Exercise 7.2

Q1. Let A(2,5), B(-1,1) and C(2,-6) find

(i) $\vec{AB}$, (ii) $2\vec{AB} - \vec{CB}$, (iii) $2\vec{CB} - 2\vec{CA}$

Solution

i. $\vec{AB} = (-1,1) - (2,5)$

$= (-1-2, 1-5)$

$\vec{AB} = -3\underline{i} - 4\underline{j}$

ii. $2\vec{AB} - \vec{CB} = 2[(-1,1) - (2,5)] - [(-1,1)-(2,-6)]$

$= 2(-3,-4)-(-3,7)$

$= (-6,-8) - (-3,7)$

$= (-6+3, -8-7)$

$= -3\underline{i} - 15\underline{j}$

iii. $2\vec{CB} - 2\vec{CA}$

Solution

$= 2\{(-1,1)-(2,-6)\} - 2\{(2,5)-(2,-6)\}$

$= 2(-1-2, 1+6)-(2-2, 5+6)$

= (-6,14)-(0,22)

= (-6-0,14-22)

= $-6\underline{i} + 8\underline{j}$

Q2. Let $\underline{U}$ = $\underline{i}$-2$\underline{j}$ -$\underline{k}$, $\underline{V}$ = 3 $\underline{i}$-2$\underline{j}$ +2$\underline{k}$, $\underline{W}$ = 5$\underline{i}$ - $\underline{j}$ +3$\underline{k}$. Find the indicated vector or number.

i. $\underline{U} + 2\underline{V} + \underline{W}$ **ii.** $\underline{V} - 3\underline{W}$ **iii.** $|3\underline{V} + \underline{W}|$

Solution

i. $\underline{U} + 2\underline{V} + \underline{W}$ $= (\underline{i}-2\underline{j}-\underline{k}) + 2(3\underline{i}-2\underline{j}+2\underline{k}) + (5\underline{i}-\underline{j}+3\underline{k})$

$= \underline{i}-2\underline{j}-\underline{k}+6\underline{i}-4\underline{j}+4\underline{k}+5\underline{i}-\underline{j}+3\underline{k}$

$= 12\underline{i}-3\underline{j}+6\underline{k}$

ii. $\underline{V} - 3\underline{W} = (3\underline{i}-2\underline{j}+2\underline{k} - 15\underline{i}+3\underline{j}-9\underline{k})$

$= 3\underline{i}-2\underline{j}+2\underline{k} - 15\underline{i}+3\underline{j}-9\underline{k}$

$= -12\underline{i}+\underline{j}-7\underline{k}$

iii. $|3\underline{V} + \underline{W}|$

Consider

$3\underline{V} + \underline{W} = 3(3\underline{i}-2\underline{j}+2\underline{k}) + (5\underline{i}-\underline{j}+3\underline{k})$

$= 9\underline{i}-6\underline{j}+6\underline{k} + 5\underline{i}-\underline{j}+3\underline{k}$

$= 14\underline{i}-7\underline{j}+9\underline{k}$

$|3\underline{V} + \underline{W}| = \sqrt{(14)^2 + (-7)^2 + (9)^2}$

$= \sqrt{196 + 49 + 81}$

$= \sqrt{326}$

Q3. Find the magnitude of the vector V and write the direction cosines of V.

Solution

(i) $\underline{V} = 2\underline{i}+3\underline{j}+4\underline{k}$

$$|\underline{V}| = \sqrt{(2)^2 + (3)^2 + (4)^2}$$

$$= \sqrt{4 + 9 + 16}$$

$$= \sqrt{29}$$

And D.C of **V** are $\left[\frac{2}{\sqrt{29}}, \frac{3}{\sqrt{29}}, \frac{4}{\sqrt{29}}\right]$

(ii) $\underline{V} = \underline{i}-\underline{j}-\underline{k}$

Solution:

$$|\underline{V}| = \sqrt{(1)^2 + (-1)^2 + (-1)^2}$$

$$= \sqrt{1 + 1 + 1}$$

$$= \sqrt{3}$$

D.Cs = $\left[\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right]$

(iii) $\underline{V} = 4\underline{i}-5\underline{j} = 4\underline{i}-5\underline{j}+0\underline{k}$

Solution

$$|\underline{V}| = \sqrt{(4)^2 + (-5)^2 + (0)^2}$$

$$= \sqrt{16 + 25} = \sqrt{41}$$

D.Cs = $\left[\frac{4}{\sqrt{41}}, \frac{-5}{\sqrt{41}}, 0\right]$

Q4. Find α, so that $\left|\alpha \underline{\mathrm{i}} + (\alpha + 1)\underline{\mathrm{j}} + 2\underline{\mathrm{k}}\right| = 3$

Solution

As, we are given that

$$\left|\alpha \underline{\mathrm{i}} + (\alpha + 1)\underline{\mathrm{j}} + 2\underline{\mathrm{k}}\right| = 3$$

$$\Rightarrow \quad \sqrt{(\alpha)^2 + (\alpha + 1)^2 + (2)^2} = 3$$

$$\Rightarrow \quad \sqrt{\alpha^2 + \alpha^2 + 2\alpha + 1 + 4} = 3$$

$$\sqrt{2\alpha^2 + 2\alpha + 5} = 3$$

Taking square both sides

$$\Rightarrow \quad 2\alpha^2 + 2\alpha + 5 = 9$$

$$\Rightarrow \quad 2\alpha^2 + 2\alpha - 4 = 0$$

$$\Rightarrow \quad \alpha^2 + \alpha - 2 = 0$$

$$\Rightarrow \quad \alpha(\alpha + 2) - 1(\alpha + 2) = 0$$

$$(\alpha + 2)(\alpha - 1) = 0$$

$$\Rightarrow \quad \alpha = -2 \quad \text{or} \quad \alpha = 1$$

Q5. Find a unit vector in the direction of $\underline{\mathrm{V}} = \underline{\mathrm{i}} + 2\underline{\mathrm{j}} - \underline{\mathrm{k}}$

Solution

$\underline{V} = \underline{i}+2\underline{j} - \underline{k}$

$|\underline{V}| = \sqrt{(1)^2 + (2)^2 + (-1)^2}$

$= \sqrt{1+4+1}$

$= \sqrt{6}$

Let $\underline{V}$ be the required unit vector in the direction of ⊥.

Then $\hat{V} = \frac{\underline{V}}{|\underline{V}|}$

$\hat{V} = \frac{1}{\sqrt{6}}(\underline{i}+2\underline{j} - \underline{k})$

$= \frac{1}{\sqrt{6}}\underline{i} + \frac{2}{\sqrt{6}}\underline{j} - \frac{1}{\sqrt{6}}\underline{k}$

Q6. If $\underline{a} = 3\underline{i} - \underline{j} - 4\underline{k}$, $\underline{b} = -2\underline{i}-4\underline{j} - 3\underline{k}$ and $\underline{c} = \underline{i}+2\underline{j} - \underline{k}$

Find a unit vector parallel to $3\underline{a} - 2\underline{b} + 4\underline{c}$

Solution

$\underline{d} = 3\underline{a} - 2\underline{b} + 4\underline{c}$

$= 3(3\underline{i} - \underline{j} - 4\underline{k}) - 2(-2\underline{i} - 4\underline{j} - 3\underline{k}) + 4(\underline{i}+2\underline{j} - \underline{k})$

$= 9\underline{i} - 3\underline{j} - 12\underline{k} + 4\underline{i} + 8\underline{j} + 6\underline{k} + 4\underline{i} + 8\underline{j} - 8\underline{k}$

$= 17\underline{i} + 13\underline{j} - 10\underline{k}$

$|\underline{d}| = \sqrt{(17)^2 + (13)^2 + (-10)^2}$

$= \sqrt{189 + 169 + 100}$

$= \sqrt{558}$

$\hat{d} = \frac{\underline{d}}{|\underline{d}|} = \frac{17\,\underline{i}+13\underline{j}-10\underline{k}}{\sqrt{558}}$

$= \frac{17}{\sqrt{558}}\,\underline{i} + \frac{13}{\sqrt{558}}\,\underline{j} - \frac{10}{\sqrt{558}}\,\underline{k}$

Q7. Find a vector whose

i. **Magnitude is 4 and is parallel to 2 $\underline{i}$-3$\underline{j}$ +6$\underline{k}$**

ii. **Magnitude is 2 and is parallel to - $\underline{i}$+$\underline{j}$ +$\underline{k}$**

Solution

(i) $|\underline{V}| = \sqrt{(2)^2 + (-3)^2 + (6)^2}$

$= \sqrt{4 + 9 + 36}$

$= \sqrt{49}$

$= 7$

$\hat{V} = \frac{\underline{V}}{|\underline{V}|}$

Required vector is

$4\hat{V} = \frac{4\underline{V}}{|\underline{V}|}$

$= \frac{4(2\,\underline{i}-3\underline{j}+6\underline{k})}{7}$

$$= \frac{8}{7}\,\underline{i} - \frac{12}{7}\,\underline{j} - \frac{24}{7}\,\underline{k}$$

(ii) $\underline{V} = -\underline{i} + \underline{j} + \underline{k}$

$$|\underline{V}| = \sqrt{(-1)^2 + (1)^2 + (1)^2}$$

$$= \sqrt{1 + 1 + 1}$$

$$= \sqrt{3}$$

Let $\underline{U}$ is the U.V parallel to $\underline{V}$

$$\underline{U} = \frac{\underline{V}}{|\underline{V}|}$$

Required vector is

$$2\underline{U} = \frac{2\underline{V}}{|\underline{V}|}$$

$$= \frac{2(-\underline{i} + \underline{j} + \underline{k})}{\sqrt{3}}$$

$$= \frac{2}{\sqrt{3}}\,\underline{i} + \frac{2}{\sqrt{3}}\,\underline{j} + \frac{2}{\sqrt{3}}\,\underline{k}$$

Q8. If $\underline{U} = 2\underline{i} + 3\underline{j} + 4\underline{k}$, $\underline{V} = -\underline{i} + 3\underline{j} - \underline{k}$ and $\underline{W} = \underline{i} + 6\underline{j} + z\underline{k}$ represent the sides of a triangle. Find the value of z.

Solution

$\Rightarrow \quad \underline{U} + \underline{V} + \underline{W} = 0$

$\Rightarrow \quad (2\underline{i}+3\underline{j}+4\underline{k}) + (-\underline{i}+3\underline{j}-\underline{k}) + (\underline{i}+6\underline{j}+z\underline{k}) = 0$

$\Rightarrow \quad 2\underline{i}+12\underline{j}+3\underline{k}+z\underline{k} = 0$

$\Rightarrow \quad 2\underline{i}+12\underline{j}+(3+z)\underline{k} = 0\underline{i}+0\underline{j}+0\underline{k}$

Comparing co-efficient of $\underline{k}$ on both sides

$\Rightarrow \quad 3+z = 0 \quad$ or $\quad z = -3$

Q9. The P.Vs of the point A , B, C and D are $2\underline{i}-\underline{j}+\underline{k}$,$3\underline{i}+\underline{j}$, $2\underline{i}+4\underline{j}-2\underline{k}$ and $-\underline{i}-2\underline{j}+\underline{k}$ respectively. Show that $\vec{AB}$ is parallel to $\vec{CD}$.

Solution

P.V of D = $-\underline{i}-2\underline{j}+\underline{k}$

$\vec{AB} = (3\underline{i}+\underline{j}) - (2\underline{i}-\underline{j}+\underline{k})$

$\vec{AB} = \underline{i}+2\underline{j}-\underline{k}$ -------------------- (1)

$\vec{CD} = (-\underline{i}-2\underline{j}+\underline{k}) - (2\underline{i}+4\underline{j}-2\underline{k})$

$= (-\underline{i}-2\underline{j}+\underline{k}-2\underline{i}-4\underline{j}+2\underline{k})$

$= -3\underline{i}-6\underline{j}+3\underline{k}$

$\vec{CD} = -3(\underline{i}+2\underline{j} -\underline{k})$

$= -3\ \vec{AB}$ by (1)

⇒ $\vec{CD}$ and $\vec{AB}$ are parallel. But they are in opposite direction.

Q10. We say that two vectors $\underline{V}$ and $\underline{W}$ in space are parallel. If there is a scalar such that $\underline{V} = c\ \underline{W}$. The vectors point in the same direction if c > 0, and the vectors point in the opposite direction if c<0.

Solution

(a) $\underline{V} = 2\underline{i}-4\underline{j} +4\underline{k}$

$|\underline{V}| = \sqrt{(2)^2 + (-4)^2 + (4)^2}$

$= \sqrt{4 + 16 + 16}$

$= \sqrt{36}$

$= 6$

if $\underline{U}$ is the required unit vector parallel to $\underline{V}$,

Then $\underline{U} = \frac{\underline{V}}{|\underline{V}|}$

⇒ $\underline{U} = \frac{1}{6}(2\underline{i}-4\underline{j} +4\underline{k})$

The two vectors whose length is 2 and parallel to $\underline{V}$ are $2\underline{U}$ and $-2\underline{U}$

⇒ $\frac{2}{6}(2\underline{i}-4\underline{j} +4\underline{k})$ and $\frac{-2}{6}(2\underline{i}-4\underline{j} +4\underline{k})$

$\Rightarrow \quad \frac{1}{3}(2\underline{i}-4\underline{j}+4\underline{k})$ and $-\frac{1}{3}(2\underline{i}-4\underline{j}+4\underline{k})$

$\Rightarrow$ these are parallel.

(b) $\underline{V} = \underline{i}-3\underline{j}+4\underline{k}$ and $\underline{W} = a\underline{i}+a\underline{j}-12\underline{k}$ are parallel then

If $\Rightarrow$ 1:a = -3:9 = 4:-12

$$\frac{1}{a} = \frac{-3}{9} = \frac{4}{-12}$$

$$\Rightarrow \frac{1}{a} = \frac{-1}{3} = a = -3$$

(c) $|\underline{V}| = \sqrt{(1)^2 + (-2)^2 + (3)^2}$

$$= \sqrt{1+4+9} = \sqrt{14}$$

If $\underline{U}$ is a vector of length one and direction of $\underline{V}$ is

$$= \frac{5(-\underline{i}+2\underline{j}-3\underline{k})}{\sqrt{14}}$$

$$= \frac{5}{\sqrt{14}}\underline{i} + \frac{10}{\sqrt{14}}\underline{j} - \frac{15}{\sqrt{14}}\underline{k}$$

(d) If $3\underline{i}-\underline{j}+4\underline{k}$ and $a\underline{i}+b\underline{j}-2\underline{k}$ are parallel

$\Rightarrow$ 3:a = -1:b = 4:-2

$$\Rightarrow \frac{3}{a} = \frac{-1}{b} = \frac{4}{-2}$$

$\Rightarrow \quad \frac{3}{a} = \frac{-2}{1}$ and $\frac{-1}{b} = -2$

$\Rightarrow \quad 3 = -2a, \quad -1 = -2b$

$-\frac{3}{2} = a, \frac{1}{2} = b$

$a = -\frac{3}{2}, \; b = \frac{1}{2}$

Q11. Find the direction cosines for the given vectors:

Solution

(i) $\underline{V} = 3\underline{i} - \underline{j} + 2\underline{k}$

$|\underline{V}| = \sqrt{(3)^2 + (-1)^2 + (2)^2}$

$= \sqrt{9 + 1 + 4}$

$= \sqrt{14}$

Direction cosines of $\underline{V}$ are $\left[\frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}\right]$

(ii) **Let** $\underline{V} = 6\underline{i} - 2\underline{j} + \underline{k}$

$|\underline{V}| = \sqrt{(6)^2 + (-2)^2 + (1)^2}$

$= \sqrt{36 + 4 + 1}$

$= \sqrt{41}$

Direction cosines of $\underline{V}$ are $\left[\frac{6}{\sqrt{41}}, \frac{2}{\sqrt{41}}, \frac{1}{\sqrt{41}}\right]$

(iii) P = (2,1,5), Q=(1,3,1)

$\vec{PQ}$ = P.V of Q – P.V of P

= [1,3,1]-[2,1,5] = [1-2,3-1,1-5]

$\vec{PQ}$ = [-1,2,4] = $-\underline{i}+2\underline{j}-4\underline{k}$

$|\vec{PQ}| = \sqrt{(-1)^2+(2)^2+(-4)^2}$

$= \sqrt{1+4+16}$

$= \sqrt{21}$

Direction cosines of $\vec{PQ}$ are $\left[\frac{1}{\sqrt{21}}, \frac{2}{\sqrt{21}}, \frac{4}{\sqrt{21}}\right]$

Q12. Which of the following triples can be the direction angles of a single vector:

Solution

If α, β, γ be the direction angles of a vector then we must have

$cos^2\alpha + cos^2\beta + cos^2\gamma = 1$

(i) Here $\alpha = 45°, \beta = 45°, \gamma = 60°$

Consider $cos^2\alpha + cos^2\beta + cos^2\gamma = 1$

L.H.S

$= cos^2 45° + cos^2 45° + cos^2 60°$

$= (\frac{1}{\sqrt{2}})^2 + (\frac{1}{\sqrt{2}})^2 + (\frac{1}{2})^2$

$= \frac{1}{2} + \frac{1}{2} + \frac{1}{4} = \frac{2+2+1}{4}$

$= \frac{5}{4} \neq 1 \neq$ **R.H.S.**

(ii) **Here $\alpha = 30°, \beta = 45°, \gamma = 60°$**

Consider $cos^2\alpha + cos^2\beta + cos^2\gamma = 1$

L.H.S

$= cos^2 30° + cos^2 45° + cos^2 60°$

$= (\frac{\sqrt{3}}{2})^2 + (\frac{1}{\sqrt{2}})^2 + (\frac{1}{2})^2$

$= \frac{3}{4} + \frac{1}{2} + \frac{1}{4} = \frac{3+2+1}{4}$

$= \frac{6}{4} = \frac{3}{2} \neq 1 \neq$ **R.H.S.**

$\Rightarrow \quad cos^2\alpha + cos^2\beta + cos^2\gamma \neq 1$

So the given angles cannot be direction angles

(iii) **Here $\alpha = 45°, \beta = 60°, \gamma = 60°$**

Consider $cos^2\alpha + cos^2\beta + cos^2\gamma = 1$

L.H.S

$= cos^2 45° + cos^2 60° + cos^2 60°$

$= (\frac{1}{\sqrt{2}})^2 + (\frac{1}{2})^2 + (\frac{1}{2})^2$

$= \frac{1}{2} + \frac{1}{4} + \frac{1}{4} = \frac{2+1+1}{4}$

$= \frac{4}{4} = 1 =$ **R.H.S.**

As $cos^2\alpha + cos^2\beta + cos^2\gamma = 1$

So given triples (angles) can be direction angles of a triangle.