PROBLEMS

5.1 Find the gravitational force of attraction between two spheres each of mass 1000 kg. The distance between the centers of the spheres is 0.5 m.

(2.67×10^{-4} N)

Solution: Mass = $m_1 = m_2 = 1000$ kg

Distance between the centers = d = 0.5 m

Gravitational constant = G = 6.673×10^{-11} Nm^{2}kg^{-2}

Gravitational force = F = ?

$$F = \frac{G\,m_1 m_2}{d_2}$$

$$F = 6.673 \times (10)^{-11} \times \frac{1000 \times 1000}{(0.5)^2}$$

$$= 6.673 \times (10)^{-11} \times \frac{(10)^6}{0.25}$$

$$= \frac{6.673 \times (10)^{-11} \times (10)^6}{0.25}$$

$$= \frac{6.673 \times (10)^{-5}}{0.25}$$

$$= 26.692 \times (10)^{-5} = 2.67 \times 10^{-4}\ N$$

5.2 The gravitational force between two identical lead spheres kept at 1 m apart is 0.006673 N. Find their masses. **(10, 000 kg each)**

Solution: Gravitational force = F = 0.006673 N

Gravitational constant = G = 6.673×10^{-11} Nm^{2}kg^{-2}

Distance between the masses = d = 1m

Mass = $m_1 = m_2 = ?$

$F = G\frac{m_1 m_2}{d_2}$

$F = G\frac{m \times m}{d_2}$ (Let $m_1 = m_2 = m$)

$F = \frac{m^2}{d_2}$

$m^2 = \frac{F \times d^2}{G}$

$m^2 = \frac{0.006673 \times (1)^2}{6.673 \times (10)^{-11}} = \frac{\frac{6673}{1000000}}{6.673 \times (10)^{-11}}$

$= \frac{6.673 \times (10)^{-3}}{6.673 \times (10)^{-11}}$

$\sqrt{m^2} = 10^8$

$m = 10^4 = 10000$ kg each

Therefore, mass of each lead sphere is 10000 kg.

5.3 Find the acceleration due to gravity on the surface of the Mars. The mass of Mars is 6.42 × 10^{23} kg and its radius is 3370 km.

Solution: Mass of Mars = $M_m = 6.42 \times 10^{23}$ kg

Radius of Mars = R_m = 3370 km = 3370 × 1000 m = 3.37×10^8 m

Acceleration due to gravity of the surface of Mars = $g_m = ?$

$g_m = G\frac{M_m}{R^2{}_m}$

or $g_m = 6.673 \times 10^{-11} \times \frac{6.42 \times 10^{23}}{(3.37 \times 10^6)^2}$

$= \frac{6.673 \times 10^{-11} \times 6.42 \times 10^{23}}{11.357}$

$= \frac{42.84}{11.357} = 3.77\ m^{-2}$

5.4 The acceleration due to gravity on the surface of moon is 1.62 ms^{-2}. The radius of Moon is 1740 km. Find the mass of moon.

Solution: Acceleration due to gravity = g_m = 1.62 ms^{-2}

Radius of the moon = R_m = 1740 km = 1740 × 1000 m = 1.74×10^6 m

Mass of moon = M_m = ?

$$g_m = G\frac{M_m}{R^2{}_m}$$

or $$M_m = \frac{gm \times R^2{}_m}{G}$$

$$M_m = \frac{1.62 \times (1.74 \times 10^6)^2}{6.673 \times 10^{-11}}$$

$$= \frac{1.62 \times 3 \times 10^{12}}{6.673 \times 10^{-11}}$$

$$= \frac{4.86 \times 10^{12} \times 10^{11}}{6.673}$$

$M_m = 7.35 \times 10^{22}$ kg

5.5 Calculate the value of g at a height of 3600 km above the surface of the Earth. (4.0 ms^{-2})

Solution: Height = h = 3600 km = 3600 × 1000 m = 3.6×10^6 m

Mass of Earth = M_e = 6.0×10^{24} kg

Gravitational acceleration = g_h =?

$$g_h = \frac{GM_e}{(R+h)^2}$$

$$g_h = 6.673 \times 10^{-11} \times \frac{6.0 \times 10^{24}}{(6.4 \times 10^6 + 3.6 \times 10^6)^2}$$

$$= 6.673 \times 10^{-11} \times \frac{6.0 \times 10^{24}}{(10.0 \times 10^6)^2}$$

$$= 6.673 \times 10^{-11} \times \frac{6.0 \times 10^{24}}{100 \times 10^{12}}$$

$= 6.673 \times 10^{-11} \times 6.0 \times 10^{10} = 40 \times 10^{-1} = 4.0\ ms^{-2}$

5.6 Find the value of g due to the Earth at geostationary satellite. The radius of the geostationary orbit is 48700 km. (0.17 ms^{-2})

Solution: Radius = R = 48700 × 1000 m = 4.87 × 10^7 m

Gravitational acceleration = g =?

$$g = \frac{GM_e}{R^2}$$

$$g = 6.673 \times 10^{-11} \times \frac{6.0 \times 10^{24}}{(4.87 \times 10^{7})^2}$$

$$= 6.673 \times 10^{-11} \times \frac{6.0 \times 10^{24}}{(23.717 \times 10^{14})^2}$$

$$= \frac{6.673 \times 6.0 \times 10^{\;1}}{23.717} = \frac{4.0038}{23.717}$$

$$= 0.17\ ms^{-2}$$

5.7 The value of g is 4.0 ms^{-2} at a distance of 10000 km from the center of the Earth. Find the mass of the Earth. (5.99 × 10^{24} kg)

Solution: Gravitational acceleration = g = 4.0 ms^{-2}

Radius of Earth = R_e = 10000 km = 10000 × 1000 m = 10^7 m

Mass of Earth = M_e =?

$$M_e = \frac{gR^2}{G}$$

$$M_e = \frac{4.0 \times (10^7)^2}{6.673 \times 10^{\;11}}$$

$$= \frac{4.0 \times 10^{14}}{6.673 \times 10^{\;11}}$$

$$= 0.599 \times 10^{25}\ kg = 5.99 \times 10^{24}\ kg$$

5.8 At what altitude the value of g would become one fourth than on the surface of the Earth? (One Earth's radius)

Solution: Mass of Earth = $M_e = 6.0 \times 10^{24}$ kg

Radius of Earth = $R_e = 6.4 \times 10^6$ m

Gravitational acceleration = $g_h = \frac{1}{4} g = \frac{1}{4} \times 10\ ms^{-2} = 2.5\ ms^{-2}$

Altitude above Earth's surface = h =?

$$g_h = \frac{GM_e}{(R+h)^2}$$

or $(R + h)^2 = \frac{GM_e}{g_h}$

Taking square root on both sides

or $\sqrt{(R + h)^2} = \sqrt{\frac{GM_e}{g_h}}$

or $R + h = \sqrt{G\frac{GM_e}{g_h}}$

or $h = \sqrt{G\frac{GM_e}{g_h} - R}$

or $h = \sqrt{\frac{6.673 \times 10^{-11} \times 6.0 \times 10^{24}}{2.5}} - 6.4 \times 10^6$

$= \sqrt{\frac{40.038 \times 10^{13}}{2.5}} - 6.4 \times 10^6$

$= \sqrt{16 \times 10^{13} m^2} - 6.4 \times 10^6 = \sqrt{0.16 \times 10^{12}} - 6.4 \times 10^6$

$= 0.4 \times 10^6 - 6.4 \times 10^6$

$= -6.0 \times 10^6$ m

As height is always taken as positive, therefore

$h = 6.0 \times 10^6$ m = One Earth's radius

5.9 A polar satellite is launched at 850 km above Earth. Find its orbital speed.

($7431\ ms^{-1}$)

Solution: Height = h = 850 km = 850 × 1000 m = 0.85×10^6 m

Orbital velocity = v_o =?

$$v_o = \sqrt{\frac{GM_e}{R+h}}$$

$$v_o = \sqrt{\frac{6.673 \times 10^{-11} \times 6.0 \times 10^{24}}{6.4 \times 10^6 + 0.85 \times 10^6}} = \sqrt{\frac{40.038 \times 10^{13}}{7.25 \times 10^6}}$$

$$= \sqrt{5.55 \times 10^7} = \sqrt{5.55 \times 10^6}$$

$$= 7.431 \times 10^3 = 7431\ ms^{-1}$$

5.10 A communication is launched at 42000 km above Earth. Find its orbital speed. ($2876\ ms^{-1}$)

Solution: Height = h = 42000 km = 42000 × 1000 m = 42×10^6 m

Orbital velocity = v_o =?

$$v_o = \sqrt{\frac{GM_e}{R+h}}$$

$$v_o = \sqrt{\frac{6.673 \times 10^{-11} \times 6.0 \times 10^{24}}{6.4 \times 10^6 + 42 \times 10^6}}$$

$$= \sqrt{\frac{40.038 \times 10^{13}}{48.4 \times 10^6}}$$

$$= \sqrt{\frac{400.38 \times 10^{12}}{48.4 \times 10^6}}$$

$$= \sqrt{8.27 \times 10^6}$$

$$= 2.876 \times 10^3 = 2876\ ms^{-1}$$