Numerical Problems

1. A 1.50cm length of pianos wire with a diameter of 0.25cm is stretched by attaching a 10kg mass to one end. How far is the wire stretched?

Given: Length L = 1.50cm = 1.50×10^{-2}m

Diameter D = 0.25cm = 0.25×10^{-2}m D   ,

Radius = r = $\frac{D}{2}$ = 0.125×10^{-2}m, Mass = m= 10kg

Young's module of steel Y = 20×10^{10} Nm'2 To find: Change in length ΔL = ?

Solution:

Young's modulus is $y = \frac{F/A}{\Delta l/l}$

or $Y = \frac{F \times L}{\Delta L \times A}$

$\Delta L = \frac{F \times L}{Y \times A}$ (i)

The factor applied is by the weight F = W = mg   (ii)

Consider the wire to be circular with cross-sectional area A given by area of circle as

$A = \pi r^2$ (iii)

Putting equation (ii) and equation (iii) in equation (i), we get

$\Delta L = \frac{mg \times L}{Y \times \pi r^2}$

Putting values: $\Delta L = \frac{10kg \times 9.8ms^2 \times 1.50 \times 10^{-2} m}{20 \times 10^{10} Nm^{-2} \times 3.14 \times (0.125 \times 10^{-2} m)^2}$

$\boxed{\Delta L = 1.498 \times 10^4 m}$

2. A cable has a length of 12m and is stretched by 1.2×10⁻⁴m when a stress of 8.0 ×10⁸ Nm⁻² is applied. What is the strain energy per unit volume in the cable when the stress is applied?

Given: Length L = 12m

Change in length ΔL = 1.2 × 10⁻⁴ m

Stress = 8.0 × 10Nm⁻²

To find:

Strain energy density = ?

Solution:

Strain energy per unit volume is strain energy density $= \frac{1}{2} \times$ stress $\times$ strain

or   energy density $= \frac{1}{2} \times$ stress $\times \frac{\Delta L}{L}$

Putting values: energy density $= \frac{1}{2} \times 8.0 \times 10^8$ Nm⁻² $\times \frac{1.2 \times 10^{-4} m}{12m}$

Therefore, | Strain energy density = 4.0 x 10³ Nm⁻² = 4.0 x 10³ Jm⁻³ |

3. A cylindrical steel rod 0.50m long and 1cm in radius is subjected to a tensile force of 1×10⁴ N.

Given: Length L = 0.50m

Radius r = 1cm = 1.0 x 10⁻²m

Force F = 1.0 × 10⁴N

Young's modulus of steel Y = 20 × 10¹⁰Nm⁻²

To find:

(a) Tensile stress = ?

(b) Change in length ΔL = ?

Solution:

The tensile stress is given by the relation

Tensile stress = $\frac{F}{A}$ (i)

Consider the wire to be circular with cross-sectional area A given by area of circle as

$A = \pi r^2$ (ii)

Putting equation (ii) in equation (i), we get

$$\text{Tensile stress} = \frac{F}{\pi r^2}$$

Putting values: Tensile strain = $\frac{1.0 \times 10^4 N}{3.14 \times (1.0 \times 10^{-2} m)}$

Tensile stress = 0.3184 x 10⁸Nm⁻²

(b) Young's modulus is $Y = \dfrac{tensile\ stress}{tensile\ strain}$

Or $= \dfrac{tensile\ stress}{Y}$

Putting values: tensile strain = $\dfrac{0.21 \times 10^8 Nm^{-2}}{20 \times 10^{10} Nm^{-2}}$

Hence,   tensile strain = 1.6 x 10⁻⁴

(c) The tensile strain is defined as

Tensile strain = $\frac{\Delta L}{L}$

ΔL = tensile strain x L

Putting values: ΔL = 1.6 x 10⁻⁴ x 0.50m

Hence ΔL = 0.8 x 10⁻⁴ m

4. A cable has an un-stretched length of 12m and it stretches by 1.2 x 10⁻⁴m when a stress of 6.4 x10⁸ Nm⁻² is applied. What is the strain energy per unit volume in the cable when this stress is applied? (3.2 x 10³ Jm⁻³)

Given: Length L= 12m

Change in length ΔL = 1.2 x 10⁻⁴ m

Stress = 6.4 x 10⁸ Nm⁻²

To find: Strain energy density = ?

Solution:

Strain energy density = $\frac{1}{2}$ × $stress$ × $\frac{\Delta L}{L}$

Putting Values:

Strain energy density = $\frac{1}{2}$ × 6.4 x 10⁸ Nm⁻² × $\frac{1.2 \times 10^{-4} m}{12 m}$

Therefore, | Strain energy density = 3.2 x 10³ Nm⁻² = 3.2 x 10³ Jm⁻³ |

5. Young's Modulus for a particular wood is 1×10¹⁰ Nm⁻². A wooden chair has four legs each of length 42cm and cross-sectional area 2×10⁻³ m². Hamza has a mass of 100Kg.

(a) What is the stress on each leg of the chair when Hamza stands on the chair?

(b) By what amount do the chair legs shrink when Hamza stands on the chair?

Given: Length L = 42cm = 0.42m

Area A = 2 x 10⁻³ m²

Mass m = 100kg

Young's modulus of wood Y = 1.0 x 10¹⁰ Nm⁻²

To find:

(a) Tensile stress = ?

(b) Change in length AL = ?

Solution:

(a) The tensile stress given by the relation

$$\text{Tensile stress} = \frac{F}{A} \qquad \text{(i)}$$

As the force is provided by the weight of Hamza,

therefore $F = W = mg$ \qquad (ii)

Putting equation (ii) in equation (i), we get

$$\text{Tensile stress} = \frac{mg}{A}$$

Putting values: $\text{Tensile stress} = \dfrac{100kg \times 9.8ms^{-2}}{2 \times 10^{-3} m^2}$

$$\boxed{\text{Tensile stress} = 4.90 \times 10^5 \ Nm^{-2}}$$

(b) Young's modulus is $Y = \dfrac{\text{tensile stress}}{\text{tensile strain}}$

Or $\text{Tensile strain} = \dfrac{\text{tensile stress}}{Y}$ \qquad (i)

The tensile strain is defined as

$$\text{Tensile strain} = \frac{\Delta L}{L} \qquad \text{(ii)}$$

Putting equation (ii) in equation (i), we get

$$\frac{\Delta L}{L} = \frac{\text{tensile stress}}{Y}$$

$$\Delta L = \frac{\text{tensile stress}}{Y} \times L$$

Putting values

$$\Delta L = \frac{4.9 \times 10^5 \ Nm^{-2}}{1.0 \times 10^{10} \ Nm^{-2}} \times 0.42m$$

Hence, $\boxed{\Delta L = 2.058 \times 10^{-4} \ m}$

6. A force of 1.5 x 10⁴N causes a strain of 1.4x10⁻⁴ in a steel cable of cross-sectional area 4.8x10⁻⁴ m².

(a) What is the Young's Modulus of the steel cable?

(b) The stress strain graph is linear for this cable. Calculate the strain energy per unit volume stored in the cable when the cable has strain of 1 x 10^{-4}.

Given: Force F = 1.5 x 10⁴N

Area A = 4.8 x 10-4 m^{-2}

Tensile strain (a) = 1.4 x 10^{-4}

Tensile strain (b) = 1.0 x 10^{-4}

To Find:

(a) Young's modulus Y = ?

(b) Strain energy density = ?

Solution:

(a) Young's modulus is $Y = \frac{Tensile\ stress}{tensile\ strain}$ (i)

Tensile by definition is

Tensile stress = $\frac{F}{A}$

Tensile stress = $\frac{1.5 \times 10^4 N}{4.8 \times 10^{-4} m^2}$

Tensile stress = 3.12 x 10⁷ Nm^{-2}

Putting values in equation (i)

$Y = \frac{3.125 \times 10^7 Nm^{-2}}{1.4 \times 10^{-4}}$

$\boxed{Y = 2.23 \times 10^{11} Nm^{-2}}$

(b) Strain energy per unit volume is

Strain energy density = $\frac{1}{2}$ x stress x strain   (ii)

When strain is 1 x 10^{-4} the stress can be calculated from Young's modulus relation as:

$$Y = \frac{tensile\ stress}{tensile\ strain}$$

tensile stress = (Y x tensile strain)

Putting values:

Tensile stress = 2.23×10^{11} Nm^{-2} x 1 x 10^4

Tensile stress = 2.23×10^7 Nm^{-2}

Putting values in equation (ii), we get

Strain energy density = $\frac{1}{2}$ x 2.23×10^7 Nm^{-2} x 1 x 10^{-4}

Strain energy density = 1.115×10^7 Nm^{-2} = $1.115 \times 10^7 Jm^{-3}$