Exercise 6.11

1. **Sum the following series up to n terms.**

i. $1 \times 1 + 2 \times 4 + 3 \times 7 + \ldots..$

ii. $1 \times 3 + 3 \times 6 + 5 \times 9 + \ldots\ldots.$

iii. $1 \times 4 + 2 \times 7 + 3 \times 10 + \ldots\ldots$

iv. $3 \times 5 + 5 \times 9 + 7 \times 13 + \ldots\ldots$

v. $1^2 + 3^2 + 5^2 + \ldots\ldots$

vi. $2^2 + 5^2 + 8^2 + \ldots\ldots$

vii. $2 \times 1^2 + 4 \times 2^2 + 6 \times 3^2 + \ldots..$

viii. $3 \times 2^2 + 5 \times 3^2 + 7 \times 4^2 + \ldots$

ix. $2 \times 4 \times 7 + 3 \times 6 \times 10 + 4 \times 8 \times 13 + \ldots$

x. $1 \times 4 \times 6 + 4 \times 7 \times 10 + 7 \times 10 \times 4 + \ldots$

xi. $1 + (1 + 2) + (1 + 2 + 3) + \ldots\ldots$

xii. $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \ldots\ldots$

xii. $2 + (2 + 5) + (2 + 5 + 8) + \ldots\ldots..$

Solution:

i. $1 \times 1 + 2 \times 4 + 3 \times 7 + \cdots$

Each term has two numbers

First numbers are

1, 2, 3, 4, ……

Here, $a = 1;$ $d = 1$

$a_n = a_1 + (n - 1)d$

$= 1 + (n - 1)(3)$

$a_n = 1 + 3n - 3 = 3n - 2$

a_n $3n - 2$

So, the product of the nth term = $n(3n - 2)$

Therefore;

$$1 \times 1 + 2 \times 4 + 3 \times 7 + \ldots\ldots\ldots\ldots = n(3n-2) = 3n^2 - 2n$$

$$= 3\sum n^2 - 2\sum n$$

$$= 3\frac{n(n+1)(2n+1)}{6} - \frac{2n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}[2n+1-2]$$

$$= \frac{n(n+1)}{2}[2n-1]$$

$$= \frac{n}{2}(n+1)(2n-1)$$

Hence, $1 \times 1 + 2 \times 4 + 3 \times 7 + \ldots\ldots = \frac{n}{2}(n+1)(2n-1)$

ii. $\mathbf{1 \times 3 + 3 \times 6 + 5 \times 9 + \ldots\ldots}$

Each term has two numbers

First numbers are

1, 3, 5, ……

Here, $a = 1;\ d = 3 - 1 = 2$

$$a_n = a_1 + (n-1)d$$

$$= 1 + (n-1)(2)$$

$$= 1 + 2n - 2 = 2n - 1$$

$$a_n = 2n - 1$$

Second number of the products are:- 3, 6, 9,…..

Have, $a = 3;\ d = 6 - 3 = 3$

$$a_n = a_1 + (n-1)d$$

$$= 3 + (n-1)(3)$$

$= 3 + 3n - 3 = 3n$

$a_n \quad = 3n$

So, the product of the nth term = $3n(2n-1) = 6n^2 - 3n$

Therefore, sum of the series = $6\sum n^2 - 3\sum n$

$= 6\frac{n(n+1)(2n+1)}{6} - 3\frac{n(n+1)}{2}$

$= \frac{n(n+1)}{2}[2(2n+1) - 3]$

$= \frac{n(n+1)}{2}[4n + 2 - 3]$

$= \frac{n}{2}(n+1)(4n-1)$

Hence, $1 \times 3 + 3 \times 6 + 5 \times 9 + \ldots\ldots = \frac{n}{2}(n+1)(4n-1)$

iii. $\mathbf{1 \times 4 + 2 \times 7 + 3 \times 10 + \ldots\ldots}$

Each term has two numbers

First numbers are 1, 2, 3, ……

Here, $a = 1;\ d = 2 - 1 = 1$

$a_n \quad = a_1 + (n-1)d$

$= 1 + (n-1)(1)$

$= 1 + n - 1 = n$

$a_n \quad = n$

Second number of the products are:- 4, 7, 10,…..

Have, $a = 4;\ d = 7 - 4 = 3$

$a_n \quad = a_1 + (n-1)d$

$= 4 + (n-1)(3)$