# Exercise 6.8

**Q1. Find an equation of each of the following curves with respect to new parallel axis obtained by shifting the origin to the indicated point.**

**i.** $\mathbf{x^2 + 16y - 16 = 0} \qquad \mathbf{O'(0, 1)}$

**Solution**

Equation of transformation are

$$x = x + 0, \qquad y = y + 1$$

Substituting these values of x,y into above we have

$$x^2 + 16(y + 1) - 16 = 0$$

$$x^2 + 16y + 16 - 16 = 0$$

$$x^2 + 16y = 0$$

**ii.** $\mathbf{4x^2 + y^2 + 16x - 10y + 37 = 0} \qquad \mathbf{O'(-2, 5)}$

**Solution**

$$4x^2 + y^2 + 16x - 10y + 37 = 0$$

Equation of transformation are

$$x = x - 2, \qquad y = y + 5$$

Substituting these values of x,y into above we have

$$4(x-2)^2 + (y+5)^2 + 16(x-2) - 10(y+5) + 37 = 0$$

$$4(x^2 - 4x + 4) + (y^2 + 10y + 25) + 16x - 32 - 10y - 50 + 37 = 0$$

$$4x^2 - 16x + 16 + y^2 + 10y + 25 + 16x - 10y - 45 = 0$$

$$4x^2 + y^2 - 4 = 0$$

**iii.** $\mathbf{9x^2 + 4y^2 + 18x - 16y - 11 = 0}$ $\mathbf{O'(-1, 2)}$

**Solution**

$$9x^2 + 4y^2 + 18x - 16y - 11 = 0$$

Equation of transformation are

$$x = x - 1, \quad y = y + 2$$

Substituting these values of x,y into above we have

$$9(x - 1)^2 + 4(y + 2)^2 + 18(x - 1) - 16(y + 2) - 11 = 0$$

$$9(x^2 - 2x + 1) + 4(y^2 + 4y + 4) + 18x - 18 - 16y - 32 - 11 = 0$$

$$9x^2 - 18x + 9 + 4y^2 + 16y + 16 + 18x - 16y - 61 = 0$$

$$49 + 4y^2 - 36 = 0$$

**iv.** $\mathbf{x^2 - y^2 + 4x + 8y - 11 = 0}$ $\mathbf{O'(-2, 4)}$

**Solution**

$$x^2 - y^2 + 4x + 8y - 11 = 0$$

Equation of transformation are

$$x = x - 2, \quad y = y + 4$$

Substituting these values of x,y into above we have

$$(x - 2)^2 - (y + 4)^2 + 4(x - 2) + 8(y + 4) - 11 = 0$$

$$(x^2 - 4x + 4) - (y^2 + 8y + 16) + 4x - 8 + 8y + 32 - 11 = 0$$

$$x^2 - 4x + 4 - y^2 - 8y - 16 + 4x + 8y + 13 = 0$$

$$x^2 - y^2 + 1 = 0$$

v. $\mathbf{9x^2 - 4y^2 + 36x + 8y - 4 = 0}$ $\mathbf{O'(-2, 1)}$

**Solution**

$$9 - 4y^2 + 36x + 8y - 4 = 0$$

Equation of transformation are

$$X = x - 2,$$

$$y = y + 1$$

Substituting these values of x,y into above we have

$$9(x - 2)^2 - 4(y + 1)^2 + 36(x - 2) + 8(y + 1) - 4 = 0$$

$$9(x^2 - 4x + 4) - 4(y^2 + 2y + 1) + 36x - 72 + 8y + 8 - 4 = 0$$

$$9x^2 - 36x + 36 - 4y^2 - 8y - 4 + 36x - 72 + 8y - 4 = 0$$

$$9x^2 - 4y^2 - 36 = 0$$

**Q2. Find coordinates of the new origin (axis remaining parallel) so that first degree terms are removed from the transformed equation of each 0f the following also find the transformed equations.**

i. $\mathbf{3x^2 - 2y^2 + 24x + 12y + 24 = 0}$

**Solution**

Let the coordinates of the new origin $O'(h, k)$ equations of transformation are

$$X = x + h, \quad y = y + k$$

Substituting these values of x,y into above we have

$$3(x + h)^2 - 2(y + k)^2 + 24(x + h) + 12(y + k) + 24 = 0$$

$$3(x^2 + 2hx + h^2) - 2(y^2 + 2yk + k^2) + 24x + 24h + 12y + 12k + 24 = 0$$

$$3x^2 + 6xh + 3h^2 - 2y^2 - 4yk - 2k^2 + 24x + 24h + 12y + 12k + 24 = 0$$

$$3x^2 - 2y^2 + x(6h + 24) - y(4k - 12) + 3h^2 - 2k^2 + 24h + 12k + 24 = 0$$

$(h, k)$ to be chosen that first degree terms are removed from the transformed equation

$$6h + 24 = 0 \qquad 4k - 12 = 0$$

$$h + 4 = 0 \qquad k - 3 = 0$$

$$h = -4 \qquad k = 3$$

thus $(h, k) = (-4,3)$

putting $(h, k)$ into above, the transformed equation

$$3x^2 - 2y^2 + x(0) - y(0) + 3(-4)^2 - 2(3)^2 + 24(-4) + 12(3) + 24 = 0$$

$$3x^2 - 2y^2 + 48 - 18 - 96 + 36 + 24 = 0$$

$$3x^2 - 2y^2 - 6 = 0$$

**ii.** $\mathbf{25x^2 + 9y^2 + 50x - 36y - 164 = 0}$

**Solution**

Let the coordinates of the new origin $O'(h, k)$ equations of transformation are

$$X = x + h, \quad y = y + k$$

Substituting these values of x,y into above we have

$$25(x + h)^2 + 9(y + k)^2 + 50(x + h) - 36(y + k) - 164 = 0$$

$$25(x^2 + 2hx + h^2) + 9(y^2 + 2yk + k^2) + 50x + 50h - 36y - 36k - 164 = 0$$

$$25x^2 + 50xh + 25h^2 + 9y^2 + 18yk + 9k^2 + 50x + 50h - 36y - 36k - 164 = 0$$

$$25x^2 + 9y^2 + x(50h + 50) + y(18k - 36) + 25h^2 + 9k^2 + 50h - 36k - 164 = 0$$

$(h, k)$ to be chosen that first degree terms are removed from the transformed equation

$$50h + 50 = 0 \qquad 18k - 36 = 0$$

$$h + 1 = 0 \qquad k - 2 = 0$$

$$h = -1 \qquad k = 2$$

thus $(h, k) = (-1,2)$

putting $(h, k)$ into above, the transformed equation is

$$25x^2 + 9y^2 + x(0) - y(0) + 25(-1)^2 + 9(2)^2 + 50(-1) - 36(2) - 164 = 0$$

$$25x^2 + 9y^2 + 25 + 36 - 50 - 72 - 164 = 0$$

$$25x^2 + 9y^2 - 225 = 0$$

**iii.** $\mathbf{x^2 - y^2 - 6x + 2y + 7 = 0}$

**Solution**

Let the coordinates of the new origin $O'(h, k)$ equations of transformation are

$$X = x + h, \quad y = y + k$$

Substituting these values of x,y into above we have

$$(x + h)^2 - (y + k)^2 - 6(x + h) + 2(y + k) + 7 = 0$$

$$(x^2 + 2hx + h^2) - (y^2 + 2yk + k^2) - 6x - 6h + 2y + 2k + 7 = 0$$

$$x^2 - y^2 + x(2h - 6) - y(2k - 2) + h^2 - k^2 - 6h + 2k + 7 = 0$$

$(h, k)$ to be chosen that first degree terms are removed from the transformed equation

$$2h - 6 = 0 \qquad 2k - 2 = 0$$

$$h - 3 = 0 \qquad k - 1 = 0$$

$$h = 3 \qquad k = 1$$

thus $(h, k) = (3,1)$

putting $(h, k)$ into above, the transformed equation

$$x^2 - y^2 + x(0) - y(0) + (3)^2 - (1)^2 - 6(3) + 2(1) + 7 = 0$$

$$x^2 - y^2 + 9 - 1 - 18 + 2 + 7 = 0$$

$$x^2 - y^2 - 1 = 0$$

**Q3. In each of the following, find an equation of the curve referred to the new axis obtained by rotation of axis about the origin through the given angle.**

i. $\mathbf{xy = 1}$ $\quad\quad\quad$ $\boldsymbol{\theta = 45^\circ}$

**Solution**

Equation o transformation are

$$x = x\cos\theta - y\sin\theta = x\cos 45^\circ - y\sin 45^\circ$$

$$= x\left(\frac{1}{\sqrt{2}}\right) - y\left(\frac{1}{\sqrt{2}}\right) = \frac{x-y}{2}$$

$$y = x\sin\theta + y\cos\theta = x\sin 45^\circ + y\cos 45^\circ$$

$$= x\left(\frac{1}{\sqrt{2}}\right) - y\left(\frac{1}{\sqrt{2}}\right) = \frac{x+y}{2}$$

Substituting these expressions for x and y into (1), we have

$$\left(\frac{x-y}{2}\right)\left(\frac{x-y}{2}\right) = 1 \quad\quad\quad \frac{x^2-y^2}{2} = 1$$

$\Rightarrow$ $\quad x^2 - y^2 = 2$ is the required equation.

ii. $\mathbf{7x^2 - 8xy + y^2 - 9 = 0}$ $\quad\quad\quad$ $\boldsymbol{\theta = \text{arc}\tan 2}$

**Solution**

$$7x^2 - 8xy + y^2 - 9 = 0 \quad\quad \text{------------ (1)}$$

$\Rightarrow$ $\quad \tan\theta = 2 \quad \sin\theta = \frac{2}{\sqrt{5}} \quad\quad\quad \cos\theta = \frac{1}{\sqrt{5}}$

Equation of transformation are

$$x = x\cos\theta - y\sin\theta = x\left(\frac{1}{\sqrt{5}}\right) - y\left(\frac{2}{\sqrt{5}}\right) = \frac{x-2y}{\sqrt{5}}$$

$$y = x\sin\theta + y\cos\theta = x\left(\frac{2}{\sqrt{5}}\right) - y\left(\frac{1}{\sqrt{5}}\right) = \frac{2x+y}{\sqrt{5}}$$

Substituting these expressions for x and y into (1), we have

$\Rightarrow$ $$7\left(\frac{x-2y}{\sqrt{5}}\right)^2 - 8\left(\frac{x-2y}{\sqrt{5}}\right)\left(\frac{2x+y}{\sqrt{5}}\right) + \left(\frac{2x+y}{\sqrt{5}}\right)^2 - 9 = 0$$

$\Rightarrow$ $$\frac{7}{5}(x^2 - 4xy + 4y^2) - \frac{8}{5}(2x^2 + xy - 4xy - 2y^2) + \frac{1}{5}(4x^2 + 4xy + y^2) - 9 = 0$$

$\Rightarrow$ $$7x^2 - 28xy + 28y^2 - 16x^2 + 24xy + 16y^2 + 4x^2 + 4xy + y^2 - 45 = 0$$

$\Rightarrow$ $$-5x^2 + 45y^2 - 45 = 0$$

$\Rightarrow$ $x^2 - 9y^2 + 9 = 0$ is the required equation.

**iii.** $\mathbf{9x^2 + 12xy + 4y^2 - x - y = 0}$ $\qquad$ $\boldsymbol{\theta = \text{arc}\tan\frac{2}{3}}$

**Solution**

$$9x^2 + 12xy + 4y^2 - x - y = 0 \qquad \text{------------ (1)}$$

$\Rightarrow$ $\tan\theta = \frac{2}{3}$ $\quad$ $\sin\theta = \frac{2}{\sqrt{13}}$ $\quad$ $\cos\theta = \frac{3}{\sqrt{13}}$

Equation of transformation are

$$x = x\cos\theta - y\sin\theta = x\left(\frac{3}{\sqrt{13}}\right) - y\left(\frac{2}{\sqrt{13}}\right) = \frac{3x-2y}{\sqrt{13}}$$

$$y = x\sin\theta + y\cos\theta = x\left(\frac{2}{\sqrt{13}}\right) - y\left(\frac{3}{\sqrt{13}}\right) = \frac{2x+3y}{\sqrt{13}}$$

Substituting these expressions for x and y into (1), we have

$\Rightarrow$ $$9\left(\frac{3x-2y}{\sqrt{13}}\right)^2 + 12\left(\frac{3x-2y}{\sqrt{13}}\right)\left(\frac{2x+3y}{\sqrt{13}}\right) + 4\left(\frac{2x+3y}{\sqrt{13}}\right)^2 - \left(\frac{3x-2y}{\sqrt{13}}\right) - \left(\frac{2x+3y}{\sqrt{13}}\right) = 0$$

$$\frac{9}{13}(9x^2 - 12xy + 4y^2) + \frac{12}{13}(6x^2 + 9xy - 4xy - 6y^2) + \frac{4}{13}(4x^2 + 12xy + 9y^2) - \frac{2x + 3y}{\sqrt{13}} - \left(\frac{3x - 2y}{\sqrt{13}}\right) = 0$$

$$81x^2 - 108xy + 36y^2 + 72x^2 + 60xy - 72y^2 + 16x^2 + 48xy + 36y^2 - 3\sqrt{13}x + 2\sqrt{13}y - 2\sqrt{13}x - 3\sqrt{13}y = 0$$

$$169x^2 - 5\sqrt{13}x - \sqrt{13}y = 0$$

Divide by $\sqrt{13}$

$\Rightarrow \quad 13\sqrt{13}x^2 - 5x - y = 0$ is the required equation.

**iv.** $\mathbf{x^2 - 2xy + y^2 - 2\sqrt{2}x - 2\sqrt{2}y + 2 = 0} \qquad \boldsymbol{\theta = 45^\circ}$

**Solution**

$x^2 - 2xy + y^2 - 2\sqrt{2}x - 2\sqrt{2}y + 2 = 0$ ------------ (1)

$\Rightarrow \quad \theta = 45^\circ \quad \sin\theta = \frac{1}{\sqrt{2}} \qquad \cos\theta = \frac{1}{\sqrt{2}}$

Equation of transformation are

$$x = x\cos\theta - y\sin\theta = x\left(\frac{1}{\sqrt{2}}\right) - y\left(\frac{1}{\sqrt{2}}\right) = \frac{x-y}{\sqrt{2}}$$

$$y = x\sin\theta + y\cos\theta = x\left(\frac{1}{\sqrt{2}}\right) - y\left(\frac{1}{\sqrt{2}}\right) = \frac{x+y}{\sqrt{2}}$$

Substituting these expressions for x and y into (1), we have

$$\Rightarrow \quad (\frac{x-y}{\sqrt{2}})^2 - 2\left(\frac{x-y}{\sqrt{2}}\right)\left(\frac{x+y}{\sqrt{2}}\right) + \left(\frac{x+y}{\sqrt{2}}\right)^2 - 2\sqrt{2}\left(\frac{x-y}{\sqrt{2}}\right) - 2\sqrt{2}\left(\frac{x+y}{\sqrt{2}}\right) + 2 = 0$$

$$\frac{1}{2}(x^2 - 2xy + y^2) - (x^2 - y^2) + \frac{1}{2}(x^2 + 2xy + y^2) - 2(x - y) - 2(x + y) + 2 = 0$$

$$x^2 - 2xy + y^2 - x^2 + 2y^2 + x^2 + 2xy + y^2 - 4x + 4y - 4x - 4y + 4 = 0$$

$\Rightarrow \quad 4y^2 - 8x + 4 = 0$

$\Rightarrow \quad y^2 - 2x + 1 = 0$ is the required equation.

**Q4. Find measure of the angle through which the axis by stated so that the product of term xy is removed from the transformed equation. Also find the transformation equation.**

**i.** $\mathbf{2x^2 + 6xy + 10y^2 - 11 = 0}$

**Solution**

Let the axis be stated through an angle θ. Equations of transformation are

$$x = x\cos\theta - y\sin\theta, \qquad y = x\sin\theta + y\cos\theta$$

substituting into the given equation, we get

$\Rightarrow \quad 2(x\cos\theta - y\sin\theta)^2 + 6(x\cos\theta - y\sin\theta)(x\sin\theta + y\cos\theta) + 10(x\sin\theta + y\cos\theta)^2 - 11 = 0$

$2(x^2\cos^2\theta - 2xy\sin\theta\cos\theta + y^2\sin^2\theta) + 6(x^2\cos\theta\sin\theta + xy\cos^2\theta - xy\sin^2\theta - y^2\sin\theta\cos\theta) + 10(x^2\sin^2\theta + 2xy\sin\theta\cos\theta + y^2\cos^2\theta) - 11 = 0$ ---------- (1)

Since this equation is to be free the product term xy, the coefficient of xy is zero. i.e.

$\Rightarrow \quad -4\sin\theta\cos\theta + 6(\cos^2\theta - \sin^2\theta) + 20\sin\theta\cos\theta = 0$

$\Rightarrow \quad 6\cos2\theta + 16\sin\theta\cos\theta = 0$

$\Rightarrow \quad 6\,cos2\theta + 8sin2\theta = 0$

$\Rightarrow \quad 6\cos2\theta = -8\sin2\theta$

$\Rightarrow \quad \tan2\theta = -\frac{3}{4} \qquad \frac{2\tan\theta}{1-\tan^2\theta} = -\frac{3}{4}$

$\Rightarrow \quad 8\tan\theta = -3(1 - \tan^2\theta) \qquad 8\tan\theta = -3 + 3\tan^2\theta$

$\Rightarrow \quad 3\tan^2\theta - 8\tan\theta - 3 = 0$

$\Rightarrow \quad 3\tan^2\theta - 9\tan\theta + \tan\theta - 3 = 0$

$\Rightarrow \quad 3\tan\theta(\tan\theta - 3) + (\tan\theta - 3) = 0$

$\Rightarrow \quad (\tan\theta - 3)(3\tan\theta + 1) = 0$

$\Rightarrow \quad (\tan\theta - 3) = 0 \qquad\qquad 3\tan\theta + 1 = 0$

$\Rightarrow \quad \tan\theta = 3 \qquad\qquad \tan\theta = -\frac{1}{3}$

since θ lies in the first quadrant,

$\tan\theta = -\frac{1}{3}$ is not admissible

$\tan\theta = 3 \quad \Rightarrow \sin\theta = \frac{3}{\sqrt{10}}, \qquad \cos\theta = \frac{1}{\sqrt{10}}$

substituting the values in (1), the transformed equation is.

$$2(x^2\left(\frac{1}{\sqrt{10}}\right)^2 + y^2\left(\frac{3}{\sqrt{10}}\right)^2) + 6(x^2(\frac{1}{\sqrt{10}})(\frac{3}{\sqrt{10}}) - y^2(\frac{3}{\sqrt{10}})(\frac{1}{\sqrt{10}})) + 10(x^2(\frac{3}{\sqrt{10}})^2 + y^2(\frac{1}{\sqrt{10}})^2) - 11 = 0$$

$$2(\frac{1}{10}x^2 + \frac{9}{10}y^2) + 6(\frac{3}{10}x^2 - \frac{3}{10}y^2) + 10(\frac{9}{10}x^2 + \frac{1}{10}y^2) - 11 = 0$$

$$\Rightarrow \quad \frac{2}{10}x^2 + \frac{18}{10}y^2 + \frac{18}{10}x^2 - \frac{18}{10}y^2 + \frac{90}{10}x^2 + \frac{10}{10}y^2 - 11 = 0$$

$$\left(\frac{2}{10} + \frac{18}{10} + \frac{90}{10}\right)x^2 + \left(\frac{18}{10} - \frac{18}{10} + \frac{10}{10}\right)y^2 - 11 = 0$$

$$\Rightarrow \quad \frac{110}{10}x^2 + y^2 - 11 = 0$$

$$\Rightarrow \quad 11x^2 + y^2 - 11 = 0$$

**ii.** $\mathbf{xy + 4x - 3y - 10 = 0}$

**Solution**

Let the axis be stated through an angle θ. Equations of transformation are

$x = x\cos\theta - y\sin\theta, \qquad y = x\sin\theta + y\cos\theta$

Substituting into the given equation, we get

$(x\cos\theta - y\sin\theta)(x\sin\theta + y\cos\theta) + 4(x\cos\theta - y\sin\theta) - 3(x\sin\theta + y\cos\theta) - 10 = 0$

$(x^2\cos\theta\sin\theta + xy\cos^2\theta - xy\sin^2\theta - y^2\sin\theta\cos\theta) + 4x\cos\theta - 4y\sin\theta - 3x\sin\theta - 3y\cos\theta - 10 = 0$ ---------- (1)

Since this equation is to be free the product term xy, the coefficient of xy is zero. i.e.

$\Rightarrow \quad \cos^2\theta - \sin^2\theta = 0 \qquad \cos^2\theta = \sin^2\theta$

$\Rightarrow \quad \tan^2\theta = 1 \qquad \tan\theta = 1$

$\Rightarrow \quad \theta = 45°$

putting value in (1), the transformed equation is

$$(x^2\cos 45°\ \sin 45° - y^2\sin 45°\ \cos 45°) + 4x\cos 45° - 4y\sin 45° - 3x\sin 45° - 3y\cos 45° - 10 = 0$$

$x^2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) - y^2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) + 4x\left(\frac{1}{\sqrt{2}}\right) - 4y\left(\frac{1}{\sqrt{2}}\right) - 3x\left(\frac{1}{\sqrt{2}}\right) - 3y\left(\frac{1}{\sqrt{2}}\right)-10=0$

$\frac{1}{2}x^2 - \frac{1}{2}y^2 + x\left(\frac{1}{\sqrt{2}}\right) - y\left(\frac{7}{\sqrt{2}}\right)-10=0$

$x^2 - y^2 + \sqrt{2}x - 7\sqrt{2}y-20=0$

**iii.** $\mathbf{5x^2 - 6xy + 5y^2 - 8 = 0}$

**Solution**

Let the axis be stated through an angle θ. Equations of transformation are

$x = x\cos\theta - y\sin\theta, \qquad y = x\sin\theta + y\cos\theta$

substituting into the given equation, we get

$\Rightarrow \quad 5(x\cos\theta - y\sin\theta)^2 - 6(x\cos\theta - y\sin\theta)(x\sin\theta + y\cos\theta) + 5(x\sin\theta + y\cos\theta)^2 - 8 = 0$

$5(x^2\cos^2\theta - 2xy\sin\theta\cos\theta + y^2\sin^2\theta) - 6(x^2\cos\theta\sin\theta + xy\cos^2\theta - xy\sin^2\theta -$
$y^2\sin\theta\cos\theta) + 5(x^2\sin^2\theta + 2xy\sin\theta\cos\theta + y^2\cos^2\theta) - 8 = 0$ ---------- (1)

Since this equation is to be free the product term xy, the coefficient of xy is zero. i.e.

$$-10\sin\theta\cos\theta + 6(\cos^2\theta - \sin^2\theta) + 10\sin\theta\cos\theta = 0$$

$$\cos^2\theta - \sin^2\theta = 0 \qquad \cos^2\theta = \sin^2\theta$$

$$\tan^2\theta = 1 \qquad \tan\theta = 1$$

$$\Rightarrow \quad \theta = 45^\circ$$

putting value in (1), the transformed equation is

$\Rightarrow \quad 5(x^2\cos^2 45^\circ + y^2\sin^2 45^\circ) - 6(x^2\cos 45^\circ \sin 45^\circ - y^2\sin 45^\circ\cos 45^\circ) +$
$5(x^2\sin^2 45^\circ + y^2\cos^2 45^\circ) - 8 = 0$

$$5\left[x^2\left(\tfrac{1}{2}\right) + y^2\left(\tfrac{1}{2}\right)\right] - 6\left[x^2\left(\tfrac{1}{2}\right) - y^2\left(\tfrac{1}{2}\right)\right] + 5\left[x^2\left(\tfrac{1}{2}\right) + y^2\left(\tfrac{1}{2}\right)\right] - 8 = 0$$

$$\frac{5}{2}x^2 + \frac{5}{2}y^2 - \frac{6}{2}x^2 + \frac{6}{2}y^2 + \frac{5}{2}x^2 + \frac{5}{2}y^2 - 8 = 0$$

$$\left(\frac{5}{2} - \frac{6}{2} + \frac{5}{2}\right)x^2 + \left(\frac{5}{2} + \frac{6}{2} + \frac{5}{2}\right)y^2 - 8 = 0$$

$$\left(\frac{5-6+5}{2}\right)x^2 + \left(\frac{5+6+5}{2}\right)y^2 - 8 = 0$$

$$\frac{4}{2}x^2 + \frac{16}{2}y^2 - 8 = 0$$

$$2x^2 + 8y^2 - 8 = 0$$

$$x^2 + 4y^2 - 4 = 0$$