Exercise 9.2

Q1. Show whether the points with vertices (5,2), (5,4) and (4,-1).are vertices of an equilateral triangle or an isosceles triangle?

Solution:

Let the points be A(5,2), B(5,4) and C(-4,1).

|AB| = $\sqrt{(5-5)^2 + (4+2)^2}$

= $\sqrt{((0)^2 + (6)^2)}$ = $\sqrt{(0+36)}$ = 6

|BC| = $\sqrt{(5+4)^2 + (4-1)^2}$

= $\sqrt{(9)^2 + (3)^2}$

= $\sqrt{(81+9)}$ = $\sqrt{90}$ = $\sqrt{9 \times 10}$ = $3\sqrt{10}$

|CA| = $\sqrt{(5+4)^2 + (-2-1)^2}$

= $\sqrt{(9)^2 + (-3)^2}$

= $\sqrt{(81+9)}$ = $\sqrt{90}$ = $\sqrt{9 \times 10}$ = $3\sqrt{10}$

As |BC| = |CA| = $3\sqrt{10}$

Since two sides are equal therefore the triangle is formed is an isosceles triangle.

Q2. Show whether or not the points with vertices (-1,1), (5,4), (2, -2) and (-4,1) from a square.

Solution:

Let the points be A(-1,1), B(5,4), C(2,2) and D (-4,1)

|AB| = $\sqrt{((5+1)^2 + (4-1)^2}$

$= \sqrt{(36 + 9)} = \sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$

$|BC| = \sqrt{((5 - 2)^2 + (4 + 2)^2}$

$= \sqrt{(9 + 36)} = \sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$

$|CD| = \sqrt{(2 + 4)^2 + (-2 - 1)^2} = \sqrt{(6)^2 + (-3)^2}$

$= \sqrt{(36 + 9)} = \sqrt{45} = 3\sqrt{5}$

$|DA| = \sqrt{(-1 + 4)^2 + (1 - 1)^2} = \sqrt{(3)^2 + 0^2} = 3$

$|AB| = |BC| = |CD| = 3\sqrt{5}$ but $|AD| = 3$

Since all sides are not equal therefore the given points did not form a square.

Q3. Show whether or not the points coordinate (1,3), (4,2) and (-2,6) are vertices of a right triangle.

Solution:

Let the given points be A (1,3), B(4,2) and C(-2,6).

$|AB| = \sqrt{(4 - 1)^2 + (2 - 3)^2}$

$= \sqrt{(3)^2 + (-1)^2} = \sqrt{(9 + 1)} = 10$

$|BC| = \sqrt{(4 + 2)^2 + (2 - 6)^2}$

$= \sqrt{(6)^2 + (-4)^2}$

$= \sqrt{(36 + 16)} = \sqrt{52}$

$|CA| = \sqrt{(1 + 2)^2 + (3 - 6)^2}$

$= \sqrt{(3)^2 + (-3)^2}$

$= \sqrt{(9 + 9)} = \sqrt{18}$

$|BC|^2 = 52$

$|AB|^2 + |CA|^2 = 10 + 18 = 28 \neq |BC|^2$

Since given points does not obey the Pythagoras theorem therefore the coordinates are not the vertices of right-angle triangle.

Q4. Use the distance formula to prove whether or not the points (1, 1), (-2,-8) and (4,10) lie on a straight line?

Solution:

A (1,1), B(-2, 8) and C(4, 10).

$|AB| = \sqrt{(2+1)^2 + (1+8)^2}$

$= \sqrt{(3)^2 + (9)^2} = \sqrt{(3)^2 + (9)^2} = \sqrt{9 + 81} = \sqrt{90} = \sqrt{(9 \times 10)} = 3\sqrt{10}$

$|BC| = \sqrt{(4+2)^2 + (10+8)^2} = \sqrt{(6)^2 + (18)^2} = \sqrt{36 + 324} = \sqrt{360} = 6\sqrt{10}$

$|AC| = \sqrt{(4-1)^2 + (10-1)^2} = \sqrt{(3)^2 + (9)^2} = \sqrt{(9 + 81)} = \sqrt{90} = 3\sqrt{10}$

By applying the condition of collinear points

As $|AB| + |AC| = 3\sqrt{10} + 3\sqrt{10} = 6\sqrt{10} = |BC|$

So, the points A, B, C are on the same straight line.

OR the given points are collinear.

Q5. Find k, given that the point (2, k) is equidistant from (3,7) and (9,1).

Solution:

Let the given points be P(2,k) and A(3, 7), B(9, 1).

As the points P is equidistant from A and B.

$\therefore \quad |PA| = |PB|$

i.e. $\sqrt{(2 \quad 3)^2 + (k \quad 7)^2} = \sqrt{(2-9) + (k-1)}$

Squaring both sides, we have

$(-1) + (5-7)^2 = (-7)^2 + (k-l)^2$

$1 + k^2 - 14k + 49 = 49 + k^2 - 2k + 1$

$50 + k^2 - 14k = 50 + k^2 - 2k$

$-14 + 2k = 0$

$-12k = 0$

=> $k = 0$

Q6. Use distance formula to verify that the points A(0,7), B(3, -5), c(-2,15) are collinear.

Solution:

Let the points be A(0,7), B(3,5) and C(-2,15).

$|AB| = \sqrt{(0 - 3)^2 + (7 + 5)^2}$

$= \sqrt{(-3)^2 + (1^2)^2} = \sqrt{(9 + 144} = \sqrt{(153} = \sqrt{(9 \times 17} = 3\sqrt{17}$

$|BC| = \sqrt{(-2 - 3)^2 + (15 + 5)^2}$

$= \sqrt{(-5)^2 + (20)^2} = \sqrt{5 + 400} = \sqrt{425} = \sqrt{25 \times 17} = \sqrt{17}$

$|CA| = \sqrt{(0 + 2)^2 + (7 - 15)^2} = \sqrt{(2)^2 + (-8)^2} = \sqrt{4 + 64} = \sqrt{68} = 2\sqrt{17}$

By applying the condition of collinear points

$|AB| + |CA| = 3\sqrt{17} + 2\sqrt{17} = (3 + 2)\sqrt{17}$

$= (3 + 2)\sqrt{17} = 5\sqrt{17} = |BC|$

the given points are collinear.

Q7. Verify whether or not the points 0 (0, 0), A($\sqrt{3}$, 1) are the vertices of an equilateral triangle.

Solution:

|0A| = $\sqrt{(\sqrt{3} \quad 0)^2 + (1 \quad 0)^2} = \sqrt{(3)^2 + (-1)^2}$

= $\sqrt{3+1} = \sqrt{4} = 2$

|0B| = $\sqrt{(\sqrt{3}-0)^2 + (-1-0)^2} = \sqrt{(3)^2 + (-1)^2}$

= $\sqrt{3+1} = \sqrt{4} = 2$

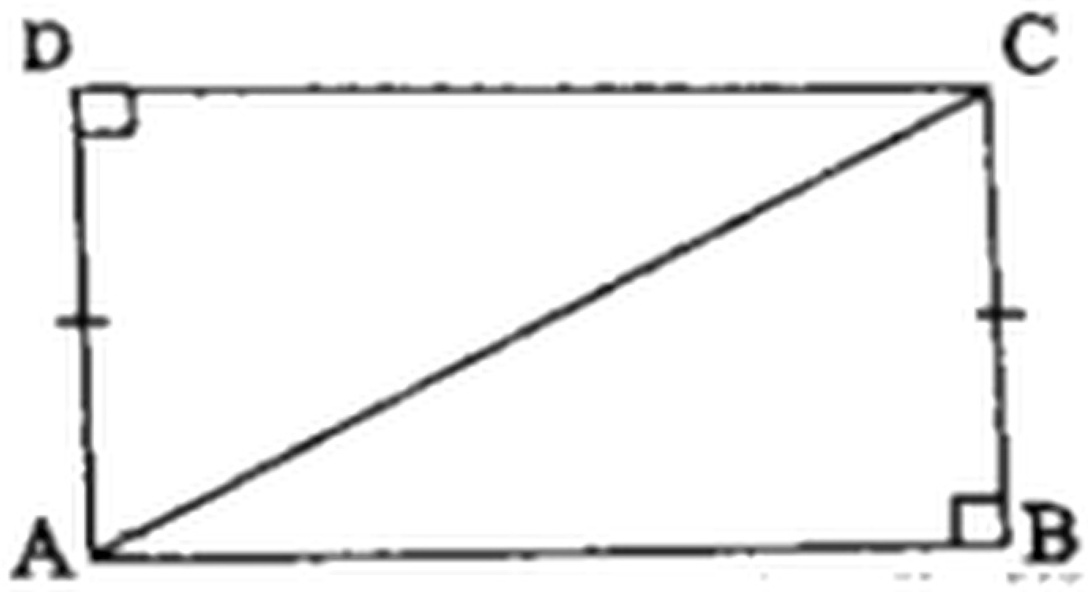
|AB| = $\sqrt{(\sqrt{3}-\sqrt{3})^2 + (-1-1)^2} = \sqrt{(0)^2 + (-2)^2}$

= $\sqrt{0+4} = \sqrt{4} = 2$

Since |0A| = |0B| = |AB| therefore ABO are vertices of an equilateral triangle.

Q8. Show that the points A(-6,-5), B(5,-5), C(5,-8) and D(-6 - 8) are vertices of a rectangle. Find the lengths of its diagonals. Are they equal?

Solution:



The points are A(-6,-5), B(5,-5), C(5,-8) and D(-6 - 8)

| AB | = $\sqrt{(-6-5)^2 + (-5+5)^2} = \sqrt{(-11)^2 + (0)^2} = \sqrt{121} = 11$

| DC | = $\sqrt{(5+6)^2 + (-8+8)^2} = \sqrt{(11)^2 + (0)^2} = \sqrt{121} = 11$

| AD | = $\sqrt{(-6+6)^2 + (-5+8)^2} = \sqrt{9} = 3$

$|BC| = \sqrt{(5 - 5)^2 + (-8 + 5)^2} = \sqrt{0 + 9} = 3$

$|AC| = \sqrt{(-6 - 5)^2 + (-5 + 8)^2} = \sqrt{(11)^2 + (3)^2} = \sqrt{121 + 9} = \sqrt{130}$

Now by applying Pythagoras theorem

$|AB|^2 + |BC|^2 = (11)^2 + (3)^2 = 121 + 9 = 130 = |AC|^2$

$\therefore \angle ABC = 90°$

and $|AB| = |DC|$ and $|AD| = |£C|$

Hence ABCD is a rectangle

For diagonals

$|AC| = \sqrt{(-6 - 5)^2 + (-5 + 8)^2} = \sqrt{(11)^2 + (3)^2} = \sqrt{121 + 9} = \sqrt{130}$

Also $|BD| = \sqrt{(-6 - 5)^2 + (-8 + 5)^2}$

$= \sqrt{(-11)^2 + (-3)^2} = \sqrt{121 + 9} = \sqrt{130}$

the two diagonals are equal in length.

Q9. Show that the points M(-l,4), N(-5,3),P(1,-3) and Q(5, -2) are the vertices of a parallelogram.

Solution:

Points are M(-l,4),N(-5,3),P(1,-3)and Q(5,-2)

$|MN| = \sqrt{(-1 + 5)^2 + (4 - 3)^2} = \sqrt{(4)^2 + (1)^2}$

$= \sqrt{16 + 1} = \sqrt{17}$

|PQ| = $\sqrt{(5-1)^2 + (4-3)^2} = \sqrt{(4)^2 + (1)^2} = \sqrt{16 + 1} = \sqrt{17}$

|NP| = $\sqrt{(1 + 5)^2 + (-3 - 3)^2} = \sqrt{(6)^2 + (-6)^2} = \sqrt{36 + 36} = \sqrt{72} = 6\sqrt{2}$

|QN| = $\sqrt{(5 + 1)^2 + (-2-4)^2} = \sqrt{(6)^2 + (-6)^2} = \sqrt{36 + 36} = 6\sqrt{2}$

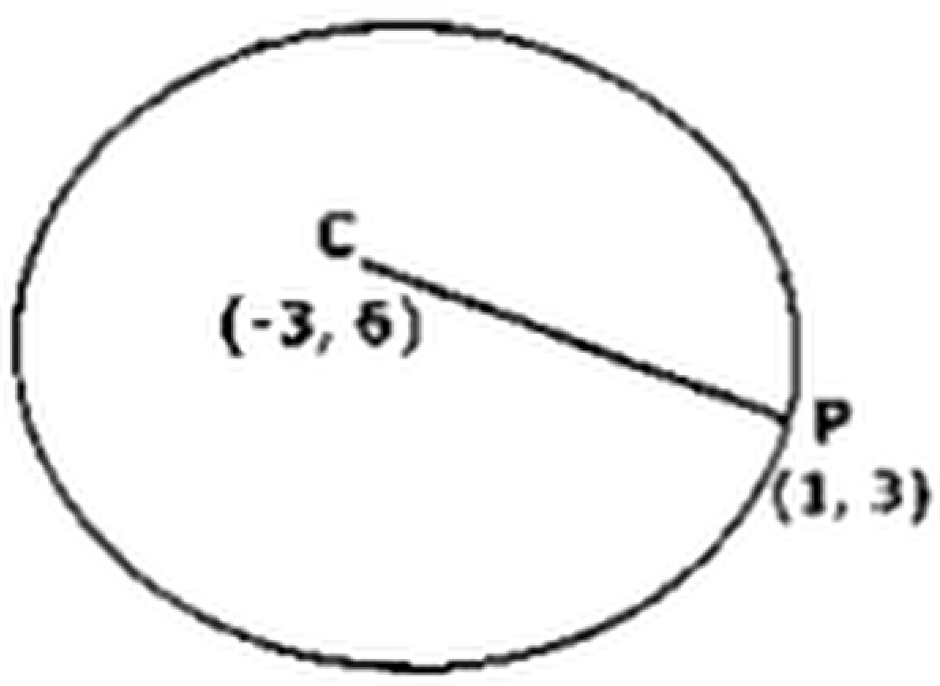
$|NP|^2 + |PQ|^2 = 7^2 + 17 = 89 \neq 125 = |QN|^2$

But |MN| = |PQ| = |NQ| = |MQ|

Hence the given points from a parallelogram.

Q10. Find the length of the diameter of the circle having center at C(-3,6) and passing through point P(1,3)

Solution:



Centre C(-3, 6) and the circle is passing through the point P(1,3)

∴ radius = m |PC |

$= \sqrt{(-3 - 1)^2 + (6-3)^2} = \sqrt{(-4)^2 + (3)^2} = \sqrt{16+9} = \sqrt{25} = 5$

Diameter = 2 x radius

= 2×5 = 10