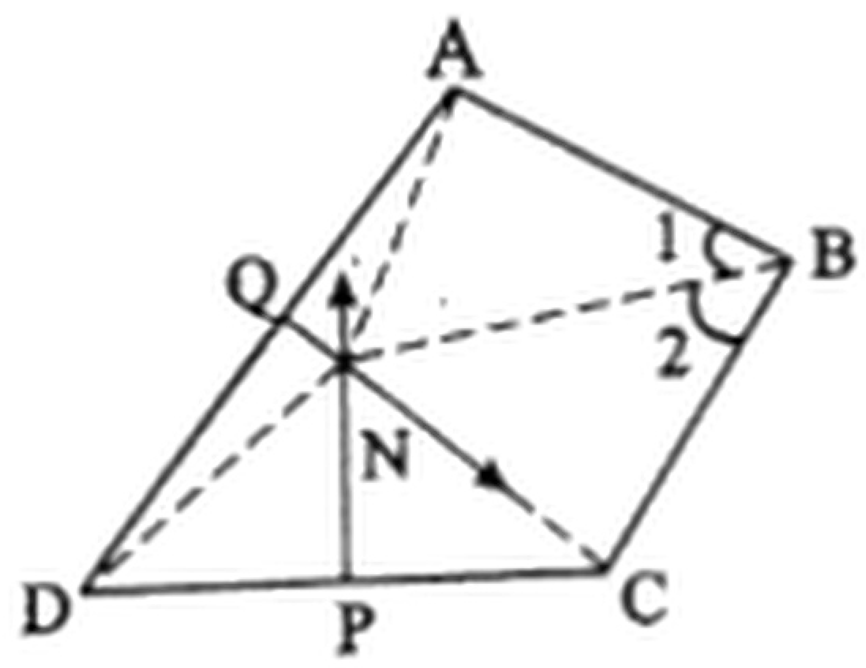
EXERCISE 12.2

Q1. In a quadrilateral ABCD, $\overline{AB} \cong \overline{BC}$ and the right bisectors of $\overline{AD}$, $\overline{CD}$ meet each other at point N. Prove that $\overline{BN}$ is a bisector of ∠ABC.

Solution



Given

In the quadrilateral ABCD, $\overline{AB} \cong \overline{BC}$

$\overline{NP}$ is right bisector of $\overline{CD}$ and $\overline{NQ}$ is right bisector of $\overline{AD}$.

They meet at N.

To Prove:

BN is a bisector of ∠ABC

Construction:

Join N to A, B, C, D.

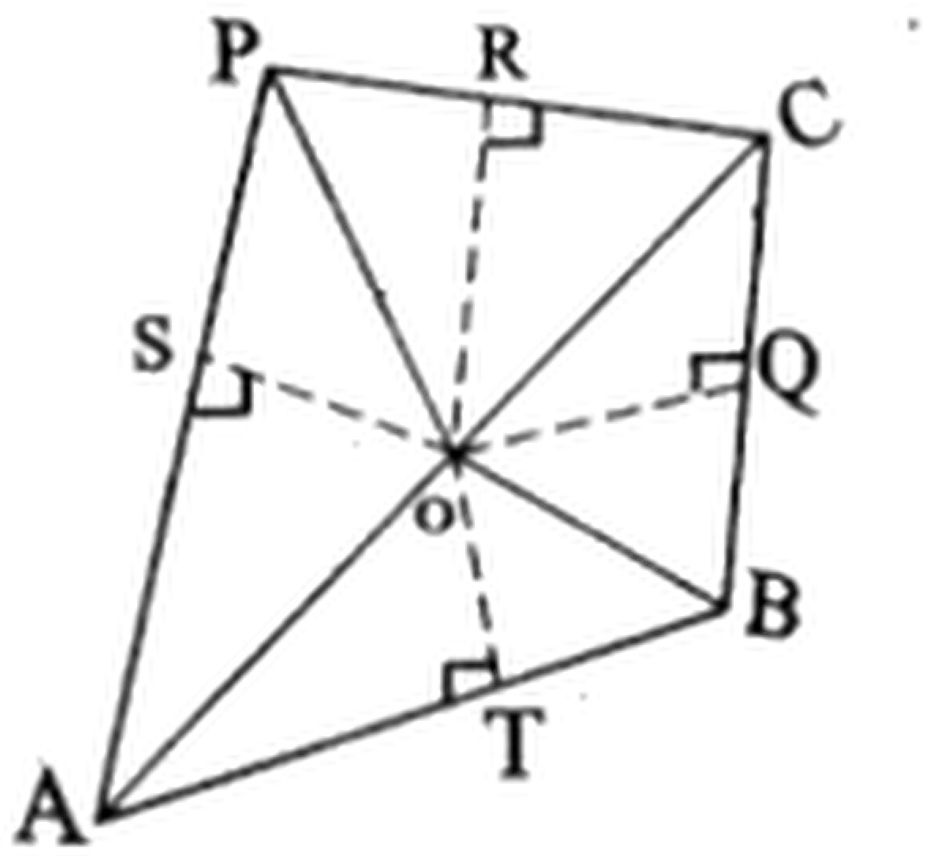
Proof:

Statements	Reasons
$\overline{ND} \cong \overline{NC}$ (i)	N is on right bisector of $\overline{DC}$
$\overline{ND} \cong \overline{NA}$ (ii)	N is on right bisector of $\overline{AC}$
$\overline{NA} \cong \overline{NC}$ (iii)	From (i), (ii)
In ΔBNA ↔ ΔBNC	
$\overline{NA} \cong \overline{NC}$	From (iii)
$\overline{AB} \cong \overline{CD}$	Given
$\overline{BN} \cong \overline{BN}$	Common
ΔBNA ↔ ΔBNC	S.S.S ≅ S.S.S.
Hence ∠1 ∠2	Corresponding angles of congruent

Hence $\overline{BN}$ is bisector of ∠ABC.	triangles.

Q2. The bisectors of ∠A, B and ∠C of a quadrilateral ABCP meet each other at point O. prove that the bisector of ∠P will also pass through the point O.

Solution:



Given:

ABCP is a quadrilateral.
$\overline{AO}$, $\overline{BO}$, $\overline{CO}$ are bisector of ∠A, ∠B, ∠C, respectively.
P is joined to O.

To prove:

$\overline{PO}$ is bisector of ∠P

Construction:

From O draw
$\overline{QT} \perp \overline{AB}$ $\overline{OQ} \perp \overline{BC}$, $\overline{OR} \perp \overline{PC}$ and $\overline{OS} \perp \overline{AP}$ respectively.

Proof:

Statements	Reasons
$\overline{OS} \cong \overline{OT}$ (i)	$\overline{AO}$, is bisector of ∠A
$\overline{OT} \cong \overline{OQ}$ (ii)	$\overline{BO}$ is bisector of ∠D
$\overline{OQ} \cong \overline{OR}$ (iii)	$\overline{CO}$ is bisector of ∠C
$\overline{OS} \cong \overline{OR}$	From (I), (ii), (iii)
∴ O is on bisector of ∠P	
Hence $\overline{PO}$ is bisector of ∠P, or Bisector of ∠P also passes through Q.	