Exercise 9.3

1. Find the mid-point of the line segment joining each of the following pairs of points

(a) A(9, 2), B(7, 2)

Solution:

Midpoint M is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$\left(\frac{9+7}{2}, \frac{2+2}{2}\right)$

or $\left(\frac{16}{2}, \frac{4}{2}\right)$

or (8, 2)

(b) A(2, −6), B(3, −6)

Midpoint M is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$\left(\frac{2+3}{2}, \frac{-6-6}{2}\right)$

or $\left(\frac{5}{2}, -6\right)$

or (2.5, -6)

(c) A(−8, 1), B(6, 1)

Midpoint M is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$\left(\frac{-8+6}{2}, \frac{1+1}{2}\right)$

or $\left(\frac{-2}{2}, \frac{2}{2}\right)$

or $(-1, 1)$

(d) A(−4, 9), B(−4, −3)

Midpoint M is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$\left(\frac{-4-4}{2}, \frac{9-3}{2}\right)$

or $\left(\frac{-8}{2}, \frac{6}{2}\right)$

or $(-4, 3)$

(e) A(3, −11), B(3, −4)

Midpoint M is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$\left(\frac{3+3}{2}, \frac{-11-4}{2}\right)$

or $\left(\frac{6}{2}, -\frac{15}{2}\right)$

or $(3, -7.5)$

(f) A(0, 0), B(0, −5)

Midpoint M is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

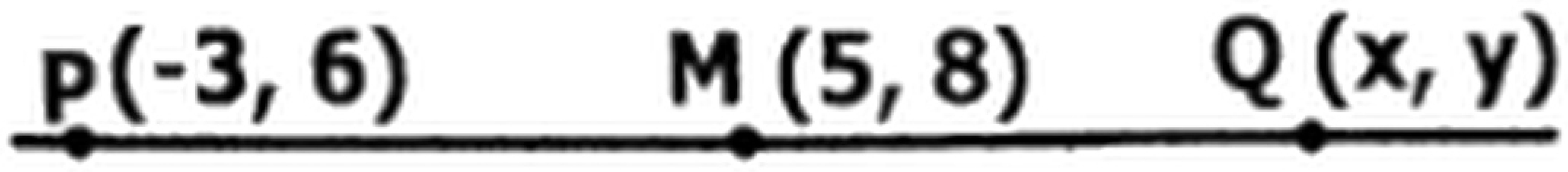
$\left(\frac{0+0}{2}, \frac{0-5}{2}\right)$

or $\left(\frac{0}{2}, \frac{-5}{2}\right)$

or $(0, -2.5)$

2. The end point P of a line segment PQ is (–3, 6) and its mid-point is (5, 8). Find the coordinates of the end point Q.

Solution:



Let Q be the point (x,y), M(5,8) is the midpoint of PQ by mind point formula we have

$x = \frac{x_1 + x_2}{2}$

$5 = \frac{-3+x}{2}$

$10 = -3 + x$

$10 + 3 = x$

$x = 13$

Now $y = \frac{y_1 + y_2}{2}$

$8 = \frac{6+y}{2}$

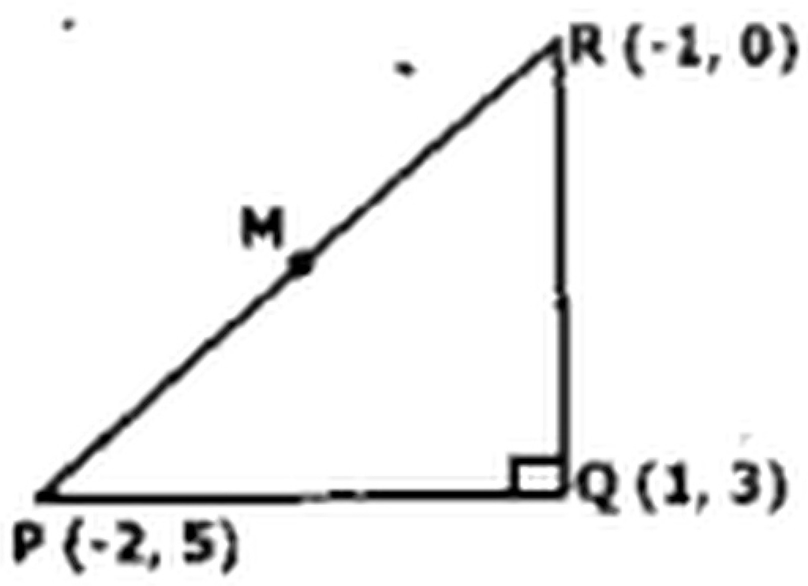
$16 = 6 + y$

$16 - 6 = y$

$y = 10$

Hence point Q is (13, 10)

3. Prove that mid-point of the hypotenuse of a right triangle is equidistant from its three vertices P(–2, 5), Q(1, 3) and R(–1, 0).



P(-2,5), Q(1,3), Q(-1,0)

IPQI = $\sqrt{(-2 \quad 1)^2 + (5-3)^2} = \sqrt{(-3)^2 + (2)^2}$

= $\sqrt{9+4} = \sqrt{13}$

IQRI = $\sqrt{(1+1)^2 + (3-0)^2} = \sqrt{(-2)^2 + (3)^2}$

= $\sqrt{4+9} = \sqrt{13}$

IPRI = $\sqrt{(-2+1)^2 + (5-0)^2} = \sqrt{(-1)^2 + (5)^2}$

= $\sqrt{1+25} = \sqrt{26} = 2$

∴. PR is hypotenuse

Midpoint of hypotenuse PR is M $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$

M$(\frac{-2-1}{2}, \frac{5+0}{2})$

or M$(-\frac{3}{2}, \frac{5}{2})$

I MP I² = I MR I²

= $\sqrt{(-\frac{3}{2}+1)^2 + (\frac{5}{2}-0)^2}$

= $\sqrt{(-\frac{1}{2})^2 + (\frac{5}{2})^2}$

= $\sqrt{(\frac{1}{4}) + (\frac{25}{4})} = \frac{\sqrt{26}}{2}$

Now I MQ I= $\sqrt{(-\frac{3}{2}-1)^2 + (\frac{5}{2}-3)^2}$

$= \sqrt{(-\frac{5}{2})^2 + (-\frac{1}{2})^2}$

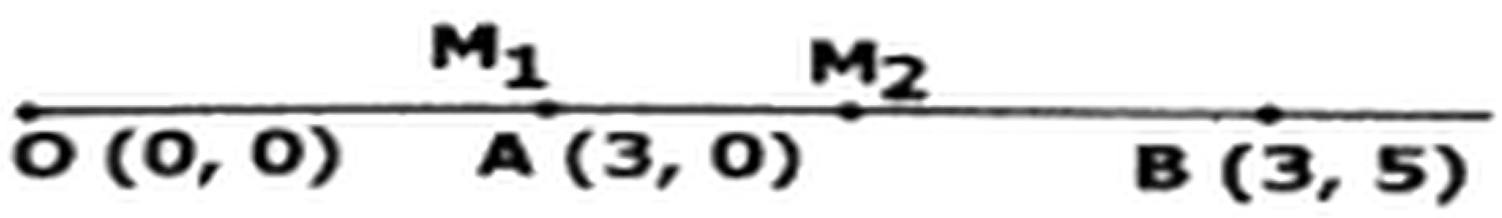
$= \sqrt{(\frac{25}{4}) + (\frac{1}{4})} = \sqrt{\frac{26}{4}} = \frac{\sqrt{26}}{2}$

| MP | = | MR |

Hence M the midpoint of hypotenuse is equidistant from the three vertices of the angle PQR.

4. If O(0, 0), A(3, 0) and B(3, 5) are three points in the plane, find M_1 and M_2 as mid-points of the line segments AB and OB respectively. Find | M_1M_2 |.

Solution:



O(0, 0), A(3, 0) and B(3, 5)

Midpoint of AB is $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$

$M_1\ (\frac{3+3}{2}, \frac{0+5}{2})$ or $M_1\ (3, \frac{5}{2})$

M_2 the mid point of OB is $M_2(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$

$M_2\ (\frac{0+3}{2}, \frac{0+5}{2})$ or $M_2\ (\frac{3}{2}, \frac{5}{2})$

Now | M_1M_2 | $= \sqrt{(3 - \frac{3}{2})^2 + (\frac{5}{2} - \frac{5}{2})^2}$

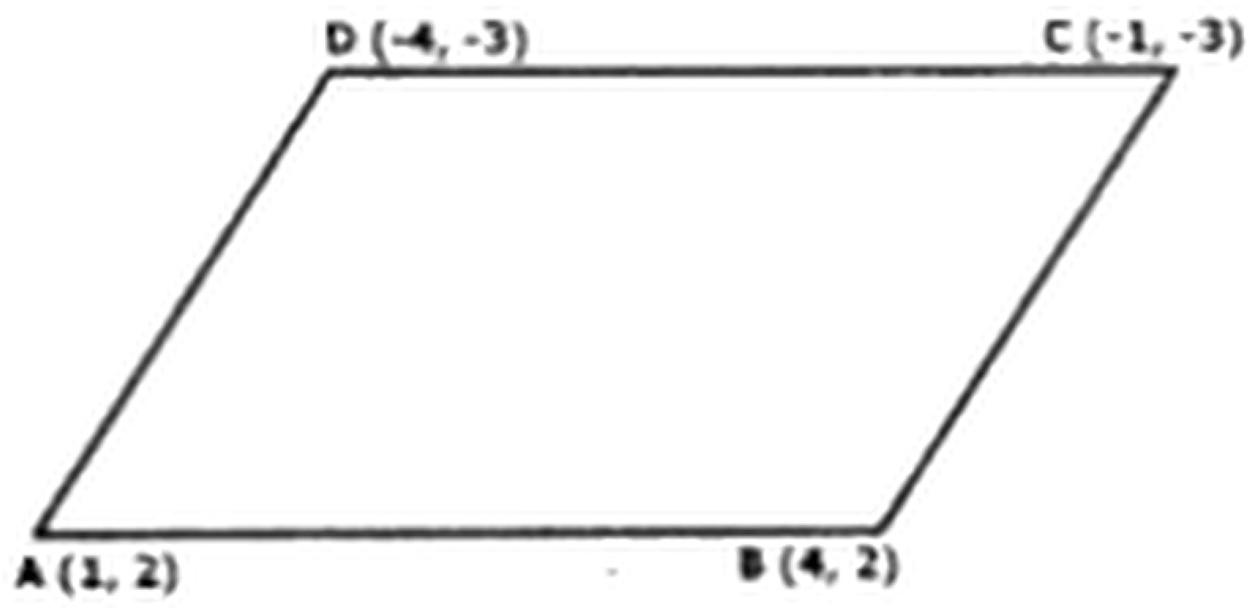
$= \sqrt{(\frac{3}{2})^2 + (0)^2}$

$= \sqrt{(\frac{9}{4}) + (0)} = \ = \frac{3}{2}$

5. Show that the diagonals of the parallelogram having vertices A(1, 2), B(4, 2), C(−1, −3) and D(−4, −3) bisect each other.

[Hint: The mid-points of the diagonals coincide]

Solution:



M_1 midpoint of AC is $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$

$(\frac{1-1}{2}, \frac{2-3}{2})$

or $(\frac{0}{2}, \frac{-1}{2})$

or $(0, \frac{-1}{2})$

M_2 midpoint of BD is $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$

$(\frac{-4+4}{2}, \frac{-3+2}{2})$

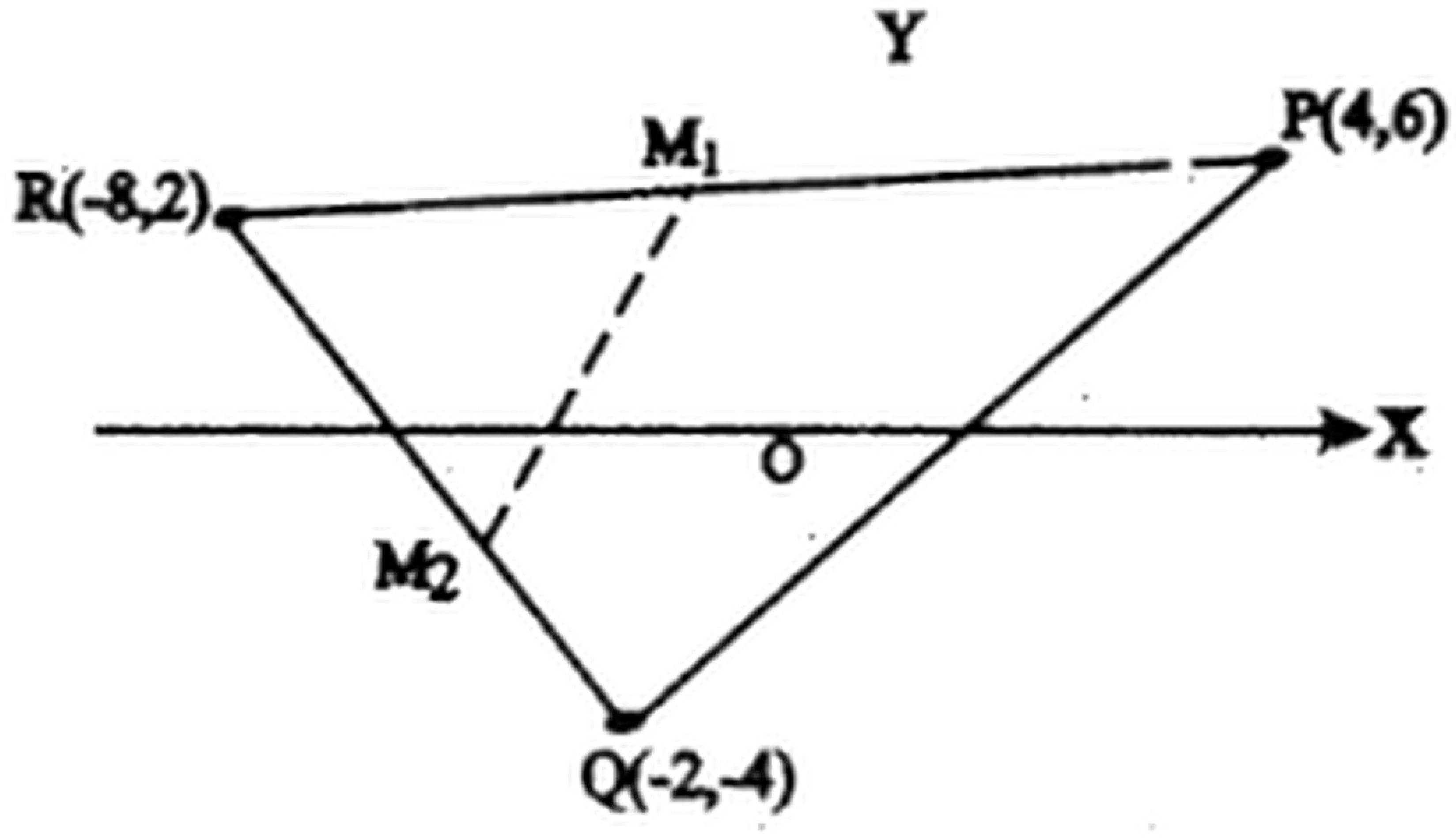
or $(\frac{0}{2}, \frac{-1}{2})$

or $(0, \frac{-1}{2})$

Since both the diagonals have same mid-point therefore, they bisect each other

Q6. The vertices of a triangle are P (4, 6), Q(−2, −4) and R(−8, 2). Show that the length of the line segment joining the mid-points of the line segments PR, QR is $\frac{1}{2}$PQ.

Solution:



Midpoint of QR is $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$

$M_1\ (\frac{-2-8}{2}, \frac{-4+2}{2})$

or $(\frac{-10}{2}, \frac{-2}{2})$ or $M_1\ (-5, -1)$

M_2 the mid point of PR is $M_2(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$

$M_2\ (\frac{4-8}{2}, \frac{6+2}{2})$ or $M_2\ (\frac{-4}{2}, \frac{8}{2})$

or $M_2\ (-2, -4)$

Now $|M_1M_2| = \sqrt{(-5+2)^2 + (4+1)^2}$

$= \sqrt{(-3)^2 + (5)^2}$

$= \sqrt{(9) + (25)} = \ = \sqrt{34}$ (i)

Now $|PQ| = \sqrt{(4+2)^2 + (6+4)^2}$

$= \sqrt{(6)^2 + (10)^2}$

$= \sqrt{36+100} = \sqrt{136}$

$= \sqrt{4 \text{ x } 34} = 2\sqrt{34}$

$\frac{1}{2}|PQ| = \sqrt{34}$ (ii)

from (i) and (ii), we get

$$| M_1M_2 | = \frac{1}{2} | PQ |$$