Exercise 5.2

Q.1 Factorize

(i) $x^4 + \frac{1}{x^4} - 3$

$= x^4 + 1/x^4 - 2 - 1$

$= x^4 - 2 + 1/x^4 - 1$

$= \left(x^2 - 1/x^2\right)^2 - 1^2$

$= \left(\left(x^2 - 1/x^2\right) + 1\right)\left(\left(x^2 - 1/x^2\right) - 1\right)$

$= \left(x^2 - 1/x^2 + 1\right)\left(x^2 - 1/x^2 - 1\right)$

(ii) $3x^4 + 12y^4$

$= 3\left(x^4 + 4y^4\right)$

$= 3\left(x^4 + 4x^2y^2 + 4y^4 - 4x^2y^2\right)$

$= 3\left(x^2 + 2y^2\right)^2 - 4x^2y^2$

$= 3\left((x^2 + 2y^2) + (2xy)^2\right)$

$= 3\left(\left(x^2 + 2y^2\right) + 2xy\right)\left(\left(x^2 + 2y^2\right) - 2xy\right)$

$= 3\left(x^2 + 2xy + 2y^2\right)\left(x^2 - 2xy + 2y^2\right)$

(iii) $a^4 + 3a^2b^2 + ab^4$

$= a^4 + 4a^2b^2 + 4b^2 - a^2b^2$

$= \left(a^2 + 2b^2\right)^2 - (ab)^2$

$= \left(a^2 + 2b^2 + ab\right)\left(a^2 + 2b^2 - ab\right)$

$= \left(a^2 + ab + 2b^2\right)\left(a^2 - ab + 2b^2\right)$

$$=\left(2x^2+9\right)^2-(6x)^2$$
$$=\left(2x^2+9+6x\right)\left(2x^2+9-6x\right)$$
$$=\left(2x^2+6x+9\right)\left(2x^2-6x+9\right)$$

(v) x^4+x^2+25

$$=x^4+10x^2+25-9x^2$$
$$=\left(x^2\right)^2+2\left(x^2\right)5-9x^2$$
$$=\left(x^2+5\right)^2-(3x)^2$$
$$=\left(x^2+5+3x\right)\left(x^2+5-3x\right)$$
$$=\left(x^2+2x+4\right)\left(x^2-2x+4\right)$$

(vi) x^4+4x^2+16

$$=x^4+8x^2+16-4x^2$$
$$=\left(x^2+4\right)^2-(2x)^2$$
$$=\left(x^2+4+2x\right)\left(x^2+4-2x\right)$$
$$=\left(x^2+2x+4\right)\left(x^2+2x-4\right)$$

Q.2

(i) $x^2+14x+48$

$$=x^2+8x+6x+48$$
$$=x\left(x+8\right)+6\left(x+8\right)$$
$$=\left(x+8\right)\left(x+6\right)$$

(ii) $x^2-21x+108$

$$=x^2-12x-9x+108$$
$$=x(x-12)-9(x-12)$$

(iii) $x^2 - 11x - 42$

$= x^2 - 14x + 3x - 42$

$= x(x-14) + 3(x-14)$

$= (x-14)(x+3)$

(iv) $x^2 + x - 132$

$= x^2 - 12x - 11x - 132$

$= x(x+12)(x-11)$

$= (x+12)(2x+1)$

Q.3

(i) $4x^2 + 12x + 5$

$= 4x^2 + 10x + 2x + 5$

$= 2x(2x+5) + 1(2x+5)$

$= (2x+1)(2x+5)$

(ii) $30x^2 + 7x - 15$

$= 30x^2 + 25x - 18x - 15$

$= 5x(6x+5) - 3(6x+5)$

$= (6x+5)(5x-3)$

(iii) $24x^2 - 65x + 21$

$= 24x^2 - 56x - 9x + 21$

$= 8x(3x-7) - 3(3x-7)$

(iv) $5x^2 - 16x - 21$

$= 5x^2 - 21x + 5x - 21$

$= x(5x - 21) + 1(5x - 21)$

$= (5x - 21)(x + 1)$

(v) $4x^2 - 17xy + 4y^2$

$= 4x^2 - 16xy - y + 4y^2$

$= 4x(x - 4y) - y(x - 4y)$

$= (x - 4y)(4x - y)$

(vi) $3x^2 - 38xy + 13y^2$

$= 3^2 - 39xy + xy - 13y^2$

$= 3x(x - 13y) + y(x - 13y)$

$= (x - 13y)(3x + y)$

(vii) $5x^2 + 33xy - 14y^2$

$= 5x^2 + 35xy - 2xy - 14y^2$

$= 5x(x + 7y) + y(x + 7y)$

$= (x + 7y)(5x + y)$

(viii) $(5x - 1/x)^2 + 4(5x - 1/x) + 4$

let $5x - \frac{1}{x} = y$

$= y^2 + 4y + 4$

$(y+2)^2 = (y+2)(y+2)$

by putting value of $y = 5x - \frac{1}{x}$

$= (5x - 1/x + 2)(5x - 1/x + 2)$

Q.4

(i) $(x^2 + 5x + 4)(x^2 + 5x + 6) - 3$

let $x^2 + 5x = y$

$(y + 4)(y + 6) - 3$

$= y^2 + 6y + 4y + 24 - 3$

$= y^2 + 10y + 21$

$= y^2 + 7y + 3y + 21$

$= y(y + 7) + 3(y + 7)$

$= (y + 7)(y + 3)$

by putting value of $y = x^2 + 5x$

$= (x^2 + 5x + 7)(x^2 + 5x + 3)$

(ii) $(x^2 - 4x)(x^2 - 4x - 1) - 20$

let $x^2 - 4x = y$

$= y(y - 1) - 20$

$= y^2 - y - 20$

$= y^2 - 5y + 4y - 20$

$= y(y - 5) + 4(y - 5)$

$= (y - 5)(y + 5)$

by putting value of $y = x^2 - 4x$

$= (x^2 - 4x - 5)(x^2 - 4x + 4)$

$= (x^2 - 5x + x - 5)(x^2 - 2(x)2 + 4)$

$= ((x(x - 5) + 1(x - 5)))(x - 2)^2$

(iii) $(x+2)(x+3)(x+4)(x+5)-15$

By using commutative property of addition

As $2+5=3+4$

$=(x^2+7x+10)(x^2+7x+12)-15$

let $x^2+7x=y$

$=(y+10)(y+12)-15$

$=y^2+22y+120-15$

$=y^2+22y+105$

$=y^2+15y+7y+105$

$=y(y+15)+7(y+15)$

$=(y+15)(y+7)$

By putting value of $y=x^2+7y$

$=(x^2+7x+15)(x^2+7x+7)$

(iv) $(x+4)(x-5)(x+6)(x-7)-504$

By using commutative property of subtraction

As $4-5=6-7$

$=(x^2-x-20)(x^2-x-42)-504$

let $x^2-x=y$

$=(y-20)(y-42)-504$

$=y^2-42y-20y+840-504$

$=y^2-62y+336$

$=y(y-56)(y-6)$

$=$ *by putting value of* $y=x^2-x$

$=(x^2-x-56)(x^2-x-6)$

$=(x^2-8x+7x-56)(x^2-3x+2x-6)$

$=(x(x-8)+7(x-8))(x(x-3)+2(x-3))$

$=(x-8)(x+7)(x-3)(x+2)$

By using commutative property of multiplication

As $(1)(6)=(2)(3)$

$=\left(x^2+7x+6\right)\left(x^2+5x+6\right)-3x^2$

$=\left(x^2+6+7x\right)\left(x^2+6+5x\right)-3x^2$

let $x^2+6=y$

$=(y+7x)(y+5x)-3x^2$

$=y^2+5xy+7xy+35x^2-3x^2$

$=y^2+12xy+32x^2$

$=y^2+8xy+4xy+32x^2$

$=y(y+8x)+4x(y+8x)$

$=(y+8x)(y+4x)$

By putting value of $y=x^2+6$

$=\left(x^2+6+8x\right)\left(x^2+6+4x\right)$

$=\left(x+8+\frac{6}{x}\right).x\left(x+4+\frac{6}{x}\right)$

$=x^2\left(x+\frac{6}{x}+8\right)\left(x+\frac{6}{x}+4\right)$

Q.5

(i) $x^3+48x-12x^2-64$

$=x^3-12x^2+48x-64$

$=x^3-3.x^2.4+3.x.4^2-4^3$

$=(x-4)^3$

(ii) $8x^3+60x^2+150x+125$

$=(2x)^3+3.(2x)^2.5+3.(2x).5^2+5^3$

$=(2x-5y)^3$

$= x^{23} - 3x^2.6 + 3.x.6^2 - 6^3$

$= (x-6)^3$

(iv) $8x^3 - 125y^3 - 60x^2y + 150xy^2$

$= 8x^3 - 60x^2y + 150xy^2 - 125y^3$

$= (2x)^3 - 3.(2x)^2.5y + 3.(2x).(5y)^2 - (5y)^3$

$= (2x-5y)^3$

Q.6

(i) $27 + 8x^3$

$= (3)^2 + (2x)^3$

$= (3+2x)\left(3^2 - 3.2x + 2(x)^2\right)$

$= (3+2x)\left(9 - 6x + 4x^2\right)$

(ii) $125x^3 - 216y^3$

$= (5x)^3 - (6y)^3$

$= (5x-6y)\left((5x)^2 + 5x.6y + (6y)^2\right)$

$= (5x-6y)\left(25x^2 + 30xy + 36y^2\right)$

(iii) $64x^3 + 27y^3$

$= (4x)^3 + (3y)^2$

$= (4x+3y)\left((4x)^2 - 4x.3y + (3y)^2\right)$

$= (4x+3y)\left(16x^2 - 12xy + 9y^2\right)$

$$= (2x)^3 + (5y)^3$$
$$= (2x + 5y)\left((4x)^2 - 4x.4y + (3y)^2\right)$$
$$= (2x + 5y)\left(4x^2 - 10xy + 25y^2\right)$$