

Exercise 5.2

Q1 Graph the feasible region of the following systems of linear inequalities and find the corner points in each case.

Solution

(i) $2x - 3y \leq 6$, $2x + 3y \leq 12$, $x \geq 0$, $y \geq 0$

$$2x - 3y \leq 6 \text{ _____(1)}$$

$$2x - 3y = 6 \text{ _____(3)}$$

Putting $x = 0$ in (3)

$$0 - 3y = 6 \Rightarrow y = -2$$

(0,-2) is a point on (3)

Putting $y = 0$ in (3)

$$2x - 0 = 6 \Rightarrow x = 3$$

(3,0) is another point on (3)

Putting $x = 0$, $y = 0$ in (1)

$$0 - 0 < 6$$

$$0 < 6$$

Which is true

The graph is

$$2x + 3y \leq 12 \text{ _____(2)}$$

$$2x + 3y = 12 \text{ _____(4)}$$

putting $x = 0$ in (4)

$$0 + 3y = 12 \Rightarrow y = 4$$

(0,4) is a point on (4)

putting $y = 0$ in (4)

$$2x + 0 = 12 \Rightarrow x = 6$$

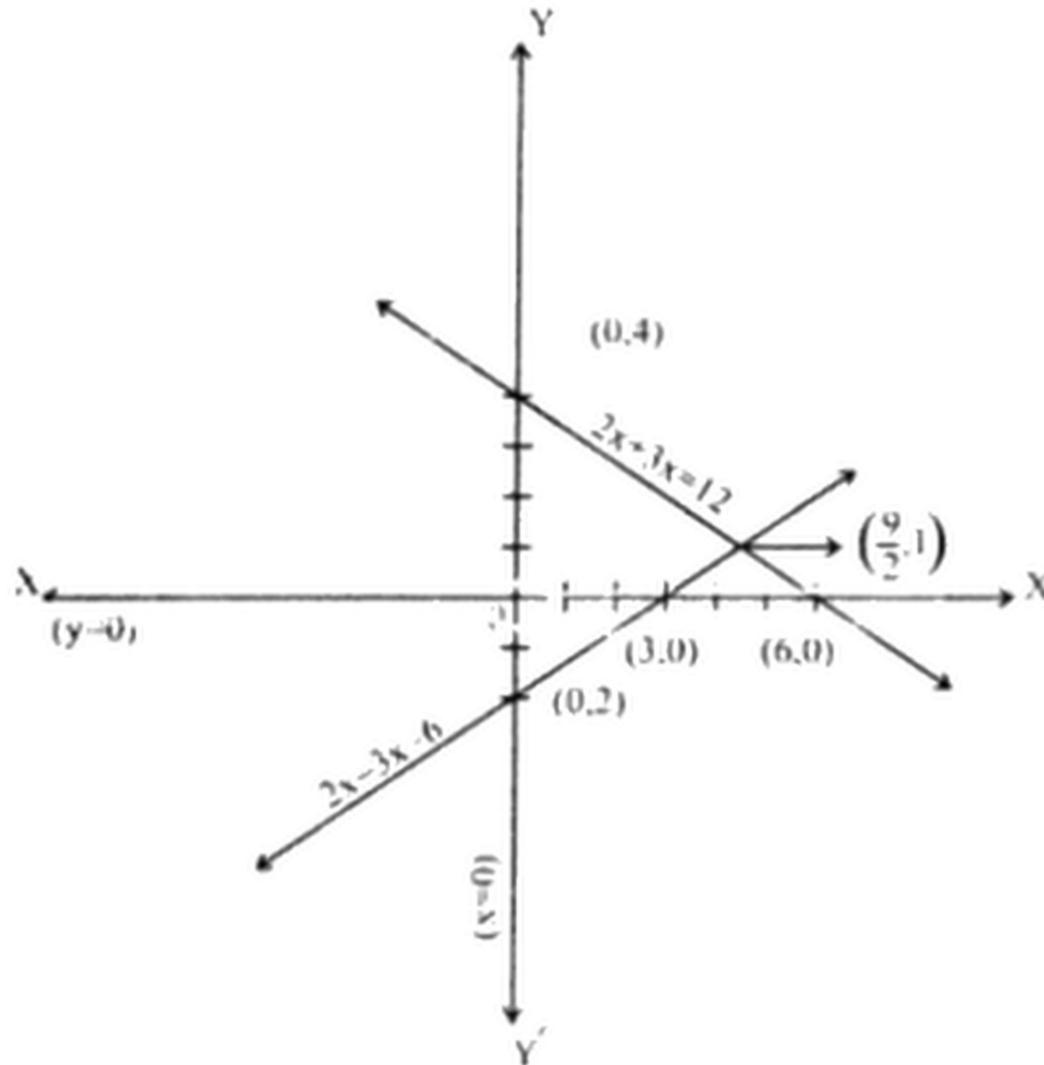
(6,0) is another point on (4)

putting $x = 0$, $y = 0$ in (2)

$$0 + 0 < 12$$

$$0 < 12$$

which is true



The intersection of four graphs in the first quadrant, i.e the shaded region is the required feasible region

Solving (3) and (4)

$$2x - 3y = 6$$

$$\begin{array}{r} 2x + 3y = 12 \\ 4x = 18 \end{array}$$

putting $x = \frac{9}{2}$ in (3)

$$2\left(\frac{9}{2}\right) - 3y = 6$$

$$-3y = -3$$

$$y = 1$$

hence $(\frac{9}{2}, \dots)$, $(4,0)$ $(0,0)$ and $(3,0)$ are the corner points

(ii) $x + y \leq 5$, $-2x + y \leq 2$, $x \geq 0$, $y \geq 0$

$$x + y \leq 5 \quad (1)$$

$$-2x + y \leq 2 \quad (2)$$

$$x + y = 5 \quad (3)$$

Putting $x = 0$ in (3)

$$0 + y = 5 \Rightarrow y = 5$$

$(0,5)$ is a point on (3)

Putting $y = 0$ in (3)

$$x + 0 = 5 \Rightarrow x = 5$$

$(5,0)$ is another point on (3)

Putting $x = 0, y = 0$ in (1)

$$0 + 0 < 5$$

$$0 < 5$$

Which is true

The graph is

$$-2x + y = 2 \quad (4)$$

putting $x = 0$ in (4)

$$-0 + y = 2 \Rightarrow y = 2$$

$(0,2)$ is a point on (4)

putting $y = 0$ in (4)

$$-2x + 0 = 2 \Rightarrow x = -1$$

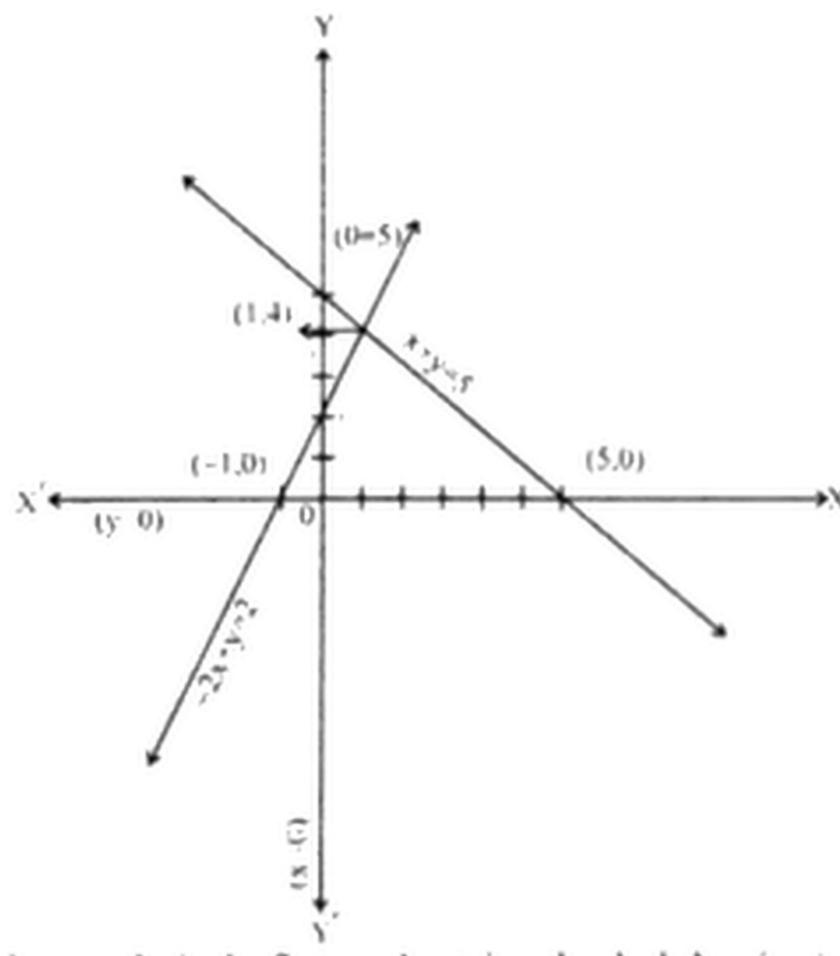
$(-1,0)$ is another point on (4)

putting $x = 0, y = 0$ in (2)

$$-0 + 0 < 2$$

$$0 < 2$$

which is true



The intersection of four graphs in the first quadrant, i.e. the shaded region is the required feasible region

Solving (3) and (4)

$$x + y = 5$$

$$\frac{-2x + y = -2}{3x = 3}$$

putting $x = 1$ in (3)

$$1 + y = 5$$

$$y = 4$$

hence (1,4), (0,2) (0,0) and (5,0) are the corner points

(iii) $x + y \leq 5$, $-2x + y \geq 2$, $x \geq 0$

$$x + y \leq 5 \text{ _____ (1)}$$

$$x + y = 5 \text{ _____ (3)}$$

Putting $x = 0$ in (3)

$$0 + y = 5 \Rightarrow y = 5$$

(0,5) is a point on (3)

Putting $y = 0$ in (3)

$$x + 0 = 5 \Rightarrow x = 5$$

(5,0) is another point on (3)

Putting $x = 0$, $y = 0$ in (1)

$$0 + 0 < 5$$

$$0 < 5$$

Which is true

The graph is

$$-2x + y \geq 2 \text{ _____ (2)}$$

$$-2x + y = 2 \text{ _____ (4)}$$

putting $x = 0$ in (4)

$$-0 + y = 2 \Rightarrow y = 2$$

(0,2) is a point on (4)

putting $y = 0$ in (4)

$$-2x + 0 = 2 \Rightarrow x = -1$$

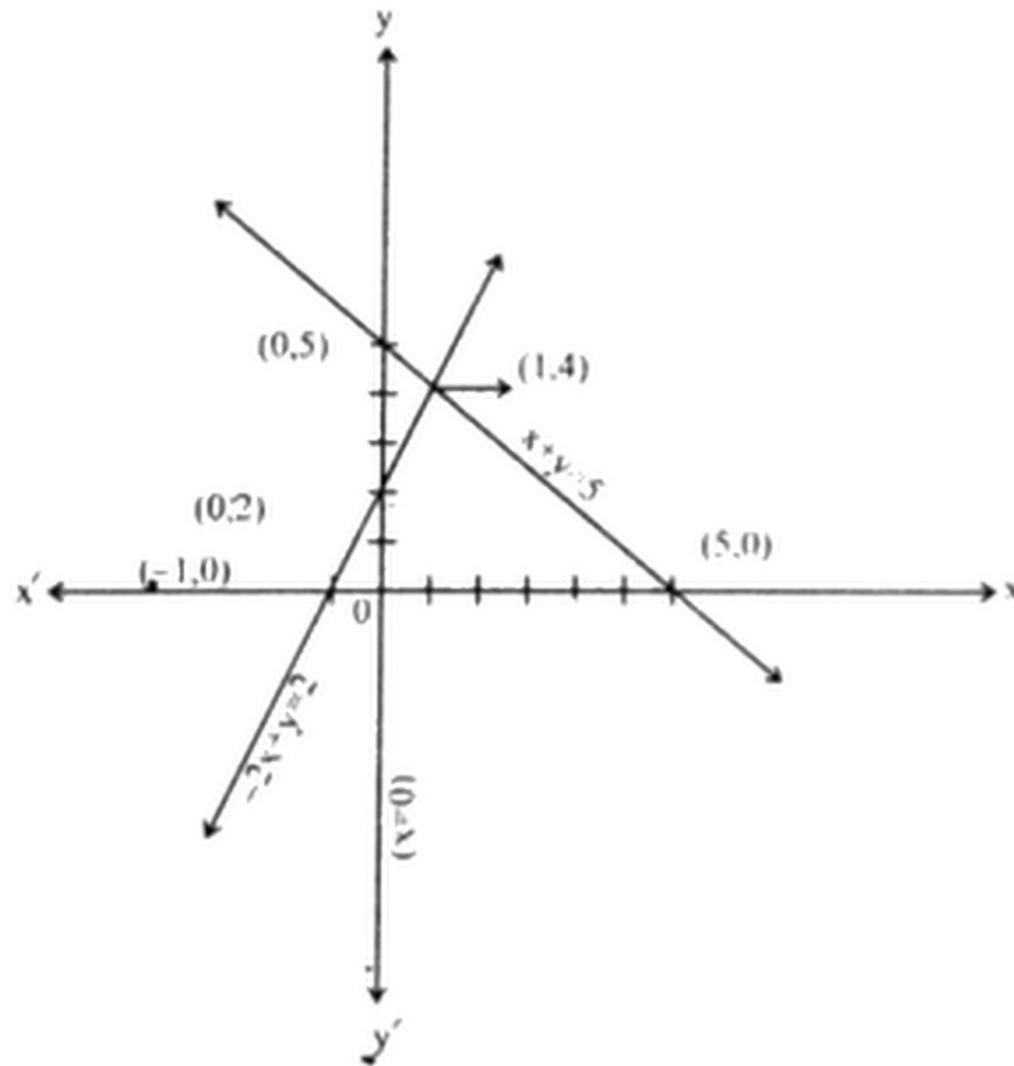
(-1,0) is another point on (4)

putting $x = 0$, $y = 0$ in (2)

$$-0 + 0 > 2$$

$$0 > 2$$

which is false



The intersection of four graphs in the first quadrant, i.e the shaded region is the required feasible region

Solving (3) and (4)

putting $x=$

1 in (3)

$$x + y = 5$$

$$1 + y = 5$$

$$\frac{-2x + y = -2}{3x = 3}$$

$$y = 4$$

hence (1,4), (0,2) and (0,5) are the corner points

(iv) $3x + 7y \leq 21, x - y \leq 3, x \geq 0, y \geq 0$

$3x + 7y \leq 21$ _____(1)

$3x + 7y = 21$ _____(3)

Putting $x = 0$ in (3)

$0 + 7y = 21 \Rightarrow y = 3$

(0,3) is a point on (3)

Putting $y = 0$ in (3)

$3x + 0 = 21 \Rightarrow x = 7$

(7,0) is another point on (3)

Putting $x = 0, y = 0$ in (1)

$0 + 0 < 21$

$0 < 21$

Which is true

The graph is

$x - y \leq 3$ _____(2)

$x - y = 3$ _____(4)

putting $x = 0$ in (4)

$0 - y = 3 \Rightarrow y = -3$

(0,-3) is a point on (4)

putting $x = 0$ in (4)

$x - 0 = 3 \Rightarrow x = 3$

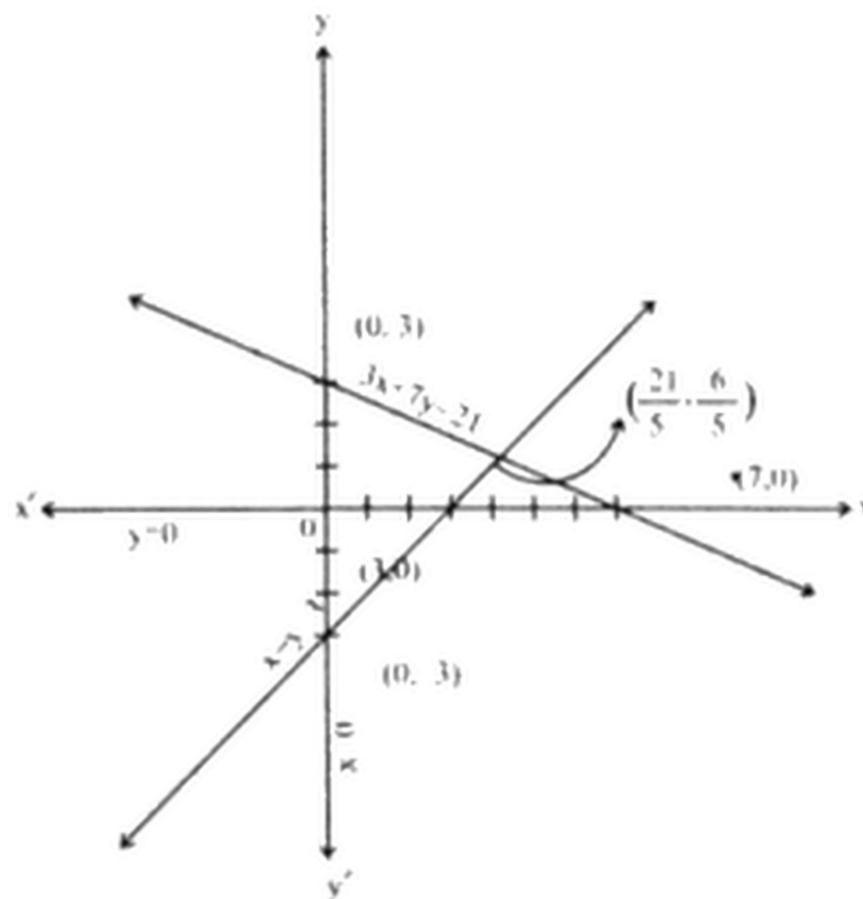
(3,0) is another point on (4)

putting $x = 0, y = 0$ in (2)

$0 + 0 < 3$

$0 < 3$

which is true



The intersection of four graphs in the first quadrant, i.e the shaded region is the required feasible region

From (3)

$$x = 3 + y \quad \text{--- (5)}$$

Putting $y = \frac{6}{5}$ in (5)

$$x = 3 + \frac{6}{5} = \frac{15+6}{5} = \frac{21}{5}$$

putting (5) in (3)

$$3(3+y) + 7y = 21$$

$$9 + 3y + 7y = 21$$

$$10y = 12$$

$$y = \frac{6}{5}$$

hence $(\frac{21}{5}, \frac{6}{5})$, (0,3), (0,0) and (3,0) are the corner points

(v) $3x + 2y \geq 6$, $x + y \leq 4$, $x \geq 0$, $y \geq 0$

$$3x + 2y \geq 6 \quad \text{--- (1)}$$

$$3x + 2y = 6 \quad \text{--- (3)}$$

Putting $x = 0$ in (3)

$$0 + 2y = 6 \Rightarrow y = 3$$

(0,3) is a point on (3)

Putting $y = 0$ in (3)

$$3x + 0 = 6 \Rightarrow x = 2$$

(2,0) is another point on (3)

Putting $x = 0$, $y = 0$ in (1)

$$0 + 0 > 6$$

$$0 > 6$$

$$x + y \leq 4 \quad \text{--- (2)}$$

$$x + y = 4 \quad \text{--- (4)}$$

putting $x = 0$ in (4)

$$0 + y = 4 \Rightarrow y = 4$$

(0,4) is a point on (4)

putting $x = 0$ in (4)

$$x + 0 = 4 \Rightarrow x = 4$$

(4,0) is another point on (4)

putting $x = 0$, $y = 0$ in (2)

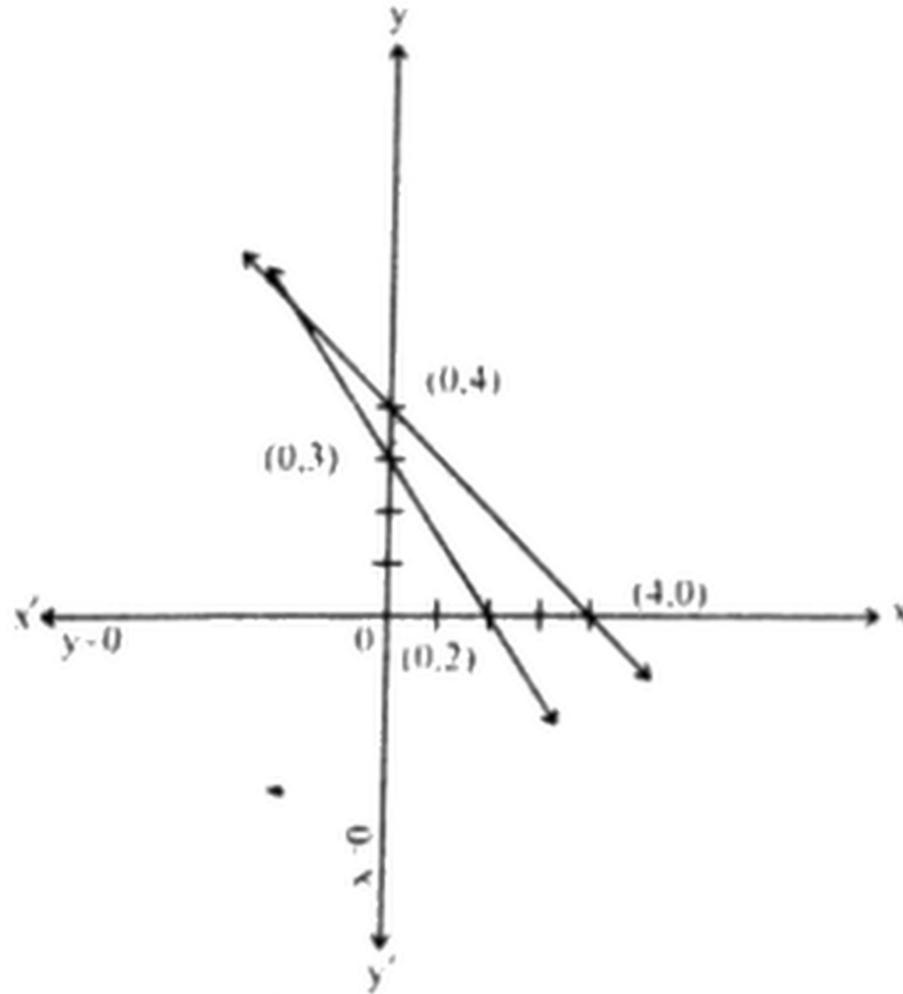
$$0 + 0 < 4$$

Which is false

$$0 < 4$$

The graph is

which is true



The intersection of four graphs in the first quadrant, i.e. the shaded region is the required feasible region. The corner points are (2,0) (4,0) (0,4) and (0,3)

(vi) $5x + 7y \leq 35, x - 2y \leq 4, x \geq 0, y \geq 0$

$$5x + 7y \leq 35 \text{ _____ (1)}$$

$$5x + 7y = 35 \text{ _____ (3)}$$

Putting $x = 0$ in (3)

$$0 + 7y = 35 \Rightarrow y = 5$$

(0,5) is a point on (3)

Putting $y = 0$ in (3)

$$x - 2y \leq 4 \text{ _____ (2)}$$

$$x - 2y = 4 \text{ _____ (4)}$$

putting $x = 0$ in (4)

$$0 - 2y = 4 \Rightarrow y = -2$$

(0,-2) is a point on (4)

$$5x + 0 = 35 \Rightarrow x = 7$$

(7,0) is another point on (3)

Putting $x = 0, y = 0$ in (1)

$$0 + 0 < 35$$

$$0 < 35$$

Which is true

The graph is

putting $x = 0$ in (4)

$$x - 0 = 4 \Rightarrow x = 4$$

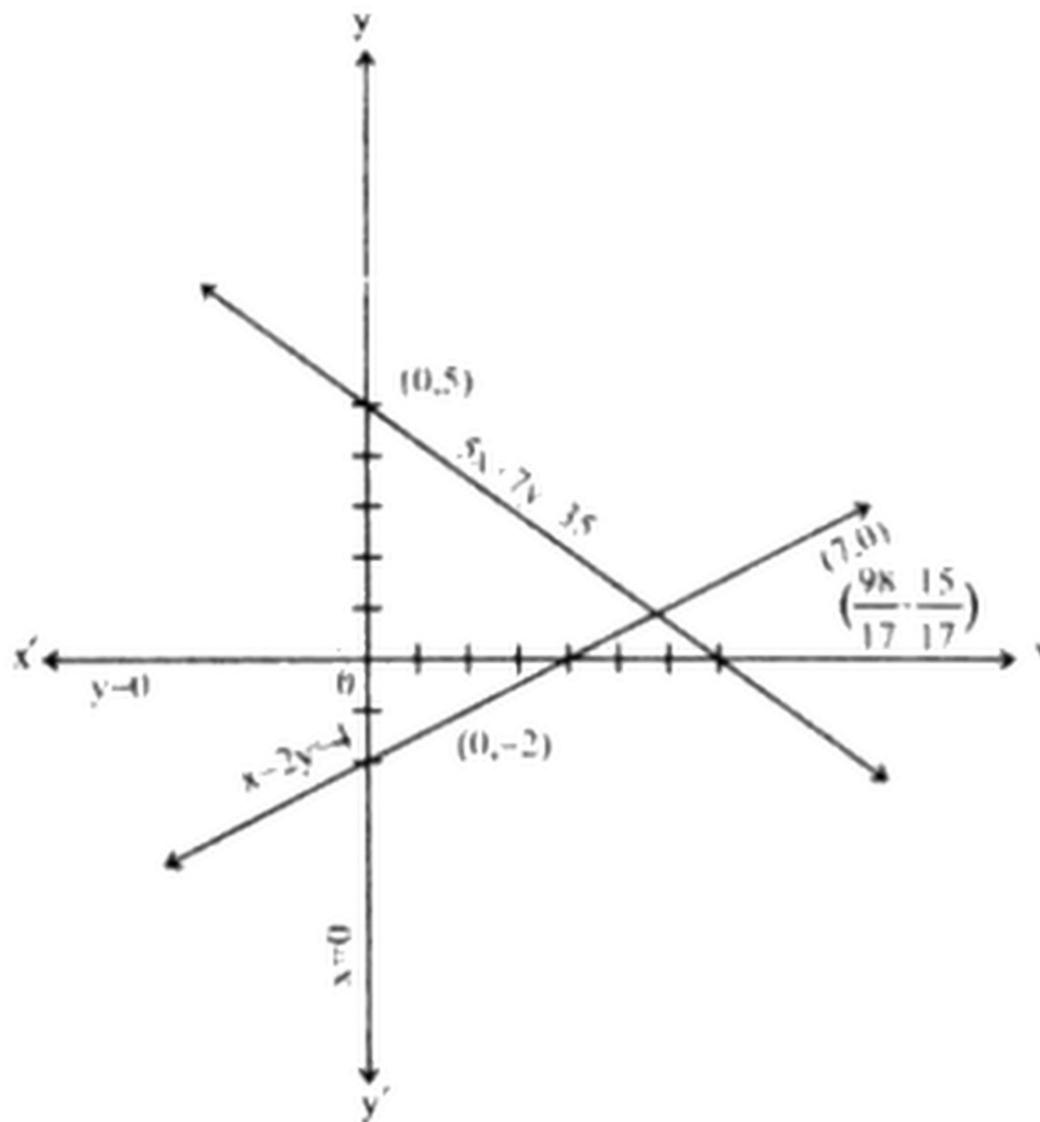
(4,0) is another point on (4)

putting $x = 0, y = 0$ in (2)

$$0 - 0 < 4$$

$$0 < 4$$

which is true



The intersection of four graphs in the first quadrant, i.e. the shaded region is the required feasible region

From (4)

$$x = 4 + 2y \quad \text{--- (5)}$$

Putting $y = \frac{15}{17}$ in (5)

$$x = 4 + 2 \times \frac{15}{17} = \frac{68 + 30}{17} = \frac{98}{17}$$

putting (5) in (3)

$$5(4 + 2y) + 7y = 35$$

$$20 + 10y + 7y = 35$$

$$17y = 15$$

$$y = \frac{15}{17}$$

hence $(\frac{98}{17}, \frac{15}{17})$, $(0, 5)$, $(0, 0)$ and $(4, 0)$ are the corner points.

Q2 Graph the feasible region of the following system of linear inequalities and find the corner points in each case

Solution

(i) $2x + y \geq 10$, $x + 4y \geq 12$, $x + 2y \leq 10$, $x \geq 0$, $y \geq 0$

$$2x + y \leq 10 \quad \text{--- (1)}$$

$$2x + y = 10 \quad \text{--- (3)}$$

Putting $x = 0$ in (3)

$$0 + y = 10 \Rightarrow y = 10$$

$\therefore (0, 10)$ is a point on (3)

Putting $y = 0$ in (3)

$$2x + 0 = 10 \Rightarrow x = 5$$

$\therefore (5, 0)$ is a point on (3)

Putting $x = 0$, $y = 0$ in (1)

$$0 + 0 < 10$$

$$0 < 10$$

$$x + 4y \leq 12 \quad \text{--- (2)}$$

$$x + 4y = 12 \quad \text{--- (4)}$$

Putting $x = 0$ in (4)

$$0 + 4y = 12 \Rightarrow y = 3$$

$\therefore (0, 3)$ is a point on (4)

Putting $y = 0$ in (4)

$$x + 0 = 12 \Rightarrow x = 12$$

$\therefore (12, 0)$ is a point on (4)

Putting $x = 0$, $y = 0$ in (2)

$$0 + 0 < 12$$

$$0 < 12$$

Which is true

$$x + 12y \leq 10 \text{ _____ (5)}$$

$$x + 12y = 10 \text{ _____ (6)}$$

Putting $x = 0$ in (6)

$$0 + 12y = 10 \Rightarrow y = \frac{5}{6}$$

$(0, \frac{5}{6})$ is a point on (6)

Putting $y = 0$ in (6)

$(10, 0)$ is another point on (6)

Putting $x = 0$, $y = 0$ in (5)

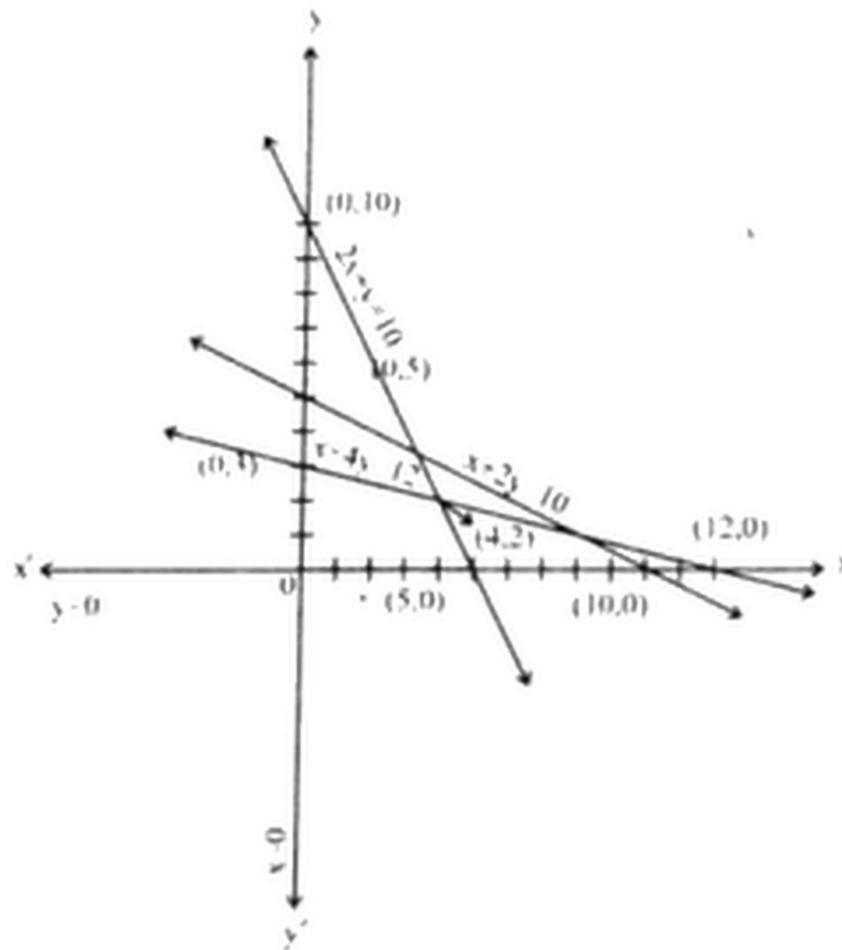
$$0 + 0 < 10$$

$$0 < 10$$

Which is true

The graph is

Which is true



The intersection of five graphs in the first quadrant, i.e. the shaded region is the required feasible region

From (4)

$$x = 12 - 4y \quad \text{---(7)}$$

putting (7) in (3)

$$2(12 - 4y) + y = 10$$

$$24 - 8y + y = 10$$

$$-7y = -14$$

$$y = 2$$

putting $y = 2$ in (7)

$$x = 12 - 4(2) = 12 - 8 = 4$$

hence (4,2), (0,3), (0,0), and (5,0) are the corner points

(ii) $2x + 3y \leq 18$, $2x + y \leq 10$, $x + 4y \leq 12$, $x \geq 0$, $y \geq 0$

$$2x + 3y \leq 18 \quad \text{---(1)}$$

$$2x + 3y = 18 \quad \text{---(3)}$$

Putting $x = 0$ in (3)

$$0 + 3y = 18 \Rightarrow y = 6$$

\therefore (0,6) is a point on (3)

Putting $y = 0$ in (3)

$$2x + 0 = 18 \Rightarrow x = 9$$

\therefore (9,0) is a point on (3)

Putting $x = 0$, $y = 0$ in (1)

$$x + 4y \geq 12 \quad \text{---(5)}$$

$$x + 4y = 12 \quad \text{---(6)}$$

Putting $x = 0$ in (6)

$$2x + 7y \leq 10 \quad \text{---(2)}$$

$$2x + 7y = 10 \quad \text{---(4)}$$

Putting $x = 0$ in (4)

$$0 + 7y = 10 \Rightarrow y = 10$$

\therefore (0, 10) is a point on (4)

Putting $y = 0$ in (4)

$$2x + 0 = 10 \Rightarrow x = 5$$

\therefore (5, 0) is a point on (4)

Putting $x = 0$, $y = 0$ in (2)

$$0 + 4y = 12 \Rightarrow y = 3$$

(0, 3) is a point on (6)

Putting $y = 0$ in (6)

$$x + 0 = 12 \Rightarrow x = 12$$

(12,0) is another point on (6)

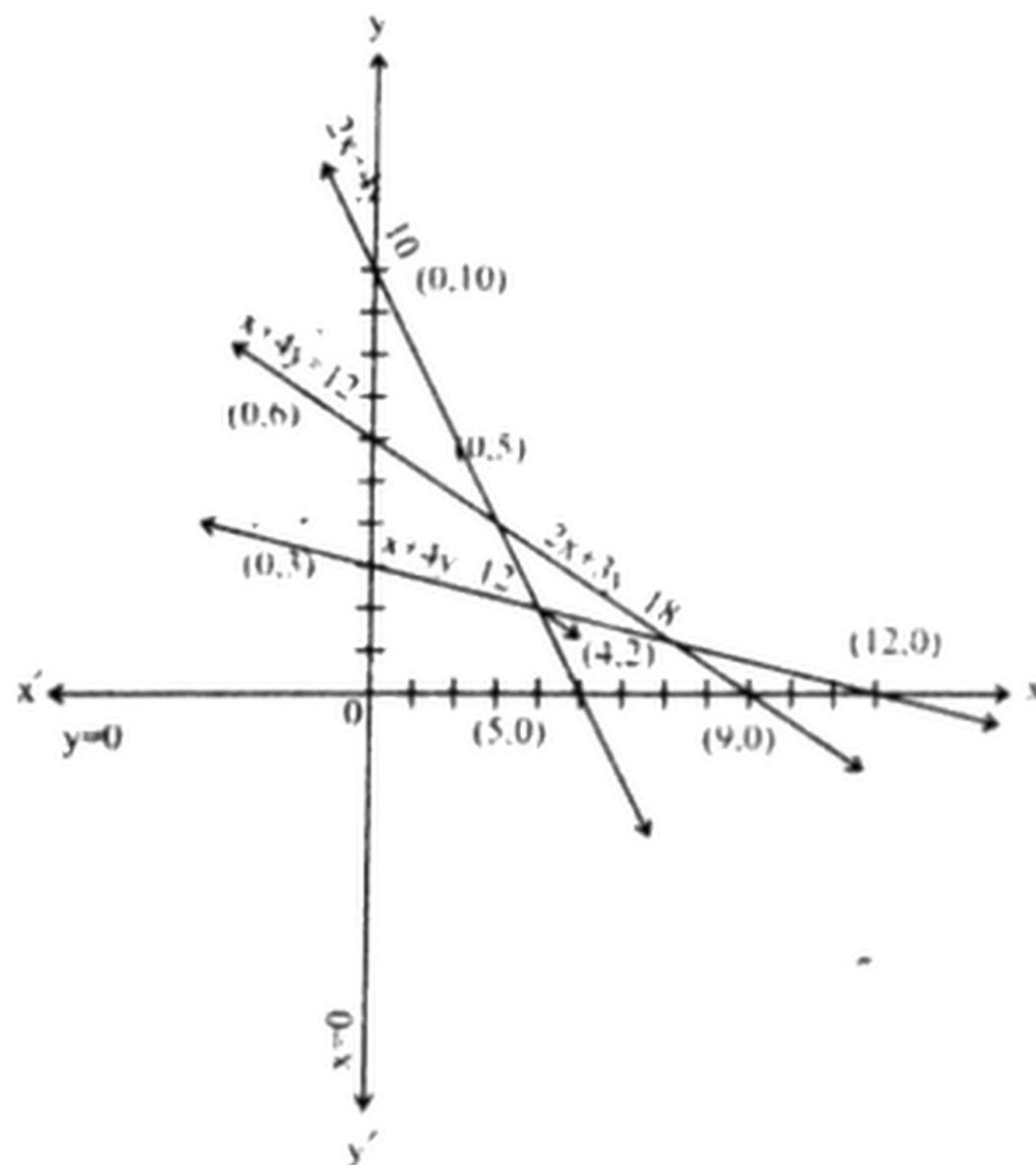
Putting $x = 0$, $y = 0$ in (5)

$$0 + 0 < 12$$

$$0 < 12$$

Which is true

The graph is



The intersection of five graphs in the first quadrant, i.e the shaded region is the required feasible region

From (4)

$$x = 12 - 4y \quad \text{---(7)}$$

putting (7) in (3)

$$2(12 - 4y) + y = 10$$

$$24 - 8y + y = 10$$

$$-7y = -14$$

$$y = 2$$

putting $y = 2$ in (7)

$$x = 12 - 4(2) = 12 - 8 = 4$$

hence $(4,2)$, $(0,3)$, $(0,0)$, and $(5,0)$ are the corner points

(iii) $2x + 3y \leq 18$, $x + 4y \leq 12$, $3x + y \leq 12$, $x \geq 0$, $y \geq 0$

$$2x + 3y \leq 18 \quad \text{---(1)}$$

$$2x + 3y = 18 \quad \text{---(3)}$$

Putting $x = 0$ in (3)

$$0 + 3y = 18 \Rightarrow y = 6$$

$\therefore (0,6)$ is a point on (3)

Putting $y = 0$ in (3)

$$2x + 0 = 18 \Rightarrow x = 9$$

$\therefore (9,0)$ is a point on (3)

Putting $x = 0$, $y = 0$ in (1)

$$0 + 0 < 18$$

$$0 < 18$$

$$x + y \leq 12 \quad \text{---(2)}$$

$$x + y = 12 \quad \text{---(4)}$$

Putting $x = 0$ in (4)

$$0 + y = 12 \Rightarrow y = 12$$

$\therefore (12,0)$ is a point on (4)

Putting $y = 0$ in (4)

$$0 + 0 < 12$$

$$0 < 12$$

which is true

Which is true

$$3x+y \leq 12 \quad \text{---(5)}$$

$$3x + y = 12 \quad \text{---(6)}$$

Putting $x = 0$ in (6)

$$0 + y = 12 \Rightarrow y = 12$$

$(0, 12)$ is a point on (6)

Putting $y = 0$ in (6)

$$3x + 0 = 12 \Rightarrow x = 4$$

$(4, 0)$ is another point on (6)

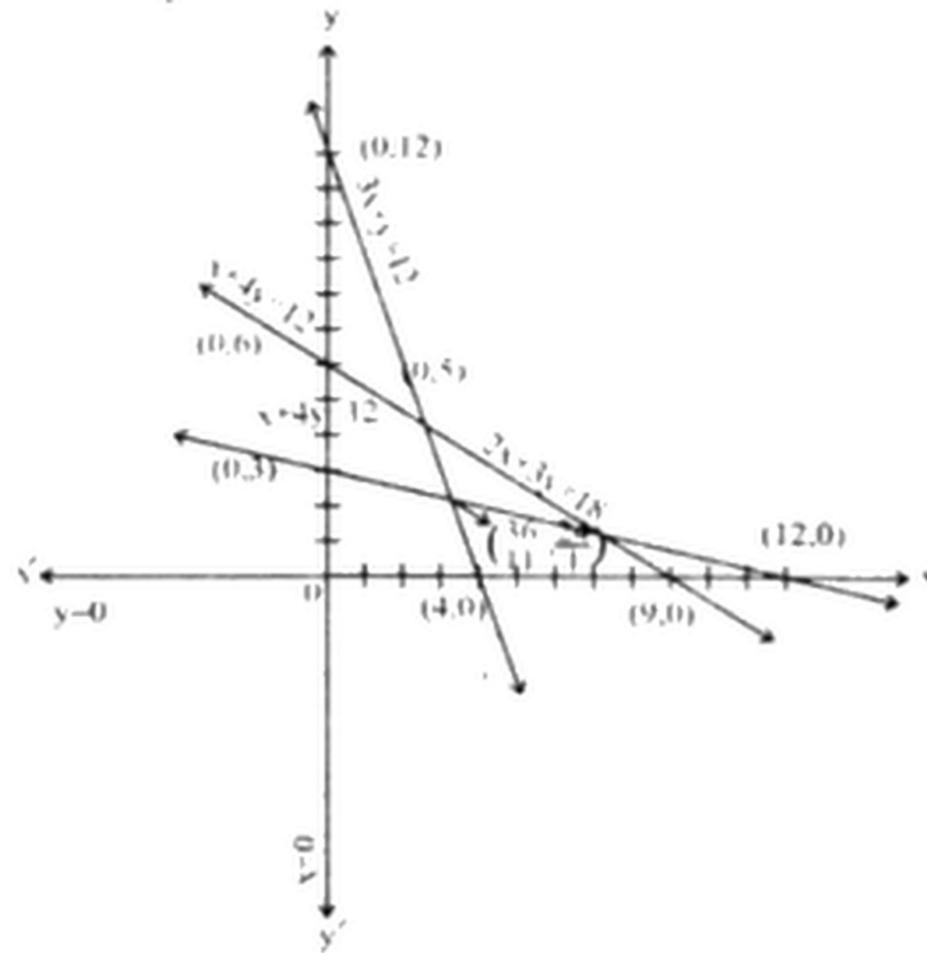
Putting $x = 0$, $y = 0$ in (5)

$$0 + 0 < 12$$

$$0 < 12$$

Which is true

The graph is



The intersection of five graphs in the first quadrant, i.e the shaded region is the required feasible region

From (4)

putting (7) in (6)

$$3(12-4y) + y = 12$$

$$36 - 12y + y = 12$$

$$-11y = -24$$

$$y = \frac{24}{11}$$

putting $y = \frac{24}{11}$ in (7)

$$x = 12 - 4\left(\frac{24}{11}\right) = 12 - \frac{96}{11} = \frac{123-96}{11} = \frac{27}{11}$$

hence $\left(\frac{27}{11}, \frac{24}{11}\right)$, $(0,3)$, $(0,0)$, and $(4,0)$ are the corner points

(iv) $x+2y \leq 14$, $3x+4y \leq 36$, $2x+y \leq 10$, $x \geq 0$, $y \geq 0$

$$x + 2y \leq 14 \text{ _____(1)}$$

$$x + 2y = 14 \text{ _____(3)}$$

Putting $x = 0$ in (3)

$$0 + 2y = 14 \Rightarrow y = 7$$

$\therefore (0, 7)$ is a point on (3)

Putting $y = 0$ in (3)

$$x + 0 = 14 \Rightarrow x = 14$$

$\therefore (14, 0)$ is a point on (3)

Putting $x = 0, y = 0$ in (1)

$$0 + 0 < 14$$

$$0 < 14$$

Which is true

$$2x + y \leq 10 \text{ _____(5)}$$

$$2x + y = 10 \text{ _____(6)}$$

Putting $x = 0$ in (6)

$$0 + y = 10 \Rightarrow y = 10$$

$(0, 10)$ is a point on (6)

Putting $y = 0$ in (6)

$$2x + 0 = 10, x = 5$$

$$0 + 0 < 10$$

$$0 < 10$$

Which is true

The graph is

$$3x + 4y \leq 36 \text{ _____(2)}$$

$$3x + 4y = 36 \text{ _____(4)}$$

Putting $x = 0$ in (4)

$$0 + 4y = 36 \Rightarrow y = 9$$

$\therefore (0, 9)$ is a point on (4)

Putting $y = 0$ in (4)

$$3x + 0 = 36 \Rightarrow x = 12$$

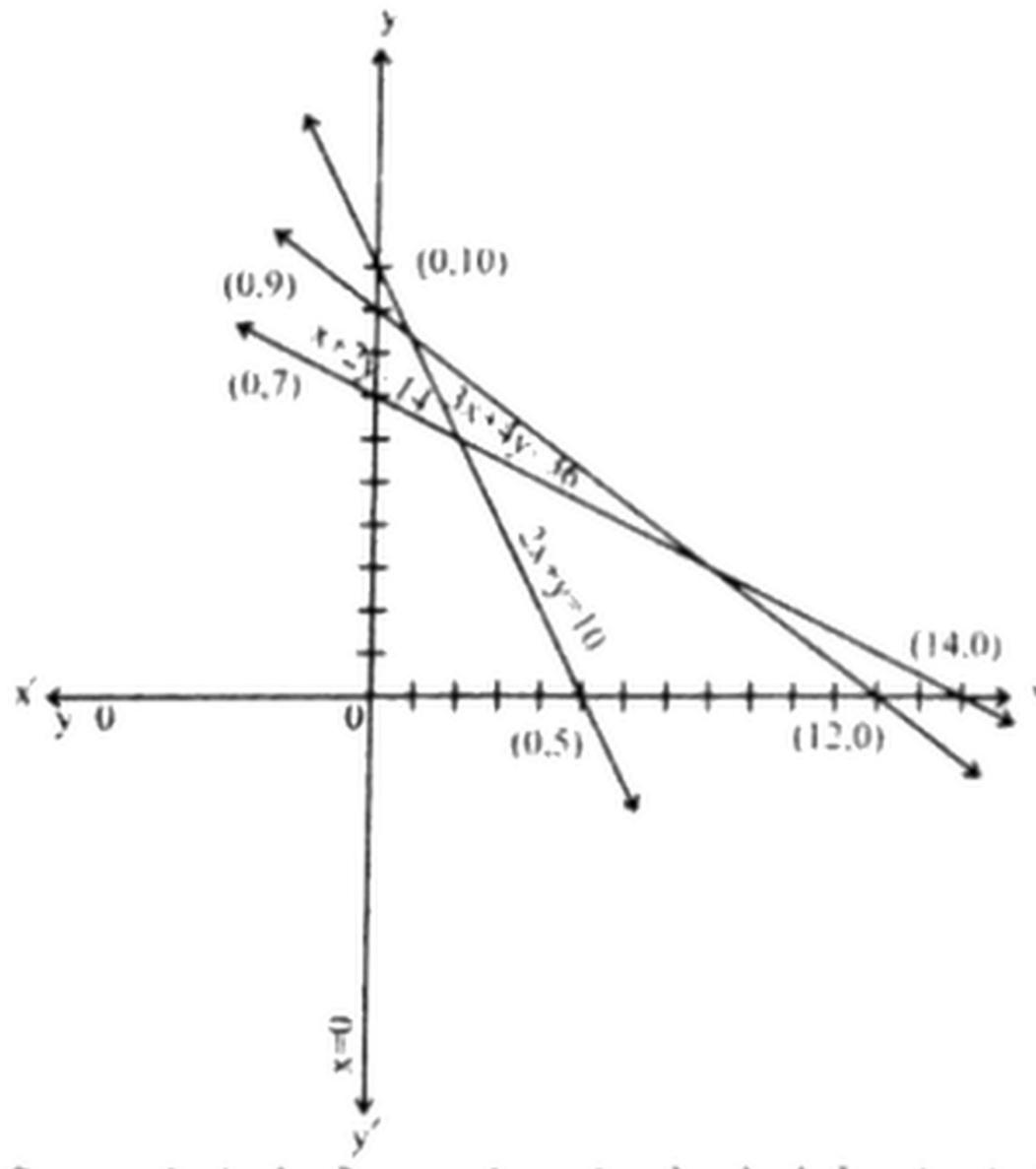
$\therefore (12, 0)$ is a point on (4)

Putting $x = 0, y = 0$ in (2)

$$0 + 0 < 36$$

$$0 < 36$$

Which is true



The intersection of five graphs in the first quadrant, i.e the shaded region is the required feasible region

From (3)

$$x = 12 - 4y \quad (7)$$

putting (7) in (6)

$$2(12 - 4y) + y = 10$$

$$24 - 8y + y = 10$$

$$-7y = -14$$

$$-y = -2$$

$$y = 2$$

putting $y = 2$ in (7)

$$x = 12 - 4(2) = 12 - 8 = 4$$

hence $(4,2)$, $(0,2)$, $(0,0)$, and $(3,0)$ are the corner points

(v) $x+3y \leq 15, 2x+y \leq 12, 4x+3y \leq 24, x \geq 0, y \geq 0$

$$x+3y \leq 15 \text{ _____(1)}$$

$$x+3y = 15 \text{ _____(3)}$$

Putting $x = 0$ in (3)

$$0+3y = 15 \Rightarrow y = 5$$

$\therefore (0,5)$ is a point on (3)

Putting $y = 0$ in (3)

$$x+0 = 15 \Rightarrow x = 15$$

$\therefore (15,0)$ is a point on (3)

Putting $x = 0, y = 0$ in (1)

$$0+0 < 15$$

$$0 < 15$$

Which is true

$$4x+3y \leq 24 \text{ _____(5)}$$

$$4x+3y = 24 \text{ _____(6)}$$

Putting $x = 0$ in (6)

$$0+3y = 24 \Rightarrow y = 8$$

$(0, 8)$ is a point on (6)

Putting $y = 0$ in (6)

$$4x+0 = 24 \Rightarrow x = 6$$

$(6,0)$ is another point on (6)

Putting $x = 0, y = 0$ in (5)

$$0+0 < 24$$

$$2x+y \leq 12 \text{ _____(2)}$$

$$2x+y = 12 \text{ _____(4)}$$

Putting $x = 0$ in (4)

$$0+y = 12 \Rightarrow y = 12$$

$\therefore (0, 12)$ is a point on (4)

Putting $y = 0$ in (4)

$$2x+0 = 12 \Rightarrow x = 6$$

$\therefore (6, 0)$ is a point on (4)

Putting $x = 0, y = 0$ in (2)

$$0+0 < 12$$

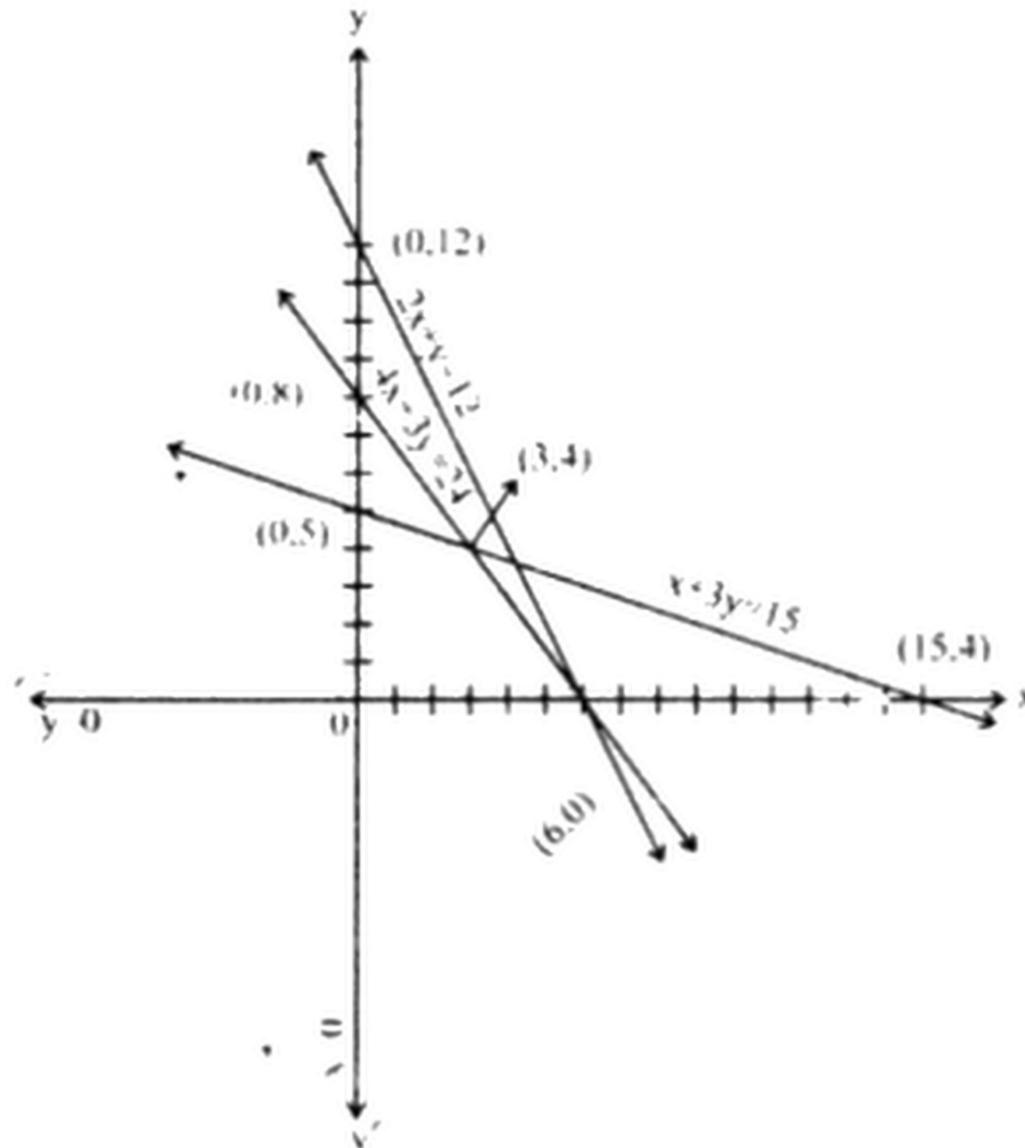
$$0 < 12$$

Which is true

$$0 < 24$$

Which is true

The graph is



The intersection of five graphs in the first quadrant, i.e. the shaded region is the required feasible region

From (3)

$$x = 15 - 3y \quad \text{---(7)}$$

putting (7) in (6)

$$4(15 - 3y) + 3y = 24$$

$$60 - 12y + 3y = 24$$

$$-9y = -36$$

$$y = 4$$

putting $y = 4$ in (7)

$$x = 15 - 3(4) = 15 - 12 = 3$$

hence (3,4), (0,5), (0,0), and (6,0) are the corner points

(vi) $2x+y \leq 20, 8x+15y \leq 120, x+y \leq 11, x \geq 0, y \geq 0$

$$2x + y \leq 20 \quad \text{_____ (1)}$$

$$2x + y = 20 \quad \text{_____ (3)}$$

Putting $x = 0$ in (3)

$$0 + y = 20 \Rightarrow y = 20$$

$\therefore (0, 20)$ is a point on (3)

Putting $y = 0$ in (3)

$$2x + 0 = 20 \Rightarrow x = 10$$

$\therefore (10, 0)$ is a point on (3)

Putting $x = 0, y = 0$ in (1)

$$0 + 0 < 20$$

$$0 < 20$$

Which is true

$$x + y \leq 11 \quad \text{_____ (5)}$$

$$x + y = 11 \quad \text{_____ (6)}$$

Putting $x = 0$ in (6)

$$0 + y = 11 \Rightarrow y = 11$$

$(0, 11)$ is a point on (6)

Putting $y = 0$ in (6)

$$x + 0 = 11 \Rightarrow x = 11$$

$$8x + 15y \leq 120 \quad \text{_____ (2)}$$

$$8x + 15y = 120 \quad \text{_____ (4)}$$

Putting $x = 0$ in (4)

$$0 + 15y = 120 \Rightarrow y = 8$$

$\therefore (0, 8)$ is a point on (4)

Putting $y = 0$ in (4)

$$8x + 0 = 120 \Rightarrow x = 15$$

$\therefore (15, 0)$ is a point on (4)

Putting $x = 0, y = 0$ in (2)

$$0 + 0 < 120$$

$$0 < 120$$

Which is true

(11,0) is another point on (6)

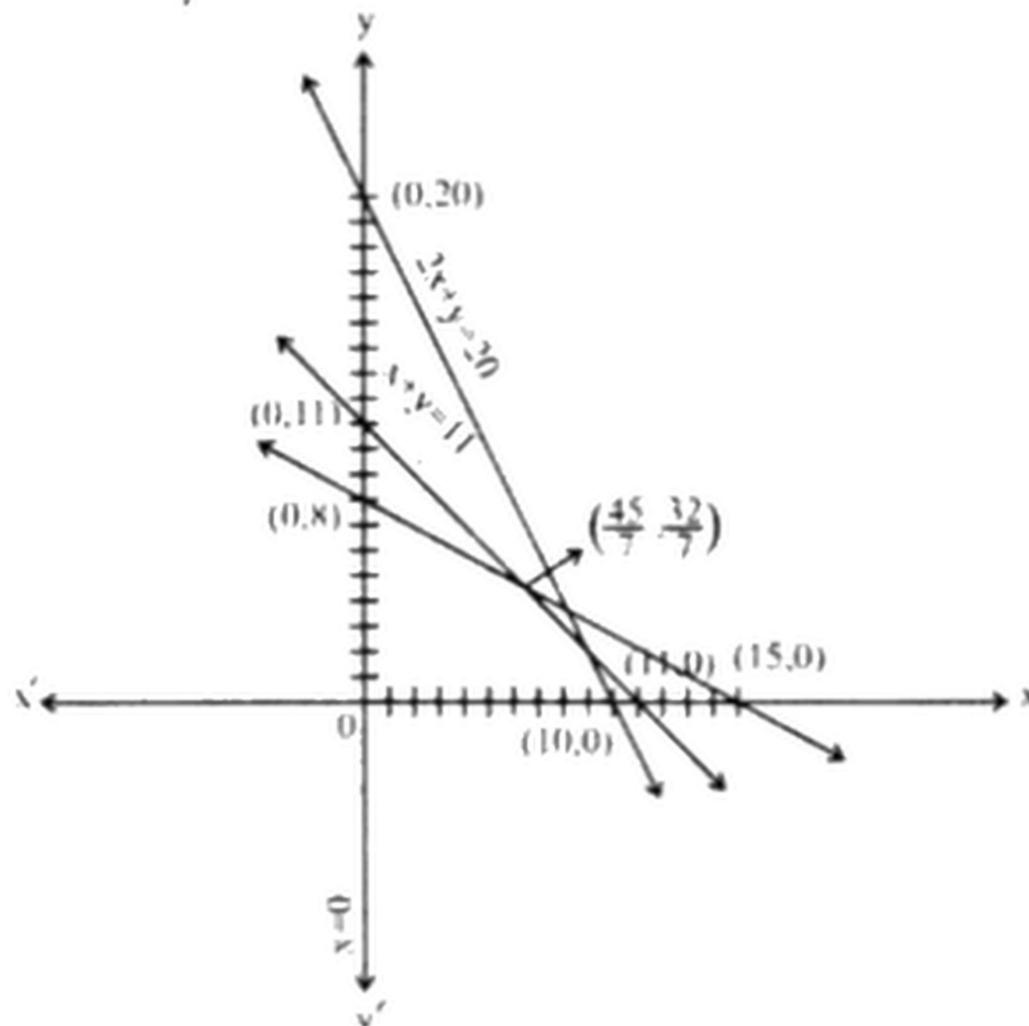
Putting $x = 0$, $y = 0$ in (5)

$$0 + 0 < 11$$

$$0 < 11$$

Which is true

The graph is



The intersection of five graphs in the first quadrant, i.e the shaded region is the required feasible region

From (6)

$$x = 11 - y \quad (7)$$

putting (7) in (4)

$$8(11 - y) + 15y = 120$$

$$88 - 8y + 15y = 120$$

$$7y = 32$$

$$y = \frac{32}{7}$$

putting $y = \frac{32}{7}$ in (7)

$$x = 11 - \frac{32}{7} = \frac{77 - 32}{7} = \frac{45}{7}$$

hence $(\frac{32}{7}, \frac{45}{7})$, $(1,0)$, $(0,8)$, $(0,0)$, and $(9,2)$ are the corner points

