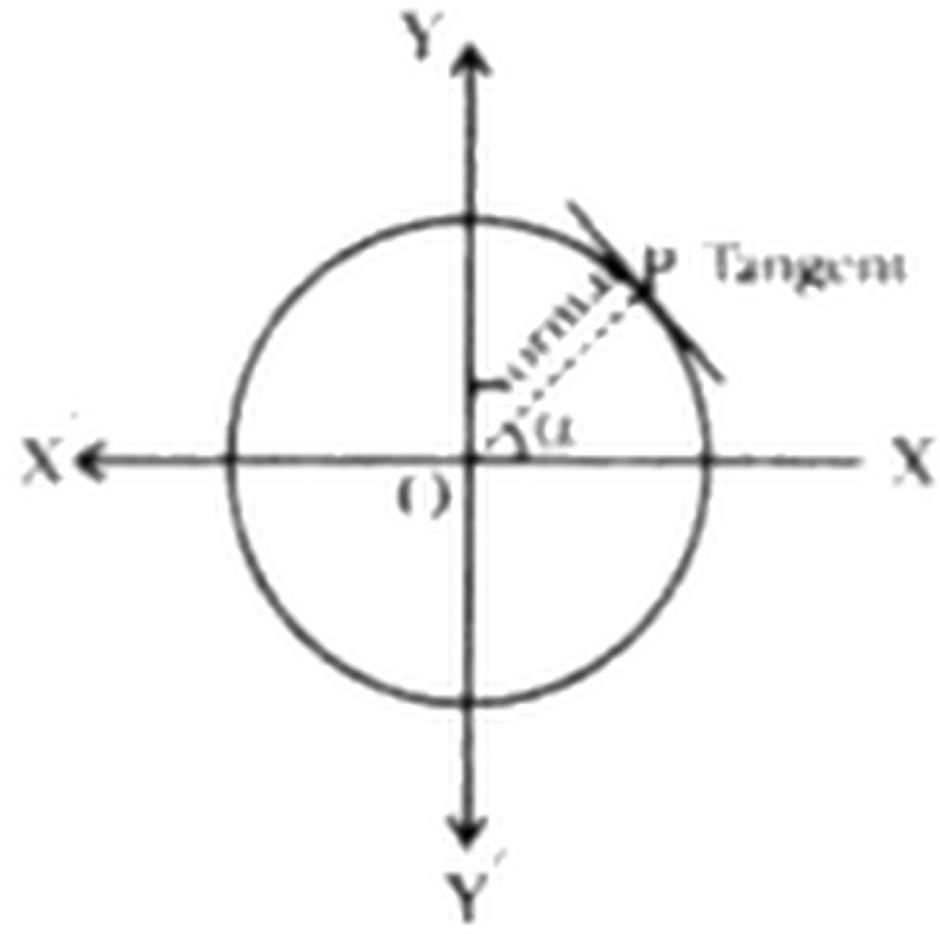
Exercise 6.3

Q1 Prove that normal lines of a circle pass through the center of the circle.

Solution:



Consider the equation of the circle $x^2 + y^2 = a^2$...............(1)

Any tangent line to the circle (1) at (x_1, y_1) is

$$xx_1 + yy_1 - a^2 = 0$$

Slope of tangent line $= -\frac{x_1}{y_1}$

Slope of normal line $= \frac{x_1}{y_1}$

Equation of the normal line passing through (x_1, y_1) is

$$y - y_1 = \frac{y_1}{x_1}(x - x_1)$$

$$yx_1 - y_1x_1 = y_1(x - x_1)$$

$$yx_1 = y_1x$$

$$y = \frac{y_1}{x_1}x$$

The above equation is satisfied by (0,0). This implies that equation of normal to circle (1) passing through the centre of circle (1) which is (0,0)

Hence the result

Q2 Prove that the straight line drawn from centre of a circle perpendicular to a tangent pass through the point of tangency.

Solution:

Consider the circle

$x^2 + y^2 = r^2$

$xy_1 + yy_1 = a^2$ be an equation of tangent to the circle.

$x^2 + y^2 = r^2$ at point (x_1, y_1)

Slope of perpendicular to tangent is $\frac{y_1}{x_1}$

Equation of line through $(0,0)$ and perpendicular to

$xx_1 + yy_1 = r^2$ is

$y - 0 = \frac{y_1}{x_1}(x - 0)$

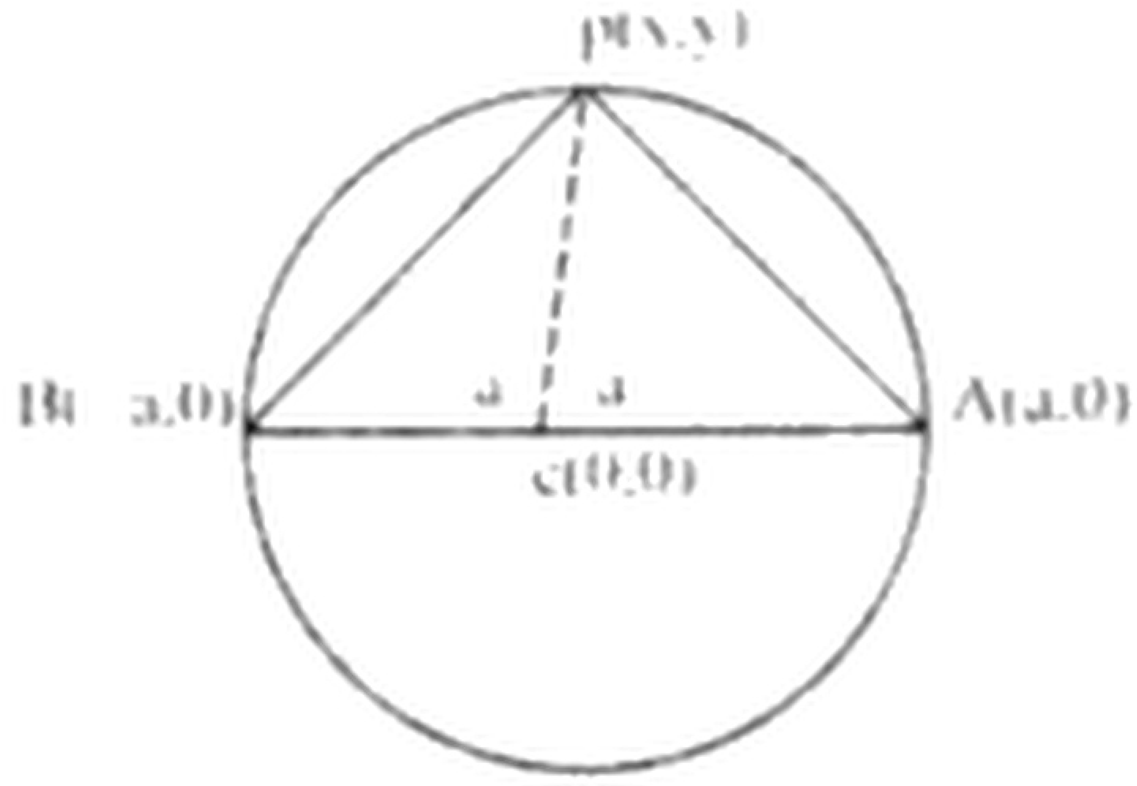
$yx_1 = y_1x$

$yx_1 - y_1x = 0$

Which passes through point of tangency (x_1, y_1). hence the result

Q3 Prove that the midpoint of the hypotenuse of a right-angled triangle is the circum centre of the triangle.

Solution:



Equation of the circle

$x^2 + y^2 = a^2$..................(1)

Slope of $AP = m_1 = \frac{y-0}{x-a} = \frac{y}{x-a}$

Slope of $BP = m_2 = \frac{y-0}{x+a} = \frac{y}{x+a}$

AP perpendicular to BP

Given $m_1 - m_2 = -1 \Rightarrow \left(\frac{y}{x-a}\right)\left(\frac{y}{x+a}\right) = -1$

$\frac{y^2}{x^2-a^2} = -1 \Rightarrow y^2 = -x^2 + a^2 \Rightarrow x^2 + y^2 = a^2$.............(2)

$|CA| = |CB| = |CP|$ (to prove)

If $|CA| = |CP|$

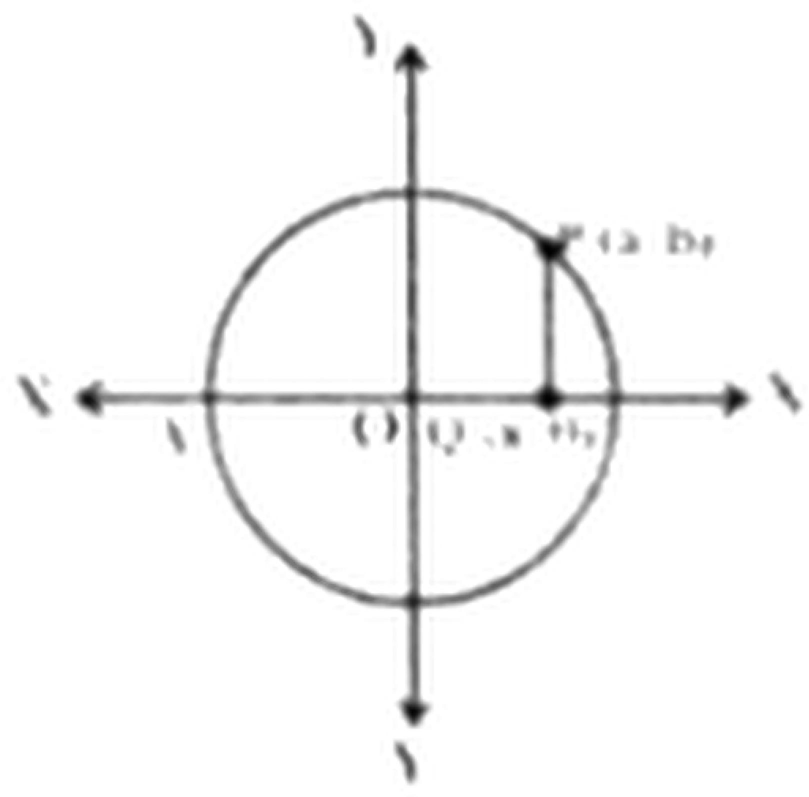
Then $a = \sqrt{(x-0)^2 + (y-0)^2} \Rightarrow x^2 + y^2 = a^2$..................(3)

If $|CB| = |CP|$

Then $-a = \sqrt{(x-0)^2 + (y-0)^2} \Rightarrow x^2 + y^2 = a^2$...............(4)

Equations (1),(2),(3) and (4) are same which proves what was required.

Q4 Prove that the perpendicular dropped from a point of a circle on a diameter is a mean proportional between the segments into which its divides the diameters.



Solution:

Let the circle be $x^2 + y^2 = r^2$..............(1)

And AB be a diameter of the circle such that if coincides with x axis.

Let $P(a, b)$ be a point on the circle and PQ must be the perpendicular dropped from P on AB.

Equation of PQ is $x = a$

Coordinates of the foot Q of the perpendicular are $(a, 0)$

$|PQ| = |b| \Rightarrow |PQ|^2 = |b|^2$

Since $P(a, b)$ lies on (1) we have

$a^2 + b^2 = r^2$ or $b^2 = r^2 - a^2$

Now $A = (-r, 0)$, $B = (r, 0)$

Therefore $|AQ| = r + a$ and $|BQ| = r - a$

Now $|AQ|.|BQ| = r^2 - a^2$

$= b^2$ from (3)

$= |PR|^2$ from (2) Hence the result