UNIT 6

Conic Section

Exercise 6.1

Q1 In each of the following, find an equation of the circle with a. Centre at (5,-2) radius 4.

Solution

Let P(x ,y) be any point on the circle, As C (h,k)

$\Rightarrow (x - h)^2 + (y - k)^2 - r^2$

$(x - 5)^2 + (y - (-2))^2 = (4)^2$

$(x - 5)^2 + (y + 2)^2 = (4)^2$

x^2-10x +25+y^2+4y +4 = 16

x^2-+y^2 $- 10x$+4y +13 = 0

Which is equation of circle

b. Centre at $(\sqrt{2}, -3\sqrt{3})$ and radius $2\sqrt{2}$

Solution

Using $(x - h)^2 + (y - k)^2 = r^2$ and c (h ,k)

Let P(x ,y) be any point on the circle then equation of circle is

$(x - \sqrt{2})^2 + (y - (-3\sqrt{3}))^2 = (2\sqrt{2})^2$

$(x - \sqrt{2})^2 + (y + 3\sqrt{3})^2 = (2\sqrt{2})^2$

x^2-$2\sqrt{2}$x +2+y^2+$6\sqrt{3}$y +27 = 8

$x^2+y^2 - 2\sqrt{2}x + 2+6\sqrt{3}y +29 = 8$

$x^2+y^2 - 2\sqrt{2}x +6\sqrt{3}y = -29 +8$

$x^2+y^2 - 2\sqrt{2}x +6\sqrt{3}y = -21$

$x^2+y^2 - 2\sqrt{2}x +6\sqrt{3}y +21 = 0$

c. Ends of a diameter at (-3,2) and (5,-6)

Solution

To find Centre of circle

We use $\left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)$ ________(1)

So plotting points (-5,6) and (-3,2)________(2)

We get

Centre $\left(\frac{5-3}{2}, \frac{-6+2}{2}\right) = (1, -2)$

Radius r $= \frac{1}{2}\sqrt{(1+3)-(-2-2)^2}$

$= \frac{1}{2}\sqrt{16+16} = = \sqrt{32}$

$= 4\sqrt{\mathbf{2}}$

Radius $= 4\sqrt{\mathbf{2}}$

So required equation of circle

$(x - 1)^2 + (y + 2)^2 = (4\sqrt{2})^2$

x^2-2x +1+y^2+4y +4 = 32

$x^2+y^2 - 2x$+4y +5-32 = 0

$x^2+y^2 - 2x$+4y -27 = 0

Q2 Find centre and radius of circle with the given equation

a. $x^2+y^2 + 12x - 10y = 0$

Solution

The general equation of circle is

$x^2+y^2 + 2gy + 2fy + c = 0$ ____(1)

Comparing it with given equation we get,

2y = 12 $\Rightarrow$ g = 6

2f = -10 $\Rightarrow$ f = -5

$\Rightarrow$ c = 0

Hence center c(-g, -f) = (-6,5)

And radius = r

$= \sqrt{g^2 + f^2 - c}$

$= \sqrt{(6)^2 + (-5)^2 - 0}$

$= \sqrt{36 + 25}$

$= \sqrt{61}$

b. $5x^2+5y^2 + 14x + 12y - 10 = 0$

Solution

The general equation of circle is

$x^2 + y^2 + 2gy + 2fy + c = 0$ ______(1)

Given equation can be reduced to general form as

$\frac{5x^2}{5} + \frac{y^2}{5} + \frac{14}{5}x + \frac{12}{5}y = 0$

$x^2 + y^2 + \frac{14}{5}x + \frac{12}{5}y - 2 = 0$

$\Rightarrow \quad x^2 + y^2 + 2(\frac{7}{5})x + 2(\frac{6}{5})y - 2 = 0$ ________(2)

$2g = \frac{14}{5} \qquad \Rightarrow g = \frac{7}{5}$

$2f = \frac{12}{5} \qquad \Rightarrow f = \frac{6}{5}$

$c = -2$

so the centre $(-g,-f) = (\frac{-7}{5}, \frac{-6}{5})$

and

radius $= \sqrt{g^2 + f^2 - c} \qquad = \sqrt{(\frac{-7}{5})^2 + (\frac{-6}{5})^2 + 2}$

$= \sqrt{\frac{49}{25} + \frac{36}{25} + 2}$

$= \sqrt{\frac{49+36+50}{25}}$

$= \sqrt{\frac{135}{25}} = \sqrt{\frac{27}{5}}$

c. $\mathbf{x^2+y^2 \quad 6x + 4y + 13 = 0}$

Solution

The general equation of circle is

$x^2 + y^2 + 2gy + 2fy + c = 0$ ________(1)

Comparing both equations, we get

$2g = -6 \qquad \Rightarrow g = -3$

$2f = 4 \quad \Rightarrow f = 2$

$c = 13$

so the centre (-g,-f) = $(3, -2)$

and

radius = $\sqrt{g^2 + f^2 - c} = \sqrt{(-3)^2 + (2)^2 - 13}$

$r = \sqrt{9 + 4 - 13} = 0$

d. $\mathbf{4x^2 + 4y^2 - 8x + 12y - 25 = 0}$

Solution

$4x^2 + 4y^2 - 8x + 12y - 25 = 0$ __________(1)

Divided by equation (1)

$x^2 + y^2 - 2x + 3y - \frac{25}{4} = 0$

Comparing it with general equation of circle I.e

$x^2 + y^2 + 2gx + 2fy + c = 0$

We get

$g = -1, f = -\frac{3}{2}$ and $c = \frac{25}{4}$

so centre c (-g,-f) = $(1, -\frac{3}{2})$

radius = $\sqrt{g^2 + f^2 - c} = \sqrt{(1)^2 + (\frac{-3}{2})^2 + \frac{25}{4}}$

$= \sqrt{1 + \frac{9}{4} + \frac{25}{4}}$

$= \sqrt{\frac{38}{4}} = \sqrt{\frac{19}{2}}$

Q3. Write an equation of circle passing through the given point

a. A(4,5), B(-4,-3), C(8,-3)

b. A(-7,7), B(5, -1),C(10,0)

c. A(a,0), B(0,b), C(10,0)

d. A(5,6), B(-3,2), C(3,-4)

Solution

a.

$$(x-h)^2+(y-k)^2 = r^2 \quad \_\_\_\_\_\_(1)$$

A(4, 5)

$$(4-h)^2+(5-k)^2 = r^2$$

$$16+ \quad h^2-8h+25+k^2-10k = r^2$$

$$h^2+k^2-8h-10k+41 = r^2 \_\_\_\_\_\_(2)$$

B(-4,-3)

$$(-4-h)^2+(-3-k)^2 = r^2$$

$$16+ \quad h^2+8h+9+k^2+6k = r^2$$

$$h^2+k^2+8h-6k+25 = r^2 \_\_\_\_\_\_(3)$$

C(8,-3)

$$(8-h)^2+(-3-k)^2 = r^2$$

$$64+ \quad h^2+8h+9+k^2+6k = r^2$$

$$h^2+k^2+8h+6k+73 = r^2 \_\_\_\_\_\_(4)$$

Solving (2) and (3)

$$h^2 + k^2 - 8h - 10k + 41 = r^2$$

$$-h^2 \pm k^2 \pm 8h \pm 6k \pm 25 = r^2$$

$$-16h - 16k + 16 = 0$$

$$-16(h+k-1) = 0$$

$$h + k - 1 = 0 \quad \_\_\_\_\_\_\_\_ (5)$$

Solving (3) and (5)

$$h^2 + k^2 + 8h - 6k + 25 = r^2$$

$$-h^2 \pm k^2 \pm 16h \pm 6k \pm 73 = r^2$$

$$24h \qquad -48 = 0$$

$$24h = 48$$

$$h = \frac{48}{24} = 2$$

putting it in (5)

$$2 + k - 1 = 0$$

$$K = -1$$

Putting the values in (2)

$$(2)^2 + (-1)^2 - 8(2) - 10(-1) + 41 = r^2$$

$$4 + 1 - 16 + 10(-1) + 41 = r^2$$

$$r^2 = 40 \Rightarrow r = \pm 2\sqrt{10}$$

Putting the values in (1)

$$(x-2)^2 + (y+1)^2 = 40$$

$$x^2 + y^2 - 4x + 2y - 35 = 0$$

b. Let the required circle

$$(x-h)^2+(y-k)^2 = r^2 \quad \_\_\_\_\_\_(1)$$

$A(-7,7)$

$$(-7-h)^2+(7-k)^2 = r^2$$

$$49+ \quad h^2+14h+49+k^2+14k = r^2$$

$$h^2+k^2+14h-14k+98 = r^2 \_\_\_\_\_\_\_\_(2)$$

B(5,-1)

$$(5-h)^2+(-1-k)^2 = r^2$$

$$25+ \quad h^2-10h+1+k^2+2k = r^2$$

$$h^2+k^2-10h+2k+26 = r^2 \_\_\_\_\_\_\_\_(3)$$

C(10,0)

$$(10-h)^2+(0-k)^2 = r^2$$

$$100+ \quad h^2-10h+9+k^2 = r^2$$

$$h^2+k^2-20h+100 = r^2 \_\_\_\_\_\_\_\_(4)$$

Solving (2) and (3)

$$h^2+k^2+14h-14k+98 = r^2$$

$$-h^2 \pm k^2 \pm 10h \pm 2k \pm 26 = r^2$$

$$24h-16k+72 = 0$$

$$8(3h-2k+9) = 0$$

$$3h-2k+9 = 0 \quad \_\_\_\_\_\_(5)$$

Solving

$$h^2+k^2-10h+2k+26 = r^2$$

$$-h^2 \pm k^2 \pm 20h \pm 100 = r^2$$

$$10h+2k-74 = 0$$

$$2(5h+k-37) = 0$$

$$5h + k - 37 = 0 \quad \_\_\_\_\_\_\_\_\_\_(6)$$

Solving (5) and (6)

$$3h - 2k + 9 = 0$$

$$10h + 2k - 74 = 0$$

$$13h \quad -65 = 0$$

$$13h = 65$$

$$h = \frac{65}{13} = 5$$

putting it in (5)

$$3(5) - 2k + 9 = 0$$

$$-2K = -24$$

$$K = \frac{24}{2} = 12$$

Putting the values in (4)

$$(5)^2 + (12)^2 - 20(5) + 100 = r^2$$

$$25 + 144 - 100 + 100 = r^2$$

$$r^2 = 169 \Rightarrow r = \pm 13$$

Putting the values in (1)

$$(x-5)^2 + (y-12)^2 = 169$$

$$x^2 + y^2 - 10x - 24y = 0$$

c. Let the required circle

$$(x-h)^2 + (y-k)^2 = r^2 \quad \_\_\_\_\_\_(1)$$

$\mathbf{A(a,0)}$

$$(a-h)^2+(0-k)^2 = r^2$$

$$a^2+ \ h^2-2ah + a^2 = r^2$$

$$h^2 + k^2 - 2ah + a^2 = r^2 \text{\_\_\_\_\_\_\_\_\_\_\_\_(2)}$$

B(0,b)

$$(0-h)^2+(b-k)^2 = r^2$$

$$h^2+b^2+k^2 - 2bk = r^2$$

$$h^2 + k^2 - 2bk + b^2 = r^2 \text{\_\_\_\_\_\_\_\_\_\_\_\_(3)}$$

C(0,0)

$$(0-h)^2+(0-k)^2 = r^2$$

$$h^2 + k^2 = r^2 \text{\_\_\_\_\_\_\_\_\_\_\_\_(4)}$$

Solving (2) and (3)

$$h^2 + k^2 - 2ah + a^2 = r^2$$

$$-h^2 \pm k^2 \pm 2bk \pm b^2 = r^2$$

-2ah+2bk +$a^2 - b^2$ =0 _______(5)

Solving (3) and (4)

$$h^2 + k^2 - 2bk + b^2 = r^2$$

$$-h^2 \pm k^2 \qquad = r^2$$

-2bk + b^2 = 0

-2bk = - b^2

K $=\frac{b^2}{2b} = \frac{b}{2}$

putting it in (5)

-2ah+2b($\frac{b}{2}$) + $a^2 - b^2$ = 0

-2ah = a^2

$$h = \frac{a^2}{2a} = \frac{a}{2}$$

Putting the values in (4)

$$(2)^2 + (-1)^2 - 8(2) - 10(-1) + 41 = r^2$$

$$4+1 -16 +10(-1) +41 = r^2$$

$$r^2 = 40 \Rightarrow r = \pm 2\sqrt{10}$$

Putting the values in (1)

$$(\frac{a}{2})^2 + (\frac{b}{2})^2 = r^2$$

$$\frac{a^2}{4} + \frac{b^2}{4} = r^2$$

$$r^2 = \frac{a^2+b^2}{4} = r = \frac{\pm\sqrt{a^2+b^2}}{2}$$

Putting values in (1)

$$(x - \frac{a}{2})^2 + (y - \frac{b}{2})^2 = \frac{a^2+b^2}{4}$$

$$x^2 - 2(x)(\frac{a}{2}) + \frac{a^2}{4} + y^2 + (y)(\frac{b}{2}) + \frac{b^2}{4} = \frac{a^2+b^2}{4}$$

$$x^2 - ax + y^2 - by = \frac{a^2+b^2}{4} - \frac{a^2+b^2}{4}$$

$$x^2 + y^2 - ax - by = 0$$

d. Let equation of required circle

$$(x-h)^2 + (y-k)^2 = r^2 \quad \_\_\_\_\_\_(1)$$

A(5, 6)

$$(5-h)^2 + (6-k)^2 = r^2$$

$$25+ \ h^2 - 10h + 36 + k^2 - 12k = r^2$$

$$h^2 + k^2 - 10h - 12k + 61 = r^2 \_\_\_\_\_\_\_\_\_\_(2)$$

B(-3,2)

$$(-3-h)^2 + (2-k)^2 = r^2$$

$$9 + h^2 + 6h + 4 + k^2 - 4k = r^2$$

$$h^2 + k^2 + 6h - 4k + 13 = r^2 \_\_\_\_\_\_\_\_\_\_(3)$$

C(3,-4)

$$(3-h)^2 + (-4-k)^2 = r^2$$

$$9+ \quad h^2 - 6h + 16 + k^2 + 8k + 25 = r^2$$

$$h^2 + k^2 - 6h + 8k + 25 = r^2 \_\_\_\_\_\_\_\_\_\_(4)$$

Solving (2) and (3)

$$h^2 + k^2 - 10h - 12k + 61 = r^2$$

$$-h^2 \pm k^2 \pm 6h \pm 4k \pm 13 = r^2$$

-16h -8k =480

-8(2h+k-6) = 0

2h +k -6 = 0 ________(5)

Solving (3) and (4)

$$h^2 + k^2 + 6h - 4k + 13 = r^2$$

$$-h^2 \pm k^2 \pm 6h \pm 8k + 25 = r^2$$

12h -12k -12 = 0

12(h-k-1) = 0

h - k – 1 = 0 _________(6)

Solving (5) and (6)

2h +k -6 = 0

h - k – 1= 0

3h -7 = 0

$$3h = 7$$

$$h = \frac{7}{3}$$

putting it in (6)

$$\frac{7}{3} - k - 1 = 0$$

$$-k = \frac{4}{4} = \frac{4}{3}$$

Putting the values in (2)

$$(\frac{7}{3})^2 + (\frac{4}{3})^2 - 10(\frac{7}{3}) - 12(\frac{4}{3}) + 61 = r^2$$

$$\frac{49}{9} + \frac{16}{9} - \frac{70}{3} -16+61 = r^2$$

$$r^2 = \frac{49+16-210+405}{9} = \frac{266}{9}$$

$$\Rightarrow r^2 = \pm\frac{2\sqrt{65}}{3}$$

Putting values in (1)

$$(x - \frac{7}{3})^2 + (y - \frac{4}{3})^2 = \frac{260}{9}$$

$$x^2 - \frac{14}{9}x + \frac{49}{9} + y^2 - \frac{8}{3}y + \frac{16}{9} - \frac{260}{9} = 0$$

$$3x^2 + 3y^2 - 14x - 8y - 65 = 0$$

Q4. In each of following find an equation of circle passing through

a. A(3,-1), B(0,1) and having circle on

4x-3y -3 = 0

Solution

Let C(h,k) be the centre of circle and r be radius of circle then equation of circle is

$$(x-h)^2 + (y-k)^2 = r \quad \_\_\_\_\_\_\_\_(1)$$

Since A(3,-1) , B(0,1) lie on (1)

For A(3,-1)

$$(3-h)^2 + (-1-k)^2 = r^2$$

$$h^2 + k^2 - 6h + 2k + 10 = r^2 \quad \_\_\_\_\_\_\_\_(2)$$

For B(0,1)

$$(0-h)^2 + (1-k)^2 = r^2$$

$$h^2 + k^2 - 2k + 1 = r^2 \quad \_\_\_\_\_\_\_\_(2)$$

AS R.H.S of both equations same so equation (2) and (3)

$$(3-h)^2 + (-1-k)^2 = (0-h)^2 + (1-k)^2$$

$$6 - 6h + h^2 + 1 + 2k + k^2 = h^2 + 1 + k^2 - 2k$$

$$-6k + 4k + 9 = 0$$

Since C(h,k) lie on line

$$4h - 3y - 3 = 0$$

We have

$$4x - 3k - 3 = 0 \quad \_\_\_\_\_\_\_\_\_\_(5)$$

Multiplying (4) by w and (5) by 3 and adding than are have

$$-12h + 8k + 18 = 0$$

$$12h - 9k - 9 = 0$$

$$-k + 9 = 0$$

$$K = 9$$

Putting k = in(5) we have

4h -3(9) – 3 = 0

4h -27 – 3 = 0

4h = 30

h $=\frac{15}{2}$

thus centre is c(h,k) = $(\frac{15}{2})$

$$r^2 \quad = (\frac{15}{12} - 0)^2 \ + (0-1)^2 \ = \ \frac{225}{4} + 64$$

$$= \frac{225+256}{4} \ = \ \frac{481}{4} \Rightarrow r \ = \ \frac{\pm\sqrt{481}}{4}$$

Hence required equation of circle is

$$(x - \frac{15}{12})^2 \ + (y-9)^2 \ = \ \frac{481}{4}$$

$$x^2 \ - \frac{15}{4}x + \frac{225}{4} + y^2 - 18y \ + 81 \ = \ \frac{481}{4}$$

$$x^2 \ + y^2 - 15x \ - 18y \ + \frac{225}{4} - \frac{481}{4} + 81 \ = \ 0$$

$$x^2 \ + y^2 - 15x \ - 18y \ + \ 17 \ = \ 0$$

b. A(-3,1) with radius 2 and centre at

2x -3y + 3 = 0__________(1)

Solution

Let C(h,k) be the centre of circle and r be radius of circle then equation of circle is

$$(x-h)^2 \ + (y-k)^2 \ = \ r \qquad \text{\_\_\_\_\_\_\_\_\_(2)}$$

Since given point lie on (1) we get

For A(-3,1)

$$(-3-h)^2 + (1-k)^2 = (2)^2$$

$$9 + 6h + h^2 + k^2 + 1 - 2k = 4$$

$$h^2 + k^2 + 6h - 2k + 6 = 0 \quad \_\_\_\_(3)$$

Since c(h,k) lie on (1)

$$2h - 3k + 3 = 0$$

$$h = \frac{3k-3}{2} \quad \_\_\_\_\_\_\_\_(4)$$

putting $h = \frac{3k-3}{2}$ in (3) we get

$$\left(\frac{3k-3}{2}\right)^2 + k^2 + 6\left(\frac{3k-3}{2}\right) - 2k + 6 = 0$$

$$\frac{9k^2-18k+9}{2} + k^2 + 9k - 9 - 2k + 6 = 0$$

$$\frac{9k^2-18k+9}{2} + k^2 + 7k - 3 = 0$$

$$13k^2 + 10k - 3 = 0$$

$$13k^2 + 13k - 3k - 3 = 0$$

$$13k(k+1) - 3(k+1) = 0$$

$$(k+1)(13k-3) = 0$$

$$K+1 = 0,\ 13k - 2 = 0$$

$$K = -1,\ k = \frac{3}{13}$$

When k = 1 putting in(4) , we get

$$h = \frac{-3-13}{2} = \frac{-6}{2} = -3$$

thus centre is C(-3,-1)

when $k = \frac{3}{13}$ $then\ h = \frac{9-39}{26}$

$$h = \frac{-30}{26} = \frac{-15}{13}$$

centre is at $(\frac{-15}{13}, \frac{3}{13})$

hence required equations of circle are

$$(x+13)^2 + (y+1)^2 = 4 \qquad (x+3)^2 + (y+1)^2 = (2)^2$$

$$x^2 + 6x + 9 + y^2 + 2y + 1 - 4 = 0$$

$$x^2 + y^2 + 6x + 2y + 6 = 0$$

And $$(x + \frac{15}{13})^2 + (y - \frac{3}{13})^2 = 4$$

$$x^2 + \frac{30}{13}x + \frac{225}{169} + y^2 - \frac{6}{13}y + \frac{9}{169} - 4 = 0$$

$$13x^2 + 13y^2 + 30x - 6y - 34 = 0$$

c. A(5,1) and tangent to the line

2x-y -10 = 0 at B(3,-4)

Solution

Let C(h,k) be the centre of circle and r be radius of circle then equation of circle is

$$(x-h)^2 + (y-k)^2 = r \quad \_\_\_\_\_\_\_\_(1)$$

Since A(5,1) , B(3,-4) lie on (1)

For A(5,1)

$$(5-h)^2 + (1-k)^2 = r^2$$

$$h^2 + k^2 - 6h + 8k + 26 = r^2 \quad \_\_\_\_\_\_\_\_(2)$$

For B(3,-4)

$$(3-h)^2 + (4-k)^2 = r^2$$

h^2 $+k^2$ -6h +8k + 25 = r^2 ________(2)

AS R.H.S of both equations same .so there L.H.S are also equal

$$(5-h)^2 + (1-k)^2 = (2-h)^2 + (4+k)^2$$

25-10h+ h^2 +1-2k $+k^2$ $= 9 - 6h + h^2$ +16+8k $+k^2$

4h +10k -1 = 0 (3)

Slope of radial line passing through h(3,-4) = $\frac{-4-k}{3-h}$

Slope of line 2x-y -10 = 0 is 2

By condition of perpendicularly , we get

$$[\frac{-4-k}{3-h}](2) = -1$$

$\Rightarrow$ 8+2k = 3 – h

$\Rightarrow$ h+2k+ 5 = 0 ________(4)

Multiplying (4) by 4 and then subtracting (3) from it we get

4h +8k + 20 = 0

- 4h $\pm$10k $\mp$ 1 = 0

-2k + 21 = 0

$\Rightarrow$ k = $\frac{21}{2}$

h = $-5-2k = -5-2\times\frac{21}{2} = -26$

thus centre is c(h,k) = $(-26, \frac{21}{2})$

$$r^2 = (-26-3)^2 + (\frac{21}{2}+a)^2 = 84 + \frac{841}{4} = \frac{4205}{4}$$

Hence required equation of circle is

$$(x+26)^2 + (y-\frac{21}{2})^2 = \frac{4205}{4}$$

$$x^2 - 52x + 676 + y^2 - 21y + \frac{441}{4} - \frac{4205}{4} = 0$$

$$x^2 + y^2 - 52x - 21y + \frac{441}{4} - \frac{4205}{4} = 0$$

$$x^2 + y^2 + 52x + 676 - 941$$

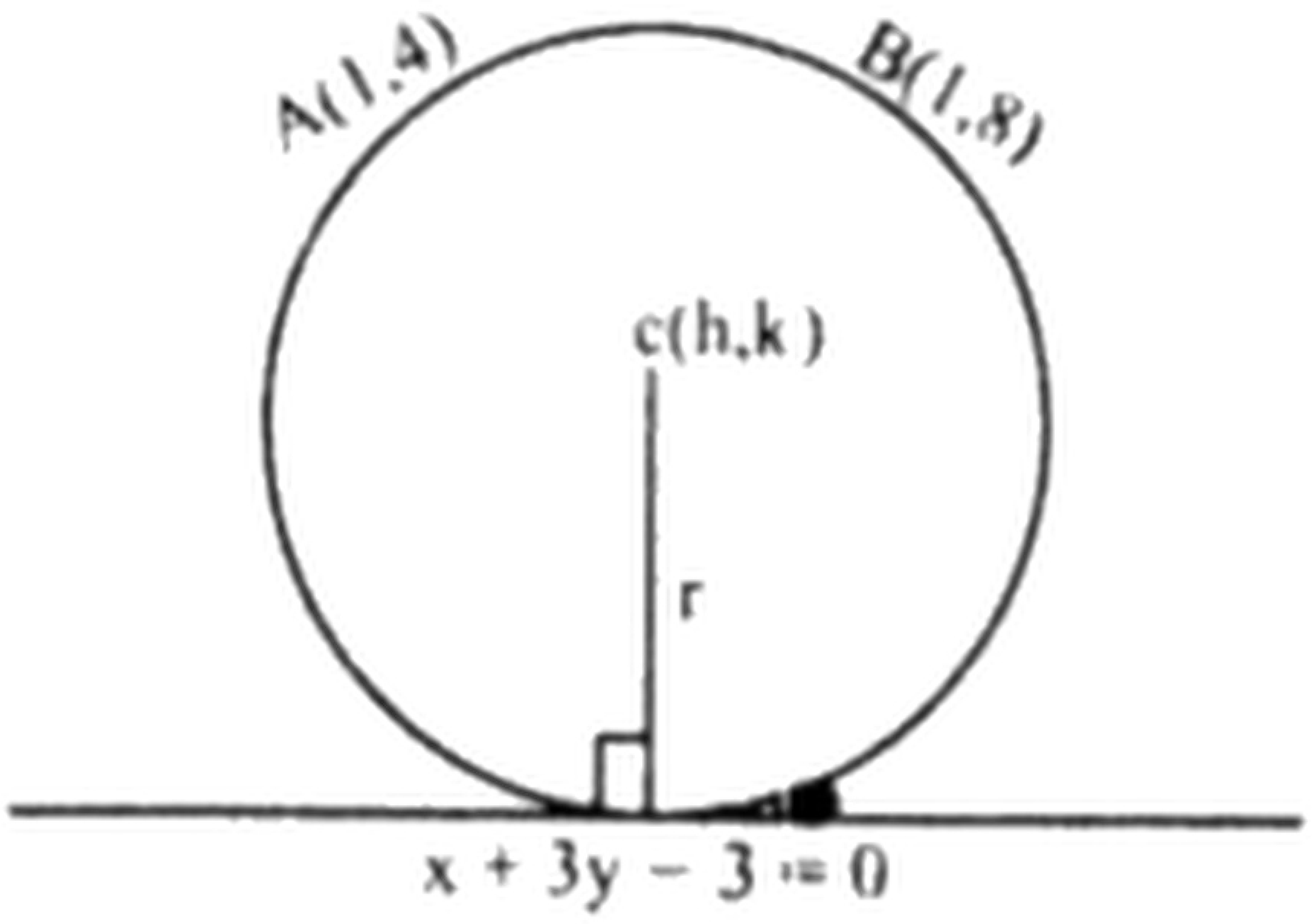
$$x^2 + y^2 + 52x - 21y - 265 = 0$$

d. A(1,4), B(-1,8) and tangent to line

x+3y -3 = 0

let equation of required circle be $(\boldsymbol{x} - \boldsymbol{h})^2 + (\boldsymbol{y} - \boldsymbol{k})^2 = \boldsymbol{r}^2$ ______(1)

since the points A and B lie on it therefore putting them in (1) one by one.



For A(1,4)

$$(1 - h)^2 + (4 - k)^2 = r^2$$

$$1 + h^2 - 2h + 16 + k^2 - 8k = r^2$$

$$h^2 + k^2 - 2h - 8k + 17 = r^2 \quad \text{\_\_\_\_\_\_\_\_\_(2)}$$

For B(-1,8)

$$(-1 - h)^2 + (4 - k)^2 = r^2$$

$$1 + h^2 - 2h + 16 + k^2 - 8k = r^2$$

$$h^2 + k^2 - 2h - 8k + 17 = r^2 \quad \_\_\_\_\_\_\_\_ (3)$$

Solving (2) and (3)

$$h^2 + k^2 - 2h - 8k + 17 = r^2$$

$$-h^2 \pm k^2 \pm 2h \pm 16k + 65 = r^2$$

$$-4h + 8k - 48 = 0$$

$$-4(h-2k+12) = 0$$

$$h-2k+12 = 0$$

radius "r" is the perpendicular distance between centre c(h,k) and the line x +3y-3 = 0

$$r = \frac{|(1)(h)+(3)(k)+(-3)|}{\sqrt{(1)^1+(3)^2}} = \frac{|h+3k-3|}{\sqrt{10}} \quad \_\_\_\_\_\_\_\_ (5)$$

$$r^2 = \frac{(h+3k-3)^2}{10} = \frac{h^2+(3k)^2+2(b)(3k)+2(3k)(-3)+2h(3)}{10}$$

$$r^2 = \frac{h^2+9k^2+9+6hk-18k-6h}{10} \quad \_\_\_\_\_\_\_\_ (6)$$

Comparing (2) and (3)

$$h^2 + k^2 - 2h - 8k + 17 = \frac{h^2+9k^2+9+6hk-18k-6h}{10}$$

$$10h^2 + 10k^2 - 20h - 80k + 170 = h^2 + 9k^2 + 6hk - 18k - 64 + 9$$

$$9h^2 + k^2 - 14h - 62k - 6hk + 161 = 0 \quad \_\_\_\_\_\_\_\_ (7)$$

From (4)

$$h = 2k - 12 \quad \_\_\_\_\_\_\_\_ (8)$$

putting (8) in (7)

$$9(2k-12)^2 + k^2 - 14(2k-12) - 62k - 6k(2k-12) + 161 = 0$$

$$9(4k^2 + 48k + 144) + k^2 - 28k + 168 - 62k - 12k^2 + 72k + 161 = 0$$

$$36k^2 - 432k + 1296 - 11k^2 - 18k + 329 = 0$$

$$25k^2 - 450k + 1625 = 0$$

$$25(k^2 - 18k + 65) = 0$$

$$k^2 - 18k + 65 = 0$$

$$K = \frac{18 \pm \sqrt{324 - 26}}{2} = \frac{18 \pm 18}{2}$$

K=13, k =5

Putting then in (8)

h = 2(13) -12 = 4 h=2(5)-12=-2

$C_1(14,13)$ *and* $C_2(-2,5)$

putting the values in (5)

$$r_1 = \frac{|14 + 3(13) - 3}{\sqrt{10}}$$

$$= \frac{50}{\sqrt{10}} = \frac{5(10)}{\sqrt{10}} = 5\sqrt{10}$$

$$r_2 = \frac{|-2 + 3(5) - 3|}{\sqrt{10}} = \frac{10}{\sqrt{10}} = \sqrt{10}$$

Putting these two pair of values in (1)

$$(3 - 14)^2 + (4 - 13)^2 = (5\sqrt{10})^2$$

$x^2 - 28x + 196$ $+y^2$ -26y +169 = 25

=0 $x^2 + y^2 -$ $28x - 26y + 15 = 0$

$$(x - 2)^2 + (y - 5)^2 = (\sqrt{10})$$

$x^2 + 4x + 4$ $+y^2$ -10y +25

$x^2 + y^2 +$ $4x - 10y + 19 = 0$

Q5. Find an equation of a circle of radius 'a' and lying in the second quadrant such that it is tangent to both the axes.

Solution

As the circle lies in second quadrant and its tangent to both the axes showing radius 'a' therefore its centre is at (-a,a)

The required equation of circle is

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x + a)^2 + (y - a)^2 = a^2$$

$$x^2 + 2ax + a^2 + y^2 - 2ay + a^2 = a^2$$

$$x^2 + y^2 + 2ax - 2ay + a^2 = 0$$

Q6. Show that the lines 3x-2y = 0 and 2x+3y -13 = 0 are tangents to circle

$$\boldsymbol{x^2 + y^2 + 6x - 4y = 0}$$

Solution

$$x^2 + y^2 + 6x - 4y = 0 \quad \_\_\_\_\_\_\_\_\_\_ (1)$$

The centre of the given circle is c(-g,-f) = (-3,2) and

$$\text{Radius} = \sqrt{(3)^2 - (-2)^2 - 0} = \sqrt{9 + 4}$$

$$\Rightarrow \quad r = \sqrt{13}$$

now we prove that ⊥ distance from centre of circle to the line 3x -2y = 0 and 2x+3y -13 = 0 are equal to radius of the circle .

perpendicular distance from (-3,2) to line 3x -2y = 0

$$= \frac{|3(-3) - 2(2) + (0)|}{\sqrt{(3)^2 - (-2)^2}}$$

$$= \frac{|1 - 9 - 4|}{\sqrt{9+4}} = \frac{|-13)|}{\sqrt{13}} = \frac{13}{\sqrt{13}}$$

$$= \frac{13}{\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}} = \frac{13\sqrt{13}}{13} = \sqrt{13} \quad \_\_\_\_\_\_\_\_\_\_ (2)$$

Also perpendicular distance from (-3,2) to line

2x+3y -13 = 0 is

$$= \frac{|2(-3)+3(2)-13|}{\sqrt{(3)^2-(-2)^2}}$$

$$= \frac{|1-6+13|}{\sqrt{9+4}} = \frac{|-13)|}{\sqrt{13}} = \frac{13}{\sqrt{13}}$$

$$= \sqrt{13} \text{ \_\_\_\_\_\_\_\_\_\_ } (3)$$

As results of equation (2) and (3) are same so the given line 3x-2y = 0 and 2x +3y -13 = 0 are tangents to the $x^2 + y^2 + 6x - 4y = 0$

Q7. Show that the circle $\mathbf{x^2 + y^2 - 3x - 2y - 7 = 0}$ and $\mathbf{x^2 + y^2 - 6x + 4y + 9 = 0}$ Touch externally.

Solution

$$x^2 + y^2 - 3x - 2y - 7 = 0 \text{ \_\_\_\_\_\_\_\_\_\_ } (1)$$

$$x^2 + y^2 - 6x + 4y + 9 = 0 \text{ \_\_\_\_\_\_\_\_\_\_ } (2)$$

The centre of circle (1) is (-1,1) and centre of circle (2) is (-3,-2)

Distance $\frac{b}{w}$ the centre = $\sqrt{(3+1)^2 + (-2-1)^2}$

$$= \sqrt{16+9} = 5$$

The radius of circle (1) = $\sqrt{(1)^2 + (-1)^2 - (-7)}$

$$= \sqrt{1+1+7} = 3$$

The radius of circle (2) = $\sqrt{(-3)^2 + (2)^2 - (9)}$

$$= \sqrt{4} = 2$$

The sum of radius = 2+3 = 5

As the sum of radius of the circles is equal to the distance $\frac{b}{w}$ their centres.

Therefore these

Circles touch externally.

Q8. Show that the circle $x^2 + y^2 + 2x - 8 = 0$ and $x^2 + y^2 - 6x + 6y - 46 = 0$ Touch internally.

Solution

$$x^2 + y^2 - 2x + 8 = 0 \text{ \_\_\_\_\_\_\_\_\_\_ (1)}$$

$$x^2 + y^2 - 6x + 6y - 46 = 0 \text{ \_\_\_\_\_\_\_\_\_\_ (2)}$$

The centre of circle (1) is (-1,0) and centre of circle (2) is (3,-3)

$$\text{Radius} = \sqrt{(1)^2 + (0)^2 - (-8)}$$

$$= \sqrt{1 + 1 + 7} = 3$$

The radius of circle (2) is (3,-3) and

$$\text{radius} = \sqrt{(-3)^2 + (3)^2 - (-46)}$$

$$= \sqrt{9 + 9 + 46} = 8 \Rightarrow 5+3$$

The radius between the centres of the given circles $= \sqrt{(3 + 1)^2 + (3 - 0)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$

The distance of radius is equal to the distance $\frac{b}{w}$ their centres. Therefore these

Circles touch internally.

Q9. Find equations of the circles of radius 2 and tangent to the line x -y – 4 = 0 at A(1,-3)

Solution

Let c(h,k) be the centre of the radius 2 and tangent to the line x -y -4 =0 at (1,-3)then

$$(h-1)^2 + (k+3)^2 = (2)^2$$

$$h^2 - 2h + 1 + k^2 + 6k + 9 = 4$$

$$h^2 + k^2 - 2h + 6k + 6 = 0 \_\_\_\_\_\_\_\_\_\_(1)$$

Slope of the line x -y -4 =0 is 1 and slope of the line through (1,-3) and (h,k) = $\frac{k+3}{h-1}$

$\Rightarrow \quad k + 3 = -h + 1 \qquad \Rightarrow k = -h - 2$

Putting k = -h -2 in (1) , we get

$$h^2 + (h^2 + 4h + 4) - 2h - 64 - 12 + 6 = 0$$

$$2h^2 - 4h - 2 = 0 \qquad \Rightarrow h^2 - 2h - 1 = 0$$

$\Rightarrow \qquad h = \frac{2\pm\sqrt{4+4}}{2} = \frac{2\pm2\sqrt{2}}{2}$

$=1 \pm\sqrt{2}$

If h=1+$\sqrt{2}$, then k = -1-$\sqrt{2} - 2 = -3 - \sqrt{2}$

If h=1-$\sqrt{2}$, then k =-1+$\sqrt{2} - 2 = -3 + \sqrt{2}$

The required equation of circle showing centre at (1+$\sqrt{2}, -3 - \sqrt{2}$) is

$(x - 1 + \sqrt{2})^2 + (y + 3 - \sqrt{2})^2 = 4$

$\Rightarrow \quad x^2 + y^2 + 2(\sqrt{2} - 1)x + 2(3 - \sqrt{2}) + 10 - 8\sqrt{2} = 0$

$(x - 1 + \sqrt{2})^2 + (y + 3 - \sqrt{2})^2 = 4$

$x^2 + 1 + 2 - 2x + 2\sqrt{2} - 2\sqrt{2}$

x +y2 +9 +2

+6y+6y+6$\sqrt{2}$

+2$\sqrt{2}$ = 4

$\Rightarrow \quad x^2 + y^2 -$

$2(\sqrt{2} - 1)x +$

$2(3 + \sqrt{2})y + 10 + 8\sqrt{2} = 0$