EXERCISE 11.3

Q1. Prove that the line-segments joining the midpoints of the opposite sides of a quadrilateral bisect each other.

Solution:

Given:

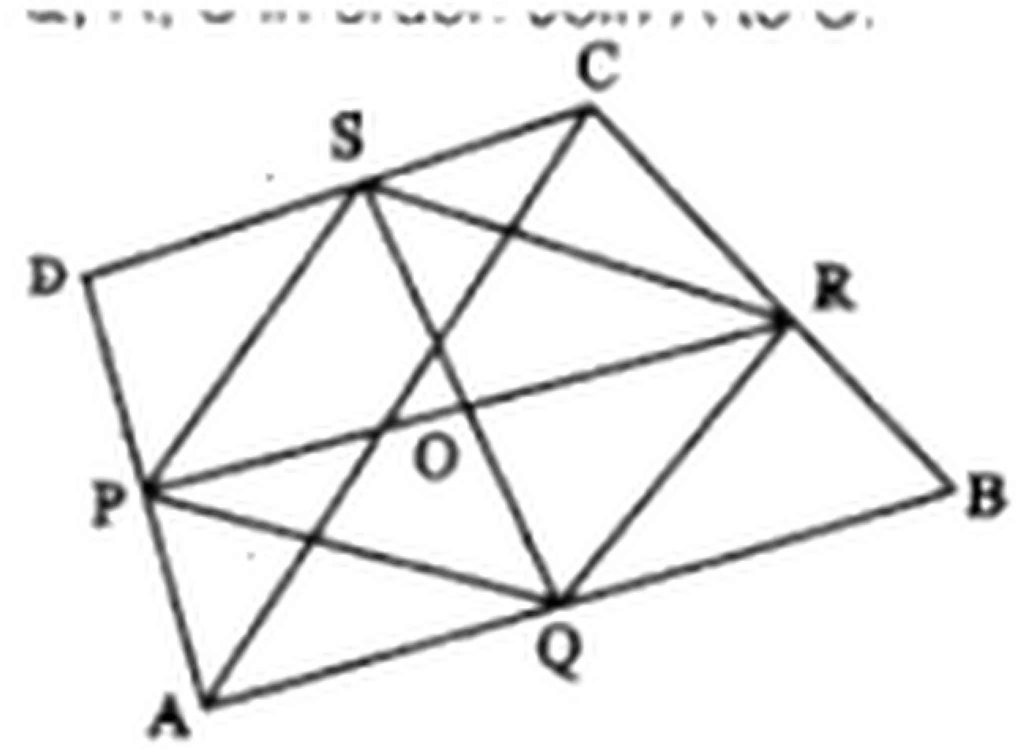
In quadrilateral ABCD, P, Q, R, S are the mid-points of the sides PR and QS are joined, they meet at O.

To prove:

$\overline{OP} \cong \overline{OR}$, $\overline{OS} \cong \overline{OQ}$

Construction:

Join P, Q, R, S in order. Join A to C.



Proof

Statements		Reasons
$\overline{SD} \cong \overline{AC}$	(i)	In ΔADC, S, P, are mid points of AD, DC.
$m\overline{SP} = \frac{1}{2} m\overline{AC}$	(ii)	In ΔABC, P, Q are mid points of AB, BC.
$\overline{RQ} \parallel \overline{AC}$	(iii)	

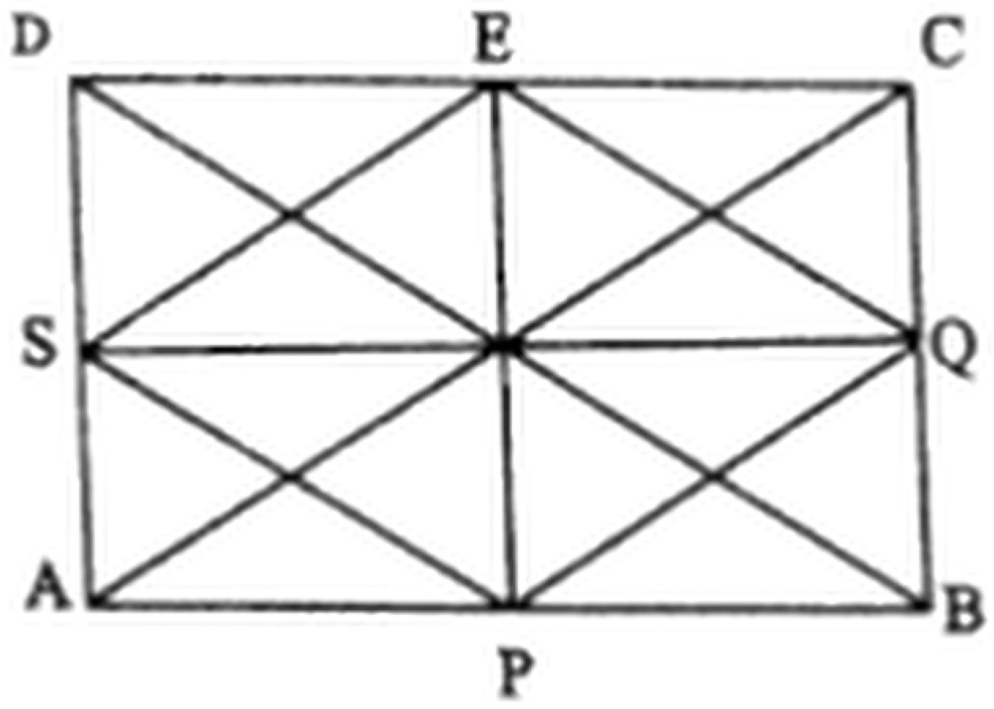
$m\overline{RQ} = \frac{1}{2} m\overline{AC}$ (iv)	
$\therefore\ \overline{SP} \parallel \overline{RQ}$ (v)	From (ii) and (iv)
and $M\overline{RQ} = \frac{1}{2} m\overline{AC}$ (vi)	
$\therefore$ PQRS is a parallelogram	From (v) and (vi)
Now $\overline{PR}$ and $\overline{SQ}$ diagonals of parallelogram PQRS intersect at O	
$\therefore\ \overline{OP} \cong \overline{OR}$	
And $\overline{OS} \cong \overline{OQ}$	Diagonals of a parallelogram bisect each other

Q2. Prove that the fine-segments joining the midpoints of the opposite sides of a rectangle are the right-bisectors of each other.

Solution:

Given:

In rectangle ABCD, P, Q, R, S are mid-point of the sides P is joined to R, Q is joined to S. $\overline{PR}$ and $\overline{QS}$ intersect at O.



To prove:

$\overline{PR}$ and $\overline{QS}$are right bisectors of each other.

Construction:

Join P, Q1 R, S in order. Join A to C and B to D.

Proof

Statements	Reasons
$\overline{SR} \cong \overline{AC}$ (i)	In ΔADC, S, R, are mid points of AD, DC.
$m\overline{SR} = \frac{1}{2} m\overline{AC}$ (ii)	In ΔABC, P, Q are mid points of AB, BC.
And $\overline{PQ} \parallel \overline{AC}$ (iii)	
$m\overline{PQ} = \frac{1}{2} m\overline{AC}$ (iv)	From (i) and (ii)
$\therefore \overline{SR} \parallel \overline{PQ}$ (v)	
and M$\overline{SR} = m\overline{PQ}$ (vi)	From (ii) and (iv)
∴ PQRS is a parallelogram	
m$\overline{AC}$ = m$\overline{BD}$	From (v) and (vi)
m$\overline{AC}$ = m$\overline{BD}$	Diagonals of a rectangle
m$\overline{PQ}$ = m$\overline{QR}$	
∴ m$\overline{PQ}$ = m$\overline{QR}$ = m$\overline{RS}$ = m$\overline{SP}$	
∴ PQRS is a rhombus.	
$\overline{PR}$ and $\overline{QS}$are diagonals of rhombus PQRS.	
∴ $\overline{PR}$ and $\overline{QS}$are right bisectors of each other.	Diagonals of a parallelogram bisect each other

Q3. Prove that the line-segment passing through the mid-points of one side and parallel to another side of a triangle also bisect the third side.

Solution:

Given:

In ΔABC, D is mid-point of AB $\overline{DE} \parallel \overline{BC}$

To prove:

$\overline{EA} \parallel \overline{EC}$

Construction:

Take $\overline{EF} \parallel \overline{AB}$

Proof

Statements	Reasons
$\overline{DE} \parallel \overline{BC}$	Given
$\overline{EF} \parallel \overline{BD}$	Construction
∴ DBEF is a parallelogram.	
$\overline{EF} \cong \overline{DB}$	Opposite sides
$\overline{AD} \cong \overline{DB}$	Given
$\overline{EF} \cong \overline{AD}$	
$\angle 1 \cong \angle B$	From (i) and (ii)
and $\angle 2 \cong \angle B$	corresponding angles
∴ $\angle 1 \cong \angle 2$	
In ΔADE ↔ ΔEFC	
$\angle 2 \cong \angle 1$	From (iv)
∴ $\angle 3 \cong \angle C$	Corresponding angle
$\overline{AD} \cong \overline{EF}$	From (iii)

Hence $\Delta ADE \cong \Delta EFC$	A.A.S s A.A.S.
$\therefore \overline{EA} \cong \overline{EC}$	Corresponding sides