Review Exercise

Q1. Select the correct answer in each of the following.

(i) **The order of matrix** $\begin{bmatrix} 2 & 1 \end{bmatrix}$ **is ....**

(a) 2-by-1 **(b)** 1-by-2

(c) 1-by-1 **(d)** 2-by-2

(ii) $\begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$ **is called .... matrix.**

(a) zero **(b)** unit

(b) scalar **(d)** singular

(iii) **Which is order of square matrix............**

(a) 2-by-1 **(b)** 1-by-2

(c) 2-by-1 **(d)** 3-by-2

(iv) **Order of transpose of** $\begin{bmatrix} 2 & 1 \\ 0 & 1 \\ 3 & 2 \end{bmatrix}$ ....

(a) 3-by-2 **(b)** 2-by-3

(c) 1-by-3 **(d)** 3-by-1

(v) **Ad joint of** $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$ **is ....**

(c) $\begin{bmatrix} -1 & 2 \\ 0 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$

(vi) **Product of** $\begin{bmatrix} x & y \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ **is............**

(a) $[2x + y]$ (b) $[x - 2y]$

c) $[2x - y]$ (d) $[x + 2y]$

(vii) **If** $\begin{vmatrix} 2 & 6 \\ 3 & x \end{vmatrix}$ **= 0, then x is equal to…a = …………**

(a) 9 (b) - 6

(c) 6 (d) - 9

(viii) **If** $X + \begin{bmatrix} -1 & -2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$**, then X is equal to…………**

(a) $\begin{bmatrix} 2 & 2 \\ 2 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 2 \\ 2 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$

Answers

(i) b	(ii) c	(iii)a	(iv) b
(v) a	(vi) c	(vii) a	(viii) d

Q2. Complete the following:

(i) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is called ………… matrix.

(ii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is called ………… matrix.

(iii) Additive inverse of $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$ is ....

(iv) In matrix multiplication, in general, AB .... BA.

(v) Matrix A + B may be found if order of A and B is ....

(vi) A matrix is called .... matrix if number of rows and columns are equal.

Answers:

(i) Null **(ii) Unit** **(iii)** $\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$

(iv) $\neq$ **(v) Same** **(vi) Square**

Q3. **If** $\begin{bmatrix} a+3 & 4 \\ 6 & b-1 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$**, then find a and b.**

Solution:

By comparing the corresponding elements, we get

$a + 3 = -3$

$a = -3 - 3 = -6$

$\boldsymbol{a = -6}$ Answer

and $b - 1 = 2$

$b = 2 + 1$

$\boldsymbol{b = 3}$ Answer

Q4. **If A =** $\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$**, B =** $\begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}$**, then find the following.**

(i) **2A + 3B** **(ii)** **-3A+ 2B**

Solution:

(i) 2A + 3B

$= 2 \times \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + 3 \times \begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}$

$= \begin{bmatrix} 2 \times 2 & 2 \times 3 \\ 2 \times 1 & 2 \times 0 \end{bmatrix} + \begin{bmatrix} 3 \times 5 & 3 \times (-4) \\ 3 \times (-2) & 3 \times (-1) \end{bmatrix}$

$= \begin{bmatrix} 4 & 6 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 15 & -12 \\ -6 & -3 \end{bmatrix}$

$= \begin{bmatrix} 4 + 15 & 6 - 12 \\ 2 - 6 & 0 - 3 \end{bmatrix}$

2A + 3B $= \begin{bmatrix} \mathbf{19} & \mathbf{-6} \\ \mathbf{-4} & \mathbf{-3} \end{bmatrix}$

(ii) -3A + 2B

$= -3 \times \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + 2 \times \begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}$

$= \begin{bmatrix} -3 \times 2 & -3 \times 3 \\ -3 \times 1 & -3 \times 0 \end{bmatrix} + \begin{bmatrix} 2 \times 5 & 2 \times (-4) \\ 2 \times (-2) & 2 \times (-1) \end{bmatrix}$

$= \begin{bmatrix} -6 & -9 \\ -3 & 0 \end{bmatrix} + \begin{bmatrix} 10 & -8 \\ -4 & -2 \end{bmatrix}$

$= \begin{bmatrix} (-6) + 10 & (-9) + (-8) \\ -3 + (-4) & 0 + (-2 \end{bmatrix}$

- 3A + 2B $= \begin{bmatrix} \mathbf{4} & \mathbf{-17} \\ \mathbf{-7} & \mathbf{-2} \end{bmatrix}$

(iii) - 3(A + 2B)

$= -3 \left(\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + 2 \times \begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix} \right)$

$= -3 \left(\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 2 \times 5 & 2 \times (-4) \\ 2 \times (-2) & 2 \times (-1) \end{bmatrix} \right)$

$= -3 \left(\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 10 & -8 \\ -4 & -2 \end{bmatrix} \right)$

$= \quad -3 \begin{bmatrix} 12 & -5 \\ -3 & -2 \end{bmatrix}$

$= \quad -\begin{bmatrix} 3 \times 12 & 3 \times (-5) \\ 3 \times (-3) & 3 \times (-2) \end{bmatrix}$

$= \quad -\begin{bmatrix} 36 & -15 \\ -9 & -6 \end{bmatrix}$

-3(A + 2B) $= \quad \begin{bmatrix} -36 & 15 \\ 9 & 6 \end{bmatrix}$

(iv) $\frac{2}{3}$**(2A – 3B)**

$= \quad \frac{2}{3}\left(2 \times \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} - 3 \times \begin{bmatrix} 5 & -4 \\ -2 & -1 \end{bmatrix}\right)$

$= \quad \frac{2}{3}\left(\begin{bmatrix} 2 \times 2 & 2 \times 3 \\ 2 \times 1 & 2 \times 0 \end{bmatrix} - \begin{bmatrix} 3 \times 5 & 3 \times (-4) \\ 3 \times (-2) & 3 \times (-1) \end{bmatrix}\right)$

$= \quad \frac{2}{3}\left(\begin{bmatrix} 4 & 6 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 15 & -12 \\ -6 & -3 \end{bmatrix}\right)$

$= \quad \begin{bmatrix} 4 - 15 & 6 + 2 \\ 2 + 6 & 0 + 3 \end{bmatrix}$

$= \quad \begin{bmatrix} -11 & 185 \\ 8 & 3 \end{bmatrix}$

$\frac{2}{3}$**(2A – 3B)** $= \quad \begin{bmatrix} -\frac{22}{3} & 12 \\ \frac{16}{3} & 2 \end{bmatrix}$

Q5. Find the value of X, if $\begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix}$ **+ X =** $\begin{bmatrix} 4 & -2 \\ -1 & -2 \end{bmatrix}$.

Solution:

$\begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix} + X = \begin{bmatrix} 4 & -2 \\ -1 & -2 \end{bmatrix}$

$X \quad = \quad \begin{bmatrix} 4 & -2 \\ -1 & -2 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 3 & -3 \end{bmatrix}$

$$= \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$$

Q6. **If A = $\begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}$, B = $\begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}$, then prove that**

(i) AB ≠ BA **(ii) A (BC) = (AB)C**

Solution:

(i) AB $= \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}$

$$= \begin{bmatrix} 0 \times (-3) + 1 \times 5 & 0 \times 4 + 1 \times (-2) \\ 2 \times (-3) + (-3) \times 5 & 2 \times 4 + (-3) \times (-2) \end{bmatrix}$$

$$= \begin{bmatrix} 0 + 5 & 0 - 2 \\ -6 - 15 & 8 + 6 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 2 \\ -21 & 14 \end{bmatrix} \quad \text{---------- (i)}$$

BA $= \begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}$

$$= \begin{bmatrix} (-3) \times 0 + 4 \times 2 & (-3) \times 1 + 4 \times (-3) \\ 5 \times 0 + (-2) \times 2 & 5 \times 1 + (-2) \times (-3) \end{bmatrix}$$

$$= \begin{bmatrix} 0 + 8 & -3 - 12 \\ 0 - 4 & 5 + 6 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & -15 \\ -4 & 11 \end{bmatrix} \quad \text{---------- (ii)}$$

From (i) and (ii), it is clear that AB ≠ BA

(ii) A (BC) = (AB)C

Solution is not possible because matrix C is not given.

Q7. If A = $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$, B= $\begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}$, then verify that

(i) $(AB)^t = B^t A^t$ **(ii) $(AB)^{-1} = B^{-1}A^{-1}$**

Solution:

(i) $(AB)^t = B^t A^t$

$$A^t = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}^t = \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}$$

$$B^t = \begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}^t = \begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \times 2 + 2 \times (-3) & 3 \times 4 + 2 \times (-5) \\ 1 \times 2 + (-1) \times (-3) & 1 \times 4 + (-1) \times (-5) \end{bmatrix}$$

$$= \begin{bmatrix} 6-6 & 12-10 \\ 2+3 & 4+5 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 5 & 9 \end{bmatrix}$$

$$(AB)^t = \begin{bmatrix} 0 & 2 \\ 5 & 9 \end{bmatrix}^t$$

$$= \begin{bmatrix} 0 & 5 \\ 2 & 9 \end{bmatrix} \quad \text{--------- (i)}$$

$$B^t A^t = \begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \times 3 + (-3) \times 2 & 2 \times 1 + (-3) \times (-1) \\ 4 \times 3 + (-5) \times 2 & 4 \times 1 + (-5) \times (-1) \end{bmatrix}$$

$$= \begin{bmatrix} 6-6 & 2+3 \\ 12-10 & 4+5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 5 \\ 2 & 9 \end{bmatrix} \quad \text{--------- (ii)}$$

From (i) and (ii), it is clear that $(AB)^t = B^t A^t$

(ii) $(AB)^{-1} = B^{-1}A^{-1}$

$A = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$|A| = 3 \times (-1) - 1 \times 2 = -3 - 2 = -5 \neq 0$

$A^{-1} = \frac{Adj\ A}{|A|}$

$= \frac{\begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}}{-5} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{1}{5} & -\frac{3}{5} \end{bmatrix}$

$B = \begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}$

$|B| = 2 \times (-5) - 4 \times (-3) = -10 + 12 = 2 \neq 0$

$B^{-1} = \frac{Adj\ B}{|B|}$

$= \frac{1}{2}\begin{bmatrix} -5 & -4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} -\frac{5}{2} & -2 \\ \frac{3}{2} & 1 \end{bmatrix}$

$AB = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}$

$= \begin{bmatrix} 3 \times 2 + 2 \times (-3) & 3 \times 4 + 2 \times (-5) \\ 1 \times 2 + (-1) \times (-3) & 1 \times 4 + (-1) \times (-5) \end{bmatrix}$

$= \begin{bmatrix} 6 - 6 & 12 - 10 \\ 2 + 3 & 4 + 5 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 5 & 9 \end{bmatrix}$

$|AB| = 0 \times 9 - 2 \times 5 = 0 - 10 = -10 \neq 0$

$(AB)^{-1} = \frac{Adj\ AB}{|AB|}$

$$= \begin{bmatrix} \frac{-9}{10} & \frac{1}{5} \\ \frac{1}{2} & 0 \end{bmatrix} \quad \text{--------------- (i)}$$

Now by solving $B^{-1}A^{-1}$

$$= \begin{bmatrix} -\frac{5}{2} & -2 \\ \frac{3}{2} & 1 \end{bmatrix} \times \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{1}{5} & -\frac{3}{5} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-5}{2} \times \frac{1}{5} + (-2) \times \frac{1}{5} & \frac{-5}{2} \times \frac{2}{5} + (-2) \times \frac{-3}{5} \\ \frac{3}{2} \times \frac{1}{5} + 1 \times \frac{1}{5} & \frac{3}{2} \times \frac{2}{5} + 1 \times \frac{-3}{5} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-1}{2} - \frac{2}{5} & -1 + \frac{6}{5} \\ \frac{3}{2} + \frac{1}{5} & \frac{3}{5} - \frac{3}{5} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-5-4}{10} & \frac{-5+6}{5} \\ \frac{3+2}{10} & \frac{3-3}{5} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-9}{10} & \frac{1}{5} \\ \frac{1}{2} & 0 \end{bmatrix} \quad \text{--------------- (ii)}$$

From (i) and (ii), it is clear that $(AB)^{-1} = B^{-1}A^{-1}$