Exercise 6.3

1. Find A.M. between

(i). $3\sqrt{5}$ *and* $5\sqrt{5}$ **(ii).** $x-3$ *and* $x+5$

(iii). $1-x+x^2$ *and* $1+x+x^2$

(i). $3\sqrt{5}$ *and* $5\sqrt{5}$

Solution:

$$\text{A.M} = \frac{a+b}{2} = \frac{3\sqrt{5} \text{ and } 5\sqrt{5}}{2} = \frac{8\sqrt{5}}{2} = 4\sqrt{5}$$

Hence $\text{A.M} = 4\sqrt{5}$

(ii). $x-3$ *and* $x+5$

Solution:

$$\text{A.M} = \frac{x-3+x+5}{2} = \frac{2x+2}{2} = x+1$$

Hence $\text{A.M} = x+1$

(iii). $1-x+x^2$ *and* $1+x+x^2$

Solution:

$$\text{A.M} = \frac{1-x+x^2+1+x+x^2}{2} = \frac{2+2x^2}{2} = 1+x^2$$

Hence $\text{A.M} = 1+x^2$

2. If 5, 8 are two A.Ms between a and b, find a and b.

Solution:

Let a, 5, 8, b is given A.P.

This mean 5 is A.M of a & b.

A.M. $= \frac{a+8}{2}$

$= \frac{a+8}{2} = 5$

$a + 8 = 10$

$a = 10 - 8 = 2$

$a = 2$

And '8' is mean of 5 and b

A.M. $= \frac{5+b}{2}$

$\frac{5+b}{2} = 8$

$5 + b = 8 \times 2 = 16$

$b = 16 - 5 = 11$

$b = 11$

Hence $a = 2 \,\&\, b = 11$

3. Find 6 A.Ms between 2 and 5.

Solution:

Let $6\ A.Ms\ ARE\ A_1, A_2, A_3, A_4, A_5, A_6$

then

$2, A_1, A_2, A_3, A_4, A_5, A_6, 5\ is\ an\ A.P.$

$a_1 = 2;$ $a_8 = 5$

$$a_8 = a_1 + 7d$$

$$5 = 2 + 7d$$

$$7d = 5 - 2 = 3$$

$$d = \frac{3}{7}$$

Therefore,

$$A_1 = a_1 + d = 2 + \frac{3}{7} = \frac{17}{7}$$

$$A_2 = a_1 + 2d = 2 + \frac{6}{7} = \frac{20}{7}$$

$$A_3 = a_1 + 3d = 2 + \frac{9}{7} = \frac{23}{7}$$

$$A_4 = a_1 + 4d = 2 + \frac{12}{7} = \frac{26}{7}$$

$$A_5 = a_1 + 5d = 2 + \frac{15}{7} = \frac{29}{7}$$

$$A_6 = a_1 + 6d = 2 + \frac{18}{7} = \frac{32}{7}$$

Hence 6, A.M. are $\frac{17}{7}, \frac{20}{7}, \frac{23}{7}, \frac{26}{7}, \frac{29}{7}, \frac{32}{7}$

4. Find four A.Ms are between $\sqrt{2}$ *and* $\frac{12}{\sqrt{2}}$

Solution:

Let 4 A.Ms are A_1, A_2, A_3, A_4

Then

$$\sqrt{2}, A_1, A_2, A_3, A_4, \frac{12}{\sqrt{2}} \text{ is an A.P.}$$

Where $a_1 = \sqrt{2}$

And $a_6 = \frac{12}{\sqrt{2}}$

$$a_1 + 5d = \frac{12}{\sqrt{2}}$$

$\sqrt{2} + 5d = \frac{12}{\sqrt{2}}$

$5d = \frac{12}{\sqrt{2}} - \sqrt{2} = \frac{12-2}{\sqrt{2}} = \frac{10}{\sqrt{2}}$

$d = \frac{10}{5\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

So,

$d = \sqrt{2}$

$A_1 = a_1 + d = \sqrt{2} + \sqrt{2} = 2\sqrt{2}$

$A_2 = a_1 + 2d = \sqrt{2} + 2\sqrt{2} = 3\sqrt{2}$

$A_3 = a_1 + 3d = \sqrt{2} + 3\sqrt{2} = 4\sqrt{2}$

$A_4 = a_1 + 4d = \sqrt{2} + 4\sqrt{2} = 5\sqrt{2}$

Hence 4 A.Ms are $2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, 5\sqrt{2}$

5. Insert 7 A.Ms between 4 and 8.

Solution:

Let 7 A.Ms are $A_1, A_2, A_3, A_4, A_5, A_6, A_7$

Then

$4, A_1, A_2, A_3, A_4, A_5, A_6, A_7, 8\ is\ an\ A.P.$

Where

$a_1 = 4; \qquad a_a = 8$

$a = a_1 + 8d$

$8d = 8 - 4 = 4$

$d = \frac{4}{8} = \frac{1}{2}$

Therefore,

$A_1 = a_1 + d = 2 + \left(\frac{1}{2}\right) = \frac{9}{2}$

$A_2 = a_1 + 2d = 2 + 2\left(\frac{1}{2}\right) = \frac{10}{7} = 5$

$A_3 = a_1 + 3d = 2 + 3\left(\frac{1}{2}\right) = \frac{11}{2}$

$A_4 = a_1 + 4d = 2 + 4\left(\frac{1}{2}\right) = \frac{12}{2} = 6$

$A_5 = a_1 + 5d = 2 + 5\left(\frac{1}{2}\right) = \frac{13}{2}$

$A_6 = a_1 + 6d = 2 + 6\left(\frac{1}{2}\right) = \frac{14}{2} = 7$

$A_7 = a_1 + 7d = 2 + 7\left(\frac{1}{2}\right) = \frac{15}{2}$

Hence 6, A.M. are $\frac{9}{2}, 5, \frac{11}{2}, 6, \frac{13}{2}, 7, \frac{15}{2}$

6. Find three A.Ms between 3 and 11.

Solution:

Let 3 A.Ms are A_1, A_2, A_3

Then

$3, A_1, A_2, A_3, 11 \text{ is an } A.P.$

Where

$a_1 = 3;$ $a_5 = 11$

$a_5 = a_1 + 4d$

$11 = 3 + 4d$

$4d = 11 - 3 = 8$

$d = \frac{8}{4} = 2$

Therefore,

$A_1 = a_1 + d = 3 + 2 = 5$

$$A_2 = a_1 + 2d = 3 + 2(2) = 7$$

$$A_3 = a_1 + 3d = 3 + 3(2) = 9$$

Hence 3 A.Ms are 5, 7, 9

7. Find n so that $\frac{a^a + a^a}{a^{a\ 1} + b^a - 1}$ may be the A.M between a and b.

Solution:

Let A.M between 'a' and 'b'

Is A.M $= \frac{a+b}{2}$

And A.M $= \frac{a^a + a^a}{a^{a\ 1} + b^a - 1}$

So,

$$\frac{a+b}{2} = \frac{a^a + a^a}{a^{a\ 1} + b^a - 1}$$

$$= (a + b)(a^n + b^n) + 2(a^n + b^n)$$

$$a^n + b^{n-1} + a^{n-1}b + b^n = 2a^n + 2b^n$$

$$-a^n + 2a^n + a^{n-1}b = 2b^n - b^n - ab^{n-1}$$

$$a^n + a^{n-1}b = b^n - ab^{n-1}$$

$$a^{n-1}(b - a) = b^{n-1}(b - a)$$

$$\frac{a^n}{b^n} = \frac{b-a}{b-a}$$

$$\left[\frac{a}{b}\right]^{n-1} = 1$$

$$\left[\frac{a}{b}\right]^{n-1} = \left[\frac{a}{b}\right]^0$$

By comparison

$$n - 1 = 0$$

$$n = 1$$

Hence, value of n = 1

8. Show that the sum of n A.Ms between a and b is equal to n time their A.M.

Solution:

Let $A_1, A_2, A_3, \ldots\ldots. A_n$ *be in A.Ms between* 'a' and 'b'

Then $a, A_1, A_2, A_3, \ldots\ldots. A_n, b$ *is an A.P*

$a + a; \qquad a_n = a + (n-1)d$

$a_n = a + (n-1)d$

$d = \frac{a^{n \; 1}}{n-1}$

$A_a = a_{a+1} = a + (n+1-1)da + nd$

Thus the sum of an A.M between 'a' and 'b' is

$S = A_1 + A_2 + A_3, + \ldots\ldots. + A_n$

$S = \frac{n}{2}[A_1 + A_n]$

$S = \frac{n}{2}[a + d + a + nd]$

$S = \frac{n}{2}[2a + (n+1)d$

$\frac{n}{2} = [2a + b - a] \qquad [\because (n+1)d = b - a$

$\frac{n}{2} = [a + b] \qquad = n\left[\frac{a+b}{2}\right]$

$= n$ [*A.Ms between* 'a' and 'b']

Hence proved

The sum of A.M is $= n$ [*A.Ms between* 'a' and 'b']