Exercise 6.2

Q1 Write down the equation of tangent and normal to the circle.

i. $x^2 + y^2 = 25 \; at \; (4, 3) and \; at \; (5\cos\theta, 5\sin\theta)$

ii. $3x^2 + 2y^2 + 5x - 13y + 2 = 0 \; at \; \left(1, \frac{10}{3}\right)$

Solution:

i. $x^2 + y^2 = 25$

$2x + 2y.\frac{dy}{dx} = 0$

$2y.\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = -\frac{2x}{2y} \quad \Rightarrow \frac{dy}{dx} = -\frac{x}{y}$

$(4,3) = \frac{dy}{dx} = -\frac{4}{3}$ = Slope of tangent at (4,3)

Slope of normal at (4,3) = $\frac{3}{4}$

Equation of tangent at (4,3) is

$y - y_1 = m(x - x_1)$

$y - 3 = -\frac{4}{3}(x - 4)$

$3y - 9 = -9x + 16$

$4x + 3y - 25 = 0$ Answer

Equation of normal at (4,3) is

$y - 3 = \frac{3}{4}(x - 4)$

$4y - 12 = 3x - 12$

$3x - 4y = 0$ Answer

At $(5\cos\theta, 5\sin\theta) = \frac{dy}{dx} = -\frac{5\cos\theta}{5\sin\theta} = -\frac{\cos\theta}{\sin\theta}$

$= $ Slope of tangent at $(5\cos\theta, 5\sin\theta)$

Slope of normal at $(5\cos\theta, 5\sin\theta) = \frac{\sin\theta}{\cos\theta}$

Equation of tangent at $(5\cos\theta, 5\sin\theta)$ is

$y - 5\sin\theta = \frac{\cos\theta}{\sin\theta}(x - 5\cos\theta)$

$y\sin\theta - 5\sin^2\theta = -x\cos\theta + 5\cos^2\theta$

$x\cos\theta + y\sin\theta = 5(\sin^2\theta + \cos^2\theta) = 5(1) = 5$

$x\cos\theta + y\sin\theta - 5 = 0$ Answer

Equation of normal

$y - 5\sin\theta = \frac{\sin\theta}{\cos\theta}(x - 5\cos\theta)$

$y\cos\theta - 5\sin\theta\cos\theta = x\sin\theta - 5\sin\theta\cos\theta$

$x\sin\theta - y\cos\theta = 0$ Answer

ii. $\mathbf{3x^2 + 2y^2 + 5x - 13y + 2 = 0}$

Differentiating w.r.t x

$$6x + 6y\frac{dy}{dx} + 5(1) - 13\frac{dy}{dx} + 0 = 0$$

$\frac{dy}{dx}(6y - 13) = -6x - 5$

$\frac{dy}{dx} = \frac{6x+5}{6y-13}$

At $\left(1, \frac{10}{3}\right) = \frac{dy}{dx} = \frac{6(1)+5}{6(\frac{10}{3})-13} = \frac{11}{7} = $ Slope of tangent at $\left(1, \frac{10}{3}\right)$

Slope of normal at $\left(1, \frac{10}{3}\right) = \frac{7}{11}$

Slope of tangent at $\left(1, \frac{10}{3}\right)$ is

$y - \frac{10}{3} = -\frac{11}{7}(x - 1)$

$7y\frac{-70}{3} = -11x + 11$

$11x + 7y\frac{-70}{3} - 11 = 0$

$11x + 7y - \frac{103}{3} = 0$

$33x + 21y - 103 = 0$ Answer

Equation of normal at $\left(1, \frac{10}{3}\right)$ is

$y - \frac{10}{3} = \frac{7}{11}(x - 1)$

$11y - \frac{110}{3} = 7x - 7$

$7x - 11y + \frac{110}{3} - 7 = 0$

$7x - 11y + \frac{89}{3} = 0$

$21x - 33y + 89 = 0$ **Answer**

Q2 Write down the equation of tangent and normal to circle

$4x^2 + 4y^2 - 16x + 24y - 117 = 0$ at the points on circle whose abscissa is -4

Solution:

$4x^2 + 4y^2 - 16x + 24y - 117 = 0$_____________(1)

Dividing by 4

$x^2 + y^2 - 4x + 6y - \frac{117}{4} = 0$_____________(2)

Put $x = -4$ in (2)

$(-4)^2 + y^2 - 4(-4) + 6y - \frac{117}{4} = 0$

$16 + y^2 - 16 + 6y - \frac{117}{4} = 0$

$y^2 + 32 + 6y - \frac{117}{4} = 0$

$y^2 + 6y - \frac{128 + 117}{4} = 0$

$y^2 + 6y + \frac{11}{4} = 0$

$4y^2 + 24y + 11 = 0$

$4y^2 + 22y + 2y + 11 = 0$

$2y(2y + 11) + 1(2y + 11) = 0$

$(2y + 1)(2y + 11) = 0$

$(2y + 1) = 0, (2y + 11) = 0$

$\Rightarrow y = \frac{-1}{2}, \quad y = \frac{-11}{2}$

The points on the circle with abscissa -4 are

$\left(-4, \frac{-11}{2}\right), \quad \left(-4, \frac{-1}{2}\right)$

Equation of tangent at $\left(-4, \frac{-11}{2}\right)$

$-4x + \left(\frac{-11}{2}\right)y + (-2)\left(x + (-4)\right) + 3\left(y + \left(\frac{-11}{2}\right)\right)\frac{-117}{4} = 0$

$-4x + \left(\frac{-11}{2}\right)y \pm 2x + 8 + 3y - \frac{33}{2} - \frac{117}{4} = 0$

$-6x + 3y - \frac{11}{2}y + 8 - \frac{33}{2} - \frac{117}{4} = 0$

$24x + 12y - 22y + 32 - 66 - 117 = 0$

$24x - 10y - 151 = 0$

$24x - 10y + 151 = 0$

Equation of tangent at $\left(-4, \frac{-1}{2}\right)$ is

$$xx_1 + I_1 + g(x + x_1) + f(y + y_1) + c = 0$$

$$-4x + \left(\frac{-1}{2}\right)y + (-2)(x-4) + 3\left(y + \left(-\frac{1}{2}\right)\right) - \frac{117}{4} = 0$$

$$-4x + \left(\frac{-1}{2}\right)y - 2x + 8 + 3y - \frac{117}{4} = 0$$

$$-6x + \left(\frac{-1}{2}\right)y - \frac{91}{4} = 0$$

$$-24x + 10y - 91 = 0$$

$$24x - 10y + 91 = 0$$

Equation of normal at $\left(-4, \frac{-11}{2}\right)$

$$y - \left(-\frac{11}{2}\right) = \frac{12}{5}\left(x - (-4)\right)$$

$$y + \left(\frac{11}{2}\right) = \frac{12}{5}(x+4)$$

$$12y + 66 = 5x + 20$$

$$15x - 12y - 46 = 0$$

Equation of normal at $\left(-4, \frac{-1}{2}\right)$ is

$$\left[y - \left(-\frac{1}{2}\right)\right]\left(-4 + (-a)\right) = (\frac{-1}{2} + 3)$$

$$\left[y + \left(\frac{1}{2}\right)\right](-6) = (x+4)(\frac{5}{2})$$

$$(2y+1)(-6) = 5x + 20$$

$$-6(2y+1) = 5x + 20$$

$$12y - 6 = 5x + 20$$

$$5x + 12y + 26 = 0$$

Is the required equation

Q3 Check the position of the point $(5,6)$ with respect to circle.

i. $\boldsymbol{x^2 + y^2 = 81}$

ii. $\boldsymbol{2x^2 + 2y^2 + 12x - 8y + 1 = 0}$

Solution:

i. $\boldsymbol{x^2 + y^2 = 81}$

Putting (5,6) in it

$L.H.S = (5)^2 + (16)^2 - 81$

$25 + 36 - 81 = -20 < 0$

Hence (5,6) lies inside the given circle.

ii. $\boldsymbol{2x^2 + 2y^2 + 12x - 8y + 1 = 0}$

$2(x^2 + y^2 + 6x - 4y + \frac{1}{2}) = 0$

$x^2 + y^2 + 6x - 4y + \frac{1}{2} = 0$

$x^2 + y^2 + (2)(3)x + (2)(-2)y + \frac{1}{2} = 0$

Putting (5,6) in it

$L.H.S. = (5)^2 + (6)^2 + (2)(3)(5) + (2)(-2)(6) + \frac{1}{2}$

$= 25 + 36 + 36 - 24 + \frac{1}{2} = 67 + \frac{1}{2} = 67.5 > 0$

Q4 Find the length of tangent drawn from the point $(-5,4)$ to the circle $5x^2 + 5y^2 - 10x + 15y - 131 = 0$

Solution:

$5x^2 + 5y^2 - 10x + 15y - 131 = 0$

Given equation can be written as

$$x^2 + y^2 - 2x + 3y - \frac{131}{5} = 0$$

Here

$$g = -1,\ f = \frac{3}{2},\ c = -\frac{131}{5}$$

The length of tangent drawn from $(-5,4)$ to the circle

$$= \sqrt{(-5)^2 + (4)^2 - 2(-5) + 3(4) - \frac{131}{5}}$$

$$= \sqrt{25 + 16 + 10 + 12 - \frac{131}{5}}$$

$$= \sqrt{63 - \frac{131}{5}}$$

$$= \sqrt{\frac{315 - 131}{5}}$$

$$= \sqrt{\frac{184}{5}}$$

Q5 Find the length of the chord cut off from the line

$2x + 3y = 13$ by circle $x^2 + y^2 = 26$

Solution:

$$3y = 13 - 2x$$

$y = \frac{13-2x}{3}$ _______________(1)

$x^2 + y^2 = 26$ _______________(2)

Putting value of y in (2) we get

$x^2 + (\frac{13-2x}{3})^2 = 26$

$x^2 + \frac{169-52x+4x^2}{9} = 26$

$9x^2 + 169 - 52x + 4x^2 = 234$

$13x^2 - 52x - 65 = 0$

Divided by 13 we get

$x^2 - 4x - 5 = 0$

$x^2 - 5x + x - 5 = 0$

$x(x-5) + 1(x-5) = 0$

$(x+1)(x-5) = 0$

$(x+1) = 0, \quad (x-5) = 0$

$x = -1, \quad x = 5$

When $x = -5$, the $y = \frac{13+2}{3} = \frac{15}{3} = 5$

When $x = 5$, the $y = \frac{13-10}{3} = \frac{3}{3} = 1$

Hence points of intersection are $(5,1), (-1,5)$ now

Length of chord

$l = \sqrt{5-(-1)^2 + (1-5)^2}$

$= \sqrt{(6)^2 + (4)^2}$

$= \sqrt{36+16}$

$= \sqrt{52}$

$= 2\sqrt{13}$

Q6 Find the coordinates of the point of intersection of the line $x + 2y = 6$ with the circle

$\boldsymbol{x^2 + y^2 - 2x - 2y - 39 = 0}$

Solution:

Given that

$x + 2y = 6$ ____________ (1)

$x^2 + y^2 - 2x - 2y - 39 = 0$ ____________ (2)

From (1) $x = 6 - 2y$ pitting it in (2)

$(6 - 2y)^2 + y^2 - 2(6 - 2y) - 2y - 39 = 0$

$5y^2 - 22y - 15 = 0$

$5y^2 - 25y - 3y - 15 = 0$

$5y(y - 5) + 3(y - 5) = 0$

$(5y + 3)(y - 5) = 0$

$y = 5, \quad and \quad y = \frac{-3}{5}$

Put $y = 5$ in (1)

$x = 6 - 2(5) = 6 - 10 = -4$

Put $y = \frac{-3}{5}$ in (1)

$x = 6 - 2\left(\frac{-3}{5}\right)$

$= 6 + \left(\frac{6}{5}\right) = \frac{30+6}{5} = \frac{36}{5}$

Here common point of intersection are (4,5) and $(\frac{36}{5}, \frac{-3}{5})$

Q7 Find the equation of the tangent to the circle $x^2 + y^2 = 2$

i. **Parallel to the line $x - 2y + 1 = 0$**

Solution:

Equation of any parallel to $x - 2y + 5 = 0$ is

$x - 2y + k = 0$____________(1)

If (1) is tangent to the give circle

Then perpendicular distance of the center (0,0) of the circle (1) is equal to the radius of the circle.

Therefore, $\frac{(0)-2(0)k}{\pm\sqrt{(1)^2+(2)^2}} = \sqrt{2}$

$\frac{k}{\pm\sqrt{1+4}} = \sqrt{2}$

$k = \pm\sqrt{5}\sqrt{2}$

$k = \pm\sqrt{10}$

Hence equation of required tangent is

$x - 2y \pm \sqrt{10} = 0$

ii. **Perpendicular to the line $3x + 2y = 6$**

Solution:

Given line is $3x + 2y = 6$

The slope of the given line $= \frac{-3}{2}$

As the tangent is perpendicular to the given line, so its slope $= -\left(\frac{-2}{3}\right) = \frac{2}{3}$

In this case radius = $\sqrt{2}$ then equation of tangents is

$x^2 + y^2 = 2$ have slope $\frac{2}{3}$

Are $y = \frac{2}{3}x \pm \sqrt{2}\sqrt{1 + \frac{4}{9}}$

$= \frac{2}{3}x \pm \sqrt{2}(\frac{\sqrt{13}}{3})$

$= \frac{2}{3}x \pm (\frac{\sqrt{26}}{3})$

$\Rightarrow \quad 3y = 2x \pm \sqrt{26}$

Thus, the equation of tangents is

$2x - 3y + \sqrt{26} = 0$ and

$2x - 3y - \sqrt{26} = 0$

Q8 Find equation of the tangents drawn from

i. $(0, 5)$ **to** $x^2 + y^2 = 16$ __________ **(1)**

Solution:

Any tangent $y = mx + c$ to circle

$x^2 + y^2 = a^2$

$y = mx + a\sqrt{1 + m^2}$ __________ (2)

$y = mx + 4\sqrt{1 + m^2}$

$y - mx = 4\sqrt{1 + m^2}$ __________ (3)

Squaring both sides we get, $\quad (y - mx)^2$

$y^2 + 2mxy + m^2x^2 = 4(1 + m^2)$

It passes through $9 = (0,5)$, then

$5^2 + 2m(0)(5) + m^2(0) \quad = 16(1 + m^2)$

$\Rightarrow \quad 25 = 16(1 + m^2)$

$\frac{25}{16} = (1 + m^2)$

$m = \sqrt{\frac{25-16}{16}} = \sqrt{\frac{9}{16}} = \pm\frac{3}{4}$ __________(4)

Hence equation of tangent from (0,5)

$y = \pm\frac{3}{4}x + 4\sqrt{1 + \frac{9}{16}}$

$= \pm\frac{3}{4}x + 4\sqrt{\frac{25}{16}}$

$= \pm\frac{3}{4}x + 4(\frac{5}{4})$

$= \pm\frac{3}{4}x + 5$

$\Rightarrow \quad 4y = \pm 3x + 20$

$\Rightarrow \quad 4y - 3x = 20$ and $\Rightarrow \quad 4y + 3x = 20$

$3x - 4y + 20 = 0$

$-4y = -3x - 20$

$\Rightarrow \quad y = \frac{3x+20}{4}$

Put $y = \frac{3x+20}{4}$ in (1) we get

$x^2 + (\frac{3x+20}{4})^2 = 16$

$x^2 + \frac{9x^2+120x+400}{16} = 16$

$\Rightarrow \quad 16x^2 + 9x^2 + 120x + 400 = 256$

$\Rightarrow \quad 25x^2 + 120x + 144 = 0$

$\Rightarrow \quad (5x + 12)^2 = 0$

$\Rightarrow \quad 5x + 12 = 0$

$\Rightarrow \quad 5x = -12$

$\Rightarrow \quad x = \frac{-12}{5}$

Now $\quad y = \frac{3(-\frac{12}{5})+20}{4}$

$y = \frac{-36+100}{26}$

$y = \frac{64}{26} = \frac{16}{5}$

i.e. $(\frac{-12}{5}, \frac{16}{5})$

when $3x - 4y + 20 = 0$

Then $(\frac{-12}{5}, \frac{16}{5})$

$3x - 4y + 20 = 0$

$-4y = 20 - 3x$

$y = \frac{20-3x}{4}$

Put $y = \frac{20-3x}{4}$ in (1) we get

$x^2 + (\frac{20-3x}{4})^2 = 16$

$$x^2 + \frac{400 - 9x^2 - 120x}{16} = 16$$

$\Rightarrow \quad 16x^2 + 9x^2 - 120x + 400 = 256$

$\Rightarrow \quad 25x^2 - 120x + 144 = 0$

$\Rightarrow \quad (5x - 12)^2 = 0$

$\Rightarrow \quad 5x - 12 = 0$

$\Rightarrow \quad 5x = 12$

$\Rightarrow \quad x = \frac{12}{5}$

Now $y = \frac{20-3(\frac{12}{5})}{4}$

$y = \frac{100-36}{20}$

$y = \frac{64}{20}$

$y = \frac{16}{5}$

i.e. $(\frac{12}{5}, \frac{16}{5})$

when $3x + 4y - 20 = 0$

Then $(\frac{12}{5}, \frac{16}{5})$

ii. Find equation of tangent drawn from

$(-1, 2)$ to $x^2 + y^2 + 4x + 2y = 0$

Solution:

$x^2 + y^2 + 4x + 2y = 0$ __________ (1)

Here $g = 2, f = 1$, i.e. center is at $(-2, -1)$

Radius $= \sqrt{(2)^2 + (1)^2 + 0}$

$= \sqrt{4+1}$

$= \sqrt{5}$

Let tangents drawn from $(-1,2)$ to the circle touch it at (x_1, y_1) then

$x^2 + y^2 + 4x_1 + 2y_1 = 0$ __________ (2)

Also by Pythagorean theorem

$(x_1 + 1)^2 + (y_1 - 2)^2 = [(-2+1)^2 + (-1-2)^2] - (\sqrt{5})^2$

$= 1 + 9 - 5 = 5$

$x_1{}^2 + 2x_1 + 1 + y_1{}^2 - 4y_1 + 4 = 5$

$x_1{}^2 + 2x_1 + y_1{}^2 - 4y_1 = 5$_____________(3)

Subtracting (2) from (3) we get

$x_1{}^2 + y_1{}^2 + 2x_1 - 4y_1 = 0$

$\underline{-x^2 \pm y^2 \pm 4x_1 \pm 2y_1 = 0}$

$-2x_1 - 6y_1 = 0$

$-2x_1 = 6y_1$

$x_1 = -3y_1$

Put $x_1 = -3y_1$ in (1) we get

$(-3y_1)^2 + y_1{}^2 + 4(-3y_1) + 2y_1 = 0$

$9y_1{}^2 + y_1{}^2 - 12y_1 + 2y_1 = 0$

$10y_1{}^2 - 10y_1 = 0$

$\Rightarrow \quad y_1(y_1 - 1) = 0$

$\Rightarrow y_1 = 0$ or $y_1 = 1$

When $y_1 = 0 \Rightarrow x_1 = 0$ i.e. $(0,0)$

When $y_1 = 1 \Rightarrow x_1 = -3$ i.e. $(-3,1)$

Slope of line passing through points

$(-1,2)$ and . $(-3,1)$

As $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{1-2}{-3+1}$

$m = \frac{-1}{-2} = \frac{1}{2}$

Equation of tangent having slope $\frac{1}{2}$ passing through $(-1,2)$

$$y - 2 = \frac{1}{2}(x + 1)$$

$\Rightarrow \quad 2y - 4 = (x + 1)$

$\Rightarrow \quad x - 2y + 5 = 0$

Slope of line passing through the points $= (-1,2)(0,0)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{2 - 0}{-1 - 0}$$

$$m = -2$$

Equation of tangent having slope -2 is

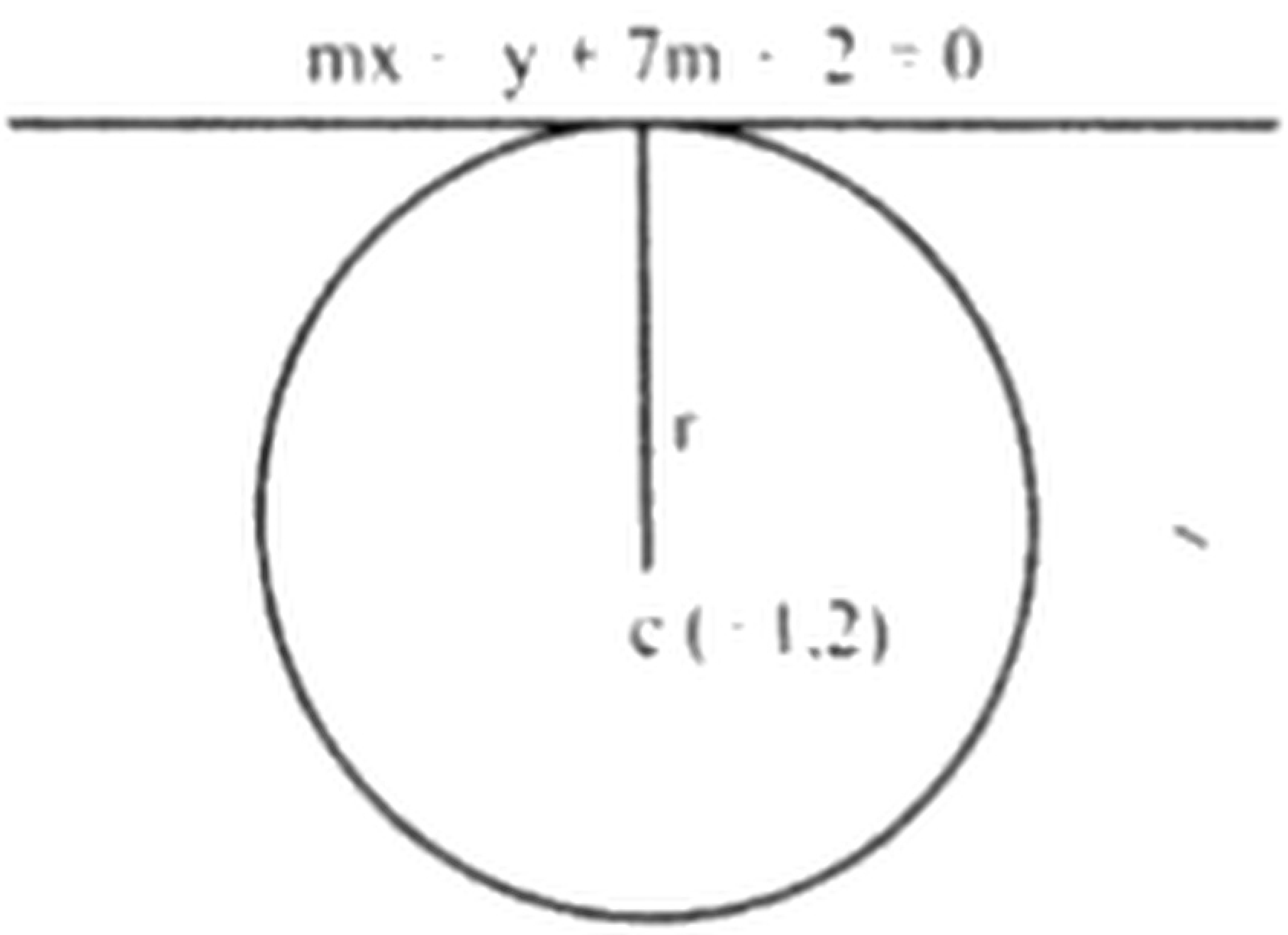
$$y - 2 = -2(x + 1)$$

$\Rightarrow \quad y - 2 = -2x - 2$

$\Rightarrow \quad y + 2x = 0$

Or $\quad 2x + y = 0$

iii. Find equation of tangents drawn from



$$(x + 1)^2 + (y - 2)^2 = 26$$

Center $= C(-1,2)$

Radius $= \sqrt{26}$

Expanding the given equation

$$x^2 + y^2 + 1 + y^2 - 4y + 4 = 26$$

$$x^2 + y^2 + 2x - 4y - 21 = 0 \_\_\_\_\_\_\_\_ (1)$$

Let equation of required tangent be

$$y - y_1 = m(x - x_1)$$

Since $(-7, -2)$ lie on it therefore

$$y + 2 = m(x + 7) = mx = 7m$$

$$mx - y + 7m - 2 = 0$$

$$r = \frac{|(m)(-1) + (-1)(2) + (7m - 2)1|}{\sqrt{m^2} + (1)^2} = \sqrt{26}$$

$$\frac{|-m - 2 + 7m - 2|}{\sqrt{m^2} + 1} = \sqrt{26}$$

$$|6m - 4| = \sqrt{26(m^2 + 1)}$$

$$(6m - 4)^2 = 26(m^2 + 1)$$

$$36m^2 - 48m + 16 = 26m^2 + 26$$

$$10m^2 - 48m - 10 = 0$$

$$2(5m^2 - 24m - 5) = 0$$

$$5m^2 - 24m - 5 = 0$$

$$m^2 = \frac{-(-24) \pm \sqrt{(-24)^2 - 4(5)(-5)}}{10}$$

$$m^2 = \frac{24 \pm \sqrt{576 + 100}}{10}$$

$$= \frac{24 \pm \sqrt{676}}{10} = \frac{24 \pm 26}{10}$$

$m = 5$ or $\quad m = \frac{-1}{5}$

Putting $m = 5$ in (2)

$5x - y + 35 - 2 = 0 \Rightarrow 5x - y + 33 = 0$

From (3)

$$y = 5x + 33$$

Putting (4) in (1)

$x^2 + (5x + 33)^2 + 2x - 4(5x + 33) - 21 = 0$

$x^2 + 25x^2 + 330x + 1089 + 2x - 20x - 132 - 21 = 0$

$26x^2 + 312x + 936 = 0$

$26(x^2 + 12x + 36) = 0$

$x^2 + 12x + 36 = 0$

$$x = \frac{-(12) \pm \sqrt{(12)^2 - 4(1)(36)}}{2(1)}$$

$$= \frac{-12 \pm \sqrt{144 - 144}}{2}$$

$$= \frac{-12}{2} = -6$$

Putting in (4)

$$y = 5(-6) + 33 = 3$$

Hence $(-6,3)$ is the point of intersection of (1) and (3)

Putting $m = \frac{-1}{5}$ in (2)

$$\frac{-1}{5}x - y + 7\left(\frac{-1}{5}\right) - 2 = 0$$

$$\frac{-x}{5}x - y - \left(\frac{17}{5}\right) = 0 \quad \Rightarrow \quad x + 5y + 17 = 0$$

From (5)

$$x = -5y - 17$$

Putting (6) in (1)

$(-5y-17)^2 + y^2 + 2(-5y-17) - 4y - 21 = 0$

$26y^2 + 156y + 234 = 0$

$26(y^2 + 6y + 9) = 0$

$y^2 + 6y + 9 = 0$

$y = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(9)}}{2(1)}$

$= \frac{-6 \pm \sqrt{36-36}}{2}$

$= \frac{-6}{2} = -3$

Putting it in (6)

$x = -5(-3) - 17 = -2$

Hence $(-2-3)$ is the point of intersection of (1) and (5)

The required equation of tangents

$5x - y + 33 = 0$ and $x + 5y + 17 = 0$

With points of intersection (6,3) and $(-2-3)$ respectively

Q9 Find an equation of chord of contact of the tangents drawn from $(4, 5)$ to the circle.

$\mathbf{2x^2 + 2y^2 - 8x + 12y + 21 = 0}$

Solution:

$2x^2 + 2y^2 - 8x + 12y + 21 = 0$

Given equation of circle is standards form is

Divided by 2

$$x^2 + y^2 - 4x + 6y + \frac{21}{2} = 0 \_\_\_\_\_\_\_\_ (1)$$

Let points of contact of the two tangents be $P(x_1, y_1)$ and $Q(x_2, y_2)$

An equation of tangents at P is

$$xx_1 + yy_1 + (-2)(x + x_1) + 3(y + y_1) + \frac{21}{2} = 0 \_\_\_\_\_\_\_\_ (2)$$

Since that (1) and (2) pass through (4,5)

$$4x_1 + 5y_1 + (-2)(4 + x_1) + 3(5 + y_1) + \frac{21}{2} = 0$$

$$2x_1 + 8y_1 + 7 + \frac{21}{2} = 0$$

$$4x_1 + 16y_1 + 35 = 0 \_\_\_\_\_\_\_\_ (3)$$

Similarly,

$$4x_2 + 16y_2 + 35 = 0$$

Equation (3) and equation (4) shows that both the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ lie on $4x + 16y + 35 = 0$ and so it is required equation of chord of contact.