Exercise 5.3

Q1 Maximize $f(x, y) = 2x + 5y$

Subject to the constraints:

$2y - x \leq 8; x - y \leq 4; x \geq 0; y \geq 0$

Solution

$2x - x \geq 8$ ______(1)

$2x - x = 8$ ______(2)

Putting $x = 0$ in (3)

$2y - 0 = 8 \Rightarrow y = 4$

(0,4) is a point on (3)

Putting $y = 0$ in (3)

$0 - x = 8 \Rightarrow x = -8$

(-8,0) is another point on (3)

Putting $x = 0, y = 0$ in (1)

$0 - 0 < 8$

$0 < 8$

Which is true

The graph is

$x - y \leq 4$______(2)

$x - y = 4$ ______(4)

putting $x = 0$ in (4)

$0 - y = 4 \Rightarrow y = -4$

(0,4) is a point on (4)

putting $y = 0$ in (4)

$x - 0 = 4 \Rightarrow x = 4$

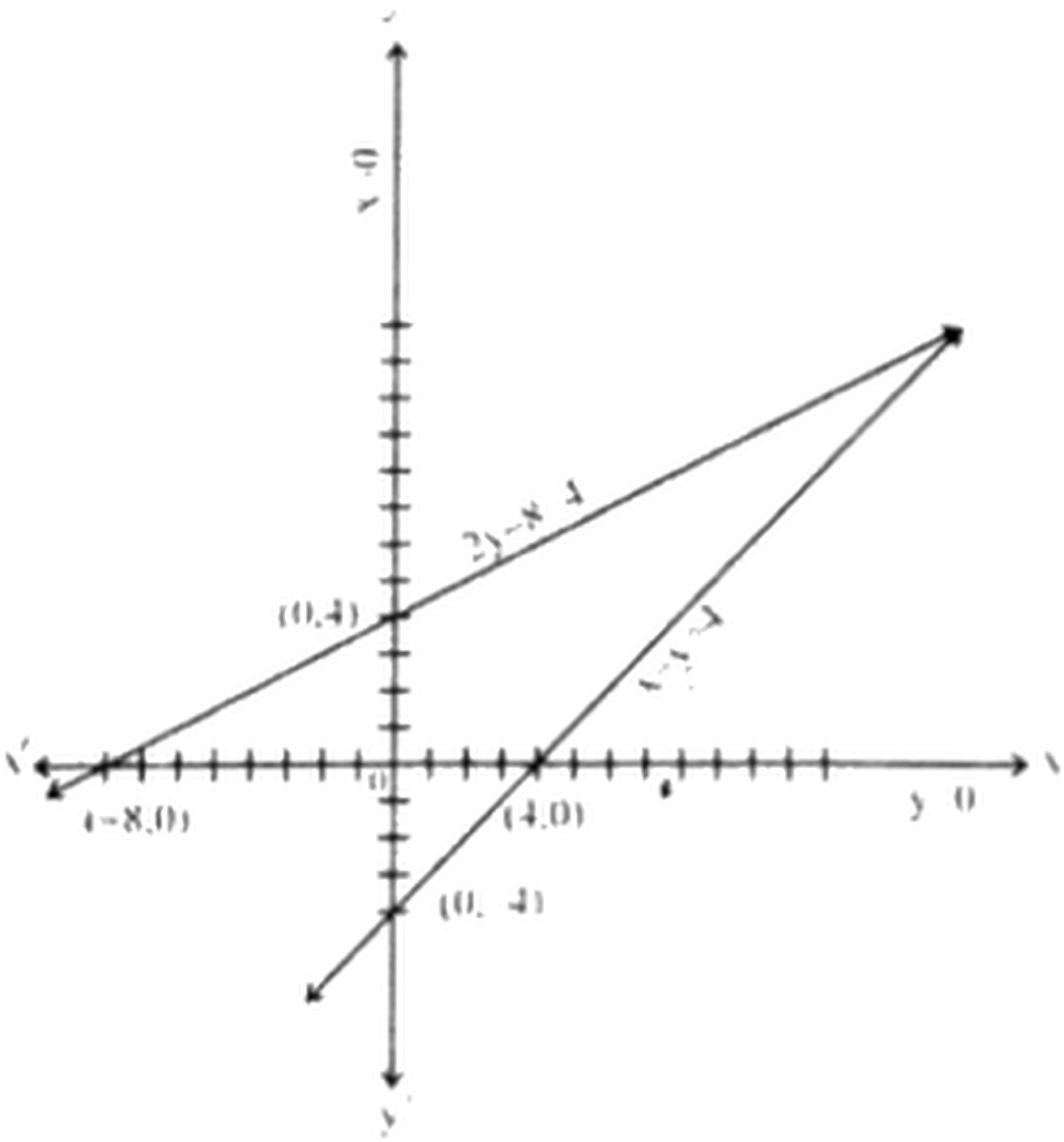
(0,-4) is a point on (4)

putting $x = 0, y = 0$ in (2)

$0 - 0 < 4$

$0 < 4$

which is true



The intersection of four graphs in the first quadrant, i.e the shaded region is the required feasible region

From (4)

x = 4 +12 = 16

putting (5) in (3)

2y – (4 +y) = 8

2y -4 – y = 8

Y = 12

Putting y =12 in (5)

x = 4 +12 = 16

hence (16,12),(0,4),(0,0) and (4,0) are the corner points. Check

Corner points	f(x ,y) = 2x +5y
(16,12)	f(16,12)=2(16)+(12)=32+60 = 92

(0,4)	f(0,4) =2(0) +5(4) = 0 +20=20
(0,0)	f(0,0)=2(0)+5(0)=0 +0 = 0
(4,0)	F(4,0)=2(4)+5(0) = 8+0= 8

The maximum value of the function is 92. Hence the corner point (16,12)

Q2 Maximize f (x ,y) = x +y

Subject to the constraints:

$2y + 5x \leq 30$; $5x + 4y \leq 20$; $x \geq 0$; $y \geq 0$

Solution

$2x - 5y \geq 30$ _______(1)

$2x - 5y = 30$ ________(2)

Putting x = 0 in (3)

$0 + 5y = 30 \Rightarrow y = 6$

(0,6) is a point on (3)

Putting y = 0 in (3)

$2x + 0 = 30 \Rightarrow x = 15$

(15,0) is another point on (3)

Putting x =0, y = 0 in (1)

$0 - 0 < 30$

$0 < 30$

Which is true

$5x - 4y \leq 20$________(2)

$5x - 4y = 20$ _________(4)

putting x =0 in (4)

$0 - 4y = 20 \Rightarrow y = 5$

(0,5) is a point on (4)

putting y =0 in (4)

$5x - 0 = 20 \Rightarrow x = 4$

(4,0) is a point on (4)

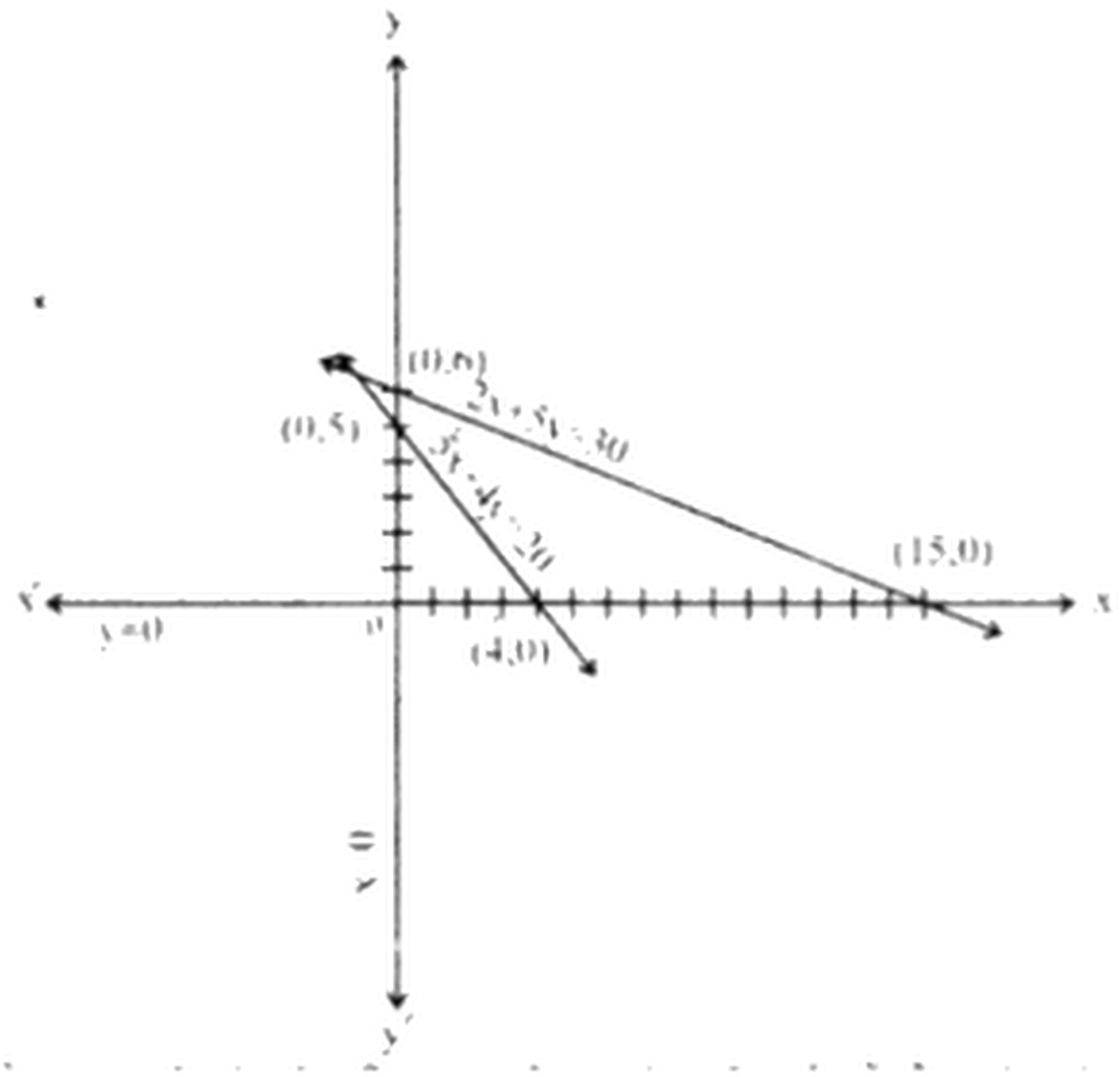
putting x =0, y =0 in (2)

$0 - 0 < 20$

$0 < 20$

which is true

The graph is



The intersection of four graphs in the first quadrant, i.e the shaded region is the required feasible region

hence (4,0),(0,0) and (0,5)are the corner points. Check

Corner points	f(x ,y) = x +3y
(4,0)	f(4,0)=(4)+3(0)=4
(0,5)	f(0,5) =(0) +3(5) = 15
(0,0)	f(0,0)=(0)+3(0)=0 +0 = 0

The maximum value of the function is 15. Hence the corner point (0,5)

Q3 Maximize z = 2x +3y

Subject to the constraints:

$3x + 4y \leq 12$; $2x + y \leq 4$; $x \geq 0$; $y \geq 0$

Solution

$3x + 4y \leq 12$ _______(1)

$2x - 4x = 12$ ________(2)

Putting $x = 0$ in (3)

$0 + 4y = 12 \Rightarrow y = 3$

(0,3) is a point on (3)

Putting $y = 0$ in (3)

$3x + 0 = 12 \Rightarrow x = 4$

(4,0) is another point on (3)

Putting $x = 0$, $y = 0$ in (1)

$0 + 0 < 12$

$0 < 12$

Which is true

The graph is

$2x + y \leq 4$________(2)

$2x + y = 4$ _________(4)

putting $x = 0$ in (4)

$0 + y = 4 \Rightarrow y = 4$

(0,4) is a point on (4)

putting $y = 0$ in (4)

$2x + 0 = 4 \Rightarrow x = 2$

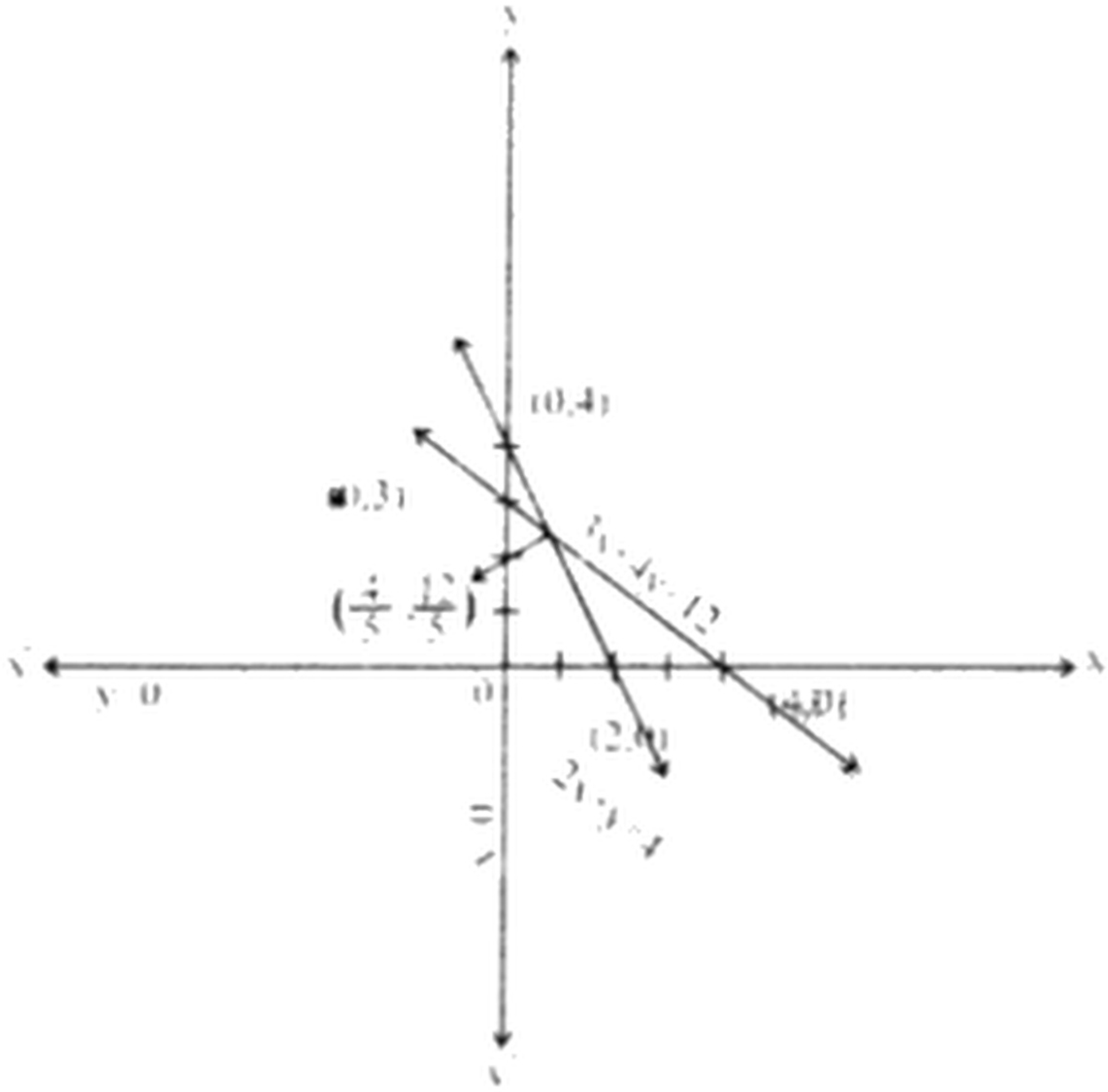
(2,0) is a point on (4)

putting $x = 0$, $y = 0$ in (2)

$0 - 0 < 4$

$0 < 4$

which is true



The intersection of four graphs in the first quadrant, i.e the shaded region is the required feasible region

From (4)

y = 4 -12x ____(5)

putting (5) in (3)

3 +4(4 -2x) = 12

3x +16 -8x= 12

-4x = -5

x = $\frac{4}{5}$

Putting x =$\frac{4}{5}$ in (5)

y = 4 -2($\frac{4}{5}$) = 4 -$\frac{8}{5}$ =$\frac{20-8}{5}$ = $\frac{12}{5}$

hence ($\frac{4}{5}$, $\frac{12}{5}$),(0,3),(0,0) and (2,0) are the corner points. Check

Corner points	f(x ,y) = 2x +3y

$(\frac{4}{5}, \frac{12}{5})$	$z=2(\frac{4}{5})+3(\frac{12}{5})= \frac{8}{5} + \frac{36}{5} = \frac{44}{5}=8.8$
(4,0)	z =2(0) +3(0) = 0 +9=9
(0,5)	z=2(0)+3(0)=0 +0 = 0
(2,0)	z=2(2)+3(0) = 4+0= 4

The maximum value of the function is 9. Hence the corner point (0,3)

Q4 Maximize z = 2x +y

Subject to the constraints:

$x + y \leq 3$; $7x + 5y \leq 35$; $x \geq 0$; $y \geq 0$

Solution

$x + y \leq 3$ ______(1)

$x + y = 3$_______(2)

Putting x = 0 in (3)

$0+y = 3 \Rightarrow y = 3$

(0,3) is a point on (3)

Putting y = 0 in (3)

$x+0 = 3 \Rightarrow x = 3$

(3,0) is another point on (3)

Putting x =0, y = 0 in (1)

$0 + 0 < 3$

$0 < 83$

Which is true

$7x + 5y \leq 35$_______(2)

$7x+5y = 35$ ________(4)

putting x =0 in (4)

$0+5y = 35 \Rightarrow y =7$

(0,7) is a point on (4)

putting y =0 in (4)

$7x +0 = 35 \Rightarrow x =5$

(5,0) is a point on (4)

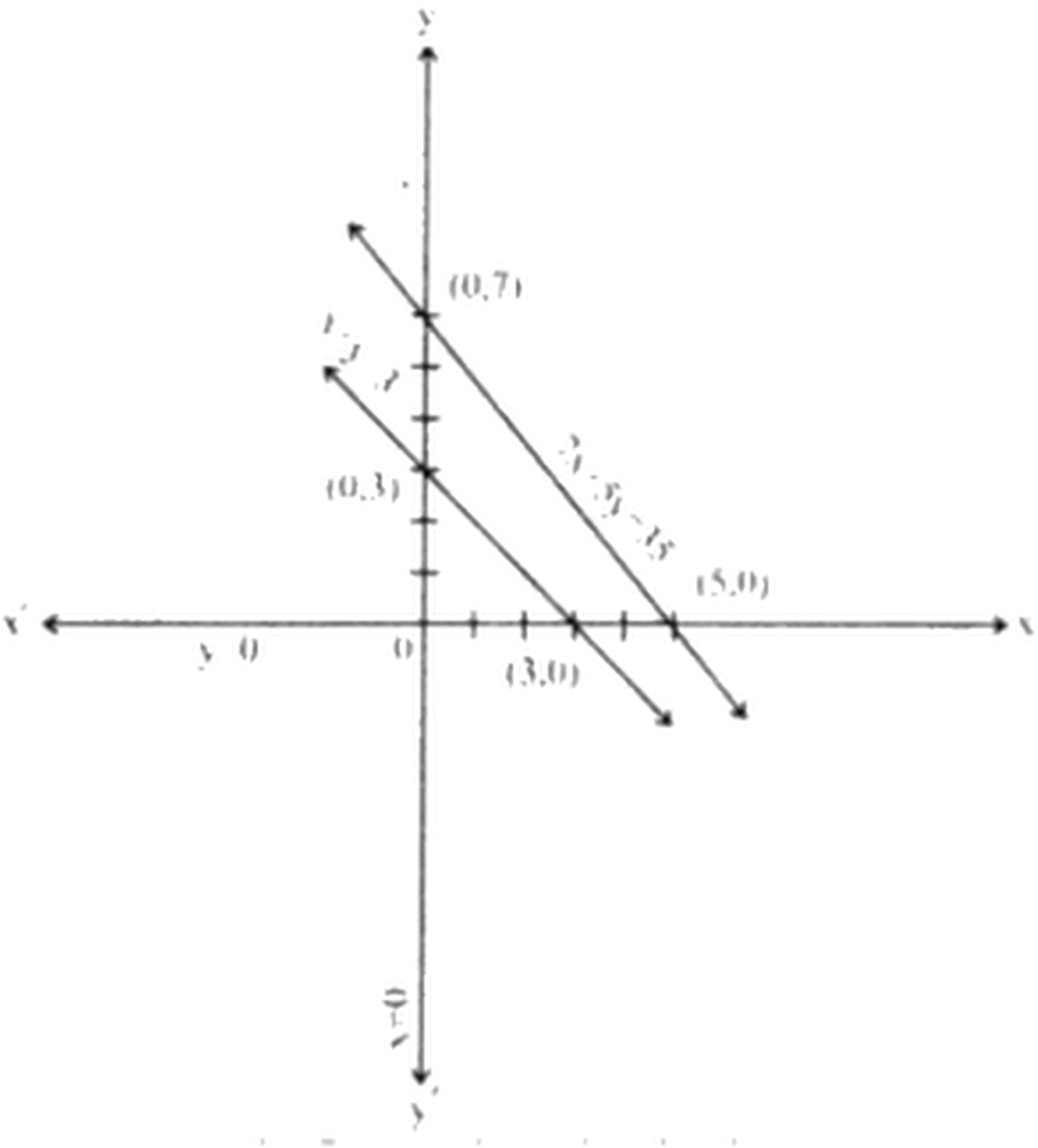
putting x =0, y =0 in (2)

$0 + 0 < 35$

$0 < 35$

which is true

The graph is



The intersection of four graphs in the first quadrant, i.e the shaded region is the required feasible region

hence (3,0),(5,0),(0,7) and (0,3) are the corner points. Check

Corner points	f(x ,y) = 2x +y
(3,0)	z=2(3)+0=6
(5,0)	z =2(5) +0 = 10
(0,7)	z=2(0)+7= 7
(0, 3)	z=2(0)+3 = 3

The minimum value of the function is 3. Hence the corner point (0,3)

Q5 Maximize the function is defined $f(x,y) = 2x + 3y$

Subject to the constraints:

$2x + y \leq 8$; $x + 2y \leq 14$; $x \geq 0$; $y \geq 0$

Solution

$2x + y \leq 8$ ______(1)

$2x + y = 8$ _______(2)

Putting x = 0 in (3)

$0 + y = 8 \Rightarrow y = 8$

(0,8) is a point on (3)

Putting y = 0 in (3)

$2x + 0 = 8 \Rightarrow x = 4$

(4,0) is another point on (3)

Putting x =0, y = 0 in (1)

$0 + 0 < 8$

$0 < 8$

Which is true

$x + 2y \leq 14$_______(2)

$x + 2y = 14$ ________(4)

putting x =0 in (4)

$0 + 2y = 14 \Rightarrow y = 7$

(0,7) is a point on (4)

putting y =0 in (4)

$x + 0 = 14 \Rightarrow x = 14$

(14,0) is a point on (4)

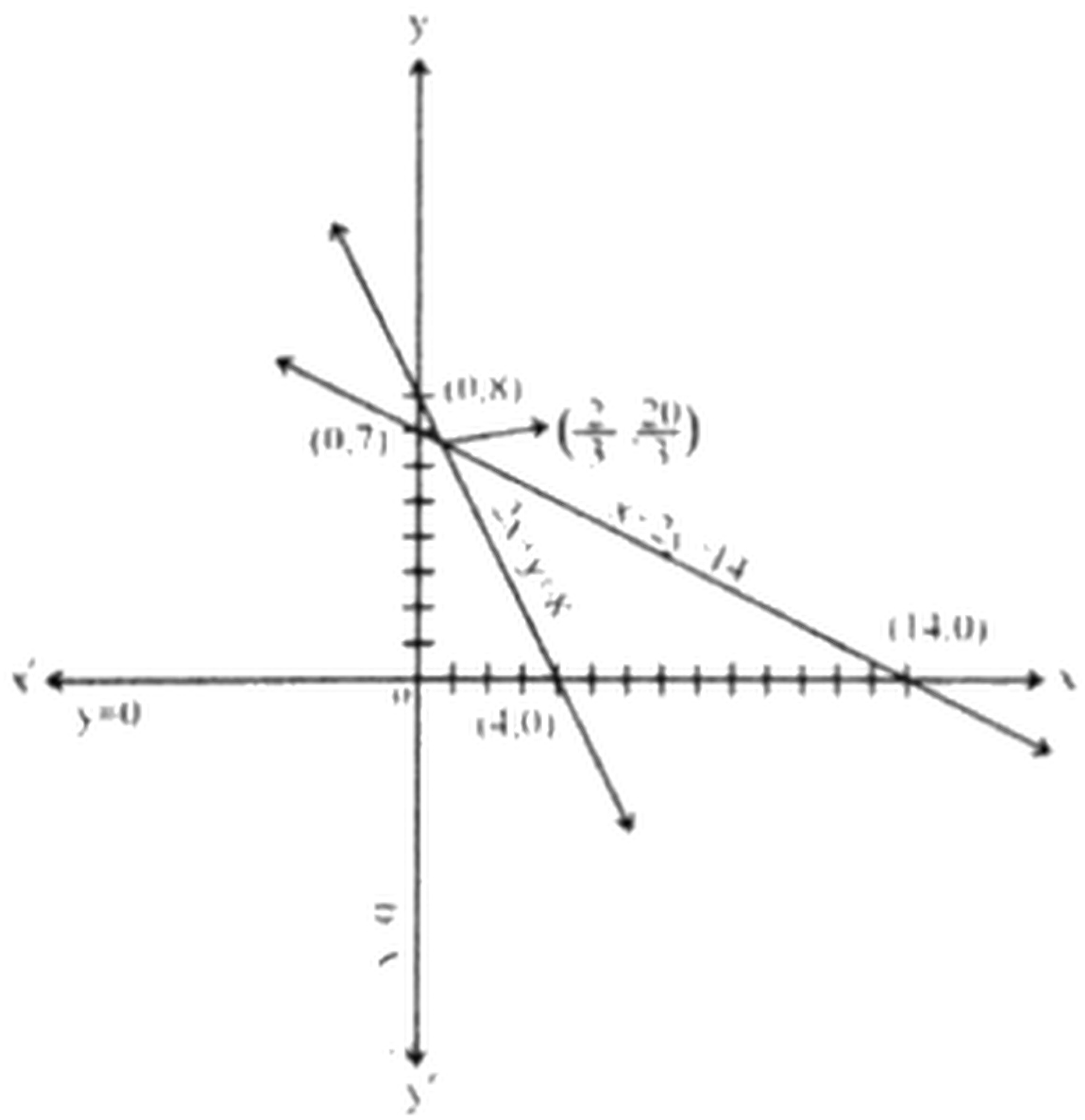
putting x =0, y =0 in (2)

$0 + 0 < 14$

$0 < 14$

which is true

The graph is



The intersection of four graphs in the first quadrant, i.e the shaded region is the required feasible region

From (4)

X = 14 -2y ____(5)

putting (5) in (3)

2(14-2y)+y= 8

28 – 4y + y= 8

-3y = -20

$y = \frac{20}{3}$

Putting $y = \frac{20}{3}$ in (5)

$x = 14 - 2y(\frac{20}{3}) = 14 - \frac{40}{3} = \frac{42-40}{3} = \frac{2}{3}$

hence $(\frac{2}{3}, \frac{20}{3})$, (0,7), (0,0) and (4,0) are the corner points. Check

Corner points	f(x ,y) = 2x +3y
$(\frac{2}{3}, \frac{20}{3})$	$f(\frac{2}{3}, \frac{20}{3})=2(\frac{2}{3})+3(\frac{20}{3})=\frac{4}{3}+20=\frac{64}{3}=2.13$
(0,7)	f(0,7) =2(0) +3(7) =21
(0,0)	f(0,0)=2(0)+3(0)= 0
(4,0)	f(4,0)=2(4)+3(0) = 8

The maximum value of the function is $\frac{64}{3}$. Hence the corner point $(\frac{2}{3}, \frac{20}{3})$

Q6 Maximize z = 3x +y

Subject to the constraints:

3x +5y ≥ 15; x +3y ≤ 9 ; x ≥ 0; y ≥ 0

Solution

3x +5y ≥ 15 _______(1)

3x +5y = 15________(2)

Putting x = 0 in (3)

0+5y = 15 ⇒ y = 3

(0,3) is a point on (3)

Putting y = 0 in (3)

3x+0 = 15 ⇒ x = 5

(5,0) is another point on (3)

Putting x =0, y = 0 in (1)

0 + 0> 15

x +3y ≤ 9________(2)

x+3y = 9 _________(4)

putting x =0 in (4)

0+y = 9 ⇒ y =3

(0,3) is a point on (4)

putting y =0 in (4)

x +0 = 9 ⇒ x =9

(9,0) is a point on (4)

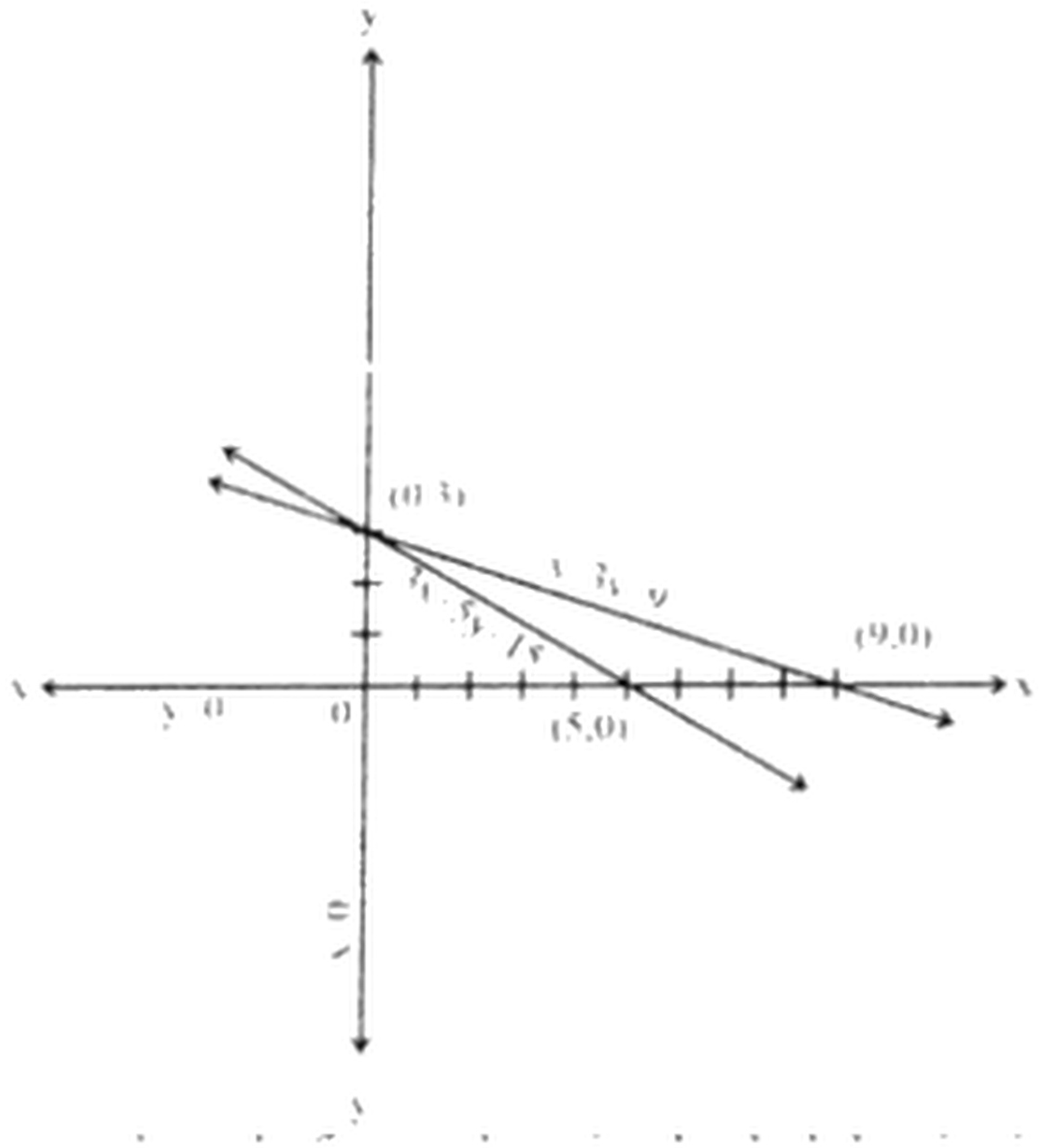
putting x =0, y =0 in (2)

0 + 0< 9

0 > 8 0 < 9

Which is true which is true

The graph is



The intersection of four graphs in the first quadrant ,i.e the shaded region is the required feasible region

hence (9,0),(5,0)and (0,3) are the corner points. Check

Corner points	Z= 3x +y
(5,0)	Z=3(5)+0=15
(9,0)	Z =3 (9) +0 = 27
(0,3)	Z=3(0)+3= 3

The minimum value of the function is 3. Hence the corner point (0,3)

Q7 Each unit of food X costs Rs. 25 and contains 2 units of protein and 4 units of iron. While each unit of food Y costs Rs. 30 and contains 3 units of protein and 2 units of iron. Each animal must receive at least 12 units of protein and 16 units of iron each dat. How many units of each food should be fed to each animal at the smallest possible cost?

Solution

Let x units of food X and y units of food y be given to each animal. The cost function will be:

$$f(x, y) = 25 \times 30y$$

Now

$$\mathbf{3x + 3y \geq 12;\ 4x + y \geq 16;\ x \geq 0;\ y \geq 0}$$

$3x + 3y \geq 12$ ______(1)

$3x + 3y = 12$ ______(2)

Putting x = 0 in (3)

$0 + 3y = 12 \Rightarrow y = 4$

(0,4) is a point on (3)

Putting y = 0 in (3)

$2x + 0 = 12 \Rightarrow x = 6$

(6,0) is another point on (3)

Putting x =0, y = 0 in (1)

$0 + 0 > 12$

$0 > 12$

Which is true

$4x + 2y \geq 16$ ______(2)

$4x + 2y = 16$ ______(4)

putting x =0 in (4)

$0 + 2y = 16 \Rightarrow y = 8$

(0,8) is a point on (4)

putting y =0 in (4)

$4x + 0 = 16 \Rightarrow x = 4$

(4,0) is a point on (4)

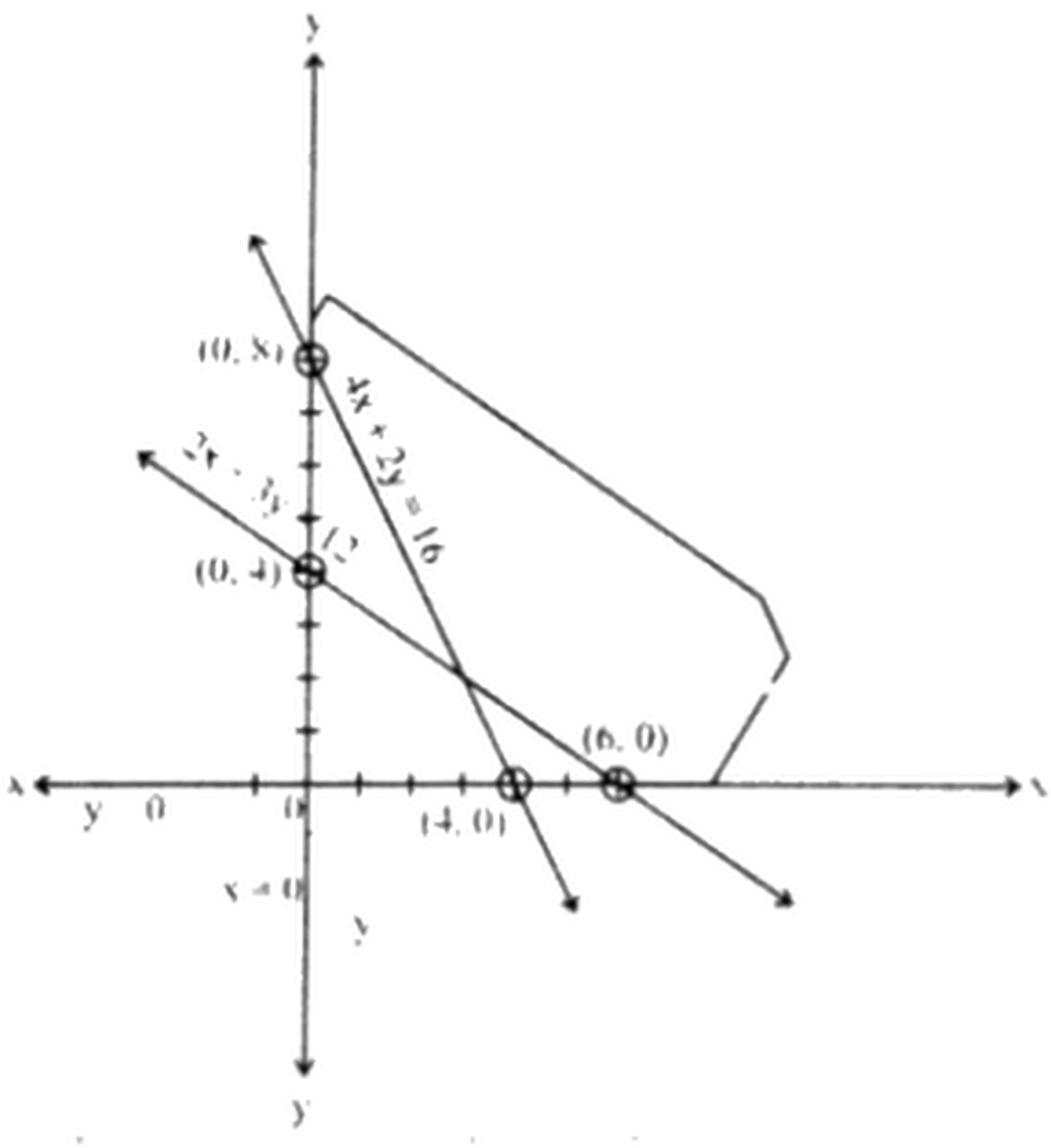
putting x =0, y =0 in (2)

$0 + 0 > 16$

$0 > 16$

which is true

The graph is



The intersection of four graphs in the first quadrant, i.e the shaded region is the required feasible region

From (3)

2x = 12 – 3y

$\frac{12-3y}{2}$ ________(5)

putting (5) in (4)

$4(\frac{12-3y}{2})$ +2y = 16

24 -6y +2y = 16

-4y = -8

y = 2

putting y = 2 in (5)

$x = \frac{12-3(2)}{2} = \frac{12-6}{2} = \frac{6}{2} = 3$

hence (3,2),(6,0)and (0,8) are the corner points. Check

Corner points	f(x ,y)= 25x +30y
(3,2)	f(3,2)=25(3)+30(2)=135
(6,0)	f(6,0) =25 (6) +30(0) = 150
(0,8)	f(0,8)=25(0)+30(8)= 240

The minimum value of the cost function is 135. Hence the corner point (3,2)

Q8. Dealer wishes to purchase a number of fans and sewing machines. He has only Rs. 5760 to invest and has space at most for 20 items. A fan cost him Rs. 360 and sewing machine costs Rs. 240. His expected is that he can sell a fan at a profit of Rs. 22 and sewing machine at a profit of Rs. 18. Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximize has profit?

Solution

Let x be the number of fans purchased and y be the number of sewing machines purchased :

$$f(x,y) = 22x + 18y$$

Now

$360x + 240y \le 5760$, $x + y \le 20$,; $x \ge 0$; $y \ge 0$

Or $120(3x + 2y) \le 5760$

or $\mathbf{3x + 2y \le 48}$; $\mathbf{x + y \le 20}$; $x \ge 0$; $y \ge 0$

$3x + 2y \le 48$ _______(1)

$3x + 2y = 48$ ________(2)

Putting x = 0 in (3)

$x + y \le 20$ ________(2)

$x + y = 20$ _________(4)

putting x =0 in (4)

$0+2y = 48 \Rightarrow y = 224$

(0,4) is a point on (3)

Putting y = 0 in (3)

$3x+0 = 48 \Rightarrow x = 16$

(16,0) is another point on (3)

Putting x =0, y = 0 in (1)

$0 + 0 < 48$

$0 < 48$

Which is true

$0+y = 20 \Rightarrow y =20$

(0,20) is a point on (4)

putting y =0 in (4)

$x +0 = 20 \Rightarrow x =20$

(20,0) is a point on (4)

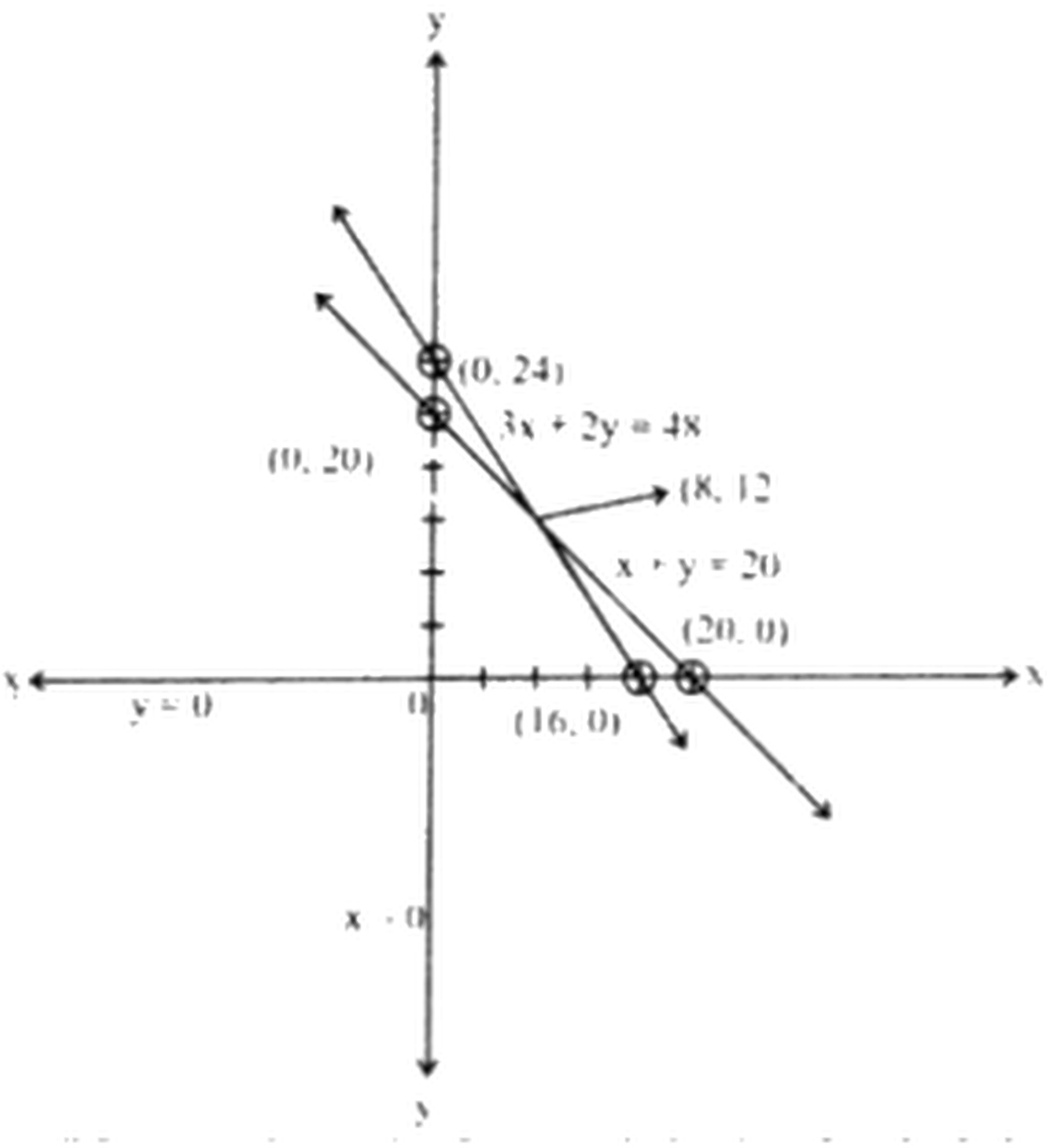
putting x =0, y =0 in (2)

$0 + 0 <20$

$0 < 20$

which is true

The graph is



The intersection of four graphs in the first quadrant, i.e the shaded region is the required feasible region

From (4)

x = 20 – 3____(5)

putting (5) in ()

3(20 -y)+2y = 48

6 -6y3 +2y = 48

-y = -12 ⇒ y =12

y = 2

putting y = 2 in (5)

x = 20-12 =8

hence (8,12,(0,20),(16,8)and (0,0) are the corner points. Check

Corner points	f(x ,y)= 22x +18y
(8,12)	f(8,12)=22(8)+18(12)= 176 +216=135
(0,20)	f(0,20) =22 (0) +18(20) = 360
(0,0)	f(0,)=22(0)+18(0)= 0
(16,0)	f(16,0) = 22(16) +18(0) = 352

The maximum value of profit function is 392. Hence the corner point (8,12)

Q9. A machine can produce product A by using 2 units of chemical and 1 unit of a compound or can produce product B by using 1 unit of chemical and 2 units of compound. Only 800 units of chemical and 1000 units of compound are available. The profits per unit of product A and B are Rs. 30 and Rs. 20 respectively. Maximize the profit function.

Solution

Let the machine produces x units of product A and y units of product B. The profit function is:

$$f(x, y) = 30x + 20y$$

Subject to the constraints:

$2xy \leq 800$; $x + 2y \leq 100$; $x \geq 0$; $y \geq 0$

$2x + y \leq 800$ ______(1)

$2x + y = 800$ ______(2)

Putting x = 0 in (3)

$0 + y = 800 \Rightarrow y = 800$

(0,800) is a point on (3)

Putting y = 0 in (3)

$2x + 0 = 800 \Rightarrow x = 400$

(400,0) is another point on (3)

Putting x = 0, y = 0 in (1)

$0 + 0 < 800$

$0 < 800$

Which is true

The graph is

$x + 2y \leq 1000$ ______(2)

$x + 2y = 1000$ ______(4)

putting x = 0 in (4)

$0 + 2y = 1000 \Rightarrow y = 500$

(0,500) is a point on (4)

putting y = 0 in (4)

$x + 0 = 1000 \Rightarrow x = 1000$

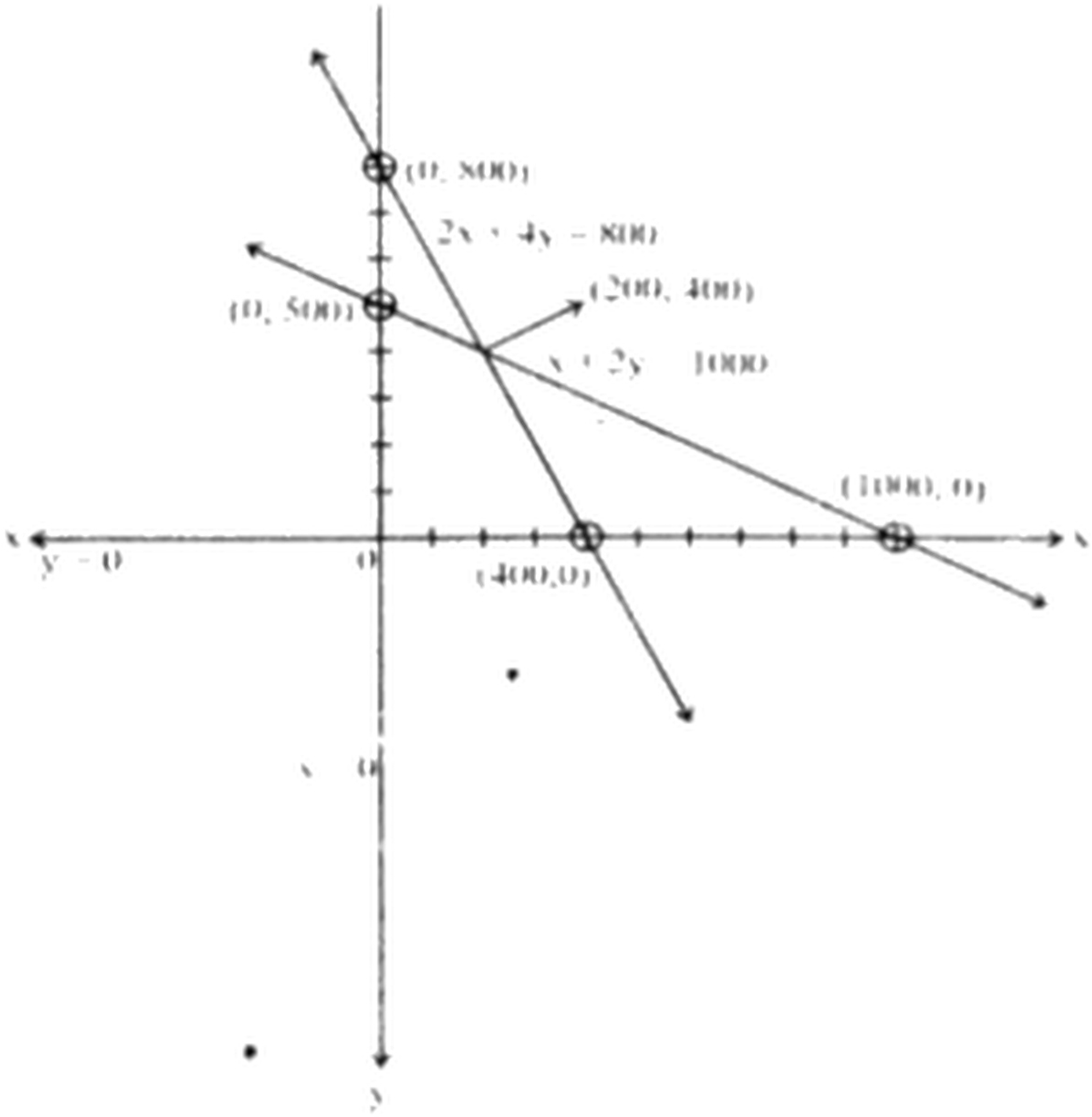
(1000,0) is a point on (4)

putting x = 0, y = 0 in (2)

$0 + 0 < 1000$

$0 < 1000$

which is true



The intersection of four graphs in the first quadrant, i.e the shaded region is the required feasible region

From (3)

y = 800-2x___(5)

putting (5) in (4)

x +1600 -4x = 1000

x +1600 -4x = 1000

-3x = -600

x = 200

putting x = 200 in (5)

y =800 – 2(200)=800 -400 = 400

hence (200,400),(0,500),(400,0)and (0,0) are the corner points. Check

Corner points	f(x ,y)= 22x +18y

(200,400)	f(200,400)=30(200)+20(400)= 6000+8000=14000
(0,500)	f(0,500) =30(0) +20(500) = 100
(0,0)	f(0,0)=30(0)+20(0)= 0
(16,0)	f(400,0) = 30(400) +20(0) = 12000

The maximum value of profit function is 1400. Hence the corner point (200,400)