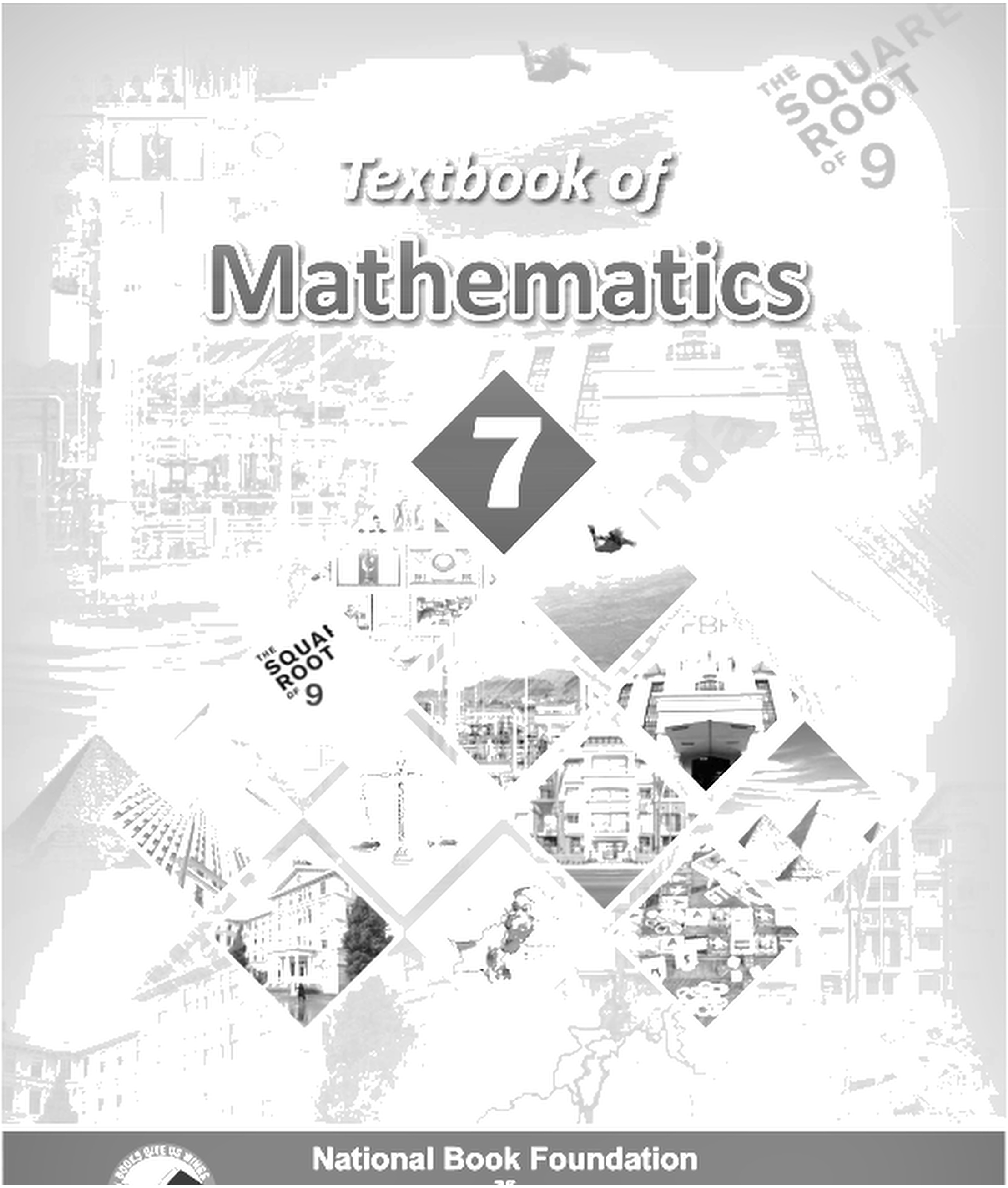

Textbook of Mathematics

7

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Textbook of

Mathematics

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Textbook of
Mathematics Grade - 7

Review Committee Members
Dr. Muhammad Irfan Ali
Mrs. Saeeda Parveen
Mr. Naveed Akmal
Mr. Shahzad Ahmed

Authors : Dr. Saleem Ullah
Dr. Khalid Mehmood
: Ms. Tayyaba Saqib

Designing/Composing : Muhammad Farooq, Shahzad Ahmed, Muhammad Zulfiqar

Desk Officer, NCC : Ms. Zehra Habib, National Curriculum Council
Management : Ishtiaq Ahmad Malik, Secretary NBF
Incharge Textbook Section : Muhammad Rafique, Assistant Director, NBF

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TEST EDITION

Preface

The Mathematics Grade 7 textbook is developed to cover the learning outcomes of the revised Mathematics curriculum 2017 comprehensively.

The Mathematics Grade 7 textbook builds a strong foundation of the subject through the use of well-researched and sound pedagogical principles. Adopting the popular Concrete- Pictorial-Abstract approach widely used in revised Mathematics Curriculum 2017-18, learners are introduced to new concepts through concrete manipulatives and engaging pictorials before they are led to see their abstract symbolic representations. This allows learners to have a deeper understating of key mathematical concepts, thus motivating them to learn.

Content is also clearly structured and spiralled across the levels to ensure a gradual build-up and review of skills as learner's progress up the grades. At the same time, emphasis is given on developing learners' problem-solving skills, critical thinking, as well as other 21st century skills.

The **Mathematics Grade 7** textbook also develops pupils' confidence for examinations through Check points, Exercises and Review Exercises for every concept. Every Chapter begins with a chapter opener which motivates learner's interest. Interesting real life situations and applications are used to develop learner's interest and curiosity so that they can appreciate the beauty of Mathematics in their environments.

The National Book Foundation endeavours to keep quality enhancement at the heart of textbooks development. Likewise, it strives to keep improving its textbooks by incorporation of feedback and suggestions from the students, teachers and the community Preface in subsequent editions of its new textbooks. As always the National Book Foundation looks forward to receiving feedback on this new textbook for Mathematics grade VII to align it more closely to the needs of the learners.

National Book Foundation

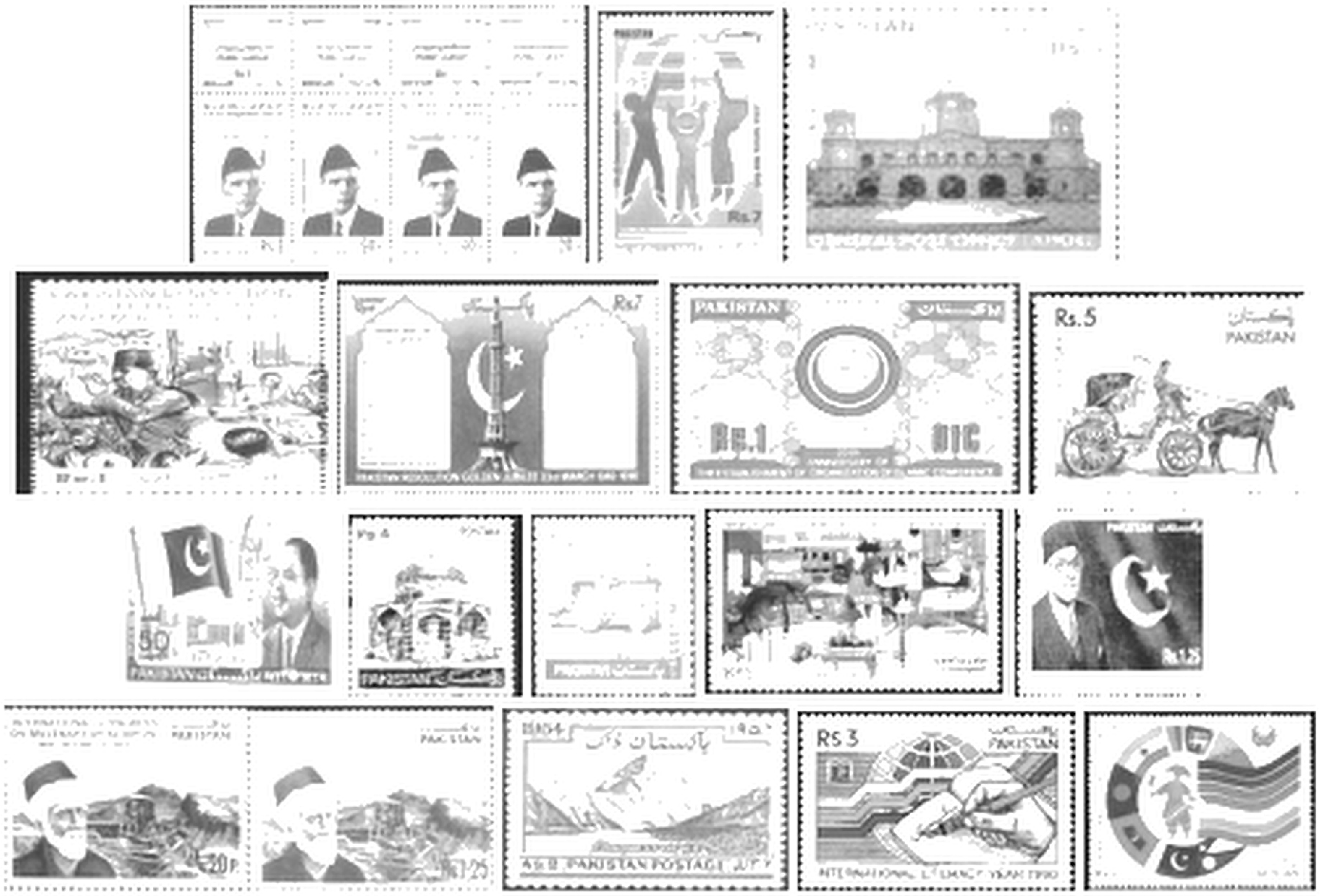
بِسۡمِ اللّٰہِ الرَّحۡمٰنِ الرَّحِیۡمِ

اللہ کے نام سے شروع جو بڑا مہربان نہایت رحم والا ہے

Contents

01

Sets



Postage stamps of different eras of Pakistan

There are various ways stamp collectors organize their stamps. A stamp collection can be organized into different themes.

A stamp collection can be considered as a set, whereas the different themes can be considered the "elements" that makes up the set.

Learning Outcomes

Students will be able to:

Use language, notation and Venn diagrams to describe:

- Subsets and supersets,
- Proper and improper subsets,
- Equal and equivalent sets,
- Disjoint and overlapping sets
- Union of two sets,
- Intersection of two sets,
- Difference of two sets,
- Complement of a set.

UNIT 1

George Cantor (1845-1918) renowned German Mathematician is regarded as the founder of "The Set Theory".

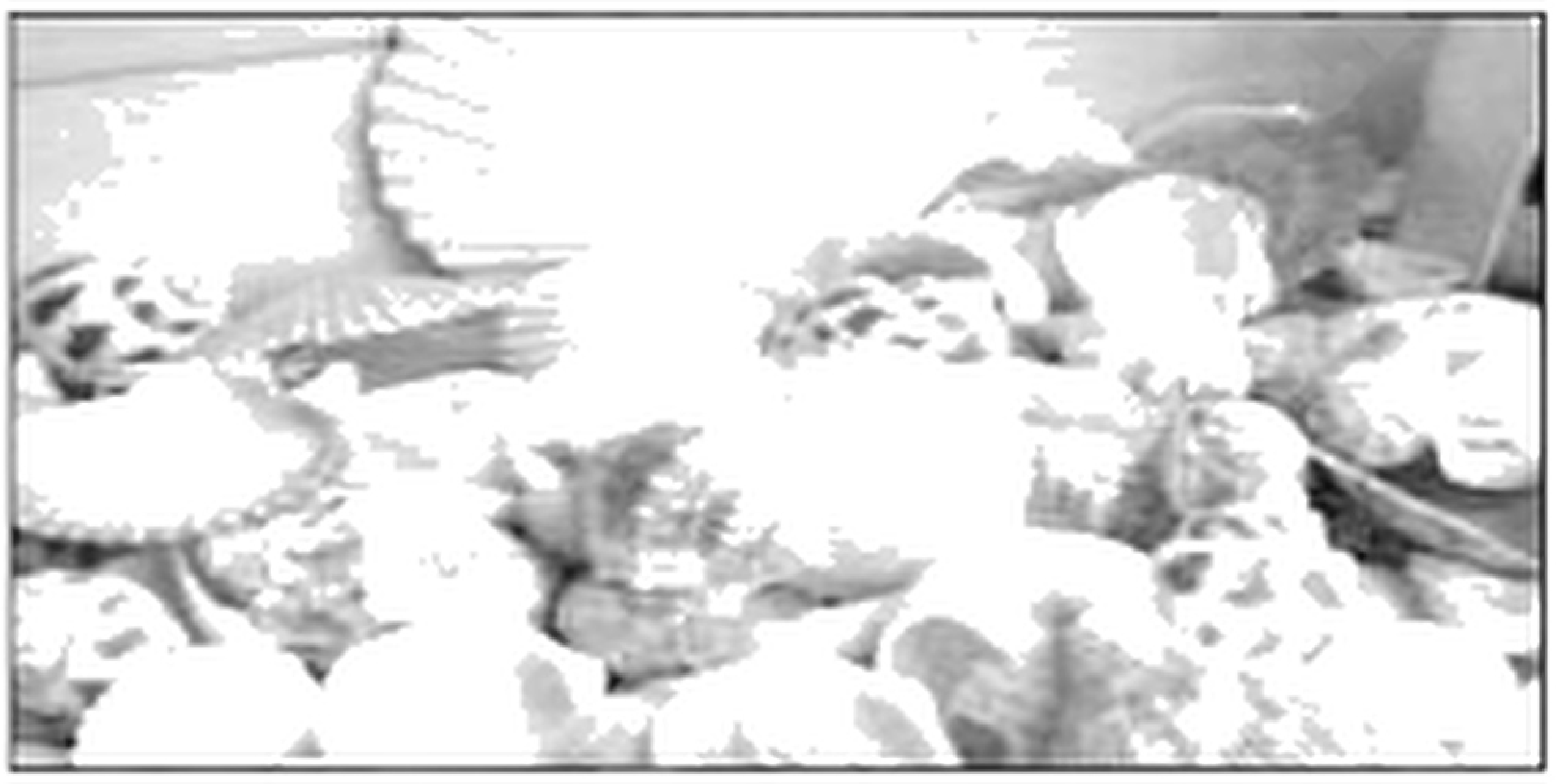
Today the set theory is the foundations of many advanced mathematical works.

Set and Its Notation

A collection of well-defined and distinct objects is called a set. The individual objects in a set are called the members or elements of the set.

For Example:

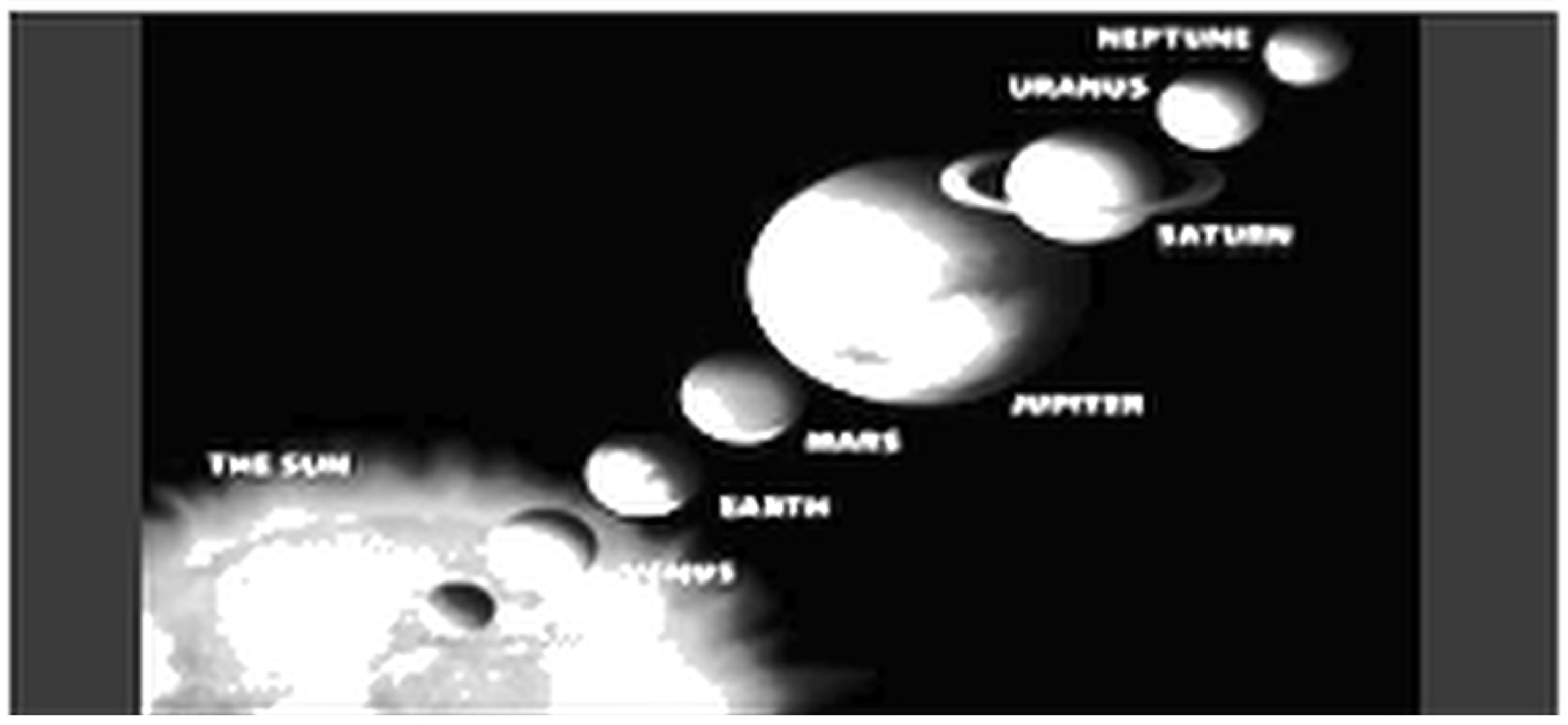
S = Set of sea shells



For Azhar's family photograph below, each person is a member of Azhar's family. The whole collection of seven people can be called the set of Azhar's Family.

A = Set of Azhar's Family

P = Set of Planets



Set Notation to List the Elements of a Set

Collection of items can be named using capital letters such as A, B or C. Consider T contains whole numbers less than 10. The element of this set will be:
T = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

T = Set of whole numbers less than 10.

Consider A = {2, 3, 5, 7, 11}

A is the set of prime numbers less than 13. The set can also be described or defined in terms of some general property of all its elements.

So, A = {All prime numbers less than 13}
or
= {First five prime numbers}

Check Point

List all the elements in each of the following sets.
The set of vowels in English alphabet.
S = {Positive odd numbers less than 10}
T = {Letters in the word "WOOD"}

Example:

For each of the following sets, list all the elements of the set.
(a) S is the set of square numbers greater than 0 but less than 10.
(b) P is the set of polygons with less than nine sides.

(a) The square of each number from 1 to 10 is less than 101. So,
S = {1, 4, 9, 16, 25, 36, 49, 64, 81, 100}
(b) Polygons are closed plane figures with three or more sides. So
P = {triangle, quadrilateral, pentagon, hexagon, heptagon, octagon}

Check Point

Describe the following sets in words.
(a) A = {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}
(b) B = {1, 2, 3, 4, 5}

UNIT 1

Using the symbol ∈ and ∉

Consider set A = {2, 3, 5, 7}.

The number 2 belongs to set A, that is, $2 \in A$.

As the number 4 does not belong to set A, therefore $4 \notin A$.

The symbol ∈ indicates that an element belongs to a particular set.

To indicate that an element does not belong to the set, we use the symbol ∉.

Example:

If D – {1, 2, 3, ……, 9, 10}.

Which of the following statements is true? State your reasons.

(a) $6 \in D$ (b) $-4 \in D$ (c) $13 \notin D$

(a) True, because the set D contains the element 6.

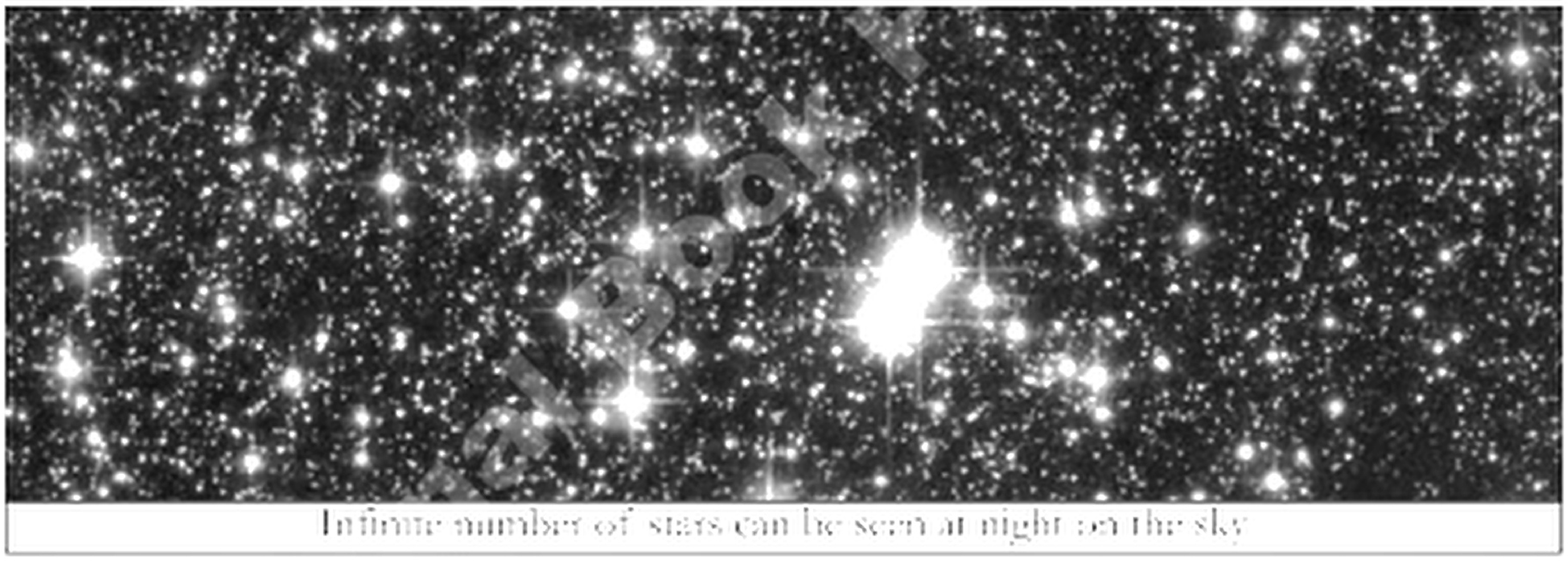
(b) False, because the set D contains only positive integers.

(c) True , because the set D does not contain 13.

Check Point

If E = {Multiples of 3}
Which of the following statement is true?
(a) $8 \in E$ (b) $2 \notin E$

Finite Set and Infinite Set



Infinite number of stars can be seen at night on the sky

A finite set has a limited number of elements. For Example:

If E represents the set of alphabet in the English language, i.e. E = {letters in the English Alphabet}, then number of elements in E are 26, we can denote the number of elements in the set as n(E) = 26.

The set of vowels V = {a, e, i, o, u}.

The set has exactly 5 elements so it is a finite set and we can denote it as n(V) = 5.

An infinite set has infinite number of elements.

For example, if W represents the set of whole numbers, then W = {0 , 1 , 2 ,……}.

Check Point

State whether the following sets are finite or infinite.
(a) The set of vowels.
(b) The set of odd numbers less than 30.
(c) The set of water drops in a river.
(d) The set of prime numbers.
(e) Set of odd numbers divisible by 2.

Empty or Null Set

Consider A is the set of vowels in the word "RHYTHM".
Then A = { }, n (A) = 0 as it contains no element.
Thus a set having no element is called an empty or a null set and is donated by the symbol ϕ.

Key Fact

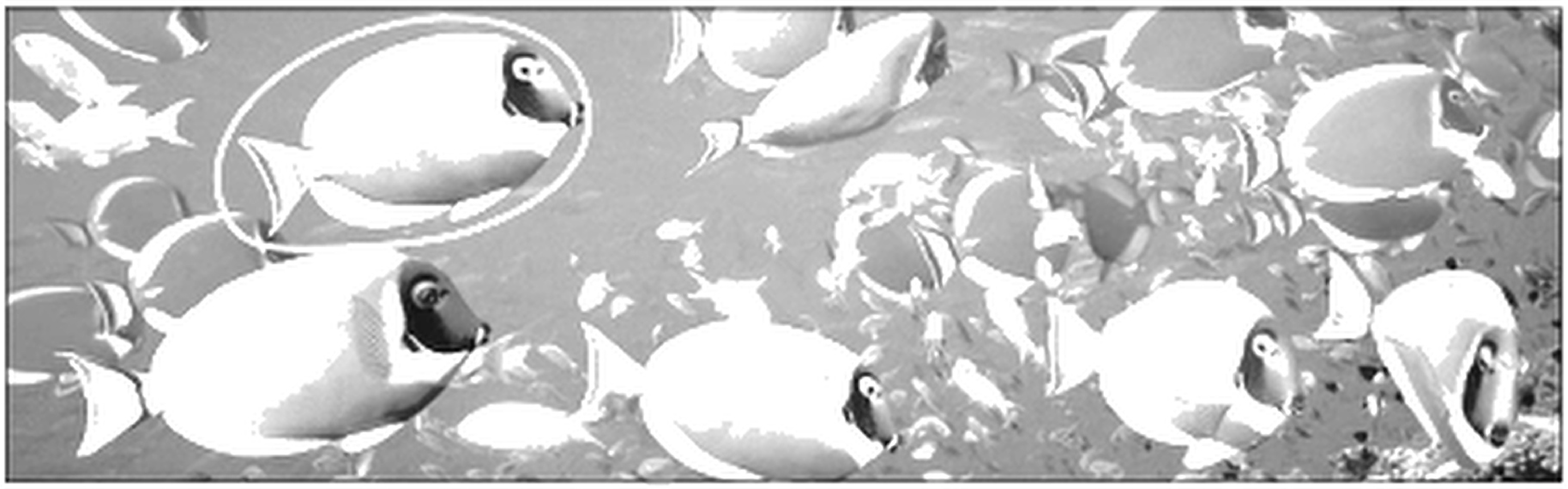
The notation ϕ was first used by Andrew Weil who took it from the Norwegan alphabets.

Key Fact

The set $\{\phi\}$ is not an empty set as it contains an element ϕ.

Subset and Superset

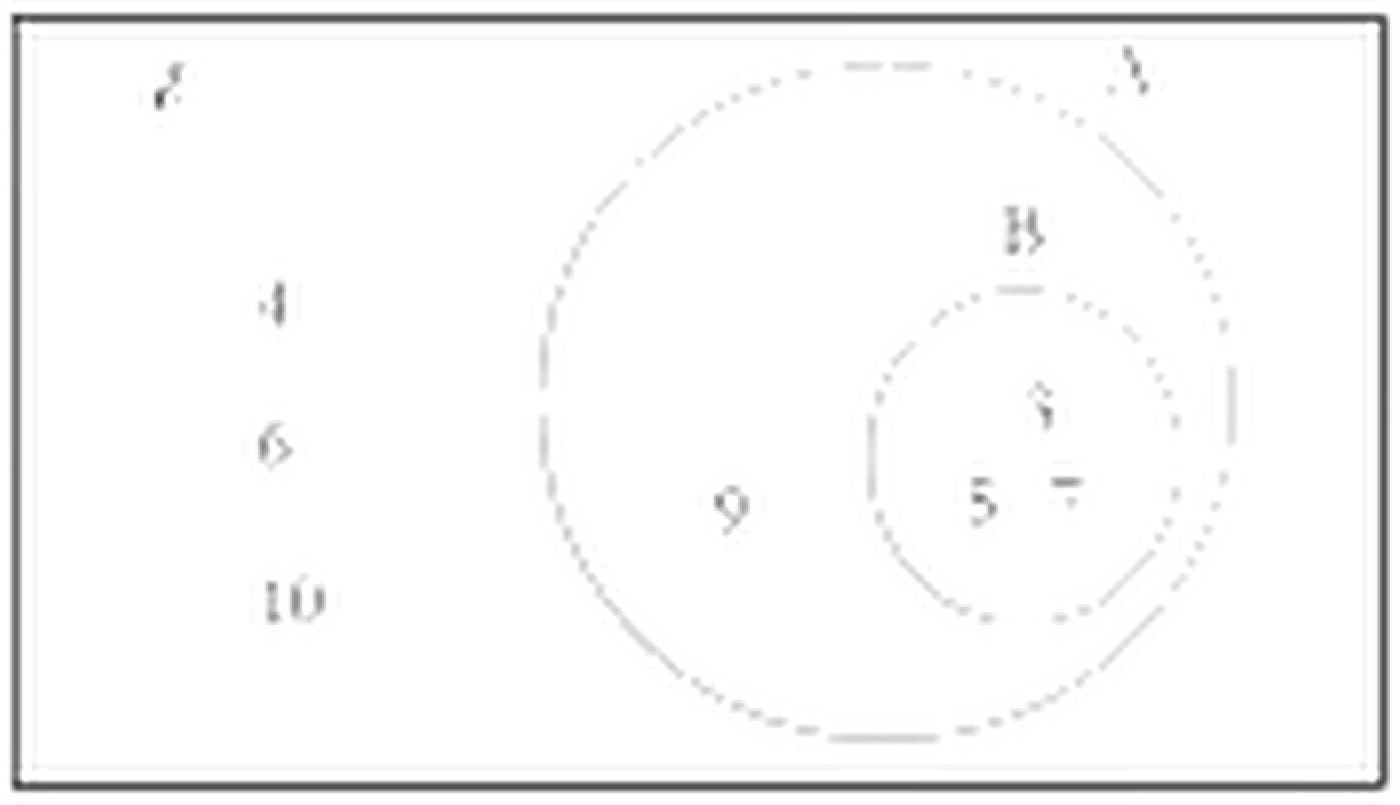
A subset is simply a set that is a part of a larger set, and thus contains the elements from the larger set.



Different types of fish are each a subset of the set of all fish. For example the set of 'salt water fish' is a subset of the set of 'all fish'.

Example:

Consider
ξ = {integers from 3 to 10},
A = {odd numbers from 3 to 10} and
B = {prime numbers from 3 to 10}.
These sets can be listed as:
$\xi = \{3, 4, 5, 6, 7, 8, 9, 10\}$
$A = \{3, 5, 7, 9\}$
$B = \{3, 5, 7\}$



B is a subset of A.

Notice that each element of B is also an element of A, therefore

- B is subset of A, denoted by $B \subseteq A$ or
- A is super set of A, denoted by $A \supseteq B$.

Both A and B are subset of ξ as well and ξ is a super set of both sets A and B.

For any two sets A and B, we say that B is a subset of A if every element of the set B is an element of the set A. We write $B \subseteq A$ and read it as 'B is subset of A'.
Every set has at least two subsets, the set itself and the null set.

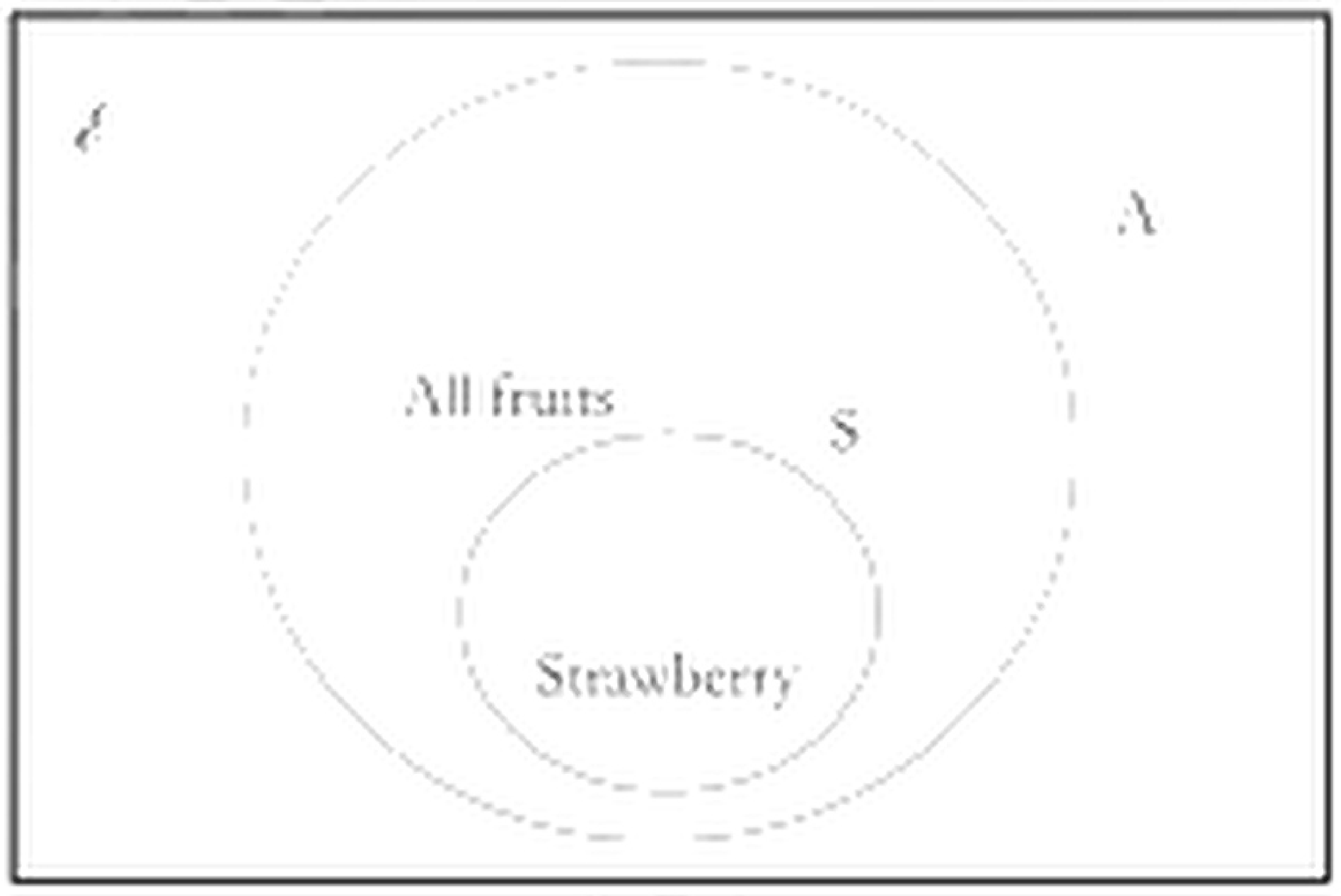
Key fact

A Venn diagram representes mathematical sets pictorially as circles within an enclosed rectangle (the universal set), common elements of the sets being represented by intersections of the circles.

Proper and Improper Subsets

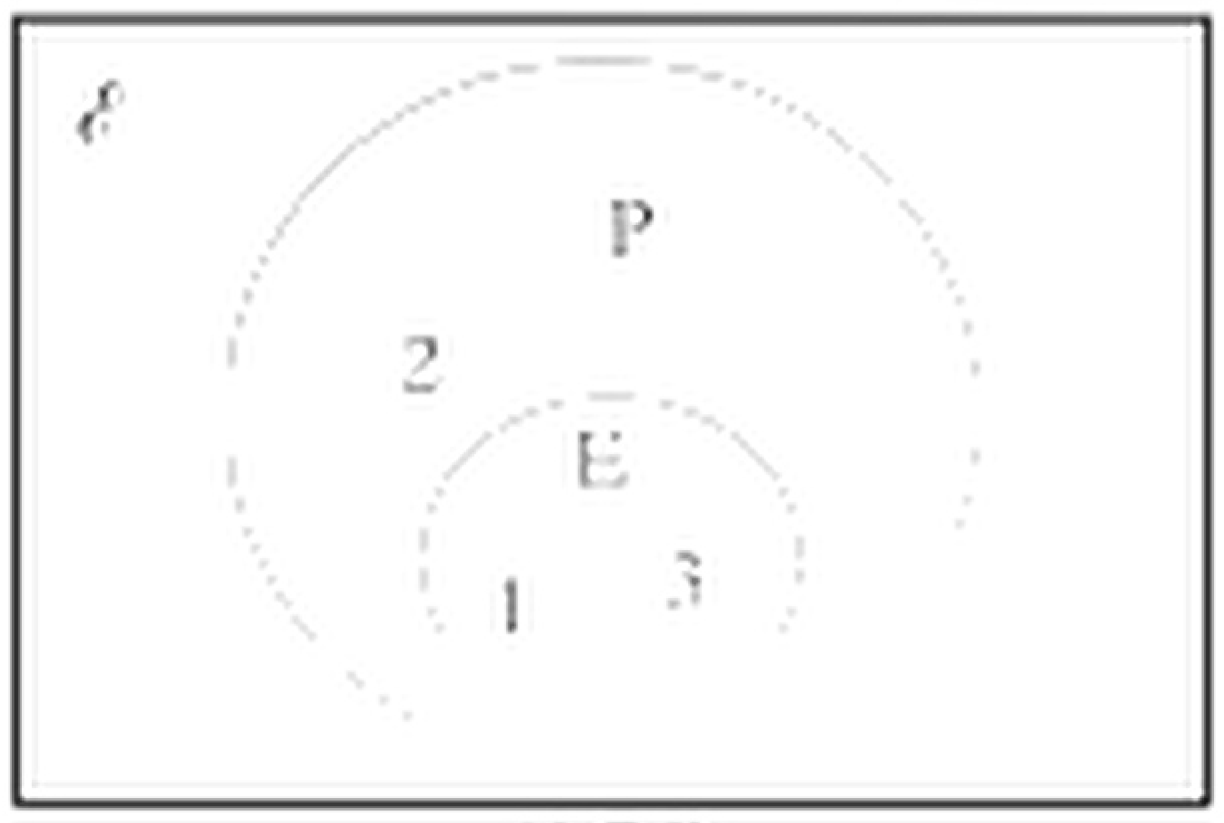
If every element of B is an element of A and there exists at least one element of A that is not in B then B is called proper subset of A denoted by $B \subset A$ and read as 'B is proper subset of A'.

The symbol "$\subseteq$" expresses the idea "includes" or contains and $\subset$ shows proper inclusion.



$S \subseteq A$

Consider the set $P = \{1, 2, 3\}$, then the set $E = \{1, 3\}$ is a subset of P since each number belong to E also belongs to P.
In a set notation, we write $E \subseteq P$.
The following diagram illustrates the relationship between the set E and P.



Besides the set $E = \{1, 3\}$, we can also form other subsets of P such as:
$A = \{1\}$, $B = \{2\}$, $C = \{3\}$, $D = \{1, 2\}$ and $F = \{2, 3\}$

Proper and Improper Subsets

A proper subset is any possible subset of a given set, except the set itself.
The set itself is called improper subset.

Example:

For $P = \{1, 2, 3\}$, we have;
$\{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{\ \}$ as proper subsets of P.
But $\{1, 2, 3\}$ is improper subset of P.

Key Fact

The subset which contains all elements of the original set is called an improper subset.

Example:

If $B = \{car, c, a, t\}$ and $A = \{c, a, r, t\}$. Is $A \subseteq B$?

Sets can have more than one 'category' of elements – depending on how it is defined.
Set B in this case includes alphabets and words.
Then B is not subset of A as the word car is an element of B but not of A. Although the individual letters of the word belong to A and we write $B \nsubseteq A$.
We also use the symbol $A \not\subset B$ to denote that A is not a proper subset of B.

Example:

If $A = \{a, b, c, d\}$ and $B = \{d, b, a, c\}$. Is $A \subseteq B$? Is $B \subseteq A$? Is $\phi \subseteq A$?

Since every element of A is also an element of B therefore $A \subseteq B$.
Similarly, every element of B is also an element of A, thus $B \subseteq A$.
Since ϕ is the subset of every set, therefore $\phi \subseteq A$.

Key Fact

Every set is a subset of itself.

Example:

Given the sets $A = \{1, 3, 6, 8, 9, 10\}$, $B = \{6, 8, 10\}$ and $C = \{1, 3, 9, 11\}$
Which of the following statements are true? State your reasons.
(i) $B \subseteq A$ (ii) $C \subseteq A$ (iii) $\phi \subseteq A$ (iv) $B \subset A$ (v) $\phi \subset B$ (vi) $\{1, 3, 9\} \subset A$

(i) $B \subseteq A$ is true since all elements of B belong to A.
(ii) $C \subseteq A$ is not true since one element of C that is 11 does not belong to A.
(iii) $\phi \subseteq A$ is true since the empty set is the subset of any set.
(iv) $B \subset A$ is true since all the elements of B also belong to A and there are elements of A that do not belong to B.
(v) $\phi \subset B$ is true since there are elements of B that do not belong to the empty set.
(vi) $\{1, 3, 9\} \subset A$ is true since all the elements of $\{1, 3, 9\}$ belong to A and there are elements of A that do not belong to $\{1, 3, 9\}$.

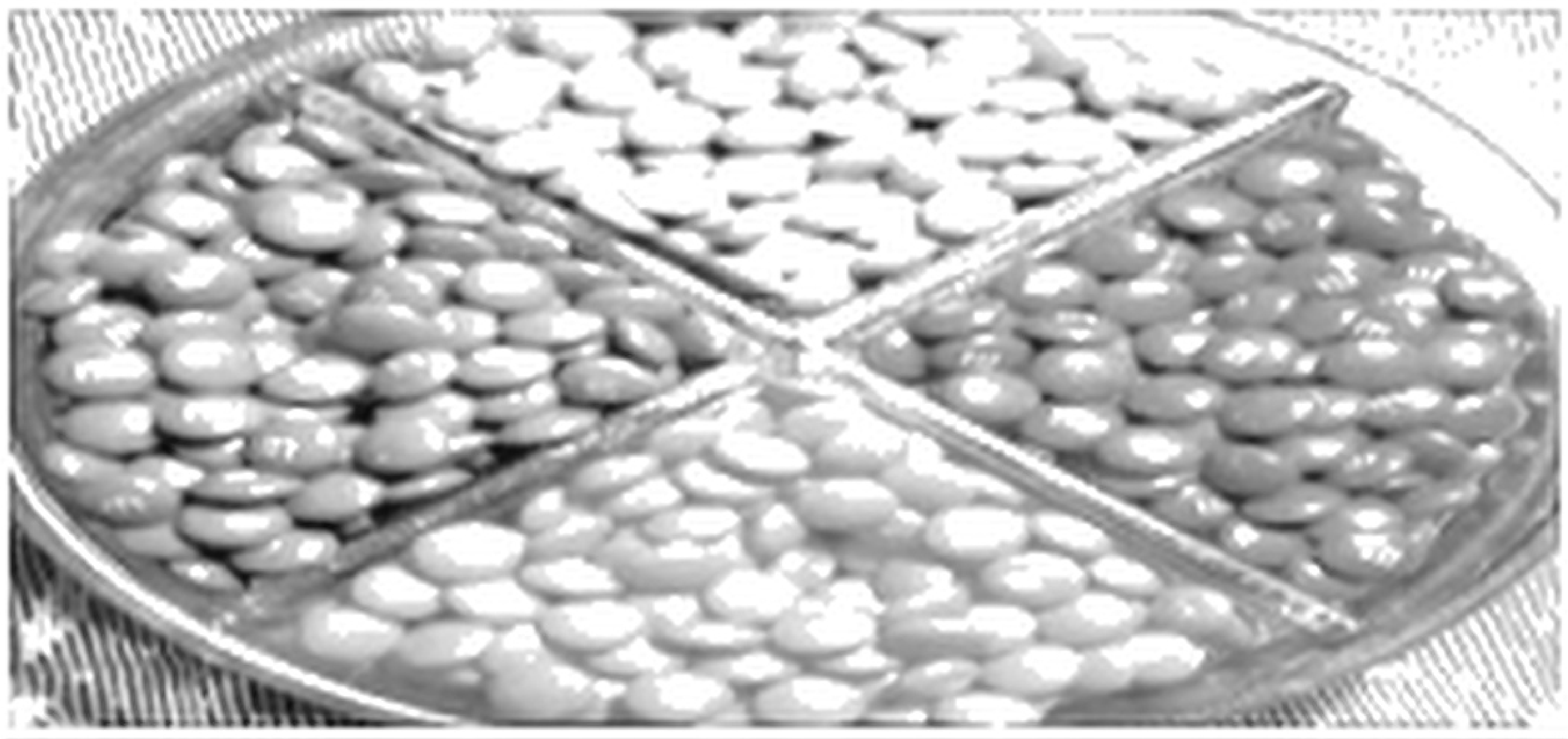
UNIT 1

Listing all Subsets and Proper Subsets of a Set

Consider the set S = {1, 2, 3}.
All subsets of a set S = {1, 2, 3} are ϕ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3} and {1, 2, 3}

All proper subsets of S = {1, 2, 3} are ϕ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3} except the set itself.

Here are 8 subsets of the given set. The given set has 3 elements and the number of subsets is 2^3. The number of subsets for a set with 'n' elements can be calculated by 2^n, where n is non negative integer denoting the number of elements in a given set.



The set of green jelly bean is a subset of colored jelly beans

Check Point

List all the subsets and proper subsets of:
R= {red, blue, green}
S = {7, 8}
T = {e, f, g, h}

Universal Set

A universal set is a set that contains all relevant elements in a particular situation.
For example, when we search for prime numbers, we sieve through the entire set of whole numbers. Thus, the universal set is the set of whole numbers for the situation.
The Symbol $\mathscr{E}$ denotes a universal set

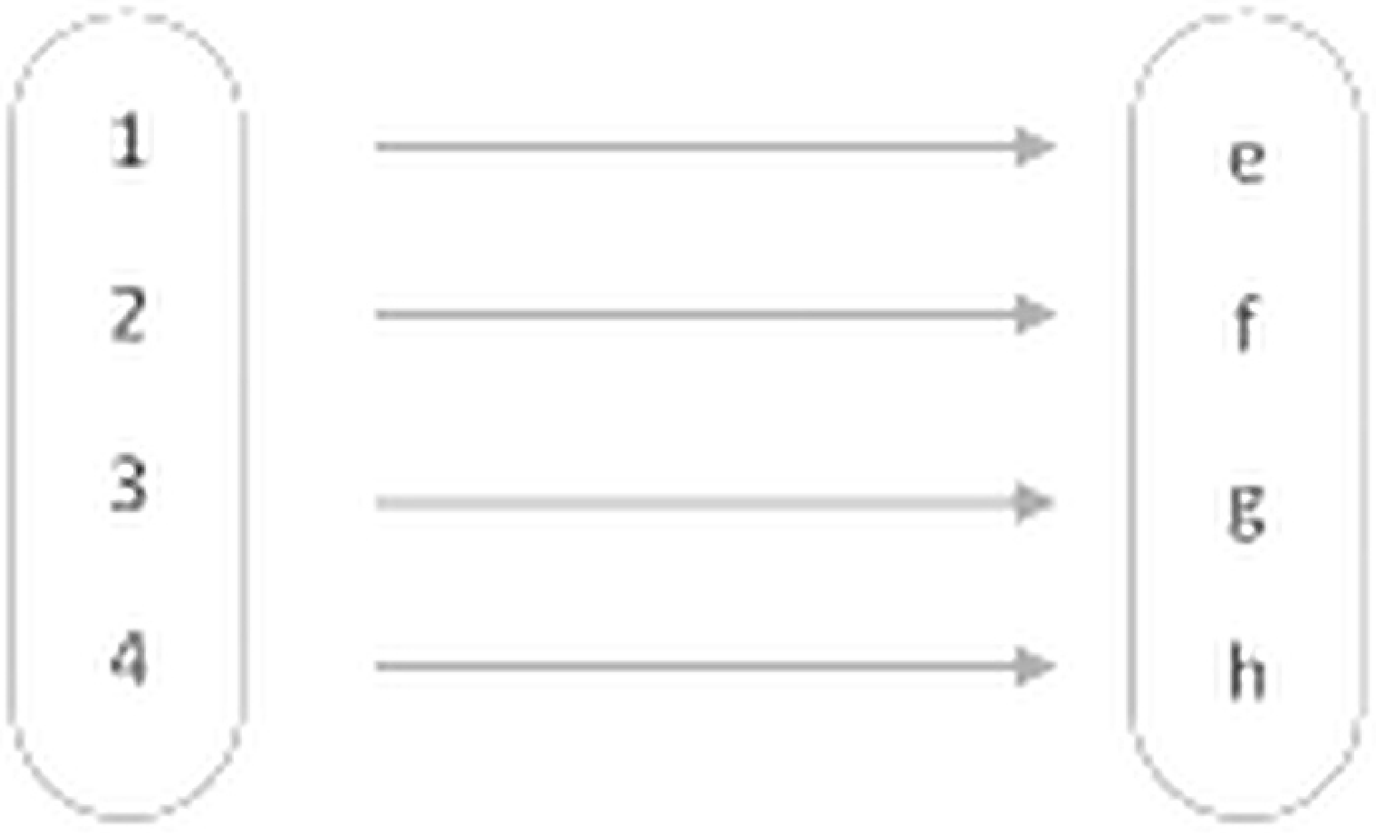
Equal and Equivalent Sets

Two sets are said to be equal if they have exactly the same elements.
Example: Consider sets A = {1, 2, 3, 4} and B = {2, 4, 1, 3}.
Since the two sets have exactly the same elements, allthough the order in which the elements are listed is different.
Thus, for two sets A and B, we have A = B if $A \subseteq B$ and $B \subseteq A$.
Conversely, if $A \subseteq B$ and $B \subseteq A$, then A = B.

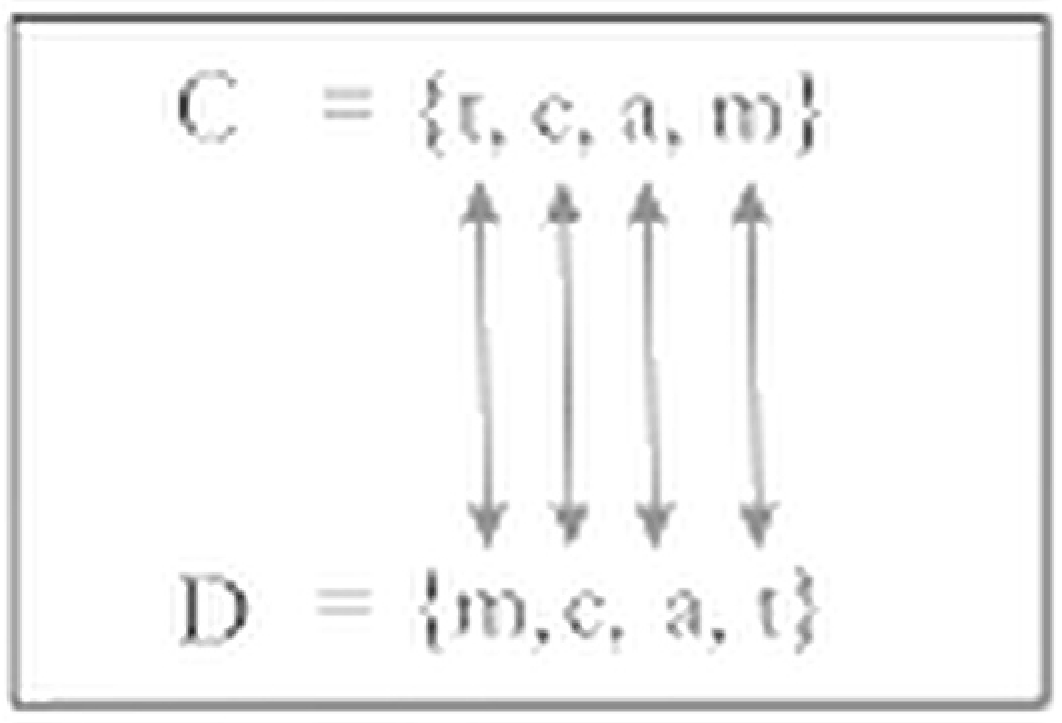
Key Fact

Two sets A and B are equal if they contain exactly the same elements.
Each element of set A is in set B and each element of set B is in set A and we write A = B.

Consider the sets A = {1, 2, 3, 4} and B = {e, f, g, h}



Notice that every element of set A can be matched with exactly one element of B, therefore A and B are **equivalent** sets as n (A) = n (B) = 4.
Let us consider another pair of equivalent sets.
C = {t, e, a, m } and D = {m, e, a, t }



Two sets A and B are equivalent if they have the same number of elements. The elements do not need to be the same.

Notice that C and D contain exactly the same elements.
Also C ⊆ D and D ⊆ C.
We say that C and D are equal sets and we write C = D.

If two sets are equal, they must be equivalent sets. However, if two sets are equivalent, they may or may not be equal sets. For example above set A and B are equivalent but not equal.

A = {even numbers less than 11}
B = {odd numbers less than 11}
State whether:
A and B are equivalent sets.
Or A and B are equal sets.

UNIT 1

Disjoint and Overlapping Sets

Consider set A = {1, 2, 3} and B = {4, 5}
Both sets have no common element.
Therefore set A and B are disjoint sets.

Key Fact Two Sets A and B are disjoint sets if both sets have no common element.

Consider the sets A = {2, 3, 5, 7, 9} and B = {1, 3, 5, 7}
As A and B are not subsets of each other.
Both sets have 3 and 5 as the common elements
Therefore set A and B are overlapping sets.

Key Fact Two sets A and B are called overlapping if:
(i) There is at least one element common in both the sets.
(ii) Neither of the sets is a subset of the other set.

Example:

Identify the following pairs of sets as disjoint or overlapping?
(i) C – {a, b, c}, D = {a, e, i, o, u}
(ii) X = {a, b, c}, Y = {5, 6, 7}

(i) C and D are overlapping sets because at least one element 'a' is common in both the sets and neither of the sets is a subset of the other set.
(ii) X and Y are disjoint sets as X and Y have no common element.

Check Point Identify the following pairs as disjoint sets, overlapping sets or none?
(i) A = {0, 2, 4, 6,...} B = Set of odd numbers
(ii) C = {0, 1, 2, 3,...} D = {1, 2, 3,...}
(iii) E = {1, 2, 3, 4, 5} F = {2, 3, 4, 5, 6}
(iv) G = {2, 3, 5, 7, 11} H = {1, 3, 5, 7, 11}
(v) I = {a, b, c, d, e} J = {f, g, h}

Exercise 1.1

1. For each of the following sets, show all the elements of the set.

(i) A is the set of positive integers between 5 and 9.
(ii) B is the set of common multiples of 2 and 3.
(iii) C is the set of factors of 66.

2. For each of the following sets, describe the set by listing all its elements.

(i) P = {names of the days of the week}
(ii) Q = {colours of a rainbow}
(iii) R = {odd numbers between 10 and 26}

3. If $A = \{1, 2, 3, 4, 5\}$, write down the value of $n(A)$.

4. Given the sets:

$\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A = \{1, 2, 3, 4, 5, 6, 7\}$
$B = \{3, 4, 5\}$
$C = \{4, 5, 6, 7, 8\}$
State true and false in each case.

(i) $B \subset A$
(ii) $C \subset A$
(iii) $C \subset B$

5. Given $A = \{2, 4, 6, 8\}$ and B = {even numbers less than 10}.
Which of the following statements is true and which is false?

(i) $A \subseteq B$ (ii) $A \subset B$ (iii) $A \not\subset B$
(iv) $A \neq B$ (v) $B \not\subset A$ (vi) $B \subseteq A$

6. List all subsets of the following sets.

(i) $P = \{3, 4\}$
(ii) Q = {pen, book, bag }
(iii) R = {Lahore, Islamabad}
(iv) $S = \{a, i, r\}$

7. The Venn diagram shows the elements in A and B.

(i) List all the elements in A and in B.
(ii) Is A a proper subset of B? Explain.

8. If A = {w, x, y, z}, B = {s, t, w}, C = {x, y, z}, then which set is subset of A?

9. If A = {−2, −1, 0, 1, 2, 3} and B = {0, 1, 2, 3}

(i) Draw a Venn diagram to represent the sets A and B.
(ii) Is B a proper subset of A? Explain.

10. Are the following sets equal?

(i)	A = {8, 6, 4, 2}	B = {6, 4, 8, 2}
(ii)	A = {r, a, t, e}	B = {e, t, a, f}
(iii)	A = {set of letters of the word "mango"}	B = {a, g, m, n, o}
(iv)	A = {letters of the word "tea"}	B = {letters of the word "eat"}
(v)	A = ϕ	B = {0}

11. If A = {even numbers less than 12}
B = {odd numbers less than 12}
C = {the first 10 natural numbers divisible by 2}
D = {multiples of 3 less than 20}

Verify which of the following statements are true or false?

(i) A and B are equivalent sets.
(ii) A and C are equivalent sets.
(iii) A and C are equal sets
(iv) B and D are equivalent sets
(v) B and D are equal sets

12. Identify equivalent and equal sets in the following:

(i) A = {a, e, i, o, u}
(ii) B = {1, 2, 3, 5, 7}
(iii) C = The set of vowels in English alphabets.
(iv) D = The set of letters of the word "remember".

Operations on Sets

Union and Intersection

Consider the following sets:
$\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8\}$
$A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4, 6, 7, 8\}$.

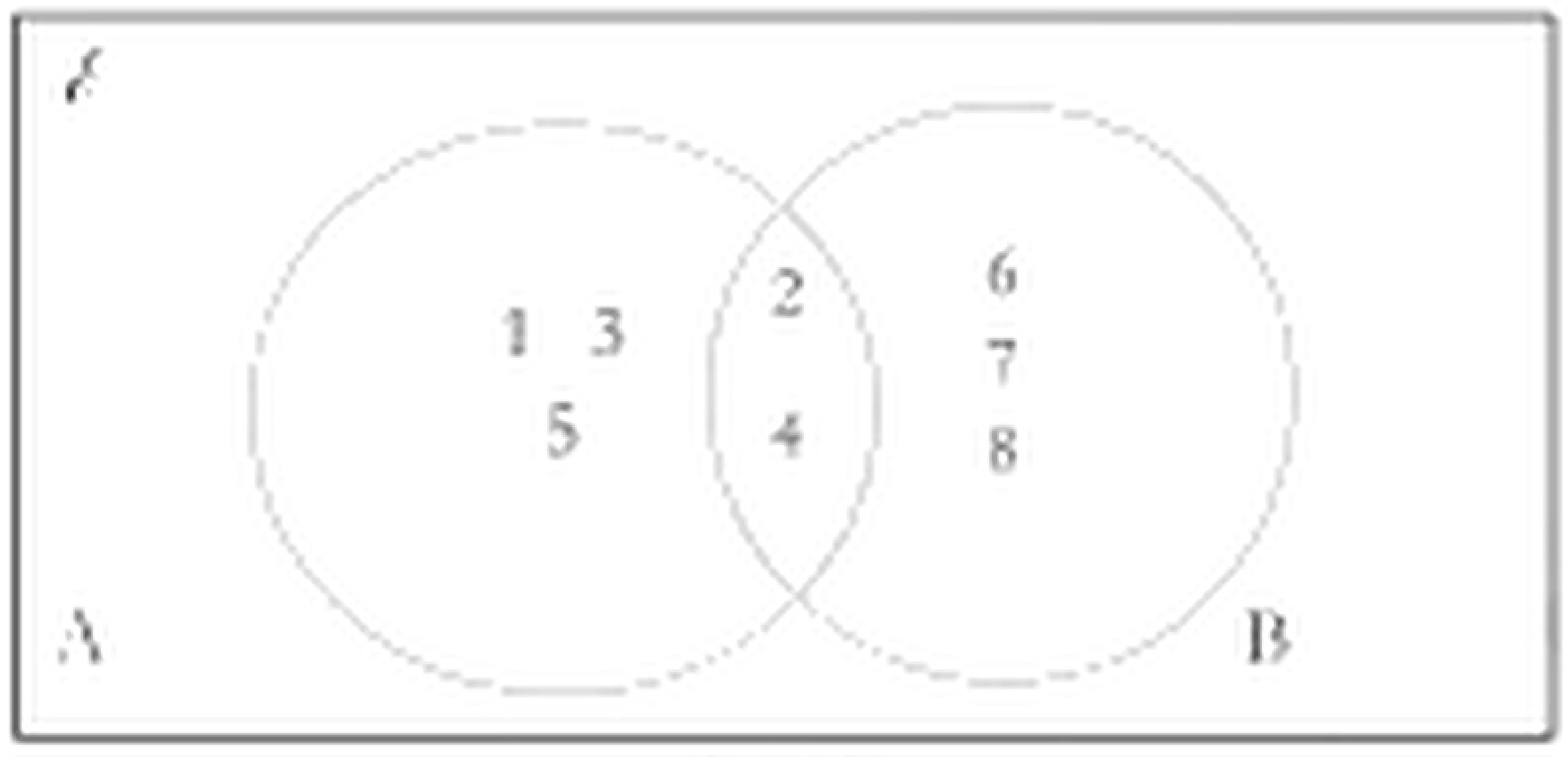
If we list all elements of a set A and A set B together in a new set without repeating common elements, then this new set is called the union of sets A and B.

It is denoted by $A \cup B$.

So, $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

Key Fact

The Union of sets A and B is the set of all the elements in A or in B together, and is denoted by $A \cup B$.



$A \cup B$

If we list common elements of the set A and the set B. It is called the intersection of A and B.

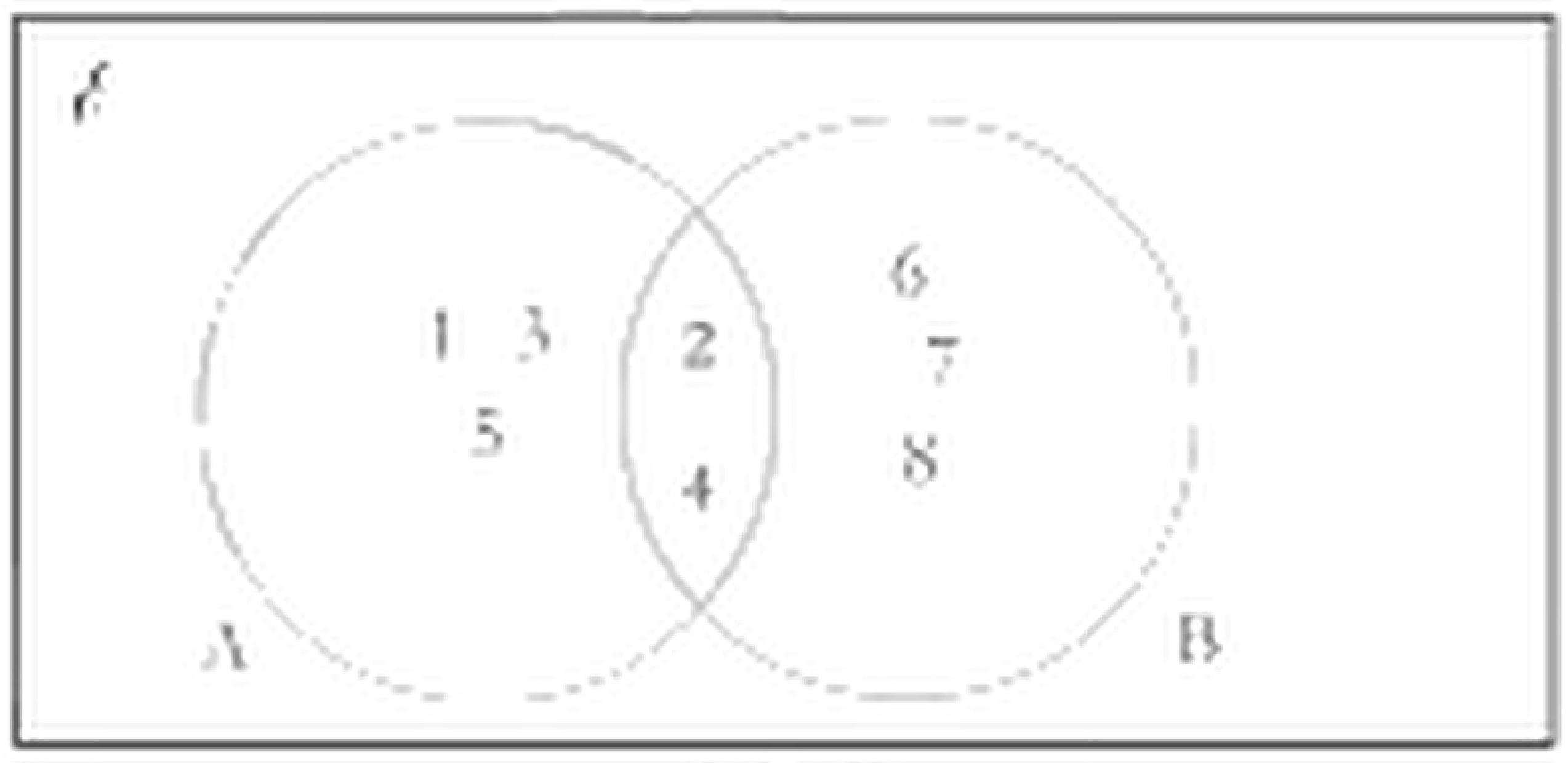
It is denoted by $A \cap B$.

So, $A \cap B = \{2, 4\}$

Since 2 and 4 belong to both A and B.
The relationship is illustrated by the Venn diagram below:

Key Fact

The intersection of sets A and B is the set of elements which are common to both A and B. It is denoted by $A \cap B$.



$A \cap B$

Note that the overlapping region represents the set $A \cap B$.

If A and B do not have any common element, then $A \cap B = \phi$

Check Point

It is given that $A = \{m, n, o, p, q, r, s\}$ and $B = \{o, q, s, t\}$
List all the elements in $A \cup B$ and $A \cap B$.

Example:

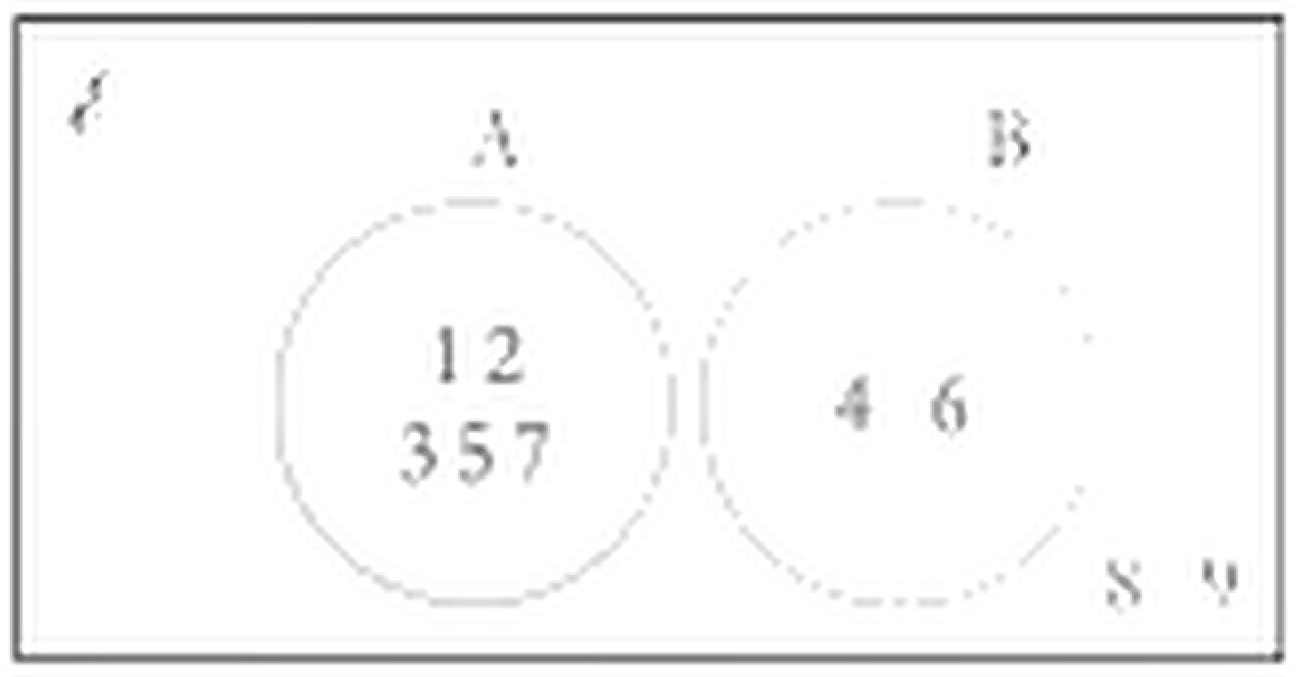
Consider the universal set $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$,

$A = \{1, 2, 3, 5, 7\}$, $B = \{4, 6\}$

The sets A and B have no common elements.

Therefore $A \cap B = \phi$

In this case $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$



Example:

Consider the Universal set $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

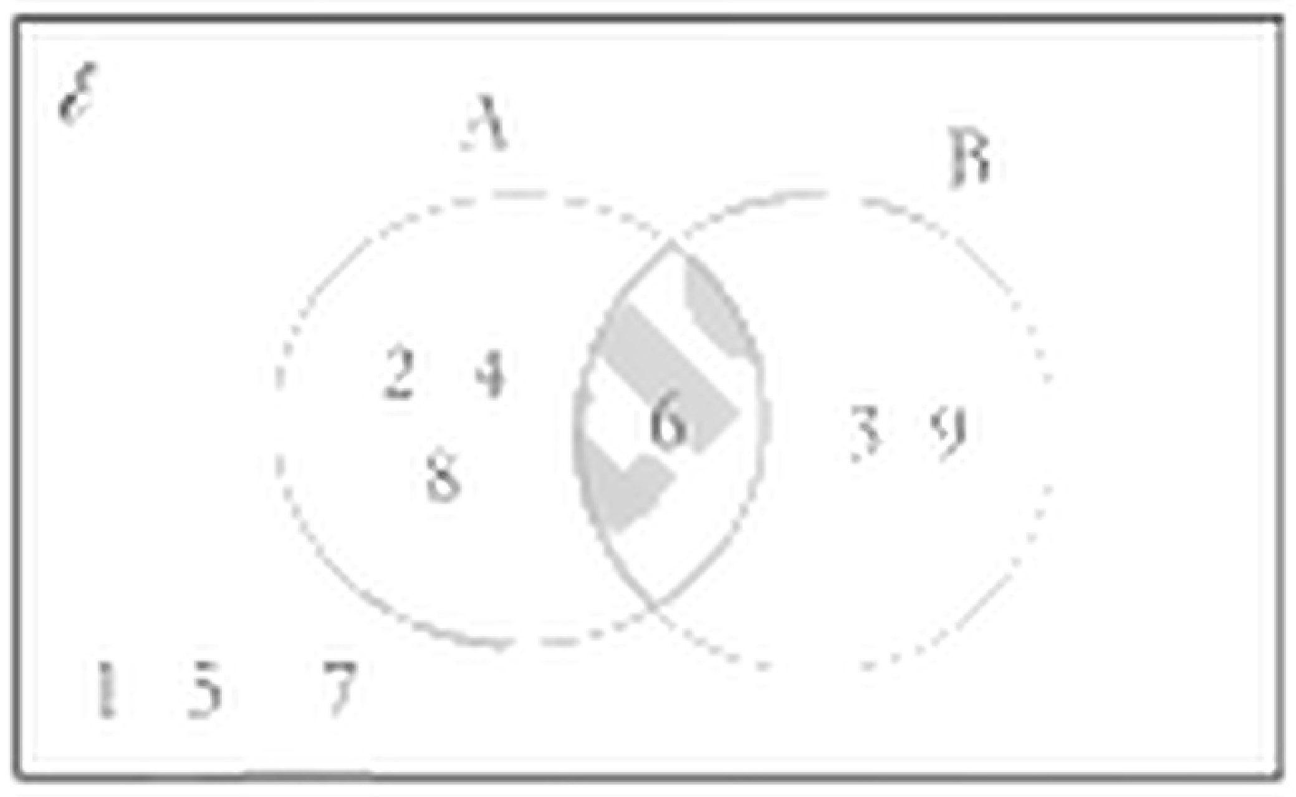
$A = \{$Multiples of 2 less than $10\} = \{2, 4, 6, 8\}$

$B = \{$Multiples of 3 less than $10\} = \{3, 6, 9\}$

The only number which is multiple of 2 and 3 is 6. Therefore

$A \cap B = \{6\}$ i.e. $n(A \cap B) = 1$ and

$A \cup B = \{2, 3, 4, 6, 8, 9\}$ i.e. $n(A \cup B) = 6$



Example:

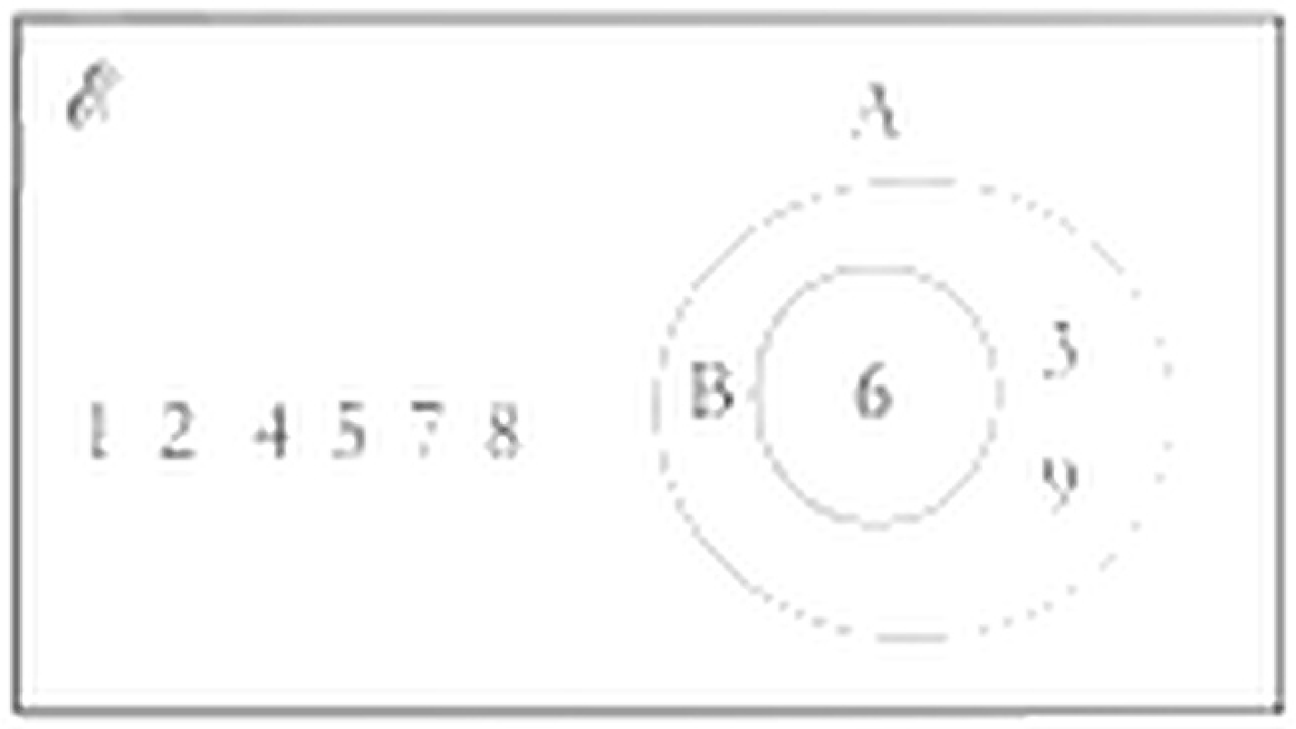
Consider the Universal set

$\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{$Multiples of 3 less than $10\} = \{3, 6, 9\}$

$B = \{$Multiples of 6 less than $10\} = \{6\}$.

Find $A \cup B$ and $A \cap B$.



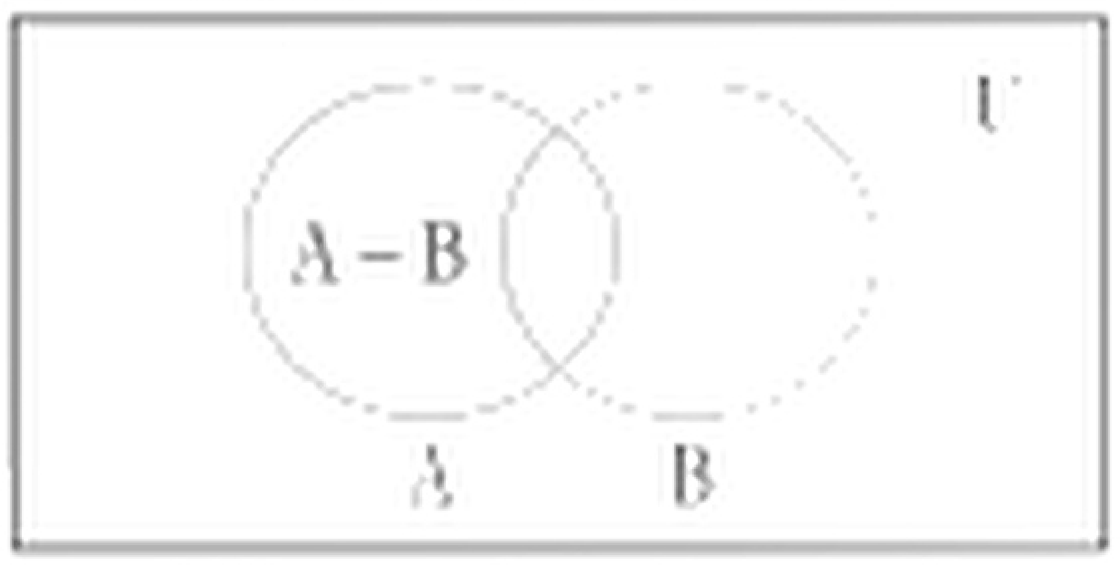
Difference of Two Sets

Difference of two sets is entirely different from the difference of two numbers. If A and B are two sets, then their difference is given by $A - B$ or $B - A$.

Consider $A = \{2, 3, 4\}$ and $B = \{4, 5, 6\}$

$A - B$ means, elements of A which are not the elements of B. i.e., in the above example

$A - B = \{2, 3\}$ and $B - A = \{5, 6\}$. So, $A - B \neq B - A$.



Different of two sets A – B

Key Fact

If A and B are any two sets then A difference B is denoted by $A - B$ or $A \setminus B$.

$A - B$ is a set of all elements of the set A which are not contained in the set B.

Similarly B difference A is denoted by $B - A$ or $B \setminus A$. $B - A$ is a set of all elements of the set B which are not present in the set A.

Also $A - B \neq B - A$.

Check Point

Find $X - Y$ and $Y - X$ if

(i) $X = \{a, e, i, o, u\}$ $Y = \{a, b, c, d, e, f\}$

(ii) $X = \{1, 3, 5, 7, ...\}$ $Y = \{2, 4, 6, ...\}$

(iii) X = Set of integers Y = Set of whole numbers

(iv) X = Set of natural numbers Y = Set of positive integers

(v) $X = \{x, y, z\}$ $Y = \{\ \}$

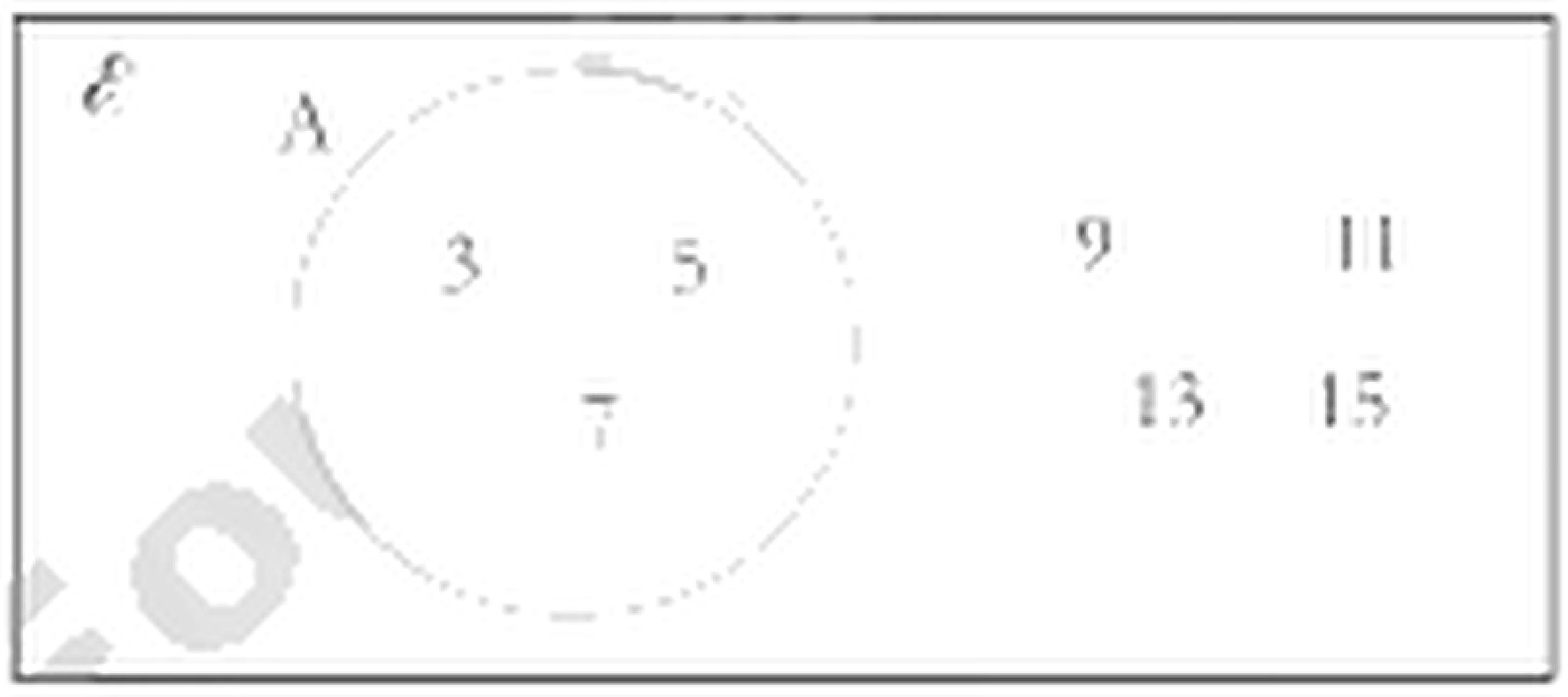
Complement of a Set

For a given set A, the complement of a set A is the set of elements that belongs to the universal set but not the set A. It is written as A′ and read as "A complement".

Let $\mathcal{E}$ = {integers from 1 to 10},
A = {odd numbers in the range 1 to 10}
Then $\mathcal{E}$ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
A = {1, 3, 5, 7, 9}
A′ = $\mathcal{E}$ − A = {2, 4, 6, 8, 10}

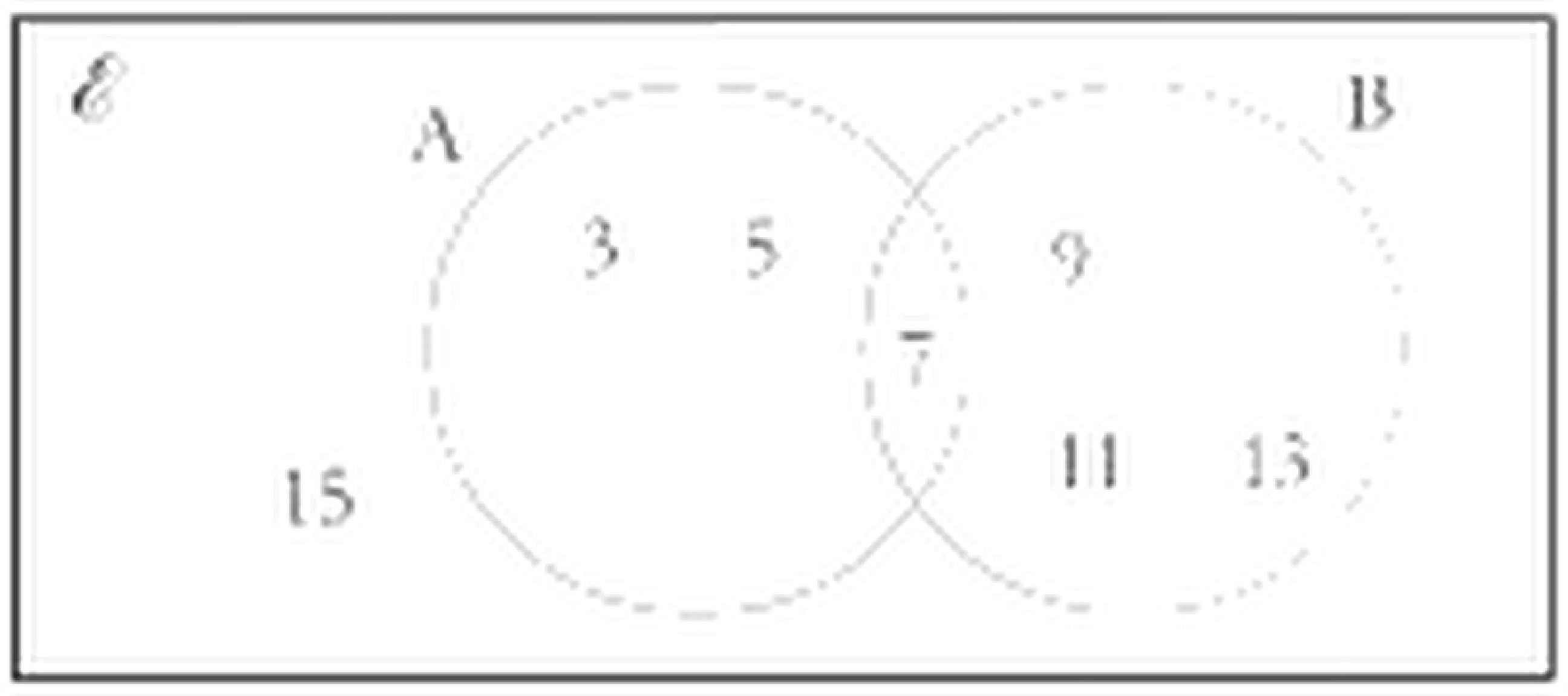
Key Fact Elements that are in $\mathcal{E}$ universal set but not in A, are members of a set called the complement of A (denoted as A′). So, A′ = $\mathcal{E}$ − A

If $\mathcal{E}$ = {3, 5, 7, 9, 11, 13, 15},
A = {3, 5, 7}, B = {7, 9, 11, 13}
Then the list of elements of the set A′= $\mathcal{E}$ − A
= {9, 11, 13, 15}

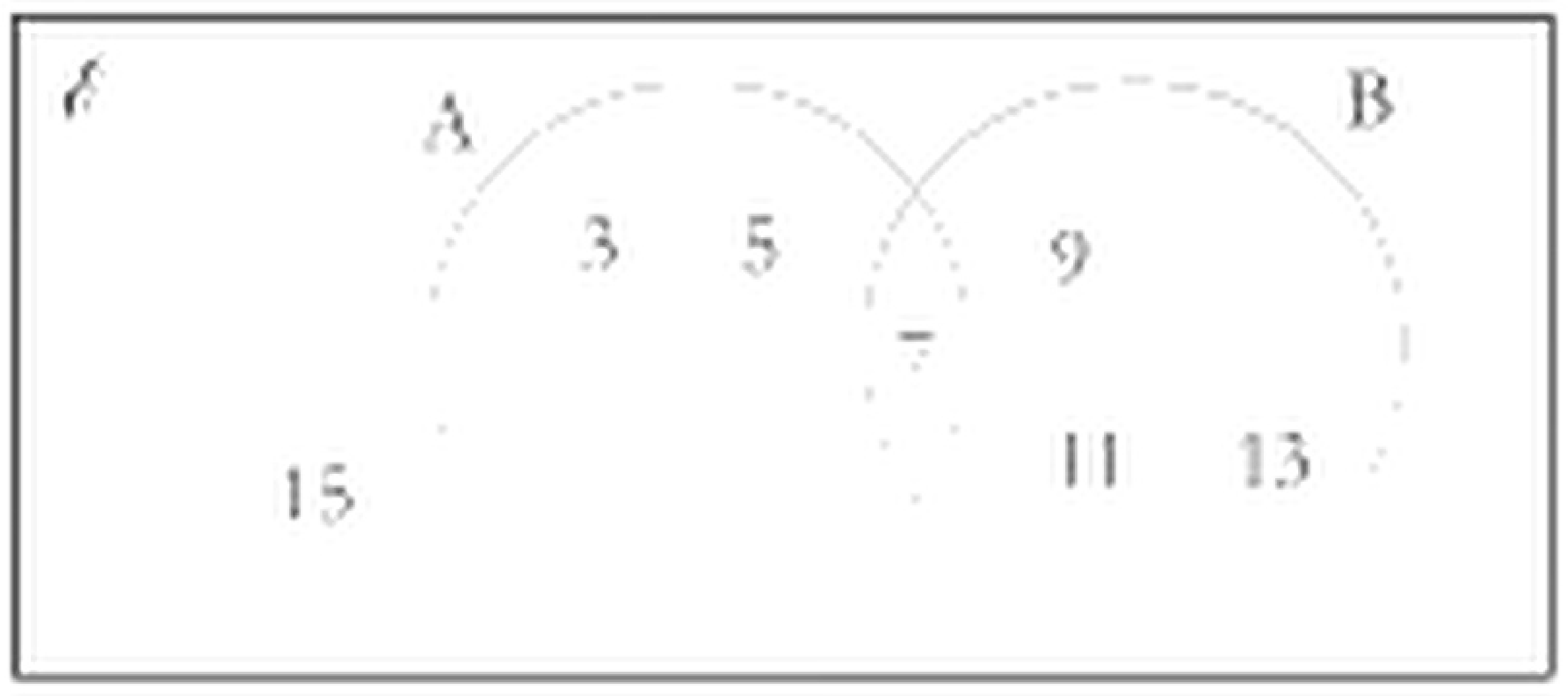


and B′ = $\mathcal{E}$ − B = {3, 5, 15}

A ∪ B = {3, 5, 7, 9, 11, 13)
(A ∪ B)′ = $\mathcal{E}$ − (A ∪ B) = {15}



A ∩ B = {7}
(A ∩ B)′ = $\mathcal{E}$ − (A∩B)= {3, 5, 9, 11, 13, 15}



Check Point It is given that
$\mathcal{E}$ = {1, 2 ,3, 4, 5, 6, 7, 8, 9, 10}
A = {2, 4, 6, 8, 10}, B = {1, 2, 3, 4, 6}
Find and represent through Venn diagrams.
(A ∪ B)′, A ∪ B′ , A′ ∩ B , (A ∩ B)′

Exercise 1.2

1. Find union and intersection of each of the following pairs of sets.

(i) $A = \{4, 8, 12, 16\}$	$B = \{8, 10, 12\}$
(ii) $C = \{e, f, g, h\}$	$D = \{q, r, s, t\}$
(iii) $E = \{\text{apple, mango, grapes}\}$	$F = \{\text{watermelon, mango}\}$
(iv) $G = \{b, n, j, z\}$	$H = \phi$

2. It is given that $A = \{r, s, t, u, v, w, x\}$ and $B = \{s, t, w, y, z\}$
   (i) List all the elements of $A \cup B$ in set notation.
   (ii) Draw a Venn diagram to represent the set $A \cup B$.

3. It is given that $A = \{1, 2, 3, 4, 7\}$and $B = \{2, 4, 8, 10\}$
   (i) List all the elements of $A \cap B$ in set notation.
   (ii) Draw a Venn diagram to represent the set $A \cap B$.

4. The universal set $\mathcal{E}$ and the set A and B are given by:
   $\mathcal{E} = \{2, 3, 4, 5, 6, 8, 11\}$, $A = \{2, 4 ,6, 8\}$, $B = \{3, 4, 6\}$
   List the elements of the following sets:
   (i) $A \cap B$ (ii) $A \cup B$ (iii) A'
   (iv) B' (v) $A' \cap B'$ (vi) $A' \cup B'$

5. It is given that $\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{2, 4, 6, 8, 10\}$, $B = \{1, 2, 3, 4, 6\}$
   Find $(A \cup B)'$ and $A \cap B'$.

6. It is given that $\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 3, 5, 7, 9\}$, $B = \{3, 6, 9\}$
   Find $(A \cap B)'$ and $A \cup B'$.

7. Find $A - B$ and $B - A$ if
   (i) A = Set of natural numbers, B = Set of even numbers
   (ii) $A = \{a, b, c, d, e\}$, $B = \{a, e, i. o, u\}$

8. $\mathcal{E} = \{p, q, r, s, t, u\}$ and $Y = \{p, r, v\}$. List the members of Y'.

9. $\mathcal{E} = \{1, 2, 3 \ldots 20\}$. P is the set of prime numbers in $\mathcal{E}$.
   (i) List the elements of P.
   (ii) List the element of the set P'.
   (iii) Draw the Venn diagram for P'.

10. Given that the universal set is set of all integers.
    What is the complement of the set of negative numbers?

UNIT 1

I have learnt

- A set is a collection of well-defined and distinct objects.
- Each object in a set is called a member or element of the set.
- Two sets A and B are equal if they have exactly the same elements and we write $A = B$ that is $A \subset B$ and $B \subset A$.
- Two sets are said to be equivalent if they have the same number of elements.
- The empty or null set is the set containing no element and is represented by ϕ.
- Two sets are overlapping if they have at least one common element.
- Two sets are disjoint sets if they do not have any common element.
- A Venn diagram can be used to represent the relationship among sets.
- The universal set, $\mathcal{E}$, is a specified set with in which the set under discussion are confined.
- A' the complement of set A, is the set of elements in $\mathcal{E}$ which are not in the set A.
- Given a set A, both A and its compliment A', are subsets of universal set.
- The intersection of set A and its complement A′, is the empty set. i.e. $A \cap A' = \phi$.
- The union of set A and its complement A′ forms the universal set. i.e. $A \cup A' = \mathcal{E}$.
- If $B \subset A'$ then $A \cap B = \phi$ and A and B are disjoint sets.

Words Board

- Distinct Objects
- Elements
- Equivalent Sets
- Equal Sets
- Empty Set
- Overlapping Sets
- Disjoint Sets
- Universal Set
- Compliment of a Set
- Union
- Intersection
- Subset

The language of Sets

Sets	Set Symbols	Venn Diagrams
Union of A and B when A and B are overlapping	$A \cup B$	 $A \cup B$
Intersection of A and B when A and B are overlapping	$A \cap B$	 $A \cap B$
Difference of A and B when	$A - B$	 $A - B$
Difference of B and A when	$B - A$	

Complement of a set A	A'	
A is a proper subset of B	$A \subset B$	
A and B are disjoint sets	$A \cup B$	
A and B are disjoint sets	$A \cap B$	

Set Language and Notation

Symbol	**Meaning**
$A = \{2, 4, 6, 8\}$	A is a set whose elements are 2, 4, 6 and 8.
$4 \in A$	4 is an element of the set A.
$3 \notin A$	3 is not an element of the set A.
$A = B$	A and B are equal sets.
$n(A)$	The number of elements in the set A.
$n(A) = n(B)$	The set A and B have an equal number of elements. They are equivalent sets (may or may not be equal sets).
ϕ or $\{\ \}$	The empty set
$A \cap B$	The intersection of sets A and B.
$A \cup B$	The union of sets A and B
$A \subseteq B$	A is a subset of B
$A \subset B$	A is a proper subset of B
$A \nsubseteq B$	A is not a subset of B
$A \not\subset B$	A is not a proper subset of B
ℰ	Universal set
A'	Complement of the set A

Review Exercise 1

1. **Encircle the correct option for the following statements.**

(i) Elements that are in the universal set but not in the set A, are members of the set called ___________ of A.

(a) subset (b) proper subset

(c) complement (d) disjoint

(ii) ___________ is the subset of every set.

(a) Universal set (b) Null set

(c) Finite set (d) Singleton set

(iii) Common elements of two sets A and B are denoted by:

(a) $A \cup B$ (b) $A \cap B$ (c) $A \subseteq B$ (d) $A \not\subset B$

(iv) __________ express the idea includes or contains.

(a) $\subset$ (b) $\subseteq$ (c) $\not\subset$ (d) $\not\subset$

(v) The set of elements which are in A, or in B, or in both A and B is called:

(a) union (b) intersection

(c) complement (d) subset

(vi) It is given that $A = \{h, I, p\}$ and $B = \{h, o, p\}$ then $A \cap B$ is equal to:

(a) $\{h, o\}$ (b) $\{h, i\}$ (c) $\{i, o\}$ (d) $\{h, p\}$

(vii) Union of a set A with itself is:

(a) Set A (b) Universal set

(c) Empty Set (d) None of the above.

(viii) Intersection of a set A with an empty set is:

(a) set A (b) universal set

(c) empty Set (d) none of the above

(ix) Set B is a subset of A can be denoted by:

(a) $B \subseteq A$ (b) $B \supseteq A$

(c) $B \cup A$ (d) $B \cap A$

UNIT 1

2. It is given that $\mathcal{E} = \{3, 4, 5, 6, 7, 8, 9, 10\}$,

$A = \{\text{set of factors of } 12\}$,
$B = \{\text{set of even numbers}\}$
Find
(a) $n(A)$ (b) $n(A \cap B)$ (c) $n(A \cup B)$ (d) $n(B')$

3. List all the subsets of $\{2, 3, 5\}$.

4. It is given that: $F = \{10, 20, 30, 40\}$, $G = \{11, 13, 17, 19\}$, $H = \{10, 11, 12, 13\}$
$J = \{11, 13\}$

(a) List (i) $G \cap H$ (ii) $F \cup H$
(b) J is a subset of two of the sets. Which are the two sets?
(c) Suggest a suitable universal set for F, G, H and J.
(d) What can you say about $F \cap J$ and represent it through Venn diagram.

5. $\mathcal{E} = \{3, 4, 5, 6, 7, 8, 9\}$, $A = \{4, 5, 6\}$, $B = \{\text{odd numbers between 2 and 8}\}$,
$C = \{\text{square numbers} < 10\}$

(i) List
(a) B (b) $A \cap B$ (c) $A \cup B$
(d) A' (e) the complement of B (f) $A' \cap B$
(g) $(A \cap B)'$
(ii) List the intersection of B and C.
(iii) List the union of A and C.
(iv) Write down $n(C)$.
(v) Find $n(A \cup C)$.
(vi) Find $n(B')$.

6. If A = {mango, banana, apples} and B = {mango, grapes, apples, strawberry}
Find $A \cup B$ and $A \cap B$ and represent the intersection through Venn diagram.

7. Find pairs of disjoint and overlapping sets.

(i) A = {3, 6, 9, 12} and B = {6, 8, 9}
(ii) C = {a, b, *x*, y} and D = {m, n, o, p}
(iii) E = {monkey, goat, lion} and F = {tiger, goat}
(iv) G = {q, r, s, t} and H = {u, v, w, *x*, y, z}

8. If $\mathcal{E}$ = {1, 2, 3, 4, 5, 6, 7, 8, 9}, A = {1, 2} and B = {1, 2, 3, 4}, draw a Venn diagram to represent the above sets and to illustrate their relationship.

9. List all proper subsets of the following sets.

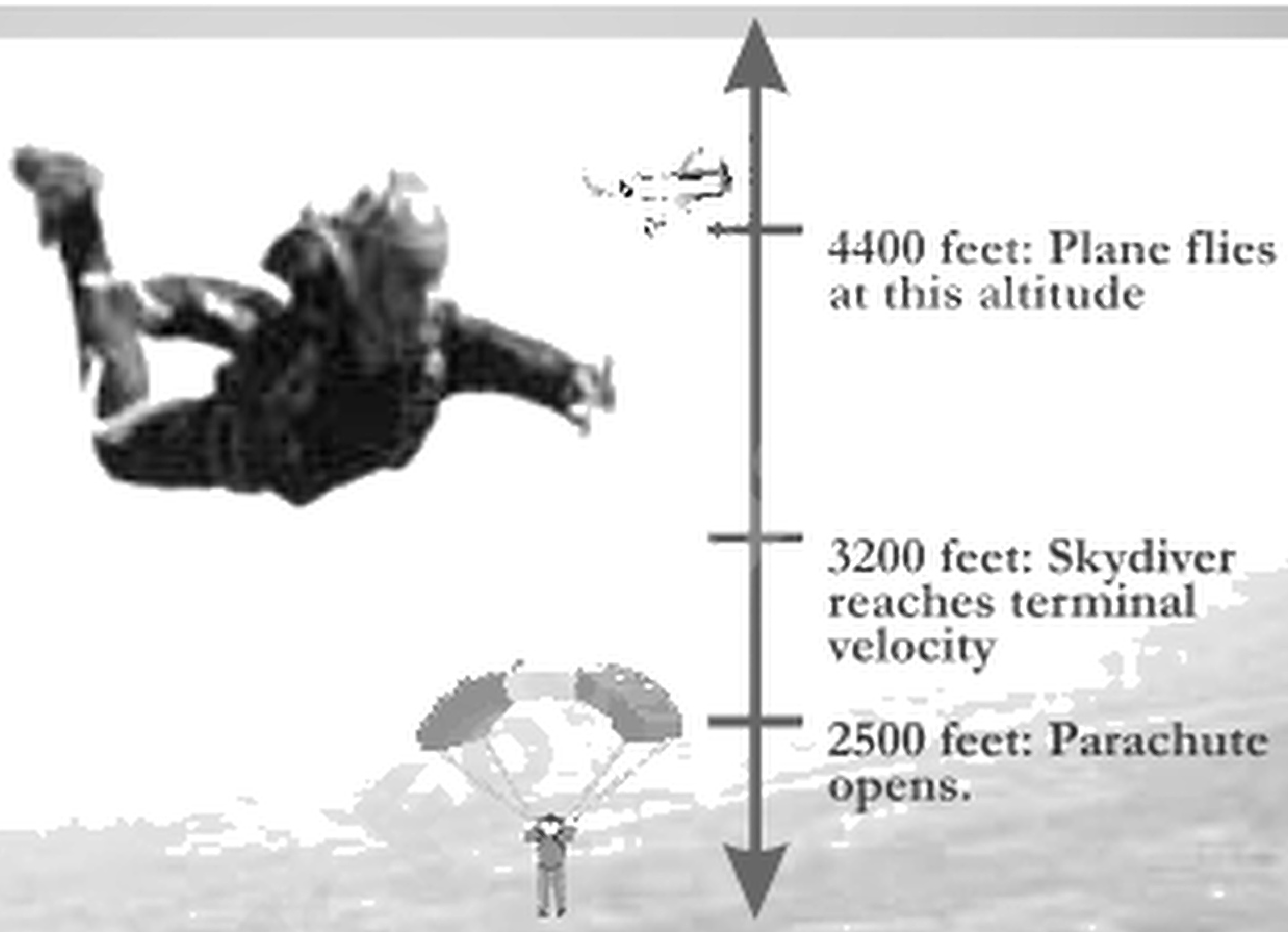
(i) X = {g, h}
(ii) Y = {5, 7, 9}
(iii) Z = {Ali, Amna}

10. Identify the following pairs as disjoint sets, overlapping sets or neither.

(i) A = {S, U, N} B = {S, T, A, R}
(ii) C = {factors of 24} D = {factors of 33}
(iii) E = {p, q, r, s} F = {2, 3, 4, 5}
(iv) G = {11, 21, 31, 41} H = {15, 25, 35, 45}
(v) I = {2, 4, 6, 8, 10} J = {1, 3, 5, 7, 11}

02

Rational Numbers



A skydiver in free fall will eventually reach a constant velocity, called terminal velocity.
A skydiver reaches a terminal velocity of 160 feet per second at an altitude of 3200 feet.
He opens the parachute at an altitude of 2500 feet.
After how many seconds should the skydiver open the parachute?

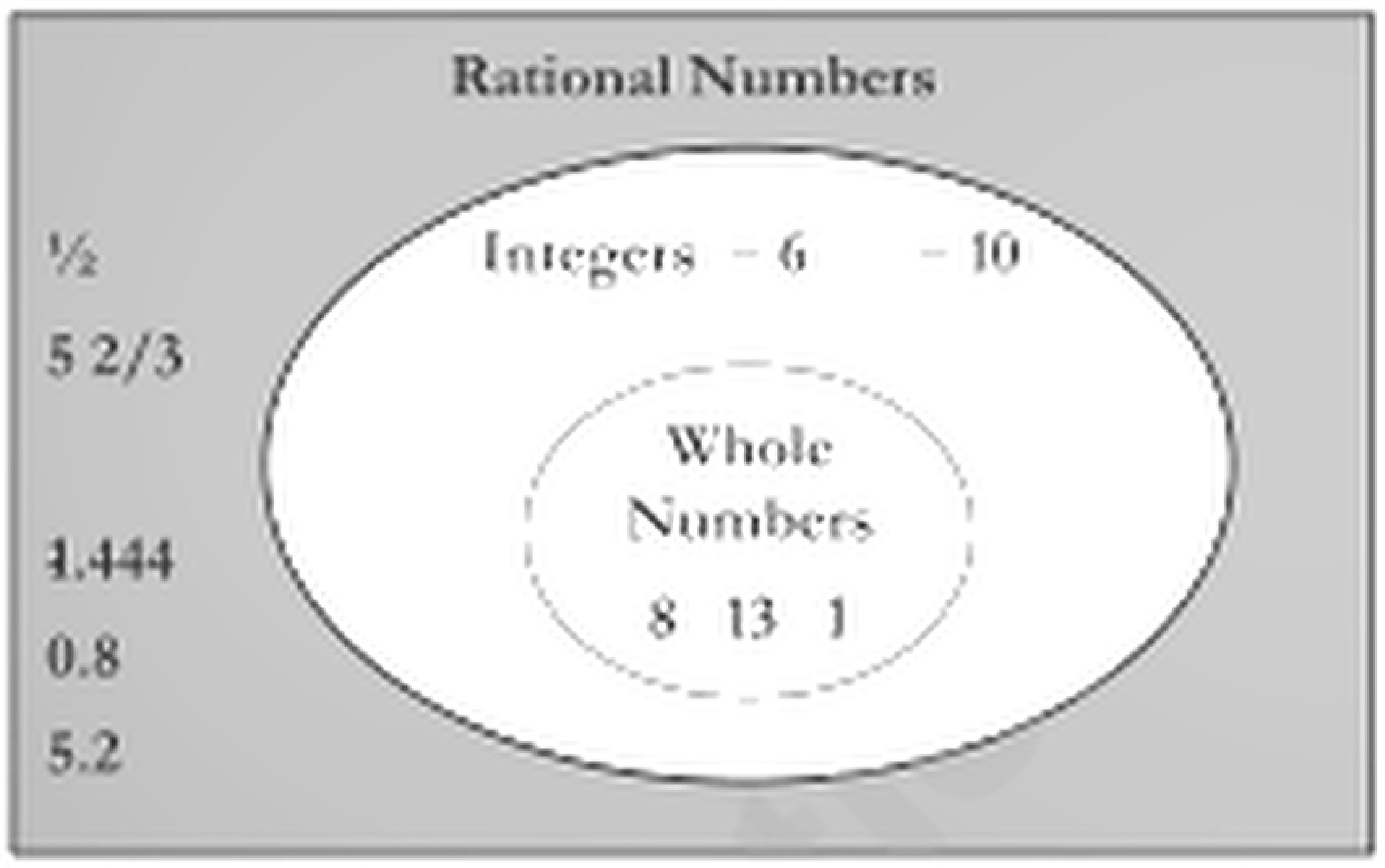
Learning Outcomes

Students will be able to:

- Understand a rational number as a number that can be expressed in the form of p/q where p and q are integers and q ≠ 0.
- Represent rational numbers on a number line.
- Compare two rational numbers.
- Arrange rational numbers in ascending or descending order.
- Add two or more rational numbers
- Subtract a rational number from another.
- Recognize 0 as additive identity
- Find additive inverse of a rational number.
- Multiply two or more rational numbers.
- Divide a rational number by a non-zero rational number.
- Recognize 1 as a multiplicative identity
- Find multiplicative inverse of a rational numbers.
- Identify multiplicative inverse of a non-zero rational numbers as the reciprocal of that number.
- Verify commutative property of rational numbers with respect to addition and multiplication.
- Verify associative property of rational numbers with respect to addition and multiplication.
- Verify distributive property of rational numbers with respect to multiplication over addition and subtraction.

UNIT 2

RATIONAL NUMBERS



When we add two whole numbers, the result is always a whole number. However, subtraction of whole number does not always produce a whole number, so negative numbers are included to form the set of integers.

For example:

$3 + 4 = 7$, where 7 is a whole number.

But $3 - 4 = -1$, where −1 is not a whole number, it is an integer.

When we add, subtract or multiply two integers, we always obtain an integer. However, division of two integers does not necessarily result in an integer. Hence there is a need to include fractions to form a bigger set of numbers known as the set of rational numbers.

For Example: $(-3) \times (4) = -12$, where −12 is an integer.

But $(-3) \div (4) = -\frac{3}{4} = -0.75$. Here – 0.75 is not an integer, it is a fraction.

Rational Numbers are the set of numbers that include integers and fractions.

A number that can be expressed as a ratio of two integers a and b, is called the rational number in the form $\frac{a}{b}$ where $b \neq 0$. For example $-\frac{1}{2}$ is a rational number because it can be written as $\frac{-1}{2}$ or $\frac{1}{-2}$.

Example: Classify following numbers as a whole number, an integer or a rational number

Number	Whole number	Integer	Rational Number
5	Yes	Yes	Yes
0.6	No	No	Yes
$-2\frac{2}{3}$	No	No	Yes
− 24	No	Yes	Yes

UNIT 2

Ordering Rational Numbers

The table shows how Ali and Amna did during a class test.

Name	Marks obtained	Total marks
Ali	7	12
Amna	8	18

Who scored better, Ali or Amna?

Ali: $\frac{7}{12} = \frac{7\times3}{12\times3} = \frac{21}{36}$ Amna: $\frac{8}{18} = \frac{8\times2}{18\times2} = \frac{16}{36}$

Since: $\frac{21}{36} > \frac{16}{36}$ therefore $\frac{7}{12} > \frac{8}{18}$

Ali has performed better then Amna.

Key Fact

To compare two rational numbers

(i) Express each rational number with positive denominators.

(ii) Find LCM of positive denominators.

(iii) Write an equivalent fraction for each rational number.

(iv) Compare the numerators.

Example

In a school, $\frac{19}{32}$ students of class 7 own tablets whereas, $\frac{16}{28}$ students of class 8 have tablets. Which class has a greater fraction of students with tablets?

Since the denominators are large, write $\frac{19}{32}$ and $\frac{16}{28}$ as decimals and then compare.

$\frac{19}{32} = 0.5938$ $\frac{16}{28} = 0.5714$

Since $0.5934 > 0.5714$, therefore $\frac{19}{32} > \frac{16}{28}$

So a greater fraction of students of class 7 owns tablets.

Example:

Eman's quiz scores for the monthly tests are $\frac{14}{20}$, 0.68, 80% and $\frac{5}{10}$

Arrange quiz scores from least to greatest.

To arrange from least to greatest, we compare the scores.

First write each number as a decimal, then compare:

$\frac{14}{20}$, 0.68, 80%, $\frac{5}{10}$

0.7, 0.68, 0.8, 0.5

Since $0.5 < 0.68 < 0.7 < 0.8$

We can write $\frac{5}{10} < 0.68 < \frac{14}{20} < 80\%$

Check Point

The amount of rain received on four consecutive days was 0.3 cm, $\frac{2}{5}$ cm, 0.75 cm and $\frac{2}{3}$ cm.
Order from least to greatest.

To arrange the following numbers from least to greatest, write each number as decimal.

$\frac{1}{5}$, -0.10, 0.25, $-\frac{1}{5}$

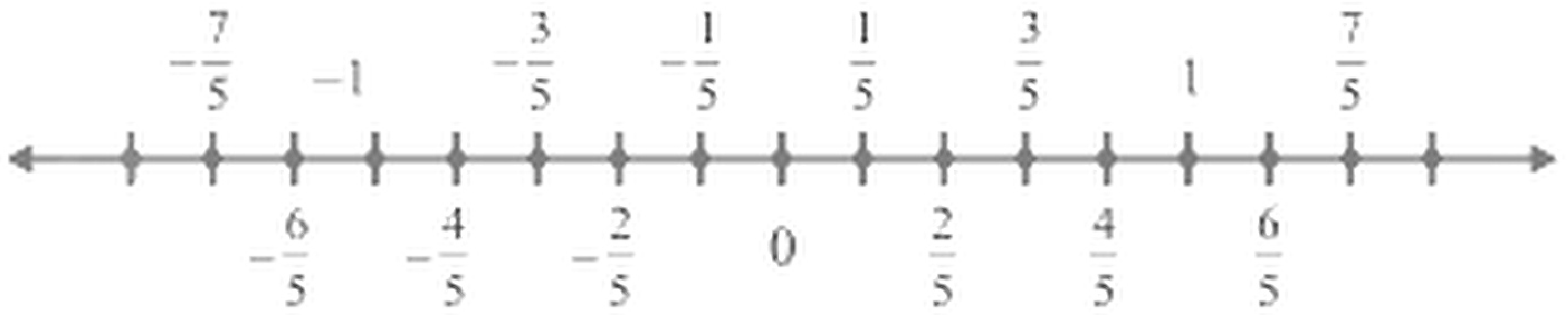
$= 0.2$, -0.10, 0.25, -0.2

From least to greatest, the numbers are:

-0.2, -0.10, 0.2, 0.25

$-\frac{1}{5}$, -0.10, $\frac{1}{5}$, 0.25

Rational numbers can be compared easily when they are represented on the number line



Any number on the number line which lies on the right side of a given number is greater than that number. Thus $\frac{3}{5} > \frac{1}{5}$; $-\frac{1}{5} > -\frac{3}{5}$ and so on. On the other hand any number on the number line which lies on the left side of a given number is less than that number.

Thus $\frac{2}{5} < \frac{4}{5}$; $-\frac{4}{5} < -\frac{2}{5}$ etc.

UNIT 2

Exercise 2.1

1. Classify the following numbers as a whole number, an integer or a rational number.

   (i) 3 , − 1.2 , − 2 , 0 (ii) 4.5 , $\frac{3}{4}$ − 2.1 , 0.5

   (iii) 3.6 , − 1.5 , − 0.31 , − 2.8 (iv) $\frac{1}{6}$, 1.75 , $-\frac{2}{3}$, 10

2. Fill in the blanks with <, >, = to make a sentence true.

   (i) 40% ------------------ $\frac{12}{25}$ (ii) 0.82 ------------------ $\frac{5}{6}$

   (iii) $\frac{3}{5}$ .... 59% (iv) $\frac{9}{20}$ ------------------ 0.45

   (v) $\frac{5}{8}$ m ------------------ $\frac{1}{16}$ m (vi) 0.25 kg ------------------ $\frac{2}{9}$ kg

3. Arrange numbers in each set from least to greatest using a number line.

   (i) 0.23 , 19% , $\frac{1}{5}$ (ii) $\frac{8}{10}$, 81% , 0.805

   (iii) 0.615 , $\frac{5}{8}$, 62% (iv) − 5.2 , $-\frac{3}{8}$, − 6 , 0.3 , $-\frac{1}{4}$

   (v) 2.1 , 0 , $-\frac{13}{10}$, − 1.38 , $\frac{3}{5}$

4. Arrange the following rational numbers in descending order.

   (i) $\frac{3}{4}$, $\frac{5}{9}$, $\frac{7}{12}$ (ii) 3 , − 1.2 , − 2 , 0

   (iii) 4.5 , $\frac{3}{4}$, − 2.0 , 0.5 (iv) 3.6 , − 1.5 , − 0.31 , − 2.8

   (v) $\frac{1}{6}$, 1.75 , $-\frac{2}{3}$, 0

5. The length of four insects are 0.02 cm, $\frac{1}{8}$ cm, 0.1 cm and $\frac{2}{3}$ cm.
   Arrange the length in ascending order.

6. The cost of four items on a menu are Rs. 9, Rs. 0.99, Rs. 9.99, and Rs. 19.99.
   Order these costs from least to greatest.

7. Ahmed and Sara played darts.
Ahmed hit the bull's eye 5 out of 18 times.
Sara missed the bull's eye 4 out of 15 times.
Who played better?

8. Three paint brushes have widths of $\frac{3}{8}\,cm$, $\frac{1}{2}\,cm$, $\frac{3}{4}\,cm$.
What is the measure of the brush with the greatest width?

9. The world's five largest deserts are shown in the table.
Order the areas from least to greatest.

Desert	Area (millions of square meters)
Sahara	$\frac{7}{2}$
Kalahari	$\frac{2}{10}$
Gobi	$\frac{2}{5}$
Australian	$1\frac{4}{10}$
Arabian	$\frac{1}{2}$

10. An investor purchases 50 shares of stock at Rs. 3.50 per share.
The next day, the change in value of a share of the stock is – 0.25.
What is the total value of the shares on the next day.

UNIT 2

Addition and Subtraction of Rational Numbers

Rational numbers with different denominators

To add or subtract two rational numbers with different denominators, take LCM and add or subtract the numerator of the equivalent fractions obtained.

> **Rational numbers with the same denominators**
>
> To add or subtract two rational numbers with the same denominator, add their numerators together and divide the sum by common denominator.
>
> For Example $\frac{5}{7}+\frac{1}{7}=\frac{6}{7}$, $\frac{5}{7}-\frac{1}{7}=\frac{4}{7}$

Example:

Sana walked $1\frac{5}{8}$ km on Saturday and $2\frac{1}{2}$ km on Sunday.

How many kilometers did she walked altogether?

This can be calculated by adding both the fractions.

$$1\frac{5}{8}\ km+\ 2\frac{1}{2}\ km$$
$$=\frac{13}{8}\ km+\ \frac{5}{2}\ km$$
$$=\frac{13}{8}\ km+\ \frac{5\times 4}{2\times 4}\ km$$
$$=\frac{13}{8}\ km+\ \frac{20}{8}\ km$$
$$=\frac{33}{8}\ km=4\frac{1}{8}\ km$$

Therefore, Sana walked $4\frac{1}{8}$ *km* altogather.

Alternatively it can be solved as:
1.6 km + 2.5 km = 4.125 km
So, Sana walked 4.125 km altogether.

Example:

How many more kilometers did Sana walk on Sunday?

This can be calculated by subtracting kilometers walked on Saturday from kilometers walked on Sunday.

$$2\frac{1}{2}\ km-1\frac{5}{8}\ km$$
$$=\frac{5}{2}\ km-\frac{13}{8}\ km$$
$$=\frac{5\times 4}{2\times 4}\ km-\frac{13}{8}\ km\ =\frac{20}{8}\ km-\frac{13}{8}\ km\ =\frac{7}{8}\ km$$

Therefore, Sana walked $\frac{7}{\ }$ *km* more on Sunday.

Alternatively it can be solved as:

2.5 km – 1.625 km = 0.875 km

So, Sana walked 0.875 km more on Sunday.

Example:

$$5\frac{1}{2}+\left(-3\frac{3}{5}\right)$$

$$=\frac{11}{2}-\left(\frac{18}{5}\right)$$

$$=\frac{11\times5}{2\times5}-\frac{18\times2}{5\times2}=\frac{55}{10}-\frac{36}{10}$$

$$=\frac{19}{10}=1\frac{9}{10}$$

Check Point

1. Evaluate
   - (a) $4\frac{1}{8}+5\frac{11}{12}$
   - (b) $6\frac{5}{8}-2\frac{1}{3}$
2. Rehan spent $2\frac{1}{2}$ hours watching a movie and $1\frac{1}{4}$ hours online. How much time did he spend on these activities?
3. Nabeel is $5\frac{1}{2}$ feet tall and his younger brother is $4\frac{3}{4}$ feet tall. How much taller is Nabeel than his brother?

Addition of Rational Numbers

The table shows the annual profits of two manufacturers.

Which manufacturer had the greater total profit for the three years?

Year	Profit (millions) for manufacturer A	Profit (millions) for manufacturer B
1	– 5.8	– 6.5
2	8.7	7.9
3	6.8	8.2

Step 1: Calculate the total profit for each manufacturer.

Manufacturer A:

Total profit = – 5.8 + 8.7 + 6.8
= – 5.8 + (8.7 + 6.8)
= – 5.8 + 15.5
= 9.7

Manufacturer B:

Total profit = – 6.5 + 7.9 + 8.2
= – 6.5 + (7.9 + 8.2)
= – 6.5 + 16.1
= 9.6

Step 2: Compare the total profits: 9.7 > 9.6.
Therefore, manufacturer A had the greater total profit.

Commutative Property:

The order in which you add two rational numbers does not change the sum.
If a and b are any two rational numbers, then

$$a + b = b + a$$

This is called the commutative property of addition.

$$3 + (-2) = -2 + 3$$

$$\frac{1}{4} + \left(-\frac{3}{8}\right) = \left(-\frac{3}{8}\right) + \frac{1}{4} = -\frac{1}{8}$$

Key Fact

In mathematics, an operation is commutative if changing the order of the operands does not change the result.
"3 + 4 = 4 + 3" or
"2 × 5 = 5 × 2".

Associative Property:

The way you group three rational numbers in a sum, does not change the sum.
If a, b and c are any three rational numbers, then

$$(a + b) + c = a + (b + c)$$

This is called the associative property of addition.

$$(-3 + 2) + 1 = -3 + (2 + 1) = 0$$

$$\left(\frac{1}{3} + \frac{5}{6}\right) + \frac{2}{12} = \frac{1}{3} + \left(\frac{5}{6} + \frac{2}{12}\right) = \frac{16}{12}$$

Check Point

Identify the property being illustrated:

(a) $7 + (-7) = 0$

(b) $-12 + 0 = -12$

(c) $4 + 8 = 8 + 4$

Identity Property:

The sum of a rational number and 0, is the same rational number.
If a is any rational number then

$$a + 0 = 0 + a = a$$

The number 0 is called the additive identity.

$$-5 + 0 = 0 + (-5) = -5$$

$$\left(-\frac{3}{5}\right) + 0 = 0 + \left(-\frac{3}{5}\right) = \left(-\frac{3}{5}\right)$$

At noon, the temperature in your city was 63°F. By midnight, the temperature fell to −21°F.

We can find the fall in temperature as follows:

= 63°F − (−21)°F

= 63°F + 21°F = 84°F

Inverse Property:

The sum of a rational number and its opposite is 0.
If a is any rational number then

$$a + (-a) = -a + a = 0$$

Then a and $-a$ are called the additive inverses of each other.

$$6 + (-6) = -6 + 6 = 0$$

$$\frac{1}{2} + \left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right) + \frac{1}{2} = 0$$

The inverse property states that the sum of a number a and its opposite is 0.

Summary

Statement	Property
$(3 + 9) + 2 = 3 + (9 + 2)$	Associative property of addition
$8.3 + (-8.3) = 0$	Inverse property of addition
$-1 + 0.7 = 0.7 + (-1)$	Commutative property of addition
$\frac{1}{3} + 0 = \frac{1}{3}$	Additive Identity

Exercise 2.2

1. Evaluate the following, expressing your answer in the simplest form.

(i) $8\frac{5}{12} + 11\frac{1}{4}$
(ii) $8\frac{3}{8} + 10\frac{1}{3}$
(iii) $9\frac{1}{5} - 2\frac{3}{5}$
(iv) $14\frac{1}{6} - 7\frac{1}{3}$
(v) $8 - 3\frac{2}{3}$
(vi) $13 - 5\frac{5}{6}$

2. Evaluate the following.

(i) $4\frac{2}{3} + 1\frac{5}{3} - 1\frac{1}{4}$
(ii) $6\frac{1}{6} + 1\frac{2}{3} + 5\frac{5}{9}$
(iii) $3\frac{1}{4} + 2\frac{5}{6} - 4\frac{1}{3}$
(iv) $4\frac{3}{5} - 1\frac{2}{15} - 2\frac{7}{9} - \frac{2}{45}$
(v) $8\frac{1}{3} - 1\frac{1}{9} - \frac{5}{18} - 2\frac{5}{6}$

3. Find the value of the following.

(i) $6\frac{1}{5} + \left(-2\frac{3}{10}\right)$
(ii) $7\frac{1}{2} + \left(-3\frac{3}{5}\right)$
(iii) $-\frac{7}{4} - \left(-\frac{5}{6}\right) + \left(-1\frac{1}{3}\right)$
(iv) $-2\frac{3}{4} + \left(-\frac{5}{6}\right) + \left(-\frac{2}{3}\right)$
(v) $-3\frac{1}{2} - \left(-7\frac{2}{5}\right) + \left(-9\frac{3}{10}\right)$
(vi) $8\frac{2}{3} - \left(-6\frac{3}{5}\right) + 3\frac{1}{4}$

UNIT 2

4. Identify the property being illustrated.

(i) $-3 + 3 = 0$ (ii) $(-6 + 1) + 7 = -6 + (1 + 7)$

(iii) $9 + (-1) = -1 + 9$ (iv) $8 + 0 = -8$

(v) $(1 + 2) + 3 = 1 + (2 + 3)$ (vi) $1 + (-4) = -4 + 1$

(vii) $8\frac{2}{3} + \left(-8\frac{2}{3}\right) = 0$ (viii) $8\frac{2}{3} + \left(-6\frac{3}{5}\right) = -6\frac{3}{5} + \left(8\frac{2}{3}\right)$

(ix) $\left(-6\frac{3}{4}\right) + 0 = \left(-6\frac{3}{4}\right)$

(x) $-3\frac{1}{2} + \left\{\left(-7\frac{2}{5}\right) + \left(-9\frac{3}{10}\right)\right\} = \left\{-3\frac{1}{2} + \left(-7\frac{2}{5}\right)\right\} + \left(-9\frac{3}{10}\right)$

5. Ahmed bought $1\frac{1}{4}$ kg beef and $2\frac{5}{8}$ kg of chicken.
How much more chicken did he buy?

6. Laiba had $4\frac{2}{3}$ cups of chopped walnuts. She used $1\frac{1}{4}$ cups in a recipe.
How many cups of chopped walnuts are left?

7. Salma wakes up at 6:00 a.m.
It takes her $1\frac{1}{4}$ hours to shower, get dressed, and comb her hair.
It takes her $\frac{1}{2}$ hour to eat breakfast, brush her teeth and make her bed.
At what time she will be ready for school?

8. Out of 160 cell phone owners, $\frac{3}{8}$ use their phone for text messaging only, $\frac{1}{4}$ prefer playing games only and remaining owners prefer only taking pictures.

What fraction of owners prefer using their cell phones for taking pictures?

9. The temperature in a city at 6 a.m. was -8° F and increased by 15° F by noon. What was the temperature at noon?

10. The temperature in a fridge is 5°C. The temperature in a freezer is -20°C.
How many degrees colder is the freezer?

Multiplication of Rational Numbers

Humans sleep about $\frac{1}{3}$ of each day. If each year is equal to 365 $\frac{1}{4}$ days, determine the number of days in a year the average human sleeps can be calculated as follows:

$$\begin{aligned}\text{The number of days in a year a human sleep} &= \frac{1}{3} \text{ of } 365\frac{1}{4}\\ &= \frac{1}{3} \times 365\frac{1}{4}\\ &= \frac{1}{3} \times \frac{1461}{4} \quad \text{(dividing 1461 by 3)}\\ &= \frac{487}{4}\\ &= 121\ \frac{3}{4}\end{aligned}$$

The average human sleeps $121\frac{3}{4}$ days each year.

Multiplication of Rational Numbers

To find the product of two rational numbers, numerator is multiplied with numerator and denominator is multiplied with denominator.

If $\frac{a}{b}, \frac{c}{d}$ are rational numbers then their product is

$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$ where a, b, c, d are integers and $b \neq 0$ and $d \neq 0$

Example:

Evaluate: $\frac{1}{2} \times 4\frac{2}{5}$

$$\frac{1}{2} \times 4\frac{2}{5} = \frac{1}{2} \times \frac{22}{5}$$

$$= \frac{1 \times 22}{2 \times 5} = \frac{11}{5} = 2\frac{1}{5}$$

Check Point

Multiply and write in simplest form.

(a) $\frac{1}{4} \times 8\frac{4}{9}$ (b) $5\frac{1}{3} \times 3$

(c) $1\frac{7}{8} \times 2\frac{2}{5}$

Example:

Find the product:

(a) $-3\ (6) = -18$ (b) $2\ (-5)\ (-4) = (-10)\ (-4) = 40$

(c) $\frac{-1}{2}\ (-4)\ (-3) = 2\ (-3) = -6$

Check Point

Find the product:

(a) $-2\ (7)$

(b) $-0.5\ (-4)(-9)$

(c) $\frac{3}{4}\ (\ 3)\ (7)$

UNIT 2

Properties of Multiplication

Commutative Property:

The order in which you multiply two numbers does not change the product.
If a and b are any two rational numbers then

$$a \times b = b \times a$$

This is called the commutative property of multiplication.

$$4 \times (-5) = (-5) \times 4 = -20$$

$$\frac{2}{5} \times (-3) = (-3) \times \frac{2}{5} = \frac{-6}{5}$$

$$\frac{3}{5} \times \left(\frac{4}{5}\right) = \left(\frac{4}{5}\right) \times \frac{3}{5} = \frac{12}{25}$$

Associative Property:

The way you group three rational numbers in a product does not change the product.
If a, b and c are two rational numbers then

$$(a \times b) \times c = a \times (b \times c)$$

This is called associative property of multiplication.

$$(-2 \times 7) \times 4 = -2 \times (7 \times 4) = -56$$

$$\frac{15}{8} \times \left(\frac{2}{3} \times \frac{3}{5}\right) = \left(\frac{15}{8} \times \frac{2}{3}\right) \times \frac{3}{5} = \frac{3}{4}$$

Identity Property:

The product of a rational number and 1 is the same original number.
If a is any rational number then

$$a \times 1 = 1 \times a = a$$

The number 1 is called the multiplicative identity.

$$(-5) \times 1 = 1 \times (-5) = -5$$

$$\frac{2}{7} \times 1 = 1 \times \frac{2}{7} = \frac{2}{7}$$

The identity property states that the product of a number a and 1 is a.

Summary

Statement	Property
$2 \times (5 \times 9) = (2 \times 5) \times 9$	Associative property of multiplication
$8 \times 0 = 0$	Multiplicative property of zero
$(-6) \times 4 = 4 \times (-6)$	Commutative property of multiplication
$9 \times -1 = -9$	Multiplicative property of -1
$1 \times 4 = 4$	Multiplicative Identity
$\frac{3}{4} \times \frac{4}{3} = 1$	Multiplicative Inverse

Check Point

Identify the properties.

(a) $2 \times (4 \times 9) = (2 \times 4) \times 9$ (b) $(-5) \times (-6) = (-6) \times (-5)$ (c) $12 \times 2 = 2 \times 12$

(d) $(-13) \times (-1) = 13$ (e) $0 \times (-41) = 0$

Distributive Property:

Junaid did 5 hours and 9 hours of community work in the first and second half of the year respectively. He was awarded 4 points for each hour of community work. Let us find the total points Junaid received for the whole year.

The points he received can be calculated in two ways as shown below:

(i) $4 \times (5 + 9) = 4 \times (14) = 56$

(ii) $(4 \times 5) + (4 \times 9) = 20 + 36 = 56$

So, $4 \times (5 + 9) = (4 \times 5) + (4 \times 9) = 56$

The product of the rational numbers a and $(b + c)$

$$a \times (b + c) = (a \times b) + (a \times c)$$

This is called the distributive property of multiplication over addition.

$$4 \times (5 + 9) = (4 \times 5) + (4 \times 9) = 56$$

$$\frac{1}{4} \times \left[\frac{3}{10} + \frac{5}{2}\right] = \left[\frac{1}{4} \times \frac{3}{10}\right] + \left[\frac{1}{4} \times \frac{5}{2}\right] = \frac{28}{40}$$

The product of the rational numbers a and $(b - c)$

$$a \times (b - c) = (a \times b) - (a \times c)$$

This is called the distributive property of multiplication over subtraction.

$$4 \times (5 - 9) = (4 \times 5) - (4 \times 9) = -16$$

$$\frac{1}{4} \times \left[\frac{3}{10} - \frac{5}{2}\right] = \left[\frac{1}{4} \times \frac{3}{10}\right] + \left[\frac{1}{4} \times \frac{5}{2}\right] = \frac{-22}{40}$$

Let us look at more examples of distributive property.

Example: Prove that $-2 \times (-3+4) = (-2) \times (-3) + (-2) \times 4$

L.H.S = $-2 \times (-3+4) = -2 \times (+1) = -2$

R.H.S = $(-2) \times (-3) + (-2) \times 4 = 6 + (-8) = -2$

Therefore, $-2 \times (-3+4) = (-2) \times (-3) + (-2) \times 4$

Example: Prove that $(-3-4) \times (-2) = (-3) \times (-2) - 4 \times (-2)$

L.H.S = $(-3-4) \times (-2) = -7 \times (-2) = 14$

R.H.S = $(-3) \times (-2) - 4 \times (-2) = 6 + 8 = 14$

So, $(-3-4) \times (-2) = (-3) \times (-2) - 4 \times (-2)$

Example: Use distributive property to find the value of $(-12 \times 10) - (-9) \times 10$

$$(-12 \times 10) - (-9) \times 10 = [-12 - (-9)] \times 10$$
$$= -3 \times 10 = -30$$

Example: Prove that $\frac{1}{6} \times \left[\frac{7}{10} + \frac{1}{2}\right] = \left[\frac{1}{6} \times \frac{7}{10}\right] + \left[\frac{1}{6} \times \frac{1}{2}\right]$

L.H.S = $\frac{1}{6} \times \left[\frac{7}{10} + \frac{1}{2}\right] = \frac{1}{6} \times \left[\frac{7}{10} + \frac{5}{10}\right]$

$$= \frac{1}{6} \times \frac{12}{10} = \frac{1}{5}$$

R.H.S = $\left[\frac{1}{6} \times \frac{7}{10}\right] + \left[\frac{1}{6} \times \frac{1}{2}\right] = \frac{7}{60} + \frac{1}{12}$

$$= \frac{12}{60} = \frac{1}{5}$$

Check Point

Use distributive property to find the values of:

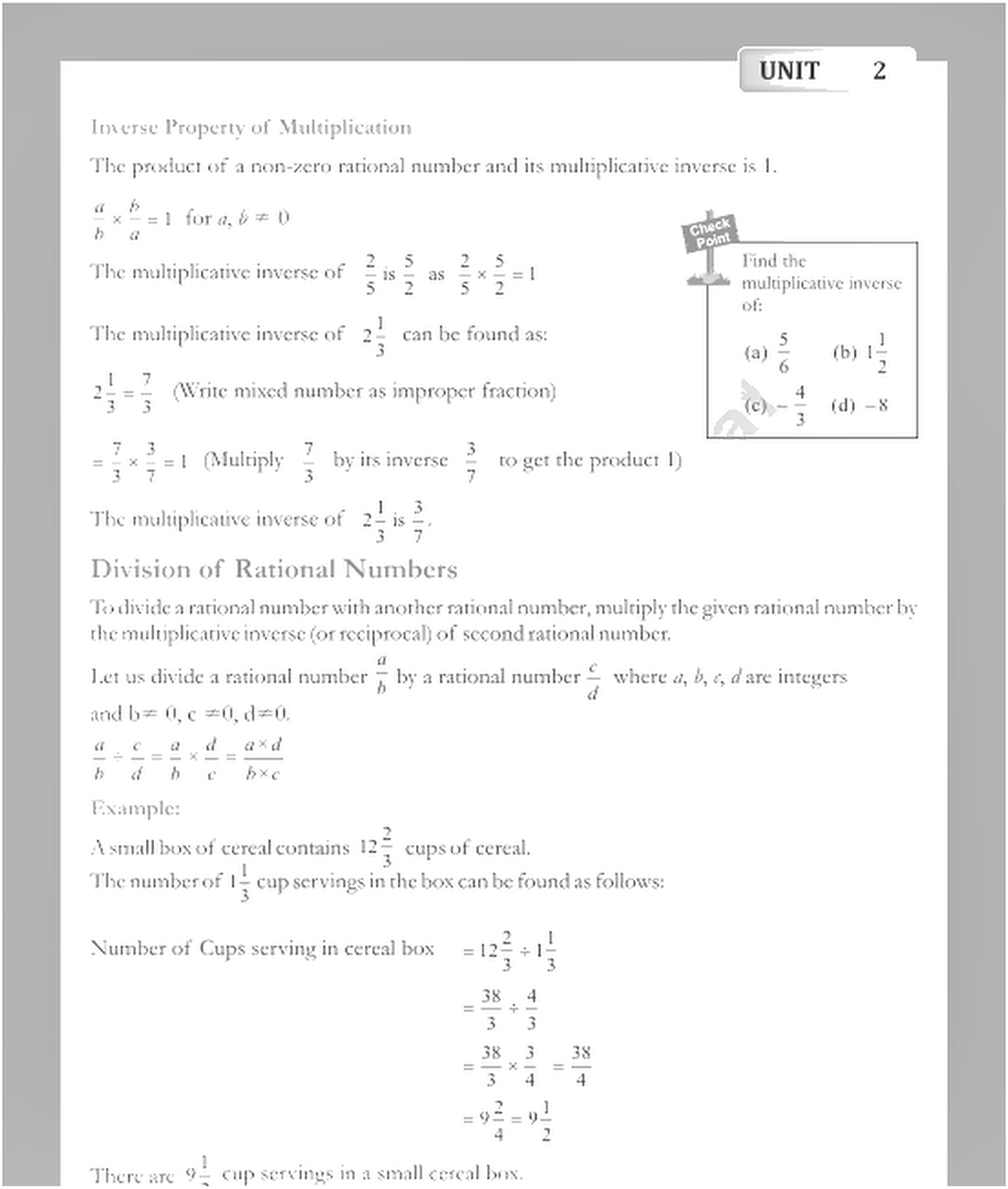
(a) $\left[\frac{1}{3} \times \frac{7}{9}\right] + \left[\frac{1}{3} \times \frac{2}{18}\right]$ (b) $\left[\frac{1}{4} \times \frac{1}{2}\right] - \left[\frac{1}{4} \times \frac{1}{2}\right]$

Multiplicative Inverse

Two non zero numbers with a product of 1 are called multiplicative inverses.

For example $\frac{3}{4} \times \frac{4}{3} = 1$

The reciprocal of a non-zero number a, written $\frac{1}{a}$ is called the multiplicative inverse of a.



Inverse Property of Multiplication

The product of a non-zero rational number and its multiplicative inverse is 1.

$\frac{a}{b} \times \frac{b}{a} = 1$ for $a, b \neq 0$

The multiplicative inverse of $\frac{2}{5}$ is $\frac{5}{2}$ as $\frac{2}{5} \times \frac{5}{2} = 1$

The multiplicative inverse of $2\frac{1}{3}$ can be found as:

$2\frac{1}{3} = \frac{7}{3}$ (Write mixed number as improper fraction)

$= \frac{7}{3} \times \frac{3}{7} = 1$ (Multiply $\frac{7}{3}$ by its inverse $\frac{3}{7}$ to get the product 1)

The multiplicative inverse of $2\frac{1}{3}$ is $\frac{3}{7}$.

Check Point

Find the multiplicative inverse of:

(a) $\frac{5}{6}$ (b) $1\frac{1}{2}$

(c) $-\frac{4}{3}$ (d) -8

Division of Rational Numbers

To divide a rational number with another rational number, multiply the given rational number by the multiplicative inverse (or reciprocal) of second rational number.

Let us divide a rational number $\frac{a}{b}$ by a rational number $\frac{c}{d}$ where a, b, c, d are integers and $b \neq 0$, $c \neq 0$, $d \neq 0$.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c}$$

Example:

A small box of cereal contains $12\frac{2}{3}$ cups of cereal.
The number of $1\frac{1}{3}$ cup servings in the box can be found as follows:

Number of Cups serving in cereal box $= 12\frac{2}{3} \div 1\frac{1}{3}$

$$= \frac{38}{3} \div \frac{4}{3}$$

$$= \frac{38}{3} \times \frac{3}{4} = \frac{38}{4}$$

$$= 9\frac{2}{4} = 9\frac{1}{2}$$

There are $9\frac{1}{2}$ cup servings in a small cereal box.

UNIT 2

- The quotient of two rational numbers with the same signs is positive.
- The quotient of two rational numbers with different signs is negative.
- The quotient of 0 and any non-zero rational number is zero.

Example:

Find the quotient of (a) $-16 \div 4$ (b) $-20 \div \left(-\frac{5}{3}\right)$

(a) $-16 \div 4 = -16 \times \frac{1}{4} = -4$

(b) $-20 \div \left(-\frac{5}{3}\right) = -20 \times \left(-\frac{3}{5}\right) = 12$

Check Point

Find the quotient

(a) $-64 \div (-4)$ (b) $\frac{3}{8} \div \left(-\frac{3}{10}\right)$

(c) $-18 \div \left(-\frac{2}{9}\right)$ (d) $\left(-\frac{2}{5}\right) \div 18$

Example:

Days	1	2	3	4	5
Temperature	– 21	– 29	– 39	– 39	– 22

To find the mean of the daily minimum temperature for 5 days, find the sum of minimum temperatures for the five days and then divide the sum by 5.

Mean: $$\frac{-21+(-29)+(-39)+(-39)+(-22)}{5} = -\frac{150}{5} = -30$$

The mean daily minimum temperature was – 30 degrees

Example:

Evaluate:

$5\frac{3}{5} \div 4\frac{2}{3}$

$= \frac{28}{5} \div \frac{14}{3} = \frac{28}{5} \times \frac{3}{14}$

$= \frac{2\times3}{5\times1} = \frac{6}{5} = 1\frac{1}{5}$

Example:

Evaluate:

$-\frac{14}{15} \div \left(-\frac{7}{3}\right)$

$-\frac{14}{15} \div \left(-\frac{7}{3}\right)$

$= -\frac{14}{15} \times \left(-\frac{3}{7}\right) = \frac{2}{5}$

Check Point

1. Divide and give your answer in simplest form.
   (a) $5 \div 1\frac{1}{3}$
   (b) $-\frac{3}{4} \div 1\frac{1}{2}$
   (c) $2\frac{1}{3} \div 5$
2. Divide $5\frac{1}{4}$ kg of cashews into $\frac{3}{4}$ kg bags. How many such bags can be made?

Exercise 2.3

1. Find the product:

(i) $-\frac{5}{6}(-12)(-4)$ (ii) $0.5(-20)(-3)$ (iii) $-1.6(-2)(-10)$

(iv) $18\left(-\frac{2}{3}\right)\left(-\frac{1}{5}\right)$ (v) $\left(-\frac{3}{4}\right)\left(-\frac{1}{3}\right)\left(-\frac{8}{9}\right)$

2. Identify the properties.

(i) $-\frac{2}{5}\times 0=0$ (ii) $-1\times(-6)=6$ (iii) $(-2\times 5)\times 4=-2\times(5\times 4)$

(iv) $-143\times 1=-143$ (v) $0\times(-76)\times 3=0$

3. Find the multiplicative inverse of the number.

(i) -18 (ii) -9 (iii) $-\frac{3}{4}$ (iv) $-\frac{5}{9}$ (v) $-4\frac{1}{3}$ (vi) $-2\frac{2}{5}$

4. Find the quotient:

(i) $-21\div(3)$ (ii) $-\frac{1}{2}\div\left(-\frac{1}{5}\right)$ (iii) $15\div\left(-\frac{3}{4}\right)$ (iv) $\left(-\frac{1}{5}\right)\div(-6)$

5. Find the mean of the following.

(i) 7, −4, 1, −9, −6 (ii) −2, 9, −3, 5 (iii) 0.25, −4, −0.75, −1, 6 (iv) − 0.6, 0.18, −2, 5, −0.5

6. Find the value of the following.

(i) $3\frac{2}{3}\div\left(-2\frac{4}{9}\right)$ (ii) $5\frac{1}{4}\div\left(-2\frac{4}{5}\right)$ (iii) $-2\frac{4}{5}\times\left(-\frac{5}{2}\right)\div\left(-2\frac{4}{3}\right)$

(iv) $1\frac{3}{4}\times\left[\left(\frac{6}{5}\right)\div\left(-\frac{1}{2}\right)\right]$ (v) $-2\frac{4}{10}\times\left(-\frac{15}{2}\right)\div\left(-4\frac{1}{3}\right)$

7. Evaluate, giving each answer as a fraction in its simplest form.

(i) $2\frac{3}{5}+3\frac{3}{4}$ (ii) $\left(5\frac{1}{3}\times\frac{1}{4}\right)-\left(3\frac{1}{3}\div 2\frac{6}{7}\right)$ (iii) $42\div(2+0.4)$ (iv) $2\frac{2}{7}\times\left(3\frac{1}{3}-1\frac{7}{12}\right)$

8. Find the multiplicative inverse of each number.

(i) $\frac{1}{2}$ (ii) $\frac{1}{3}$ (iii) -1 (iv) -0.9

9. To attend the class field trip, 24 students brought their permission slips.
If this represented $\frac{4}{5}$ of the class, how many students are there in the class?

10. An average person spends $\frac{1}{3}$ of his life in sleep.
According to this, if a person spent 26 years in sleep, how old is he?

UNIT 2

I have learnt

- A rational number can be expressed as the ratio of two integers a and b, that is in the form $\frac{a}{b}$, where $b \neq 0$.
- To add or subtract two rational numbers with different denominators, take LCM and add or subtract the numerator of the equivalent fractions obtained.
- To find the product of two rational numbers, numerator is multiplied with numerator and denominator is multiplied with denominator.
- To divide a rational number with another rational number, multiply the given rational number by the multiplicative inverse (or reciprocal) of second rational number.
- To compare two rational numbers
  (i) Express each rational number with positive denominators.
  (ii) Find LCM of positive denominators.
  (iii) Write an equivalent fraction for each rational number.
  (iv) Compare the numerators.

Properties of Addition and Multiplication

Property	Addition	Multiplication
Commutative Property	$a + b = b + a$	$a \times b = b \times a$
Associative Property	$(a + b) + c = a + (b + c)$	$(a \times b) \times c = a \times (b \times c)$
Identity Property	$a + 1 = 1 + a = a$	$a \times 1 = 1 \times a = a$
Inverse Property	$a + (-a) = -a + a = 0$	$\frac{a}{1} \times \frac{1}{a} = 1, a \neq 0$
Property of Zero	$a + 0 - 0 + a - a$	$a \times 0 - 0 \times a - 0$
Distributive Property	$a \times (b + c) = (a \times b) + (a \times c)$	

Words Board

- Rational Number
- Addition of Rational Numbers
- Subtraction of Ration Numbers
- Multiplication of Rational Numbers
- Division of Rational Numbers
- Multiplicative Inverse
- Additive Identity
- Multiplicative Identity
- Distributive Property
  Commutative Property

Review Exercise 2

1. Choose the correct option.

(i) $-(-6) =$

(a) $\frac{1}{6}$ (b) -6 (c) $+6$ (d) 0

(ii) $8 - [-6 - (-10)] =$

(a) 4 (b) -8 (c) 12 (d) -4

(iii) $-3.6 + (-5.5) =$

(a) 9.1 (b) -9.1 (c) 1.9 (d) -1.9

(iv) $16.1 + (-9.3) =$

(a) 25.4 (b) -25.4 (c) 6.8 (d) -6.8

(v) $-\frac{4}{7} + \left(-\frac{9}{14}\right) =$

(a) $\frac{17}{14}$ (b) $-\frac{17}{14}$ (c) $\frac{5}{14}$ (d) $-\frac{5}{14}$

(vi) $\frac{1}{2}(-6)(-3) =$

(a) 4.5 (b) -9 (c) 9 (d) -4.5

(vii) $\frac{3}{2} - 2 - (-3) =$

(a) $\frac{4}{2}$ (b) $-\frac{1}{2}$ (c) $-\frac{4}{2}$ (d) $\frac{1}{2}$

(viii) $-6 \div \left(\frac{3}{13}\right) =$

(a) $\frac{18}{13}$ (b) $-\frac{18}{13}$ (c) $\frac{78}{3}$ (d) $-\frac{78}{3}$

(ix) $\left(\frac{24-40}{8}\right) =$

(a) $\frac{16}{8}$ (b) $-\frac{16}{8}$ (c) $\frac{64}{8}$ (d) $-\frac{64}{8}$

(x) $\left(\frac{-18-9}{-9}\right) =$

(a) $\frac{-9}{-9}$ (b) $\frac{-27}{-9}$ (c) $\frac{9}{-9}$ (d) $\frac{27}{-9}$

UNIT 2

2. Identify the properties:

(i) $3 + (-3) = 0$ (ii) $-1 + 0 = -1$

(iii) $5 + 7 = 7 + 5$ (iv) $3 \times (5 \times 8) = (3 \times 5) \times 8$

(v) $-4 \times (-7) = -7 \times (-4)$ (vi) $25 \times x = x \times 25$

(vii) $-3 \times (-1) = 3$ (viii) $0 \times (-15) = 0$

3. Complete the following.

(i) -6 ____ $4 = 4$ ____ $-6 = -24$ (ii) $22 \times$ ____ $=$ ____ $\times 22 = -264$

(iii) $(-12) \times (8 - 9) = [-12 \times$ ____$] - [-12 \times$ ____$] =$ ____

4. Find the sum or difference for the following.

(i) $-5 + 2$ (ii) $1.3 + (10.4)$ (iii) $\frac{1}{-3} + \frac{1}{6}$ (iv) $-\frac{2}{7} - \frac{5}{14}$ (v) $-\frac{5}{8} + \frac{1}{5}$

5. Find the product or quotient for the following.

(i) $11\,(-7)$ (ii) $-4.5\,(20)\,(2)$ (iii) $-\frac{1}{5}\,(-20)\,(-5)$

(iv) $-\frac{3}{5} \div 12$ (v) $5 \div \left(-\frac{10}{11}\right)$

6. Find the additive and multiplicative inverse of the following rational numbers.

(i) -12 (ii) $\frac{1}{4}$ (iii) $-\frac{3}{4}$ (iv) $-\frac{13}{29}$

7. Simplify the following:

(i) $8\frac{2}{5} + \left(-6\frac{3}{4}\right) + \left(3\frac{1}{6}\right)$ (ii) $5\frac{2}{3} - \left(-2\frac{5}{3}\right) + \left(7\frac{1}{4}\right)$ (iii) $2\frac{2}{3} + \left(-3\frac{2}{3}\right) + \left(-1\frac{1}{2}\right)$

(iv) $7\frac{2}{3} + \left(-5\frac{1}{6}\right) + \left(-3\frac{1}{12}\right)$ (v) $9\frac{2}{3} + \left(5\frac{1}{2}\right) + \left(3\frac{5}{6}\right)$

8. Use distributive law to solve the following.

(i) $\frac{1}{2} \times \frac{1}{3} + \frac{1}{2} \times \frac{1}{4}$ (ii) $-5 \times \left(-\frac{1}{6}\right) - 7 \times \left(-\frac{1}{6}\right)$

9. Mahad bought $4\frac{1}{2}$ liters of ice cream to serve at his birthday party. If a scoop is $\frac{1}{16}$ of a liter, how many scoops can be served?

10. Nahyan has 8 cups of popcorns to divide into $\frac{2}{3}$ cup portions. How many portions will there be?

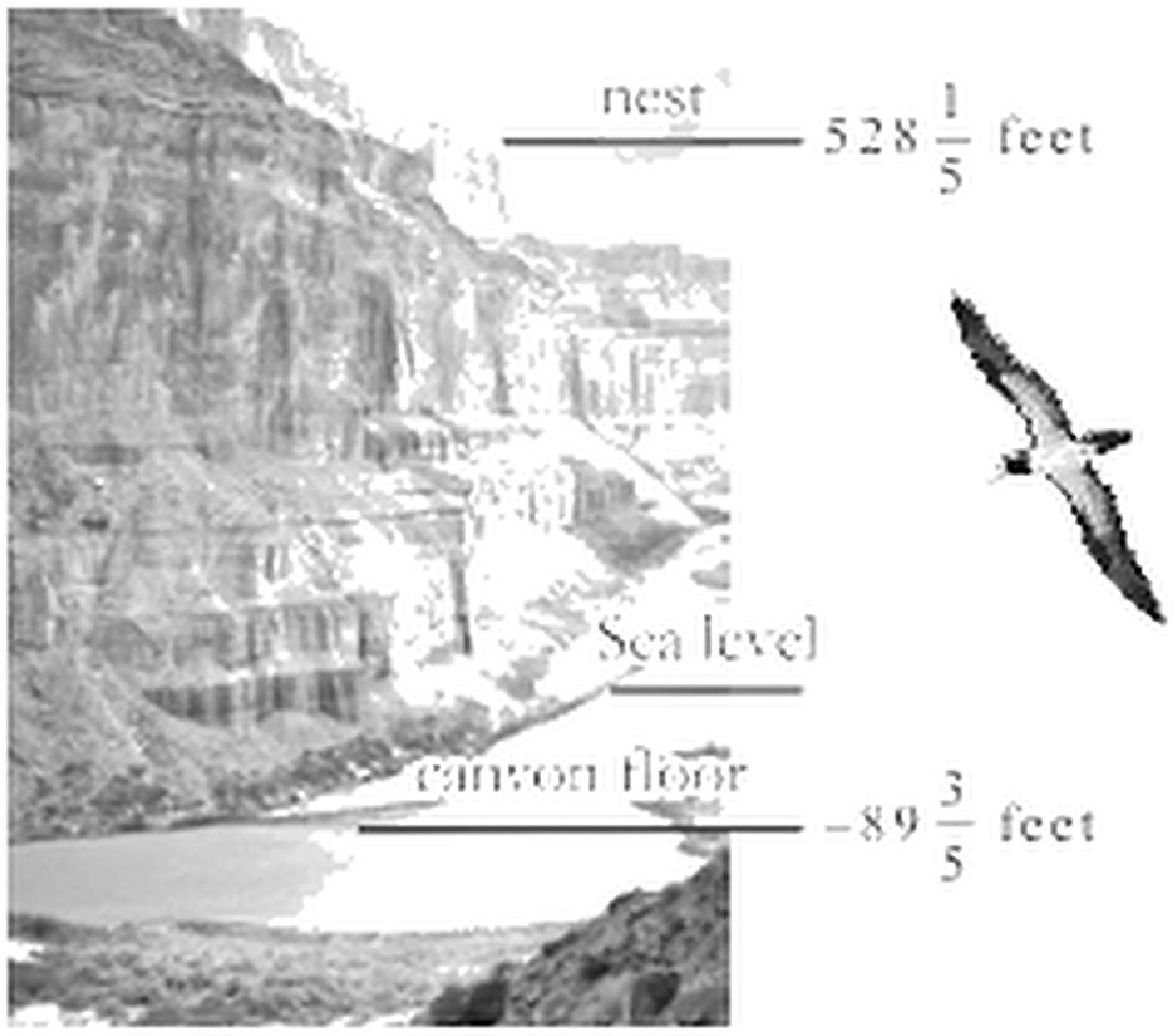
11. Wasiq has $8\frac{1}{4}$ cups of fruit juice. If he divides the juice into $\frac{3}{4}$ cups serving, how many servings are possible?

12. Amna says that $2\frac{5}{8}$ is same as 2.58 .

(a) Convert $2\frac{5}{8}$ to a decimal.

(b) Was Amna correct in saying that $2\frac{5}{8}$ same as 2.58?

13. A bird flies from its nest to the bottom of the canyon. How far did the bird fly?



03

Square and Square Roots

Square roots are important because they have real world applications such as computing areas. Suppose, you are renting an apartment, that has a square floor plan that covers 400 square feet. You know by taking the square root that this must be a 20-foot by 20 - foot room.

Learning Outcomes

Students will be able to:

- Identify base and exponent.
- Recognize square of a number.
- Find the square of a number up to three digits.
- Find the square of a proper fraction and decimal.
- Find square roots of perfect squares upto to six digits by factorization.
- Find the square root of natural numbers, common fractions and decimal fractions by factorization.
- Solve word problems on square root.

UNIT 3

Index Notation

Base and Exponent

exponent

The prime factorization of $125 = 5 \times 5 \times 5$, which can be written as $5 \times 5 \times 5 = 5^3$

base

Here, base is 5 and exponent is 3.

5	125
5	25
5	5
	1

Example: Find the base and exponent of 3^4.

Base is 3 and exponent is 4.

3^4 can be written as $3^4 = 3 \times 3 \times 3 \times 3$

Similarly, in 6^5, base = 6, exponent = 5.
6^5 is read as "6 raised to the power 5". This means that 6 is being multiplied by itself 5 times i.e.
$6^5 = 6 \times 6 \times 6 \times 6 \times 6$.

Key Fact

The base number tells what number is being multiplied, the exponent indicates that how many times the base number is being multiplied.

Check Point

Find the base and exponent of the following.

(i) 3^8

(ii) 7^2

(iii) x^y

Key Point

When a number x is multiplied n times it can be written in index notation as

$x \times x \times x \times$ .... n times $= x^n$

Here, x is base and exponent is n.

Perfect Square

Square of a Natural Number

The area of a square of side 6cm is given by

Area of a square = Length × Length
$= 6 \times 6$
$= 6^2$
$= 36cm^2$

①	2	3	4	5	6
2	④	6	8	10	12
3	6	⑨	12	15	18
4	8	12	⑯	20	24
5	10	15	20	㉕	30
6	12	18	24	30	㊱

Therefore, 36 is said to be the square of 6. We write $6^2 = 36$

36cm²

When exponent of a number is 2, we call it a square of that number.

Key Point

When a number x is multiplied by itself, that is $x \times x = x^2$, then "x^2" is called the square of "x".

UNIT 3

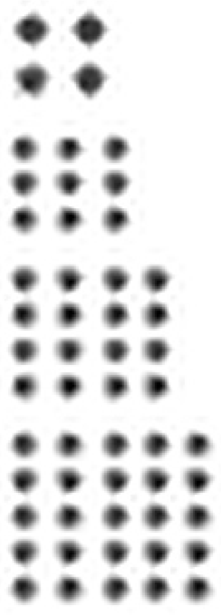
Perfect Square of a Number

$2\,squared = 2^2 = 2\times2 = 4$

$3\,squared = 3^2 = 3\times3 = 9$

$4\,squared = 4^2 = 4\times4 = 16$

$5\,squared = 5^2 = 5\times5 = 25$



$9\,squared = 9^2 = 9\times9 = 81$

The numbers 4, 9, 16, 25 and 81 are squares of 2, 3, 4, 5 and 9 respectively. These .can be written as $4 = 2^2$, $9 = 3^2$, $16 = 4^2$, $25 = 5^2$, $81 = 9^2$.

The square of an even number is always even. i.e.

$2^2 = 4,\ 6^2 = 36$

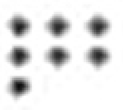
$14^2 = 196,\ 20^2 = 400$

The square of an odd number is always odd i.e

$1^2 = 1,\ 5^2 = 25$

$11^2 = 121,\ 17^2 = 289$

Key Fact

- The numbers which can be represented in the form of squares are perfect squares. The numbers 4, 9, 16, 25, 49, 64, 81, 100, 121, and 144 can be arranged in squares.
- The number 7 cannot be arranged in the form of square,  so 7 is not a perfect square. The numbers 3, 5, 6, 7, 8, 11, 12, 13, 14, 15, 17 and 19 etc. are not perfect squares. These numbers cannot be arranged in the form of squares.

Example:

Find out which of the following numbers are perfect squares.

(i) 144 (ii) 80 (iii) 256 (iv) 190

(i) $144 = 2\times2\times2\times2\times3\times3$

2	144
2	72
2	36
2	18
3	9
	3

(Grouping the factors into the pairs of equal factors)

$144 = 2^2\times2^2\times3^2$

$= (2\times2\times3)^2 \quad = (12)^2$

Hence, 144 is a perfect square.

Key Fact

Product of two or more perfect squares results in a perfect square.

(ii) 80 $80 = 2^2\times2^2\times5$

Hence, 80 is not a perfect square.

2	80
2	40
2	20
2	10
5	5
	1

If a number cannot be written as a product of square is not a perfect square.

(iii) 256 $256 = 2^2 \times 2^2 \times 2^2 \times 2^2 = (2 \times 2 \times 2 \times 2)^2 = (16)^2$

Hence, 256 is a perfect square of 16.

(iv) 190 $190 = 2 \times 5 \times 19$

Hence, 190 is not a perfect square.

If a number cannot be written as a product of squares it is not a perfect square.

Check Point

Check the whether the following are perfect square or not.

(i) 200 (ii) 324 (iii) 400

The Square of Proper Fraction is Less than Itself.

$\frac{3}{5}, \frac{4}{15}, \frac{16}{20}, \frac{4}{15}, \frac{16}{20}$ etc. are proper fractions because numerator is less than its denominator.

The decimal representation of $\frac{3}{5}$ is 0.6. The square of $\frac{3}{5}$ is

$\left(\frac{3}{5}\right)^2 = \frac{3}{5} \times \frac{3}{5} = \frac{3^2}{5^2} = \frac{9}{25} = 0.36$ we note that $\frac{9}{25} < \frac{3}{5}$ i.e. the square of $\frac{3}{5}$ is smaller than $\frac{3}{5}$. Similarly, the square of $\frac{1}{2}$ is $\frac{1}{4}$ which is less than $\frac{1}{2}$.

Key Fact

The square of a proper fraction is less than itself.

If we take decimal number which is less than 1, then its square is also less than the decimal. We consider some decimals less than 1 as: 0.2, 0.6, 0.01, 0.025 etc.

Now the square of $(0.2)^2 = 0.2 \times 0.2 = 0.04$, which is less than 0.2.

Similarly, the square of 0.01 is 0.0001, which is less than 0.01.

We conclude that:

Key Fact

The square of a decimal less than 1 is smaller than the decimal.

Brain Buster

Simplify: $\sqrt{9 + 16}$

Why $\sqrt{9 + 16} \neq \sqrt{9} + \sqrt{16}$

UNIT 3

Exercise 3.1

1. Find the squares of the following numbers.

(i) 7 (ii) 9 (iii) 10 (iv) 13 (v) 16 (vi) 19

(vii) 30 (viii) 100 (ix) 150 (x) 21 (xi) 23 (xii) 25

2. Find the square of the following numbers.

(i) $\frac{2}{3}$ (ii) $\frac{16}{13}$ (iii) $\frac{9}{16}$ (iv) $\frac{7}{23}$

(v) 0.3 (vi) 0.5 (vii) .07 (viii) .09

3. Tell which of the following are perfect squares.

(i) 16 (ii) 27 (iii) 48 (iv) 121

(v) 64 (vi) 72 (vii) 1000 (viii) 625

4. Find all the perfect squares between 1 and 101.

Square Root of a Natural Number

Total students of a college is 625 stand in rows in such a way that the number of rows is equal to the number of students in a row. How many students are there in each row?

Since the number of students in a row is the same as the number of rows, square root of 625 can be found as follows.

We find prime factors of 625

5	625
5	125
5	25
5	5
	1

$$625 = 5 \times 5 \times 5 \times 5$$

$$\sqrt{625} = \sqrt{5 \times 5 \times 5 \times 5}$$

$$= \sqrt{5^2 \times 5^2} = 5 \times 5 = 25$$

Thus, the number of students in each row is 25.

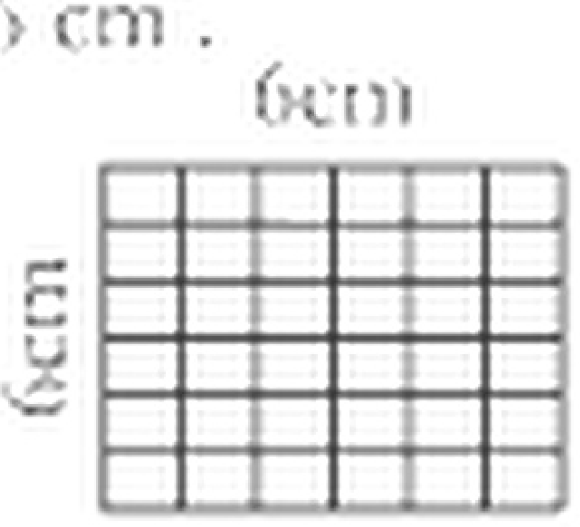
Similarly, to find side of a square whose area is 36 cm^2.

We find a positive number, whose square is 36.

Hence, $36 = 6 \times 6 = 6^2$

we say that 6 is the positive square root of 36

So the length of the square = 6 cm



Here, 36 is a perfect square while 6 is called the square root.

The square root of 36 can be written as:

$\sqrt{36} = 6$ Read: "Square root of 36 is 6"

The sign $\sqrt{\ }$ is used for square root of a number.

The exponent $\frac{1}{2}$ represents square root. Instead of this exponent, we use the notation "$\sqrt{\ }$".

i.e. $\sqrt{36} = (36)^{\frac{1}{2}} = (6^2)^{\frac{1}{2}} = 6$

Key Fact

- If $x^2 = y$ then x is called a square root of y, $x = \sqrt{y}$.
- The square root of a number is another number which when multiplied by itself gives the original number.

Example: Find the square root of 81, 121, 400.

$$\sqrt{81} = \sqrt{9\times 9} = \sqrt{9^2} = (9^2)^{\frac{1}{2}} = 9$$

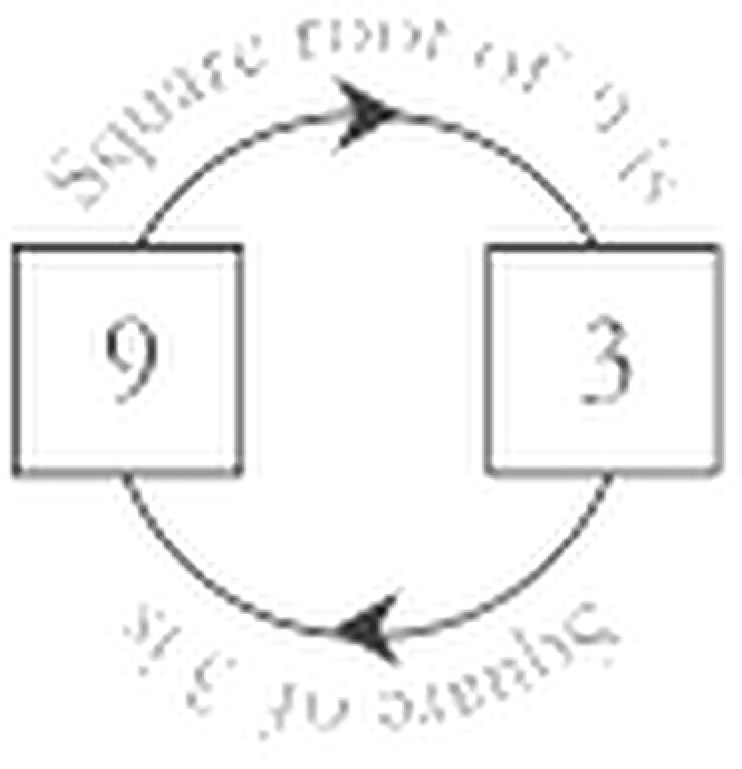
$\sqrt{81} = 9$, here, 9 is called square root of 81.

$$\sqrt{121} = \sqrt{11\times 11} = \sqrt{11^2} = (11^2)^{\frac{1}{2}} = 11$$

Therefore, $\sqrt{121} = 11$, here, 11 is called square root of 121.

$$\sqrt{400} = \sqrt{20\times 20} = \sqrt{20^2} = 20$$

is 20 a square root of 400?



Key Fact

Now look at the following.

$(-4)\,(-4) = 16$; $(4)\,(4) = 16$

$(-5)\,(-5) = 25$; $(5)\,(5) = 25$

$(-6)\,(-6) = 36$; $(6)\,(6) = 36$

We see that 16 is square of both 4 and −4.

Therefore, 16 has two square roots i.e. 4 and −4. Therefore $\sqrt{16} = \pm 4$

Similarly, 25 has two square roots 5 and −5. Thus $\sqrt{25} = \pm 5$ etc

We will study only positive square roots in this grade.

The following are perfect squares.

$\sqrt{1} = \sqrt{1\times 1} = 1$

$\sqrt{4} = \sqrt{2\times 2} = 2$

$\sqrt{9} = \sqrt{3\times 3} = 3$

$\sqrt{16} = \sqrt{4\times 4} = 4$

$\sqrt{25} = \sqrt{5\times 5} = 5$

$\sqrt{36} = \sqrt{6\times 6} = 6$

$\sqrt{49} = \sqrt{7\times 7} = 7$

$\sqrt{64} = \sqrt{8\times 8} = 8$

$\sqrt{81} = \sqrt{9\times 9} = 9$

$\sqrt{100} = \sqrt{10\times 10} = 10$

$\sqrt{121} = \sqrt{11\times 11} = 11$

$\sqrt{144} = \sqrt{12\times 12} = 12$

$\sqrt{169} = \sqrt{13\times 13} = 13$

$\sqrt{225} = \sqrt{15\times 15} = 15$

Check Point

Find the square root of the following:
(i) 441 (ii) 254 (iii) 900

The numbers whose square roots are integers are called perfect squares.

UNIT 3

Square Root by Factorization

Let us find the square root of 256.

The prime factors of 256 are as under.

$256 = 2\times2\times2\times2\times2\times2\times2\times2$

Taking square root of both sides

$$\sqrt{256} = \sqrt{2\times2\times2\times2\times2\times2\times2\times2}$$
$$= \sqrt{2^2\times2^2\times2^2\times2^2}$$
$$= 2\times2\times2\times2$$
$$= 16$$
$$\therefore\ \sqrt{256} = 16$$

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Example: Find the square root of 576.

First find prime factors of 576.

$576 = 2\times2\times2\times2\times2\times2\times3\times3$

Taking square root of both sides

$$\sqrt{576} = \sqrt{2\times2\times2\times2\times2\times2\times3\times3}$$
$$= \sqrt{2^2\times2^2\times2^2\times3^2}$$
$$= 2\times2\times2\times3$$
$$= 24$$
$$\therefore\ \sqrt{576} = 24$$

2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

Example: Find square root of 11025.

$$\sqrt{11025} = \sqrt{3\times3\times5\times5\times7\times7}$$
$$= \sqrt{3^2\times5^2\times7^2}$$
$$= 3\times5\times7$$
$$= 105$$

We have

$$\sqrt{11025} = 105$$

3	11025
3	3675
5	1225
5	245
7	245
7	49
7	7
	1

Example: Verify that

$$\sqrt{36\times25} = \sqrt{36}\times\sqrt{25}$$

L.H.S $= \sqrt{36\times25}$

$= \sqrt{2\times2\times3\times3\times5\times5}$

$= \sqrt{2^2\times3^2\times5^2}$

$= 2\times3\times5 = 30$

R.H.S $= \sqrt{36}\times\sqrt{25}$

$= \sqrt{2\times2\times3\times3}\times\sqrt{5\times5}$

$= 2\times3\times5 = 30$

Hence, $\sqrt{36\times25} = \sqrt{36}\times\sqrt{25}$

Brain Buster

Is $\sqrt{36}\times\sqrt{49} = \sqrt{36\times49}$?

Example: Area of a square is 4225 m^2.

Find the length of its side.

Let the length of square be y metre.

As we know that:

(Length of side)^2 = Area of the square

$(y)^2 = 4225$

Taking square root of both the sides

$$\sqrt{y^2} = \sqrt{4225}$$
$$y = \sqrt{5\times5\times13\times13}$$
$$= 5\times13 = 65$$

Length of the side of the square = 65 m

5	4225
5	845
13	169
13	13
	1

Example:

Area of a rectangular piece of carpet is $98cm^2$. Its length is twice of the width of rectangular field. Find length and width of the piece of carpet.

Let the width of the piece of carpet $= x$ cm

Then, its length $= 2x$ cm

Area $= 98$ cm^2

According to the given condition of question

As length × width = area

$$\therefore \quad (2x)(x) = 98$$

$$2x^2 = 98$$

$$x^2 = \frac{98}{2} = 49$$

Taking square root of both the sides

$$\sqrt{x^2} = \sqrt{49} \text{ or } x = 7$$

Width $= 7$ cm

Length $= 2 \times 7 = 14$ cm

Example: Arrange 1764 soldiers in such a way as the number of rows and number of soldiers in each row should be equal. Find the number of soldiers in each row.

Let the number of soldiers in each row be x then, according to given condition, number of rows is also x.

So, $x \times x = 1764$

$$(x)^2 = 1764$$

Taking square root of both sides

$$\sqrt{x^2} = \sqrt{1764}$$

$$x = \sqrt{2\times2\times3\times3\times7\times7}$$

$$= 2\times3\times7$$

Number of soldiers in each row = 42

CHALLENGE

Describe and correct the error.

$\sqrt{361} = 17$ ✗

Exercise 3.2

1. **Find the square root of the following.**

(i) 196 (ii) 225 (iii) 144 (iv) 625

2. **Compute the following.**

(i) $\sqrt{81}$ (ii) $\sqrt{169}$ (iii) $\sqrt{289}$ (iv) $\sqrt{400}$ (v) $\sqrt{2^2 \times 3^2}$ (vi) $\sqrt{7^2 \times 8^2}$

3. **Find the square root of the following by factorization.**

(i) 36 (ii) 64 (iii) 196 (iv) 324 (v) 441 (vi) 484
(vii) 625 (viii) 841 (ix) 961 (x) 1089 (xi) 1225 (xii) 1521
(xiii) 1024 (xiv) 1764 (xv) 2025 (xvi) 4096 (xvii) 4225 (xviii) 2116
(xix) 4356 (xx) 6400 (xxi) 9801 (xxii) 8100 (xxiii) 12100 (xxiv) 102400

4. **Verify.**

(i) $\sqrt{25\times16} = \sqrt{25}\times\sqrt{16}$ (ii) $\sqrt{36\times4} = \sqrt{36}\times\sqrt{4}$
(iii) $\sqrt{49\times81} = \sqrt{49}\times\sqrt{81}$ (iv) $\sqrt{100\times121} = \sqrt{100}\times\sqrt{21}$
(v) $\sqrt{144\times169} = \sqrt{144}\times\sqrt{169}$

5. **Area of a square region is 121 m^2. Find the length of its side.**
6. **The area of a square photo frame is 256 cm^2. Find the perimeter of the photo frame.**
Product of a number and its half is 72. Find the number.
7. **Area of a square field is 2500 m^2. Find the perimeter of the square field.**
8. **If 5 litres of paint will cover 16 m^2 and you are painting a square pattern, what is the**
9. **length of square the paint will cover?**
10. **Area of a square field is 900 m^2. Find the cost of cementing boundary wall at a rate of Rs. 50 per metre.**
11. **Length of rectangular region is double of its width. Find perimeter if its area is 128 cm^2.**

Finding the Square Root of Common Fractions

Consider the square root of $\frac{16}{25}$

$$\sqrt{\frac{16}{25}} = \frac{\sqrt{2\times2\times2\times2}}{\sqrt{5\times5}} \quad \text{(Write in prime factorization)}$$

$$= \frac{\sqrt{2^2\times2^2}}{\sqrt{5^2}}$$

$$= \frac{2\times2}{5} = \frac{4}{5}$$

Rule: If a and b are any two numbers, then

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Similarly $\sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}}$

Square root of common fraction = $\frac{\text{Square root of numerator}}{\text{Square root of denominator}}$

Key Fact

- To find square root of proper and improper fractions, we take square root of the numerator and the denominator separately. Mixed fractions are converted into improper fraction while finding the square root.

Example: Compute $\sqrt{\dfrac{196}{256}}$

$$\sqrt{\frac{196}{256}} = \frac{\sqrt{196}}{\sqrt{256}}$$
$$= \frac{\sqrt{2\times2\times7\times7}}{\sqrt{2\times2\times2\times2\times2\times2\times2\times2}}$$
$$= \frac{\sqrt{2^2\times7^2}}{\sqrt{2^2\times2^2\times2^2\times2^2}}$$
$$= \frac{2\times7}{2\times2\times2\times2}$$
$$= \frac{14}{16} = \frac{7}{8}$$

Example: Compute $\sqrt{14\dfrac{1}{16}}$

Convert the mixed fraction into common fraction

$$14\frac{1}{16} = \frac{225}{16}$$

Now taking square root $\sqrt{14\dfrac{1}{16}} = \sqrt{\dfrac{225}{16}} = \dfrac{\sqrt{225}}{\sqrt{16}}$

$$= \frac{\sqrt{3\times3\times5\times5}}{\sqrt{2\times2\times2\times2}} = \frac{\sqrt{3^2\times5^2}}{\sqrt{2^2\times2^2}}$$
$$= \frac{3\times5}{2\times2} = \frac{15}{4}$$
$$= 3\frac{3}{4}$$

Example: Find $\sqrt{5\dfrac{41}{64}}$

$$\sqrt{5\frac{41}{64}} = \sqrt{\frac{361}{64}}$$
$$= \frac{\sqrt{19\times19}}{\sqrt{8\times8}} = \frac{\sqrt{19^2}}{\sqrt{8^2}}$$
$$= \frac{19}{8} = 2\frac{3}{8}$$

CHALLENGE

Describe and correct the error.

$$\sqrt{2\frac{7}{9}} = \sqrt{\frac{36}{9}}$$
$$= \frac{6}{3}$$

✗

Exercise 3.3

Find the square root of the following.

(1) $\dfrac{64}{81}$

(2) $\dfrac{361}{529}$

(3) $\dfrac{256}{625}$

(4) $\dfrac{1225}{169}$

(5) $\dfrac{400}{121}$

(6) $\dfrac{144}{49}$

(7) $\dfrac{3025}{4096}$

(8) $\dfrac{1296}{324}$

(9) $\dfrac{196}{729}$

(10) $\dfrac{4761}{841}$

(11) $3\dfrac{22}{49}$

(12) $3\dfrac{13}{81}$

(13) $5\dfrac{41}{64}$

(14) $40\dfrac{41}{64}$

(15) $26\dfrac{25}{36}$

UNIT 3

Square Root of Decimal Fractions by Factorization

Let us find the square root of 1.69.

Change decimal fraction into fraction.

$$\sqrt{1.69} = \sqrt{\frac{169}{100}} = \frac{\sqrt{169}}{\sqrt{100}}$$

$$= \frac{\sqrt{13\times13}}{\sqrt{2\times2\times5\times5}} = \frac{\sqrt{13^2}}{\sqrt{2^2\times5^2}}$$

$$= \frac{13}{2\times5} = \frac{13}{10}$$

$$= 1.3$$

Hence, $\sqrt{1.69} = 1.3$

Key fact

- To find square root of a decimal fraction, first we change it into a common fraction. After finding the square root, the result is changed into decimal fraction again.

Example: Find the square root of 0.0144

$$\sqrt{0.0144} = \sqrt{\frac{144}{10000}} = \frac{\sqrt{144}}{\sqrt{10000}}$$

$$= \frac{\sqrt{12\times12}}{\sqrt{100\times100}} = \frac{12}{100} = 0.12$$

$$= \frac{\sqrt{12\times12}}{\sqrt{100\times100}} = \frac{12}{100} = 0.12$$

$$\sqrt{0.0144} = 0.12$$

Example: The area of a square field is 144.9616 m^2. Find its perimeter.

Solution: To find its perimeter of the field, first we will find the length of one side. To find the length of one side we will take square root of 144.9616.

$$\sqrt{144.9616} = \sqrt{\frac{1449616}{10000}} = \frac{\sqrt{1449616}}{\sqrt{10000}}$$

$$= \frac{\sqrt{1204\times1204}}{\sqrt{100\times100}} = \frac{1204}{100} = 12.04$$

$$\sqrt{144.9616} = 12.04$$

Length of one side = 12.04 m

Perimeter of square = 4(12.04) m = 48.16 m

We know that perimeter of square = 4 (length of a square)

Exercise 3.4

1. Find the square root by factorization for the following:

   (i) 0.09 (ii) 0.16 (iii) 12.25 (iv) 1.44 (v) 0.0256 (vi) 0.1936 (vii) 19.36
   (viii) 11.56 (ix) 30.25 (x) 0.0441 (xi) 0.1225 (xii) 0.0064 (xiii) 2.89 (xiv) 0.000081

2. Area of a square region is 43.56 km^2. Find the length of its side.
3. Find the positive number which when multiplied by itself gives 110.25.
4. What is the length of a side of a square whose area is 23.04 cm^2.
5. Evaluate:

   (i) $\left(\frac{1}{2}\right)^2 + \sqrt{0.25}$ (ii) $\sqrt{(.5)^2 + (1.2)^2}$ (iii) $\sqrt{(0.5)^2 - (0.4)^2}$

I have learnt

- The numbers which can be represented in the form of squares are called perfect squares. e.g. 4, 9, 16, 25, 49, 64, 81, 100, 121, and 144 etc. all are perfect squares.
- The numbers 3, 5, 6, 7, 8, 11, 12, 13, 14, 15, 17 and 19 etc. are not perfect squares. These numbers cannot be written in the form of a perfect square.
- The square root of a given number is the number whose square is equal to the given number.
- The sign "$\sqrt{}$" is used for square root of a number.
- The square of an odd integer is always odd.
- The square of an even integer is always even.
- The square of a proper fraction is less than itself.
- Square root of a number can be found by factorization or by division.
- To find square root of a common fraction, we take the square roots of numerator and denominator separately.
- Following rule is applied to find the square root of common fractions.

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Or Square root of common fraction = $\frac{\text{Square root of numerator}}{\text{Square root of denominator}}$

Words Board

- Index Notation
- Base and Exponents
- Perfect Square
- Square Root
- Square Root of Common Fraction
- Square Root of Decimal Fractions
- Square Root of Natual Number

UNIT 3

Review Exercise 3

1. **Encircle the correct answer for the following questions.**

(i) The square of an odd positive integer is:

(a) odd (b) even (c) odd or even (d) negative number

(ii) The square of an even positive integer is:

(a) odd (b) even (c) odd or even (d) negative number

(iii) $\sqrt{5\times5\times7\times7}$ **is equal to:**

(a) 5 (b) 7 (c) 12 (d) 35

(iv) $\sqrt{121}\times\sqrt{144}$ **is equal to:**

(a) 11 (b) 12 (c) 132 (d) 23

(v) $\sqrt{2\frac{1}{4}}$ **is equal to:**

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

(vi) $\sqrt{1.21}$ **is equal to:**

(a) 1.1 (b) 2.1 (c) 1.2 (d) 0.1

(vii) $\sqrt{\frac{169}{196}}$ **is equal to:**

(a) $\frac{12}{13}$ (b) $\frac{13}{15}$ (c) $\frac{13}{14}$ (d) $\frac{14}{13}$

(viii) $\sqrt{1\frac{7}{9}}$ **is equal to:**

(a) $\frac{4}{5}$ (b) $\frac{4}{3}$ (c) $\frac{3}{4}$ (d) $\frac{5}{3}$

(ix) $\left(\frac{9}{4}\right)^2$ **is equal to:**

(a) $\frac{81}{16}$ (b) $\frac{81}{25}$ (c) $\frac{100}{16}$ (d) $\frac{100}{16}$

(x) $\sqrt{5^2 \times 6^2}$ **is equal to:**

(a) 30 (b) 36 (c) 25 (d) 24

2. Evaluate. $\sqrt{\frac{2809}{4096}}$
3. Find. $\sqrt{2\frac{1337}{3844}}$
4. Find square root of 4900 using factorization.
5. Find square root of 167281 using factorization.
6. Find square root of 19.36.
7. The product of two positive numbers is 2400. One of them is six times the other. Find the numbers.
8. 3600 students are asked to stand in different rows. Every row has as many students as there are rows. Find the number of rows.
9. Find the perimeter of a square whose area is 6889 m^2.
10. A society collected Rs. 8836. Each member contributed as many rupees as there were members. Find the number of members of the society.

Brain Buster

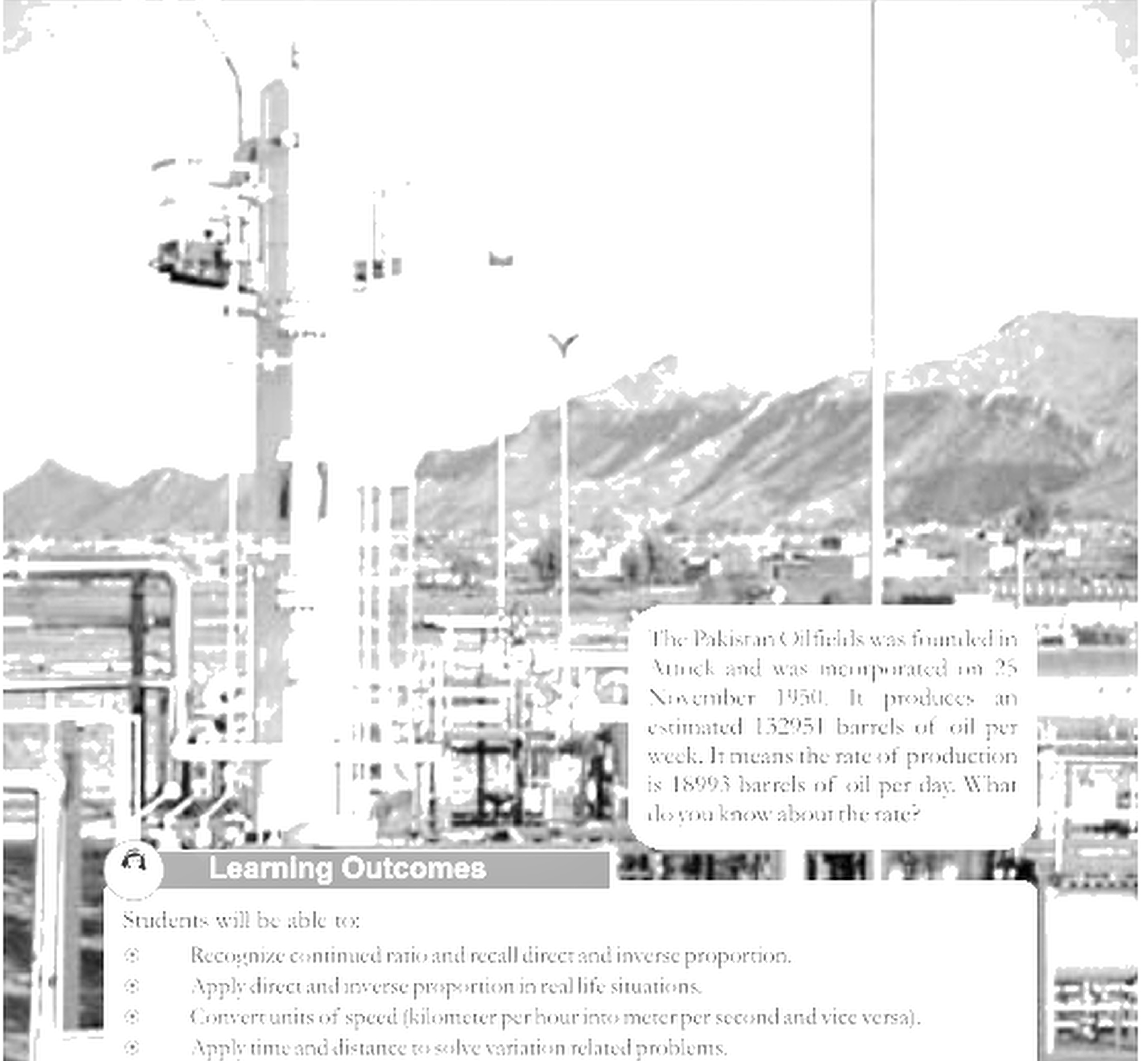
$\sqrt{7-2\sqrt{6}}$ is equal to:

(a) $\sqrt{6}+1$ (b) $\sqrt{6}-1$

(c) $\sqrt{3}+1$ (d) $\sqrt{3}-1$

04

Variation



The Pakistan Oilfields was founded in Attock and was incorporated on 25 November 1950. It produces an estimated 132951 barrels of oil per week. It means the rate of production is 18993 barrels of oil per day. What do you know about the rate?

Learning Outcomes

Students will be able to:

- Recognize continued ratio and recall direct and inverse proportion.
- Apply direct and inverse proportion in real life situations.
- Convert units of speed (kilometer per hour into meter per second and vice versa).
- Apply time and distance to solve variation related problems.

UNIT 4

Ratio

In a school the total number of students is 510 and the total number of teachers is 30. The student teacher ratio in the above situation is 510 : 30 which can be written in simplest form as 51 : 3 or 17 : 1 .

A ratio is a comparison of like quantities measured in like units or quantities of the same kind. The ratio of two quantities a and b, where a and b represent two quantities and b is not equal to zero, can be written as :

$$a \text{ to } b \quad \text{or} \quad a : b \quad \text{or} \quad \frac{a}{b}$$

Check Point

Write each ratio as a fraction in the simplest form.

(a) 32 : 18

(b) 0.84 : 1.12

(c) $2.4 : 1\frac{1}{5}$

A ratio is said to be in simplest form when a and b are integers with no common factors (other than 1).

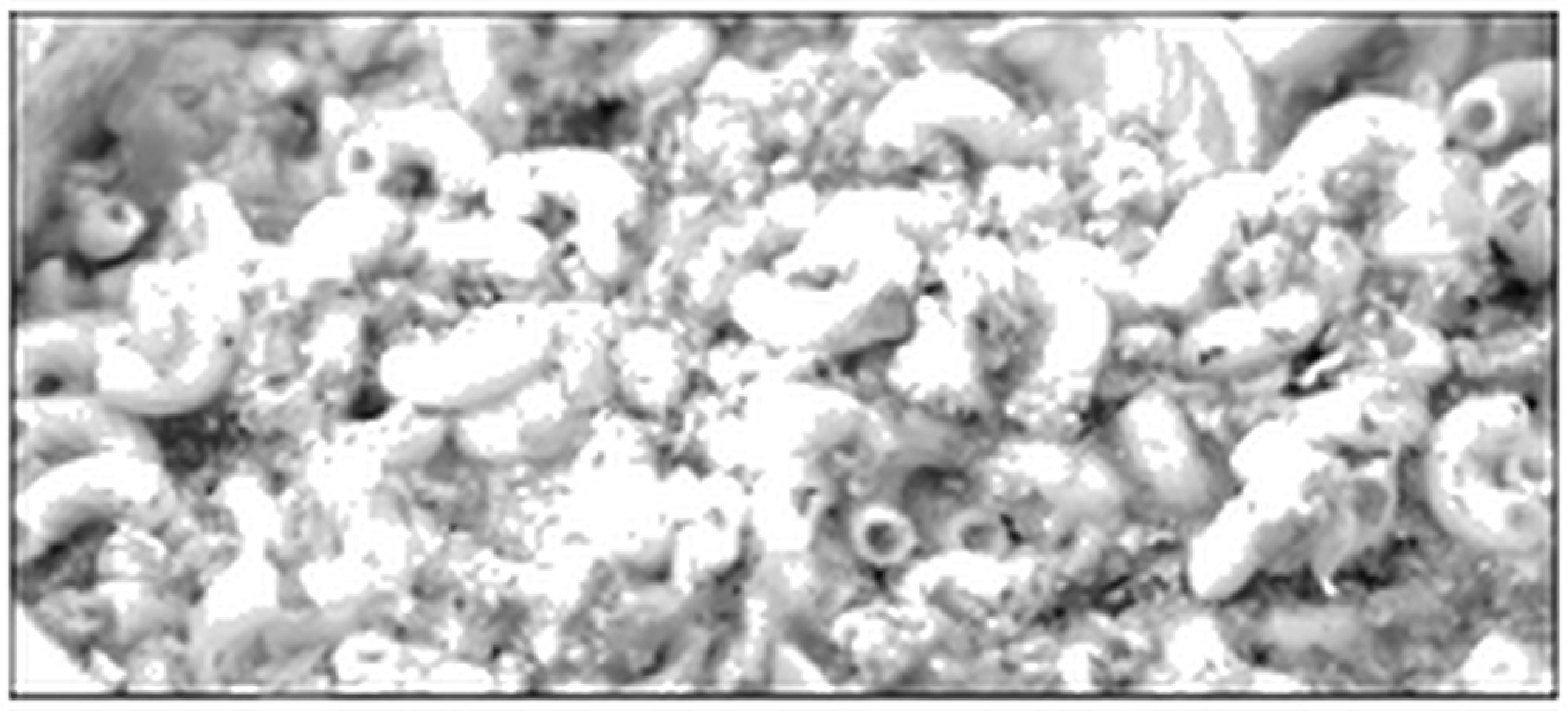
Key Fact

A ratio is expressed as a fraction of the first quantity over the second. To find the ratio of two quantities, we must express them in the same units. A ratio has no unit.

We can write:

$$\frac{\text{number of students}}{\text{number of teachers}} = \frac{510}{30} = \frac{51}{3} = \frac{17}{1}$$

thus 51 : 3 or 17 : 1 are equivalent ratios.



Check Point

In a recipe, the following ingredients are used:
4 tsp garlic powder
6 tsp dried oregano
2 tsp pepper

Use the recipe to write each ratio as a fraction in simplest form.

(a) pepper : garlic powder

(b) oregano: pepper

Continued Ratio

Three friends, A, B and C share the profit of a business. They receive Rs. 4000, Rs. 3000 and Rs. 1000 respectively. The ratio of their share in the profit is then 4000 : 3000 : 1000 or 4 : 3 : 1.

Check Point

Express the following ratios in their simplest forms.
3 : 6 : 10
60 : 96 : 24
9 : 27 : 63

UNIT 4

Example:

The interior angles of a quadrilateral are 40° , 60° , 120° , 140°

The ratio of these angles is 40 : 60 : 120 : 140

which can be written as 4 : 6 : 12 : 14

or 2 : 3 : 6 : 7

The above ratios can be written as

x	:	y		
		y	:	z
x	:	y	:	z

Key Fact

If x, y, z are three quantities and the ratios x : y and y : z are given, then the ratio x : y : z is called continued ratio.

Here y is present in both the ratios and is called the common element.

To find a continued ratio, it is necessary that one element of both the ratios should be the same.

If there is no common (same) element, we can obtain the common element by using idea of equivalent fractions or by multiplying both ratios by different numbers, we can get the same result.

To calculate x : y : z, we find the equivalent ratios of x : y and y : z such that y has the same value in both ratios.

Example:

If x : y = 5 : 7 and y : z = 6 : 8 find

i. x : y : z
ii. x : z

Since $\frac{x}{y} = \frac{5}{7}$ and $\frac{y}{z} = \frac{6}{8}$

i. Now $\frac{x}{z} = \frac{5\times6}{6\times6} = \frac{30}{42}$ and $\frac{y}{z} = \frac{6\times7}{8\times7} = \frac{42}{56}$

y is the common part in the ratio x : y and y : z.
So the LCM of 7 and 6 is 42.

We have x : y : z = 30 : 42 : 56 = 15 : 21 : 28

ii. x : z = 15 : 28

Example:

Find x : y : z, when x : y = 1 : 4 and y : z = 4 : 9

Here y is the common element

x	:	y	:	z
1	:	4		
		4	:	9
1	:	4	:	9

∴ x : y : z = 1 : 4 : 9

Example:

Ratio between the sides of a triangle ABC is such that AB : BC = 4 : 3 and AB : AC – 4 : 5. Find the length of each side if the perimeter of the triangle is 24 cm.

Here perimeter = 24 cm

BC	:	AB	:	AC
3	:	4		
		4	:	5
12	:	16	:	20
3	:	4	:	5

$$\therefore \quad AB = \frac{4}{12}\times 24 = 8 \text{ cm} \quad , \quad BC = \frac{3}{12}\times 24 = 6 \text{ cm} \quad , \quad AC = \frac{5}{12}\times 24 = 10 \text{ cm}$$

Example:

The sum of ratio is divided among 3 people in the ratio 2 : 3 : 7.
If the sum of money is Rs.192, how much will each of them receive?

Consider the three people are X, Y and Z.
The sum of money is divided into 2 + 3 + 7 = 12 equal parts.

$$\text{X's share} = \frac{2}{12}\times \text{ Rs. } 192 = \text{Rs. } 32 \qquad \text{X's share} = \frac{3}{12}\times \text{ Rs. } 192 = \text{Rs. } 48$$

$$\text{Z's share} = \frac{7}{12}\times \text{ Rs. } 192 = \text{Rs. } 112$$

Check:

X's share + Y's share +Z's share
= Rs. (32 + 48 + 112)
= Rs. 192

Example:

Divide 200 kg in the ratio 1 : 3 : 4

Continued ratio = 1 : 3 : 4

Sum of elements of the ratio = 1 + 3 + 4
= 8

The parts are:

$$\text{1st part} = \frac{1}{8} \times 200 \text{ kg} = 25 \text{ kg}$$

$$\text{2nd part} = \frac{3}{8} \times 200 \text{ kg} = 75 \text{ kg}$$

$$\text{3rd part} = \frac{4}{8} \times 200 \text{ kg} = 100 \text{ kg}$$

Check Point

In a quadrilateral, ABCD, A, B and C are in the ratio 1 : 3 : 7. Find (a) ∠D (b) ∠C, Given that the sum of angles in a quadrilateral is 360°.

UNIT 4

Exercise 4.1

1. Express each of the following ratios in its simplest form.

(i) 192 : 75 (ii) 1.44 : 0.48 (iii) $\frac{7}{15}:\frac{14}{9}$ (iv) 108 : 36 : 60

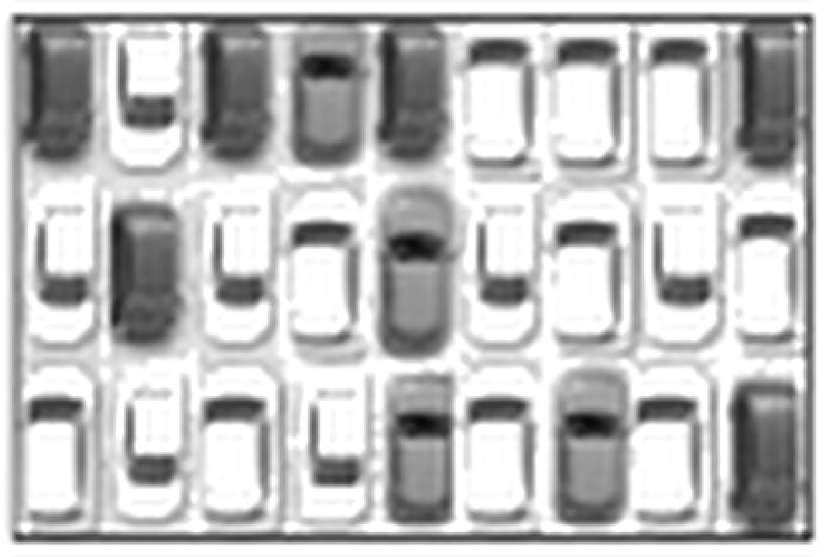
(v) 64 : 96 : 224 (vi) 57 : 19 : 133

2. Find $x:y:z$ and $x:z$ if:

(i) $x:y = 4:5$, $y:z = 5:7$ (ii) $x:y = 3:2$, $y:z = 4:9$

(iii) $x:y = 6:7$, $y:z = 8:11$ (iv) $x:y = \frac{2}{3}:1$, $y:z = \frac{3}{4}:\frac{1}{2}$

(v) $x:y = \frac{2}{3}:\frac{5}{2}$, $y:z = 1:\frac{3}{2}$



3. In a car park, the ratio of black cars to white cars is 5 : 6 and that of white cars to silver cars is 3 : 10.
Find the ratio of black cars to white cars to silver cars in simplest form.

4. The salaries of A and B are in the ratio 8 : 3.
The salaries of B and C are in the ratio 5 : 12.
Express the salaries of A, B and C in the form of a continued ratio.

5. In a farm there are 847 cattle.
The ratio between goats and sheep is 4 : 5.
The ratio between cows and sheep is 10 : 3.
Find the ratio of goats, sheep and cows.

6. Find the difference between the largest and the smallest shares, when Rs. 160 is shared among three friends in the ratio 1 : 6 : 9.

7. A sum of money is divided among three people in the ratio 15 : 18 : 7.
Find the total sum of money when the smallest share is Rs. 84.

8. A sum of money is divided in the ratio 3 : 5 : 9.
Calculate the smallest share given the largest share is Rs. 369.

9. An alloy consists of three metals X, Y and Z.
Calculate the ratio X : Z given that X : Y = 2 : 3 and Y : Z = 5 : 4.

10. A sum of money is divided among Fiza, Hadia, and Maha in the ratio 13 : 12 : 7. Calculate how much Hadia gets if the amount that Fiza gets is Rs. 360 more than Maha.

Direct Variation / Proportion

Laiba spent Rs. 200 to make 10 prints from her digital camera. Later she spent Rs. 600 to make 30 prints.

In the situation, there are two related quantities: the number of prints and the cost for these prints. Notice that both quantities change, but in the same way.

Cost of Prints in Rupees	200	600
Number of Prints	10	30

As the number of prints triples, the cost also triples.
By comparing these quantities as rates in the simplest form, you can see that the relationship between the two quantities stays the same.

$$\frac{\text{Rs. } 200}{10 \text{ prints}} = \frac{\text{Rs. } 20}{1 \text{ print}} \quad \text{and} \quad \frac{\text{Rs. } 600}{30 \text{ prints}} = \frac{\text{Rs. } 20}{1 \text{ print}}$$

Two quantities are proportional if they have a constant ratio or rate.
In the above situation, the cost of making prints is proportional to the number of prints because each quantity has a constant rate of 1 print for Rs. 20.

A proportional relationship is often expressed by writing a proportion.

A proportion is an equation stating that two ratios or rates are equivalent.

The general form of a proportion is $\frac{a}{b} = \frac{c}{d}$ where $b \neq 0, d \neq 0$.

For Example: $\frac{2}{5} = \frac{6}{15}$

If one of the numbers in a proportion is unknown, you can solve the proportion to find the unknown number.

Key Fact

A proportion is a statement showing that two ratios are equivalent.
If a, b, c, d are in proportion then a : b : : c : d .

Example:

The table shows the amount of money Hadia earns based on number of hours she baby sits.

Number of baby-sitting hours (x)	Earnings (Rs) (y)
1	50
2	100
3	150
4	200

The two quantities x (number of baby-sitting hours) and y (earning) are related in such a way that when x is changed in any ratio, y is changed in the same ratio.

That is if x is doubled, then y is also doubled or if x is halved, then y is also halved.

We say that y varies directly as x or y is directly proportional to x.

If y varies directly as x, then we write $y \propto x$.

The sign $\propto$ is read as 'varies as' and is called the sign of variation.

In the above example we further notice that:

$$\frac{y}{x} = \frac{50}{1} = \frac{100}{2} = \frac{150}{3} = \frac{200}{4} = 50$$

i.e $$\frac{\text{corresponding value of } y}{\text{value of } x} = \text{constant}$$

Hence, when $y \propto x$. the ratio $\frac{y}{x}$ is a constant, known as the constant of variation.

If this constant is represented by k, then $\frac{y}{x} = k$

or $y = kx, \quad k \neq 0$

In our example, the relationship between the number of hours she baby sits and her earnings.

$$k = 50$$
$$y = 50x$$

Using this equation, we can determine the amount Hadia earns for babysitting for 8 hours as

$$y = 50\,(8)$$
$$y = 400$$

Key Fact

Direct proportion is a relation between two quantities such that when one quantity is increased (decreased), the other quantity is also increased (decreased) in the same ratio. If a, b, c, d are in direct proportion then a : b :: c : d or $\frac{a}{b} = \frac{c}{d}$ or ad = bc

Example:

If y is directly proportional to x and y = 12 when $x = 4$, find

(i) an equation connecting x and y,
(ii) the value of y when $x = 8$,
(iii) the value of x when $y = 21$.

(i) Since y is directly proportional to x,

$\therefore\ y = k\,x$ where k is a constant.

When $x = 4$, $y = 12$

$$12 = k \times 4$$
$$k = 3$$
$$y = 3x$$

(ii) Substitute $x = 8$ into $y = 3\,x$

$$y = 3 \times 8$$
$$y = 24$$

(iii) Substitute $y = 24$ into $y = 3x$

$$24 = 3x$$
$$x = 24 \div 3$$
$$x = 8$$

Check Point

A variable T is directly proportional to a variable x.
If T = 100 when $x = 25$,
Express T in terms of x,
Find T when $x = 8.5$.

Check Point

A man took 15 minutes to assemble 40 boxes. How many boxes can he assemble in?
a. 9 minutes
b. 27 minutes
If 9 muffins cost Rs. 405, what would you expect to pay for?
a. 6 muffins
b. 28 muffins?

Example:

The expenses (E) at a tea party vary directly with the number of Guests (G) present.
For 40 guests, the expenses are Rs. 3200.
Find the expenses of 120 guests.

E α G i.e. E = k G, where k is a constant.

$$3200 = k\,(40)$$
$$k = \frac{3200}{40} = 80$$

Therefore, E = 80 G

E = 80 (120) = 9600

The expense for 120 guests are Rs. 9600.

Example:

M is directly proportional to the square of y.

If M = 30 when y = 10,

(a) express M in terms of y,

(b) find y when M = 750

(a) Since M is directly proportional to y^2.

$\therefore M = ky^2$ where k is a constant.

When M = 30, y = 10

So, $30 = k \times (10)^2$

$$k = \frac{30}{100} = \frac{3}{10}$$

$$\therefore M = \frac{3}{10} y^2$$

(b) When M = 750, we have

$$750 = \frac{3}{10} (y^2)$$

$$y^2 = \frac{750 \times 10}{3}$$

$$y^2 = 2500$$

$$y = 50$$

Check Point

The capacity C of a container is directly proportional to the square of the length of a side *l*, of its base. If C = 108 cm^3 when *l* = 3 cm, find the capacity when *l* = 5 cm.

The symbol α for "is proportional to" was first used in 1768 by the English Mathematician, William Emerson, in a text book called Doctrine of Fluxions.

Exercise 4.2

1. Find the cost of 10 kg of tea leaves when 3 kg of tea leaves cost Rs. 18.
2. If y is directly proportional to *x* and y = 5 when *x* = 2. Find the value of y when *x* = 7.
3. If z is directly proportional to *x* and z = 12 when *x* = 3. Find the value of *x* when z = 18.
4. If B is directly proportional to A and B = 3 when A = 18. Find the value of B when A = 24.
5. If Q is directly proportional to P and Q = 28 when P = 4,
   - (i) express Q in terms of P,
   - (ii) find the value of Q when P = 5,
   - (iii) calculate the value of P when Q = 42.

6. The expenses "E" of a tea party are directly proportional to the number of guests "N" present.

When there are 30 guests present at the tea party, the expenses incurred are Rs. 210.

(i) Find the equation containing E and N.

(ii) Calculate the expense incurred when there are 80 guests present at the tea party.

7. In each of the following tables, the quantities given are in direct proportion. Complete the tables.

(i)

x	1	3		7
y	9		45	

(ii)

M	10	20	50	
E	5			40

8. Zeb works 36 hours for Rs. 17280. If he is paid at the same rate, how long will it take him to earn:

(i) Rs. 96000? (ii) Rs. 28800?

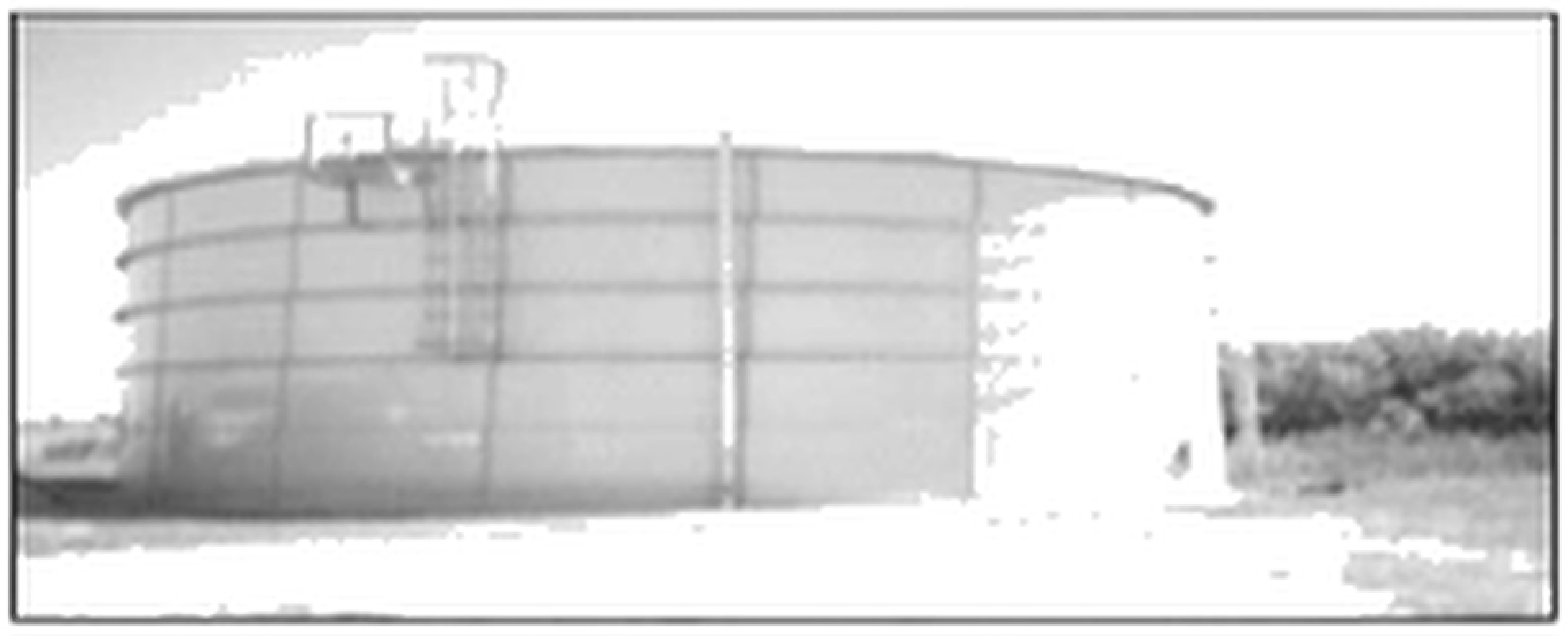
9. It is given that y is directly proportional to x^2 and $y = 16$ when $x = 2$.

(i) find an equation connecting y and x. (ii) find the value of y when $x = -3$.

10. If y is directly proportional to $(x + 1)^2$, and $y = 32.4$ when $x = 2$,

(i) express y in terms of x, (ii) the values of x when $y = 144$.

Inverse Variation / Proportion



10 identical taps can fill a tank in 4 hours.
Calculate the time taken for 8 such taps to fill the same tank.

Time taken to fill the tank is inversely proportional to the number of taps used because as the number of taps increases the time taken to fill the tank decreases.

10 taps can fill a tank in 4 hours.

1 tap can fill a tank in (10 × 4) hours

8 taps can fill a tank in $\frac{10 \times 4}{8} = 5$ hours

The table shows the time taken for a car to travel a distance of 120 km at different speeds.

Speed x km/h	10	20	30	40	60	120
Time taken y hours	12	6	4	3	2	1

UNIT 4

We notice that the two quantities, x the speed of car and y the time taken are related in such a way that when one quantity increases, the other decreases proportionally.
Similarly as the speed of car decreases the time taken increases proportionally.
This relationship is known as **inverse proportion.**

We notice that in the table, when x is doubled, y is halved and when x is halved, y is doubled.

Further more, $x\,y = 10 \times 12 = 20 \times 6 = 30 \times 4 = \ldots.. = 120$

i.e. Product of corresponding values of x and y = constant.

We say y varies inversely as x and this relationship is written as $y \; \alpha \; \frac{1}{x}$.

If y varies inversely as x then y varies directly as $\frac{1}{x}$.

i.e. $\dfrac{y}{\frac{1}{x}} = k$, or $xy = k$ or $y = \frac{k}{x}$, k is a non-zero constant

Key Fact

Inverse proportion is the relation between two quantities such that when one quantity is increased (decreased), the other quantity is decreased (increased) in the same ratio. If a, b, c, d are in inverse proportion then a : b : : d : c or $\frac{a}{b} = \frac{d}{c}$ or ac = bd

If y is inversely proportional to x,

then $x\,y = k$ or $y = \frac{k}{x}$, where k is a constant and $k \neq 0$.

Example:

If y is inversely proportional to x and y = 3 when x = 4, find

(a) An equation connecting x and y,
(b) The value of x when y = 48.

Since y is inversely proportional to x, then $y = \frac{k}{x}$ where k is a constant.

(a) When $x = 4$, y = 3

then $y = \frac{k}{x}$

$\Rightarrow$ $3 = \frac{k}{4}$

or $k = 12$

$\therefore$ $y = \frac{12}{x}$ .... (i)

(b) When y = 48, then from (i)

$48 = \frac{12}{x}$

$x = \frac{12}{48}$

$x = \frac{1}{4}$

Example:

Six men can complete a certain job in 8 hours.
Suppose all men work at the same speed,
how long will 18 men take to complete the same job?

Let x men take y hours to finish the job.
It is obvious that more men will take less time to complete a given job.
Then y varies inversely as x.

So, $y = \frac{k}{x}$ where k is a constant.

When $x = 6$, $y = 8$ then, $y = \frac{k}{x}$

$$\Rightarrow \quad 8 = \frac{k}{6}$$

$$\text{or} \quad k = 48$$

$$\therefore \quad y = \frac{48}{x}$$

When $x = 18$, $y = \frac{48}{18} = 2\frac{2}{3}$

So, 18 men will take $2\frac{2}{3}$ hours or 2 hours 40 minutes to finish the job.

Check Point

A farmer has enough food to feed 60 cows for 48 days. For how many days can the same food feed?

a. 24 cows

b. 72 cows

Example:

Given that y is inversely proportional to the square of x, and that $y = 4$ when $x = 1.5$.
Find the two values of x for which $y = 0.25$.
Given y is inversely proportional to x^2, then $y = \frac{k}{x^2}$ where k is a constant.
Substitute $y = 4$ and $x = 1.5$, we have:

$$4 = \frac{k}{(1.5)^2} \quad \Rightarrow \quad k = 9$$

So, $y = \frac{9}{x^2}$

Substituting $y = 0.25$, we have

$$0.25 = \frac{9}{x^2}$$

$$x^2 = \frac{9}{0.25} = 36$$

$$x = \pm 6$$

Check Point

If x is inversely proportional to the square root of y, and $y = 9$ when $x = 2$, find y when $x = 18$.

UNIT 4

Exercise 4.3

1. If y is inversely proportional to x and $y = 3$ when $x = 4$. Find:
    - (i) an equation connecting x and y,
    - (ii) the value of y when $x = 8$,
    - (iii) the value of x when $y = 48$.

2. Given that y is inversely proportional to x, and $y = 16$ when $x = 3$. Find the value of y when $x = 24$.

3. Given that x is inversely proportional to y, and $x = 40$ when $y = 5$. Find:
    - (i) a relationship between x and y,
    - (ii) the value of x when $y = 25$,
    - (iii) the value of y when $x = 400$.

4. If P is inversely proportional to Q and P = 9 when Q = 2, find the value of P when Q = 3.

5. Given that y is inversely proportional to x, and $y = 5$ when $x = 7$. Find the value of x when $y = 70$.

6. Given that y is inversely proportional to x. Complete the table below:

(i)

x	2	4		6
y	2		4	

(ii)

x	2	3		8
y		4	6	

7. Eight men can build a bridge in 12 days. Find the time taken for 6 men to build the same bridge.

8. If 12 workers can paint a building in 20 days, how many workers would it take to complete in
    - (i) 15 days?
    - (ii) 48 days?

9. 3 identical pumps can fill a tank in 40 minutes. What is the time taken to fill the tank if there are:

(i) 2 pumps? (ii) 8 pumps?

10. If a soldier's ration is 650g/day and there are enough supplies to last 12 days, how long would the supplies last if his ration is 780g/day?

Rate and Speed:

A comparison of two quantities with different kinds of units is called a rate. For example:
Ali read 100 words of his textbook in 2 minutes.

$$\therefore \text{Rate} = \frac{\text{words}}{\text{minutes}} = \frac{100}{2} = \frac{50}{1}$$

So, the rate is 50 words per minute.
The table below shows some common unit rates.

Key Fact

- A rate is a ratio comparing two quantities with different kinds of units.
- In the speed, kilometers and hours are different kinds of units that are compared.
- The rate for one unit of a given quantity is called a unit rate.

Rate	Unit rate	abbrevation	Name
$\frac{\text{number of kilimeters}}{\text{1hour}}$	Kms per hour	Km/h	Speed
$\frac{\text{number of kilimeters}}{\text{1liter}}$	Km per liter	Km/l	Petrol mileage
$\frac{\text{number of rupees}}{\text{1kg}}$	Price per kg	Rs/ kg	Unit price
$\frac{\text{number of rupees}}{\text{1 month}}$	Rupees per month	Rs/month	monthly wage

Speed is the rate of distance traveled per unit time.

$$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

For a given distance, speed and time or inversely related.

For Example:

If a car travels at a speed of 40 km/h, the car travels:

40 km in 1 hour

$40 \times 2 = 80$ km in 2 hour

$40 \times 3 = 120$ km in 3 hour, and so on.

So distance = speed × time

The time the car takes to travel 120 km at 40 km/h is $\frac{120}{40} = 3$ hours

Example:

If it had traveled the 200 km at the same speed for the whole 4 hours, what is the speed of car?

$$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}} = \frac{200 \text{ km}}{4 \text{ h}} = 50 \text{ km / h}$$

50 km per hour is the speed of the car.

Speed can be measured in kilometers per hour or meters per second.

Example:

Junaid drove 100 km from Islamabad to Mardan in $2\frac{1}{2}$ hour.

What was his speed for the journey?

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{100}{2.5} = 40\text{km / h}$$

This distance is in kilometers and the time in hours, so the speed is km per hour.

Example:

The aerial distance from Karachi Airport to Lahore Airport is 1035 km.
An aero plane flew the distance at a speed of 300 km/h
Work out the flight time, in hours and minutes.

$$\text{time} = \frac{\text{distance}}{\text{time}} = \frac{1035}{300} = 3.45 \text{ hours, which is 3 hour 27 minutes}$$

To change 0.45 hours to minute, multiply by 60, as there are 60 minutes in 1 hour.

Example:

Ahmed is riding a bike at a speed of 72 km/h.
Express his speed in m/s.

$$72 \text{ km} = 72 \times 1000 \text{ m}, \quad (1\text{km} = 1000\text{m})$$

$$1\text{h} = 60 \times 60 \text{ s}$$

$$72\text{km/ h} = \frac{72 \text{ km}}{1\text{hour}} = \frac{72 \times 1000 \text{ m}}{60 \times 60 \text{ s}} = 20 \text{ m/s}$$

Example:

A bus travels at a speed of 15 m/s. Express its speed in km/h.

$$15\text{m} = 15 \div 1000 \text{ km}, \quad (1\text{m} = 1 \div 1000\text{km})$$

$$1\text{h} = 3600 \text{ s}$$

$$15\text{m/s} = \frac{\frac{15}{1000}\text{km}}{\frac{1}{3600}\text{hour}} = \frac{15 \times 3600\,\text{km}}{1000} = 54\text{km/h}$$

Exercise 4.4

1. Express the following in m/s.
   (i) 18 km/ h (ii) 90 km/h (iii) 367 km/h
2. Express the following in km /h.
   (i) 10 m/s (ii) 35 m/s (iii) 315 m/s
3. A journey of 65 km takes $2\frac{1}{2}$ hours.
   What is the speed of this journey?
4. Harris jogged 3.8 km in 40 minutes.
   Work out his speed in km/h.
5. An athlete runs a 1000 m race in 3 minutes 20 seconds.
   Find his speed in meters per second.
6. A train journey covered 205 km at a speed of 100 km/h.
   Find the time taken in hours and minute.
7. Junaid drove 85 km from Wazirabad to Sialkot at a speed of 51 km/h.
   How long did his journey take in hours and minutes?
8. Amir drove 275 km from Islamabad to Lahore at a speed of 50 km/h.
   Work out the time his journey took.
9. Bader rode his bike for 4 hours 15 minutes at a speed of 20 km/h.
   What distance did he ride?
10. A car is traveling at a speed of 30 m/s.
    Work out the distance the car travels in 0.4 seconds.
11. The flight time from Islamabad Airport to London Airport is 8 hours 20 minutes. The speed of the jumbo jet is 540 km / h.
    What is the distance between Islamabad and London Airport?
12. A man walks at a rate of 1.25 m/s.
    Find the time he takes to walk 3.75 km
13. Convert a speed of 85 m/s to km/h.

UNIT 4

I have learnt

- Ratio is a comparison of two quantities by division.
- Equivalent ratios express the same relationship between two quantities.
- To find the equivalent ratio, multiply or divide each quantity in the ratio by the same number.
- Rate is a ratio comparing two quantities with different kinds of units.
- A unit rate is a rate for one unit of a given quantity.
- Proportions are equations stating that two ratios are equivalent.
- To solve a proportion, use either an equivalent fraction or a unit rate.
- Direct proportion is a relation between two quantities such that when one quantity is increased (decreased), the other quantity is also increased (decreased) in the same ratio. If a, b, c, d are in direct proportion then a : b : : c : d or $\frac{a}{b} = \frac{c}{d}$ or ad = bc
- Inverse proportion is the relation between two quantities such that when one quantity is increased (decreased), the other quantity is decreased (increased) in the same ratio. If a, b, c, d are in inverse proportion then a : b : : d : c
  or $\frac{a}{b} = \frac{d}{c}$ or ac = bd
- If y is directly proportional to x, then $y = kx$, where k is a constant and $k \neq 0$
  If y is inversely proportional to x, then $x y = k$ or $y = \frac{k}{x}$ where k is a constant and $k \neq 0$
- Speed is the rate of distance traveled per unit time.
  $$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}}$$
  For a given distance, speed and time or inversely related.

Words Board

- Variation
- Direct Variation
- Inverse Variation
- Continued Ratio
- Speed
- Distance
- Time
- Rate
- Ratio
- Proportion

Review Exercise 4

1. Choose the correct option.

(i) The ratio 16 seconds : 2 minutes as a single fraction in its lowest terms is:

(a) 7.5 : 1 (b) 1 : 6 (c) 8 : 1 (d) 1 : 8

(ii) 8 workers can paint a building in 24 days. How many days will 18 workers take?

(a) 54 (b) $10\frac{2}{3}$ (c) $10\frac{1}{3}$ (d) 6

(iii) A car is traveling at a speed of 25 metres per second.
Then the speed in kilometres per hour is:

(iv) Sara's class was sorting books in the library. The class sorted 45 books in 90 minutes. If they continue sorting at this rate, how long will it take them to sort 120 books?

(a) 5 hours (b) $4\frac{1}{2}$ hours (c) 4 hours (d) $3\frac{1}{2}$ hours

(v) The ratio of cats to dogs seen by a vet in a day is about 2 to 5. If a vet saw 40 animals in one day, about how many were dogs?

(a) 5 (b) 12 (c) 29 (d) 40

(vi) At a sports camp, there is 1 counselor for every 12 campers. If there are 156 campers attending the camp, which proportion can be used to find x, the number of counselors?

(a) $\frac{x}{12} = \frac{1}{156}$ (b) $\frac{12}{1} = \frac{x}{156}$ (c) $\frac{1}{12} = \frac{x}{156}$ (d) $\frac{x}{1} = \frac{12}{156}$

(vii) Ahmed buys 12 cans of orange juice for Rs. 600.
At this rate, how much would she pay for 48 cans of orange juice?

(a) Rs. 2000 (b) Rs. 2200 (c) Rs. 2400 (d) Rs. 3000

(viii) Annika can run 2 km in 15 minutes. At this rate, about how long will it take her to run $3\frac{1}{2}$ km?

(a) 26 minutes (b) 36 minutes (c) 32 minutes (d) 42 minutes

2. When Rs. 143 is divided in the ratio 2 : 4 : 5.
What is the difference between the largest share and the smallest share?

3. Divide 180 kg in the ratio 1 : 2 : 3 : 4.

4. Divide Rs. 4000 in the ratio 2 : 5 : 8.

5. A man and a woman share a bingo prize of Rs. 1000 between them in the ratio 1 : 4. The woman shares her part among herself, her mother and her daughter in the ratio 2 : 1: 1.
How much does her daughter receive?

6. A brother and sister shared their collection of 5000 stamps in the ratio 5 : 3. The brother then shared his stamps with 2 friends in the ratio 3 : 1 : 1 keeping most for himself. How many stamps did each of his friend receive?

7. If a car travels 36 km on 1.5 liter of petrol, how far can the car travel on 2.4 liters of petrol?

8. 8 workers are hired to build a house in 15 days. How many days are required if 2 additional workers are hired?

9. The kinetic energy E of the body is directly proportional to the square of its velocity V. If the difference in the value E when V = 15 m/s and V = 20 m/s is 14 Joules, find

(i) The expression of E in terms of V.

(ii) The velocity of the body when energy is 8 Joules.

10. Six men produce 240 loaves of bread in 3 hours.

(i) How many loaves can six men produce in $4\frac{1}{2}$?

(ii) How many men are needed to produce 240 loaves in 2 hours?

05

Financial Arithmetic

Learning Outcomes

- Explain property tax and general sales tax.
- Solve tax related problems.
- Explain profit and markup.
- Find the rate of profit/markup per annum.
- Solve real life problems involving profit/markup.
- Define Zakat and Ushr.
- Solve real life problems related to Zakat and Ushr

Government needs money to run the country. For this purpose, government imposes different types of taxes. The tax collected by the government is used to bear the cost of various public services such as education, health and defense. Some taxes are imposed directly and some indirectly. Federal Board of Revenue (FBR) is the main institution which is responsible for tax collection in Pakistan.

Property Tax

Mr. Arshad buys a house of worth Rs. 3,500,000. He paid a property tax @ 1.2 % on the property worth.

Let us calculate:

(i) Property tax paid by him.
(ii) Total amount paid by him.

Rate of property tax = 1.2 %
Value of house = Rs. 3500000
Property tax on the house = Rs. $3500000 \times \frac{12}{10} \times \frac{1}{100}$
= Rs.42000
Total amount paid by him = Rs. 3500000 + Rs. 42000
= Rs. 3542000

Key Fact

Financial years starts from first July and ends on 30th June of next year.

(i) The tax which is paid on a property is called the *property tax*. Property tax is a provincial tax paid on the value of a property.

(ii) Property tax is paid on real estate such as houses, shops, buildings, plazas at the end of each financial year. The government assesses the value of property and receives tax according to specified rate.

(iii) The rate of property tax is different for different categories. It is also different for residential and commercial areas. Moreover, rate of property tax is different for on road and off-road property.

Example:

Find the property tax for a plot worth Rs.250000 @ 2 % and 3 shops worth Rs.200000 each @ 3%.

Value of plot = Rs. 250000
Rate of property tax for plot = 2 %
Property tax for plot @ 2 % = Rs. $250000 \times \frac{2}{100}$
= Rs. 5000
Value of each shop = Rs. 200000
Rate of property tax for shop = 3 %
Property tax for shop @ 3% = Rs. $200000 \times \frac{3}{100}$

Key Fact

The word tax is derived from the Latin word taxo. It is amount of money that public pays to the government so that it can use for public services.

Property tax for three shops = 3×6000 = Rs. 18000
Net property tax = Rs.5000 + Rs.18000
= Rs. 23000

Example:

Tariq paid property tax worth Rs.25800 for two plots @ 2.4%. Find the value of each plot.

Let value of both plots be Rs. x.
Amount of tax = Rs. 25800
Rate of tax = 2.4 %
2.4% of x = Rs. 25800

$$\frac{24}{1000}x = \text{Rs. } 25800$$

$$x = \text{Rs. } 25800 \times \frac{1000}{24} = \text{Rs. } 1075000$$

Value of both plots = Rs. 1075000

$$\text{Value of each plot} = \text{Rs. } \frac{1075000}{2} = \text{Rs. } 537500$$

Key Fact

Direct taxes are levied on income or wealth and indirect taxes are levied on expenditures. Moreover, direct taxes are levied directly on people while indirect taxes are levied against goods and services.
Can you guess whether a property tax is a direct or indirect tax?

Check Point

Abdullah paid Rs. 8340 as a property tax at the rate of 4%. Find the worth of his property.

General Sales Tax

A manufacturer has the following transactions during the financial year.

Sales of chairs = Rs. 1020000
Sales of tables = Rs. 850000

Let us calculate the GST (General Sales Tax) and total amount paid by him at the rate of 17%.

Sale price of chairs = Rs. 1020000
Sale price of tables = Rs. 850000
Net sale price = Rs. 1020000 + Rs. 850000
= Rs.1870000
Rate of general sales tax = 17 %

$$\text{Therefore, GST paid} = \text{Rs. } 1870000 \times \frac{17}{100} = \text{Rs. } 317900$$

Total amount paid = Rs.(1870000 + 317900) = Rs. 2187900

Key Fact

(i) GST is the abbreviation of general sales tax.
(ii) Current GST rate in Pakistan is 17%.

General sales tax is a tax charged at the time of purchase for certain goods and services. The tax is usually set as a percentage by the government. The tax is included in the price or added at the time of sale. Therefore,

Original price + GST = Sale price

Key Fact

The tax a buyer pays to the seller at the time of buying things is called general sales tax. The rate of GST varies from 0% to 25% in Pakistan depending on types of industry.

Example:

Sale price of 5 juice packs including GST is Rs. 232. Find the factory price of 1 juice pack, if rate of GST is 16 %.

Sales Price of 5 juice packs = Rs. 232

Rate of GST = 16 %

Let Factory price of 5 juice packs = Rs. x

GST = 16% of $x = \frac{16}{100}x$

Now factory price + GST = sale price

$$x + \frac{16}{100}x = 232 \Rightarrow \frac{116}{100}x = 232$$

$$x = \frac{232 \times 100}{116} = \text{Rs. } 200$$

Factory price of 1 juice pack $= \text{Rs. } \frac{200}{5} = \text{Rs. } 40$

Key Fact

70% of government revenue is collected by different taxes.
However, only 16-20% people in Pakistan pay direct tax.

Check Point

The marked price of a mobile is Rs. 16500 including a 10% GST. What is the original price of the mobile?

Brain Buster

Hamna bought a house for Rs.5,000,000. She paid property tax @ 2% on it. After one year she sold it to his brother Asim by getting a profit of Rs. 200000. Asim paid transfer fee @ 1% on the worth of house.

(i) Find the property tax paid by Hamna.
(ii) Find the transfer fee paid by Asim.

UNIT 5

Exercise 5.1

1. Find the property tax @ 3% on a shop worth Rs. 475000. Also find the net amount including property tax.

2. Value of a shopping centre is Rs. 5600000. Find the property tax paid by the owner @ 3.5% if there is a rebate of Rs. 50000 in tax for maintenance.

3. Ammara paid property tax Rs. 35000 @ 2.5% for a house. Calculate the value of the property.

4. Tariq paid property tax Rs. 10800 for a property worth Rs. 180000. What is the rate of property tax?

5. Factory price of a bicycle is Rs. 10000. Calculate general sales tax @ 16 % and sales price of bicycle.

6. Rustam runs 5 shopping centres worth Rs. 50000000. Last Month he paid a property tax of Rs. 1000000 in total.
   (i) Find the rate of property tax.
   (ii) Find the property tax of each shopping centre at the same rate.

7. The sale price of a table is Rs. 9360 including sales tax @ 17 %. Find the original price of table.

8. General sales tax @ 17 % written on an electricity bill is Rs.170. Calculate the total electricity bill.

9. Original (factory) price of a motor car is Rs.800000. Amount of GST is Rs.95000. Find the rate of GST.

10. Asma bought the following items from a super store:
    (i) 10 litres cooking oil @ Rs. 250 per litre.
    (ii) 5 kg sugar @ Rs. 80 per kg.
    (iii) 2 bottles of tomato ketchup @ Rs. 120 per bottle.

    Calculate GST @ 17 % on each item paid by her. Also find net GST paid by her.

Profit / Markup

Abid deposited Rs. 50000 in a bank account. The bank offers a profit of 10% annually on the amount deposited. Let us find the profit and amount in the account after one year.

Amount deposited in the bank account = Rs. 50000

Rate of profit per annum = 10%

Profit on Rs. 50000 = deposited amount × rate

$$= \text{Rs. } 50000 \times \frac{10}{100}$$

$$= \text{Rs. } 5000$$

$$\text{Amount after 1 year} = \text{Rs. } 50000 + \text{Rs. } 5000$$

$$= \text{Rs. } 55000$$

Profit:

When we deposit money in a bank for a certain period of time, the bank pays us some extra amount, along with the original amount. This extra amount is called *profit*.

The original amount deposited in the bank is called *principal*.

The total money received from the bank is called *amount*.

$$\therefore \quad \text{Amount} = \text{Principal} + \text{Profit}$$

Markup:

When we borrow money from the bank, the bank charges some extra money for a certain period of time. This extra money is called *markup*.

The total amount given to bank is calculated as

$$\text{Amount} = \text{Principal} + \text{Markup}$$

Formula for Finding Profit/Markup

If the principal (P) is deposited or borrowed at the rate (R%) per annum, then the profit/markup can be calculated by the following formula:

$$\text{Profit} = \text{principle} \times \text{rate}$$

$$\text{Markup} = \text{principle} \times \text{rate}$$

$$\text{Rate of profit / markup} = \frac{\text{profit / markup}}{\text{principal}} \times 100$$

Key Fact

Profit or markup is usually calculated on yearly basis called rate per annum. But sometimes, it is calculated half yearly, quarterly, monthly or even daily basis.

Example:

Umar deposited Rs. 50000 in a bank and received Rs. 56000 after one year. Calculate the rate of profit per annum.

Principal = Rs. 50000

Amount = Rs. 56000

Profit = Amount – Principal

= Rs. 56000 – Rs. 50000 = Rs. 6000

$$\text{Rate of profit per annum} = \frac{6000}{50000} \times 100$$

$$= 12\ \%$$

Check Point

Amna bought a washing machine on installments costing Rs. 17000 at a markup rate of 12 % annually. Find the amount of markup and amount paid by her.

Example:

A man borrowed Rs. 80000 and agreed to repay the amount at the markup rate of 12.5 % in 15 equal monthly installments Find the total amount to be repaid and amount of each installment.

Principal = Rs. 80000

Rate of markup = 12.5 %

$$\text{Markup} = \text{Rs. } 80000 \times \frac{12.5}{100}$$

$$= \text{Rs. } 80000 \times \frac{125}{1000} = \text{Rs. } 10000$$

Total amount repaid = Rs. (80000 + 10000)

= Rs. 90000

$$\text{Amount of each installment} = \text{Rs. } \frac{90000}{15}$$

$$= \text{Rs. } 6000$$

Brain Buster

If 60% discount is offered on the marked price so that the selling price becomes equal to the cost price, then what was the percentage markup?

Exercise 5.2

1. **Complete the following table.**

S. No.	Principal	Profit / Markup	Amount = P + Profit/Markup
i.	Rs. 1000	Rs. 500	
ii.	Rs. 7000		Rs. 7800
iii.		Rs. 1200	Rs.18000
iv.	Rs. 4000		Rs. 4600

2. **Complete the following table:**

S. No.	Principal (P)	Rate (R)	Profit / Markup	Amount = P + Profit/Markup
i.	Rs. 2000	5 %		
ii.	Rs. 15000	10 %		
iii.	Rs. 25000	12 %		
iv.	Rs. 50000	15 %		

3. **Faiza borrowed Rs. 20000 and agreed to repay with a markup of Rs. 2000 in two installments.**

4. **Asif got a scholarship of Rs. 35000 from the education board and deposited in the post office at the profit rate of 7%. What is the amount of profit got by him?**

5. **Saeeda deposited Rs. 50000 in her account in a bank. She is paid Rs. 55000 after one year by the bank. Find the rate of profit per annum.**

6. **Mr. Shaheen bought a motorbike through a bank. The original price of motorbike was Rs. 65000 while he paid Rs. 71825 to the bank. Find the markup and the rate of markup.**

7. **Abdullah got a profit of Rs. 12000 on a principal amount of Rs. 15000. Find the rate of profit.**

8. **Amina bought a freezer from the market at the markup rate of 10%. The retail price of freezer is Rs. 45000. Find the markup and amount paid to the shopkeeper after one year.**

UNIT 5

Zakat

Mrs. Nadia has Rs. 200000 in her bank account. She wants to arrange bags for needy students from Zakat money. Let us find the Zakat on Rs. 200000 @ 2.5%.

Total Amount = Rs. 200000
Rate of Zakat = 2.5%
Zakat on Rs. 200000 = Rs. 200000 × 2.5%
= Rs. 200000 × $\frac{2.5}{100}$
= Rs. 5000

Therefore, she will give Rs. 5000 as Zakat.

Zakat

Zakat is the fundamental pillar of Islam. Zakat is a specific amount of savings given to the needy at the end of one Lunar year.

Nisab of Zakat

It is the minimum limit of assets on which Zakat is due.
All assets are liable to Zakat according to the following rates:

Key Fact

Lunar Year is the beginning of a calendar year whose months are coordinated by the cycles of the Moon.

(i) Gold: $7\frac{1}{2}$ tolas or 87.4 grams (ii) Silver: $52\frac{1}{2}$ tolas or 612.32 grams

So, if the aggregate value of all assets is equal to 87.4 g gold or 612.32 g silver whichever is less, these assets are liable to Zakat provided the amount remains in one's possession for one complete Lunar year.

Key Fact

- Zakat money could be given as a gift or present on a suitable occasion, but the intension (niyyat) of the given must be of Zakat.
- The amount of Zakat given to any person should not be less than his needs for at least one day.
- 1 tola = 11.65 grams

Rate of Zakat

Rate of Zakat is $2\frac{1}{2}$ % (or $\frac{1}{40}$ part) of the total amount possessed by a Muslim.

Key Fact

- Zakat on minerals and buried wealth is 5%.
- There is no Zakat on things that are of daily use.
- Nisab for cash is the same as that for gold or silver whichever is less.

Example:

A person having a capital of Rs. 170000 earns a profit of Rs. 75000 on it. Will he pay Zakat on profit only? What is amount of zakat paid by him?.

No! he will pay Zakat on the total amount.

Total amount = Rs. 170000 + Rs. 75000

= Rs. 245000

$$\text{Zakat} = 2\frac{1}{2}\ \%\ \text{of Rs. } 245000$$

$$= \frac{5}{200} \times \text{Rs. } 245000$$

$$= \text{Rs. } 6125$$

Example:

Rashida paid Zakat worth Rs. 4625 on gold ornaments and savings. Find the price of ornaments, if the savings are Rs. 80000.

Let total amount of gold ornaments and savings be x.

$$\text{Therefore, } 2\frac{1}{2}\ \%\ \text{of } x = \text{Rs. } 4625$$

$$\frac{5}{200} \times x = \text{Rs. } 4625$$

$$x = \text{Rs. } 4625 \times \frac{200}{5}$$

$$= \text{Rs. } 185000$$

Price of ornaments = Rs. 185000 – Rs. 80000

= Rs. 105000

Check Point

Rida has 90 gram gold and 800 gram silver with her. Find zakat given by her in terms of gold and silver.

UNIT 5

Ushr

Ushr is Zakat on agricultural produce (grains, fruits, vegetables). Ushr means 'one-tenth' or 10 %. Ushr is to be paid at the time of harvest.

Rate of Ushr

(i) If the land is irrigated by natural resources of water, then one-tenth (10 %) of the product is to be paid. Natural resources mean rain water, natural water canal, streams, rivers etc.

(ii) If the land is irrigated by artificial resources of water, then one-twentieth (5 %) of the product is to be paid. Artificial resources are wells and artificial canals etc.

Key Fact

There is no Nisab for Ushr. However, according to some scholars, the Nisab for grain is '5 Wasaq' which is equal to 600 kg (or 612 kg). Note that one Wasaq is the weight of one camel.

Example:

Find Ushr on 850 kg wheat irrigated by artificial resources and 740 kg wheat irrigated by natural resources.

Ushr on 850 kg wheat = 5% of 850 kg

$$= \frac{5}{100} \times 850 \text{ kg}$$

$$= 42.5 \text{ kg}$$

Ushr on 740 kg wheat = 10% of 740 kg

$$= \frac{10}{100} \times 740 \text{ kg} = 74 \text{ kg}$$

Total Ushr = 42.5 kg + 74 kg

= 116.5 kg

Check Point

Calculate Ushr on 1250 kg mangoes @ Rs. 120 per kg and 2500 kg potatoes @ Rs. 40 per kg.

Exercise 5.3

1. **Find Zakat on the following:**
    (i) 8 tola gold @ Rs. 80000 per tola.
    (ii) 70 tola silver @ Rs. 800 per tola.
    (iii) 100 g gold @ Rs. 8000 per gram.
    (iv) 712 g silver @ Rs. 80 per gram.

2. **Ahmed has the following assets:**
40 gram silver @ Rs. 750 per gram, shares worth Rs. 75000 and bank balance worth Rs. 47800. Find Zakat paid by him.

3. **Find Zakat if a person has the following assets:**
Income from rented house = Rs. 124000
2 prize bonds worth = Rs. 100000
Savings = Rs. 68000

4. **Asma paid Zakat worth Rs.15000 on gold. Find the value of gold.**

5. **Mr. Akbar is jeweller. He purchased 60 gm gold and 600 gm silver. If the market value for gold is Rs. 8000 per gram and Rs. 80 per gram for silver, find Zakat paid by him.**

6. **Find Usher separately on the following crops when the land is irrigated by rain water:**
1000 kg rice, 80 kg fruit, 140 kg vegetables.

7. **A person gives Ushr Rs. 3750 on crop worth Rs. 75000. What is rate of Ushr given? Is the land irrigated by artificial resources?**

8. **Aamir has a poultry farm. At the end of lunar year, he paid Zakat Rs. 8250. Find number of hens in the poultry farm provided that each hen is sold for Rs. 330.**

9. **A land is irrigated by a river. Find value of cotton produced if Ushr given is Rs. 9786.5.**

UNIT 5

I have learnt

- General sales tax is a consumption tax charged at the point of purchase for certain goods and services.
- The tax which is charged on property is called property tax. It is paid on houses, shops, buildings and plazas according to specified rate.
- When we deposit money in a bank for a certain period of time, the bank pays us some extra amount, along with the original amount called profit.
- When we borrow money from the bank (company), we have to pay some extra money after a certain period of time. This extra money is called markup.
- If the principal (P) is deposited or borrowed at the rate (R%) per annum then the profit/markup can be calculated by the formula:

  Profit / Markup = Principle × Rate
- Zakat is fundamental pillar of Islam. The Nisab for Zakat is:

  (a) Gold : $7\frac{1}{2}$ totals are 87.4 grams (b) Silver $52\frac{1}{2}$ tola are 612.32 gram
- Rate of Zakat is 2.50%.
- Zakat is due after completion of a lunar year if a person is 'Sahib-i-Nisab'.
- Ushr is Zakat on agricultural produce.
- Rate of Ushr is 10%, if land is irrigated by natural resources and 5 %, if land is irrigated by artificial resources.

Words Board

- Property Tax
- General Sales Tax
- Ushr
- Rate of Ushr
- Rate of Zakat
- Nisab of Zakat
- Ushr
- Rate of Ushr

Review Exercise 5

1. Choose the correct option in the following.

(i) General sales tax is paid by:

(a) Salesman (b) Buyer (c) Importer (d) Exporter

(ii) General sales tax is a:

(a) Direct tax (b) Indirect tax (c) Wealth tax (d) Penalty

(iii) Rate of sales tax on most of the goods in Pakistan is:

(a) 10 % (b) 15 % (c) 17 % (d) 25 %

(iv) GST on the milk powder worth Rs. 500 @ 17 % is

(a) Rs. 85 (b) Rs. 75 (c) Rs. 78 (d) Rs. 100

(v) Aleena deposited Rs. 15000 in her saving account. She got a profit of Rs. 3000 after one year. What is rate of the profit?

(a) 2 % (b) 10 % (c) 20 % (d) 30 %

(vi) What will be the markup for the principal of Rs.100 @ 10 % ?

(a) Rs. 10000 (b) Rs. 1000 (c) Rs. 100 (d) Rs. 10

(vii) Nisab of Zakat for the gold in grams is:

(a) 80 gm (b) 82.3 gm (c) 87.4 gm (d) 89.1 gm

(viii) Rate of Zakat on minerals or buried wealth is:

(a) 2.50 % (b) 3 % (c) 4 % (d) 5 %

(ix) Zakat on Rs. x is :

(a) $\frac{1}{2.50}x$ (b) $\frac{5}{2}x$ (c) $\frac{1}{40}x$ (d) $\frac{1}{20}x$

(x) Zakat is due after completion of one:

(a) financial year (b) lunar year (c) solar year (d) tax year

(xi) Ushr means:

(a) 10 % (b) 5 % (c) $\frac{1}{10}$ % (d) $\frac{1}{5}$ %

(xii) According to some scholars, the Nisab of Ushr for grain is:

(a) 650 kg (b) 450 kg (c) 500 kg (d) 600 kg

(xiii) Ushr is paid at the time of:

(a) Eid-ul-Fitr (b) Harvest (c) Cultivation (d) First Ramadan

2. **Calculate general sales tax @ 17 % on an electricity bill when:**

Previous meter reading = 4841
Present meter reading = 4941
Rate for first 100 units is Rs. 8 per unit.

3. **Calculate general sales tax @ 16 % on a gas bill when:**

Gas charges = 1180
Meter rent = 20
Hint: GST is applied on the sum of gas charges and meter rent.

4. **Asad paid property tax Rs. 40000 on the property worth Rs. 800000. What is the rate of property tax?**

5. **Laiba has Rs. 80000 in her bank account. After 1 year, she got Rs. 90000 from the bank. Calculate the rate of profit.**

6. **Find Zakat on the following assets at the end of lunar year.**

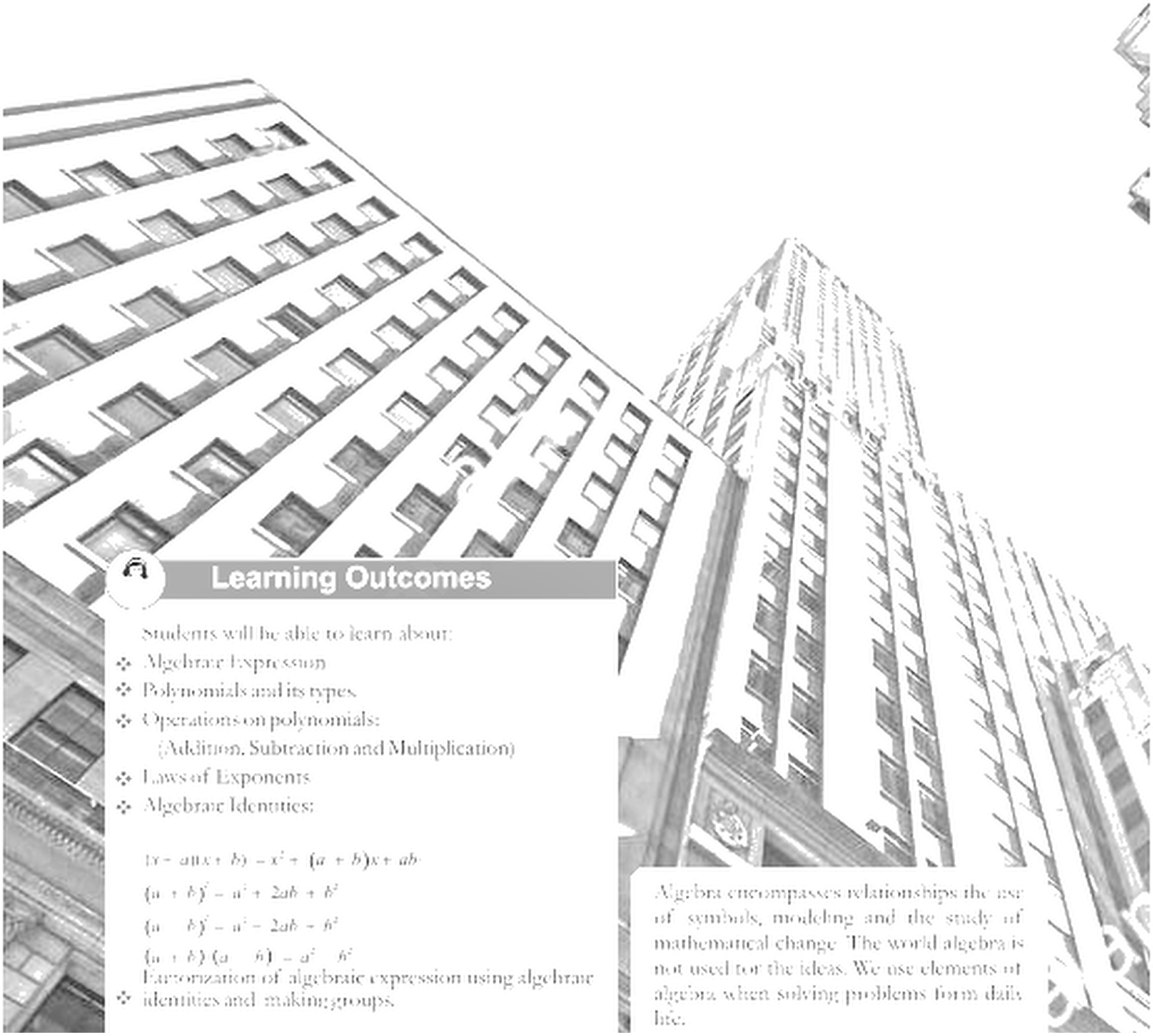
Savings = Rs. 350000, 100 gm gold @ Rs. 7500 per gram

7. **A general store uses a 40% markup on cost of a certain item. Find the cost of item that sells for Rs. 2800.**

8. **Ali invested Rs. 12000 at the profit rate of 12% for a period of one year. Find the monthly profit.**

06

Introduction To Algebra



Learning Outcomes

Students will be able to learn about:

- Algebraic Expression
- Polynomials and its types.
- Operations on polynomials:
  (Addition, Subtraction and Multiplication)
- Laws of Exponents
- Algebraic Identities:

$(x + a)(x + b) = x^2 + (a + b)x + ab$

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

$(a + b)(a - b) = a^2 - b^2$

- Factorization of algebraic expression using algebraic identities and making groups.

Algebra encompasses relationships the use of symbols, modeling and the study of mathematical change. The world algebra is not used for the ideas. We use elements of algebra when solving problems form daily life.

UNIT 6

Algebraic Expressions

Do you know how old Qasim is?

I don't know. I only know his brother, Furqan , is 3 years older than him.

Qasim's age is unknown. We can express it using a letter such as "y".
That is, Qasim is y years old.
Hence, Furqan is $(y + 3)$ years old.
$y + 3$ means adding 3 to the number "y".

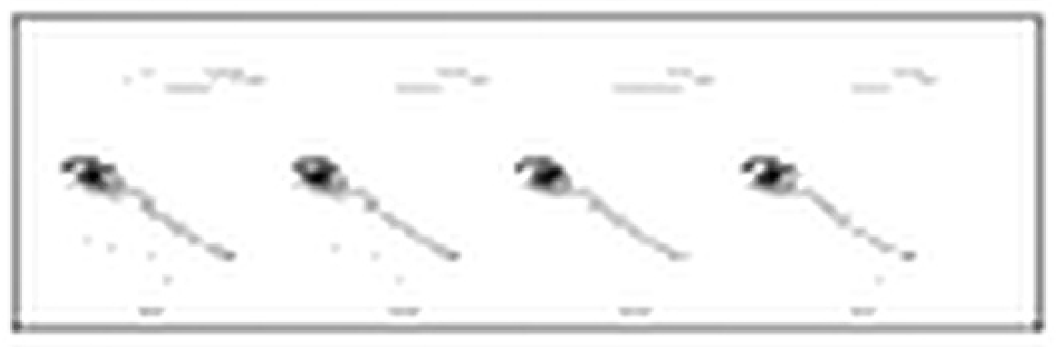
In the above situation, we can also express Furqan's age using a letter such as "x".

That is, Furqan is x years old.

Hence, Qasim's is $(x - 3)$ years old.

$x - 3$ means subtracting 3 from the number "x".

How much tea is there in all these cups.



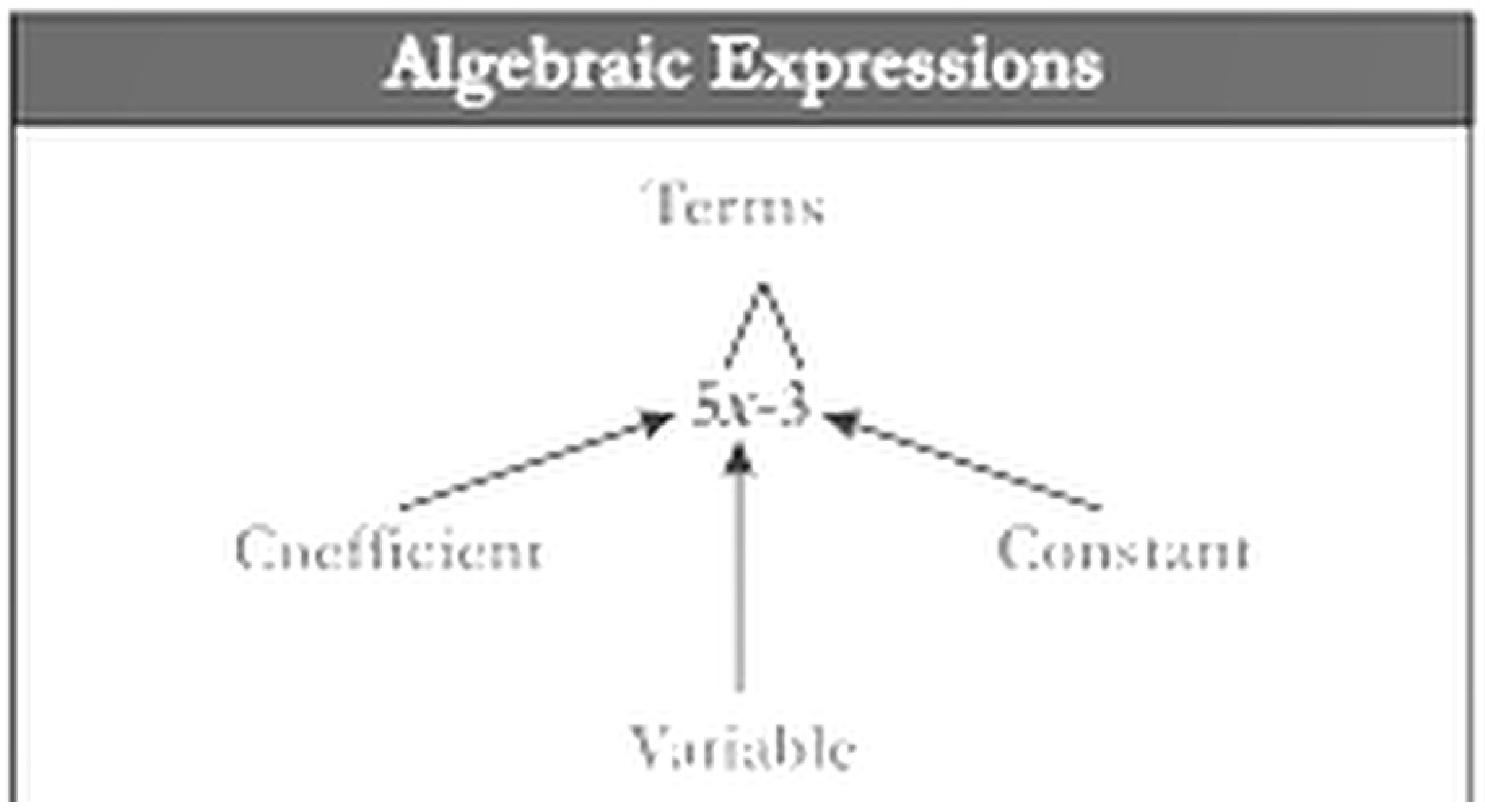
There are four cups of tea. How much tea is there in one cup?
The amount of tea in one cup is unknown.
We can express it using the letter z.
That is, there is z ml of tea in one cup.
The amount of tea in 4 cups is then

$$z + z + z + z = 4 \times z, \quad 4 \times z \text{ means 4 times the number called } z, \text{ that is } 4 \times z = 4z$$

We write $4 \times z$ as $4z$.

We use letters like x, y, z,….. to represent unknown quantities. These letters are called variables. $4z$, $y + 3$, $x + 6$ are examples of algebraic expression.

Algebra is a generalized arithmetic. Algebra deals with numbers, as does arithmetic, but uses variables-letters which stands for numbers. This permits more complicated problems to be solved. An algebraic expression is made up of three things; numbers, variables and operation signs. For example $3x - 3$, $7y + 9$, $x + 4y$, $8z + 6y + 5$



6 UNIT

A variable is a letter or symbol such as x, y, z,... that stands for a number. For example the height of a student is a variable, as it varies from student to student.

A quantity which has a fixed numerical value is called a constant e.g. 2, 6, 0, −3 are constants. In the expression $3x+4$, 3 and 4 are constants and x is variable.

The constant appearing before a variable is called a coefficient. In the term 9y, the constant 9 is the coefficient of the term.

i. In the expression $x+y+12$ the two numbers in x and y are variables, 12 is a constant, and 1 is coefficient of x and y.

ii. In x^2+y+5z , x , y and z all are variables and 1 and 5 are coefficients.

iii. In the algebraic equation $7+3y=15$, y is a variable, 7 and 15 are constants and 3 is a coefficient.

Example: Write an algebraic expression for each of the following:

(i) Add $3x$ to $7y$ (ii) Subtract $7x$ from 12

(iii) Multiply y by 3z (iv) Divide 4a by 9b

(v) Subtract the product of 2x and 5y from the sum of a and b

(i) $3x+7y$ (ii) $12-7x$

(iii) $y\times 3z = 3yz$ (iv) $4a \div 9b = \dfrac{4a}{9b}$

(v) $a+b-2x\times 5y = a+b-10xy$

Key Fact

Algebraic expressions consist of numbers, variables and arithmetic operations (addition, subtraction, multiplication, division).
In 10y, 10 is number, y is variable and multiplication is operation.

Example: Asma is three times as old as Saima and Saima is five years older than Nosheen. If Saima is x years old, write an algebraic expression for each of the following:

(i) Saima's age today.
(ii) Asma's age today.
(iii) Nosheen's age today.
(iv) Asma's age in 5 years time.
(v) The sum of Asma's and Saima's ages in 3 years' time.
(vi) The sum of Saima's and Nosheen's ages 4 years ago.

(i) Suppose Saima's age is x years old.
(ii) Asma's age $3x$.
(iii) Nosheen's age is $x-5$ years old.
(iv) Asma's will be $3x+5$ years old.

(v) In 3 years' time, Asma will be $3x+3$ years old and Saima will be $x+3$ years old. The sum of their ages is $(3x+3)+(x+3)=4x+6$.

(vi) Four years ago, Saima was $x-4$ years' old and Nosheen was $x-5-4$, i.e., $x-9$ years old. The sum of their ages then was $(x-4)+(x-9)=2x-13$ years old.

Terms of Algebraic Expression

The parts of the expressions connected by the signs of "+" and "–" are called the terms of the expression. In $x+y$, there are two terms x and y of the expression. In $x^2-3xz+yz$ there are three terms x^2, $-3xz$ and yz.

Polynomial

A monomial is a number, a variable or the product of a number and one or more variables with whole number exponents. The degree of a monomial is the sum of the exponents of the variables in the monomial.

Monomial	Degree	Not Monomial	Reason
12	0	$2 + y$	A sum is not a monomial.
$4x$	1	$\frac{y}{x}$	A monomial can not have a variable in the denominator.
$7y^2$	2	$x^{\frac{2}{3}}$	The variable must be a whole number exponent.
$-15z^6$	6	x^{-2}	The variable must have a whole number exponent.

A **polynomial** is a monomial or sum of monomials, each called term of the polynomial. The **degree of a polynomial** is the greatest degree of its terms.

Key Fact

The degree of a nonzero constant term is 0. The constant 0 does not have degree.

6 UNIT

When a polynomial is written in such a way that the exponents of a variable decrease from left to right, the coefficient of the first term is called the **leading coefficient**.

$6x^3 + x^2 - 7x + 13$

6 leading coefficient, degree 3, constant 13.

Key Fact

(i) A constant is also a polynomial of degree zero.

(ii) An algebraic expression is NOT a polynomial if the exponent of any variable is

- Not a whole number • A fraction or a negative integers.

Example:

Write $18x - x^3 + 3$, so that the exponents decrease from left to right. Identify the degree and leading coefficients of the polynomial.

Consider the degree of each of the polynomial's terms.

$18x - x^3 + 3$

Degree is 1. Degree is 3. Degree is 0. The polynomial can be written as $-x^3 + 18x + 3$. The greatest degree is 3, so degree of the polynomial is 3 and the leading coefficient is -1.

Example:

Indicate whether the expression is a polynomial. If it is a polynomial, find its degree and classify it by the number of its terms. Otherwise explain why it is not a polynomial.

Expression	Is it a polynomial	Classify by degree and number of terms
8	Yes	0 degree monomial(no variable)
$5x^2 + x - 6$	Yes	Second degree trinomial (one variable)
$6y^4 - 7^y$	No, variable exponent	
$3x^{-2} - 9$	No, negative exponent	
$7x^2y^3 + 4x^2z$	Yes	5th(2+3)degree binomial (Three variable)
$p^2 + pq + 8$	Yes	Second degree trinomial (Two Variable)
$x^2 - x^3 + 6x + \frac{1}{4}$	Yes	Third degree containing four terms
$x^2 + y^{\frac{1}{2}} + 6$	No, fractional exponent	
$9x^2y - 6x^2y^3 - xy - x^2 + 4$	Yes	Fifth (2+3) degree containing five terms

Monomial

Monomial is a polynomial consisting of a single term.

For example $4x, 3xy, y^2, 7, -7xy$

Binomial

A polynomial with two terms is called a binomial. e.g. $x^2+4,\ 3xz-y$ are binomials.

Trinomial

A polynomial of three terms is called a trinomial. e.g. $x-y+z,\ 2y+7xy+7z^2,\ x^3-9xy+yz$ etc. are trinomials.

Addition and Subtraction of Two or More Polynomials

Like Terms

Terms having the same variables with the same exponents are called like terms. For example $4xy^2, -10xy^2$ are like terms.

Unlike Terms

If the terms are not like, they are called unlike terms. For example $4xy^2, -9x$ and $2y$ are unlike terms.

Key Fact

In algebra the terms with the same kind of variables are called like terms. Like terms can be combined to a single term by addition or subtraction of coefficients.

Addition of Polynomials

Like terms can be combined to a single term by addition or subtraction of coefficients.

Key Fact

Rules of addition

(i) The sum of two positive like terms is positive.

For example: $3x + 8x = (3+8)x = 11x$

(ii) The sum of two negative like terms is negative.

For example: $(-5x) + (-8x) = -5x - 8x$

$= (-5 - 8)x$

$= -13x$

(iii) If two like terms having different signs are added then the result is difference of the terms with the sign of greater term.

For example: $-5x + 8x = +3x$ and $5x - 8x = -3x$

There are two methods used to find the sum of algebraic terms.

Horizontal Method: Group like terms and simplify.

Vertical Method: Align like terms in vertical columns. The expressions are arranged vertically so that like terms are written in the same column, then each column is added.

Example: Find the sum of $3x^2$, $7x^2$

Horizontal Method

$3x^2 + 7x^2$

$=(3 + 7)\,x^2$

$=10x^2$

Vertical Method

$$\begin{array}{l} 3x^2 \\ \underline{7x^2} \\ \underline{10x^2} \end{array}$$

Example: Find the sum of $x^2 - 2x$, $2x^2 + 3x + 5$

Horizontal Method

$(x^2 - 2x) + (2x^2 + 3x + 5)$

$= x^2 - 2x + 2x^2 + 3x + 5$

$- x^2 + 2x^2 - 2x + 3x + 5$

$= (x^2 + 2x^2) + (-2x + 3x) + 5$

$= (1 + 2)\,x^2 + (-2+3)\,x + 5$

$= 3x^2 + x + 5$

Vertical Method

$$\begin{array}{l} x^2 \; - \; 2x \\ \underline{2x^2 + 3x \; + \; 5} \\ \underline{3x^2 \; + x + 5} \end{array}$$

Example: Find the sum of 5a + 2b – 8, 6a + 9 , 7b + 2a – 4

Horizontal Method

(5a + 2b – 8) + (6a + 9) + (7b + 2a – 4)

= 5a + 2b – 8 + 6a + 9 + 7b + 2a – 4

= 5a + 6a + 2a + 2b + 7b – 8 + 9 – 4

= (5 + 6 + 2)a + (2 + 7)b + (–8 + 9 – 4)

= 13a + 9b – 3

Vertical Method

$$\begin{array}{l} 5a + 2b - 8 \\ 6a \qquad + 9 \\ \underline{2a + 7b - 4} \\ \underline{13a + 9b - 3} \end{array}$$

Example: Find the sum of $2x^2 + y^2 + 3z^2$, $6x^2 - 3y^2 + 9z + 6$

$(2x^2 + y^2 + 3z^2) + (6x^2 - 3y^2 + 9z + 6)$

$= (2x^2 + y^2 + 3z^2) + (6x^2 - 3y^2 + 9z + 6)$

$= 2x^2 + y^2 + 3z^2 + 6x^2 - 3y^2 + 9z + 6$

$= 2x^2 + 6x^2 + y^2 - 3y^2 + 3z^2 + 9z + 6$

$= (2 + 6)\,x^2 + (1 - 3)\,y^2 + 3z^2 + 9z + 6$

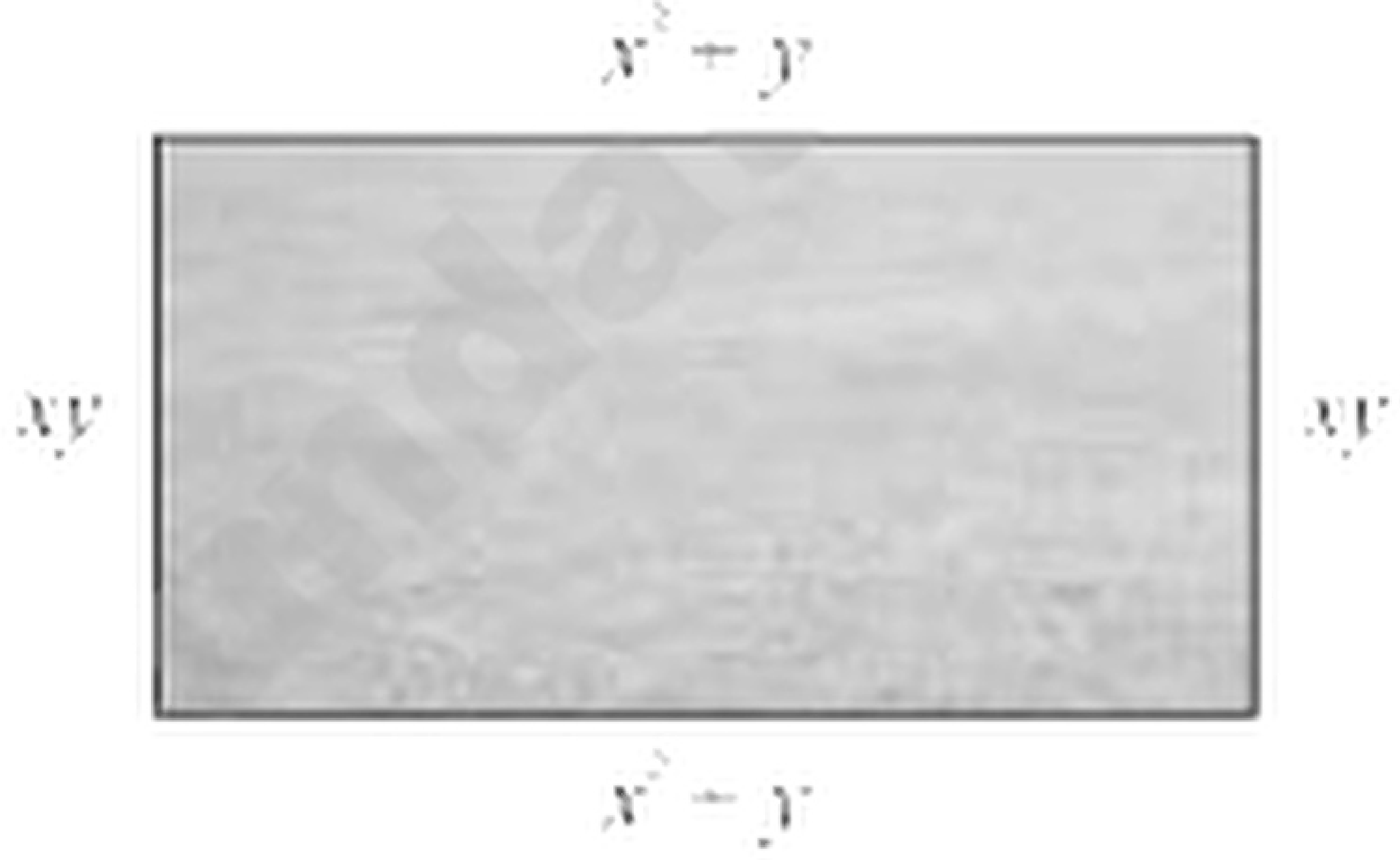
$= 8x^2 - 2y^2 + 3z^2 + 9z + 6$

Example: Find the sum of $4x^2 - 5xy - 6y^2$, $11xy - 7x^2 + 10y^2$, $6y^2 - 5x^2 + 10xy$

$$(4x^2 - 5xy - 6y^2) + (11xy - 7x^2 + 10y^2) + (6y^2 - 5x^2 + 10xy)$$
$$= (4x^2 - 5xy - 6y^2)+(11xy - 7x^2 + 10y^2) + (6y^2 - 5x^2 + 10xy)$$
$$= 4x^2 - 7x^2 - 5x^2 - 5xy + 11xy + 10xy - 6y^2 + 10y^2 + 6y^2$$
$$= (4 - 7 - 5)\,x^2 + (-5 + 11 + 10)\,xy + (-6 + 10 + 6)y^2$$
$$=- 8x^2 + 16xy + 10y^2$$

Example:

Find the perimeter of the square



Perimeter = Sum of all sides of a square

$$= (xy) + (x^2 + y) + (xy) + (x^2 + y)$$
$$= xy + x^2 + y + xy + x^2 + y$$
$$= x^2 + x^2 + xy + xy + y + y$$
$$= 2x^2 + 2xy + 2y$$

Subtraction of Polynomials

To subtract a polynomial from another polynomial, we change the sign of the polynomial to be subtracted. To change the sign of polynomial, each of its terms is multiplied by − 1.

Key Fact

In the subtraction of polynomials, we remember the following points.

(i) Change the sign of the terms to be subtracted. i.e. "+" into "−" and "−" into "+"

(ii) After changing the sign add the like terms.

Example: Subtract $4x - 10$ from $8x - 12$

Horizontal Method

$$(8x - 12) - (4x - 10)$$
$$= 8x - 12 - 4x + 10$$
$$= 8x - 4x - 12 + 10$$
$$= (8 - 4)\,x - 2$$
$$= 4x - 2$$

Vertical Method

$$\begin{array}{r} 8x - 12 \\ \underline{\pm\ \ 4x \mp 10} \\ \underline{4x - \ \ 2} \end{array}$$

Example: Subtract $4x^2 - 7x$ from $3x^2 - 9x - 6$

Horizontal Method

$$\begin{aligned}
&(3x^2 - 9x - 6) - (4x^2 - 7x)\\
&= 3x^2 - 9x - 6 - 4x^2 + 7x\\
&= 3x^2 - 4x^2 - 9x + 7x - 6\\
&= (3 - 4)\, x^2 + (-9 + 7)\, x - 6\\
&= -x^2 - 2x - 6
\end{aligned}$$

Vertical Method

$$\begin{array}{r}
3x^2 - 9x - 6\\
\underline{\pm 4x^2 \mp 7x}\\
\underline{-x^2 - 2x - 6}
\end{array}$$

Example: Subtract $2a^5 - 2a^4 + 5a^3 - 8$ from $5a^5 + 3a^4 - 2a^3 + 3a + 1$

Horizontal Method

$$\begin{aligned}
&(5a^5 + 3a^4 - 2a^3 + 3a + 1) - (2a^5 - 2a^4 + 5a^3 - 8)\\
&= 5a^5 + 3a^4 - 2a^3 + 3a + 1 - 2a^5 + 2a^4 - 5a^3 + 8\\
&= 5a^5 - 2a^5 + 3a^4 + 2a^4 - 2a^3 - 5a^3 + 3a + 1 + 8\\
&= 3a^5 + 5a^4 - 7a^3 + 3a + 9
\end{aligned}$$

Vertical Method

$$\begin{array}{r}
5a^5 + 3a^4 - 2a^3 + 3a + 1\\
\underline{\pm 2a^5 \mp 2a^4 \pm 5a^3 \qquad \mp 8}\\
\underline{3a^5 + 5a^4 - 7a^3 + 3a + 9}
\end{array}$$

Brain Buster

(i) Think of a number. Add 7 and then double the answer. Subtract 10, halve the result, and then subtract the number you originally thought of. Algebra can show you why the answer is always 2.

(ii) Investigate what happens when you substitute various values (positive or negative) for x in these expressions.

$$x + 1 \text{ and } \frac{x^3 + x^2 + x + 1}{x^2 + 1}$$

What is you conclusion? Which expression would you rather use?

Exercise 6.1

1. Indicate whether each of the following expressions are polynomials or not.
   (b) If it is a polynomial, find its degree and classify it by the number of terms.
   (c) Identify the leading coefficient in the case of the polynomial.

(i) $6x + 4$ (ii) $x^2 - 2x + 1$ (iii) $y^2 - 2 + \frac{1}{y^2}$

(iv) $x - 3 + \frac{1}{x}$ (v) $\frac{5}{2}x^2 - 2x^4$ (vi) $6 + \frac{7}{y}$

(vii) $y + 6y^3$ (viii) $x + \frac{1}{x}$ (ix) $\frac{1}{x^2} + \frac{1}{x^3}$

(x) $\frac{3}{7} - 4x + 6x^3 - 3x^2$

2. Categorize the following polynomials as monomial, binomial or trinomial.

(i) $7x$ (ii) $6x^2 + 3x + 2$ (iii) $6x^2 + x - 10x^3$

(iv) -10 (v) $x^3 - 10xy^2$ (vi) $6x + 7xy$

(vii) $y^3 - 6x^2y^2 + y^3$ (viii) $x^2yz + 5y^2z + 6x^2z^2$ (ix) $x^4 - 3$

(x) $3 - x^3 + 10x$

3. Find the sum of the following polynomials.

(i) $7x + 4,$ $9x - 3$

(ii) $a^2 - 8,$ $a^2 + 9$

(iii) $9x^2 + x - 3,$ $6x^2 - 3x + 4$

(iv) $a^2 + 2ab + b^2,$ $a^2 - 2ab + b^2$

(v) $x^3 + 5x^2 - 6x + 7,$ $2x^3 + 7x^2 - 10x + 7$

(vi) $2a + 9b + c - 4d,$ $-a - 7b - 2c + 5d$

(vii) $4a^2 - 7b - 4b^2,$ $2a^2 + 10b + 3b^2$

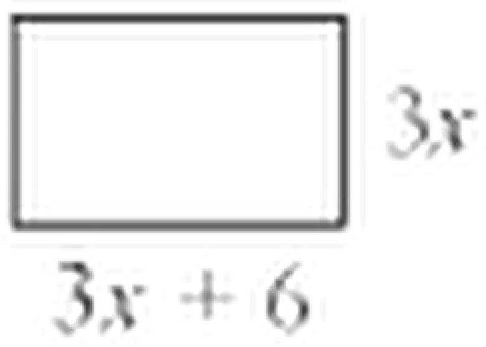
(viii) $3x^2 + x - 2,$ $2x^2 - 2x + 3$, $x^2 + 4x + 2$

(ix) $s^2 + 3t^2 + 4st,$ $2s^2 + 4t^2 - 3st$, $4s^2 - 2t^2 + 9st$

4. Find the perimeter of the rectangle.

(i)

(ii)



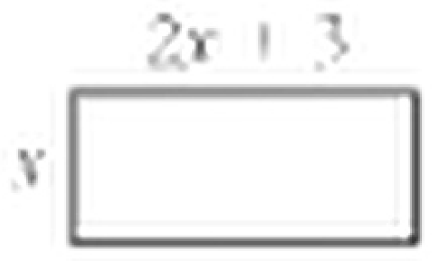
(iii)

5. Subtract.

(i)	$a + b + c$	from	$2a + 2b - c$
(ii)	$5x^2 + x - 9$	from	$7x^2 - 2x + 10$
(iii)	$6a^2 - 10ab - b^2$	from	$-2a^2 + 5ab - 3b^2$
(iv)	$4a^2 + 3b^2 - 4ab$	from	$4a^2 + 3b^2 - 6ab$
(v)	$10x^3 - 8x^2 + 4x + 3$	from	$8x^3 + 4x^2 - x + 5$
(vi)	$x^2 + y^2 - xy$	from	$3x^2 - 2y^2 + 4xy$

Multiplication of Polynomials

The diagram shows that a rectangle with width x and length $2x+3$ has an area of $x(2x+3) = 2x^2 + 3x$. We can also find product by using the distributive property.



In addition and subtraction of algebraic expressions, we add or subtract coefficient of like terms but in multiplication like terms can be multiplied with unlike terms as well.

Key Fact

While multiplying the signs change in the following way.

(i) The product of two terms with positive signs is positive.
$(+x)(+y) = +(xy)$

(ii) The product of two terms with negative signs is positive.
$(-x)(-y) = +(xy)$

(iii) The product of two terms with different signs (one positive and other negative is negative.
$(+x)(-y) = -(xy)$
$(-x)(+y) = -(xy)$

(a) Multiply of a monomial with a polynomial

Example: Find the product $2x^3$, $x^3 + 3x^2 - 2x + 4$

$$2x^3(x^3 + 3x^2 - 2x + 4) \quad \text{(write product)}$$
$$= 2x^3(x^3) + 2x^3(3x^2) - 2x^3(2x) + 2x^3(4) \quad \text{(distributive property)}$$
$$= 2x^6 + 6x^5 - 6x^4 + 8x^3 \quad (a^m \times a^n = a^{m+n})$$

The exponent of the product of two terms with the same base will be equal to the sum of the exponents of the same variable. For this we apply the following rule.

$$a^m \times a^n = a^{n+m}$$

Example: Multiply $8x^2y^2z$ by $2xyz$

$$(8x^2y^2z)\ (2xyz) = 16x^{2+1}y^{2+1}z^{1+1} = 16x^3y^3z^2$$

(b) Multiplication of a monomial with a binomial and a trinomial

To mulitply a monomial with polynomial, we apply the following rule.

$$a\ (b + c + d) = a\ b + a\ c + a\ d$$

Example:

(i) Multiply $3x - 2y$ by $6xy$

(ii) Mulitply $8x^3 - 3x^2 + x$ by $6x$

(i) $6xy\ (3x - 2y) = 6xy \times 3x - 6xy \times 2y$ (distributive property)

$= 18x^2y - 12xy^2$ (product of power property)

(ii) $6x\ (8x^3 - 3x^2 + x)$

$= 6x \times 8x^3 - 6x \times 3x^2 + 6x \times x^1$ (distributive property)

$= (6 \times 8)x^{1+3} - (6 \times 3)x^{1+2} + (6 \times 1)x^{1+1}$ (product of power property

$= 48x^4 - 18x^3 + 6x^2$

(c) Multiplication of a binomial with a binomial and a trinomial

Find the product of $(x - 4)$ and $(3x + 2)$

Method I: Use distributive law.

$(x - 4)\ (3x + 2)$

$= x\ (3x + 2) - 4\ (3x + 2)$

$= 3x^2 + 2x - 12x - 8$

$= 3x^2 - 10x - 8$

Method 2: Make a table of product.

$(x-4)(3x+2) = (x+(-4))(3x+2)$

	$3x$	2
x	$3x^2$	
-4		

	$3x$	2
x	$3x^2$	$2x$
-4	$-12x$	-8

$$\begin{aligned}(x-4)(3x+2) &= 3x^2 + 2x - 12x - 8 \\ &= 3x^2 - 10x - 8\end{aligned}$$

To multiply a binomial with another binomial, we apply the following rule.
$(a+b)\ (c+d) = ac + ad + bc + bd$
To multiply a binomial with a trinomial, we apply the following rule.
$(a+b)\ (c+d+e) = ac + ad + ae + bc + bd + be$

Example:

Multiply $(2x+3)$ by $(4x-3)$

$(2x+3)(4x-3)$

$(2x+3)(4x-3) = 2x\,(4x-3) + 3\,(4x-3)$

$= 2x \times 4x + 2x\,(-3) + 3 \times 4x + 3\ (-3)$ (distributive property)

$= 8x^2 - 6x + 12x - 9$ (multiply)

$= 8x^2 + 6x - 9$ (combine like terms)

Example:

Multiply $(3x^2 + 4x - 2)$ by $(4x-3)$

$(3x^2 + 4x - 2)\,(4x-3)$

$= 3x^2\,(4x-3) + 4x\,(4x-3) - 2\,(4x-3)$ (distributive property)

$= (3x^2)\,(4x) - 3x^2\,(3) + 4x\,(4x) - 4x\,(3) - 2\,(4x) + 2(3)$ (distributive property)

$= 12x^3 - 9x^2 + 16x^2 - 12x - 8x + 6$ (multiply)

$= 12x^3 + 7x^2 - 20x + 6$ (combine like terms)

CHALLENGE

Describe and correct the error in finding the product of the polynominals.

$(x-5)\,(3x+1)$

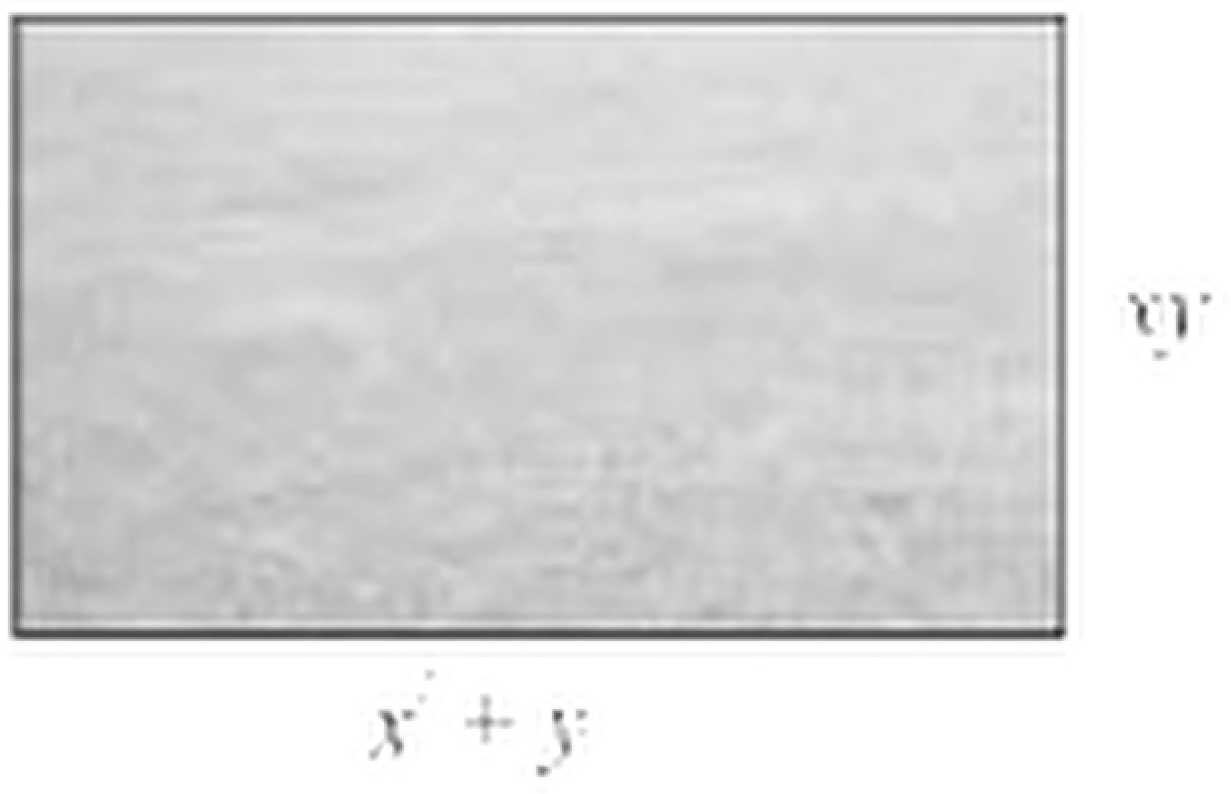
	$3x$	1
x	$3x^2$	x
5	$15x$	5

✗

$(x-5)\,(3x+1) = 3x^2 + 16x + 5$

Example:

Find the area of the rectangle.



Area $= $ (length of the rectangle) × (width of rectangle)

$= xy\,(x^2 + y)$

$= x^{2+1}y + xy^{2+1}$

$= x^3y + xy^2$

Exercise 6.2

1. **Find the product of the following.**

(i) $3x,\ 9x^2$ (ii) $6x,\ 4\,xy$ (iii) $7x^2,\ 7x + 3$

(iv) $-3x,\ 6x^2 + 4x$ (v) $4x,\ 3x^2 + 7x - 6$

2. **Evaluate the following.**

(i) $(2x - 3)\,(3x - 4)$ (ii) $(2x - 7)\,(4x + 8)$

(iii) $(6a - 2)\,(6a^2 + a - 5)$ (iv) $(4a^2 - 3a + 6)\,(2a + 5)$

(v) $(x + y)\,(x^2 - xy)$ (vi) $(x - y)\,(x^2 + xy + y^2)$

(vii) $(4x^3 + 6x^2 - 3x + 7)\,(x + 2)$ (viii) $(5x^2 - 6x + 8)\,(3x + 2)$

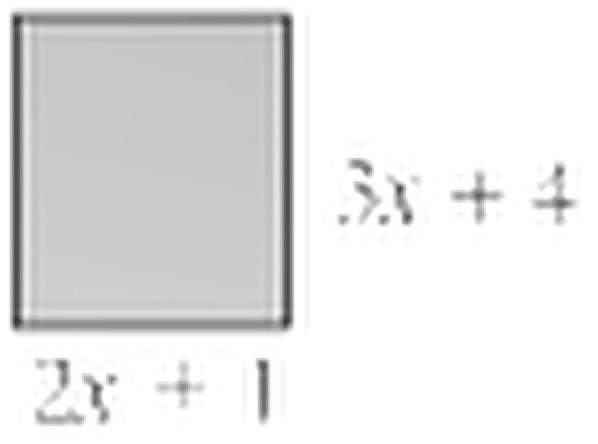
3. **The dimensions of a rectangle are $x + 3$ and $x + 2$. Find the area of the rectangle in term of x.**

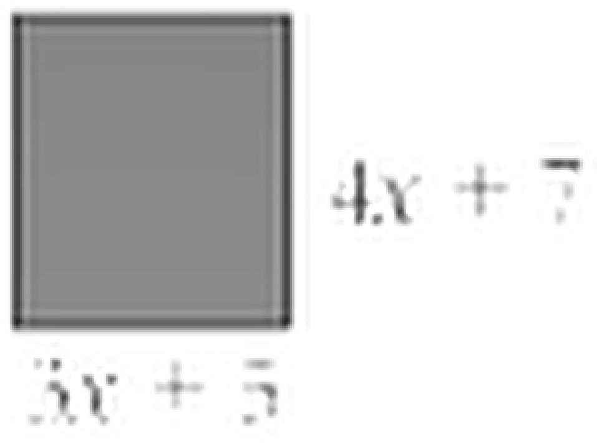
4. **Simplify.**

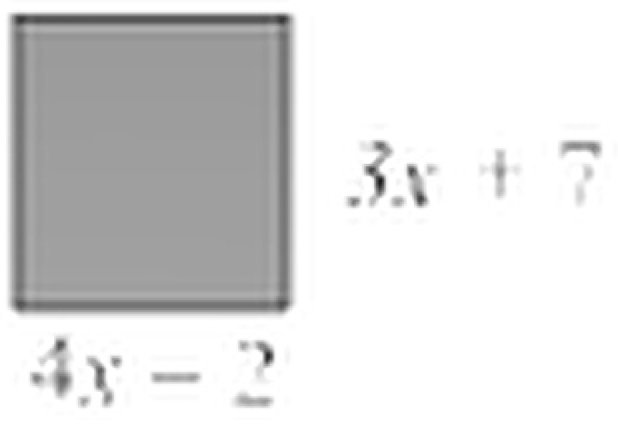
(i) $(x + 2)\,(x - 5) + (3x + 1)\,(x - 3)$

(ii) $(2x - 1)\,(x + 3) - (4x - 2)\,(6x + 4)$

5. **Find the area and the perimeter of the following figures.**

(i)


(ii)


(iii)


6 UNIT

Laws of Exponents

Law 1: Product Law

Consider the following:

(i) $4^2 \times 4^5 = (4\times4)\times(4\times4\times4\times4\times4) = 4\times4\times4\times4\times4\times4\times4 = 4^7$

(ii) $(-5)^3 \times (-5)^5 = (-5)\times(-5)\times(-5)\times(-5)\times(-5)\times(-5)\times(-5)\times(-5) = (-5)^8$

(iii) $\left(\frac{2}{5}\right)^4 \times \left(\frac{2}{5}\right)^2 = \left(\frac{2}{5}\times\frac{2}{5}\times\frac{2}{5}\times\frac{2}{5}\right)\times\left(\frac{2}{5}\times\frac{2}{5}\right) = \left(\frac{2}{5}\right)^6$

(iv) $\left(-\frac{4}{7}\right)^7 \times \left(-\frac{4}{7}\right)^4 = \left(\left(-\frac{4}{7}\right)\times\left(-\frac{4}{7}\right)\times\text{.....7times}\right)\times\left(\left(-\frac{4}{7}\right)\times\left(-\frac{4}{7}\right)\text{....4times}\right)$

$$= \left(\frac{-4}{7}\right)\times\left(\frac{-4}{7}\right)\text{.....11 times} = \left(\frac{-4}{7}\right)^{11}$$

In general, if x is a rational number and m and n are positive integers, then

$$x^m \times x^n = \underbrace{(x \times x \times \text{.....} \; x)}_{\text{m times}} \times \underbrace{(x \times x \times \text{.....} \; x)}_{\text{n times}}$$

$$= \underbrace{(x \times x \times \text{.....} \; x)}_{\text{(m+n) times}}$$

$$= x^{m+n}$$

Law 1 (a):

If x is any rational number and m and n are positive integers, then

$$x^m \times x^n = x^{m+n}$$

Now consider power of a product for two or more variables. If x and y are two rational numbers, then

(i) $(5\times7)^2 = (5\times7)\times(5\times7)$

$= 5\times5\times7\times7 = (5\times5)\times(7\times7)$

$= 5^2 \times 7^2$

(ii) Consider the following.

$$(x \times y)^n = \underbrace{(x \times y) \times (x \times y) \times (x \times y) \text{.....} (x \times y)}_{\text{n times}}$$

$$= \underbrace{(x \times x \times x \text{.....} \; x)}_{\text{n times}} \times \underbrace{(y \times y \times y \text{.......} \; y)}_{\text{n times}}$$

$$= x^n \times y^n$$

Law 1 (b):

If x and y are two different rational numbers and n is any integer, then

$$x^n \times y^n = (xy)^n$$

We apply this law when bases are different but exponents are the same.

Brain Buster

Product rule of powers cannot be applied to $2^3 \times 3^2$, why?

Example: Simplify.

(i) $3^5 \times 3^8$ (ii) $4^8 \times 4^5$ (iii) $\left(-\frac{9}{11}\right)^7 \times \left(-\frac{9}{11}\right)^{-3}$

(i) $3^5 \times 3^8$

$= 3^{5+8} = 3^{13}$

(ii) $4^8 \times 4^5$

$= 4^{8+5} = 4^{13}$

(iii) $\left(-\frac{9}{11}\right)^7 \times \left(-\frac{9}{11}\right)^3 = \left(-\frac{9}{11}\right)^{7+3} = \left(-\frac{9}{11}\right)^{10}$

Example: Simplify $\left(\frac{2}{3} \times \frac{5}{7}\right)^4$.

$$\left(\frac{2}{3} \times \frac{5}{7}\right)^4 = \left(\frac{2}{3} \times \frac{5}{7}\right) \times \left(\frac{2}{3} \times \frac{5}{7}\right) \times \left(\frac{2}{3} \times \frac{5}{7}\right) \times \left(\frac{2}{3} \times \frac{5}{7}\right)$$

$$= \left(\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}\right) \times \left(\frac{5}{7} \times \frac{5}{7} \times \frac{5}{7} \times \frac{5}{7}\right)$$

$$= \left(\frac{2}{3}\right)^4 \times \left(\frac{5}{7}\right)^4$$

Law 2: Quotient Law

Look at the following:

(i) $5^5 \div 5^3 = \frac{5 \times 5 \times \cancel{5} \times \cancel{5} \times \cancel{5}}{\cancel{5} \times \cancel{5} \times \cancel{5}} = 5^2$ or $5^5 \div 5^3 = 5^{5-3} = 5^2$

Law 2: (a)

If x is any non zero number and m and n are positive integers (m > n), then

$$x^m \div x^n = \frac{x^m}{x^n} = x^{m-n}$$

The division of two rational numbers with the same base can be performed by substracting their exponents.

Consider the following.

$$\frac{16}{81} = \frac{2\times 2\times 2\times 2}{3\times 3\times 3\times 3} = \frac{2^4}{3^4} = \left(\frac{2}{3}\right)^4$$

Law 2: (b)

If x and y are two different non zero rational numbers and n is any integer, then

$$x^n \div y^n = \frac{x^n}{y^n} = \left(\frac{x}{y}\right)^n$$

Example: Simplify the following by using the appropriate law.

(i) $(-2)^6 \div (-2)^3$

(ii) $6^3 \div 5^3$

(iii) $4^5 \div 4^3$

(iv) $\left(\frac{-2}{3}\right)^7 \div \left(\frac{-2}{3}\right)^5$

(i) $(-2)^6 \div (-2)^3$

$$(-2)^6 \div (-2)^3 = \frac{(-2)^6}{(-2)^3}$$
$$= (-2)^{6-3}$$
$$= (-2)^3$$

(ii) $6^3 \div 5^3 = \frac{6^3}{5^3}$

$$= \left(\frac{6}{5}\right)^3$$

(iii) $4^5 \div 4^3$

$$= \frac{4^5}{4^3} = 4^{5-3}$$
$$= 4^2$$

(iv) $\left(\frac{-2}{3}\right)^7 \div \left(\frac{-2}{3}\right)^5$

$$= \frac{\left(\frac{-2}{3}\right)^7}{\left(\frac{-2}{3}\right)^5}$$
$$= \left(\frac{-2}{3}\right)^{7-5} = \left(\frac{-2}{3}\right)^2$$

Law 3: Power Law

Consider the following

$$(4^5)^2 = 4^5 \times 4^5 = 4^{5+5} = 4^{10}$$

Which can be written as: $(4^5)^2 = 4^{5\times2} = 4^{10}$

Let us recall Law 1 (a), which says that $x^m \times x^n = x^{m+n}$

If m = n , then

$$x^m \times x^m = x^{m+m}$$

$$(x^m)^2 = x^{2m}.$$

Similarly,

$$(x^m)^3 = x^m \times x^m \times x^m$$

$$= x^{m+m+m}$$

$$= x^{3m}.$$

We generalize the power laws as follows:

If x is any rational number and m and n are integers, then

$$(x^m)^n = x^{mn}.$$

Note: Consider $\frac{2^4}{2^4} = \frac{\cancel{2}\times\cancel{2}\times\cancel{2}\times\cancel{2}}{\cancel{2}\times\cancel{2}\times\cancel{2}\times\cancel{2}} = 1$

Using quotient law this can be written as:

$$\frac{2^4}{2^4} = 2^{4-4} = 2^0$$

The above example illustrates the following law.

Zero Exponent

If x is any non zero real number with zero exponent, then

$x^0 = 1$

Key Fact

Negative Exponent

If x is any number having negative exponent, then

$$x^{-n} = \frac{1}{x^n}$$

Example:

(i) $(3^5)^2$ (ii) $(x^5)^2$ (iii) $\left(\left(\frac{3}{4}\right)^5\right)^2$

(i) $(3^5)^2$

$(3^5)^2 = (3)^{10} = 3^{10}$

(ii) $(x^7)^4$

$(x^7)^4 = x^{28}$

(iii) $\left(\left(\frac{3}{4}\right)^5\right)^2$

$$\left(\left(\frac{3}{4}\right)^5\right)^2 = \left(\frac{3}{4}\right)^{5\times2} = \left(\frac{3}{4}\right)^{10}$$

Example:

Express each of the following with a positive exponent and simplify where possible.

(i) x^{-3} (ii) $\frac{1}{y^{-5}}$ (iii) $\frac{a^6}{a^6}$ (iv) $\frac{2^2 3^{-2} x^4}{y^{-3}}$

(i) x^{-3}

$= \frac{1}{x^3}$

(ii) $\frac{1}{y^{-5}}$

$= y^5$

(iii) $\frac{a^6}{a^6}$

$= a^{6-6}$

$= a^0$

$= 1$

(iv) $\frac{2^2 3^{-2} x^4}{x^3 y^{-5}}$

$= \frac{2^2 x^{4-3} y^3}{3^2}$

$= \frac{4xy^3}{9}$

Concept of power of an integer when n is even or odd integer

Consider (i) $(-2)^3 = (-2)\times(-2)\times(-2) = -8$

(ii) $(-2)^5 = (-2)\times(-2)\times(-2)\times(-2)\times(-2) = -32$

If the base is negative and its exponent is an odd number then the result will be a negative integers.

Similarly,

(iii) $(-2)^4 = (-2)\times(-2)\times(-2)\times(-2) = 16$

(iv) and also $(-2)^6 = 64$

If exponent is even number of negative base then result is positive integer.

The laws of exponents help us simplify numerical and algebraic expressions. Note that more than one law can be applied to simply an expression.

Example: Simplify: $\frac{2a^4}{b^2} \div \frac{4a}{a^2b}$

$$\frac{2a^4}{b^2} \div \frac{4a}{a^2b} = \frac{2a^4}{b^2} \times \frac{a^2b}{4a}$$

$$= \frac{2a^4}{b^2} \times \frac{a^2b}{4a} = \frac{2 \times a^4 \times a^2 \times b}{b^2 \times 4 \times a}$$

$$= \frac{a^{4+2}}{b^{2-1} \times 2 \times a} = \frac{a^6}{b^1 \times 2 \times a} = \frac{a^{6-1}}{b^1 \times 2}$$

$$= \frac{a^5}{2b}$$

Exercise 6.3

1. Simplify by using the product law and express the result in power notation?

(i) $7^4 \times 7^5$ (ii) $(-x)^{10} \times (-x)^{-6}$ (iii) $5^{n+1} \times 5^{n-2}$

(iv) $y^{4-x} \times y^{3x+5}$ (v) $5^4 \times 3^4$ (vi) $\left(\frac{7}{5}\right)^7 \times \left(\frac{7}{5}\right)^4$

(vii) $\left(\frac{11}{4}\right)^{24} \times \left(\frac{11}{4}\right)^{-7}$ (viii) $x^{9+n} \times y^{9+x}$ (ix) $\left(\frac{-9}{11}\right)^{3x+6} \times \left(\frac{-9}{11}\right)^{-x-4}$

(x) $z^{2n-3} \times y^{2n-3}$ (xi) $b^5 \times b^4 \times b^5$ (xii) $3^5 \times 3^5 \times 3^2$

2. Simplify by using the quotient law.

(i) $2^{14} \div 2^{10}$ (ii) $x^{10} \div x^5$ (iii) $x^3y^5 \div x^2y^3$

(iv) $(-4)^7 \div (-4)^5$ (v) $x^9 \div y^9$ (vi) $\left(-\frac{11}{3}\right)^{12} \div \left(-\frac{11}{3}\right)^{10}$

(vii) $\left(\frac{-4}{5}\right)^{10} \div \left(\frac{-4}{5}\right)^5$ (viii) $(-xy)^6 \div (-xy)^4$ (ix) $(12c^4) \div (4c^2)$

3. **Simplify by using the power law and express the result in positive power notation.**

(i) $(5^4)^3$ (ii) $(x^{10})^5$ (iii) x^2y^{-4}

(iv) $\left(\left(\frac{3}{5}\right)^2\right)^4$ (v) $\frac{5^4}{5^4}$ (vi) $\frac{3x^4y^6}{6x^6y^5}$

(vii) $\frac{(a^3)^5}{a^{-16}}$ (viii) $\frac{x^6}{(x^{-3})^2}$ (ix) $(-3)^{-4}$

4. **Simplify the following.**

(i) $x^4 \times x^5 \div x^3$ (ii) $(x^6 \div x^4)^2$ (iii) $(xy)^4 \div (3xy)^3$

(iv) $(5x^2y^3)^3 \div 5x^4y^7$ (v) $\frac{(2a^5)^5}{3ab} \div \frac{a}{2b^2}$ (vi) $\frac{2x^5}{y^2} \div \frac{4x^2}{x^2y}$

(vii) $\frac{5y^3}{xy} \div \frac{2xy^3}{3y^2}$ (viii) $\frac{(2ab^2)^5}{(4a^2b)^2(ab^3)}$ (ix) $\frac{2a^8 \times (2a)^8}{2a\,(2a)^8}$

Algebraic Identities

Find Special Product of Polynomials

An algebraic identity simplifies answering a mathematical problem by using a pattern of calculations instead of performing the whole process.

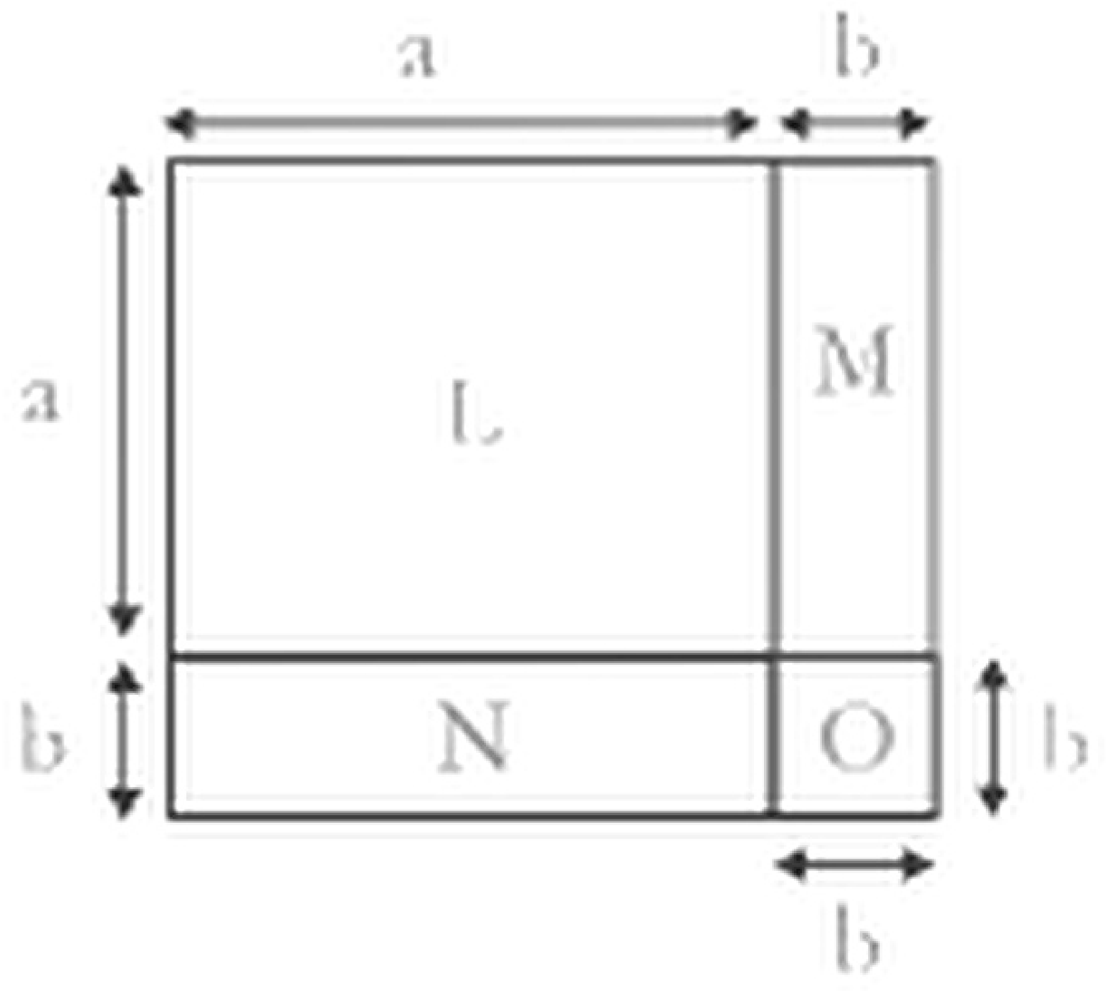
We will learn about three basic identities.

(i) $(a + b)^2 = a^2 + 2ab + b^2$

(ii) $(a - b)^2 = a^2 - 2ab + b^2$

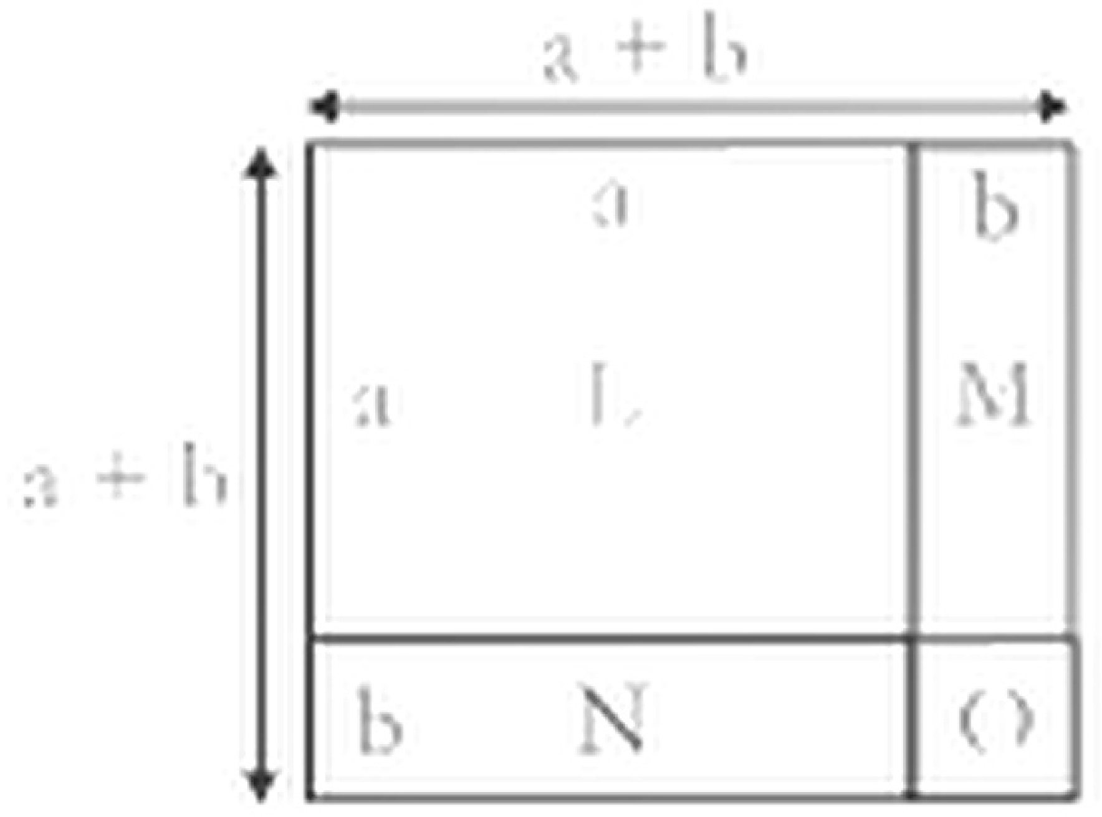
(iii) $a^2 - b^2 = (a + b)(a - b)$

(iv) Consider the area of a square whose sides are (a + b).



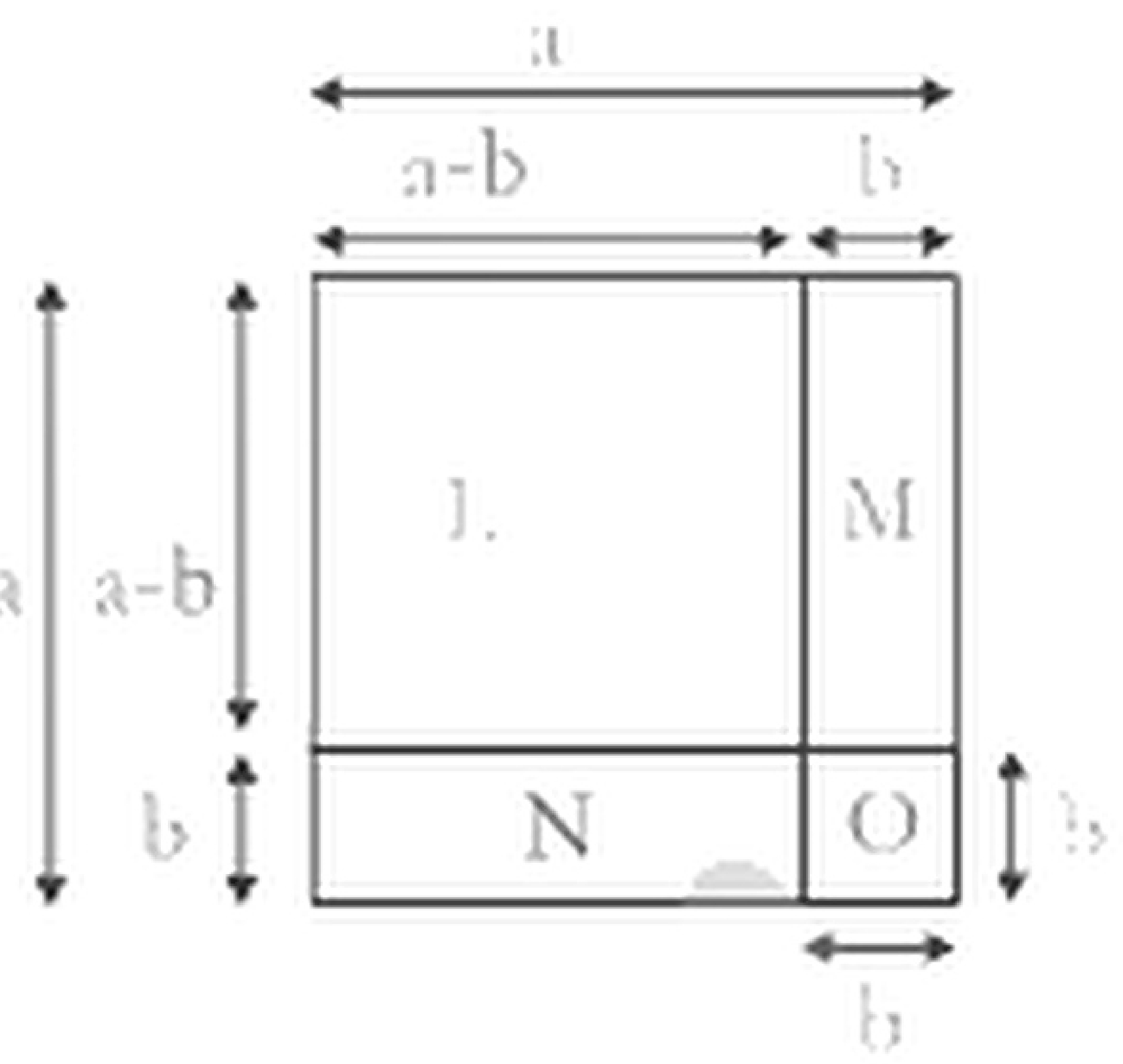
$$\text{Area of a square} = (a+b)\,(a+b)$$

The area of the square whose sides are (a + b) is equal to the sum of the areas of regions L, M, N and O.



$$(a + b)(a + b) = a^2 + ab + ab + b^2$$
$$a^2 + 2ab + b^2$$
$$\therefore \quad (a + b)^2 = a^2 + 2ab + b^2$$

(ii) Consider the area of a square whose sides are of length "a" and the area of two rectangles whose length and width are a and b respectively.



Area of the unshaded square is equal to $(a - b)\ (a - b)$

$(a - b\ )\ (a - b)$ = Area of original square of side a

− Area of two rectangles of sides a and b

+ Area of small square of side b

$= a^2 - ab - ab + b^2$

$= a^2 - 2ab + b^2$

(because you subtract the area of the small square twice, you must add one area).

$\therefore\quad (a - b)^2 = a^2 - 2ab + b^2$

Example:

Find the square of the following using an identity.

(i) $x + 2y$ (ii) $2m + 5n$ (iii) $x - 4$ (iv) $\frac{3}{5}x - \frac{3}{5}y$

(i) $(x + 2y)^2 = (x)^2 + 2\ (x)(2y) + (2y)^2$

$= x^2 + 4xy + 4y^2$

(ii) $(2m + 5n)^2 = (2m)^2 + 2\ (2m)\ (5n) + (5n)^2$

$= 4m^2 + 20mn + 25n^2$

(iii) $(x - 4)^2 = (x)^2 - 2(x)\ (4) + (4)^2$

$= x^2 - 8x + 16$

(iv) $\left(\frac{3}{5}x - \frac{3}{5}y\right)^2 = \left(\frac{3}{5}x\right)^2 - 2\left(\frac{3}{5}x\right)\left(\frac{3}{5}y\right) + \left(\frac{3}{5}y\right)^2$

$= \frac{9}{25}x^2 - \frac{18}{25}xy + \frac{9}{25}y^2$

Example:

Simplify the following.

(i) $(2x + 3y)^2 - (\ x + 2y)^2$ (ii) $(3a - 4b)^2 + (2a - 3b)^2$ (iii) $(7a + 6b)^2 - (2a - 3b)^2$

(i) $(2x + 3y)^2 - (\ x + 2y)^2$

$= (2x)^2 + 2(2x)(3y) + (3y)^2 - [(x)^2 + 2\ (x)(2y) + (2y)^2]$

$= 4x^2 + 12xy + 9y^2 - [x^2 + 4xy + 4y^2]$

$= 4x^2 + 12xy + 9y^2 - x^2 - 4xy - 4y^2$

$= 3x^2 + 8xy + 5y^2$

(ii) $(3a - 4b)^2 + (2a - 3b)^2$

$= (3a)^2 - 2(3a)(4b) + (4b^2) + (2a)^2 - 2(2a)(3b) + (3b)^2$

$= 9a^2 - 24ab + 16b^2 + 4a^2 - 12ab + 9b^2$

$= 13a^2 - 36ab + 25b^2$

(iii) $(7a + 6b)^2 - (2a - 3b)^2$

$= (7a + 6b)^2 - (2a - 3b)^2$

$= [(7a)^2 + 2(7a)(6b)+(6b)^2] - [(2a)^2 2 - (2a)(3b)+(2b)^2]$

$= 49a^2 + 84ab + 36b^2 - [4a^2 - 12ab + 9b^2]$

$= 49a^2 + 84ab + 36b^2 - 4a^2 + 12ab - 9b^2$

$= 45a^2 + 96ab + 27b^2$

Example:

Find the values of the following by using an appropriate formula.

(i) $(102)^2$ (ii) $(297)^2$ (iii) $(1.98)^2$

(i) $(102)^2 = (100 + 2)^2$

$= (100)^2 + 2(100)(2) + (2)^2$

$= 10000 + 400 + 4$

$= 10404$

(ii) $(297)^2 = (300 - 3)^2$

$= (300)^2 - 2(300)(3) + (3)^2$

$= 90000 - 1800 + 9$

$= 88209$

(iii) $(1.98)^2 = (2 - .02)^2$

$= (2)^2 - 2(2)(.02) + (.02)^2$

$= 4 - 0.08 + .0004$

$= 3.9196$

Exercise 6.4

1. Fill in the blanks.

(i) $(2a + b)^2 = (2a)^2 + 2\,(\_\_\_\_\_)\,(b) + (b)^2$ (ii) $(4x + 6y)^2 = 16x^2 + (\_\_\_\_)\,xy + 36y^2$

(iii) $\left(x^2 + \frac{1}{x^2}\right)^2 = x^4 + \_\_\_\_\_\_\_\_\_\_ + \frac{1}{x^4}$ (iv) $\left(2a + \frac{1}{2b}\right)^2 = 4a^2 + \_\_\_\_\_\_\_\_\_ + \frac{1}{4b^2}$

(v) $(2x - 3y)^2 = (2a)^2 - 2\,(\quad)\,(\quad) + (3y)^2$ (vi) $(4a - \_\_)^2 = (4a)^2 - \_\_\_\_\_\_\_\_\_ + (6b)^2$

(vii) $(\_\_\_\_ - 4y)^2 = (\_\_\_\_\_)^2 - 16xy + (4y)^2$ (viii) $(x - 5)^2 = x^2 - 10x + \_\_\_\_\_\_$

2. Expand the following by using the appropriate formula.

(i) $(2a + 7)^2$ (ii) $(3x + 1)^2$ (iii) $(8a + 3b)^2$ (iv) $(x^2 + y^2)^2$ (v) $(3x + 4y)^2$

(vi) $(7x + 8y)^2$ (vii) $\left(\frac{3}{4}x + \frac{4}{3x}\right)^2$ (viii) $\left(\frac{2}{3}a + \frac{3}{2}b\right)^2$ (ix) $(3a - 7b)^2$ (x) $\left(3a - \frac{1}{3a}\right)^2$

(xi) $(3x - 11y)^2$ (xii) $\left(\frac{5}{6}x - \frac{3}{4}y\right)^2$ (xiii) $\left(\frac{x}{2} - \frac{3}{4}y\right)^2$ (xiv) $\left(\frac{2}{3}x - \frac{3}{2}y\right)^2$

3. Simplify the following.

(i) $(a + b)^2 + (2a + 2b)^2$ (ii) $(2a + 4b)^2 - (a + 3b)^2$

(iii) $(3x + 4y)^2 - (2x + 3y)^2$ (iv) $(3x - 4y)^2 + (2x - 3y)^2$

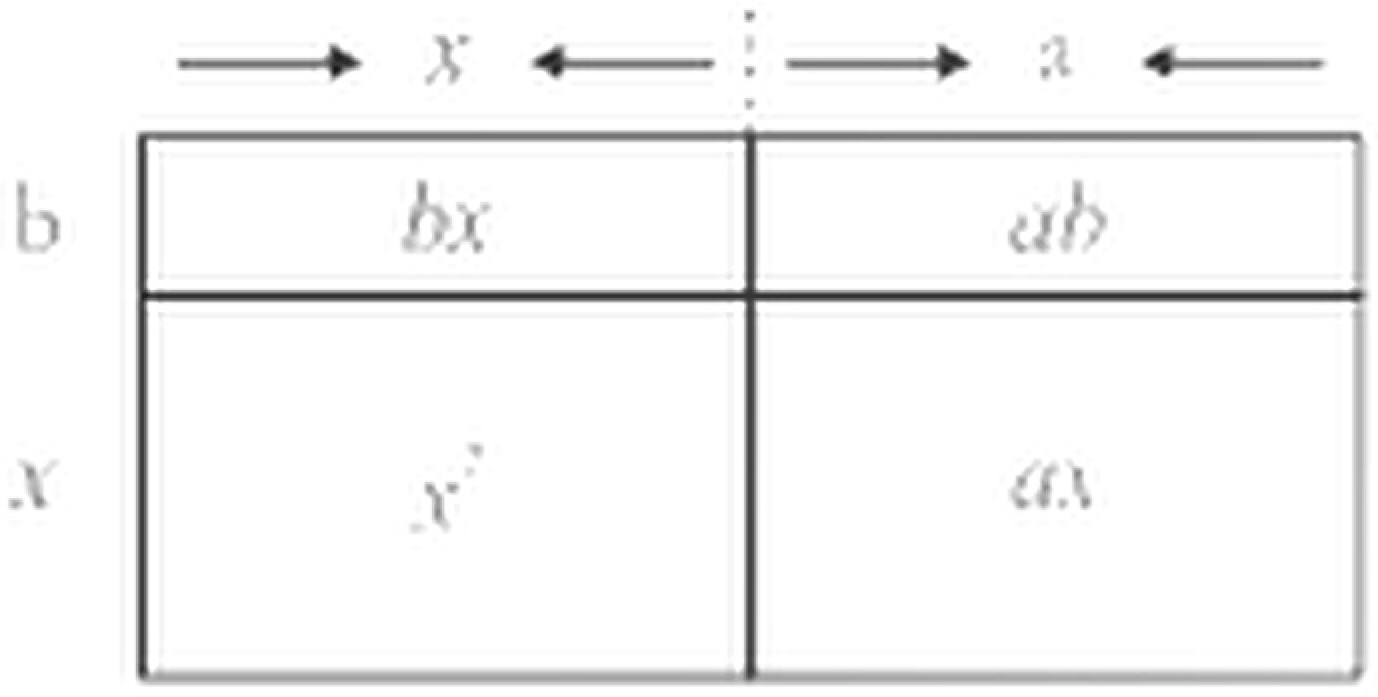
(v) $(5x - 4y)^2 + (4x - 2y)^2$ (vi) $(8x - 9y)^2 + (6x - 4y)^2$

4. Evaluate by using formula.

(i) $(48)^2$ (ii) $(103)^2$ (iii) $(196)^2$

(iv) $(504)^2$ (v) $(999)^2$ (vi) $(703)^2$

Identity III:



Prove the identity

$(x + a)\,(x + b) = x^2 + (a + b)\,x + ab$

$(x - a)\,(x + b) = x^2 - (a + b)\,x + ab$

L.H.S $= (x + a)\,(x + b) = x\,(x + b) + a\,(x + b)$

$= x^2 + xb - ax + ab$

$= x^2 + (a - b)x + ab$ = R.H.S

Example:

Find the product by using an appropriate formula.

(i) $(x + 1)(x + 2)$ (ii) $(x + 3)(x + 7)$

(iii) $(x - 2)(x + 3)$ (iv) $(2x + 5)(2x - 6)$

(i) $(x + 1)(x + 2) = x^2 + (1 + 2)x + 2$

$= x^2 + 3x + 2$

(ii) $(x + 3)(x + 7) = x^2 + (3 + 7)x + 21$

$= x^2 + 10x + 21$

(iii) $(x - 2)(x + 3) = x^2 + (2 + 3)x + (-2)(3)$

$= x^2 + x - 6$

(iv) $(2x + 5)(2x - 6) = (2x)^2 + [5 + (-6)](2x) + (5)(-6)$

$= 4x^2 - 2x - 30$

Example:

Evaluate the following by using the formula:

(i) 103×102 (ii) 102×97

Use the formula

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

(i) $103 \times 102 = (100 + 3)(100 + 2)$

$= 100 \times 100 + (3 + 2)\,100 + 3 \times 2$

$= 10000 + 5\,(100) + 6$

$= 10000 + 500 + 6$

$= 10506$

(ii) $102 \times 97 = (100 + 2)(100 - 3)$

$= 100 \times 100 + \{2 + (-3)\}\,100 + 2\;(-3)$

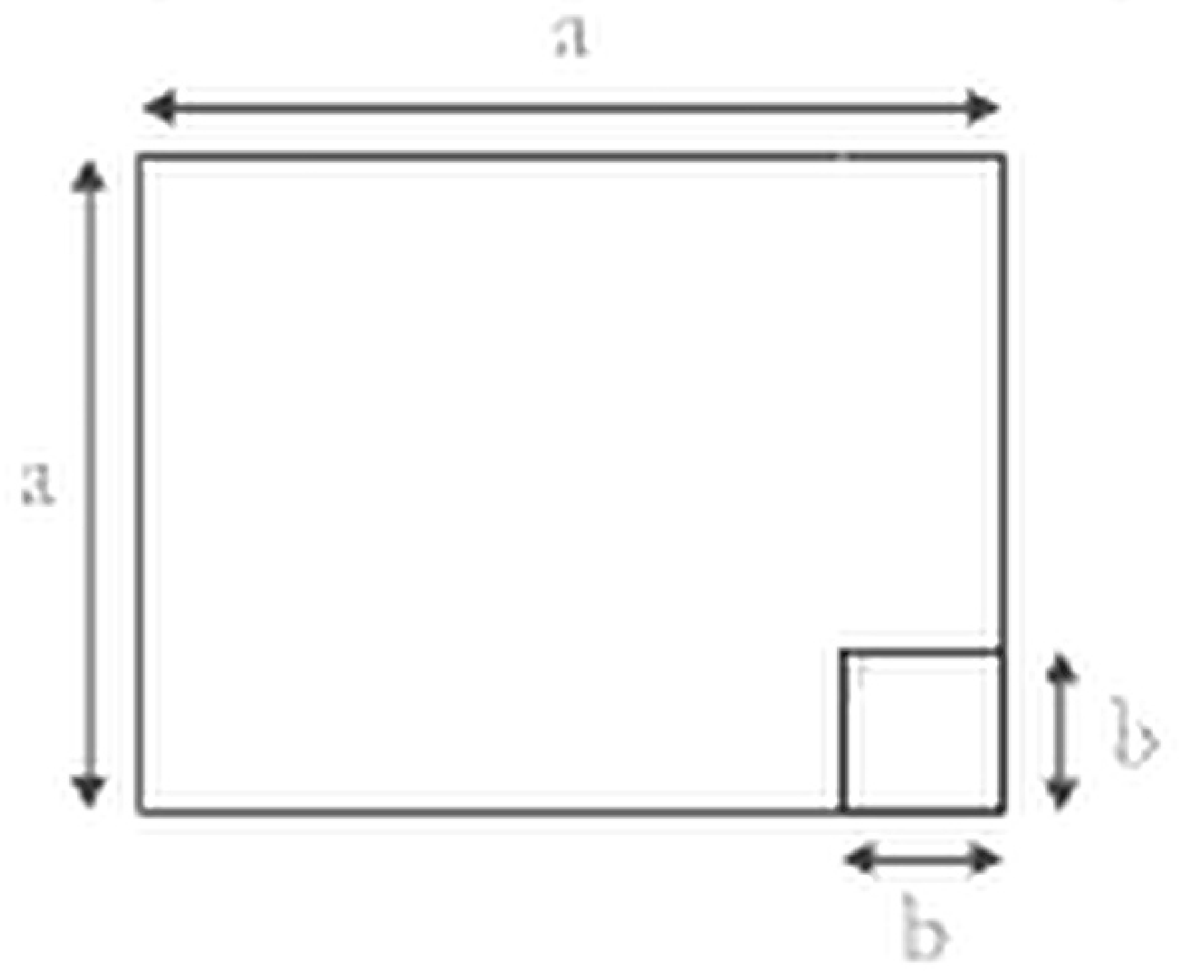
$= 10000 + (-1)\,100 + (-6)$

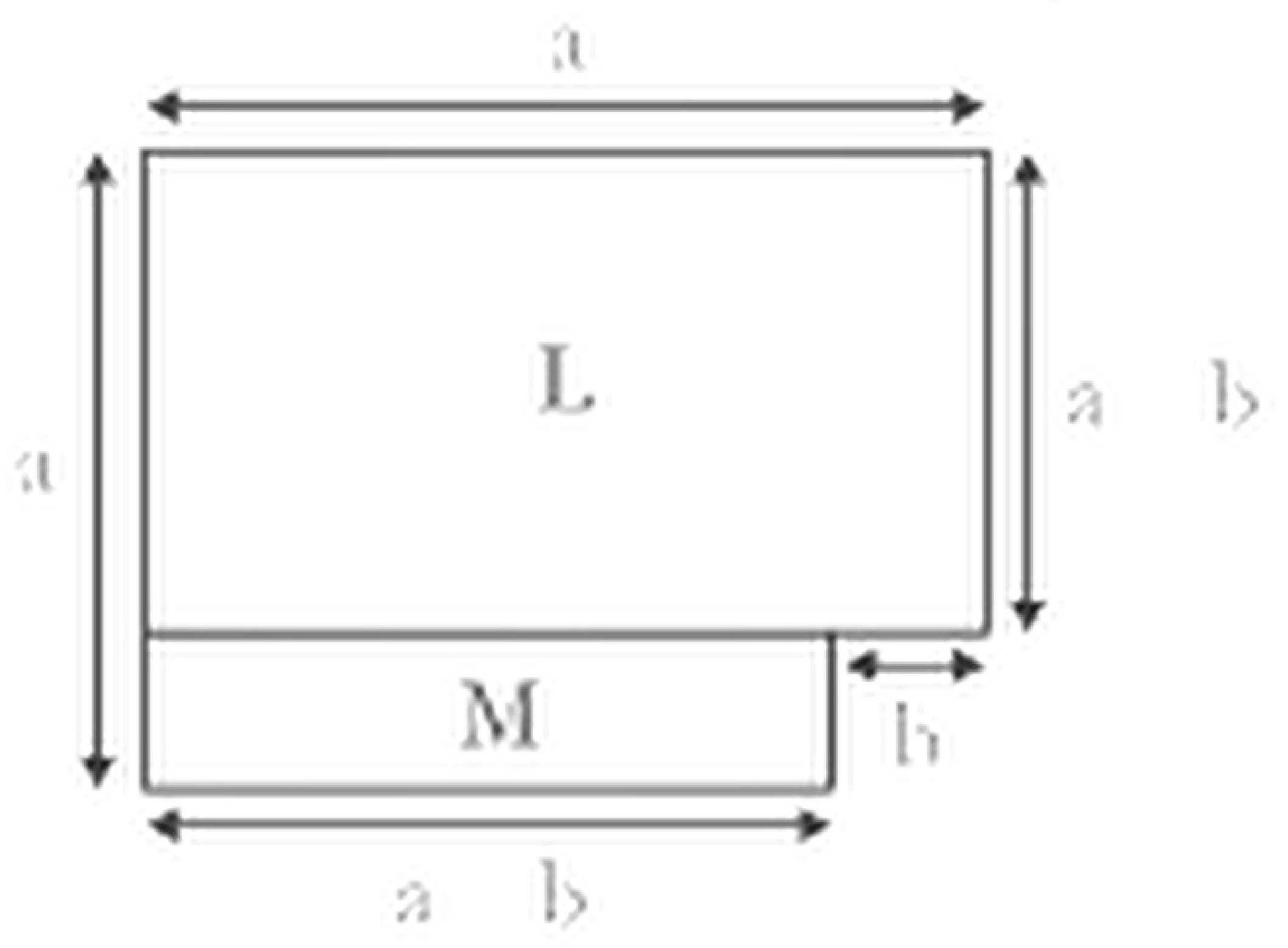
$= 10000 - 100 - 6$

$= 9894$

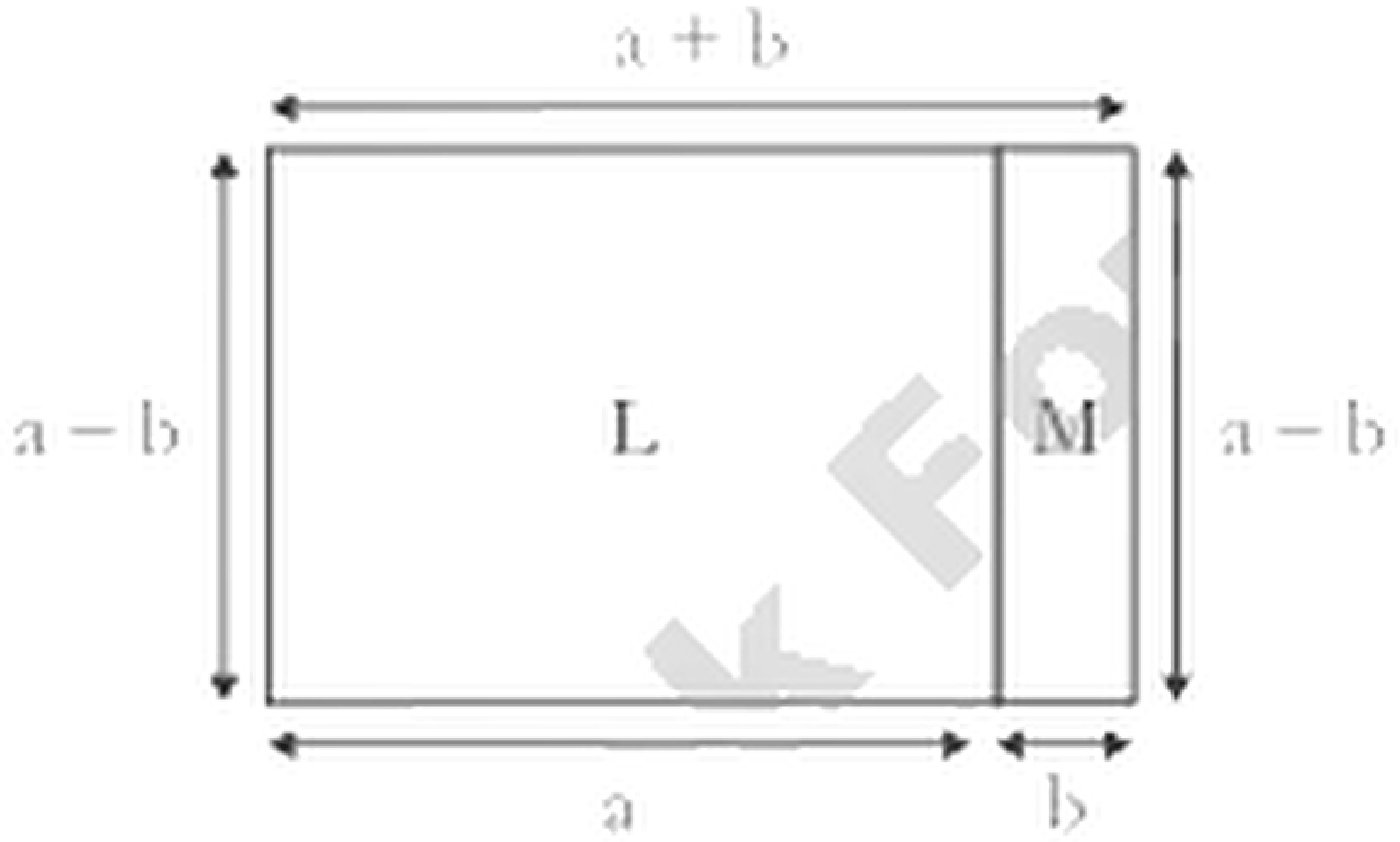
Identity IV:

Consider the square of sides a. A small square of sides b is to be cut out in the square of sides a.





The rectangle M is then cut out and placed next to rectangle L as shown below.



The area of the shaded rectangle is equal to $(a - b)\ (a \quad b)$.

$= $ Area of rectangle L + Area of rectangle M

$= a\,(a - b) + b\,(a - b)$

$= a^2 - ab + ab - b^2$

$= a^2 - b^2$

$\therefore\ (a + b)\,(a - b) = a^2 - b^2$

CHALLENGE

Describe and correct the error in factoring

$(4x^2 - 9) = (2x)^2 - 3^2$

$= (2x - 3)^2$ ✗

Example:

Find the product without multiplication.

(i) $(x + 6)\,(x - 6)$ (ii) $\left(x+\frac{4}{3}\right)\left(x-\frac{4}{3}\right)$

(i) $(x + 6)\,(x - 6)$

$= (x)^2 - (6)^2$

$= x^2 - 36$

(ii) $\left(x+\frac{4}{3}\right)\left(x-\frac{4}{3}\right)$

$= (x)^2 - \left(\frac{4}{3}\right)^2$

$= x^2 - \frac{16}{9}$

Example:

Find the product 52×48 with the help of the formula.

$52 \times 48 = (50 + 2)\,(50 - 2)$

$= (50)^2 - (2)^2$

$= 2500 - 4 = 2496$

Example:

Simplify: $(3x + 1)(3x - 1) - (2x - 3)^2$

$$(3x + 1)(3x - 1) - (2x - 3)^2$$
$$= (3x)^2 - (1)^2 - [(2x - 3)^2]$$
$$= 9x^2 - 1 - [(2x)^2 - 2(2x)(3) + (3)^2]$$
$$= 9x^2 - 1 - [4x^2 - 12x + 9]$$
$$= 9x^2 - 1 - 4x^2 + 12x - 9$$
$$= 5x^2 + 12x - 10$$

CHALLENGE

Describe and correct the error in simplifying the expression.

$5y - (2y - 8) = 5y - 2y - 8$
$= 5y - 8$ ✗

$8 + 2(4 + 3x) = 8 + 8 + 6x$
$= 22x$ ✗

Exercise 6.5

1. **Find the following products by using the appropriate formula.**

(i)	$(x + 3)(x + 4)$	(ii)	$(x + 5)(x + 7)$	(iii)	$(p - 4)(p + 6)$
(iv)	$(z + 7)(z + 9)$	(v)	$(2x - 4)(2x + 5)$	(vi)	$(3x + 7)(3x - 2)$

2. **Evaluate the following by using formula.**

(i)	103 × 96	(ii)	104 × 105	(iii)	998 × 1002

3. **Find the product without actual multiplication.**

(i)	$(x + 4)(x - 4)$	(ii)	$(7x + 8)(7x - 8)$	(iii)	$\left(6x+\frac{3}{8}\right)\left(6x-\frac{3}{8}\right)$
(iv)	$(x+2y)(x-2y)$	(v)	$(4x+7y)(4x-7y)$	(vi)	$(2a + 9b)(2a - 9b)$

4. **Find the continuous product of the following.**

(i)	$(a+b)(a-b)(a^2+b^2)$	(ii)	$(x + 2y)(x - 2y)(x^2 + 4y^2)$
(iii)	$(4a + b)(4a - b)(16a^2 + b^2)$	(iv)	$(a + 3)(a - 3)(a^2 + 9)$

5. **Evaluate with the help of formula.**

(i)	102 × 98	(ii)	65 × 55
(iii)	(1.01) × (0.99)	(iv)	202 × 198

6. **Simplify the following.**

(i) $(x + 2)(x - 2) + (x + 2)^2$

(ii) $(3a - 2)(3a + 2) - (a - 4)^2$

(iii) $(2x - y)(2x + y) - (x + 2y)(x - 2y)$

UNIT 6

Factorization

Consider the factor of 12 as 3 × 4 and the factors of polynomial $x^2 - 4$ as $(x - 2)(x + 2)$. In both cases, a product of objects is obtained. The aim of factoring is usually to reduce a number to prime numbers or polynomials to irreducible polynomials.

Factorization by making Groups

Key Fact

To factorize the expression, write it as a term inside brackets, with common factors taken outside the brackets.

Factorize: $5x - 15 = 5x - 5 \times 3$

$= 5(x - 3)$, 5 is a common factor.

Factorize: $9xy + 33y z$

$9xy + 33y z = 3y \times 3x + 3y \times 11z$

$= 3y(3x + 11z)$, 3y is a common factor.

Factorization is of different kind. Here, we will learn the following cases.

Factorization by using Algebraic Identities

(a) Case I: $a^2 \pm 2ab + b^2$

(i) $(a + b)^2 = a^2 + 2ab + b^2$

(ii) $(a - b)^2 = a^2 - 2ab + b^2$

Example: Factorize: $x^2 + 4xy + 4y^2$

$x^2 + 4xy + 4y^2$

(i) Two terms i.e. x^2 and $4y^2$ are perfect squares. i.e. $(x)^2$ and $(2y)^2$

(ii) Third term is twice the product of square roots of the other two terms.

$4xy = 2(x)(2y)$

We may write as:

$$x^2 + 4xy + 4y^2 = (x)^2 + 2(x)(2y) + (2y)^2$$
$$= (x + 2y)^2$$
$$= (x + 2y)(x + 2y)$$

Example: Factorize: $25x^2 - 10xy + y^2$

$25x^2 - 10xy + y^2$

$$25x^2 - 10xy + y^2 = (5x)^2 - 2(5x)(y) + (y)^2$$
$$= (5x - y)^2$$
$$= (5x - y)(5x - y)$$

Example: Factorize: $x^2 - x + \frac{1}{4}$

$$x^2 - x + \frac{1}{4} = (x)^2 - 2(x)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2$$

$$= \left(x - \frac{1}{2}\right)^2 = \left(x - \frac{1}{2}\right)\left(x - \frac{1}{2}\right)$$

(b) Case II: $a^2 - b^2$

$$a^2 - b^2 = (a + b)(a - b)$$

Example: Factorize: $x^2 - 16y^2$

$x^2 - 16y^2$

As square root of x^2 is x and $16y^2$ is $4y$. The sum and difference of their square roots are $(x + 4y)$ and $(x - 4y)$

$$\therefore x^2 - 16y^2 = (x)^2 - (4y)^2$$

$$= (x + 4y)(x - 4y)$$

Example: Factorize: $36a^2 - b^4$

$36a^2 - b^4$

$$36a^2 - b^4 = (6a)^2 - (b^2)^2$$

$$= (6a + b^2)(6a - b^2)$$

Example: Factorize: $\frac{a^2}{9} - \frac{b^2}{16}$

$\frac{a^2}{9} - \frac{b^2}{16}$

$$\frac{a^2}{9} - \frac{b^2}{16} = \left(\frac{a}{3}\right)^2 - \left(\frac{b}{4}\right)^2$$

$$= \left(\frac{a}{3} + \frac{b}{4}\right)\left(\frac{a}{3} - \frac{b}{4}\right)$$

Example:

Factorize: $49(x + y)^2 - 16(x - y)^2$

$$49(x + y)^2 - 16(x - y)^2$$

$$= [7(x + y)]^2 - [4(x - y)]^2$$

$$= [7(x + y) + 4(x - y)][7(x + y) - 4(x - y)]$$

$$= [7x + 7y + 4x - 4y][7x + 7y - 4x + 4y]$$

$$= [11x + 3y][3x + 11y]$$

$$= (3x + 11y)(11x + 3y)$$

Example:

Evaluate: $x^2 - 2x + 1 - y^2$

$$x^2 - 2x + 1 - y^2$$

$$= (x - 1)^2 - (y)^2$$

$$= (x - 1 + y)(x \quad 1 \quad y)$$

CHALLENGE

Describe and correct the error in multiplying

$(2d - 10)^2 = 4d^2 - 20d + 100$ ✗

$(s + 3)^2 = s^2 + 9$ ✗

Exercise 6.6

1. **Factorize the following (make groups).**

(i) $4x^2 - 6$ (ii) $5xy + 10\,xy^2$ (iii) $x^2y^2 - 16x^2$

(iv) $50x^3 - 8xb^2$ (v) $a^2b^2 - b^2$ (vi) $(a + b)^3 + (a + b)\,(a - b)^2$

2. **Factorize the following.**

(i) $4a^2 + 4ab + b^2$ (ii) $1 + 14y^2 + 49y^4$

(iii) $4x^2 + 12xy + 9y^2$ (iv) $25x^2 + 30x + 9$

(v) $x^2 - 6xy + 9y^2$ (vi) $4a^2 - 8ab + 4b^2$

(vii) $9x^2 - 12xy + 4y^2$ (viii) $\frac{4}{9}\,x^2 - xy + \frac{9}{16}\,y^2$

(ix) $16x^2 + 4xy + \frac{1}{4}\,y^2$ (x) $\frac{9}{16}\,x^2 + 2xy + \frac{16}{9}\,y^2$

(xi) $18a^2 - 84ab + 98b^2$ (xii) $243x^2 + 54x + 3$

(xiii) $x^4 + 2x^2 + 1$ (xiv) $x^2 - 22xy + 121y^2$

3. **Factorize the following.**

(i) $x^2 - 81$ (ii) $4x^2 - y^2$

(iii) $x^2y^2 - 1$ (iv) $x^2 - (y + 3)^2$

(v) $a^2b^2 - 36b^2$ (vi) $(a + b)^2 - (a - b)^2$

(vii) $4a^2 - \frac{9}{16}$ (viii) $x^4 - y^4$

(ix) $49\,(2x - 2y)^2 - 16\,(x - y)^2$ (x) $15\,(a + b)^2 - 60\,(a - b)^2$

6 UNIT

I have learnt

- Algebraic expressions consist of numbers, variables and operations.
- The parts of the expressions connected by the signs of " + " and " – " are called the terms of the expression.
- A polynomial is an algebraic expression in which power of variables are whole numbers.
- A polynomial consisting of a single term is known as a monomial.
- A polynomial with two terms is called a binomial.
- A polynomial of three terms is called a trinomial.
- Addition of polynomials means to add or subtract the coefficients of like terms.
- The sum of two positive terms is positive.
- The sum of two negative terms is negative.
- If two terms with different signs are added then the results is the difference of the terms with the sign of the greater term.
- To subtract a polynomial from another polynomial, we change the sign of the first polynomial and add it to the second one.
- The product of two terms with positive signs is positive.
- The product of two terms with negative signs is positive.
- The product of two terms with different signs is negative.
- Factorization is the decomposition of an expression into a product of other expressions or factors, which when multiplied together gives the original expression.

If x and y are any numbers and m and n are integers, then

(i) Product Laws: (a) $x^m \times x^n = x^{m+n}$ (b) $x^n \times y^n = (xy)^n$	(ii) Quotient Laws: (a) $x^m \div x^n = x^{m-n}$ (b) $x^n \div y^n = \left(\frac{x}{y}\right)^n$	(iii) Power Law: (a) $\left(x^m\right)^n = x^{mn}$.
(iv) Zero Exponent: (a) $x^0 = 1$	(v) Negative Exponent: (a) $x^{-n} = \frac{1}{x^n}$	

- To simplify and factorize the algebraic expressions, the following identities are used.

(i) $(x + a)(x + b) = x^2 + (a + b)x + ab$ (ii) $(a + b)^2 = a^2 + 2ab + b^2$

UNIT 6

Review Exercise 6

1. **Encircle the correct option in the following statements.**

(i) $2^5 \times 2^{-3} =$

(a) 2^8 (b) 2^{-2} (c) 2^2 (d) 2^{-15}

(ii) $\left(-\frac{3}{4}\right)^6 \times \left(-\frac{3}{4}\right)^9 =$

(a) $\left(-\frac{3}{4}\right)^3$ (b) $\left(-\frac{3}{4}\right)^{54}$ (c) $\left(-\frac{3}{4}\right)^{-3}$ (d) $\left(-\frac{3}{4}\right)^{15}$

(iii) $\left(\frac{2}{5} \times \frac{3}{7}\right)^5 =$

(a) $\left(\frac{2}{5}\right)^5$ (b) $\left(\frac{2}{5}\right)^5 \times \left(\frac{3}{7}\right)^5$ (c) $\left(\frac{2}{5}\right)^5 \times \left(\frac{3}{7}\right)$ (d) $\left(\frac{3}{7}\right)^5$

(iv) $7^9 \div 7^6 =$

(a) 7^3 (b) 7^{-3} (c) 7^9 (d) 7^6

(v) $a^4b^4 \div a^3b^2 =$

(a) $\frac{a}{b^2}$ (b) $\frac{a}{b}$ (c) $\frac{b^2}{a}$ (d) ab^2

(vi) $\left(x^4\right)^3 \div x^{-4} =$

(a) x^{12} (b) x^{16} (c) x^8 (d) x^{11}

(vii) $y^5 \div \left(y^2\right)^3 =$

(a) y (b) 1 (c) y^{11} (d) $\frac{1}{y}$

(viii) $(-4)^6 =$

(a) -4^6 (b) $(-6)^4$ (c) 4^6 (d) $-(-4)^6$

(ix) $\left(\frac{x}{y}\right)^{3} \div \left(\frac{y}{x}\right)^{2} =$

(a) $\frac{x}{y}$ (b) $\frac{x^5}{y^5}$ (c) $\frac{x^5}{y}$ (d) $\frac{x}{y^5}$

(x) $(-2)^5 =$

(a) 2^5 (b) 32 (c) –32 (d) –25

(xi) $3x^3 + 2x^2 + 1$ is polynomial of degree.

(a) 2 (b) 3 (c) 1 (d) 4

(xii) Sum of $3x + 4$ and $4x - 2$ is.

(a) $8x+6$ (b) $7x$ (c) $7x-2$ (d) $8x+2$

(xiii) Subtracting $6x - 4$ from $4x - 3$, we get:

(a) $10x+1$ (b) $-2x-7$ (c) $-2x+1$ (d) $8x+7$

(xiv) The product $\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)$ is

(a) $a+b$ (b) a^2-b^2 (c) a^2+b^2 (d) $a-b$

(xv) The square of $x+2y$ is

(a) x^2+4y^2 (b) x^2+4y^2+4xy (c) x^2+4y^2-4xy (d) x^2-4y^2

(xvi) The square of $y-2x$ is

(a) y^2+4x^2-4xy (b) y^2+4x^2+4xy (c) y^2+4x^2 (d) y^2-4x^2

(xvii) The product of $2x+3$ and x is

(a) $2x^2+3x$ (b) $2x^2-3x$ (c) $2x^2-6$ (d) $2x^2+3x+6$

(xviii) The simplified form of $(a+b)-(a-b)$ is

(a) $4ab$ (b) $2a+2b$ (c) $2b$ (d) $2a$

(xix) The product of $(x+2)(x-2)$ is

(a) x^2+4 (b) x^2-4 (c) x^2 (d) $2x$

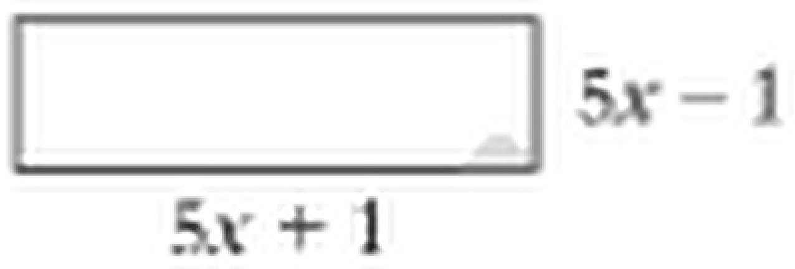
(xx) Factors of 6^2-x^2 are

(a) $(x-6)(6+x)$ (b) $(6+x)^2$ (c) $(6-x)(6-x)$ (d) $(6+x)(6-x)$

2. Find the perimeter of a square of side $x + y$.

3. Find the area of a square of side 4a − b.

4. Find the area and perimeter of the rectangle.



5. Simplify $(3^9 \div 3^4)\times 3^{-4}$

6. Simplify $\frac{6a^6}{a^2b}\div\frac{12ab^3}{4b^2}$

7. Simplify $(2x + 1)\ (2x - 1) - (2x - 1)^2$

8. Simplify $60\ (a + b)^2 - 45\ (a - b)^2$

9. Simplify $(\ y - z)^2 - (\ y + z)^2$

10. Factorize $x^8 - y^8$

ABOUT AUTHORS

Dr. Saleem Ullah

Dr. Saleem Ullah possesses Ph.D (Maths) along with different professional teaching degrees. He has experience of two decades in teaching of mathematics at various levels. He authored and reviewed many books. He has been involved in the development of curriculum for mathematics upto higher level. As a resource person, he has conducted training programmes, workshops and seminars on the subject. Email: saleemullah314@hotmail.com

Dr. Khalid Mahmood

He has a long teaching experience of more than two decades at school and college level. He holds the degree of Ph.D (Maths) along with professional degree of M.Ed. He has authored and reviewed many books in the subject of mathematics at different level. He has worked as a resource person for imparting training to teachers at elementary and secondary levels. Email: khalidmeh2012@gmail.com

Mrs. Tayyaba Saqib

She has done M.Sc in the subject of Mathematics and has twenty-five years of experience in the field of education particularly teaching mathematics up to O-level. She has been involved in the development of curriculum 2006, 2016 and 2019. She has been remained to import training to the teachers at various levels. She has reviewed a number of mathematics books. Email: tayyaba_saqib@hotmail.com

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