

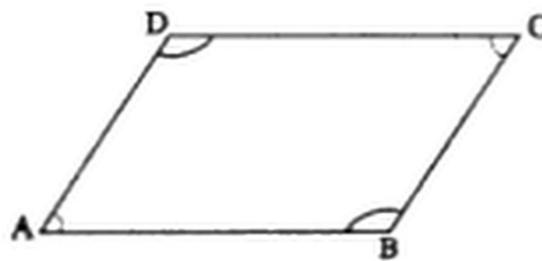
## EXERCISE 11.2

**Q1. Prove that a quadrilateral is a parallelogram if its**

- Opposite angles are congruent.
- Diagonals bisect each other.

**Solution:**

**(a) Opposite angles are congruent.**



**Given:**

In a quadrilateral ABCD

$$m\angle A = m\angle C$$

$$m\angle B = m\angle D$$

**To Prove:**

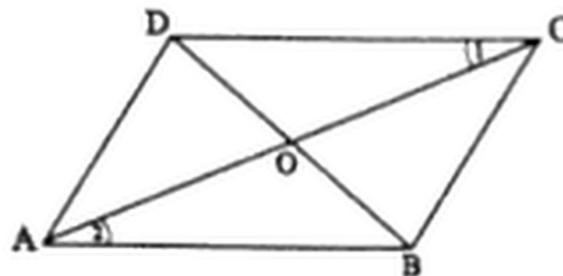
ABCD is a parallelogram

**Proof:**

Statements	Reasons
$m\angle A = m\angle C$ (i)	Given
$m\angle B = m\angle D$ (ii)	Given
$m\angle A + m\angle B + m\angle C + m\angle D = 360^\circ$	Angles of quadrilateral
$m\angle A + m\angle B + m\angle A + m\angle B = 360^\circ$	From (i) and (ii)

$2m\angle A + 2m\angle B = 360^\circ$ $\therefore m\angle A + m\angle B = 180^\circ$ $\therefore \overline{AD} \parallel \overline{BC}$ <p>Similarly, it can be proved that <math>\overline{AB} \parallel \overline{CD}</math></p> <p>Hence ABCD is a parallelogram.</p>	<p>Sum of internal angles</p>
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**(b) Diagonals bisect each other.**



**Solution:**

**Given:**

In quadrilateral ABCD, diagonals  $\overline{AC}$  and  $\overline{BD}$  bisect each other, i.e.  $\overline{OA} \cong \overline{OC}$ ,  $\overline{OB} \cong \overline{OD}$

**To prove:**

ABCD is a parallelogram

**Proof:**

Statements	Reasons
<p>In <math>\triangle ABO \leftrightarrow \triangle CDO</math></p> <p><math>\overline{OA} \cong \overline{OC}</math>,</p> <p><math>\overline{OB} \cong \overline{OD}</math></p>	<p>Given</p> <p>Given</p>

<p><math>\therefore \angle AOB \cong \angle COD</math></p> <p><math>\therefore \angle AOB \cong \angle COD</math></p> <p><math>\angle 1 \cong \angle 2</math></p> <p>Hence <math>\overline{AB} \parallel \overline{DC}</math> (i)</p> <p>By taking <math>\Delta</math>s AOD and BOC we can prove that</p> <p><math>\overline{AD} \parallel \overline{BC}</math> (ii)</p> <p>Hence ABCD is a parallelogram.</p>	<p>Vertical opposite angles</p> <p>S.A.S <math>\cong</math> S.A.S.</p> <p>Corresponding angles of congruent triangles</p> <p><math>\angle 1 = \angle 2</math></p> <p>From (i) and (ii)</p>
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**Q2. Prove that a quadrilateral is a parallelogram if its opposite sides are congruent.**

**Solution:**



**Given:**

In quadrilateral ABCD

$\overline{AB} \cong \overline{DC}$ , and  $\overline{AD} \cong \overline{BC}$

**To prove:**

ABCD is a parallelogram

**Construction:**

Join B to D

**Proof:**

Statements	Reasons
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<p>In <math>\triangle ABD \leftrightarrow \triangle CBD</math></p> <p><math>\overline{AD} \cong \overline{CB}</math>,</p> <p><math>\overline{AB} \cong \overline{CD}</math></p> <p><math>\overline{BD} \cong \overline{BD}</math></p> <p><math>\therefore \angle ABD \cong \angle CDB</math></p> <p><math>\angle 1 \cong \angle 2</math> (i)</p> <p>And <math>\angle 4 \cong \angle 3</math> (ii)</p> <p>Hence <math>\overline{AB} \parallel \overline{DC}</math></p> <p>And <math>\overline{BC} \parallel \overline{AD}</math></p> <p>Hence ABCD is a parallelogram.</p>	<p>Given</p> <p>Given</p> <p>Common</p> <p>S.A.S <math>\cong</math> S.A.S.</p> <p>Corresponding angles of congruent triangles</p> <p>(i) alternate angles</p> <p>(ii) alternate angles</p>
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