)$ ways.

Similarly,

The third place can be filled in $(n-2)$ ways, and so on. This continues until

The r^{th} place which can be filled in $n-(r-1)=n-r+1$ ways.

Therefore, by the **Fundamental Principle of Counting**, r places can be filled by n different objects in $n(n-1)(n-2)\cdots(n-r+1)$ ways.

$$^nP_r = n(n-1)(n-2)\dots(n-r+1)$$

$$=\frac{n(n-1)(n-2)\dots(n-r+1)(n-r)!}{(n-r)!}$$

$$^nP_r=\frac{n!}{(n-r)!}$$

Example 4: How many different 4-digit numbers can be formed out from the digits 1, 2, 3, 4, 5, 6, when no digit is repeated?

Solution: The total number of digits = 6

The digits forming each number = 4

So, the required number of 4-digit numbers is given by:

$$^6P_4=\frac{6!}{(6-4)!}=\frac{6!}{2!}=\frac{6\cdot5\cdot4\cdot3\cdot2\cdot1}{2.1}=6\cdot5\cdot4\cdot3=360$$

Example 5: In how many ways can a set of 4 different mathematics books, 3 different physics books and 2 different chemistry books be placed on a shelf with a space for 9 books, if:

(a) all the books are kept without any restriction.
(b) all the books of the same subject are kept together.
(c) only the mathematics books are kept together.

Solution:

(a) all the books are kept without any restriction.

Total number of books = 4 + 3 + 2 = 9

$^9P_9 = 9!=9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1$

$= 362880$ ways

Reason for defining 0! = 1

If $n=r$, then $^nP_n=\frac{n!}{(n-n)!}=\frac{n!}{0!}$

and $^nP_n=n(n-1)\dots3\cdot2\cdot1=n!$

On Comparing: $\frac{n!}{0!}=n! \Rightarrow \boxed{0!=1}$

(b) all the books of the same subject are kept together.

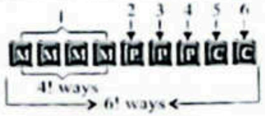
$^4P_4\cdot{}^3P_3\cdot{}^2P_2\cdot{}^3P_3 = 4!\cdot3!\cdot2!\cdot3!$

$= 3!\cdot24\cdot6\cdot2\cdot6$

$= 1728$ ways

(c) only the mathematics books are kept together.

$^4P_4 \cdot {}^6P_6 = 4!\,6!$
$= 24 \cdot 720$
$= 17280$ ways



Challenge!
Find the number of ways if only physics books are kept together.

Example 6: In how many ways 5 people are to be seated on a bench if:
(a) there are no restrictions
(b) two people can sit next to each other
(c) two people cannot sit next to each other.

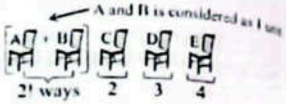
Solution:

(a) when there is no restriction, then

Number of ways $= {}^5P_5 = 5! = 120$

(b) when two people can sit next to each other, then

$= {}^4P_4 \cdot {}^2P_2$
$= 4!\,2! = 24 \cdot 2$
$= 48$ ways



(c) when two people cannot sit next to each other, then

$= {}^5P_5 - [2 \text{ can sit next to each other}]$
$= 5! - 48 = 120 - 48$
$= 72$ ways

Try yourself!
In how many ways 6 people are to seated on a table if 3 cannot sit next to each other?

Exercise 7.2

1. Evaluate the following:

(i) $^{10}P_5$

Solution:

$$^{10}P_5 = \frac{10!}{(10-5)!} = \frac{10!}{5!}$$
$$= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{5!} = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 30240$$

(ii) $^5P_2$

Solution:

$$^5P_2 = \frac{5!}{(5-2)!} = \frac{5!}{3!}$$
$$= \frac{5 \cdot 4 \cdot 3!}{3!} = 5 \cdot 4 = 20$$

(iii) $^7P_7$

Solution:

$$^7P_7 = \frac{7!}{(7-7)!} = \frac{7!}{0!}$$
$$= \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1} = 5040$$

(iv) $^{10}P_3$

Solution:

$$^{10}P_3 = \frac{10!}{(10-3)!} = \frac{10!}{7!}$$
$$= \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7!} = 10 \cdot 9 \cdot 8 = 720$$

2. Find the value of n when:

(i) $^nP_3 = 504$

Solution:

$$nP_3 = 504$$
$$\frac{n!}{(n-3)!} = 504$$
$$\frac{n(n-1)(n-2)(n-3)!}{(n-3)!} = 504$$
$$n(n-1)(n-2) = 9.8.7$$
$$n(n-1)(n-2) = 9(9-1)(9-2)$$

Comparing both sides, we have

$\boxed{n = 9}$

(ii) $^{15}P_n = 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11$

Solution:

$$15P_n = 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11$$
$$\frac{15!}{(15-n)!} = 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11$$
$$\frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10!}{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11} = (15-n)!$$
$$10! = (15-n)! \text{ we have}$$

Comparing both sides, we have

$$10 = 15 - n$$
$$n = 15 - 10$$
$$\Rightarrow \boxed{n = 5}$$

(iii) $^nP_5 : {}^{n-2}P_2 = 540 : 1$

Solution:

$$^nP_5 : {}^{n-2}P_2 = 540 : 1$$
$$^nP_5 \div {}^{n-2}P_2 = \frac{540}{1}$$
$$\frac{n!}{(n-5)!} \div \frac{(n-2)!}{(n-2-2)!} = 540$$
$$\frac{n!}{(n-5)!} \times \frac{(n-4)!}{(n-2)!} = 540$$
$$\frac{n(n-1)(n-2)!}{(n-5)!} \times \frac{(n-4)(n-5)!}{(n-2)!} = 540$$
$$n(n-1)(n-4) = 540$$
$$n(n-1)(n-4) = 10 \cdot 9 \cdot 6$$
$$n(n-1)(n-4) = 10(10-1)(10-4)$$

Comparing both sides, we have

$\boxed{n = 10}$

3. Prove from the first principle that:

(i) $^nP_r = n \cdot {}^{n-1}P_{r-1}$

Solution:

$$^nP_r = n \cdot {}^{n-1}P_{r-1}$$
$$\text{R.H.S} = n \cdot {}^{n-1}P_{r-1}$$
$$= n\frac{(n-1)!}{(n-1-r+1)!} = \frac{n(n-1)!}{(n-r)!}$$
$$= \frac{n!}{(n-r)!} \qquad \because n(n-1)! = n!$$
$$= {}^nP_r = \text{L.H.S (Proved)}$$

(ii) $^nP_r = {}^{n-1}P_r + r \cdot {}^{n-1}P_{r-1}$

Solution:

$$^nP_r = {}^{n-1}P_r + r \cdot {}^{n-1}P_{r-1}$$
$$\text{R.H.S} = {}^{n-1}P_r + r \cdot {}^{n-1}P_{r-1}$$
$$= \frac{(n-1)!}{(n-1-r)!} + \frac{r(n-1)!}{(n-1-r+1)!}$$
$$= \frac{(n-1)!}{(n-r-1)!} + \frac{r(n-1)!}{(n-r)!}$$
$$= \frac{(n-1)!}{(n-r-1)!} + \frac{r(n-1)!}{(n-r)(n-r-1)!}$$
$$= \frac{(n-1)!}{(n-r-1)!}\left\{1 + \frac{r}{(n-r)}\right\}$$
$$= \frac{(n-1)!}{(n-r-1)!}\left\{\frac{n-r+r}{(n-r)}\right\}$$
$$= \frac{n(n-1)!}{(n-r)(n-r-1)!}$$
$$= \frac{n!}{(n-r)!} = {}^nP_r = \text{L.H.S (Proved)}$$

4. How many words can be formed from the letters of the following words using all letters when no letter is to be repeated:

(i) PYTHON

Solution:

PYTHON

No. of letters in the wand 'PYTHON' = 6

No. of words formed using all the letters $= {}^6P_6 = 6!$
$= 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

(ii) NETWORK

Solution:

NETWORK

No. of letters in The word 'NETWORK' = 7

No. of words formed using all the letters $= {}^7P_7 = 7!$
$= 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$

(iii) COMPUTER

Solution:

COMPUTER

No. of letters in the word 'COMPUTER' = 8

No. of words formed using all the letters $= {}^8P_8 = 8!$
$= 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40320$

5. How many signals can be given by 6 flags of different colours, using 2 flags at a time?

Solution:

Total no. of flags = 6

Number of signals using 2 flags $= {}^6P_2$
$$= \frac{6!}{(6-2)!} = \frac{6.5.4!}{4!} = 6.5 = 30$$

6. How many signals can be given by 5 flags of different colours, when any number of flags are used at a time.

Solution:

Total no. of flags = 5

Number of signals using 1 flag
$$= {}^5P_1 = \frac{5!}{(5-1)!} = \frac{5.4!}{4!} = 5$$

Number of signals using 2 flags
$$= {}^5P_2 = \frac{5!}{(5-2)!} = \frac{5.4.3!}{3!} = 5.4 = 20$$

Number of signals using 3 flags
$$= {}^5P_3 = \frac{5!}{(5-3)!} = \frac{5.4.3.2!}{2!} = 5.4.3 = 60$$

Number of signals using 4 flags

$$= {}^5P_4 = \frac{5!}{(5-4)!} = \frac{5\cdot4\cdot3\cdot2\cdot1}{1!} = 120$$

Number of signals using 5 flags $= {}^5P_5 = 5! = 120$

Total no. of signals $= 5 + 20 + 60 + 120 + 120 = 325$

7. How many 4 digit numbers can be formed, with distinct digits, with each digit odd?

Solution:

Odd digits: 1, 3, 5, 7, 9

Total number of digits = 5

No. of 4-digit numbers $= {}^5P_4$

$$= \frac{5!}{(5-4)!} = \frac{5\cdot4\cdot3\cdot2\cdot1}{1!} = 120$$

8. How many numbers between 100 and 1000 can be formed by using the digits 0, 1, 2, 3, 4, 5 without repetition? how many of them are divisible by 5?

Solution:

Digits: 0, 1, 2, 3, 4, 5

Numbers between 100 and 1000 are the 3 -digit numbers.

For 3-digit numbers '0' should not be at extreme left position. Total no. of arrangements of three disits $= {}^6P_3$

$$= \frac{6!}{(6-3)!} = \frac{6\cdot5\cdot4\cdot3!}{3!} = 120$$

No. of arrangements of three digits, when '0' is at The extreme left position $= {}^5P_2 = \frac{5!}{(5-2)!} = \frac{5\cdot4\cdot3!}{3!} = 20$

Required no. of 3-digit numbers $= 120 - 20 = 100$

For the numbers divisible by 5 digits 0 or 5 at unit place. i.e.,

CASE-I:

If 0 is at unit place, then

×	×	0

No. of ways $= 5 \times 4 \times 1 = 20$

CASE-II:

If 5 is at unit place, Then

×	×	5

No. of ways $= 4 \times 4 \times 1 = 16$

No. of 3-digit numbers divisible by $5 = 20 + 16 = 36$

9. Find the numbers greater than 35000 that can be formed from the digits 1, 2, 3, 4, 5, 6, without repeating any digit.

Solution:

Digit: 1, 2, 3, 4, 5, 6

We want to find the 5-digit numbers greats than 35000 by using given digits.

Numbers greater than 35000 are of the form.

$$35\otimes\otimes\otimes \quad {}^4P_3 = \frac{4!}{(4-3)!} = \frac{4\cdot3\cdot2\cdot1}{1!} = 24$$

$$36\otimes\otimes\otimes \quad {}^4P_3 = \frac{4!}{(4-3)!} = \frac{4\cdot3\cdot2\cdot1}{1!} = 24$$

$$4\otimes\otimes\otimes\otimes \quad {}^5P_4 = \frac{5!}{(5-4)!} = 5\cdot4\cdot3\cdot2\cdot1 = 120$$

$$5\otimes\otimes\otimes\otimes \quad {}^5P_4 = \frac{5!}{(5-4)!} = 5\cdot4\cdot3\cdot2\cdot1 = 120$$

$$6\otimes\otimes\otimes\otimes \quad {}^5P_4 \frac{5!}{(5-4)!} = 5.4.3.2.1 = 120$$

Total required numbers $= 24 + 24 + 120 + 120 + 120 = 408$

10. Find the number of 5-digit numbers that can be formed from the digits 1, 2, 4, 6, 8 (when no digit is repeated), but

(i) the digits 2 and 8 are next to each others;

(ii) the digits 2 and 8 are not next to each other.

(i) the digits 2 and 8 are next to each others;

Solution:

Digits: 1, 2, 4, 6, 8

When 2 and 8 are next to each other, we consider them a single digit.

Arrangements of 2 and 8 are [82], [28] = 2!

No. of arrangements when 2 and 8 are next to each other

$= {}^4P_4 \times 2! = 4! \times 2!$

$= 24 \times 2 = 48$

(ii) the digits 2 and 8 are not next to each other.

Solution:

Total number of arrangements $= {}^5P_5 = 5!$

$$= \frac{5.4.3.2.1}{1} = 120$$

No. of arrangements when 2 and 8 are not next to each other

$= 120 - 48 = 72$

11. How many 6-digit numbers can be formed, without repeating any digit from the digits 0, 1, 2, 3, 4, 5? In how many of them will 0 be at the tens place?

Solution:

Digits: 0, 1, 2, 3, 4, 5

For 6-digit numbers '0' should not be at extreme left position, i.e.,

Total no. of arrangements of six digits $= {}^6P_6 = 6! = 720$

No. of arrangements of six digits when '0' at the extreme left place $= {}^5P_5 = 5! = 120$

Required no. of 6-digit numbers $= 720 - 120 = 600$

If we fix '0' at tens place then numbers formed

$= \otimes\ \otimes\ \otimes\ \otimes\ 0\ \otimes = {}^5P_5 = 5! = 120$

12. How many 5-digit multiples of 5 can be formed from the digits 2, 3, 5, 7, 9, when no digit is repeated.

Solution:

Digits: 2, 3, 5, 7, 9

For multiples of 5, 5 should be at the unit place, i.e. $\otimes\ \otimes\ \otimes\ \otimes\ 5$

No. of 5-digit numbers multiples of $5 = {}^4P_4 = 4! = 24$

13. In how many ways can 8 different books including 2 on English be arranged on a shelf in such a way that the English books are never together?

Solution:

No. of English books = 2

No. of other books = 6

Total No. of books = 8

Total arrangements of 8 books $= {}^8P_8 = 8! = 40320$

When two English books come together, we consider them as single book.

Arrangements of two English books $[E_1E_2]$ and $[E_2E_1] = 2!$

No. of arrangements when English books E_1 and E_2 are together $= {}^7P_7 \times 2! = 7! \times 2! = 10080$

No. of arrangements when English books are never together $= 40320 - 10800 = 30240$

14. Find the number of arrangements of 3 different books on English and 5 different books on Urdu for placing them on a shelf such that the books on the same subject are together.

Solution:

No. of English books = 3

No. of Urdu books = 5

Total No. of books = 8

Arrangements of three English books E_1, E_2, E_3 and five Urdu books U_1, U_2, U_3, U_4, U_5 are of the forms

$[E_1E_2E_3U_1U_2U_3U_4U_5] = {}^3P_3 \times {}^5P_5 = 3! \times 5! = 720$

$[U_1U_2U_3U_4U_5E_1E_2E_3] = {}^5P_5 \times {}^3P_3 = 5! \times 3! = 720$

Total required number of arrangements $= 720 + 720 = 1440$

15. In how many ways can 5 boys and 4 girls be seated on a bench so that the girls and the boys occupy alternate seats?

Solution:

No. of boys = 5

No. of girls = 4

Let the five boys be B_1, B_2, B_3, B_4, B_5 and the four girls be G_1, G_2, G_3, G_4

Arrangements of 5 boys and 4 girls occupy alternate seats are

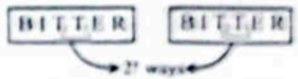
$B_1\ G_1\ B_2\ G_2\ B_3\ G_3\ B_4\ G_4\ B_5$

$= {}^5P_5 \times {}^4P_4 = 5! \times 4! = 120 \times 24 = 2880$

Permutation of Objects Not All Different:

Suppose we have to find the permutations of the letters of the word BITTER using all the letters. The word BIT_1T_2ER consists of 6 different letters which can be permuted among themselves in 6! ways.

We can see that all the letters of the word BITTER are not different. It has 2Ts in it. We can see there are 2! ways of replacing two T_s.



The replacement of the two T_s by T_1 and T_2 in any other permutation will give rise to 2 permutations.

Hence, the number of permutations of the letters of the word BITTER taken all at a time

$$\frac{6!}{2!} = \frac{6\cdot5\cdot4\cdot3\cdot2\cdot1}{2.1} = 360 \text{ ways}$$

> **Remember!**
>
> If there are n_1 alike objects of one kind, n_2 alike objects of second kind and n_3 alike objects are of third kind, then the number of permutations of n objects taken all at a time is given by
>
> $$\frac{n!}{n_1!n_2!n_3!} = \binom{n}{n_1, n_2, n_3}$$

Example 7: In how many ways can the letters of the word MISSISSIPPI be arranged when all the letters are to be used?

Solution: Total number of letters in the word = 11

MISSISSIPPI

I is repeated 4 times = 4! ways

S is repeated 4 times = 4! ways

P is repeated 2 times = 2 ! ways

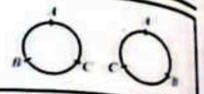
M comes once only = 1 ! ways

$$\text{Required number of permutations} = \frac{11!}{4!\,4!\,2!\,1!} = 34650 \text{ ways}$$

Circular Permutations:

The permutations in which the object are arranged in a circular order are known as circular permutations.

Note:
The following circular arrangements are reflection of each other and considered same when anticlockwise and clockwise arrangements are considered identical.



Circular permutations can occur in two cases:

Case-I: When clockwise and anticlockwise arrangements are considered different

In a linear arrangement, changing the order of objects results in a new arrangement. However, in a circular arrangement, rotating the entire circle does not produce a new, distinct arrangement.

For example, suppose three people A, B and C are sitting around a round table. The following three linear arrangements $A-B-C, B-C-A$ and $C-A-B$ are considered the same in circular permutations because each one is simply a rotation of the other.

We conclude that:

3 linear permutations gives 1 circular permutation.

1 linear permutations gives $\frac{1}{3}$ circular permutation.

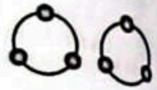
3! linear permutations gives $\frac{1}{3}\cdot 3!=\frac{3!}{3}=2!$ permutations.

Generalizing the above idea if n objects are arranged in a circle, the number of distinct circular permutations is $\frac{n!}{n}=(n-1)!$.

Case-II: When clockwise and anticlockwise arrangements are considered identical

In many real-life situations, a circular permutation and its mirror image are not considered different.

For example, if three beads red, blue, and black are arranged in a ring, then an arrangement and its reflection (as shown in the figure) are considered the same.



In such cases, we divide the total number of circular permutations by 2 to eliminate symmetrical duplicates.

Thus, in this case the number of distinct circular permutations is: $\frac{(n-1)!}{2}$

Example 8: In how many ways can 4 persons be seated at a round table, while:

(i) clockwise and anticlockwise orders are different

(ii) clockwise and anticlockwise orders are identical.

Solution: Let A, B, C and D be the 4 persons.

(i) If clockwise and anticlockwise orders are different

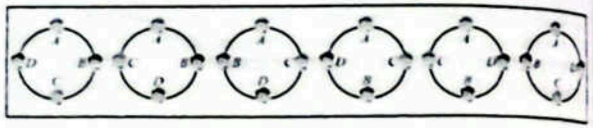
According to Case-I:

The possible number of ways are:

$= (n-1)!$ ways

$= (4-1)! = 3!$

$= 3\cdot 2\cdot 1 = 6$ ways



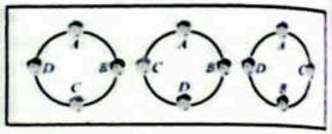
(ii) If clockwise and anticlockwise orders are identical

According to Case-II:

The possible number of ways are $= \frac{(n-1)!}{2}$ ways

$= \frac{(4-1)!}{2} = \frac{3!}{2}$

$= \frac{3\cdot 2}{2} = 3$ ways



Exercise 7.3

1. How many arrangements of the letters of the following words, taken all together can be made?

(i) PAKISTAN

Solution:

Total no. of letters = 8
A is repeated 2 times
P comes 1 time
K comes 1 time
I comes 1 time
T comes 1 time
S comes 1 time
N comes 1 time

Total number of arrangements $= \binom{8}{2,1,1,1,1,1,1}$

$= \frac{8!}{2!.1!.1!.1!.1!.1!}$

$= \frac{8.7.6.5.4.3.2!}{2!}$

$= 20160$

(ii) CURRICULUM

Solution:

CURRICULUM

Total no. of letters	= 10
C is repeated 2 times	= 2! ways
U is repeated 3 times	= 3! ways
R is repeated 2 times	= 2! ways
I comes only	= 1! ways
L comes once only	= 1! ways
M comes once only	= 1! ways

Required number of permutations $= \frac{10!}{2!\cdot 3!\cdot 2!\cdot 1!\cdot 1!\cdot 1!}$

$= \frac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3!}{2\cdot 3!\cdot 2}$

$= 151200$

(iii) PROBABILITY

Solution:

PROBABILITY
Total no. of letters = 11
B is repeated 2 times = 2! ways
I is repeated 2 times = 2! ways
P comes once only = 1! ways
R comes once only = 1! ways
0 comes once only = 1! way
A comes once only = 1! way
L comes once only = 1! ways
T comes once only = 1! ways
Y comes once only = 1! ways

Required number of permutations are

$= \frac{11!}{2!\cdot 2!\cdot 1!\cdot 1!\cdot 1!\cdot 1!\cdot 1!\cdot 1!\cdot 1!}$

$= \frac{11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2!}{2!\cdot 2!}$

$= 9979200$

2. How many permutations of the letters of the word "BANANA" can be made, if B must be the first letter in each arrangement?

Solution:

If B is the first letter of each arrangement, Then words of the form

$B \circ \circ \circ \circ \circ$

Total no. of letters other than $B = 5$
A is repeated 3 times
N is repeated 2 times

Total number of arrangements $= \frac{5!}{3!\cdot 2!}$

$= \frac{5\cdot 4\cdot 3!}{3!(2\cdot 1)} = 5\cdot 2 = 10$

3. How many arrangements of the letters of the word TRIGONOMETRY can be made, if each arrangement begins with T and ends with Y?

Solution:

If each arrangement begins with T and ends with Y, then the words of the form.

$T \circ \circ \circ \circ \circ \circ \circ \circ \circ \circ Y$

Total no. of letters other than T and $Y = 10$
R is repeated 2 times
O is repealed 2 times
I, G, N, M, E and T comes only once.

Total no. of arrangements $= \frac{10!}{2!\cdot 2!\cdot 1!\cdot 1!\cdot 1!\cdot 1!\cdot 1!\cdot 1!}$

$= \frac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2!}{2!(2\cdot 1)}$

$= 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 2\cdot 3$

$= 907200$

4. Abdullah has a collection of 9 marbles consisting of 4 identical red marbles, 3 identical blue marbles and 2 identical green marbles. If he wants to arrange all of them is a straight row, how many distinct arrangements are possible?

Solution:

Total no. of marbles = 9
4 identical red marbles
3 identical blue marbles
2 identical green marbles

Total no. of distinct arrangements $= \frac{9!}{4!\,3!\,2!}$

$= \frac{9.8.7.6.5.4!}{4!\,(3.2.1)\cdot(2.1)}$

$= 9\cdot4\cdot7\cdot5 = 1260$

5. In how many different ways can the following persons sit in a round table?

(a) 8 persons

Solution:

Total no. of persons $= n = 8$

possible number of ways $= (n-1)!$

$= (8-1)!$

$= 7!$

$= 7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1$

$= 5040$ ways

(b) 7 persons

Solution:

Total no. of persons $= n = 7$

Possible number of ways $= (n-1)!$

$= (7-1)!$

$= 6!$

$= 6\cdot5\cdot4\cdot3\cdot2\cdot1$

$= 720$ ways.

(c) 6 persons

Solution:

Total no. of persons $= n = 6$

possible number of ways $= (n-1)!$

$= (6-1)!$

$= 5!$

$= 5\cdot4\cdot3\cdot2\cdot1$

$= 120$ ways

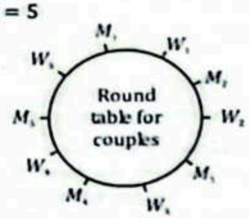
6. In how many ways can 5 couples sit around a round table if no two women are sitting together?

Solution:

Total no. of persons = 10

No. of men = 5

No. of women = 5



Since no two women sit together, therefore

Possible no. of ways $= (5-1)!\cdot5!$

$= 4!\cdot5!$

$= (4\cdot3\cdot2\cdot1)(5\cdot4\cdot3\cdot2\cdot1)$

$= 24\cdot120$

$= 2880$ ways.

7. How many 6-digit numbers can be formed from the digits 7, 7, 8, 8, 9, 9?

Solution:

Digits: 7, 7, 8, 8, 9, 9

No. of 6-digit numbers $= \frac{6!}{2!\,2!\,2!}$

$= \frac{6\cdot5\cdot4\cdot3\cdot2!}{2!\cdot2\cdot2}$

$= 6\cdot5\cdot3$

$= 90$

8. 15 members of a club form 4 committees of 3, 5, 4, 3 members so that no member is a member of more than one committee. Find the number of committees.

Solution:

Total members of a club = 15

Members in 1[st] committee = 3

Members in 2[nd] committee = 5

Members in 3[rd] committee = 4

Members in 4[th] committee = 3

No. of committees $= \frac{15!}{3!\,5!\,4!\,3!}$

$= \frac{15\cdot14\cdot13\cdot12\cdot11\cdot10\cdot9\cdot8\cdot7\cdot6\cdot5!}{5!\,(3\cdot2\cdot1)\cdot(4\cdot3\cdot2\cdot1)\cdot(3\cdot2\cdot1)}$

$= 15\cdot14\cdot13\cdot11\cdot10\cdot3\cdot2\cdot7$

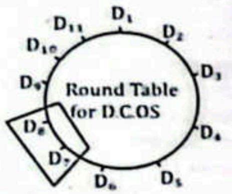
$= 12{,}612{,}600$

9. The D.C.Os of 11 districts meet to discuss the law-and-order situation in their districts. In how many ways can they be seated at a round table, when two particular D.C.Os insist on sitting together?

Solution:

Total no. of D.C.Os $= 11 = n$

Since two particular D.C.Os insist on sitting together, so consider them as one D.C.O.



Two particular D.C.Os can be seated in 2! ways since D.C.Os sit along a round table, so

Required no. of arrangements $= (n-1)!\times2!$

$= (10-1)!\times2!$

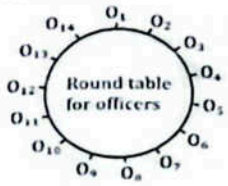
$= 9!\times2!$

$= 362880\times2 = 725760$

10. The Governor of the Punjab calls a meeting of 14 officers. In how many ways can they be seated at a round table?

Solution:

Total no. of officers $= 14 = n$



Since all officers sit along a round table, so

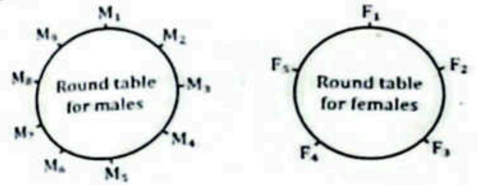
Required no. of permutations $= (n-1)!$

$= (14-1)!$

$= 13!$

11. Fatima invites 14 people to a dinner. There are 9 males and 5 females who are seated at two different tables. Guests of one sex sit at one round table and the guests of the other sex sit at the second table. Find the number of ways in which all guests are seated.

Solution:



No. of males $= 9 = n$

No. of females $= 5 = n'$

Since males and females sit at two different round table, so

Required no. of permutations $= (n-1)!\times(n'-1)!$

$= (9-1)!\times(5-1)!$

$= 8!\times4!$

$= (8.7.6.5\cdot4.3.2\cdot1)(4.3\cdot2.1)$

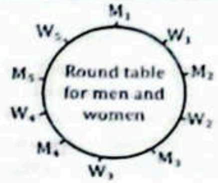
$= (40.320)(24) = 967680$

12. Find the number of ways in which 5 men and 5 women can be seated at a round table in such a way that no two persons of the same sex sit together.

Solution:

No. of men = 5

No. of women = 5



Since men and women sit along a round table and no two persons of same sex sit together, so

Required no. of permutations $= (5-1)!\times5!$

$= 4!\times5!$

$= (4.3.2.1)(5.4.3.2.1)$

$= 24\times120$

$= 2880$

13. In how many ways can 8 keys be arranged in a circular key ring?

Solution:

No. of keys $= n = 8$

Both clockwise and anticlockwise arrangements are identical, then

Possible no. of ways $= \frac{(n-1)!}{2}$

$= \frac{(8-1)!}{2}$

$= \frac{7!}{2}$

$= \frac{5040}{2}$

$= 2520$ ways

14. How many necklaces can be made from 10 beads of different colours?

Solution:

No. of beads $= n = 10$

Both clockwise and anticlockwise arrangements are identical, then

Possible no. of ways $= \frac{(n-1)!}{2}$

$= \frac{(10-1)!}{2}$

$= \frac{9!}{2}$

$= \frac{362880}{2}$

$= 181440$

Combinations:

Suppose, a teacher uses the names of few students to make a team for a writing competition. Such as Ahmad, Sana, Hamza and Danish. As a combination of team members, (Ahmad, Sana, Hamza and Danish) is equivalent to (Hamza, Ahmad, Danish and Sana). Because same students are in the combination. Consequently, you have the same team because the order of the name of the students does not matter.

So, we are interested in the membership of the team and not in the ways the students are listed (arranged).

Ahmad	Sana	Hamza	Danish
Hamza	Ahmad	Danish	Sana

Definition:

A combination of r objects taken out of n objects is a subset of r objects of a set of n objects.

The number of combinations of n different objects taken r at a time is denoted by ^nC_r or $C(n,r)$ or $\binom{n}{r}$ and is given by $^nC_r = \dfrac{n!}{r!(n-r)!}$.

Theorem: Prove that $^nC_r = \dfrac{n!}{r!(n-r)!}$.

Proof: Elements of a subset of r objects of a set of n objects can be arranged among themselves in $r!$ ways. So, each combination will give rise to $r!$ permutation. Thus, there will be $^nC_r \times r!$ permutations of n different objects taken r at a time that is:

$$^nC_r \times r! = {^nP_r}$$

$$\Rightarrow \quad ^nC_r \times r! = \frac{n!}{(n-r)!} \qquad \therefore \quad ^nC_r = \frac{n!}{r!(n-r)!}$$

Which completes the proof.

Corollary:

(i) If $r = n$, then $^nC_n = \dfrac{n!}{n!(n-n)!} = \dfrac{n!}{n!\,0!} = 1$

(ii) If $r = 0$, then $^nC_0 = \dfrac{n!}{0!(n-0)!} = \dfrac{n!}{0!\,n!} = 1$

> **Remember!**
> The formulae ^nP_r and ^nC_r are also known as counting formulae. Because, they are used to count the possible number of ways without listing them all.

Applications of Combination in Real Life:

Example 9: Zain has 8 different fruits. He wants to select 5 fruits out of 8 fruits to make a fruit chat. How many combinations of fruits he can select?

Solution: To solve this problem, we have to find the number of combinations of 5 fruits out of 8 fruits. In this situation, $n = 8$ and $r = 5$.

$$^nC_r = \frac{n!}{r!(n-r)!}$$

After putting values

$$^8C_5 = \frac{8!}{5!(8-5)!} = \frac{8!}{5!\,3!}$$

$$= \frac{8\times7\times6\times5!}{5!\,3!} = \frac{8\times7\times\not{6}}{\not{3}\cdot\not{2}\cdot1}$$

$$= 8\times7 = 56 \text{ ways}$$

Zain has 56 different ways to select 5 different fruits to make a fruit chat.

Example 10: In a school, a class consists of 12 girls and 8 boys. The teacher wants to select 5 students for an activity. In how many ways can the students be selected including? (i) 2 girls (ii) 5 boys (iii) 2 boys

Solution:

Number of girls = 12

Number of boys = 8

> **Challenge!**
> A restaurant offers 6 flavours of pizza. How many ways are there to select 2 flavours of pizza?

(i) Now let's find the total number of ways to select students when exactly 2 are girls.

$$\text{Total no. of ways} = {^{12}C_2}\cdot{^8C_3} = \frac{12!}{2!\,10!}\cdot\frac{8!}{3!\,5!} = \frac{12\cdot11\cdot10!}{2\cdot10!}\cdot\frac{8\cdot7\cdot6\cdot5!}{3\cdot2\cdot1\cdot5!} = 3696$$

(ii) Let's find total number of ways to select students when exactly 5 students are boys.

$$\text{Total no. of ways} = {^8C_5} = \frac{8!}{5!(8-5)!} = \frac{8!}{5!\,3!} = \frac{8\cdot7\cdot6\cdot5!}{5!\,3\cdot2\cdot1} = 56$$

(iii) Let's find total number of ways to select students when exactly 2 students are boys.

$$\text{Total no. of ways} = {^8C_2}\cdot{^{12}C_3} = \frac{8!}{2!\,6!}\cdot\frac{12!}{3!\,9!} = \frac{8\cdot7\cdot6!}{2\cdot6!}\cdot\frac{12\cdot11\cdot10\cdot9!}{3\cdot2\cdot1\cdot9!} = 36960$$

Complementary Combinations:

Theorem: Prove that $^nC_r = {^nC_{n-r}}$

Proof:

To Prove: $^nC_r = {^nC_{n-r}}$

$$\text{L.H.S} = {^nC_{n-r}}$$

$$= \frac{n!}{(n-r)!(n-n+r)!}$$

$$= \frac{n!}{r!(n-r)!}$$

$$= {^nC_r} = \text{R.H.S (Proved)}$$

> **Note:**
> This result will be found useful in evaluating ^nC_r when $r > \frac{n}{2}$.
> For example,
> $^{12}C_{10} = {^{12}C_{12-10}} = {^{12}C_2} = \frac{(12)\cdot(11)}{2} = 6\cdot11 = 66$

> **Note:** ^nC_r and $^nC_{n-r}$ are known as complementary combinations.

Example 11: Find the number of the diagonals of a 6-sided figure.

Solution: A 6-sided figure has 6 vertices. By joining any two vertices, we get a line segment.

$\therefore$ Number of line segments $= {^6C_2} = \dfrac{6!}{2!\,4!} = 15$

But these line segments include 6 sides of the figure

$\therefore$ Number of diagonals $= 15 - 6 = 9$

Difference between permutation and combination:

Permutation	Combination
• Order is important. e.g., *ab* and *ba* are different (because order of any object is matter) • Arrangement of objects e.g. arrangement of: • ball of different colours • English alphabet (letters) • people while sitting on chairs	• Order is not important e.g., *ab* and *ba* are same (because order does not matter) • Selection of objects e.g. selection of: • different colours • members in a team • food items

Application of Permutations and Combinations in Cryptography:

Example 12: Zain wants to generate a password for his laptop to secure the data. He can take only 6 characters to generate a password. Each character can either be an upper case letter $(A - Z)$ or digits from $(0 - 9)$.
Can you tell how many passwords can be generated by using the above letters and digits:
(i) if repetition of characters is not allowed
(ii) if repetition of characters is allowed

Solution:

Total number of letters = 26
Total number of digits = 10
Total number of letters and digits = 26 + 10 = 36
n = total number of characters = 36
r = required number of characters = 6

(i) If repetition of characters is not allowed, we find out total possible permutations as.

$$^nP_r = {}^{36}P_6 = \frac{36!}{(36-6)!} = \frac{36!}{30!}$$
$$= \frac{36\cdot35\cdot34\cdot33\cdot32\cdot31\cdot30!}{30!}$$
$$= 36\cdot35\cdot34\cdot33\cdot32\cdot31$$
$$= 1,402,410,240 \text{ ways}$$

Hence, 1,402,410,240 passwords can be generated by using the 26 alphabet and 10 digits. (If repetition of the characters is not allowed)

(ii) If the repetition of the characters is allowed. Using fundamental principle of counting:
The total number of possible combinations = $36 \times 36 \times 36 \times 36 \times 36 \times 36 = 36^6$
Hence, 36^6 passwords can be generated by using the 26 alphabets and 10 digits. If repetition of characters is allowed.

Application of permutations to estimate the odd of winning the lottery:

Example 13: A box contains 15 cards from (1-15). Danish is to select 5 cards. If all the selected cards are the first five multiples of 2 then Danish will win the game. Find Danish's chance of winning the game, when
(i) order is important
(ii) order is not important

Solution:

n = total number of cards = 15
r = required number of cards = 5

(i) When order is important,

$$\text{Total possible ways} = {}^nP_r = {}^{15}P_5 = \frac{15!}{(15-5)!}$$
$$= \frac{15!}{10!} = 360,360 \text{ ways}$$

$$\text{Hence Danish's chance to win the game} = \frac{1}{360,360} = 0.000002775$$

(ii) When order is not important
n = Total number of cards = 15
r = Required number of cards = 5

$$\text{Total possible ways} = {}^nC_r = {}^{15}C_5 = \frac{15!}{5!(15-5)!}$$
$$= \frac{15!}{5!10!} = \frac{15\times14\times13\times12\times11\times10!}{5!\cdot10!}$$

$$= \frac{15\times14\times13\times12\times11}{5\times4\times3\times2\times1} = 3003 \text{ ways}$$

$$\text{Hence, Danish's chance to win the game} = \frac{1}{3003} = 0.00033$$

Application of Permutation and Combination to choose different sets of songs for Certain Occasions:

Example 14: On Independence Day, a DJ has a list of ten different national songs. He wants to select any five national songs for the day. Find how many ways he can select and play the songs:
(i) if the order of playing the songs matters
(ii) if the order of playing the songs does not matter

Solution

(i) When order matters
n = total number of national songs = 10
r = required number of national songs = 5

$$\text{Total number of ways} = {}^nP_r = {}^{10}P_5$$
$$= \frac{10!}{(10-5)!} = \frac{10!}{5!} = 30,240 \text{ ways}$$

Hence, the DJ can play the five national songs in 30,240 different ways.

(ii) When order is not matter
n = total number of national songs = 10
r = total number of selected national songs = 5

$$\text{Total number of ways} = {}^nC_r = {}^{10}C_5 = \frac{10!}{5!(10-5)!}$$
$$= \frac{10!}{5!\cdot5!} = 252 \text{ ways}$$

Hence, the DJ can play the five national songs in 252 different ways.

Exercise 7.4

1. Evaluate the following:

(i) $^{50}C_{50}$

Solution:

$$^{50}C_{50} = \frac{50!}{50!(50-50)!} = \frac{1}{0!} = \frac{1}{1} = 1$$

(ii) $^{1000}C_0$

Solution:

$$^{1000}C_0 = \frac{1000!}{0!(1000-0)!}$$
$$= \frac{1000!}{1\cdot1000!} = \frac{1}{1} = 1$$

(iii) $^{10}C_7$

Solution:

$$^{10}C_7 = \frac{10!}{7!(10-7)!} = \frac{10\cdot9\cdot8\cdot7!}{7!\cdot3!}$$
$$= \frac{10\cdot9\cdot8}{3\cdot2\cdot1} = 10\cdot3\cdot4 = 120$$

(iv) $^{20}C_{17}$

Solution:

$$^{20}C_{17} = \frac{20!}{17!(20-17)!}$$
$$= \frac{20\cdot19\cdot18\cdot17!}{17!\cdot3!} = \frac{20\cdot19\cdot18}{3\cdot2\cdot1}$$
$$= 20\cdot19\cdot3 = 1140$$

2. (i) If $^{3n}C_2 : {}^nC_2 = 15:1$, find n.

(ii) If $^nP_r = 120$ and $^nC_r = 20$, find r.

Solution:

(i) Given that: $^{3n}C_2 : {}^nC_2 = 15:1$

$$^{3n}C_2 \div {}^nC_2 = \frac{15}{1}$$

$$\frac{(3n)!}{2!(3n-2)!} \div \frac{n!}{2!(n-2)!} = 15$$

$$\frac{3n(3n-1)(3n-2)!}{2(3n-2)!} \div \frac{n(n-1)(n-2)!}{2(n-2)!} = 15$$

$$\frac{3n(3n-1)}{2} \div \frac{n(n-1)}{2} = 15$$

$$\frac{3n(3n-1)}{2} \times \frac{2}{n(n-1)} = 15$$

$$\frac{3(3n-1)}{n-1} = 15$$

$$3n - 1 = \frac{15}{3}(n-1)$$

$$3n - 1 = 5n - 5$$

$$-1 + 5 = 5n - 3n$$

$$2n = 4$$

$$n = 2$$

(ii) Given that:

$$^nP_r = 120 \qquad \ldots(1)$$

$$^nC_r = 20$$

$$\frac{1}{r!} \cdot {}^nP_r = 20$$

$$\frac{1}{r!}(120) = 20 \quad \text{using (1)}$$

$$\frac{120}{20} = r!$$

$$r! = 6$$

$$r! = 3!$$

Comparing both sides, we have

$$r = 3$$

3. Find the values of n and r, when:

(i) $^nC_r = 56, {}^nP_r = 336$

Solution:

Given that: $\quad ^nP_r = 336 \qquad \ldots(1)$

and $\quad ^nC_r = 56$

$$\frac{1}{r!} \cdot {}^nP_r = 56$$

$$\frac{1}{r!}(336) = 56 \qquad \text{Using (1)}$$

$$\frac{336}{56} = r!$$

$$r! = 6$$

$$r! = 3!$$

Comparing both sides, we have $r = 3$ put in eq (1)

$$^nP_3 = 336$$

$$\frac{n!}{(n-3)!} = 336$$

$$\frac{n(n-1)(n-2)(n-3)!}{(n-3)!} = 336$$

$$n(n-1)(n-2) = 8 \cdot 7 \cdot 6$$

$$n(n-1)(n-2) = 8(8-1)(8-2)$$

Comparing both sides, we have

$$n = 8$$

Hence, $n = 8$ and $r = 3$

(ii) $^{n-1}C_{r-1} : {}^nC_r : {}^nC_r = 1:3:7$

Solution:

Given that:

$$^{n-1}C_{r-1} : {}^nC_r : {}^{n+1}C_{r+1} = 1:3:7$$

Take $\quad ^{n-1}C_{r-1} : {}^nC_r = 1:3$

$$\frac{(n-1)!}{(r-1)!(n-1-r+1)!} \div \frac{n!}{r!(n-r)!} = \frac{1}{3}$$

$$\frac{(n-1)!}{(r-1)!(n-r)!} \div \frac{n(n-1)!}{r(r-1)!(n-r)!} = \frac{1}{3}$$

$$\frac{(n-1)!}{(r-1)!(n-r)!} \times \frac{r(r-1)!(n-r)!}{n(n-1)!} = \frac{1}{3}$$

$$\frac{r}{n} = \frac{1}{3}$$

$$n = 3r \qquad \ldots(1)$$

and $\quad ^nC_r : {}^{n+1}C_{r+1} = 3:7$

$$\frac{n!}{r!(n-r)!} \div \frac{(n+1)!}{(r+1)!(n+1-r-1)!} = \frac{3}{7}$$

$$\frac{n!}{r!(n-r)!} \div \frac{(n+1) \cdot n!}{(r+1)r!(n-r)!} = \frac{3}{7}$$

$$\frac{n!}{r!(n-r)!} \times \frac{(r+1) \cdot r!(n-r)!}{(n+1) \cdot n!} = \frac{3}{7}$$

$$\frac{r+1}{n+1} = \frac{3}{7}$$

$$7r + 7 = 3n + 3$$

$$7r = 3(3r) + 3 - 7 \qquad \because n = 3r \text{ from (1)}$$

$$7r = 9r - 4$$

$$4 = 9r - 7r$$

$$2r = 4$$

$$r = 2 \qquad \text{Put in eq. (1), we have}$$

$$n = 3(2) = 6$$

Hence, $n = 6$ and $r = 2$

4. Prove that:

(i) $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

Solution:

$$\text{L.H.S} = {}^nC_r + {}^nC_{r-1}$$

$$= \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!}$$

$$= \frac{n!}{r(r-1)!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)(n-r)!}$$

$$= \frac{n!}{(r-1)!(n-r)!}\left\{\frac{1}{r} + \frac{1}{(n-r+1)}\right\}$$

$$= \frac{n!}{(r-1)!(n-r)!}\left\{\frac{r+n-r+1}{r(n-r+1)}\right\}$$

$$= \frac{n!}{(r-1)!(n-r)!}\left\{\frac{n+1}{r(n-r+1)}\right\}$$

$$= \frac{(n+1)n!}{r(r-1)!(n-r+1)(n-r)!}$$

$$= \frac{(n+1)!}{r!(n-r+1)!} = \frac{(n+1)!}{r!(n+1-r)!}$$

$$= {}^{n+1}C_r = \text{R.H.S (Proved)}$$

(ii) $r \cdot {}^nC_r = (n-r+1)\, {}^nC_{r-1}$

Solution:

$$\text{R.H.S} = (n-r+1)\, {}^nC_{r-1}$$

$$= (n-r+1) \cdot \frac{n!}{(r-1)!(n-r+1)!}$$

$$= (n-r+1) \cdot \frac{n!}{(r-1)!(n-r+1)(n-r)!}$$

$$= \frac{n!}{(r-1)!(n-r)!}$$

$$= r \cdot \frac{n!}{r(r-1)!(n-r)!} \qquad \text{Multiply and divide by '}r\text{'}$$

$$= r \cdot \frac{n!}{r!(n-r)!}$$

$$= r \cdot {}^nC_r$$

$$= \text{L.H.S (Proved)}$$

5. Prove that product of r consecutive integers is divisible by $r!$.

Solution:

Let $n, n+1, n+2, \cdots, n+r-1$ be the r consecutive integers.

$$\text{Product} = n(n+1)(n+2)\cdots(n+rr-1)$$

$$= (n+r-1)\cdots(n+2)(n+1) \cdot n$$

$$= \frac{(n+r-1)\cdots(n+2)(n+1) \cdot n \cdot (n-1)!}{(n-1)!}$$

$$= \frac{(n+r-1)!}{(n-1)!}$$

Multiply and divide by $r!$

$$= r! . \frac{(n+r-1)!}{r!(n-1)!}$$

$$= r! . \frac{(n+r-1)!}{r!(n+r-1-r)!}$$

Product $= r! \cdot {}^{n+r-1}C_r$, where $^{n+r-1}C_r$ is a +ve integer.

⇒ Product is a multiple of $r!$

Hence, the product of 'r' consecutive integers is divisible by $r!$.

6. In how many ways can five subjects be selected out of eight subjects to select a course programme?

Solution:

Total no. of subjects = 8

No. of Subjects in a course programme = 5

No. of course programmes = $^8C_5$

$$= \frac{8!}{5!(8-5)!} = \frac{8!}{5!\,3!}$$

$$= \frac{8 \cdot 7 \cdot 6 \cdot 5!}{5!\,(3 \cdot 2 \cdot 1)} = 8 \cdot 7 = 56$$

7. Find the number of possible arrangements of vowel letters from the English alphabet?

Solution:

Vowel letters: a, e, i, o, u

Possible no. of arrangements $= {}^5P_5$

$$= 5!$$

$$= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

$$= 120$$

8. In how many ways 3 dishes of Desi foods and 2 dishes of Chinese foods be selected from 6 dishes of desi foods and 8 dishes of Chinese foods?

Solution:

Total no. of dishes of Desi foods = 6

Total no. of dishes of Chinese foods = 8

If we want to select 3 dishes of Desi food and 2 dishes of Chinese food, then

No. of ways $= {}^6C_3 \times {}^8C_2$

$$= \frac{6!}{3!(6-3)!} \times \frac{8!}{2!(8-2)!}$$

$$= \frac{6 \cdot 5 \cdot 4 \cdot 3!}{3!(3 \cdot 2 \cdot 1)} \times \frac{8 \cdot 7 \cdot 6!}{(2 \cdot 1) \cdot 6!}$$

$$= (5.4)(4.7) = (20)(28) = 560$$

9. From a standard desk of 52 playing cards, there are 26 black cards and 26 red cards. How many different ways can eight cards be selected if 3 are black and the remaining 5 are red?

Solution:

Total no. of playing cards = 52

No. of black cards = 26

No. of red cards = 26

If we want to select 8 cards, 3 from black ad remaining 5 from red, then

No. of ways $= {}^{26}C_3 \times {}^{26}C_5$

$$= \frac{26!}{3!(26-3)!} \times \frac{26!}{5!(26-5)!}$$

$$= \frac{26!}{3!\,23!} \times \frac{26!}{5!\,21!}$$

$= \frac{26\cdot25\cdot24\cdot23!}{(3\cdot2\cdot1)\cdot23!}\times\frac{26\cdot25\cdot24\cdot23\cdot22\cdot21!}{(5\cdot4\cdot3\cdot2\cdot1)\cdot21!}$

$= (26\cdot25\cdot4)(26\cdot5\cdot23\cdot22)$

$= (2600)(65780) = 171{,}028{,}000$

10. A bag contains 8 red balls, 7 green balls. Find the total number of possible ways in which five balls are selected in a way:

(i) 3 red and 2 green (ii) 1 red and 4 green
(iii) 4 red and 1 green (iv) All the red balls

Solution:

Total no. of red balls = 8
Total no. of green balls = 7
No. of selected balls = 5

(i) If we want to select 5 ball in which 3 red and 2 green, then

No. of ways $= {}^8C_3 \times {}^7C_2$

$= \frac{8!}{3!(8-3)!}\times\frac{7!}{2!(7-2)!}$

$= \frac{8\cdot7\cdot6\cdot5!}{(3\cdot2\cdot1)\cdot5!}\times\frac{7\cdot6\cdot5!}{(2\cdot1)\cdot5!}$

$= (8\cdot7)(7\cdot3)$

$= (56)(21) = 1176$

(ii) If we want to select 5 balls in which red and 4 green, then

No. of ways $= {}^8C_1 \times {}^7C_4$

$= \frac{8!}{1!(8-1)!}\times\frac{7!}{4!(7-4)!}$

$= \frac{8\cdot7!}{1\cdot7!}\times\frac{7\cdot6\cdot5\cdot4!}{4!\cdot(3\cdot2\cdot1)}$

$= (8)(7\cdot5)$

$= 8(35) = 280$

(iii) If we want to select 5 balls in which 4 red and 1 green, then

No. of ways $= {}^8C_4 \times {}^7C_1$

$= \frac{8!}{4!(8-4)!}\times\frac{7!}{1!(7-1)!}$

$= \frac{8\cdot7\cdot6\cdot5\cdot4!}{(4\cdot3\cdot2\cdot1)\cdot4!}\times\frac{7\cdot6!}{1\cdot6!}$

$= (2.7.5)(7) = (70)(7) = 490$

(iv) If we want to select 5 balls, all are red, Then

No. of ways $= {}^8C_5$

$= \frac{8!}{5!(8-5)!} = \frac{8\cdot7\cdot6\cdot5!}{5!\cdot(3\cdot2\cdot1)}$

$= 8\cdot7 = 56$

11. How many diagonals and triangles can be formed by joining the vertices of the polygon having 15 sides.

Solution:

No. of vertices of 15-sided polygon $= n = 15$

No. of diagonals formed $= {}^nC_2 - n$

$= {}^{15}C_2 - 15$

$= \frac{15!}{2!(15-2)!} - 15$

$= \frac{15.14.13!}{2.13!} - 15$

$= 15\cdot7 - 15 = 105 - 15 = 90$

No. of triangles formed $= {}^nC_3 = {}^{15}C_3$

$= \frac{15!}{3!(15-3)!} = \frac{15\cdot14\cdot13\cdot12!}{(3\cdot2\cdot1)\cdot12!}$

$= 5\cdot7\cdot13 = 455$

12. Find the number of sides of a polygon if the number of its diagonals is 104.

Solution:

Let Number of sides of polygon = n.

As No. of diagonals formed = 104

$\Rightarrow \quad {}^nC_2 - n = 104$

$\frac{n!}{2!(n-2)!} - n = 104$

$\frac{n(n-1)(n-2)!}{2(n-2)!} - n = 104$

$\frac{n(n-1)}{2} - n = 104$

$n\left(\frac{n-1}{2} - 1\right) = 104$

$\frac{n(n-1-2)}{2} = 104$

$n(n-3) = 208$

$n(n-3) = 16\times13$

$n(n-3) = 16(16-3)$

Comparing both sides, we have

$n = 16$

16-sided polygon has 104 diagonals.

13. How many triangles can be formed by joining 15 points, 6 of which lie on the same straight line?

Solution:

Total no. of points = 15

No. of collinear points = 6

Total no. of ways to choose 3 points $= {}^{15}C_3$

$= \frac{15!}{3!(15-3)!}$

$= \frac{15\cdot14\cdot13\cdot12!}{(3\cdot2\cdot1)\cdot12!}$

$= 5\cdot7\cdot13 = 455$

No. of ways to choose 3 points from the 6 collinear points $= {}^6C_3$

$= \frac{6!}{3!(6-3)!}$

$= \frac{6\cdot5\cdot4\cdot3!}{3!\,3!}$

$= 5\cdot4 = 20$

No. of triangle formed = 455 − 20 = 435

14. The members of a club are 10 boys and 8 girls. In how many ways can a committee of 6 boys and 3 girls be formed?

Solution:

No. of boys = 10

No. of girls = 8

Members in each committee = 6 boys and 3 girls

Total no. of committee $= {}^{10}C_6 \times {}^8C_3$

$= \frac{10!}{6!(10-6)!}\times\frac{8!}{3!(8-3)!}$

$= \frac{10.9.8.7.6!}{6!.(4.3.2.1)}\times\frac{8.7.6.5!}{(3.2.1).5!}$

$= (10.3.7)(8.7)$

$= (210)(56) = 11760$

15. How many committees of 7 members can be chosen from a group of 10 persons when each committee must include 2 particular persons?

Solution:

Total no. of persons = 10

No. of persons in a committee = 7

Since each committee must include 2 particular persons, so remaining 5 members are selected out of 8 persons.

Total no. of committees $= {}^8C_5$

$= \frac{8!}{5!(8-5)!} = \frac{8\cdot7\cdot6\cdot5!}{5!\cdot(3\cdot2\cdot1)}$

$= 8\cdot7 = 56$

16. In how many ways can a cricket team of 11 players be selected out of 17 players? How many of them will include a particular player?

Solution:

Total no. of players = 17

No. of players in a team = 11

Total no. of teams $= {}^{17}C_{11}$

$= \frac{17!}{11!(17-11)!}$

$= \frac{17\cdot16\cdot15\cdot14\cdot13\cdot12\cdot11!}{11!(6\cdot5\cdot4\cdot3\cdot2\cdot1)}$

$= 17\cdot4\cdot14\cdot13 = 12376$

Since each team include one particular player so remaining 10 players are selected out of 16 players

Total no. of teams $= {}^{16}C_{10}$

$= \frac{16!}{10!(16-10)!}$

$= \frac{16\cdot15\cdot14\cdot13\cdot12\cdot11\cdot10!}{10!(6\cdot5\cdot4\cdot3\cdot2\cdot1)}$

$= 4\cdot14\cdot13\cdot11$

$= 8008$

17. There are 6 men and 8 women members of a club. How many committees of seven can be formed:

(i) with 3 women (ii) with at most 3 women
(iii) with at least 5 women

(i) with 3 women

Solution:

No. of men = 6

No. of women = 8

No. of members in a committee = 7

As each committee has 3 women, then remaining 4 members are selected from 6 men.

No. of ways $= {}^8C_3 \times {}^6C_4$

$= \frac{8!}{3!(8-3)!}\times\frac{6!}{4!(6-4)!}$

$= \frac{8\cdot7\cdot6\cdot5!}{(3\cdot2\cdot1)\cdot5!}\times\frac{6\cdot5\cdot4!}{4!(2\cdot1)}$

$= (8.7)(3.5)$

$= (56)(15) = 840$

(ii) with at most 3 women

Solution:

At the most 3 women means that $(W \le 3)$

No. of ways = (3W, 4M) or (2W, 5M) or (1W, 6M) or (0W, 7M)

$= ({}^8C_3\times{}^6C_4) + ({}^8C_2\times{}^6C_5) + ({}^8C_1\times{}^6C_6) + ({}^8C_0\times{}^6C_7)$

$= \left(\frac{8!}{3!\,5!}\times\frac{6!}{4!\,2!}\right) + \left(\frac{8!}{2!\,6!}\times\frac{6!}{5!\,1!}\right) + \left(\frac{8!}{1!\,7!}\times\frac{6!}{6!\,0!}\right) + \left(\frac{8!}{0!\,8!}\times0\right)$

$= (56\times15) + (28\times6) + (8\times1) + (1\times0)$

$= 840 + 168 + 8 + 0 = 1016$

(iii) with at least 5 women

Solution:

At least 5 women means that $(W \ge 5)$

No. of ways = (5W, 2M) or (6W, 1M) or (7W, 0M)

$= ({}^8C_5\times{}^6C_2) + ({}^8C_6\times{}^6C_1) + ({}^8C_7\times{}^6C_0)$

$= \left(\frac{8!}{5!\,3!}\times\frac{6!}{2!\,4!}\right) + \left(\frac{8!}{6!\,2!}\times\frac{6!}{1!\,5!}\right) + \left(\frac{8!}{7!\,1!}\times\frac{6!}{0!\times6!}\right)$

$= (56\times15) + (28\times6) + (8\times1)$

$= 840 + 168 + 8 = 1016$

18. There are three sections in a question paper; each section has 3 questions. A student has to solve all 5 questions, choosing at least one question from each section. In how many ways can be student make his choice?

Solution:

Possible distributions of questions.

Section-A	Section -B	Section-C
1	1	3
1	3	1
3	1	1
1	2	2
2	1	2
2	2	1

Total no. of ways $= 3({}^3C_1 \times {}^3C_1 \times {}^3C_3) + 3({}^3C_1 \times {}^3C_2 \times {}^3C_2)$

$= 3(3\times3\times1) + 3(3\times3\times3)$

$= 3(9) + 3(27)$

$= 27 + 81$

$= 108$ ways

Hence, the student can make his choice in 108 ways

19. Using a cryptographic system, a password is generated with 8 characters. Each character can either be a lowercase letter (a-f) or a digit (0-5). How many passwords can be generated if each password must contain exactly 5 lowercase letters and 3 digits?

(a) With repetition allowed.

(b) Without repetition.

Solution:

Password length = 8 characters (5 lowercase letters and 3 digits)

No. of lowercase letters $(a - f) = 6$

No. of digits $(0 - 5) = 6$

Form of password: ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗ ⊗

Out of 8 positions, choose any 5 to place the letters.

No. of ways $= {}^8C_5 = 56$

(a) With repetition allowed:

Total no. of passwords formed $= {}^8C_5 \times 6^5 \times 6^3 = 56 \times 6^8$

$= 56 \times 1679616$

$= 94066496$

(b) Without repetition:

Total no. of passwords formed $= {}^8C_5 \times {}^6P_5 \times {}^6P_3$

$= 56 \times 720 \times 120$

$= 4838400$

20. On Defense Day, Teacher I compiles a list of 10 distinct national songs, while Teacher II prepares a separate list of 10 different national songs (with no overlap between the two lists). The principal needs to select 3 songs from Teacher I's list, and 3 songs from Teacher II's list.

(i) the order/sequence of the selected songs is important.

(ii) the order/sequence of the selected songs is not important.

Solution:

No. of national songs prepared by Teacher-I = 10

No. of national songs prepared by Teacher-II = 10

No. of Selected songs = 3 from Teacher-I and 3 from Teacher-II

(i) When the sequence of selected songs matter.

Total possible ways $= {}^{10}P_3 \times {}^{10}P_3$

$= \dfrac{10!}{(10-3)!} \times \dfrac{10!}{(10-3)!}$

$= \dfrac{10\cdot9\cdot8\cdot7!}{7!} \times \dfrac{10.9.8.7!}{7!}$

$= 720 \times 720 = 518{,}400$

(ii) When the sequence of selected songs does not matter

Total possible ways $= {}^{10}C_3 \times {}^{10}C_3$

$= ({}^{10}C_3)^2$

$= \left(\dfrac{10!}{3!(10-3)!}\right)^2$

$= \left(\dfrac{10.9.8.7!}{(3.2.1).7!}\right)^2$

$= (10.3.4)^2 = (120)^2$

$= 14400$

Formula Sheet

1. $n! = \underline{|n} = \begin{cases} 1 & \text{if } n = 0 \\ n(n-1)(n-2)\ldots3\cdot2\cdot1 & \text{if } n \geq 1 \end{cases}$

2. ${}^nP_r = P(n,r) = \dfrac{n!}{(n-r)!}$ when $r \leq n$

3. Permutation of objects not all different is: $\dfrac{n!}{n_1!\cdot n_2!\cdot n_3!} = \begin{pmatrix} n \\ n_1, n_2, n_3 \end{pmatrix}$

4. Case-I: When clockwise and anticlockwise arrangements are considered different

   No. of distinct circular permutations is: $(n-1)!$

   Case-II: When clockwise and anticlockwise arrangements are considered identical

   No. of distinct circular permutations is: $\dfrac{(n-1)!}{2}$

5. ${}^nC_r = C(n,r) = \dfrac{n!}{r!(n-r)!}$ 6. ${}^nC_r \times r! = {}^nP_r$ 7. ${}^nC_r = {}^nC_{n-r}$

Multiple Choice Questions (MCQs)

Exercise 7.1

1. For a natural number n, we define the factorial of n as:

(A) $n(n-1)!$ (B) $(n-1)(n-2)$ (C) $n(n+1)$ (D) $n(n+1)!$

2. $0! =$ -----

(A) 0 (B) 1 (C) –1 (D) 2

3. Simplify form of $\dfrac{10!}{7!}$ is equal to -----

(A) 720 (B) 620 (C) 520 (D) 420

4. $\underline{|-4} =$ -----

(A) 24 (B) – 24 (C) 0 (D) not defined

5. Factorial form of $(n + 2)(n + 1)(n)$ is -----

(A) $\dfrac{(n+2)!}{(n-1)!}$ (B) $\dfrac{(n+2)!}{n!}$ (C) $\dfrac{n!}{(n+2)!}$ (D) $(n + 2)!$

Exercise 7.2

6. An ordering arrangement of n-objects is called a -----

(A) permutation (B) combination (C) both (A) & (B) (D) none of these

7. ${}^nP_r$ equal to -----, where $n, r > 0$.

(A) $\dfrac{\underline{|n}}{\underline{|r}}$ (B) $\dfrac{\underline{|n}}{\underline{|n-r}}$ (C) $\underline{|n-r}$ (D) $\dfrac{\underline{|n}}{\underline{|n}\,\underline{|n-r}}$

8. With usual notation ${}^6P_4$ equals -----

(A) 160 (B) 260 (C) 360 (D) 340

9. Number of signals can be made with 4 flags when one flag is used at a time are:

(A) ${}^4C_0$ (B) ${}^4C_1$ (C) ${}^4C_2$ (D) ${}^4C_3$

Exercise 7.3

10. The number of permutations of word PANAMA are:

(A) 10 (B) 60 (C) 20 (D) 120

11. The permutation of things on a circle is called -----

(A) combination (B) circular permutation (C) both (A) & (B) (D) none of these

12. The number of ways in which 5 person can be seated at a round table are -----

(A) 2! (B) 3! (C) 4! (D) 5!

13. Number of ways of arranging 5 keys in a circular ring is -----

(A) 24 (B) 12 (C) 6 (D) 5