Exercise 4.4

- **Find the point of intersection of the lines**

a. x – 2y +1 = 0 and 2x-y +12 = 0

Solution

x-2y +1 = 0 ________(1)

2x-y+2 = 0 _________(2)

Slope of (1) = $-\left(\frac{1}{2}\right) = \frac{1}{2} = m_1$

Slope of (2) = $-\left(\frac{-2}{1}\right) = 2 = m_2$

Hence the lines (1) and (2) are not parallel so they will intersect.

If (x ,y) is the point of intersection of (1) and (2) then solving the two n equations simultaneously , we get

$\Rightarrow \quad \frac{x}{-4+1} = \frac{y}{2-2} = \frac{1}{-1+4}$

$\frac{x}{-3} = \frac{4}{0} = \frac{1}{3}$ i.e $x = \frac{-3}{3} = -1, y = \frac{0}{3} = 0$

i.e (-1,0) is the required point of intersection.

b. 3x+y+12 = 0 and x+2y -1 = 0

Solution

3x+y +12 = 0 ________(1)

X+2y-1 = 0 _________(2)

Slope of (2) = $\left(\frac{-1}{2}\right)$ = m_2

Slope of line (1) ≠ Slope of line (2)

Hence the lines (1) and (2) are not parallel so they will intersect.

If (x ,y) is the point of intersection of (1) and (2) then solving the two n equations simultaneously , we get

$$\Rightarrow \quad \frac{x}{-1-24} = \frac{y}{12+3} = \frac{1}{6-1}$$

$$\frac{x}{25} = \frac{y}{15} = \frac{1}{5} \quad \text{i.e } x = \frac{-25}{5} = -5, y = \frac{15}{5} = 3$$

i.e (-5,3) is the required point of intersection.

c. x+4y-12 = 0 and x-3y+3 = 0

Solution

x+4y -12 = 0 ________(1)

x-3y+3 = 0 _________(2)

Slope of (1) = - $\left(\frac{1}{4}\right)$ = m_1

Slope of (2) = $\left(\frac{-1}{3}\right)$ = m_2

Slope of line (1) ≠ Slope of line (2)

Hence the lines (1) and (2) are not parallel ,so they will intersect .

If (x ,y) is the point of intersection of (1) and (2) then solving the two n equations simultaneously , we get

$$\Rightarrow \quad \frac{x}{12-36} = \frac{y}{-12-3} = \frac{1}{-3-4}$$

$$\frac{x}{-24} = \frac{y}{-15} = \frac{1}{-7} \quad \text{i.e } x = \frac{-24}{7} = \frac{24}{7}, y = \frac{-15}{7} = \frac{15}{7}$$

Q2. Find an equation of the line through.

(i) The point (2,-9) and the intersection of lines 2x+5y -8 = 0 and 3x-4y-6 = 0

Solution

First we will find the point of intersection of lines

2x+5y -8= 0 ________(1)

3x-4y-6 = 0 ________(2)

$\Rightarrow \quad \frac{x}{-30-32} = \frac{-y}{-12-124} = \frac{1}{-8-15}$

$\frac{x}{-62} = \frac{y}{-12} = \frac{1}{-23}$

i.e $x = \frac{-62}{-23} = \frac{62}{23}, y = \frac{-12}{-13} = \frac{12}{23}$

i.e $\left[\frac{62}{23}, \frac{12}{23}\right]$ is the required point of intersection.

Now slope of the line passing through the point (2,9), $\left[\frac{62}{23}, \frac{12}{23}\right]$ is

$m = \frac{\frac{12}{23}-(-9)}{\frac{62}{23}-2} = \frac{12+207}{62-46} = \frac{219}{16}$

Thus required line is $y - y_1 = m(x-x_1)$

$y - (-9) = \frac{219}{16}(x-2)$

16(y+9) = 219x- 428

219x – 16y -438-144 = 0

219x – 16y – 582 = 0

ii. The intersection of the lines x -y – 4 = 0 and 7x +y +20 = 0

i. Parallel and

Solution

Any line through the intersection of lines.

$x - y - 4 = 0$ and $7x + y + 20 = 0$ is

$(x - y - 4) + K(7x + y + 20) = 0$

$(1+7k)x + (K - 1)y - 4 - 120K = 0$ ______(1)

Its slope is $-\frac{7K+1}{K-1}$ ______(2)

Line (1) is parallel to $6x + y - 14 = 0$ if ______(3)

$$\frac{7K+1}{K-1} = -6$$

$$7K + 1 = 6K - 6$$

$K = -7$ Put in (1)

$$(1+7)(-7)(-7-1)\,Y - 4 + 20(-7) = 0$$

$$(1-49)x + (-8)y - 4 - 140 = 0$$

$$-48x - 8y - 144 = 0$$

Divided by -1

$$6x + y + 18 = 0$$

b. Line(1) is perpendicular to $6x+y-14=0$ ______(4)

Solution

If $\left[\frac{7K+1}{K-1}\right](-6) = -1$

$$6(7K+1) = -(K-1)$$

$$12K + 6 = -5$$

$$43K = -5$$

$$\left[1+\ldots\left(\frac{5}{43}\right)\right]x + \left[\frac{-5}{43} - 1\right]y - 4 + 20\left[-\frac{5}{43}\right] = 0$$

$$\left[\frac{43-35}{43}\right]x + \left[\frac{-5-43}{43}\right]y - 4\ \frac{100}{43} = 0$$

Multiply both sides by 43

8x-48y=172 = 0

8x – 48 – 272 = 0

X -6y -34 = 0

iii. Through the intersection of the lines x +2y+3 = 0 and 3x+4y+7 = 0 and making eq intercepts on the axes

Solution

Any line through the intersection of the lines

x +2y+3 = 0; 3x+4y+7 = 9 is

(1+3K)x+ (2+4K)y+ 2 +7K = 0

To find x-intercept, put = 0

(1+3K)x+0+3+7K = 0

$$x = \frac{3+1K}{1+3K}$$

To find y-intercept put x = 0

0+(2+4K)y+3+7K = 0

$$x = \frac{-3+7K}{2+4K}$$

Since x-intercept and y-intercept are equal we have

$$\frac{-3+7K}{1+3K} = \frac{-3+7K}{2+4K}$$

(1-3)x+(2-4)y+3-7 = 0

-2x-2y -4 = 0

Divide through out by -2

x +y +2 = 0

Q3 Find an equation of the line through the intercepts of 16x-10y-33 = 0 and 12x+14+29 = 0 and the intersection of x-y+4 = 0 and x-7y +2 = 0

Solution

First we will find the point of intersection of

16x-10y -33= 0 ________(1)

12x+14y+29 = 0 _________(2)

$$\Rightarrow \quad \frac{x}{-290+462} = \frac{-y}{464+396} = \frac{1}{224+120}$$

$$\frac{x}{172} = \frac{y}{-860} = \frac{1}{344}$$

$$x = \frac{172}{344} = \frac{1}{2}, y = \frac{-860}{344} = -\frac{5}{2}$$

Point of intersection is $\left[\frac{1}{2}, -\frac{5}{2}\right]$

Now any line through the intersection of

x-y+4 = 0, x-7y+2 = 0 is

(x-y+4)+K(x-7y+2) = 0 ______(1)

This line will pass through the point $\left[\frac{1}{2}, \frac{5}{2}\right]$ the

$$\left[\frac{1}{2} - \frac{5}{2} + 4\right] + K\left[\frac{1}{2} - 7 - \frac{5}{2} + 2\right] + \left[\frac{4}{2}\right] + K\left[-\frac{30}{2}\right] = 0$$

$$2 - 15K = 0 \Rightarrow 15K = +2$$

Putting $K = -\frac{2}{15}$ in (3)

$(x-y+4)+\frac{2}{15}(x-7y+2) = 0$

$15(x-y+4)+2(x-7y+2) = 0$

$15x+15y+60-2x+14y+6 = 0$

$13x+29y+66 = 0$ is the required line.

Q4. Find the condition that the lines $y=m_1x + c_1$, $y = m_2x+c_2$, $y = m_3x+c_3$ are concurrent.

Solution

Given $= m_1x - y + C_1 = 0$ ________(1)

$m_2 = x-y+C_2 = 0$ ________(2)

$m_3 = x-y+C_3 = 0$________(3)

We proceed to solve the first two equation

$= \frac{-y}{m_1C_2 \; m_2C_1} = \frac{1}{-m_1+m_2}$ ________(4)

$x = \frac{C_1-C_2}{m_2-m_1} = \frac{m_2C_1-m_1C_2}{m_2-m_1}$

Substituting the values of x, y in (3)

$m_3 = \left[\frac{m_1-C_2}{m_2-m_1}\right] - \left[\frac{m_2C_1-m_1C_2}{m_2-m_1}\right] + C_3 = 0$

$\Rightarrow m_3(C_1 - C_2)\text{-} (m_2 - m_1C_2)\, C_3(m_2 - m_1) = 0$

$\Rightarrow m_3(C_1 - C_2)\text{-}\, m_2C_1 - m_1C_2+m_2C_3 - m_1C_3 = 0$

$\Rightarrow m_3(C_1 - C_2)+ m_2(C_3 - C_1)+m_1 (C_2 - C_3) = 0$

$\Rightarrow m_3(C_2 - C_3)+ m_2(C_3 - C_1)+m_3 (C_1 - C_2) = 0$

$$(m_2 - m_1)(C_3 - C_1) \quad = \qquad (m_3 - m_1)(C_2 - C_1)$$

Q5. Determine the value of p such that the line 2x-3-1 = 0, 3x-y-5 = 0 and 3x+py + 8 = 0 meet at a point.

Solution

2x-3-1 = 0 __________(1)

3x-y-5 = 0 __________(2)

3x+py + 8 = 0 __________(3)

Given lines are concurrent if

$$\begin{vmatrix} 2 & -3 & -1 \\ 3 & -1 & -5 \\ 3 & p & 8 \end{vmatrix} = 0$$

= 2(-8+5P)+3(25+15)-1(3P+3) = 0

-16 +10P+3(39)-3P-3 = 0

-16+10P+117-3P-3 = 0

$\Rightarrow$ 7P+98 = 0

$\Rightarrow$ 7P = -98

$\Rightarrow$ P = $\frac{-98}{7}$.

= -14

Which is the required value of P

Q6. Show that the lines 4x-3y-8 = 0, 3x-4y-6 = 0 and x-y-2 = 0 are concurrent and the third line bisects the angle formed by the first two lines.

Solution

3x-4y-6 = 0 ________(2)

x-y-2 = 0 ________(3)

The lines (1),(2),(3), will be concurrent if

$$\begin{vmatrix} 4 & -3 & -8 \\ 3 & -4 & -6 \\ 1 & -1 & -2 \end{vmatrix} = 0$$

$$\Delta = \begin{vmatrix} 4 & -3 & -8 \\ 3 & -4 & -6 \\ 1 & -1 & -2 \end{vmatrix}$$

= 4(8-6)+3(-6+6)-8(3+4)

= 4(2)+3(1)-8(1) = 8+0 -8 = 0

Thus the lines are concurrent

Slope of line (1) = $m_1 = \frac{-4}{3} = \frac{4}{3}$

Slope of line (2) = $m_2 = \frac{-3}{4} = \frac{3}{4}$

Slope of line (3) = $m_3 = \frac{-1}{-1} = 1$

Angle between line (1) and line (3) is given by

$$\tan\Theta = \frac{m_1 - m_3}{1 + m_{1m_3}} = \frac{\frac{4}{3} - 1}{1 + \frac{4}{3}}$$

$$= \frac{\frac{3}{7}}{\frac{}{3}} = \frac{1}{7}$$

Angle between line (2) and line (3) is given by

$$\tan\Theta = \frac{m_3 - m_2}{1 + m_{3m_2}} = \frac{1 - \frac{4}{3}}{1 + \frac{4}{3} \cdot 1}$$

$$= \frac{\frac{1}{4}}{\frac{7}{4}}$$

$$= \frac{1}{7} \qquad \text{________(5)}$$

[Angle between have (1) and (3)] = [Angle between have (2) and (3)]

Hence prove the result

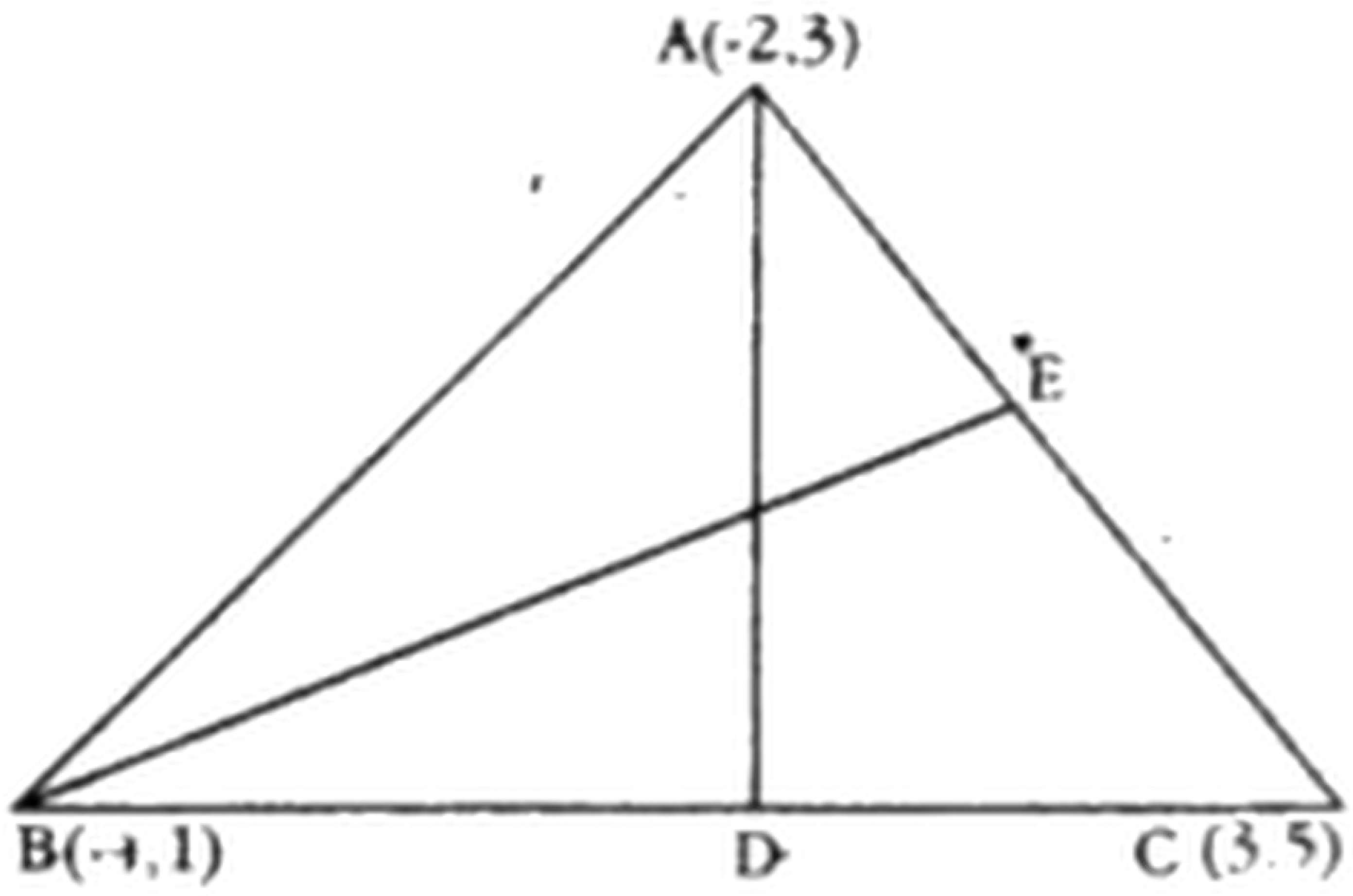
Q7. The vertices of a triangle are A(-2,3) B(-4,1) and C(3,5).Find coordinates of the

i. centroid ii. orthocenter iii. Circumcenter of the triangle

Are these three points collinear?

Solution

We know that centroid a point of all concurrency of medians of a triangle.



Mid point of $\overline{BC}$ is D$\left[-\frac{1}{2},3\right]$

Mid point of $\overline{AC}$ is E$\left[\frac{1}{2},4\right]$

Equation of median $\overline{AD}$ is

$$\frac{y-3}{3-3}=\frac{x-(-2)}{-\frac{1}{2}-(-2)}$$

$$\Rightarrow \quad \frac{y-0}{0}=\frac{x-2}{\frac{1}{2}+1}$$

$$\frac{3}{2}(y-3)=0(x+2)$$

$\Rightarrow \quad y = 3$

Equation of median $\overline{BE}$ is

$$\frac{y-1}{4} = \frac{x-(-4)}{-\frac{1}{2}-(-4)}$$

$\Rightarrow \quad \frac{y-1}{3} = \frac{x+4}{\frac{1}{2}-(-4)}$

$\frac{9}{2}(y-1) = 3(x+49)$

$\Rightarrow \quad 9y - 9 = 6x+24$

$6x-9y-\ldots y +9 = 0$

$\Rightarrow \quad 9y -9y +33 = 0$

$2x -3y +11 = 0$ __________(2)

Put $\quad y = 3$

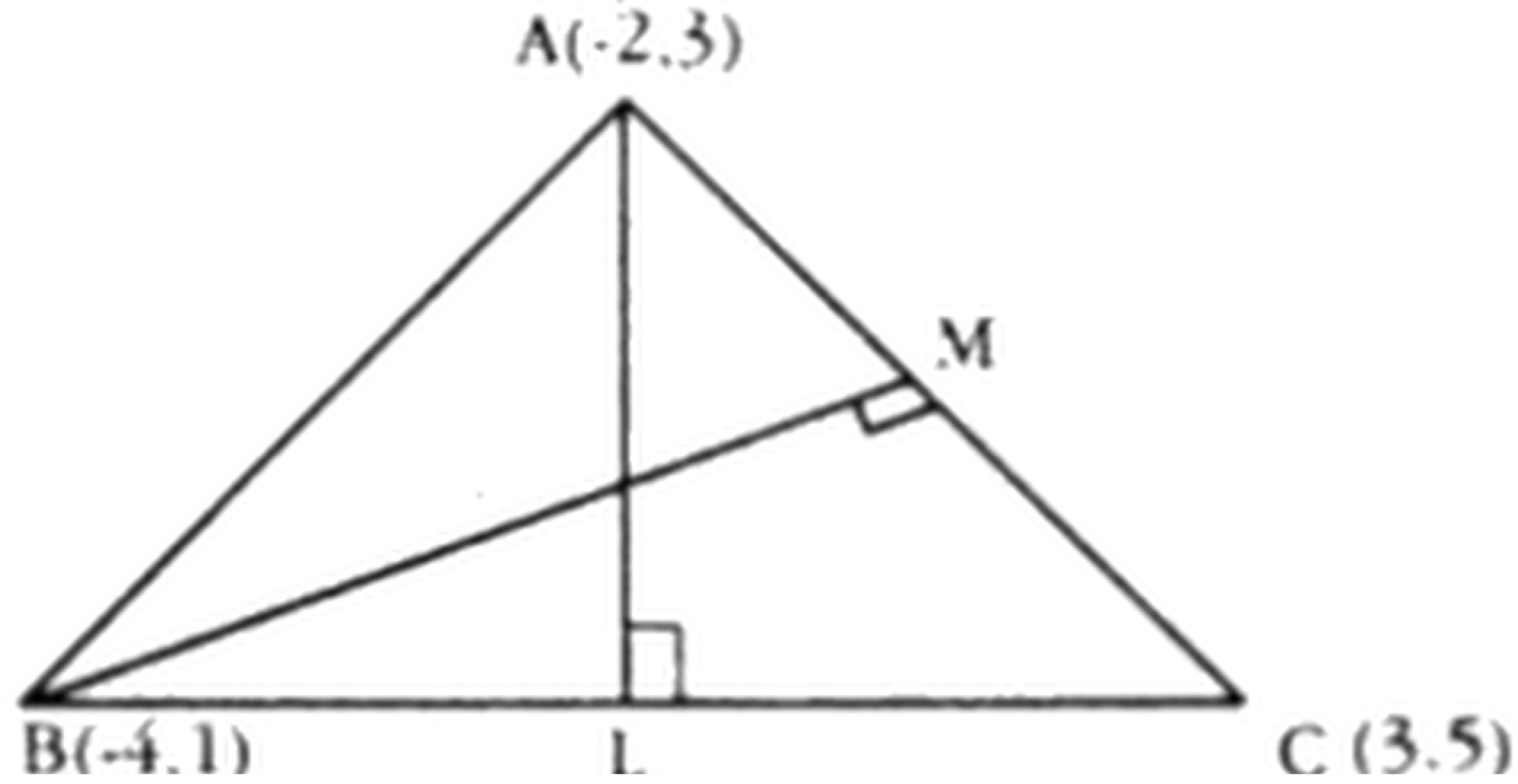
$2x-9+11 = 0$

$2x+2 = 0 \Rightarrow 2x = -2$

$\Rightarrow \quad x = -1$

(-1,3) is the centroid of triangle

i. **We know that ortho center is the point of concurrency of all the altitudes of a triangle.**



Slope of $\overline{BP}$ is $\frac{5-1}{3-(-4)} = \frac{4}{3+4} = \frac{4}{7}$

Slope of $\overline{AL} = -\frac{7}{4}$

Equation of altitude $\overline{AL}$ is

$$y-3 = -\frac{7}{4}[x-(-2)]$$

$$4(y-3) = -7(x+2)$$

$$4y-12 = -7x-14$$

$$7x+4y-12-14 = 0$$

$$7x+4y+2 = 0 \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_}(3)$$

Slope of $\overline{AC}$ is $\frac{5-3}{3-(-2)} = \frac{2}{2+3} = \frac{2}{5}$

Slope of altitude $\overline{BM} = \frac{5}{2}$

Equation of altitude $\overline{BM}$ is

$$y-1 = \frac{5}{2}[x-(-4)]$$

$$2(y-1) = -5(x+4)$$

$$2y-2 = 5x-20$$

$$5x+2y+18 = 0$$

Now we will solve equation (3) and (4) simultaneously

$$7x+4y+2 = 0$$

$$5x+2y+18 = 0$$

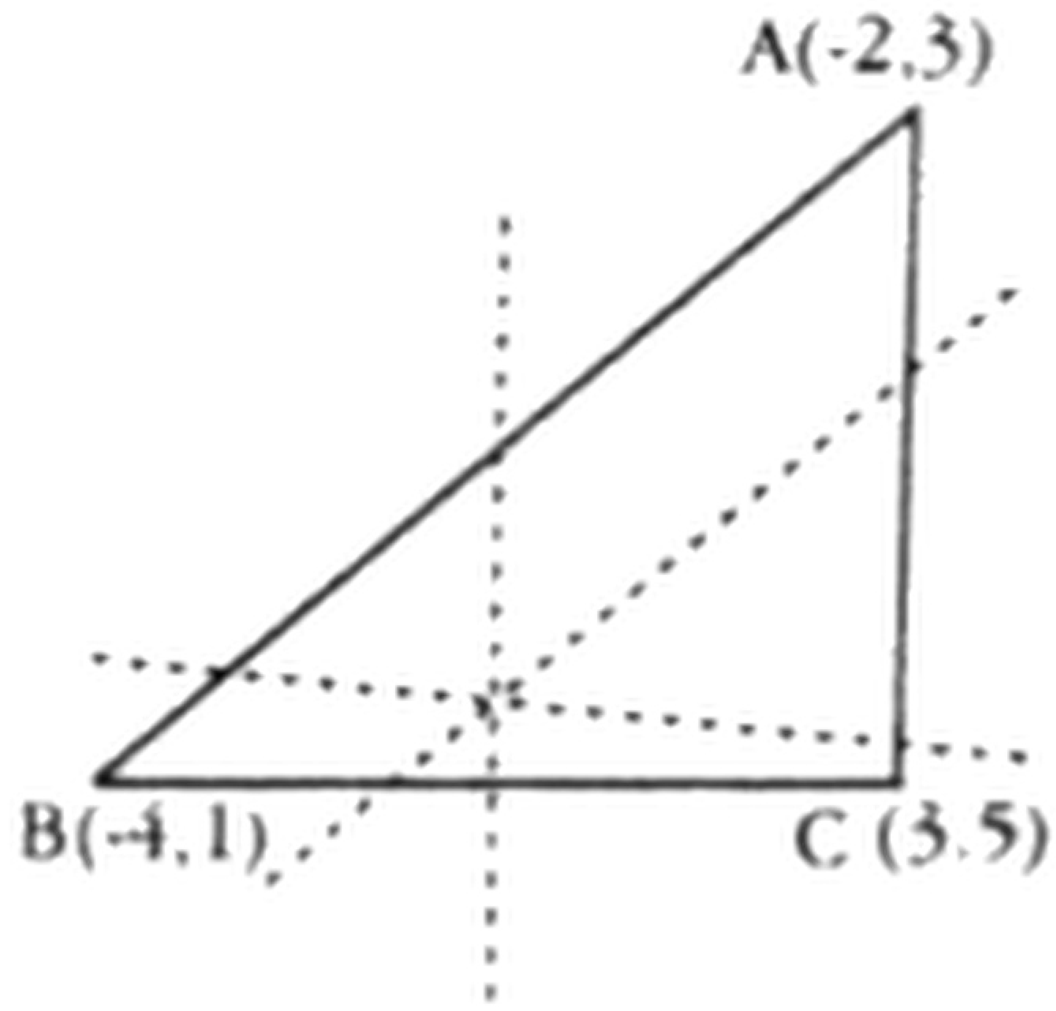
$$\frac{x}{72-4} = \frac{-y}{-116} = \frac{1}{-6}$$

x __ y __ 1

$$x = \frac{-68}{6} = -\frac{34}{3}, y = \frac{-116}{-6} = \frac{58}{3}$$

$\left[\frac{-34}{3}, -\frac{58}{3}\right]$ is the ortho center of triangle

iii We know that the circumcenter is the point of concurrency of all the right bisectors of the side a triangle.



Mid point of $\overline{BC}$ is P $\left[\frac{-1}{2}, 3\right]$

Slope of $\overline{BC}$ is $\frac{4}{7}$

Slope of the corresponding bisector is $\frac{-7}{4}$

Equation of right bisector of side $\overline{BC}$ is

$$y-3 = -\frac{7}{4}[x-(-\frac{-1}{2})]$$

$$4(y-3) = -7(x+\frac{1}{2}) = 0$$

$$4y- 12 = 7x- \frac{7}{2}$$

$$7x+4y -12+\frac{7}{2} = 0$$

$$7x+4y - \frac{24-7}{2} = 0$$

$14x + 8x - 17 = 0$

Mid point of $\overline{AC}$ is P $\left[\frac{1}{2}, 4\right]$

Slope of $\overline{AC}$ is $\frac{2}{5}$

Slope of the corresponding bisector is $\frac{5}{2}$

Equation of right bisector of side $\overline{AC}$ is

$$y-4 = -\frac{5}{2}\,[\,x-\frac{1}{2}]$$

$$2(y-4) = -5+\frac{1}{2}$$

$$5x-8y - 8 - \frac{5}{2}$$

$$5x+2y - \frac{21}{2} \quad = 0$$

$$10x + 4y - 21 = 0$$

Now we will solve equation

$$10x + 8y - 17 = 0$$

$$10x + 4y - 21 = 0$$

$$\frac{x}{-168+68} = \frac{-y}{-294+170} = \frac{1}{56-80}$$

$$\frac{x}{-100} = \frac{y}{124} = \frac{1}{-124}$$

$$x = \frac{-100}{24} = \frac{25}{6}, y = \frac{124}{24} = \frac{-31}{6}$$

$\left[\frac{25}{6}, \frac{31}{-6}\right]$ is the circumcenter of triangle.

Now we will check the centroid orthocenter circumcenter are collinear or not

Consider $\begin{vmatrix} -1 & 3 & 1 \\ \frac{-34}{3} & \frac{58}{3} & 1 \\ \frac{25}{6} & -\frac{31}{6} & 1 \end{vmatrix}$

$$= -1\left[\frac{58}{3}+\frac{13}{6}\right] - 3\left[\frac{-34}{3}-\frac{25}{6}\right] + 1\left[\frac{34}{3}\times\frac{31}{6}-\frac{58}{2}\times\frac{25}{6}\right]$$

$$= -\left[\frac{58+31}{6}\right] + 3\left[\frac{68+24}{6}\right] + \left[\frac{1054-1450}{18}\right]$$

$$= \frac{-147}{6}+\frac{279}{6}-\frac{396}{18} = \frac{-276}{6}+\frac{279}{6} = 0$$

Hence three points are collinear.

Q8. Check whether the lines

4x-3y-8 = 0 ____________(1)

3x-4y – 6 = 0 ___________(2)

x – y – x = 0 ____________(3)

are concurrent. If so, find the point where they meet.

Solution

The determinant of the coefficients of the given equation if

$$\begin{vmatrix} 4 & -3 & -8 \\ 3 & 4 & -6 \\ 1 & -1 & -2 \end{vmatrix} = 4(8-6)+3(-6+6)-8(-3+4)$$

$$= 4(2)+3(0)-8(1)$$

Thus lines are concurrent.

The point of intersection of any two lines is the required point of concurrency.

From (1) and (2) we have

$$\frac{x}{-14} = \frac{y}{0} = \frac{1}{-7}$$

$$x = \frac{-14}{-7} = 2, y = \frac{0}{7} = 0$$

Thus (2,0) is the required point of concurrency

Q9. Find the coordinates of the triangle formed by the lines

x-2y-6 = 0 _____ (1)

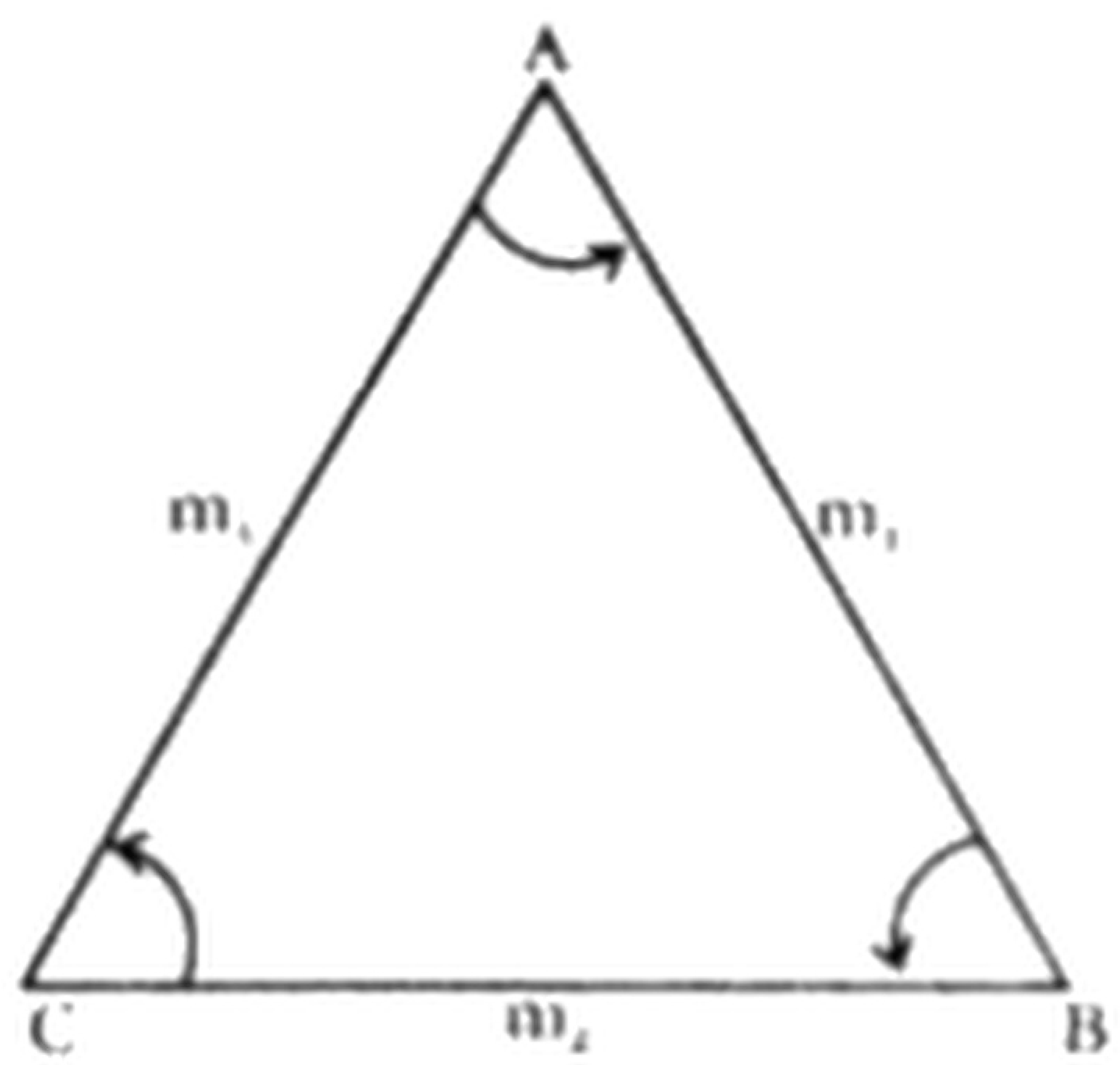
3x-y+3 = 0 _____ (2)

2x+y-4 = 0 _____ (3)

Also find measures of the angles of the triangles

Solution

Vertices of a triangle are the points of intersection of its sides.



From (1) and (2) we have.

$$\frac{x}{-6-6} = \frac{-y}{3+18} = \frac{1}{-1+6}$$

$$\frac{x}{-12} = \frac{y}{-21} = \frac{1}{5}$$

$$x = \frac{-12}{5}, y = \frac{-21}{5}$$

From (2) and (3) we have

$$\frac{x}{4-3} = \frac{-y}{-12-6} = \frac{1}{3+2}$$

$$\frac{x}{1} = \frac{y}{18} = \frac{1}{5}$$

$$x = \frac{1}{5} \;, y = \frac{18}{5}$$

$\left[\frac{1}{5}, \frac{18}{5}\right]$ is the vertex formed by the lines (2) and (3)

From (1) and (3) we have

$$\frac{x}{8+6} = \frac{-y}{-4+12} = \frac{1}{1+4}$$

$$\frac{x}{14} = \frac{y}{-8} = \frac{1}{5}$$

$$x = \frac{14}{5} \;, y = \frac{-8}{5}$$

$\left[\frac{14}{5}, \frac{-8}{5}\right]$ is the vertex formed by the lines (1) and (2)

Now we will find measures of the angles of the triangle with vertex A $\left[\frac{12}{5}, \frac{21}{5}\right]$

B$\left[\frac{1}{5}, \frac{18}{5}\right]$ C$\left[\frac{14}{5}, \frac{-8}{5}\right]$

$$m_1 = Slope\ of\ \overline{AB} = \frac{\frac{18}{5} - \left[\frac{21}{5}\right]}{\frac{21}{5} - \left[\frac{21}{5}\right]}$$

$$= \frac{\frac{18}{5} + \frac{21}{5}}{\frac{21}{5} + \frac{12}{5}} = \frac{39}{13} = 3$$

$$m_2 = Slope\ of\ \overline{BC} = \frac{-\frac{8}{5} - \frac{18}{5}}{\frac{14}{5}\,1\,\frac{1}{5}} = \frac{-8-18}{14-1}$$

$$= \frac{-26}{13} = -2$$

$$m_3 = Slope\ of\ \overline{AC} = \frac{\frac{8}{5} - \left[\frac{21}{5}\right]}{\frac{14}{5} - \left[\frac{21}{5}\right]}$$

$\frac{8}{\;} + \frac{21}{\;}$ $\quad -8+21$

$$= \frac{13}{26} = \frac{1}{2}$$

$$\Rightarrow \quad m\angle A = \text{Tan}^{-1}\left[\frac{m_1 - m_2}{1 + m_1 m_2}\right]$$

$$\Rightarrow \quad \text{Tan}^{-1}\left[\frac{3-\frac{1}{2}}{1+3.\frac{1}{2}}\right] = \text{Tan}^{-1}\left[\frac{6-1}{2+3}\right]$$

$$\Rightarrow \quad \tan^{-1}\left[\frac{5}{5}\right] = \tan^{-1} 1 = 45^\circ$$

$$\angle B = \text{Tan}^{-1}\left[\frac{-2-3}{1+(-2)(3)}\right]$$

$$= \text{Tan}^{-1}\left[\frac{-5}{1-6}\right] = \tan^{-1}\left[\frac{-5}{-5}\right]$$

$$= \text{Tan}^{-1} 1 = 45^\circ$$

$$\angle C = \text{Tan}^{-1}\left[\frac{m_3 - m_2}{1 + m_3 \quad m_2}\right]$$

$$\Rightarrow \quad \text{Tan}^{-1}\left[\frac{\frac{1}{2}-(-2)}{1+\frac{1}{2}.(-2)}\right] = \text{Tan}^{-1}\left[\frac{\frac{1}{2}+2}{1-1}\right]$$

$$\Rightarrow \quad \text{Tan}^{-1}\left[\frac{\frac{1}{2}+2}{0}\right] = \text{Tan}^{-1}(\infty) = 90^\circ$$

Thus measure of the angles of the triangle are $45^\circ, 45^\circ, 90^\circ$.

Hence ΔABC is an isosceles right-angled triangle.

Q10. Find the angle measured from ℓ_1 to the line ℓ_2 where (a)

ℓ_1 = Joining (2,7) and (7,10)

ℓ_2 = Joining (1,1) and (-5,3)

Also find acute angle in each case

Solution

$$m_1 = \text{Slope of } \ell_1 = \frac{10-7}{7-2} = \frac{3}{5}$$

Angles measure from ℓ_1 to ℓ_2 ***given*** by

$$\text{Tan}\,\theta = \frac{m_1 - m_2}{1 + m_1\ m_2} = \frac{\frac{3}{5} + \frac{1}{3}}{1 + (\frac{3}{5}).(\frac{3}{5})}$$

$$= \left[\frac{\frac{14}{15}}{\frac{12}{15}}\right] = \frac{7}{6}$$

Required acute angle is

$$\theta = \tan^{-1}\left[\frac{7}{6}\right] = 49.4^{\circ}$$

b $\quad$ **ℓ_1 = Joining (3,-1) and (5,7)**

ℓ_2 = Joining (2,4) and (-8,2)

Solution

$$m_1 = \, = \frac{7+1}{2} = \frac{8}{2} = 4$$

$$m_2 = \text{Slope of } \ell_2 = \frac{2-4}{-8-2} = \frac{-2}{-10} = \frac{1}{5}$$

Angles measure from ℓ_1 to ℓ_2 *given* by

$$\text{Tan}\,\theta = \frac{m_1 - m_2}{1 + m_1\ m_2} = \frac{4 - 1/5}{1 + 4.(\frac{1}{5})}$$

$$= \left[\frac{19/15}{9/15}\right] = \frac{19}{9}$$

Required acute angle is

$$\theta = \tan^{-1}\left[\frac{19}{9}\right] = 64.65^{\circ}$$

c. $\quad$ **ℓ_1 = Joining (1,-7) and (6,4)**

ℓ_2 = Joining (-1,2) and (-6,-1)

$m_1 \;=\;$ Slope of $\ell_1 = \frac{-4-(-7)}{6-1}$

$= \frac{-4+7}{5} \;=\; \frac{3}{5}$

$m_2 \;=\;$ Slope of $\ell_2 = \frac{-1-2}{-6-(-1)}$

$= \frac{-3}{-6+1} \;=\; \frac{3}{5}$

$$\theta \quad = \quad \tan^{-1}\left[\frac{\frac{3}{5}-\frac{3}{5}}{1+\frac{3}{5}\times\frac{3}{5}}\right] = \tan^{-1}(0) \;=\; 0^{\circ}$$

Acute angle is $= 0^{\circ}$

c ℓ_1 = Joining (-9,-1) and (3,-5)

ℓ_2 = Joining (2,7) and (-6,-7)

Solution

$m_1 \;=\;$ Slope of $\ell_1 = \frac{-5-(-1)}{3-(-9)} \qquad = \frac{-5+1}{9} \;=\; \frac{-4}{12} \;=\; \frac{-1}{3}$

$m_2 \;=\;$ Slope of $\ell_2 = \frac{-7-7}{-6-} \;=\; \frac{14}{-8} \;=\; \frac{7}{4}$

Angles from ℓ_1 to ℓ_2 is

$$\theta \quad = \quad \tan^{-1}\left[\frac{\frac{7}{4}-\left[-\frac{3}{5}\right]}{1+\left[\frac{7}{4}\right]\left[\frac{-1}{3}\right]}\right] \;=\; \tan^{-1}\left[\frac{\frac{7}{4}+\frac{3}{5}}{1-\frac{7}{12}}\right]$$

$$= \;\tan^{-1}\left[\frac{\frac{21+4}{2}}{\frac{12\quad 7}{4}}\right] = \tan^{-1}\left[\frac{\frac{25}{12}}{\frac{5}{12}}\right] = \tan^{-1}(5) \;=\; 78.69^{\circ}$$

Q11. Find the interior angles of the triangle whose vertices are (a). A(-2,11), B(-6,-3),C(4,-9)

Solution

$m_1 = \text{Slope of } \overline{AB}$

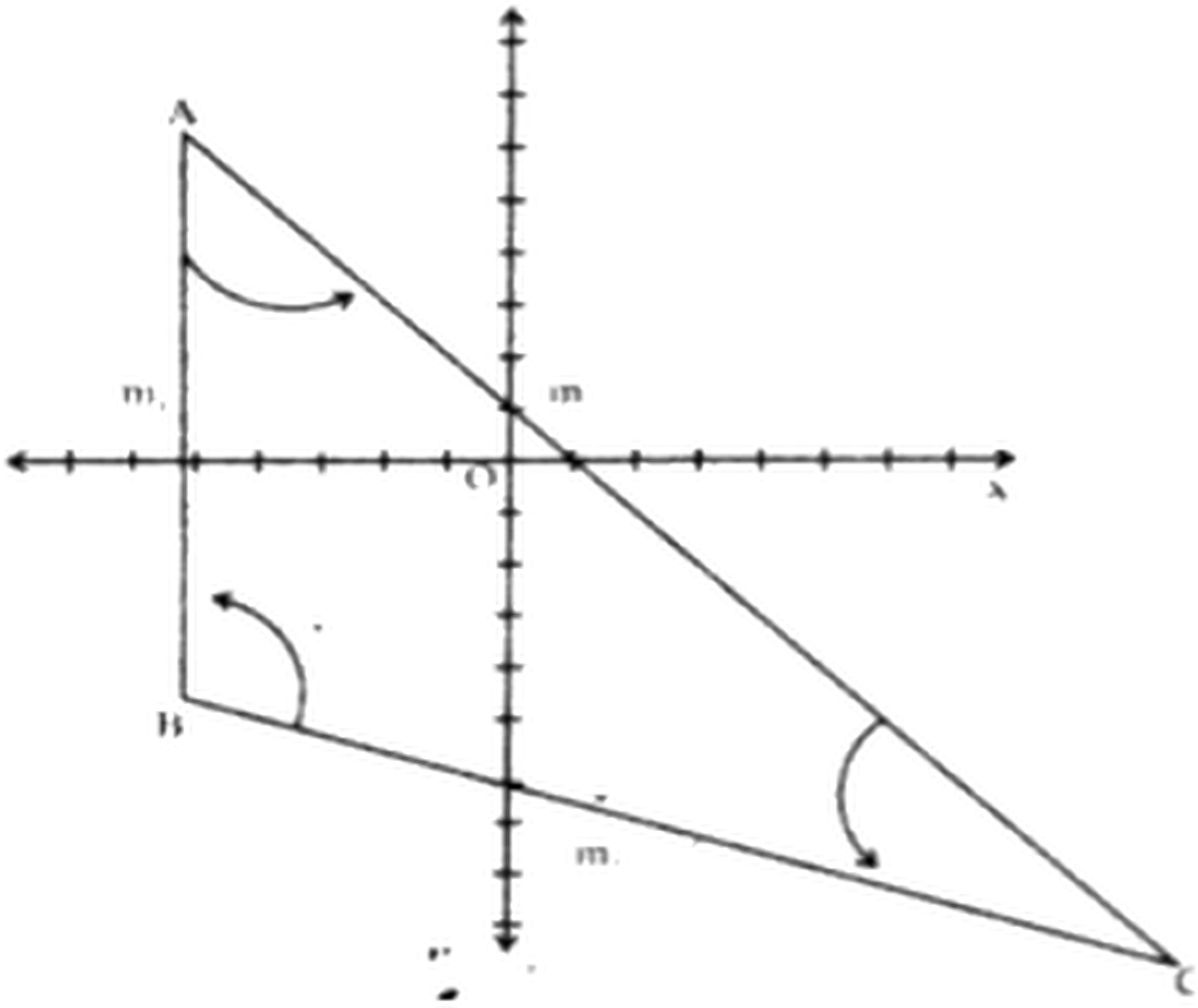
$$= \frac{-3-4}{-6-(-1)} \qquad = \frac{-14}{-6+2} = \frac{-14}{4} = \frac{7}{2}$$

$m_2 = \text{Slope of } \overline{BC}$

$$= \frac{-9-(-3)}{4-(-6)} \qquad = \frac{-9+3}{4+6} = \frac{-6}{10} = -\frac{3}{3}$$

$m_3 = \text{Slope of } \overline{AC}$

$$= \frac{-9-11}{4-(-2)} \qquad = \frac{-20}{4+2} = \frac{-20}{6} = -\frac{10}{3}$$



Now angle A is measured from $\overline{AB}$ to $\overline{AC}$

$$\text{Tan}\,(m\angle A) = \left[\frac{m_1 - m_2}{1 + m_1 m_2}\right] = \frac{-\frac{10}{3} - \frac{7}{2}}{1 + \left(-\frac{10}{3}\right)\cdot\left(\frac{7}{12}\right)}$$

$$= \frac{-20-21}{6-70} = \frac{-41}{-64} = \frac{41}{64}$$

$$\Rightarrow \quad m\angle A = \tan^{-1}\left[\frac{41}{64}\right] = 32.64^\circ$$

$$\text{Tan } (m\angle B) = \left[\frac{m_1 - m_2}{1+ m_1 m_2}\right] = \frac{\frac{7}{2}+\frac{7}{35}}{1+[\frac{7}{2}].[-\frac{3}{5}]} = \tan^{-1}\left[-\frac{41}{11}\right] = 105.02^{\circ}$$

Similarly

$$\Rightarrow \quad m\angle C = \tan^{-1}\left[\frac{41}{25}\right] = 42.34^{\circ}$$

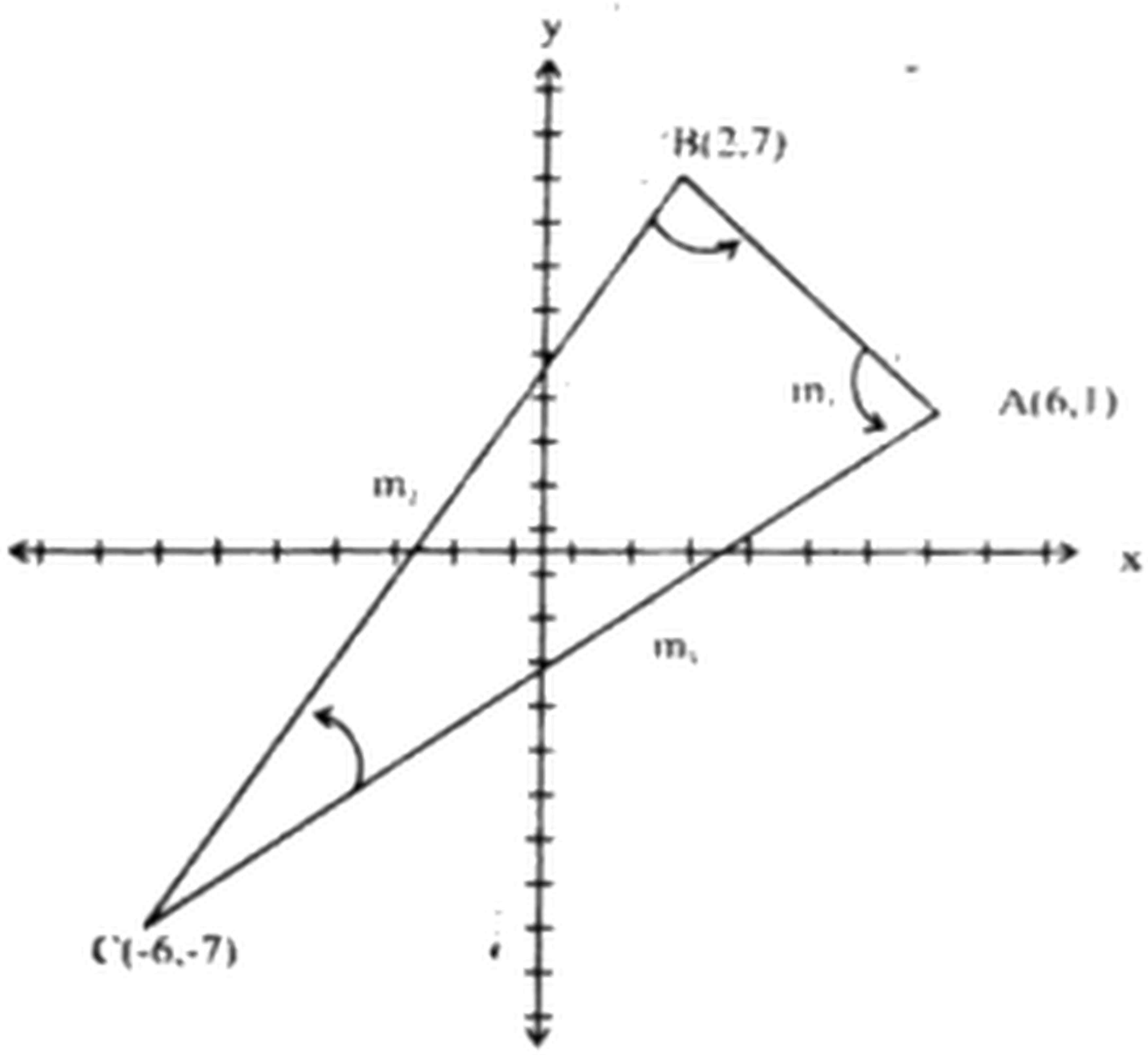
b A(6,1)B(2,7) C(-6,-7)

Solution

$$m_1 = \text{Slope of } \overline{AB} = \frac{7-1}{2-6} = \frac{6}{-4} = \frac{-3}{2}$$

$$m_2 = \text{Slope of } \overline{BC} = \frac{-7-7}{-6-2} = \frac{-4}{-8} = \frac{7}{-8}$$

$$m_3 = \text{Slope of } \overline{AC} = \frac{-7-1}{-6-6} = \frac{-8}{-12} = \frac{2}{3}$$



$$\text{Tan } (m\angle A) = \left[\frac{m_3 - m_1}{1+ m_{13} m_1}\right] = \frac{\frac{2}{3}-[-\frac{7}{2}]}{1+[\frac{2}{3}].[\frac{3}{2}]}$$

$$\Rightarrow \quad m\angle A = \tan^{-1}\infty = 90^\circ$$

$$\text{Tan}\,(m\angle B) = \left[\frac{m_1 - m_2}{1+ m_1 m_2}\right] = \frac{-\frac{3}{2}-\frac{7}{4}}{1+[-\frac{3}{2}].[-\frac{7}{4}]}$$

$$= \frac{\frac{6\quad 7}{4}}{\frac{8\quad 21}{8}} = -\frac{13}{4} \times \left[\frac{-8}{12}\right] = 2$$

$$m\angle B = \tan^{-1} 2 = 63.4^\circ$$

$$\text{Tan}\,(m\angle C) = \left[\frac{m_2 - m_3}{1+ m_2 m_3}\right] = \frac{\frac{7}{4}-\frac{2}{3}}{1+[\frac{7}{4}].[-\frac{2}{3}]}$$

$$= \frac{\frac{21\quad 8}{12}}{\frac{12+14}{12}} = \frac{13}{26} = \frac{1}{2}$$

$$m\angle C = \tan^{-1}\left[\frac{1}{2}\right] = 26.5^\circ$$

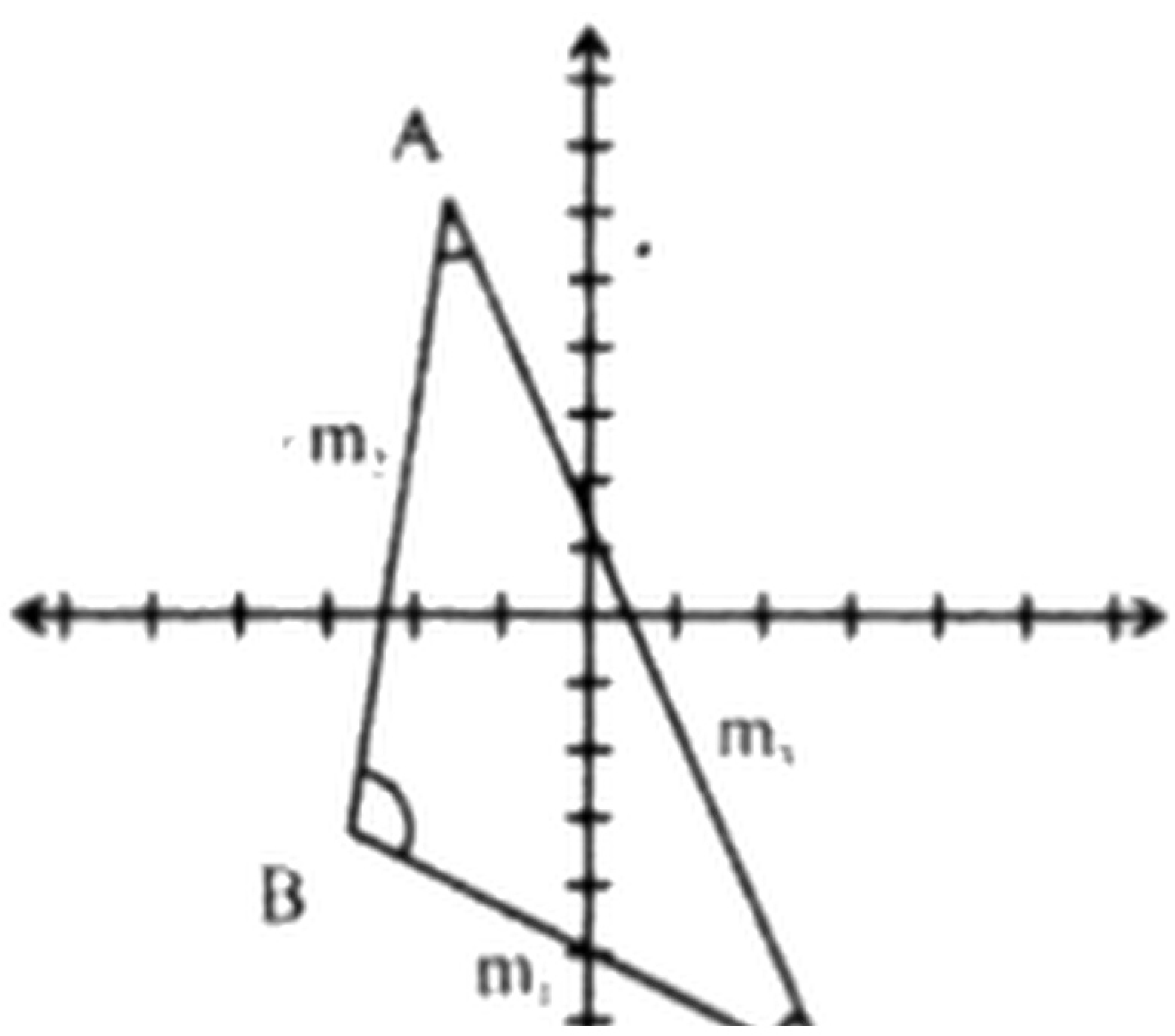
c A(2,-5)B(-4,-3) C(-1,5)

Solution

$$m_1 = \text{Slope of } \overline{AB} = \frac{-3-(-5)}{-4-2} = \frac{-2}{6} = -\frac{1}{2}$$

$$m_2 = \text{Slope of } \overline{BC} = \frac{5-(-3)}{-(-4)} = \frac{5+3}{-1+4} = \frac{8}{3}$$

$$m_3 = \text{Slope of } \overline{AC} = \frac{5-(-5)}{-1-2} = \frac{5+5}{-1-2} = -\frac{10}{3}$$



$$\text{Tan}\,(m\angle A) = \left[\frac{m_1 - m_3}{1 + m_1\, m_3}\right] = \frac{-\frac{1}{3} - [-\frac{10}{3}]}{1 + [-\frac{1}{3}].[\frac{10}{3}]}$$

$$= \frac{\frac{1}{3} + \frac{10}{3}}{1 + \frac{10}{9}} = \frac{\frac{1+10}{3}}{\frac{9\ \ 10}{9}} = \frac{9}{3} \times \frac{9}{19}$$

$$m\angle A = \text{Tan}^{-1}\left[\frac{27}{19}\right] = 54.8^\circ$$

$$\text{Tan}\,(m\angle B) = \left[\frac{m_2 - m_1}{1 + m_2\, m_1}\right] = \frac{\frac{8}{3} - [-\frac{1}{3}]}{1 + [\frac{8}{3}].[-\frac{1}{3}]}$$

$$= \frac{\frac{8+1}{3}}{\frac{9\ \ 8}{9}} = \frac{9}{3} \times \left[\frac{9}{1}\right] = 27$$

$$m\angle B = \tan^{-1}(2) = 87.9^\circ$$

$$\text{Tan}\,(m\angle C) = \left[\frac{m_3 - m_2}{1 + m_2\, m_3}\right] = \frac{-\frac{10}{3} - \frac{8}{3}}{1 + [-\frac{10}{3}].[\frac{8}{3}]}$$

$$= \frac{\frac{10\ \ 8}{3}}{1 - \frac{80}{12}} = 6 \times \frac{9}{-71} = \frac{54}{71}$$

$$m\angle C = \tan^{-1}\left[\frac{54}{71}\right] = 37.2^\circ$$

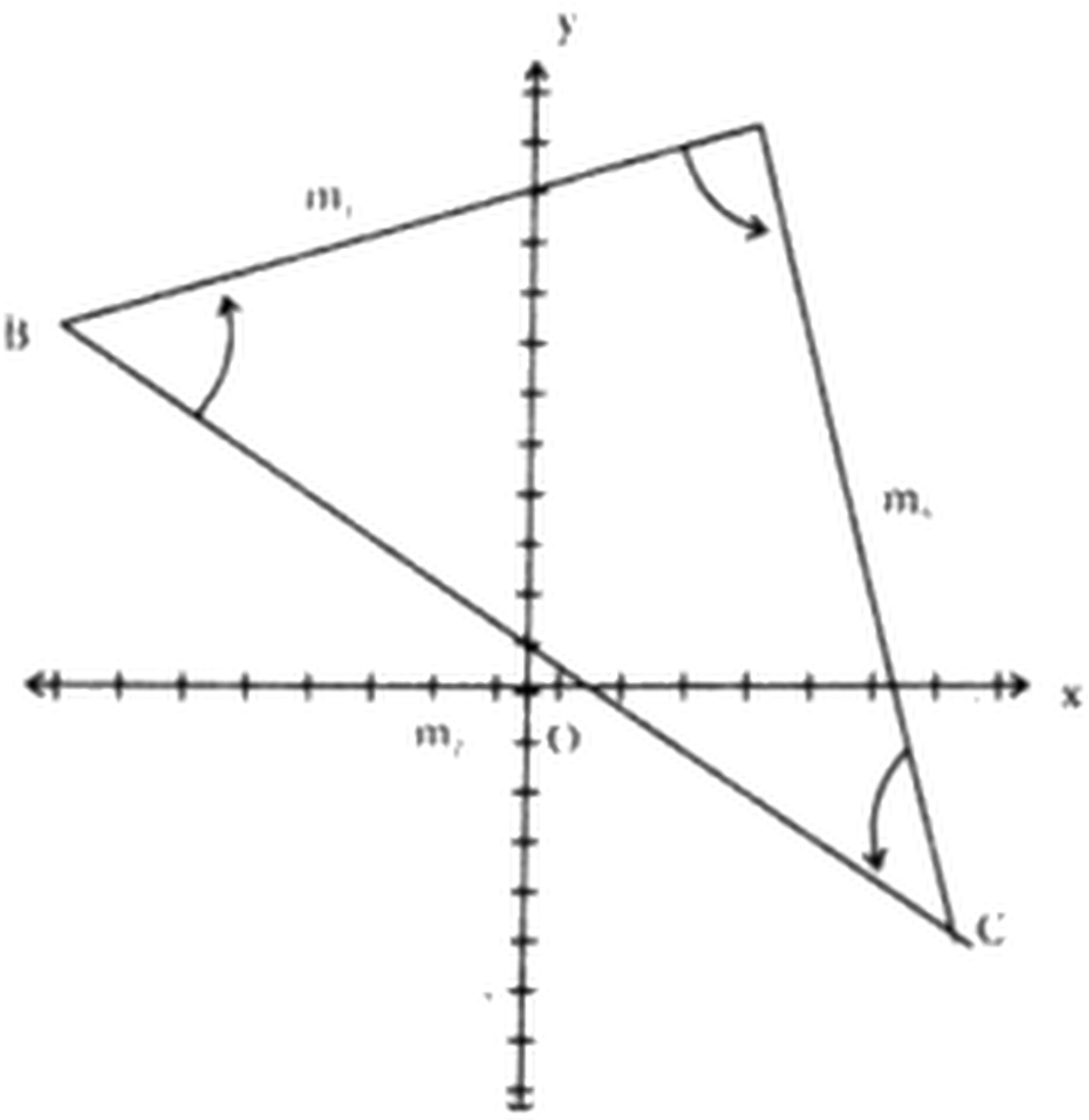
d. A(2,8)B(-5,4) C(4,-9)

Solution

$$m_1 = \text{Slope of } \overline{AB} = \frac{4-8}{-5-2} = \frac{-4}{-7} = \frac{4}{7}$$

$$m_2 = \text{Slope of } \overline{BC} = \frac{-9-4}{4-(-5)} = \frac{-9-4}{4+5} = \frac{-13}{9}$$

$$m_3 = \text{Slope of } \overline{AC} = \frac{9-8}{4-2} = \frac{-17}{2}$$



$$\text{Tan }(m\angle A) = \left[\frac{m_3 - m_1}{1+ m_{13}\, m_1}\right] = \frac{-\frac{17}{2}-\frac{7}{4}}{1+[\frac{2}{3}].[-\frac{3}{2}]}$$

$$= \frac{\frac{117\ \ 8}{14}}{1-\frac{68}{14}} = \frac{\frac{127}{4}}{\frac{14\ \ 68}{14}} = \frac{-127}{-5} = 67^{\circ}$$

$$\text{Tan }(m\angle B) = \left[\frac{m_1 - m_2}{1+ m_1\, m_2}\right] = \frac{\frac{4}{7}-[-\frac{13}{9}]}{1+[\frac{4}{7}].[-\frac{17}{9}]} = \frac{\frac{4}{7}+\frac{13}{9}}{1-\frac{52}{63}}$$

$$= \frac{\frac{36+91}{63}}{\frac{63\ \ 52}{63}} = \frac{127}{11}$$

$$m\angle B = \tan^{-1}\left[\frac{127}{11}\right] = \tan^{-1}(11.54) = 85^{\circ}$$

$$\text{Tan }(m\angle C) = \left[\frac{m_2 - m_3}{1+ m_2\, m_3}\right] = \frac{-\frac{13}{9}+\frac{17}{2}}{1+[-\frac{13}{9}].[-\frac{17}{2}]}$$

$$m\angle C = \tan^{-1}\left[\frac{127}{239}\right] = \tan^{-1}(0.531) = 28^{\circ}$$

Q12. Find the interior angles of the quadrilateral whose vertices are A(5,2),B(-2,3)C(-3,-4)D(4,-5).

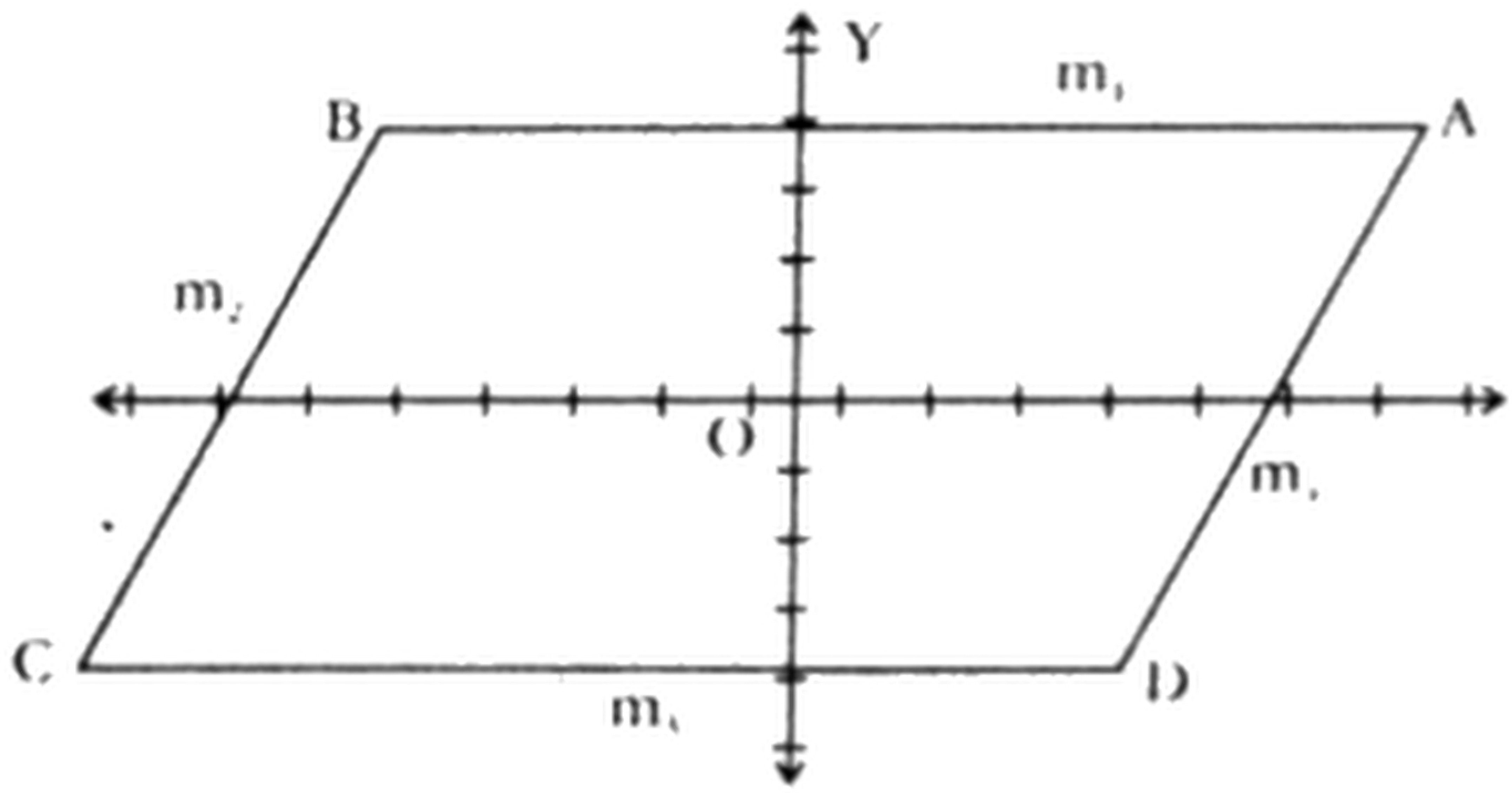
Solution

$m_1 = \text{Slope of } \overline{AB} = \frac{-3-2}{-2-5} = \frac{1}{-7}$

$m_2 = \text{Slope of } \overline{BC} = \frac{-4-3}{-3-(-2)} = \frac{-4-3}{-3+2} = \frac{-7}{-1} = 7$

$m_3 = \text{Slope of } \overline{CD} = \frac{-5(-4)}{4-(3)} = \frac{-5+4}{4+3} = \frac{1}{-7}$

$m_4 = \text{Slope of } \overline{AD} = \frac{-5-2}{4-5} = \frac{-7}{-1} = 7$



$\text{Tan } (m\angle A) = \left[\frac{m_4 - m_1}{1+ m_4 m_1}\right] = \frac{7-(-\frac{1}{7})}{1+7(\frac{1}{7})} = \frac{7+\frac{1}{7}}{1-1} = \infty$

$m\angle A = \tan^{-1}(\infty) = 90^\circ$

$\text{Tan } (m\angle B) = \left[\frac{m_1 - m_2}{1+ m_1 m_2}\right] = \frac{-\frac{1}{7}-7}{1+(\frac{1}{7})(7)} = \frac{-\frac{1}{7}-7}{1-1} = \infty$

$m\angle B = \tan^{-1}\infty = 90^\circ$

$\text{Tan } (m\angle C) = \left[\frac{m_2 - m_3}{1+ m_2 m_3}\right] = \frac{7-(-\frac{1}{7})}{1+7(\frac{1}{7})} = \frac{7+\frac{1}{7}}{1-1} = \infty$

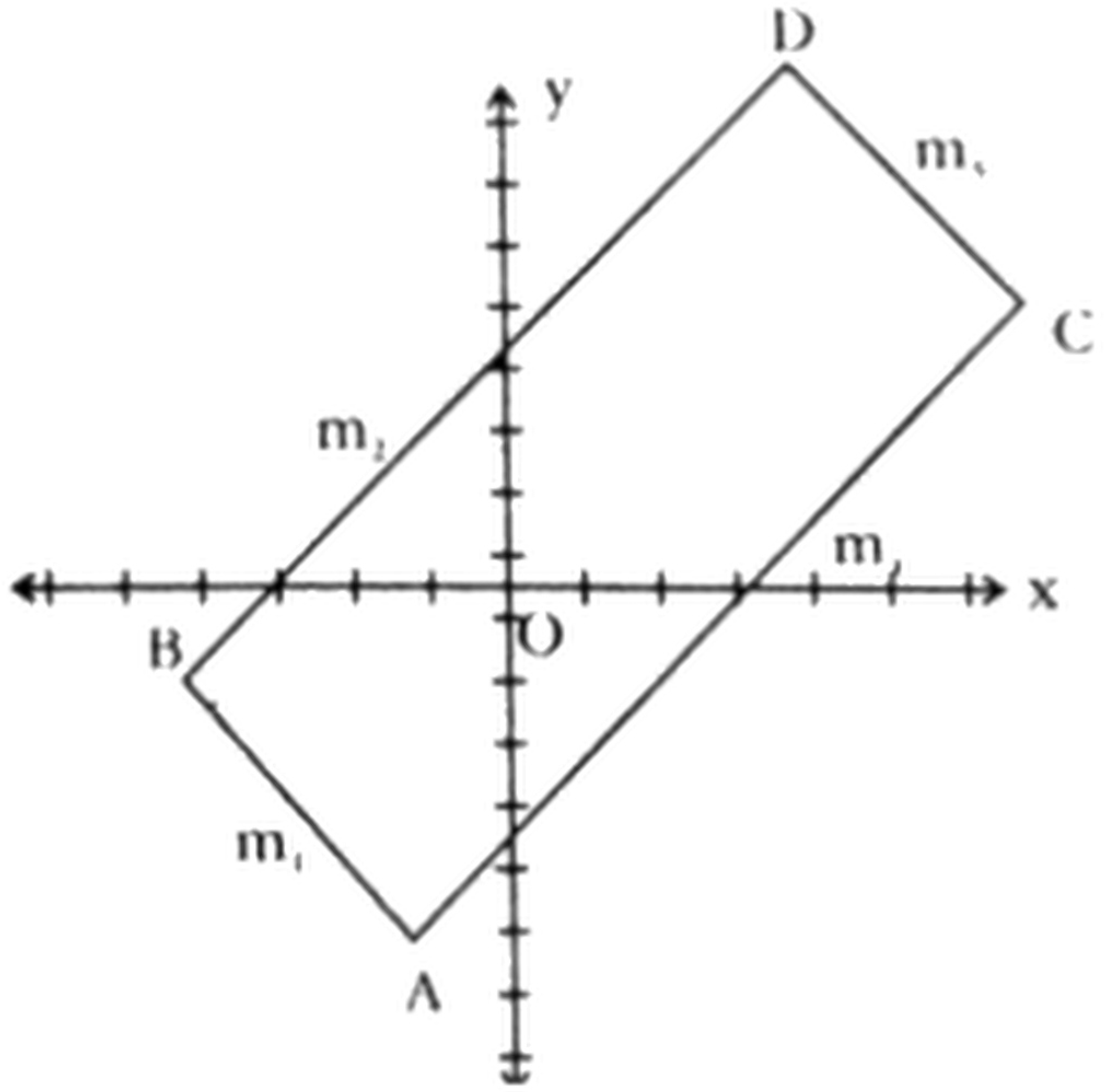
$m\angle C = \tan^{-1}\infty = 90^\circ$

$\text{Tan } (m\angle D) = \left[\frac{m_3 - m_4}{1+ m_3 m_4}\right] = \frac{-\frac{1}{7}-7}{1+(\frac{1}{7})(7)} = \frac{-\frac{1}{7}-7}{1-1} = \infty$

$m\angle D = \tan^{-1}\infty = 90^\circ$

Q13. Show that the points A(-1,-1), B(-3,0), C(3,7) and D(1,8) vertices of a trapezium. Find its interior angles.

Solution:



A trapezium is a four sided figure with a pair of sides parallel

m_1 = Slope of $\overline{AB} = \frac{-0-(-1)}{-3-1(-1)} = \frac{1}{-3+1} = -1/2$

m_2 = Slope of $\overline{BD} = \frac{8-0}{-1-(-3)} = \frac{8}{1+3} = 2$

m_3 = Slope of $\overline{CD} = \frac{8-7}{1-3} = \frac{1}{-2}$

m_4 = Slope of $\overline{AC} = \frac{7-(-1)}{-3-(-1)} = \frac{7+1}{3+1} = 2$

Opposite sides are parallel, so ABCD is a parallelogram (in particular trapezium)

Tan (m∠A) $= \left[\frac{m_3 - m_4}{1+ m_3\ m_4}\right] = \frac{-\frac{1}{2}-2}{1+(-\frac{1}{2})(2)} = \frac{-\frac{1}{2}-2}{1-1} = \infty$

m∠A $= \tan^{-1}\infty = 90°$

7x-y-10 = 0 __________(1)

10x +y -41 = 0 __________(2)

3x +2y + 3 = 0 __________(3)

Solution

From (1) and (2)

$$\frac{x}{(-1)(-41)-(1)(-10)} = \frac{-y}{(7)(-41)-(10)(-10)}$$

$$= \frac{1}{(7)(1)-(10)(-1)}$$

$$\frac{x}{41+40} = \frac{y}{187} = \frac{1}{7+10}$$

$$x = \frac{51}{17} = 3\,, y = \frac{187}{17} = 11$$

The intersection of (1) and (2) is (3,11) from (2),(3)

$$\frac{x}{(1)(3)-(2)(-41)} = \frac{-y}{(10)(3)-(3)(-41)}$$

$$= \frac{1}{(10)(2)-(3)(1)}$$

$$\frac{x}{3+82} = \frac{-x}{30+123} = \frac{1}{20-3}$$

$$\frac{x}{85} = \frac{y}{-153} = \frac{1}{17}$$

$$x = \frac{85}{17} = 5\,, y = \frac{153}{17} = -9$$

Thus intersection of (1) and (2) is (5,-9)

From (1),(3)

$$\frac{x}{(-1)(3)-(2)(-10)} = \frac{-y}{(7)(3)-(3)(-10)}$$

$$= \frac{1}{(7)(2)-(3)(-1)}$$

$$\frac{x}{17} = \frac{y}{-51} = \frac{1}{17}$$

$$x = \frac{17}{17} = 1 , y = \frac{-51}{17} = -3$$

Thus intersection of (1) and (3) is (1,-3).

Thus the triangle formed by intersection of the given lines has its vertices as A(3,11),B(5,-9),C(1,-3)

$$\text{Area of } \Delta \text{ ABC} \begin{vmatrix} 3 & 11 & 1 \\ 5 & -9 & 1 \\ 1 & -3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[3(-9+3) - 11(5-1) + 1(-15+9)]$$

$$= \frac{1}{2}[3(-68) - 11(4) + 1(-6)]$$

$$= \frac{1}{2}[-18 - 44 - 6]$$

$$= \frac{1}{2}(-68)$$

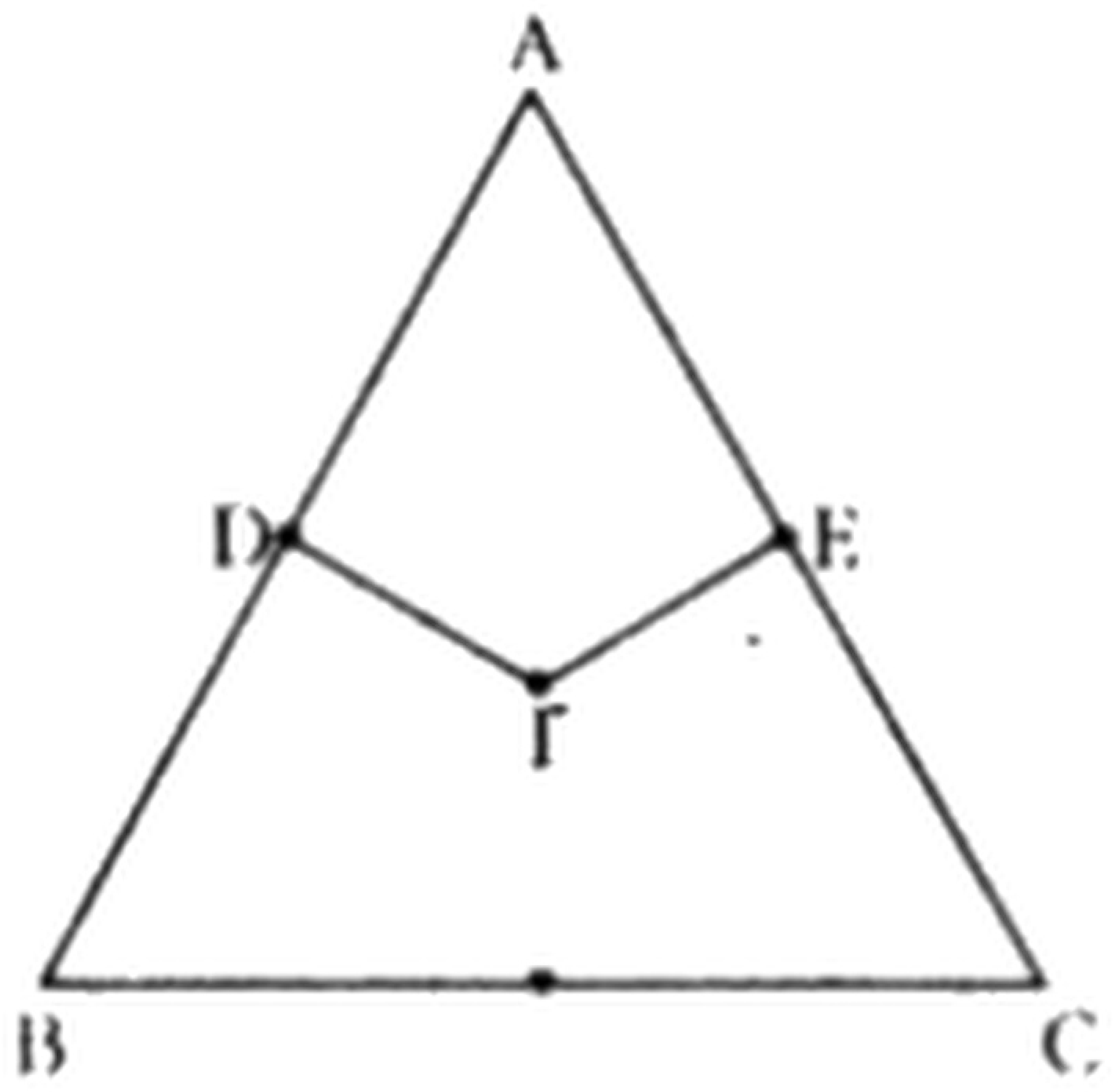
$$= -34$$

Or Area of Δ ABC = 34 (omitting the negative sign).

Q15. The vertices of a triangle are A(-2,3),B(-4,1) , C(3,5). Find the center of circum circle of a triangle as the point of intersection of the perpendicular bisectors of the sides.

Solution

Given the vertices A(-2,3), B(-4,1) , C(3,5)



Mid Point of $\overline{BC}$ IS D $\left(\frac{-4+32}{2}, \frac{1+5}{2}\right) = D\left(-\frac{1}{2}, 3\right)$

Slope of $\overline{BC}$ is $\frac{5-1}{3-(-4)} = \frac{4}{3+4} = \frac{4}{7}$

Eq of perpendicular bisector of $\overline{BC}$ is

$$y - 3 = -\frac{7}{4}\left(x - \left(-\frac{1}{2}\right)\right)$$

$$4\ (y-3) = -7\left(x + \frac{1}{2}\right)$$

$$4y-12 = -7x\frac{1}{2}$$

$$7x+4y-12+\frac{7}{2} = 0$$

$$7x+4y\ 24 - 7 = 0$$

$$14x + 8y-17 = 0$$

Mid Point of $\overline{AC}$ IS $\left(\frac{-2+3}{2}, \frac{3+5}{2}\right) = E\left(\frac{1}{2}, 4\right)$

Slope of $\overline{AC}$ is $\frac{5-3}{3-(-2)} = \frac{2}{3+2} = \frac{2}{5}$

Eq of perpendicular bisector of $\overline{AC}$ is

$$y - 4 = -\frac{5}{2}\left(x - \left(\frac{1}{2}\right)\right)$$

$2y-8 = -5x+\frac{5}{2}$

$5x+2y-8\frac{5}{2} = 0$

$7x+2y \frac{16+5}{2} = 0$

$10x + 4y-21 = 0$ ________(2)

The centre of circumference of a triangle ABC is the intersection of (1) and (2)

$14x + 8y-1 = 0$

$10x + 4y - 21 = 0$

$$\frac{x}{-168+68} = \frac{-y}{-294+170} = \frac{1}{56-80}$$

$$\frac{x}{-100} = \frac{y}{124} = \frac{1}{-24}$$

$$x = \frac{-100}{-24} = \frac{25}{6}, y = \frac{124}{24} = -\frac{31}{6}$$

i.e $\left(\frac{25}{6}, -\frac{31}{6}\right)$ is required circumference

Q16. Expression the given system of equation in matrix form. Find each case whether the lines are concurrent

a. $x+3y-2 = 0$

$2x-y+4 = 0$

$x- 11y +14 = 0$

Solution

The matrix eq of system is

$$\begin{vmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{vmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{vmatrix}$$

$= 1(-14+44)-3(28-4)-2(-22+1)$

$= 30-3(24-2(-21)$

$= 30-72+42$

$= 72-72$

$= 0$

Thus A is a singular matrix and so the lines are concurrent.

b. **2 x+3y+4 = 0**

x-2y-3 = 0

3x+y -8 = 0

Solution

The matrix eq of system is

$$\begin{bmatrix} 2 & 3 & 4 \\ 1 & -2 & -3 \\ 3 & 1 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Coefficient matrix of the system is

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -2 & -3 \\ 3 & 1 & -8 \end{bmatrix}$$

$= 2(16+3)-3(-8+9)-4(1+6)$

$= (19)-3(1)-4(7)$

$= 38-3-28$

$= 7 \neq 0$

Thus A is non-singular matrix and so the lines are not concurrent.

C 3x-4y-2 = 0

x+2y-4 = 0

3x- 2y +5 = 0

Solution

The matrix form of system is

$$\begin{vmatrix} 3 & -4 & -2 \\ 1 & 2 & -4 \\ 3 & -2 & 5 \end{vmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Coefficient matrix of the system is

$$A = \begin{bmatrix} 3 & -4 & -2 \\ 1 & 2 & -4 \\ 3 & -2 & 5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & -4 & -2 \\ 1 & 2 & -4 \\ 3 & -2 & 5 \end{vmatrix}$$

$= 3(10\text{-}8)+4(5+12)\text{-}2(\text{-}2\text{-}6)$

$= 3(2)+4(17)\text{-}2(\text{-}8)$

$= 6+68+16$

$= 90 \neq 0$

As Δ is non singular matrix so the lines are not concurrent.

Q17. Find a system of linear equations corresponding to the given matrix form. Check whether the lines represent by a sys are concurrent.

a: $$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & 1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solution

Multiplying the matrix on the L.H.S of (1), we have

$$\begin{bmatrix} x & +0y & -1 \\ 2x & +0y & +2 \\ 0x & -0 & +2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Therefore using the definition of equality of two matrix.

We have form (2),x-1 = 0, 2x +1 – 0, -y+2 = 0 are required sys of eq the co-efficient matrix of sys is

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & 1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 0 & -1 \\ 2 & 0 & 1 \\ 0 & -1 & 2 \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix}$$

$$= 1+2 = 3 \neq 0$$

Hence the lines of systems are not concurrent

b: $$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solution

Multiplying the matrix on the L.H.S of (1) , we have

$$\begin{bmatrix} x & +y & +2 \\ 2x & +4y & -3 \\ 3x & +6y & -5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

x+y+2 = 0

2x+4y-3 = 0

Are required sys of eq.

The co-efficient Sys is

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{vmatrix}$$

$= 1(-20+18)-1(-10+9)+2(12-12)$

$= (-2)-1(-1)+2(0)$

$= -2+1$

$= -1 \neq 0$

Thus A is not singular matrix

Hence the lines of systems are not concurrent.