Exercise 6.2

Q1. Write the first four terms of the following arithmetic sequence, if

$i.\ a_1 = 5\ and$ **other three consecutive are** $23, 26, 29$

$ii.\ a_5 = 17\ and\ a_9 = 37 \quad iii.\ 3a_7 = 7a_4\ and\ a_{10} = 33$

Solution:

$i.\ a_1 = 5$

Three consecutive terms are 23, 26, 29

Then common difference $(d) = 26 - 23\ and\ a_1 = 5$

$a_2 = a_1 + d = 5 + 3 = 8$

$a_3 = a_2 + d = 8 + 3 = 11$

$a_4 = a_3 + d = 11 + 3 = 14$

Thus the first four terms 5, 8, 11, 14.

Hence required first four terms 5, 8, 11, 14.

$ii.\ a_5 = 17\ and\ a_9 = 37$

we know that

$a_5 = a + 4d$ & $a_9 = a + 8d$

$17 = a + 4d$ $\quad 37 = a + 8d$

$a + 8d + 37$

$\frac{\pm a \pm 4d = -17}{4d = 20}$

$d = \frac{20}{4} = 5$

Put the value of 'd=5' in equation

$a + 4d = 17$

$a + 4(5) = 17$

$a + 20 = 17$

$a = 17 - 20 = -3$

$a = -3$

So $a_1 = -3$

$a_2 = a_1 + d = -3 + 5 = 2$

$a_3 = a_2 + d = 2 + 5 = 7$

$a_4 = a_3 + d = 7 + 5 = 12$

Thus the first four terms -3,2, 7,12

Hence required terms -3,2, 7,12

$iii.$ $\boldsymbol{3a_7 = 7a_4 \text{ and } a_{10} = 33}$

$3[a_1 + 6d] = 7[a_1 + 3d]$

$3a_1 + 18d = 7a_1 + 21d$

$7a_1 - 3a_1 + 21d - 18d = 0$

$4a_1 + 3d = 0$

$a_1 = \frac{-3}{4}d$ and

$a_{10} = a_1 + 9d$

$a_1 + 9d = 33$

Put the value of a_1

$\frac{-3}{4} + 9d = 33$

$-3d + 36d = 132$

$33d + 132$

$d = \frac{132}{33} = 4$

Therefore, $a_1 = \frac{-3}{4}(4) = -3$

So $a_1 = -3$

$a_2 = a_1 + d = -3 + 4 = 1$

$a_3 = a_2 + d = 1 + 4 = 5$

$a_4 = a_3 + d = 5 + 4 = 9$

Thus the first four terms -3,1, 5,9

Hence required terms --3,1, 5,9

Q2. If $a_{n-3} = 2n - 5$, find he nth term of the sequence.

Solution:

$\boldsymbol{a_{n-3} = 2n - 5}$

Let n 3 = n

$n = n + 3$

So $n = n + 3$

becomes $a_n = 2(n + 3) = n$

$= 2n + 6 - 5$

$= 2n + 1$

Therefore $a_n = 2n + 1$

Hence $a_n = 2n + 1$

Q3. If the 5th term of an A.P. is 16 and the 20th term is 46, what is it's 12th term?

Solution:

$\boldsymbol{a_5 = 16}$ **&** $\boldsymbol{a_{20} = 46}$

$a_5 = a_1 + 4d$

$16 = a_1 + 4d$

$a_1 + 4d = 16$ ………………. (i)

$a_{20} = a_1 + 19d$

$46 = a_1 + 19d$

$a_1 + 4d = 16$ ………………. (ii)

$a_1 + 19d = 46$

$\frac{\pm a_1 \pm 4d = \pm 16}{15d = 30}$ (by subtracting)

$d = \frac{30}{15} = 2$

Put the value of d in equation (i)

$a_1 + 4(2) = 16$

$a_1 + 8 = 16$

$a_1 = 16 - 8 = 8$

Therefore

$a_{12} = a_1 + 11d = 8 + 11(2)$

$8 + 22 = 30$

$a_{12} = 30$

Hence $a_{12} = 30$

Q4. Find the 13th term of the sequence x, 1, 2-x,3-2x, …..

Solution:

Let **x, 1, 2-x,3-2x,** ….. is an A.P.

Then common difference (d) =1-x

And $a_1 = x$

Then

$a_{13} = a_1 + 12d$

$= x + 12(1 - x)$

$= x + 12 - 12x$

$a_{13} = 12 - 11x$

Hence; $a_{13} = 12 - 11x$

Q5. Find the 18th term of the A.P if its 6th term is 19 and the 9th term is 31.

Solution:

$\boldsymbol{a_6 = 19}$ **&** $\boldsymbol{a_9 = 31}$

$a_1 + 5d = 19$ & $a_1 + 8d = 31$

$a_1 + 8d = 31$

$\frac{\pm a_1 . \pm 5d = \pm 19}{3d = 22}$ (by subtracting)

$d = \frac{12}{3} = 4$

by using the value of d

$a_1 + 5(4) = 19$

$a_1 + 20 = 19$

$a_1 + 19 - 20 = -1$

$a_1 = -1$ & $d = 4$

Therefore:

$a_{18} = a_1 + 17d$

$= -1 + 17(4)$

$= -1 + 68 = 67$

$a_{18} = 67$

Hence, $a_{18} = 67$

Q6. Which term of the A.P. 5,2, -1… is -85?

Solution:

Let 5,2, -1… is A.P.

Then $a_1 = 5$

$d = 2 - 5 = -3$

We know that

$a_1 + (n-1)d = -85$

$5+(n-1)(-3) = -85$

$(n-1)(-3) \quad = -85 - 5$

$-3n + 3 \qquad = -90$

$-3n = -90 - 3 = -93$

$-3n = -93$

$n = \frac{-93}{-3} = 31$

$n = 31$

Hence '-85' is the 31th term of give A.P.

Q7. Which term of the A.P. -2, 4, 10, … is 148?

Solution:

Let -2, 4, 10, … is A.P.

Then $a_1 = -2$

And $d = 4 \quad 2 = 4 + 2 = 6$

$a_1 + (n-1)d = 148$

$-2+(n-1)6 \ = 148$

$6n - 6 = 148 + 2 = 150$

$6n = 150 + 6 = 156$

$n = \frac{156}{6} = 26$

Therefore;

$n = 26$ Hence; '128' is the 26th term of give A.P.

Q8. How many terms are there in the A.P.in which $a_1 = 11, a_n = 68, d = 3$?

Solution:

Let $a_1 = 11$; $d = 3$; $a_n = 68$

$a_n = a_1 + (n-1)d$

$16 = 11 + (n-1)3$

$68 - 11 = 3n - 3$

$57 = 3n - 3$

$3n = 57 + 3 = 60$

$3_n = \frac{60}{3} = 20$

Hence: $a_n = 68$ is the 20th term.

Q9. If the nth term of the A.P is 3n-1, FIND THE A.P.

Solution:

Let $a\ n = 3n - 1$

Put n = 1, 2, 3, ...

$a_1 = 3(1) - 1 = 3 - 1 = 2$

$a_2 = 3(2) - 1 = 6 - 1 = 5$

$a_3 = 3(3) - 1 = 9 - 1 = 8$

Thus A.P. 2, 5, 8, ...

Hence; 2, 5, 8, ...

Q10. Determine whether (i) -19, (ii) 2 are the terms of A.P. 17, 13, 19, ... or not.

Solution:

Let 17, 13, 19, ...is A.P.

$a_1 = 17; \qquad d = 13 - 17 = -4$

Therefore

$a_n = a_1 + (n-1)d$

$-19 = 17 + (n-1)(4)$

$-19 = 17 - 4n + 4$

$-19 = 21 - 4n$

$-4n = -19 - 21 = -40$

$n = \frac{-40}{-4} = 10$

Hence; −19 is the 10^{th} term of the given A.P.

Q11. If l, m, n are the pth, qth, and rth term of an A.P, show that

(i) l (q-r)+m(r-p)+n(p-q) = 0 **(ii) p(m-n)+q(n-1)+r(l-m) = 0**

Solution:

Let l= a_p; m= a_q & n= a_r

$$l = a_p = a + (p-1)d = a + pd - d$$

$$m = a_q = a + (q-1)d = a + qd - d$$

$$n = q_r = a + (r-1)d = a + rd - d$$

l (q-r)+m(r-p)+n(p-q) = 0

L.H.S.

$$= (a + pd - d)[q - r] + (a + qd - d)[r - p] + [a + rd - d][p - q]$$

$$= aq + pqd - dq - ar + pdr + dr + ar + qrd - rd - ap - pqd + dp$$

$$+ap + prd - pd - aq - qrd + dp$$

$$= 0$$

R.H.S.

Hence proved

$$l(q-r) + m(r-p) + n(p-q) = 0$$

(ii) p(m-n)+q(n-1)+r(l-m) = 0

L.H.S

$$p(m-n) + q(n-1) + r(l-m)$$

$$= p[a + qd - d - a - rd + d] + q[a + rd - d - a - pd + d] +$$

$$r[a + pd - d - a - qd + d]$$

$$= p[qd - rd] + q[rd - pd] + r[pd - qd]$$

$$= pqd - prd + qrd - qpd + rpd - rqd$$

$$= 0$$

$$= \boldsymbol{R.H.S}$$

Hence proved

$$p(m-n) + q(n-1) + r(l-m) = 0$$

Q12. Find the nth term of the sequence,

$$\left(\frac{4}{3}\right)^2, \left(\frac{7}{3}\right)^2, \left(\frac{10}{3}\right)^2, \ldots.$$

Solution:

So, 4, 7, 10, ... Is an A.P.

Where $a_1 = 4$; $d = 7 - 4 = 3$

$$a_2 = a_1 + d = 4 + 3 + 7$$

$$a_3 = a_1 + 2d = 4 + 6 = 11$$

$$a_r = a_1 + (r - d)d = 4 + 3r - 3 = 1 + 3r$$

$$a_n = a_1(n-1)d = 4 + (n-1)3 = 4 + 3n - 3$$

$$= 3n + 1$$

Thus $a_n = \left[\frac{3n+1}{3}\right]^2$

Hence the nth term of A.P. is $a_n = \left[\frac{3n+1}{3}\right]^2$

Q13. If $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ are in A.P. SHOW THAT $b = \frac{2ac}{a+c}$.

Solution:

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$, ……. **Is A.P.**

Then

$d = \frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$

$\frac{1}{b} + \frac{1}{b} = \frac{1}{c} + \frac{1}{a}$

$\frac{1+1}{b} = \frac{a+c}{ac}$

$\frac{2}{b} = \frac{a+c}{ac}$

$\frac{1}{b} = \frac{a+c}{2ac}$

$b = \frac{2ac}{a+c}$

Hence proved

$b = \frac{2ac}{a+c}$

Q14. If $\frac{1}{a}, \frac{1}{b}$ $and \frac{1}{c}$ are in A.P. show that the common difference is $\frac{a-c}{2ac}$

Solution:

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$, ……. **Is A.P.**

Then the common difference (d) $= \frac{1}{b} - \frac{1}{a}$

And $(d) = \frac{1}{c}\frac{1}{b}$

$2d = \frac{1}{c} + \frac{1}{b}$ (adding)

$2d = \frac{a+c}{ac}$

$d = \frac{a+c}{2ac}$

Hence proved

$$\boldsymbol{d = \frac{a+c}{2ac}}$$