# Exercise 6.9

**Q1. By a rotation of axis, elimination the xy-term in each of the following equations.**

**i.** $4x^2 - 4xy + y^2 - 6 = 0$

**Solution**

Here $a = 4$, $b = 1$, $2h = -4$ the angle θ through which axi be rotated is given by

$\tan 2\theta = \frac{2h}{a-b}$ $\Rightarrow \frac{2\tan\theta}{1-\tan^2\theta} = \frac{-4}{4-1}$

$\Rightarrow$ $\frac{2\tan\theta}{1-\tan^2\theta} = -\frac{4}{3}$

$\Rightarrow$ $6\tan\theta = -4(1-\tan^2\theta)$ $6\tan\theta = -4 + 4\tan^2\theta$

$\Rightarrow$ $4\tan^2\theta - 6\tan\theta - 4 = 0$

$\Rightarrow$ $2\tan^2\theta - 3\tan\theta - 2 = 0$ $2\tan^2\theta - 4\tan\theta + \tan\theta - 2 = 0$

$\Rightarrow$ $2\tan\theta(\tan\theta - 2) + 1(\tan\theta - 2) = 0$

$\Rightarrow$ $(\tan\theta - 2)(2\tan\theta + 1) = 0$

$\Rightarrow$ $(\tan\theta - 2) = 0$ $2\tan\theta + 1 = 0$

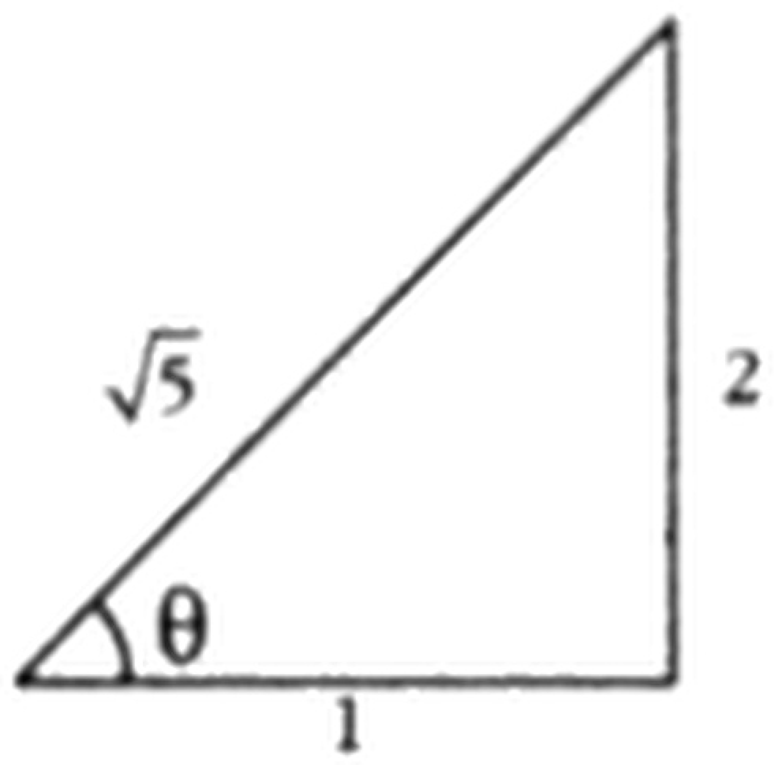
$\Rightarrow$ $\tan\theta = 2$ $\tan\theta = -\frac{1}{2}$

since θ lies in the first quadrant,

$\tan\theta = -\frac{1}{2}$ is not admissible

$\tan\theta = 2$ $\Rightarrow \sin\theta = \frac{2}{\sqrt{5}}$, $\cos\theta = \frac{1}{\sqrt{5}}$

Equation of transformation become




$$x = x\cos\theta - y\sin\theta = x\left(\frac{1}{\sqrt{5}}\right) - y\left(\frac{2}{\sqrt{5}}\right) = \frac{x-2y}{\sqrt{5}}$$

$$y = x\sin\theta + y\cos\theta = x\left(\frac{2}{\sqrt{5}}\right) + y\left(\frac{1}{\sqrt{5}}\right) = \frac{2x+y}{\sqrt{5}}$$

Substituting these expressions for x and y into (1), we have

$\Rightarrow$ $$4\left(\frac{x-2y}{\sqrt{5}}\right)^2 - 4\left(\frac{x-2y}{\sqrt{5}}\right)\left(\frac{2x+y}{\sqrt{5}}\right) + \left(\frac{2x+y}{\sqrt{5}}\right)^2 - 6 = 0$$

$$\frac{4}{5}(x^2 - 4xy + 4y^2) - \frac{4}{5}(2x^2 + xy - 4xy - 2y^2) + \frac{1}{5}(4x^2 + 4xy + y^2) - 6 = 0$$

$$4x^2 - 16xy + 16y^2 - 8x^2 + 12xy + 8y^2 + 4x^2 + 4xy + y^2 - 30 = 0$$

$$25y^2 - 30 = 0 \qquad 25y^2 = 30$$

$\Rightarrow$ $$y^2 = \frac{30}{25} \qquad \Rightarrow \qquad y^2 = \frac{6}{5}$$

$\Rightarrow$ $y = \pm\sqrt{\frac{6}{5}}$ which represents a pair of lines

from (2), $x - 2y = \sqrt{5}x, \quad 2x + y = \sqrt{5}y$

$$2x - 4y = 2\sqrt{5}x$$

$$\pm 2x \pm y = \pm\sqrt{5}y$$

$$-5y = \sqrt{5}(2x - y)$$

$\Rightarrow$ $$y = -\frac{1}{\sqrt{5}}(2x - y)$$

$$\pm\sqrt{\frac{6}{5}} = -\frac{1}{\sqrt{5}}(2x - y)$$

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$$\pm\sqrt{6} = -2x + y$$

$$-2x + y = \sqrt{6} \qquad -2x + y = -\sqrt{6}$$

$$\Rightarrow \qquad -2x + y - \sqrt{6} = 0 \qquad -2x + y + \sqrt{6} = 0$$

**ii.** $\mathbf{x^2 - 2xy - 8x - 8y = 0}$

**Solution**

Here $a = 1, \quad b = 1, \quad 2h = -2$

the angle θ through which axi be rotated is given by

$$\Rightarrow \quad \tan 2\theta = \frac{2h}{a-b} \quad \Rightarrow \quad \frac{2\tan\theta}{1-\tan^2\theta} = \frac{-2}{1-1}$$

$$\Rightarrow \quad \frac{2\tan\theta}{1-\tan^2\theta} = \frac{1}{0}$$

$$\Rightarrow \quad 1 - \tan^2\theta = 0 \qquad \Rightarrow \tan^2\theta = 1$$

$$\Rightarrow \quad (\tan\theta) = \pm 1$$

$$\Rightarrow \quad (\tan\theta) = 1 \qquad \Rightarrow \quad \tan\theta = -1$$

since θ lies in the first quadrant, $\tan\theta = -1$ is not admissible

$$\tan\theta = 1 \qquad \Rightarrow \theta = 45^\circ,$$

Equation of transformation become

$$x = x\cos\theta - y\sin\theta = x\cos 45^\circ - y\sin 45^\circ = \frac{x-y}{\sqrt{2}}$$

$$y = x\sin\theta + y\cos\theta = x\sin 45^\circ + y\cos 45^\circ = \frac{x+y}{\sqrt{}}$$

Substituting these expressions for x and y into (1), we have

$$\Rightarrow \quad \left(\frac{x-y}{\sqrt{2}}\right)^2 - 2\left(\frac{x-y}{\sqrt{2}}\right)\left(\frac{x+y}{\sqrt{2}}\right) + \left(\frac{x+y}{\sqrt{2}}\right)^2 - 8\left(\frac{x-y}{\sqrt{2}}\right) - 8\left(\frac{x+y}{\sqrt{2}}\right) = 0$$

$$\frac{1}{2}(x^2 - 2xy + y^2) - (x^2 - y^2) + \frac{1}{2}(x^2 + 2xy + y^2) - 8\left(\frac{x-y}{\sqrt{2}}\right) - 8\left(\frac{x+y}{\sqrt{2}}\right) = 0$$

Multiplying by 2 we have

$$x^2 - 2xy + y^2 - 2(x^2 - y^2) + x^2 + 2xy + y^2 - 8\sqrt{2}(x-y) - 8\sqrt{2}(x+y) = 0$$

$$x^2 - 2xy + y^2 - 2x^2 + 2y^2 + x^2 + 2xy + y^2 - 8\sqrt{2}x + 8\sqrt{2}y - 8\sqrt{2}x - 8\sqrt{2}y = 0$$

$$4y^2 - 16\sqrt{2}x = 0$$

$$y^2 - 4\sqrt{2}x = 0$$

$y^2 = 4\sqrt{2}x$nwhich represents a parabola

Now we will solve (2),for x and y

$$X = \frac{x-y}{\sqrt{2}} \quad , \quad y = \frac{x+y}{\sqrt{2}}$$

$$\sqrt{2}x = x - y, \quad \sqrt{2}y = x + y$$

By adding

$$x - y = \sqrt{2}x$$

$$\underline{x + y = +\sqrt{2}y}$$

$$2x = \sqrt{2}(x+y)$$

$$\Rightarrow \quad x = \frac{1}{\sqrt{2}}(x+y)$$

by subtracting

$$x - y = \sqrt{2}x$$

$$\underline{\pm x \pm y = \pm\sqrt{2}y}$$

$$-2y = \sqrt{2}(x-y)$$

$$y = -\frac{1}{\sqrt{2}}(x-y)$$

**elements of parabola are:**

**focus**=0, $x = \sqrt{2}$

$$-\frac{1}{\sqrt{2}}(x-y) = 0 \qquad \frac{1}{\sqrt{2}}(x+y) = \sqrt{2}$$

$$(x-y) = 0 \qquad (x+y) = 2$$

By adding we have $2x = 2$ $x = 1$

By subtracting we have $2y = 2$ $y = 1$

i.e. focus(1,1)

**vertex**, $x = 0$ $y = 0$

$\frac{1}{\sqrt{2}}(x + y) = 0$ $-\frac{1}{\sqrt{2}}(x - y) = 0$

$(x + y) = 0$ $(x - y) = 0$

, $x = 0$ $y = 0$

Vertex (0,0)

**Axis** $y = 0$

$-\frac{1}{\sqrt{2}}(x - y) = 0$

$(x - y) = 0$ $x = y$

**iii.** $\mathbf{x^2 + 2xy + y^2 + 2\sqrt{2}x - 2\sqrt{2}y + 2 = 0}$

**Solution**

Here $a = 1,$ $b = 1,$ $2h = 2$

the angle θ through which axi be rotated is given by

$\tan 2\theta = \frac{2h}{a-b}$ $\Rightarrow$ $\frac{2\tan\theta}{1-\tan^2\theta} = \frac{2}{1-1}$

$\Rightarrow$ $\frac{2\tan\theta}{1-\tan^2\theta} = \frac{2}{0}$

$\Rightarrow$ $1 - \tan^2\theta = 0$ $\Rightarrow \tan^2\theta = 1$

$\Rightarrow$ $(\tan\theta) = \pm 1$

$\Rightarrow$ $(\tan\theta) = 1$ $\Rightarrow$ $\tan\theta = -1$

since θ lies in the first quadrant, $\tan\theta = -1$ is not admissible

$\tan\theta = 1 \quad \Rightarrow \theta = 45°$ ,

Equation of transformation become

$$x = x\cos\theta - y\sin\theta = x\cos45° - y\sin45° = \frac{x-y}{\sqrt{2}}$$

$$y = x\sin\theta + y\cos\theta = x\sin45° + y\cos45° = \frac{x+y}{\sqrt{2}}$$

Substituting these expressions for x and y into (1), we have

$$(\frac{x-y}{\sqrt{2}})^2 + 2\left(\frac{x-y}{\sqrt{2}}\right)\left(\frac{x+y}{\sqrt{2}}\right) + (\frac{x+y}{\sqrt{2}})^2 + 2\sqrt{2}\left(\frac{x-y}{\sqrt{2}}\right) - 2\sqrt{2}\left(\frac{x+y}{\sqrt{2}}\right) + 2 = 0$$

$$\frac{1}{2}(x^2 - 2xy + y^2) + (x^2 - y^2) + \frac{1}{2}(x^2 + 2xy + y^2) + 2(x - y) - 2(x + y) + 2 = 0$$

$$x^2 - 2xy + y^2 + 2(x^2 - y^2) + x^2 + 2xy + y^2 + 4(x - y) - 4(x + y) = 0$$

$$x^2 - 2xy + y^2 - 2x^2 + 2y^2 + x^2 + 2xy + y^2 + 4x - 4y - 4x - 4y = 0$$

$$4x^2 - 8y + 4 = 0$$

Dividing throughout by 4

$$x^2 - 2y + 1 = 0$$

$x^2 = 2y - 1$ which represents a parabola

Now we will solve (2),for x and y

$$X = \frac{x-y}{\sqrt{2}} \quad , \quad y = \frac{x+y}{\sqrt{2}}$$

$$\sqrt{2}x = x - y, \quad \sqrt{2}y = x + y$$

By adding

$$x - y = \sqrt{2}x$$

$$\underline{x + y = +\sqrt{2}y}$$

$$2x = \sqrt{2}(x + y)$$

$$x = \frac{1}{\sqrt{2}}(x + y)$$

by subtracting

$$x - y = \sqrt{2}x$$

$$\underline{\pm x \pm y = \pm\sqrt{2}y}$$

$$-2y = \sqrt{2}(x - y)$$

$$y = -\frac{1}{\sqrt{2}}(x - y)$$

**elements of parabola are:**

⇒ **focus**, $x = 0 \quad y - \frac{1}{2} = \frac{1}{2}, \ y = 1$

$\frac{1}{\sqrt{2}}(x + y) = 0 \qquad -\frac{1}{\sqrt{2}}(x - y) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

$(x + y) = 0 \qquad (x - y) = \sqrt{2}$

By adding we have $\quad 2x = -\sqrt{2} \qquad x = -\frac{1}{\sqrt{2}}$

By subtracting we have $\quad 2y = \sqrt{2} \qquad y = \frac{1}{\sqrt{2}}$

i.e. focus$(-\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}})$

⇒ **vertex** , $x = 0 \quad y - \frac{1}{2} = 0$

$\frac{1}{\sqrt{2}}(x + y) = 0 \qquad -\frac{1}{\sqrt{2}}(x - y) = \frac{1}{2}$

$(x + y) = 0 \qquad (x - y) = -\frac{1}{\sqrt{2}}$

By adding we have $\quad 2x = -\frac{1}{\sqrt{2}} \qquad x = -\frac{1}{2\sqrt{2}}$

By subtracting we have $\quad 2y = \frac{1}{\sqrt{2}} \qquad y = \frac{1}{2\sqrt{2}}$

i.e. vertex $(-\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}})$

**Axis** $\quad x = 0$

$$\frac{1}{\sqrt{2}}(x + y) = 0$$

$\Rightarrow \qquad (x + y) = 0 \qquad \Rightarrow \qquad x = -y$

To find x-intercept, put $y = 0$

$X^2 + 2\sqrt{2}x + 2 = 0 \qquad \left(x + \sqrt{2}\right)^2 = 0$

$\Rightarrow \qquad x + \sqrt{2} = 0 \quad \Rightarrow \qquad x = -\sqrt{2}$

i.e. $\quad (-\sqrt{2}, 0)$

To find y-intercept, put $x = 0$

$y^2 - 2\sqrt{2} + 2 = 0 \qquad \left(y - \sqrt{2}\right)^2 = 0$

$\Rightarrow \qquad y - \sqrt{2} = 0 \quad \Rightarrow \qquad y = \sqrt{2}$

i.e. $\quad (0, \sqrt{2})$ is y-intercept

eq. of directrix of (3) is

$y - \frac{1}{2} = -a$

$-x + y = 0 \qquad x - y = 0$

$\Rightarrow \qquad \frac{x+y}{\sqrt{2}} = 0 \qquad\qquad \Rightarrow x + y = 0$

**iv.** $\quad \mathbf{x^2 + xy + y^2 - 4 = 0}$

**Solution**

Here $a = 1, \qquad b = 1, \quad 2h = 1$

the angle θ through which axi be rotated is given by

$\Rightarrow \qquad \tan 2\theta = \frac{2h}{a-b} \qquad\qquad \Rightarrow \frac{2\tan\theta}{1-\tan^2\theta} = \frac{1}{1-1}$

$\Rightarrow \qquad\qquad \frac{2\tan\theta}{1-\tan^2\theta} = \frac{1}{0}$

$\Rightarrow \qquad 1 - \tan^2\theta = 0 \qquad\qquad \tan^2\theta = 1$

$\Rightarrow \qquad (\tan\theta) = \pm 1$

$(\tan\theta) = 1$ $\tan\theta = -1$

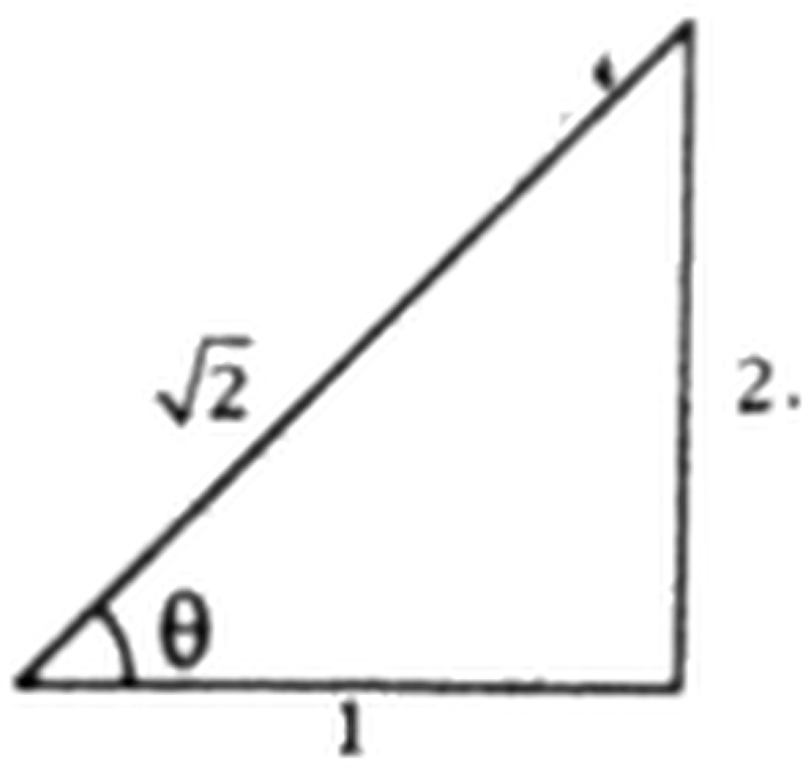
since θ lies in the first quadrant, $\tan\theta = -1$ is not admissible

$\tan\theta = 1 \quad \Rightarrow \theta = 45°$ ,

Equation of transformation become

$$x = x\cos\theta - y\sin\theta = x\cos45° - y\sin45° = \frac{x-y}{\sqrt{2}}$$

$$y = x\sin\theta + y\cos\theta = x\sin45° + y\cos45° = \frac{x+y}{\sqrt{2}}$$




Substituting these expressions for x and y into (1), we have

$$\Rightarrow \quad \left(\frac{x-y}{\sqrt{2}}\right)^2 + \left(\frac{x-y}{\sqrt{2}}\right)\left(\frac{x+y}{\sqrt{2}}\right) - 4 = 0$$

$$\frac{1}{2}(x^2 - 2xy + y^2) + \frac{1}{2}(x^2 - y^2) + \frac{1}{2}(x^2 + 2xy + y^2) - 4 = 0$$

$$x^2 - 2xy + y^2 + (x^2 - y^2) + x^2 + 2xy + y^2 - 8 = 0$$

$$3x^2 + y^2 - 8 = 0$$

$$3x^2 + y^2 = 8$$

$$\Rightarrow \quad \frac{3x^2}{8} + \frac{y^2}{8} = 1$$ is an ellipse.

Now we will solve (2), for x and y

$$X = \frac{x-y}{\sqrt{2}}x + \frac{1}{\sqrt{2}}$$

By adding

$$2x = \sqrt{2}(x + y)$$

$\Rightarrow$ $$x = \frac{1}{\sqrt{2}}(x + y)$$

by subtracting

$\Rightarrow$ $$-2y = \sqrt{2}(x - y)$$

$\Rightarrow$ $$y = -\frac{1}{\sqrt{2}}(x - y)$$

$\Rightarrow$ centre of ellipse is $x = 0$ $y = 0$

$\frac{1}{\sqrt{2}}(x + y) = 0$ $-\frac{1}{\sqrt{2}}(x - y) = 0$

$(x + y) = 0$ $(x - y) =$

$x = 0$ $y = 0$

thus centre of (1) is$(0,0)$

length of major axis = 8

length of minor axis = $\frac{1}{8}$

vertex , $x = 0$ $y = \pm 2\sqrt{2}$

$\frac{1}{\sqrt{2}}(x + y) = 0$ $-\frac{1}{\sqrt{2}}(x - y) = \pm 2\sqrt{2}$

$(x + y) = 0$ $(x - y) = \pm 4$

By adding we have $2x = \pm 4$ $x = \pm 2$

By subtracting we have $-2y = \pm 4$ $y = \pm 2$

i.e. vertex $(\pm 2, \pm 2)$

$\Rightarrow$ **ends of minor axis are** $x = \pm \frac{2\sqrt{2}}{\sqrt{3}}$, $y = 0$

$\frac{1}{\sqrt{2}}(x + y) = \pm \frac{2\sqrt{2}}{\sqrt{3}}$ $-\frac{1}{\sqrt{2}}(x - y) = 0$

$(X + y) = \pm \frac{4}{\sqrt{3}}$ $x - y = 0$

Gives $\left[\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right]$ gives $\left[-\frac{2}{\sqrt{3}}, -\frac{2}{\sqrt{3}}\right]$

Thus $\left[\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right]$ $\left[-\frac{2}{\sqrt{3}}, -\frac{2}{\sqrt{3}}\right]$ are ends o minor axis

Equation of major axis $x = 0$ i.e. $x + y = 0$

Equation of minor axis $y = 0$ i.e. $x - y = 0$

Eccentricity= $\sqrt{\frac{a^2-b^2}{a^2}} = \sqrt{\frac{8-\frac{8}{3}}{8}}$

$= \sqrt{\frac{\frac{24\ \ 8}{3}}{8}} = \sqrt{\frac{16}{24}} == \sqrt{\frac{4}{6}} = \frac{2}{\sqrt{6}}$

Foci $x = 0 \quad y = \pm\sqrt{8}[\frac{2}{\sqrt{6}}]$

$\frac{1}{\sqrt{2}}(x + y) = 0 \qquad -\frac{1}{\sqrt{2}}(x - y) = \pm\sqrt{8}[\frac{2}{\sqrt{6}}]$

$(X + y) = 0 \qquad x - y = \pm\sqrt{8}[\frac{2}{\sqrt{3}}]$

Solving these equation

$(X + y) = 0 \qquad -(x - y) = \sqrt{8}[\frac{2}{\sqrt{3}}]$

$(X + y) = 0 \qquad -(x - y) = \sqrt{8}[\frac{2}{\sqrt{3}}]$

$\left[\frac{-2\sqrt{2}}{\sqrt{3}}, \frac{2\sqrt{2}}{\sqrt{3}}\right] \left[\frac{2\sqrt{2}}{\sqrt{3}}, \frac{2\sqrt{2}}{\sqrt{3}}\right]$

**v.** $\mathbf{7x^2 - 6\sqrt{3}xy + 13y^2 - 16 = 0}$

**Solution**

Here $a = 7, \qquad b = 13, \qquad 2h = -6\sqrt{3}$

the angle θ through which axi be rotated is given by

$\tan 2\theta = \frac{2h}{a-b}$

$\Rightarrow \quad \frac{2\tan\theta}{1-\tan^2\theta} = \frac{-6\sqrt{3}}{7-13}$

$\Rightarrow \quad \frac{2\tan\theta}{1-\tan^2\theta} = \frac{\sqrt{3}}{1}$

$\Rightarrow \quad 2\tan\theta = \sqrt{3} - \sqrt{3}\tan^2\theta$

$\Rightarrow \quad \sqrt{3}\tan^2\theta + 2\tan\theta - \sqrt{3} = 0$

$\Rightarrow \quad \sqrt{3}\tan^2\theta + 3\tan\theta - \tan\theta - \sqrt{3} = 0$

$\Rightarrow \quad \sqrt{3}\tan\theta(\tan\theta + \sqrt{3}) - 1(\tan\theta + \sqrt{3}) = 0$

$\Rightarrow \quad (\sqrt{3}\tan\theta - 1)(\tan\theta + \sqrt{3}) = 0$

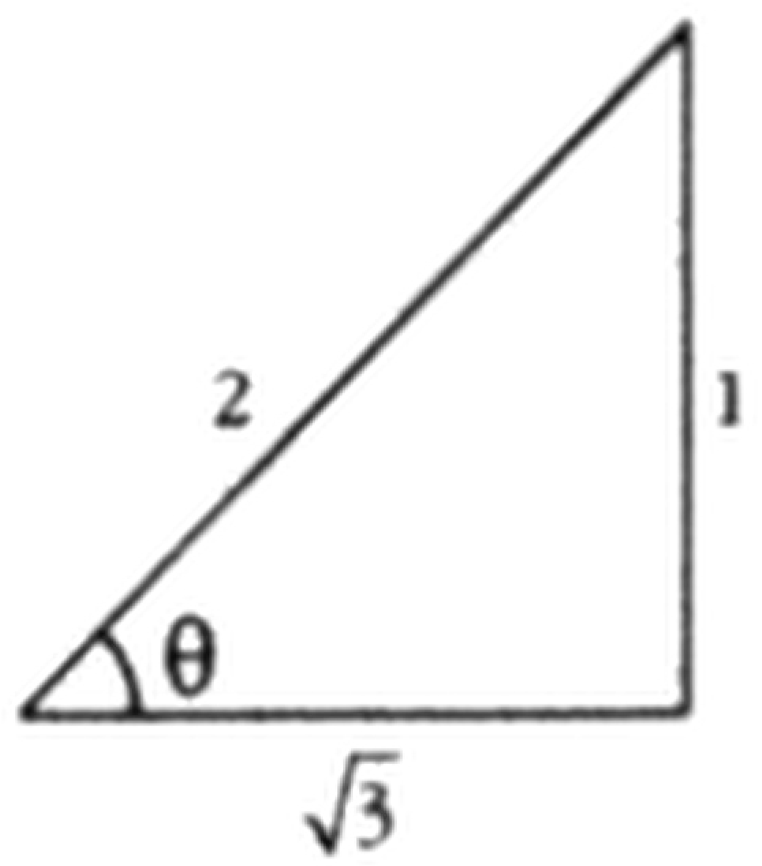
$\Rightarrow \quad (\sqrt{3}\tan\theta - 1) = 0 \qquad \tan\theta + \sqrt{3} = 0$

$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}} \qquad \tan\theta = -\sqrt{3}$

since θ lies in the first quadrant, $\tan\theta = -\sqrt{3}$ is not admissible

$\tan\theta = \frac{1}{\sqrt{3}} \quad \Rightarrow \sin\theta = \frac{1}{2},$

$\cos\theta = \frac{\sqrt{3}}{2}$




Equation of transformation become

$x = x\cos\theta - y\sin\theta = x\left[\frac{\sqrt{3}}{2}\right] - y\left[\frac{1}{2}\right] = \frac{\sqrt{3}x-y}{2}$

$y = x\sin\theta + y\cos\theta = x\left[\frac{1}{2}\right] + y\left[\frac{\sqrt{3}}{2}\right] = \frac{\sqrt{3}x+y}{2}$

Substituting these expressions for x and y into (1), we have

$$\Rightarrow \qquad 7\left(\frac{\sqrt{3}x-y}{2}\right)^2 - 6\sqrt{3}\left(\frac{\sqrt{3}x-y}{2}\right)\left(\frac{\sqrt{3}x+y}{2}\right) + 13\left(\frac{x+\sqrt{3}y}{2}\right)^2 - 16 = 0$$

$$\frac{7}{4}\left(3x^2 - 2\sqrt{3}xy + y^2\right) - \frac{3\sqrt{3}}{2}\left(\sqrt{3}x^2 + 2xy - \sqrt{3}y^2\right) + \frac{13}{4}\left(x^2 + 2\sqrt{3}xy + \sqrt{3}y^2\right) - 16 = 0$$

$$7(3x^2 - 2\sqrt{3}xy + y^2) - 6\sqrt{3}\left(\sqrt{3}x^2 + 2xy - \sqrt{3}y^2\right) + 13(x^2 + 2\sqrt{3}xy + \sqrt{3}y^2) - 16 = 0$$

$$16x^2 + 64y^2 - 64 = 0$$

$$x^2 + 4y^2 = 4$$

$$\frac{x^2}{4} + \frac{y^2}{1} = 1 \qquad \text{is an ellipse.}$$

Now we will solve (2),for x and y

$$2x = \sqrt{3}x - y \qquad 2y = x + \sqrt{3}y$$

$$x = \frac{1}{2}\left(\sqrt{3}x + y\right) \qquad y = \frac{1}{2}(x - \sqrt{3}y)$$

for centre

$$x = 0 \qquad y = 0$$

$$\frac{1}{2}\left(\sqrt{3}x + y\right) = 0 \qquad \frac{1}{2}\left(\sqrt{3}x - y\right) = 0$$

$$\left(\sqrt{3}x + y\right) = 0 \qquad \left(-x + \sqrt{3}y\right) = 0$$

Solving these equations, we have (0,0) as the coordinates of centre.

Eccentricity= $\sqrt{\frac{a^2-b^2}{a^2}} = \sqrt{\frac{4-1}{4}}$

$$= \frac{\sqrt{3}}{2}$$

Foci $x = \pm\sqrt{3}\ \ y = 0$

$$\frac{1}{2}\left(\sqrt{3}x + y\right) = \pm\sqrt{3} \qquad \frac{1}{2}\left(-x + \sqrt{3}y\right) = 0$$

$$\left(\sqrt{3}X + y\right) = \pm 2\sqrt{3} \qquad -x + \sqrt{3}\,y = 0$$

Solving these equation

$\left(\sqrt{3}X + y\right) = 2\sqrt{3} \qquad -\left(x - \sqrt{3}\,y\right) = 0$

$\left(\sqrt{3}X + y\right) = -2\sqrt{3} \qquad -\left(x - \sqrt{3}\,y\right) = 0$

Which gives as a solution

$\left[\frac{3}{2}, \frac{\sqrt{3}}{2}\right]$ and $\left[-\frac{3}{2}, -\frac{\sqrt{3}}{2}\right]$

Vertices

, $x = \pm 2 \qquad y = 0$

$\frac{1}{2}\left(\sqrt{3}x + y\right) = \pm 2 \qquad \frac{1}{2}\left(\sqrt{3}x - y\right) = 0$

$\left(\sqrt{3}x + y\right) = \pm 4 \qquad -\left(x - \sqrt{3}y\right) = 0$

Solving these equations

$\left(\sqrt{3}X + y\right) = 4 \qquad -\left(x - \sqrt{3}\,y\right) = 0$

$\left(\sqrt{3}X + y\right) = -4 \qquad -\left(x - \sqrt{3}\,y\right) = 0$

Which gives as a solution

$\left[\sqrt{3}, 1\right]$ and $\left[-\sqrt{3}, -1\right]$

Equation of major axis

$y = 0$

$-\frac{1}{2}\left(x - \sqrt{3}y\right) = 0$

$x - \sqrt{3}\,y = 0$

Equation of minor axis $x = 0$

$\frac{1}{2}(\sqrt{3}\,x + y) = 0$

$\sqrt{3}\,x - y = 0$

**vi.** $\mathbf{4x^2 - 4xy + 7y^2 + 12x + 6y - 9 = 0}$

**Solution**

Here $a = 4$, $b = 7$, $2h = -4$

the angle θ through which axi be rotated is given by

$$\tan 2\theta = \frac{2h}{a-b} \qquad \frac{2\tan\theta}{1-\tan^2\theta} = \frac{-4}{4-7}$$

$\Rightarrow \quad \frac{2\tan\theta}{1-\tan^2\theta} = -\frac{4}{3} \Rightarrow \quad \frac{\tan\theta}{1-\tan^2\theta} = -\frac{2}{3}$

$\Rightarrow \quad 3\tan\theta = 2 - 2\tan^2\theta$

$\Rightarrow \quad 2\tan^2\theta + 3\tan\theta - 2 = 0$

$\Rightarrow \quad 2\tan^2\theta + 4\tan\theta - \tan\theta - 2 = 0$

$\Rightarrow \quad 2\tan\theta(\tan\theta + 2) - 1(\tan\theta + 2) = 0$

$\Rightarrow \quad (2\tan\theta - 1)(\tan\theta + 2) = 0$

$\Rightarrow \quad (2\tan\theta - 1) = 0 \qquad \tan\theta + 2 = 0$

$\Rightarrow \quad \tan\theta = \frac{1}{2} \qquad \tan\theta = -2$

since θ lies in the first quadrant, $\tan\theta = -2$ is not admissible

$\tan\theta = \frac{1}{2} \quad \Rightarrow \sin\theta = \frac{1}{\sqrt{5}},$

$\cos\theta = \frac{2}{\sqrt{5}}$

Equation of transformation become

$$X = x\cos\theta - y\sin\theta = x\left[\frac{2}{\sqrt{5}}\right] - y\left[\frac{1}{\sqrt{5}}\right]$$

$$= \frac{2x - y}{\sqrt{5}}$$

$$Y = x\sin\theta + y\cos\theta = x\left[\frac{1}{\sqrt{5}}\right] + y\left[\frac{2}{\sqrt{5}}\right]$$

$$= \frac{x + 2y}{\sqrt{5}}$$

Substituting these expressions for x and y into (1), we have

$$\Rightarrow \quad 4\left(\frac{2x-y}{\sqrt{5}}\right)^2 - 4\left(\frac{2x-y}{\sqrt{5}}\right)\left(\frac{x+2y}{\sqrt{5}}\right) + 7\left(\frac{x+2y}{\sqrt{5}}\right)^2 + 12\left(\frac{2x-y}{\sqrt{5}}\right) + 6\left(\frac{x+2y}{\sqrt{5}}\right) - 9 = 0$$

$$\frac{4}{5}(4x^2 - 4xy + y^2) - \frac{4}{5}(2x^2 + 4xy - xy - 2y^2) + \frac{7}{5}(x^2 + 24xy + 4y^2) + 12\left(\frac{2x-y}{\sqrt{5}}\right) + 6\left(\frac{x+2y}{\sqrt{5}}\right) - 9 = 0$$

Multiplying both sides by 5, we have

$$4(4x^2 - 4xy + y^2) - 4(2x^2 + 3xy - 2y^2) + 17(x^2 + 4xy + 4y^2) + 12\sqrt{5}(2x - y) + 6\sqrt{5}(x + 2y) - 16 = 0$$

$$15x^2 + 40y^2 + 30\sqrt{5}x - 45 = 0$$

$$3(x^2 + 2\sqrt{5}x) + 8y^2 = 9$$

$$3(x^2 + 2\sqrt{5}x + (\sqrt{5})^2) + 8y^2 = 9 + 15$$

$$3(x + \sqrt{5})^2 + 8y^2 = 24$$

$$\frac{3(x+\sqrt{5})^2}{24} + \frac{8y^2}{24} = 1$$

$$\Rightarrow \quad \frac{(x+\sqrt{5})^2}{3} + \frac{y^2}{3} = 1$$

This is an ellipse.

Equation above can be written as

$$x = \frac{2x+y}{\sqrt{5}} \qquad y = \frac{2x-y}{\sqrt{5}}$$

for centre

$$x + \sqrt{5} = 0 \qquad y = 0$$

$$x = -\sqrt{5} \qquad y = 0$$

$$\frac{(2x+y)}{\sqrt{5}} = -\sqrt{5} \qquad \frac{(2x-y)}{\sqrt{5}} = 0$$

$$(2x + y) = -5 \qquad (-x + 2y) = 0$$

Solving these equations we have (-2,-1) as the coordinates of centre.

Eccentricity= $\sqrt{\frac{a^2-b^2}{a^2}} = \sqrt{\frac{8-3}{8}}$

$= \sqrt{\frac{5}{8}}$

Foci $x = \pm\sqrt{5}\ y = 0$

$x = 0 \qquad y = 0$

$\frac{(2X+y)}{\sqrt{5}} = 0 \qquad \frac{2x-y}{\sqrt{5}} = 0$

$2x + y = 0 \qquad 2y - x = 0$

Solving these equation

$X = -\sqrt{5} \qquad y = 0$

$x = -2\sqrt{5} \qquad y = 0$

$\frac{(2X+y)}{\sqrt{5}} = 2\sqrt{5} \qquad \frac{(2x-y)}{\sqrt{5}} = 0$

$(2X + y) = -10 \qquad 2y - x = 0$

Solving

$[-4, -2]$

Vertices

$. x + \sqrt{5} = \pm\sqrt{8} \qquad y = 0$

$\frac{(2x+y)}{\sqrt{5}} + \sqrt{5} = \pm\sqrt{8} \qquad \frac{(2x-y)}{\sqrt{5}} = 0$

$(2x + y + 5) = \pm\sqrt{40} \qquad -(x - 2y) = 0$

$(2X + y + 5) = \sqrt{40} \qquad (2y - x) = 0$

$\left[\sqrt{\frac{32}{5}} - 2, \sqrt{\frac{8}{5}} - 1\right]$

$(2X + y + 5) = -\sqrt{40}$ $\quad\quad$ $(2y - x) = 0$

$$\left[-\sqrt{\frac{32}{5}} - 2, \sqrt{\frac{8}{5}} - 1\right]$$

Equation of major axis

$$y = 0$$

$$\frac{(2y-x)}{\sqrt{5}} = 0$$

$$2\,y - x = 0$$

Equation of minor axis $x + \sqrt{5} = 0$

$$X = -\sqrt{5}$$

$$\frac{2\,x+y}{\sqrt{5}} = -\sqrt{5}$$

$$2\,x + y = -5$$

$$2x + y + 5 = 0$$

**vii.** $\mathbf{xy - 4x - 2y = 0}$

**Solution**

Here $a = 0,\quad b = 0,\quad 2h = 1$

the angle θ through which axi be rotated is given by

$$\tan 2\theta = \frac{2h}{a-b} \quad\quad \frac{2\tan\theta}{1-\tan^2\theta} = \frac{1}{0-0}$$

$$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{1}{0}$$

$$1 - \tan^2\theta = 0 \quad\quad \tan^2\theta = 1$$

$$(\tan\theta) = \pm 1$$

$$(\tan\theta) = 1 \quad\quad \tan\theta = -1$$

since θ lies in the first quadrant, $\tan\theta = -1$ is not admissible

$\tan\theta = 1 \qquad \theta = 45°$ ,

Equation of transformation become

$$x = x\cos\theta - y\sin\theta = x\cos45° - y\sin45° = \frac{x-y}{\sqrt{2}}$$

$$y = x\sin\theta + y\cos\theta = x\sin45° + y\cos45° = \frac{x+y}{\sqrt{2}}$$

Substituting these expressions for x and y into (1), we have

$$\left(\frac{x-y}{\sqrt{2}}\right)\left(\frac{x+y}{\sqrt{2}}\right) - 4\left(\frac{x-y}{\sqrt{2}}\right) - 2\left(\frac{x-y}{\sqrt{2}}\right) = 0$$

$$\frac{1}{2}(x^2 - y^2) - \frac{4}{\sqrt{2}}(x-y) - \frac{2}{\sqrt{2}}(x+y) = 0$$

$$x^2 - y^2 - 4\sqrt{2}x + 4\sqrt{2}y - 2\sqrt{2}x - 2\sqrt{2}y = 0$$

$$(x^2 - 6\sqrt{2}x) - (y^2 - 2\sqrt{2}y) = 0$$

$$(x^2 - 6\sqrt{2}x + 18 - 18) - (y^2 - 2\sqrt{2}y + 2 - 2) = 0$$

$$(x^2 - 6\sqrt{2}x + 18) - (y^2 - 2\sqrt{2}y + 2) = 18 - 2$$

$$(x - 3\sqrt{2})^2 - (y - \sqrt{2})^2 = 16$$

$$\frac{(x-3\sqrt{2})^2}{16} - \frac{(y-\sqrt{2})^2}{16} = 1$$

this is hyperbola.

Now we will solve (2), for x and y

$X = \frac{x+y}{\sqrt{2}} \qquad y = -\frac{x-y}{\sqrt{2}}$

for centre

$x - 3\sqrt{2} = 0 \qquad y - \sqrt{2} = 0$

$x = 3\sqrt{2} \qquad y = \sqrt{2}$

$\frac{(x+y)}{\sqrt{2}} = 3\sqrt{2} \qquad -\frac{(x-y)}{\sqrt{2}} = \sqrt{2}$

$(x+y)=6 \qquad (x-y)=-2$

Solving these equations we have (2,4) as the coordinates of centre.

Equation of foci axis $y-\sqrt{2}=0$

$-\frac{(x-y)}{\sqrt{2}}=\sqrt{2}$

$(X-y)=-2$

$x-y+6=0$

Equation of conjugate axis

$x-3\sqrt{2}=0$

$\frac{(x+y)}{\sqrt{2}}=3\sqrt{2}$

$(x+y)=6 \qquad x+y-6=0$

Eccentricity= $\sqrt{\frac{a^2-b^2}{a^2}}=\sqrt{\frac{16+16}{32}}=\sqrt{\frac{32}{16}}=\sqrt{2}$

Foci of (3) is

$x-3\sqrt{2}=\pm4\sqrt{2} \qquad y-\sqrt{2}=0$

$x=3\sqrt{2}\pm4\sqrt{2} \qquad y=\sqrt{2}$

$\frac{(X+y)}{\sqrt{2}}=3\sqrt{2}\pm4\sqrt{2} \qquad -\frac{x-y}{\sqrt{2}}=\sqrt{2}$

$x+y=6\pm8 \qquad x-y=-2$

Solving these equation

$(X+y)=14 \qquad x-y=-2$

Gives [6,8] give $(-2,0)$

Vertices of (3) are

. $x-3\sqrt{2}=\pm4 \qquad y-\sqrt{2}=0$

$x=3\sqrt{2}\pm4 \qquad y=\sqrt{2}$

$\frac{(X+y)}{\sqrt{2}} = 3\sqrt{2} \pm 4 \qquad -\frac{x-y}{\sqrt{2}} = \sqrt{2}$

$(x+y) = 6 \pm 4\sqrt{2} \qquad (x-y) = -2$

$(X+y) = 6 + 4\sqrt{2} \qquad x - y = -2$

Give $[2 + 2\sqrt{2}, 4 + 2\sqrt{2}]$

$(X+y) = 6 - 4\sqrt{2} \qquad x - y = -2$

Give $[2 - 2\sqrt{2}, 4 - 2\sqrt{2}]$

**viii.** $\mathbf{x^2 + 4xy - 2y^2 - 6 = 0}$

**Solution**

Here $a = 1, \quad b = -2, \quad 2h = 4$

the angle θ through which axi be rotated is given by

$\tan 2\theta = \frac{2h}{a-b} \qquad \frac{2\tan\theta}{1-\tan^2\theta} = \frac{4}{1-(-2)}$

$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{4}{3} \qquad \frac{\tan\theta}{1-\tan^2\theta} = \frac{2}{3}$

$3\tan\theta = 2 - 2\tan^2\theta$

$2\tan^2\theta + 3\tan\theta - 2 = 0$

$2\tan^2\theta + 4\tan\theta - \tan\theta - 2 = 0$

$2\tan\theta(\tan\theta + 2) - 1(\tan\theta + 2) = 0$

$(2\tan\theta - 1)(\tan\theta + 2) = 0$

$(2\tan\theta - 1) = 0 \qquad \tan\theta + 2 = 0$

$\tan\theta = \frac{1}{2} \qquad \tan\theta = -2$

since θ lies in the first quadrant, $\tan\theta = -2$ is not admissible

$\tan\theta = \frac{1}{2} \qquad \Rightarrow \sin\theta = \frac{1}{\sqrt{5}},$

$\cos\theta = \frac{2}{\sqrt{5}}$

Equation of transformation become

$$X = x\cos\theta - y\sin\theta = x\left[\frac{2}{\sqrt{5}}\right] - y\left[\frac{1}{\sqrt{5}}\right]$$

$$= \frac{2x - y}{\sqrt{5}}$$

$$Y = x\sin\theta + y\cos\theta = x\left[\frac{1}{\sqrt{5}}\right] + y\left[\frac{2}{\sqrt{5}}\right]$$

$$= \frac{x + 2y}{\sqrt{5}}$$

Substituting these expressions for x and y into (1), we have

$$\left(\frac{2x-y}{\sqrt{5}}\right)^2 + 4\left(\frac{2x-y}{\sqrt{5}}\right)\left(\frac{x+2y}{\sqrt{5}}\right) - 2\left(\frac{x+2y}{\sqrt{5}}\right)^2 - 6 = 0$$

$$\frac{1}{5}(4x^2 - 4xy + y^2) + \frac{4}{5}(2x^2 + 4xy - xy - 2y^2) - \frac{2}{5}(x^2 + 24xy + 4y^2) - 6 = 0$$

Multiplying both sides by 5, we have

$$(4x^2 - 4xy + y^2) + 4\,(2x^2 + 3xy - 2y^2) - 2(x^2 + 4xy + 4y^2) - 30 = 0$$

$$10x^2 - 15y^2 - 30 = 0$$

$$10x^2 - 15y^2 = 30$$

$$\frac{10x^2}{30} - \frac{25y^2}{30} = 1$$

$$\frac{x^2}{3} - \frac{y^2}{2} = 1$$

This is a hyperbola.

Equation above can be written as

$$x = \frac{2x+y}{\sqrt{5}} \qquad y = \frac{2x-y}{\sqrt{5}}$$

for centre

$$x = 0 \qquad y = 0$$

$\frac{(2x+y)}{\sqrt{5}} = 0$ $\frac{(2x-y)}{\sqrt{5}} = 0$

$(2x + y) = 0$ $(-x + 2y) = 0$

$x = 0$ $y = 0$

Solving these equations, we have (0,0) as the coordinates of centre.

Equation of foci axis $y = 0$

$$\frac{(2x-y)}{\sqrt{5}} = 0$$

$$2y - x = 0$$

$$x - 2y = 0$$

Equation of conjugate axis

$$x = 0$$

$$\frac{(2x+y)}{\sqrt{5}} = 0$$

$$(2x + y) = 0$$

Eccentricity=$e = \sqrt{\frac{a^2-b^2}{a^2}} = \sqrt{\frac{3+2}{3}}$

$$= \sqrt{\frac{5}{3}}$$

Foci 0f (3) are

$x = \pm\sqrt{3}.\frac{\sqrt{5}}{\sqrt{3}}$ $y = 0$

$x = \pm\sqrt{5}$ $y = 0$

$\frac{(2X+y)}{\sqrt{5}} = \pm\sqrt{5}$ $\frac{2x-y}{\sqrt{5}} = 0$

$2x + y = \pm 5$ $2y - x = 0$

Solving these equation

$(2X + y) = 5$ , $2y - x = 0$ and $\sqrt{5}2x + y = -5$ , $2y - x = 0$

$\Rightarrow \quad [2,1] \qquad \Rightarrow \quad (-2,-1)$

Vertices of (3) are

$, x = \pm\sqrt{3} \qquad y = 0$

$\frac{(2x+y)}{\sqrt{5}} = \pm\sqrt{3} \qquad \frac{(2x-y)}{\sqrt{5}} = 0$

$(2x + y) = \pm\sqrt{15} \qquad -(x - 2y) = 0$

We solved

$(2X + y) = \sqrt{15} \qquad (2y - x) = 0$

$(2X + y) = -\sqrt{15} \qquad (2y - x) = 0$

We got

$\left[2\sqrt{\frac{3}{5}}, \sqrt{\frac{3}{5}}\right] \left[-2\sqrt{\frac{3}{5}}, -\sqrt{\frac{3}{5}}\right]$

These are vertices.

Asymptotes of hyperbola (3) are given by

$\frac{x^2}{3} - \frac{y^2}{2} = 0$

$\left[\frac{x}{\sqrt{3}}\right]^2 - \left[\frac{y}{\sqrt{2}}\right]^2 = 0$

$\left[\frac{x}{\sqrt{3}} + \frac{y}{\sqrt{2}}\right]\left[\frac{x}{\sqrt{3}} - \frac{y}{\sqrt{2}}\right] = 0$

$\frac{x}{\sqrt{3}} + \frac{y}{\sqrt{2}} = 0 \qquad \frac{x}{\sqrt{3}} - \frac{y}{\sqrt{2}} = 0$

$\sqrt{2}x + \sqrt{3}y = 0 \qquad \sqrt{2}x - \sqrt{3}y = 0$

$\sqrt{2}\left(\frac{(2x+y)}{\sqrt{5}}\right) + \sqrt{3}\left(\frac{(2x-y)}{\sqrt{5}}\right) = 0$

$\sqrt{2}\left(\frac{(2x+y)}{\sqrt{5}}\right) - \sqrt{3}\left(\frac{(2x+y)}{\sqrt{5}}\right) = 0$

$2\sqrt{2}x + \sqrt{2}y + 2\sqrt{3}y - \sqrt{3}x = 0$

$2\sqrt{2}x + \sqrt{2}y - 2\sqrt{3}y + \sqrt{3}x = 0$

$(2\sqrt{2} - \sqrt{3})x + (\sqrt{2} + 2\sqrt{3})y = 0$

$(2\sqrt{2} + \sqrt{3})x + (\sqrt{2} - 2\sqrt{3})y = 0$

**ix.** $\mathbf{x^2 - 4xy - 2y^2 + 10x + 4y = 0}$

**Solution**

Here $a = 1, \quad b = -2, \quad 2h = -4$

the angle θ through which axi be rotated is given by

$\tan 2\theta = \frac{2h}{a-b} \qquad \frac{2\tan\theta}{1-\tan^2\theta} = \frac{-4}{1-(-2)}$

$\frac{2\tan\theta}{1-\tan^2\theta} = -\frac{4}{3} \qquad \frac{\tan\theta}{1-\tan^2\theta} = -\frac{2}{3}$

$3\tan\theta = -2 + 2\tan^2\theta$

$2\tan^2\theta - 3\tan\theta - 2 = 0$

$2\tan^2\theta - 4\tan\theta + \tan\theta - 2 = 0$

$2\tan\theta(\tan\theta - 2) + 1(\tan\theta - 2) = 0$

$(2\tan\theta + 1)(\tan\theta - 2) = 0$

$(2\tan\theta + 1) = 0 \qquad \tan\theta - 2 = 0$

$\tan\theta = -\frac{1}{2} \qquad \tan\theta = 2$

since θ lies in the first quadrant, $\tan\theta = -\frac{1}{2}$ is not admissible

$\tan\theta = 2 \qquad \Rightarrow \sin\theta = \frac{2}{\sqrt{5}},$

$\Rightarrow \qquad \cos\theta = \frac{1}{\sqrt{5}}$

Equation of transformation become

$X = x\cos\theta - y\sin\theta = x\left[\frac{1}{\sqrt{5}}\right] - y\left[\frac{2}{\sqrt{5}}\right]$

$$= \frac{x - 2y}{\sqrt{5}}$$

$$Y = x\sin\theta + y\cos\theta = x\left[\frac{2}{\sqrt{5}}\right] + y\left[\frac{1}{\sqrt{5}}\right]$$

$$= \frac{2x + y}{\sqrt{5}}$$

Substituting these expressions for x and y into (1), we have

$$\left(\frac{x-2y}{\sqrt{5}}\right)^2 - 4\left(\frac{x-2y}{\sqrt{5}}\right)\left(\frac{2x+y}{\sqrt{5}}\right) - 2\left(\frac{2x+y}{\sqrt{5}}\right)^2 + 10\left(\frac{x-2y}{\sqrt{5}}\right) + 4\left(\frac{2x+2y}{\sqrt{5}}\right) = 0$$

$$\frac{1}{5}(x^2 - 4xy + 4y^2) - \frac{4}{5}(2x^2 + 4xy - xy - 2y^2) - \frac{2}{5}(4x^2 + 4xy + y^2) + 10\left(\frac{x - 2y}{\sqrt{5}}\right) + 4\left(\frac{2x + y}{\sqrt{5}}\right) = 0$$

Multiplying both sides by 5, we have

$$(x^2 - 4xy + 4y^2) + 4\,(2x^2 + 3xy - 2y^2) - 2(4x^2 + 4xy + y^2) + 10\sqrt{5}(x - 2y) + 4\sqrt{5}(2x + y) = 0$$

$$-15x^2 + 10y^2 + 18\sqrt{5}x + 16\sqrt{5}y = 0$$

$$(10y^2 - 15\sqrt{5}y) - (15x^2 - 18\sqrt{5}x) = 0$$

$$10\left[y^2 - \frac{8}{\sqrt{5}}y\right] - 15\left[x^2 - \frac{6}{\sqrt{5}}x\right] = 0$$

$$10\left[y^2 - \frac{8}{\sqrt{5}}y + \left(\frac{4}{\sqrt{5}}\right)^2\right] - 15\left[x^2 - \frac{6}{\sqrt{5}}x + \left(\frac{3}{\sqrt{5}}\right)^2\right] = 10\left[\frac{16}{5}\right] - 15\left[\frac{9}{5}\right]$$

$$10\left[y - \frac{4}{\sqrt{5}}\right]^2 - 15\left[x - \frac{3}{\sqrt{5}}\right]^2 = \frac{160}{5} - \frac{135}{5} = \frac{25}{5} = 5$$

$$2\left[y - \frac{4}{\sqrt{5}}\right]^2 - 3\left[x - \frac{3}{\sqrt{5}}\right]^2 = 1$$

$$\frac{\left[y-\frac{4}{\sqrt{5}}\right]^2}{\left[\frac{1}{2}\right]} - \frac{\left[x-\frac{3}{\sqrt{5}}\right]^2}{\left[\frac{1}{3}\right]} = 1 \qquad \text{------------- (3)}$$

This is a hyperbola.

for centre

$$x = \frac{x+2y}{\sqrt{5}} \qquad y = \frac{x-2y}{\sqrt{5}}$$

$$x - \frac{3}{\sqrt{5}} = 0 \qquad y - \frac{4}{\sqrt{5}} = 0$$

$$x = \frac{3}{\sqrt{5}} \qquad y = \frac{4}{\sqrt{5}}$$

$$\frac{(x+2y)}{\sqrt{5}} = \frac{3}{\sqrt{5}} \qquad \frac{(y-2x)}{\sqrt{5}} = \frac{4}{\sqrt{5}}$$

$$(x + 2y) = 3 \qquad y - 2x = 4$$

Solving these equations, we have (-1,2) as the coordinates of centre.

$$\text{Eccentricity}=e = \sqrt{\frac{a^2-b^2}{a^2}} = \sqrt{\frac{\left(\frac{1}{2}\right)+\left(\frac{1}{3}\right)}{\left(\frac{1}{2}\right)}} = \sqrt{\frac{\left(\frac{3+2}{6}\right)}{\left(\frac{1}{2}\right)}} = \sqrt{\frac{5}{6}.\frac{2}{1}} = \sqrt{\frac{10}{6}}$$

$$= \sqrt{\frac{5}{3}}$$

Foci 0f (3) are

$$x - \frac{3}{\sqrt{5}} = 0 \qquad y - \frac{4}{\sqrt{5}} = \pm\frac{1}{\sqrt{2}}$$

$$x = \frac{3}{\sqrt{5}} \qquad y = \frac{4}{\sqrt{5}} \pm \frac{1}{\sqrt{2}}$$

$$\frac{(X+2y)}{\sqrt{5}} = \frac{3}{\sqrt{5}} \qquad \frac{x-2y}{\sqrt{5}} = \frac{4}{\sqrt{5}} \pm \frac{1}{\sqrt{2}}$$

$$x + 2y = 3 \qquad y - 2x = 4 \pm \frac{\sqrt{5}}{\sqrt{2}}$$

Solving these equation

$(X + 2y) = 3$ , $y - 2x = 4 + \frac{\sqrt{5}}{\sqrt{2}}$ and $x + 2y = 3$ , $y - 2x = 4 - \frac{\sqrt{5}}{\sqrt{2}}$

$$\left[-1 - \frac{2}{\sqrt{10}}, 2 + \frac{1}{\sqrt{10}}\right] \qquad \left(-1 + \frac{2}{\sqrt{10}}, 2 - \frac{1}{\sqrt{10}}\right)$$

Equation of foci axis

$$x - \frac{3}{\sqrt{5}} = 0$$

$\Rightarrow \quad \frac{(x+2y)}{\sqrt{5}} = \frac{3}{\sqrt{5}}$

$\Rightarrow \quad (x + 2y) = 3$

$\Rightarrow \quad x + 2y - 3 = 0$

Equation of conjugate axis

$y - \frac{4}{\sqrt{5}} = 0$

$y = \frac{4}{\sqrt{5}}$

$\frac{(y-2x)}{\sqrt{5}} = \frac{4}{\sqrt{5}}$

$\Rightarrow \quad y - 2x = 4$

$2x - y + 4 = 0$ conjugate axis.

**Q2. Show that (i) $10xy + 8x - 15y - 12 = 0$ and**

**(ii) $6x^2 + xy - y^2 - 21x - 8y + 9 = 0$ each represents a pair of straight lines and find an equation of each line.**

**Solution**

**i.** $10xy + 8x - 15y - 12 = 0$

$10xy - 15y + 8x - 12 = 0$

$5y(2x - 3) + 4(2x - 3) = 0$

$(2x - 3)(5y + 4) = 0$

$(2x - 3) = 0 \qquad 5y + 4 = 0$

Required two straight lines.

**(ii)** $\mathbf{6x^2 + xy - y^2 - 21x - 8y + 9 = 0}$

**Solution**

$$6x^2 + xy - y^2 - 21x - 8y + 9 = 0$$

$$6x^2 + (y - 21)x + (-y^2 - 8y + 9) = 0$$

That is quadratic in x

$$x = \frac{-(y-21) \pm \sqrt{(y-21)^2 - 4(6)(-y^2-8y+9)}}{2(6)}$$

$$= \frac{-(y-21) \pm \sqrt{y^2-42y+441-24y^2+192y-216}}{12}$$

$$= \frac{-(y-21) \pm \sqrt{(5y+12)^2}}{12}$$

$$= \frac{-(y-21)+(5y+21)}{12} = \frac{-(y-21)-(5y+21)}{12}$$

$$= \frac{-y+21+5y+15}{12} = \frac{-y+21+5y+15}{12}$$

$$12x = 4y + 36, \qquad 12x = -6y + 6$$

$$12x - 4y - 36 = 0, \quad 12x + 6y + 6 = 0$$

$$3x - y - 9 = 0, \qquad 2x + y - 1 = 0$$

**Q3. Find an equation of the tangent to each of the given conics at the indicated point.**

**i.** $\mathbf{3x^2 - 7y^2 + 2x - y - 48 = 0}$ **at (4,1)**

**Solution**

Differentiating both sides

$$6x + 14y\frac{dy}{dx} + 2 - \frac{dy}{dx} = 0$$

$$14y\frac{dy}{dx} + \frac{dy}{dx} = 6x + 2$$

$$\frac{dy}{dx}(1 + 14y) = 2 + 6x$$

$$\frac{dy}{dx} = \frac{2+6x}{1+14y}$$

$$\frac{dy}{dx}\ (4,1) = \frac{2+6(4)}{1+14(1)} = \frac{2+24}{2+14} = \frac{26}{15} = m \quad \text{(Slope)}$$

The equation of tangent at this point is

$$y - 1 = \frac{26}{15}(x - 4)$$

$$15(y - 1) = 26(x - 4)$$

$$15y - 15 = 26x - 140$$

$$26x - 15y - 89 = 0$$

**ii.** $\mathbf{x^2 + 5xy - 4y^2 + 4 = 0}$ **at y = -1**

**Solution**

$$x^2 + 5xy - 4y^2 + 4 = 0 \qquad \text{------ (1)}$$

Substitute y = - 1 in eq. (1), we have

$$x^2 + 5x(-1) - 4(-1)^2 + 4 = 0$$

$$x^2 - 5x - 4 + 4 = 0$$

$$x^2 - 5x = 0$$

$$x(x - 5) = 0 \qquad x = 0 \qquad x = 5 \quad \text{(two points)}$$

Differentiating both sides

$$\frac{dy}{dx}(x^2 + 5xy - 4y^2 + 4) = \frac{dy}{dx}(0)$$

$$\frac{dy}{dx}(x^2) + 5\frac{dy}{dx}(xy) - 4\frac{d}{dx}(y^2) + \frac{dy}{dx}(4) = 0$$

$$2x + 5\left\{x\frac{dy}{dx} + y(1)\right\} - 4(2y)\frac{dy}{dx} + 0 = 0$$

$2x + 5x\frac{dy}{dx} + 5y + 8y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(5x - 8y) = -(2x + 5y)$

$\frac{dy}{dx} = -\frac{(2x+5y)}{5x-8y}$

$\frac{dy}{dx}\,(0, -1) = -\frac{0-5}{0+8}$

$= \frac{5}{8} = m_1$ (Slope)

$\frac{dy}{dx}\,(5, -1) = -\frac{10-5}{25+8}$

$= -\frac{5}{33} = m_2$ (Slope)

Now the equation of tangent at (0,-1) point is

$y - (-1) = \frac{5}{8}(x - 0)$

$(y + 1) = \frac{5}{8}x$

$8y + 8 = 5x$

$5x - 8y - 8 = 0$

Now the equation of tangent at (8,-1) point is

$y - (-1) = \frac{5}{33}(x - 5)$

$(y + 1) = -\frac{5}{33}(x - 5)$

$33y + 33 = -5x + 25$

$5x + 33y + 8 = 0$ at (5,-1)

**iii.** $\mathbf{x^2 + 4xy - 3y^2 - 5x - 9y + 6 = 0}$ **at x = 3**

**Solution**

$x^2 + 4xy - 3y^2 - 5x - 9y + 6 = 0$ ------ (1)

Substitute x= 3 in eq. (1), we have

$$(3)^2 + 4(3)y - 3y^2 - 5(3) - 9y + 6 = 0$$

$$9 + 12y - 3y^2 - 15 - 9y + 6 = 0$$

$$-3y^2 - 3y = 0$$

$$-3y(y-1) = 0 \qquad y = 0 \qquad y = 1$$

Thus (3,0),(3,1)two points.

Differentiating both sides

$$\frac{dy}{dx}(x^2 + 4xy - 3y^2 - 5x - 9y + 6) = \frac{dy}{dx}(0)$$

$$\frac{dy}{dx}(x^2) + 4\frac{dy}{dx}(xy) - 3\frac{d}{dx}(y^2) - 5\frac{dx}{dx} - 9\frac{dy}{dx} + \frac{dy}{dx}(6) = 0$$

$$2x + 4\left\{x\frac{dy}{dx} + y(1)\right\} - 3(2y)\frac{dy}{dx} - 5 - 9\frac{dy}{dx} + 0 = 0$$

$$2x + 4x\frac{dy}{dx} + 4y - 6y\frac{dy}{dx} - 9\frac{dy}{dx} - 5 = 0$$

$$\frac{dy}{dx}(4x - 6y - 9) = 5 - 2x - 4y)$$

$$\frac{dy}{dx} = \frac{5-2x-4y}{4x-6y-9}$$

$$\frac{dy}{dx}\ (3,0) = -\frac{5-6-0}{12-0-9} = -\frac{1}{3} = m_1 \quad \text{(Slope)}$$

$$\frac{dy}{dx}\ (3,1) = \frac{5-6-4}{12-6-9} = \frac{-5}{-3} = \frac{5}{3} = m_2 \quad \text{(Slope)}$$

Now the equation of tangent at (3,0) point is

$$y - 0 = -\frac{1}{3}(x-3)$$

$$3y = -x + 3$$

$$x + 3y - 3 = 0$$

Now the equation of tangent at (3,1) point is

$$y - 1 = \frac{5}{3}(x-3)$$

$$3y - 3 = 5x - 15$$

$$5x - 3y - 12 = 0$$