Unit 7

Vectors

Exercise 7.1

Q1. Write the vectors $\vec{PQ}$ in the form $x\underline{i} + y\underline{j}$.

(i) P(2,3) , Q(6,-2)

Solution

$\vec{PQ} = (6,-2) - (2,3)$

$= (6\underline{i} - 2\underline{j}) - (2\underline{i} + 3\underline{j})$

$= 6\underline{i} - 2\underline{j} - (2\underline{i} + 3\underline{j})$

$= 6\underline{i} - 2\underline{j} - 2\underline{i} - 3\underline{j}$

$= 4\underline{i} - 5\underline{j}$

(ii) P(0,5) , Q(-1,-6)

Solution

$\vec{PQ} = (-1,-6) - (0,5)$

$= (-1-0, -6-5)$

$= (-1,-11)$

$= \underline{i} - 11\underline{j}$

Q2. Find the magnitude of vector of U.

(i) $\underline{U} = 2\underline{i} - 7\underline{j}$

Solution

We are given

$\underline{U} = 2\underline{i} - 7\underline{j}$

$\Rightarrow \quad |\underline{U}| = 2\underline{i} - 7\underline{j}$

$= \sqrt{(2)^2 + (-7)^2} = \sqrt{4 + 49} = \sqrt{53}$

(ii) $\underline{U} = \underline{i} + \underline{j}$

$|\underline{U}| = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$

(iii) $\underline{U} = [3, -4]$

$\Rightarrow \quad |\underline{U}| = 3\underline{i} + (-4\underline{j})$

$\Rightarrow \quad |\underline{U}| = \sqrt{(3)^2 + (-4)^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

$= 5$

Q3. **If $\underline{U} = 2\underline{i} - 7\underline{j}$, $\underline{V} = \underline{i} - 6\underline{j}$ and $\underline{W} = -\underline{i} + \underline{j}$. Find the following vectors.**

(i) $\underline{U} + \underline{V} - \underline{W}$ **(ii)** $2\underline{U} - 3\underline{V} + 4\underline{W}$ **(iii)** $\frac{1}{2}\underline{U} + \frac{1}{2}\underline{V} + \frac{1}{2}\underline{W}$

Solution

(i) $\underline{U} + \underline{V} - \underline{W}$

$= (2\underline{i} - 7\underline{j}) + (\underline{i} - 6\underline{j}) - (-\underline{i} + \underline{j})$

$= 2\underline{i} - 7\underline{j} + \underline{i} - 6\underline{j} + \underline{i} - \underline{j}$

$= 2\underline{i} + \underline{i} + \underline{i} - 7\underline{j} - 6\underline{j} - \underline{j}$

$= 4\underline{i} - 14\underline{j}$

(ii) $2\underline{U} - 3\underline{V} + 4\underline{W}$

$= 2(2\underline{i}-7\underline{j}) - 3(\underline{i}-6\underline{j}) + 4(-\underline{i}+\underline{j})$

$= 4\underline{i}-14\underline{j} - 3\underline{i}+18\underline{j} - 4\underline{i} + 4\underline{j}$

$= 4\underline{i} - 3\underline{i} - 4\underline{i} - 14\underline{j} + 18\underline{j} + 4\underline{j}$

$= -3\underline{i}+8\underline{j}$

(iii) $\frac{1}{2}\underline{U} + \frac{1}{2}\underline{V} - \frac{1}{2}\underline{W}$

$= \frac{1}{2}(2\underline{i}-7\underline{j}) + \frac{1}{2}(\underline{i}-6\underline{j}) - \frac{1}{2}(-\underline{i}+\underline{j})$

$= \frac{1}{2}[(2\underline{i}-7\underline{j}) + (\underline{i}-6\underline{j}) + (-\underline{i}+\underline{j})]$

$= \frac{1}{2}[2\underline{i}-7\underline{j} + \underline{i}-6\underline{j} - \underline{i}+\underline{j}]$

$= \frac{1}{2}[2\underline{i}-12\underline{j}]$

$= \underline{i}-6\underline{j}$

Q4. Find the sum of vectors $\vec{AB}$ and $\vec{CD}$, given the four points, A(1,-1), B(2,0), C(-1,3) and D(-2,2).

Solution

We find $\vec{AB}$ and $\vec{CD}$

$\vec{AB} = (2,0) - (1,-1)$

$= (2-1, 0+1)$

$= \underline{i}+\underline{j}$ ------------- (1)

Now $\vec{CD} = (-2,2) - (-1,3)$

$= (-2+1, 2-3)$

$= -\underline{i}-\underline{j}$ --------------- (2)

Adding (1) and (2)⇒

$\vec{AB} + \vec{CD} = (\underline{i}+\underline{j})+(-\underline{i}-\underline{j})$

$= \underline{i} - \underline{i}+\underline{j}-\underline{j}$

= 0 (null vector)

Q5. Find the vector from the point A to the origin where $\vec{AB} = 4\underline{i}-2\underline{j}$ and B is the point (-2,5)

Solution

$\vec{AB}$ = P.V of B – P.V of A

⇒ P.V of A = P.V of B - $\vec{AB}$

$= (-2,5) - (4\underline{i}-2\underline{j})$

$= -2\underline{i}+5\underline{j} - 4\underline{i}+2\underline{j}$

P.V of A = $-6\underline{i}+7\underline{j}$

Now vector from the origin

$\vec{HO} = (0,0) - (-6,7)$

$= 6\underline{i} - 7\underline{j}$

Q6. Find a unit vector in the direction of a vector given below.

(i) $\underline{V} = 2\underline{i} - \underline{j}$

Solution

$\Rightarrow \quad \underline{U} = \frac{\underline{v}}{|\underline{v}|}$

$= \frac{2\underline{i} - \underline{j}}{\sqrt{4+1}}$

$= \frac{2\underline{i} - \underline{j}}{\sqrt{5}}$

$= \frac{2}{\sqrt{5}}\underline{i} - \frac{1}{\sqrt{5}}\underline{j}$

(ii) $\underline{V} = \frac{1}{2}\underline{i} - \frac{\sqrt{3}}{2}\underline{j}$

$|\underline{V}| = \sqrt{(\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}$

$= 1$

Let $\underline{U}$ be the required unit vector

Then $\underline{U} = \frac{\underline{v}}{|\underline{v}|}$

$\Rightarrow \quad \underline{U} = \frac{\frac{1}{2}\underline{i} - \frac{\sqrt{3}}{2}\underline{j}}{1}$

$\underline{U} = \frac{1}{2}\underline{i} - \frac{\sqrt{3}}{2}\underline{j}$

(iii) $\underline{V} = -\frac{\sqrt{3}}{2}\underline{i} - \frac{1}{2}\underline{j}$

$|\underline{V}| = \sqrt{(-\frac{\sqrt{3}}{2})^2 + (-\frac{1}{2})^2}$

$= 1$

Let $\underline{U}$ be the required unit vector

Then $\underline{U} = \frac{\underline{v}}{|\underline{v}|}$

$$\Rightarrow \quad \underline{U} = \frac{-\frac{\sqrt{3}}{2}\underline{i} - \frac{1}{2}\underline{j}}{1}$$

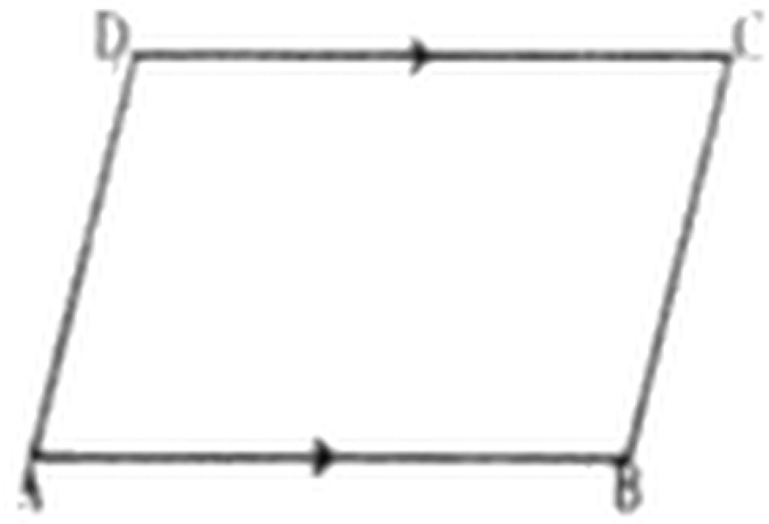
$$\Rightarrow \quad \underline{U} = -\frac{\sqrt{3}}{2}\underline{i} - \frac{1}{2}\underline{j}$$

Q7. If A, B, C are respectively the points (2,-4),(4,0) and (1,6). Use vector method to find the co-ordinates of the point D if:

i. ABCD is a parallelogram ii. ADBC is a parallelogram

Solution

i) Let the co-ordinates of D are (x, y).s



∵ ABCD is a parallelogram

⇒ AB = DC

⇒ (4,0) – (2,-4) = [(1,6) – (x , y)]

(4-2,0+4) = [(1-x,6-y)]

On comparing unit vectors

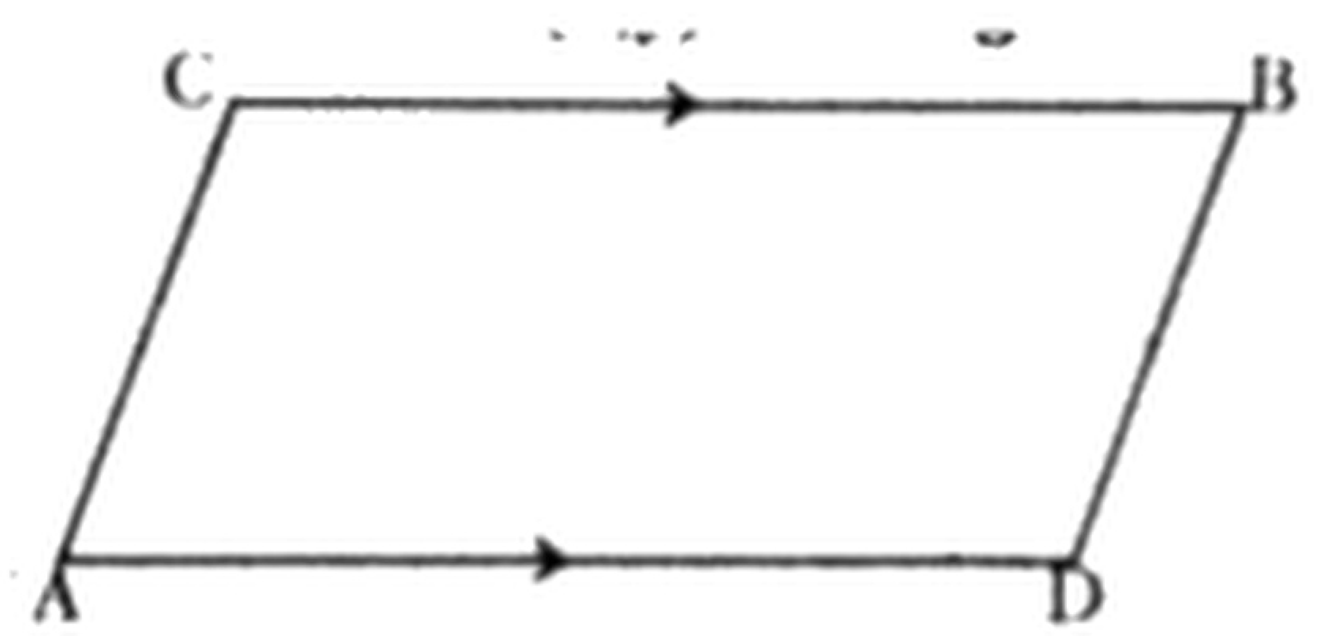
⇒ 1-x = 4-2 6-y = 4

-x=2-1 -y=4-6

x = -1 y = 2

so the co-ordinates of D(-1,2)

(ii) Let the co-ordinates of D are (x,y).



$\vec{AD} = \vec{CB}$ and $\vec{AD} \parallel \vec{CD}$

$\Rightarrow$ $(x,y) - (2,-4) = (4,0) - (1,6)$

$[(x-2),(y+4) = [4-1,0-6]$

On comparing

$x-2 = 3$ $y+4 = -6$

$x = 3+2$ $y = -6-4$

$x=5$ $y = -10$

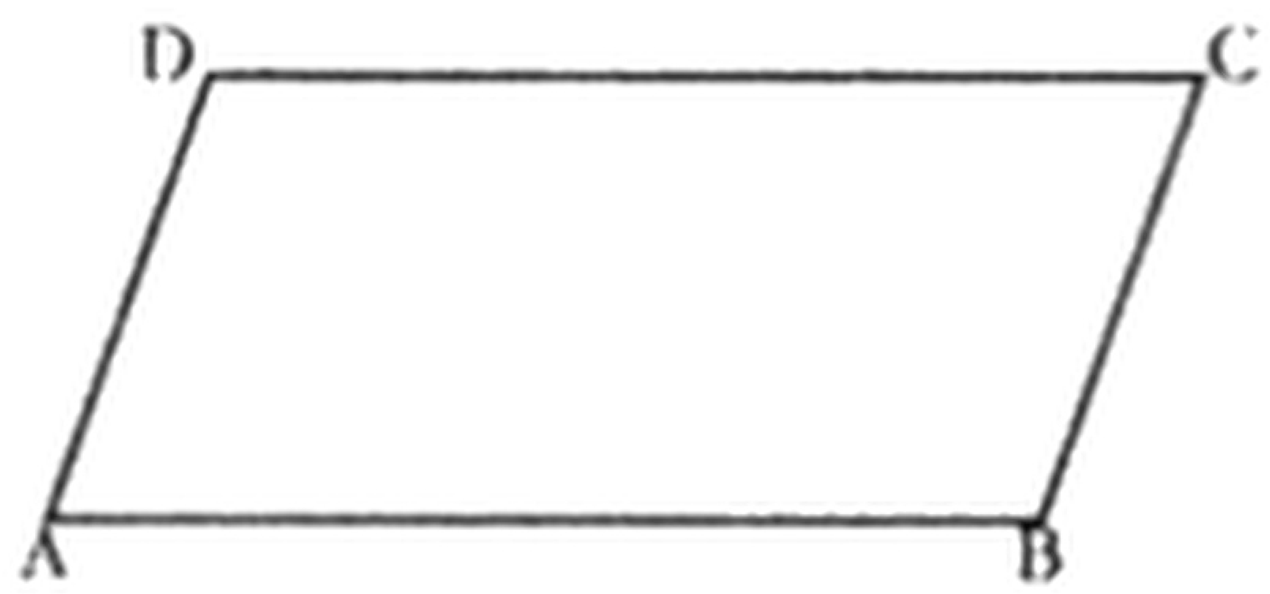
$\Rightarrow$ $D(5,-10)$

Q8. If B,C and D are respectively (4,1),(-2,3) and (-8,0). Use vector method to find the co-ordinates of the point.

(i) A if ABCD is a parallelogram. (ii) E if AEBD is a parallelogram.

Solution

(i) Let A = (x,y)



$\vec{AB} = \vec{DC}$ and $\vec{AB} \parallel \vec{DC}$

Now $\vec{AB} = \vec{DC}$

$\Rightarrow$ (4,1) – (x,y) = (-2,3) – (-8,0)

(4-x,1-y) = (-2+8,3-0)

4-x,1-y = (6,3)

$\Rightarrow$ 4-x = 6 1-y = 3

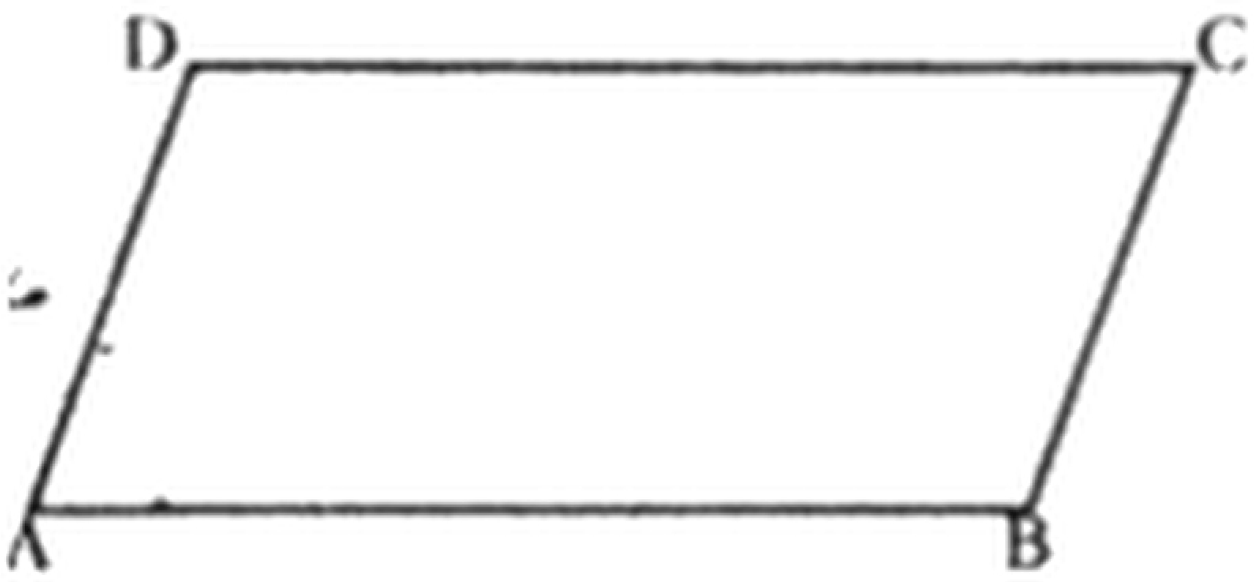
x = 4-6 y = 1-3

x = -2 y = -2

$\Rightarrow$ A(x,y) = (-2,-2)

(ii) Let E (x,y)

$\because$ AEBD is a parallelogram



$\Rightarrow$ $\vec{AE} = \vec{DB}$ and $\vec{AE} \parallel \vec{DB}$

$\Rightarrow$ (x,y) – (-2,-2) = (4,1) – (-8,0)

(x+2,y+2) = (4+8,1-0)

$\Rightarrow$ (x+2,y+2) = (12,1)

x+2 = 12 y+2 = 1

x = 12 – 2 y = 1-2

x = 10 y = -1

$\Rightarrow$ E(X,Y) = (10,-1)

Q9. If O is the origin and $\vec{OP} = \vec{AB}$, find the point P when A and B are (-3,7) and (1,0) respectively.

Solution

Let the co-ordinates of P are (x,y)

P(x,y), O(0,0)

A(-3,7), B(1,0)

$\vec{OP} = \vec{AB}$ **(given)**

$\Rightarrow$ (x,y) −(0,0) = (1,0) − (-3,7)

(x-0),(y-0) = (1+3, 0-7)

(x,y) = (4,-7)

$\Rightarrow$ P(4,-7)

Q10. Use vectors to show that ABCD is a parallelogram when the points A,B,C and D are (0,0),(a,0),(b,c) and (b-a,c).

Solution

In order to prove ABCD a parallelogram we have to show then $\vec{AB} = \vec{DC}$ and $\vec{AD} = \vec{BC}$

Now $\vec{AB}$ = (a,0) – (0,0)

= (a,0) ---------------- (1)

Now $\vec{DC}$ = (b,c)-(b-a,c)

= (b-b+a,c-c)

= (a,0) -------------------- (2)

(1) and (2) ⇒ $\vec{AB} = \vec{DC}$

now we find

$\vec{AD}$ = (b-a,c) – (0,0)

= (b-a-0,c-0)

= (b-a,c) ------------------ (3)

Also

$\vec{BC}$ = (b,c)-(a,0)

= (b-a,c-0)

= (b-a,c) -------------------- (4)

From (3) and (4) $\vec{AD} = \vec{BC}$

$\therefore$ $\vec{AB} \parallel \vec{DC}$ $\therefore$ $\vec{AD} \parallel \vec{BC}$

From A and B we assume that

ABCD is a parallelogram.

Q11. If $\vec{AB} = \vec{CD}$. Find the co-ordinates of point A when points B,C,D are (1,2), (-2,5),(4,11).

Solution

Let A = (x,y)

We are given

$$\vec{AB} = \vec{CD}$$

$\Rightarrow$ (1,2) – (x,y) = (4,11) – (-2,5)

(1-x,2-y) = (4+2, 11-5)

1-x = 6 2-y = 6

$\Rightarrow$ x = -5 y = -4

So A = (-5,-4)

Q12. Find the position vector of the point of division of the line segments joining the following pair of points, in the given ratio.

i. **Point C with position vector $2\underline{i} - 3\underline{j}$ and point D with position vector $3\underline{i} + 2\underline{j}$ in the ratio 4:3.**

ii. **Point E with position vector $5\underline{i}$ and point F with position vector $4\underline{i} + \underline{j}$ in the ratio 2:5.**

Solution

i. **Let position vector of C is**

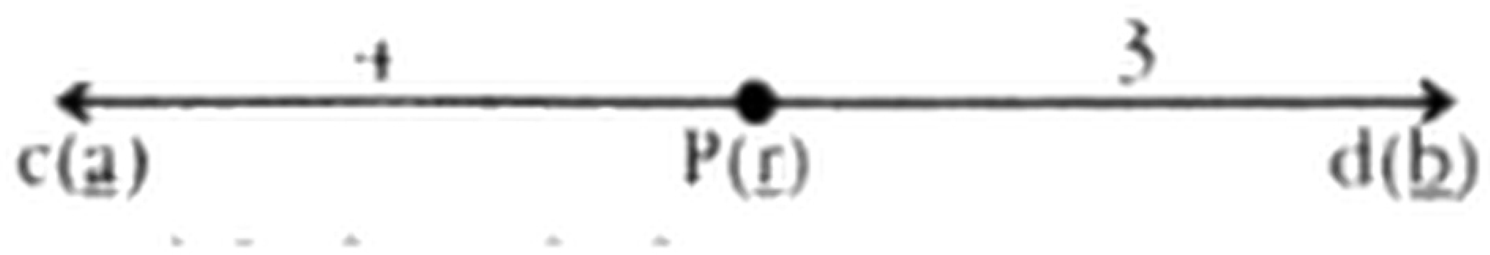
$$\underline{a} = 2\underline{i} - 3\,\underline{j}$$

and P.V of D is

$$\underline{b} = 3\underline{i} + 2\,\underline{j}$$

Let be P.V of required point which divide CD in ratio 4:3

By ratio formula



$$\underline{r} = \frac{3\underline{a}+4\underline{b}}{4+3} = \frac{3\left(2\underline{i} - 3\,\underline{j}\right)+4(3\underline{i} + 2\,\underline{j})}{7}$$

$$= \frac{\left(6\underline{i} - 9j\right)+12\underline{i} + 8\,\underline{j}}{7}$$

$$= \frac{18}{7}\underline{i} \quad \frac{1}{7}\underline{j}$$

ii. Let P.V of E = $\underline{a}$ = 5 $\underline{i}$

P.V of F = $\underline{b} = 4\underline{i} + \underline{j}$

Let P be the required point which divide EF in 2:5.

$$\underline{r} = \frac{5\underline{a}+2\underline{b}}{5+2}$$

$$= \frac{5\left(5\,\underline{i}\right)+2(4\underline{i} + \underline{j})}{7}S$$

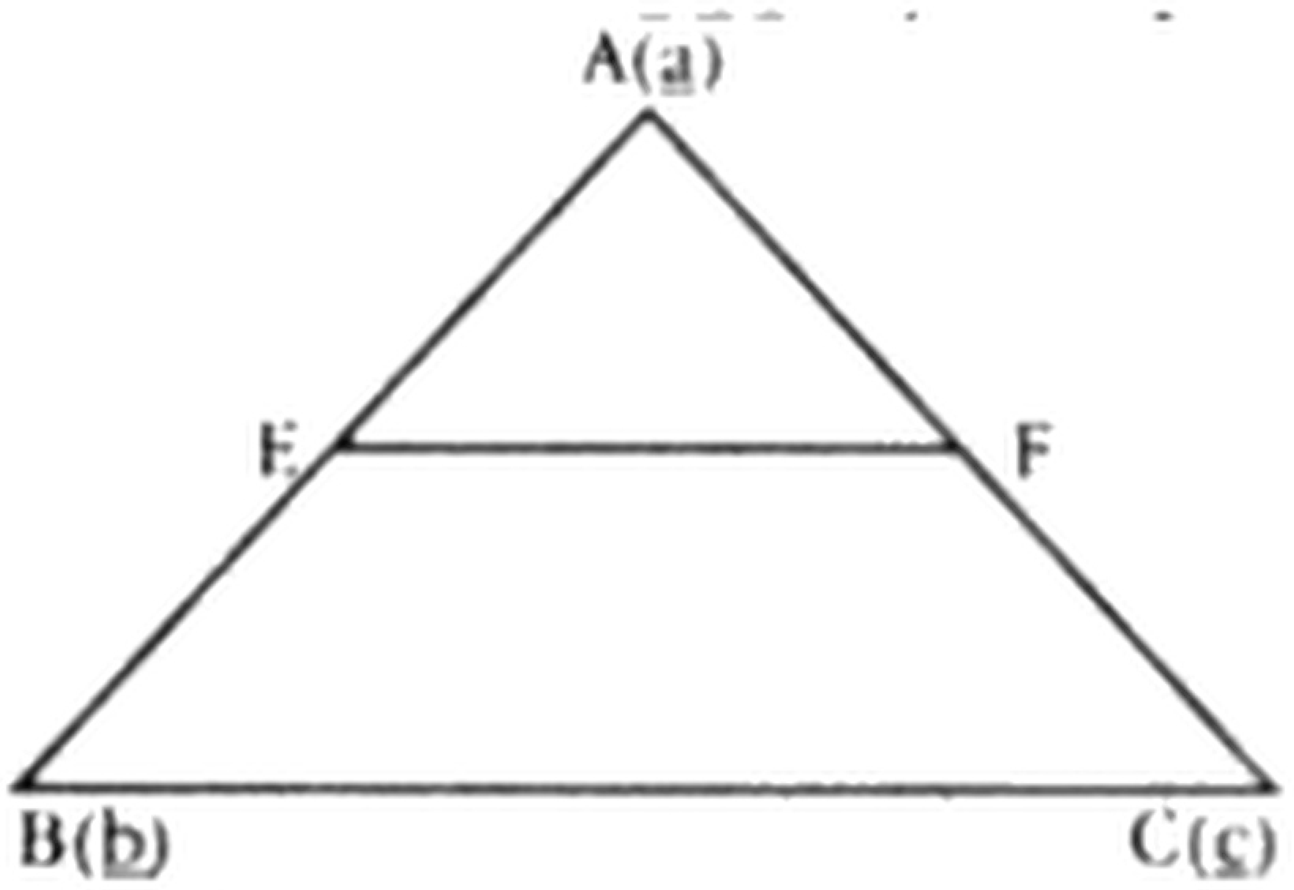
$$= \frac{\left(25\,\underline{i}\right)+(8\underline{i} + 2\,\underline{j})}{7}$$

$$= \frac{33}{7}\underline{i} \quad \frac{2}{7}\underline{j}$$

Q13. Prove that line segment joining the midpoint of two sides of a triangle is parallel to third side and half as long.

Solution

Let A,B,C be the vertices of a triangle with P.Vs $\underline{a}$, $\underline{b}$, $\underline{c}$ repectively.



Let E and F are the mid points of AB and AC.

P.V of E = $\frac{\underline{a}+\underline{b}}{2}$

P.V of F = $\frac{\underline{a}+\underline{c}}{2}$

$\vec{EF} = F - E$

$= \frac{\underline{a}+\underline{c}}{2} - \frac{\underline{a}+\underline{b}}{2}$

$= \frac{\underline{a}+\underline{c}-\underline{a}+\underline{b}}{2}$

$= \frac{\underline{c}-\underline{b}}{2}$

$= \frac{1}{2}\overrightarrow{BC}$

Hence $\vec{EF} \parallel \vec{BC}$ and half of its length.

Q14. Prove that line segments joining the mid-point of the side of a quadrilateral taken in order form a parallelogram

Solution

Let A, B, C, D be the vertices of a parallelogram with P.Vs $\underline{a}, \underline{b}, \underline{c}, \underline{d}$ respectively,

Let E, F ,G and H be the md-points of sides $\vec{AB}$, $\vec{BC}$, $\vec{CD}$, $\vec{DE}$ respectively.

P.V of E = $\frac{\underline{a}+\underline{b}}{2}$

P.V of F = $\frac{\underline{b}+\underline{c}}{2}$

P.V of G = $\frac{\underline{c}+\underline{d}}{2}$

P.V of H = $\frac{\underline{a}+\underline{d}}{2}$

EFGH will form a parallelogram. If $\vec{EF} = \vec{HG}$ and $\vec{HE} = \vec{GF}$.

Now $\vec{EF} = \frac{\underline{b}+\underline{c}}{2} \quad \frac{\underline{a}+\underline{b}}{2}$

$= \frac{\underline{c}-\underline{a}}{2}$

$\vec{HG} = \frac{\underline{c}+\underline{d}}{2} \quad \frac{\underline{a}+\underline{d}}{2}$

$= \frac{\underline{c}-\underline{a}}{2}$

$$\vec{EF} = \vec{HG}$$

Now $$\vec{HE} = \frac{\underline{a}+\underline{b}}{2} \quad \frac{\underline{a}+\underline{d}}{2}$$

$$= \frac{\underline{b}-\underline{d}}{2}$$

Also $$\vec{GF} = \frac{\underline{b}+\underline{c}}{2} \quad \frac{\underline{c}+\underline{d}}{2}$$

$$= \frac{\underline{b}-\underline{d}}{2}$$

$\Rightarrow$ $$\vec{HE} = \vec{GF}$$

So, $\vec{EF} \parallel \vec{HG}$ and $\vec{EH} \parallel \vec{FG}$.

From it we conclude that EFGH is a parallelogram.