Exercise 4.3

Q1. Find the slope and angle of Inclination of the line joining the points. Sketch the line in each case.

i. **(-2,4), (5,11)**

Solution

Let A(-2,4) and B(5,11)

Slope of AB = m $\frac{11-4}{5+2} = \frac{7}{7} = 1$

m $= \tan\theta = 1$ ⇒ inclination = θ = $\tan^{-1} 1 = 45^{\circ}$

Let A(-2,3), B(2,7) be the given point,

Now A(3,-2) and B(2,7)

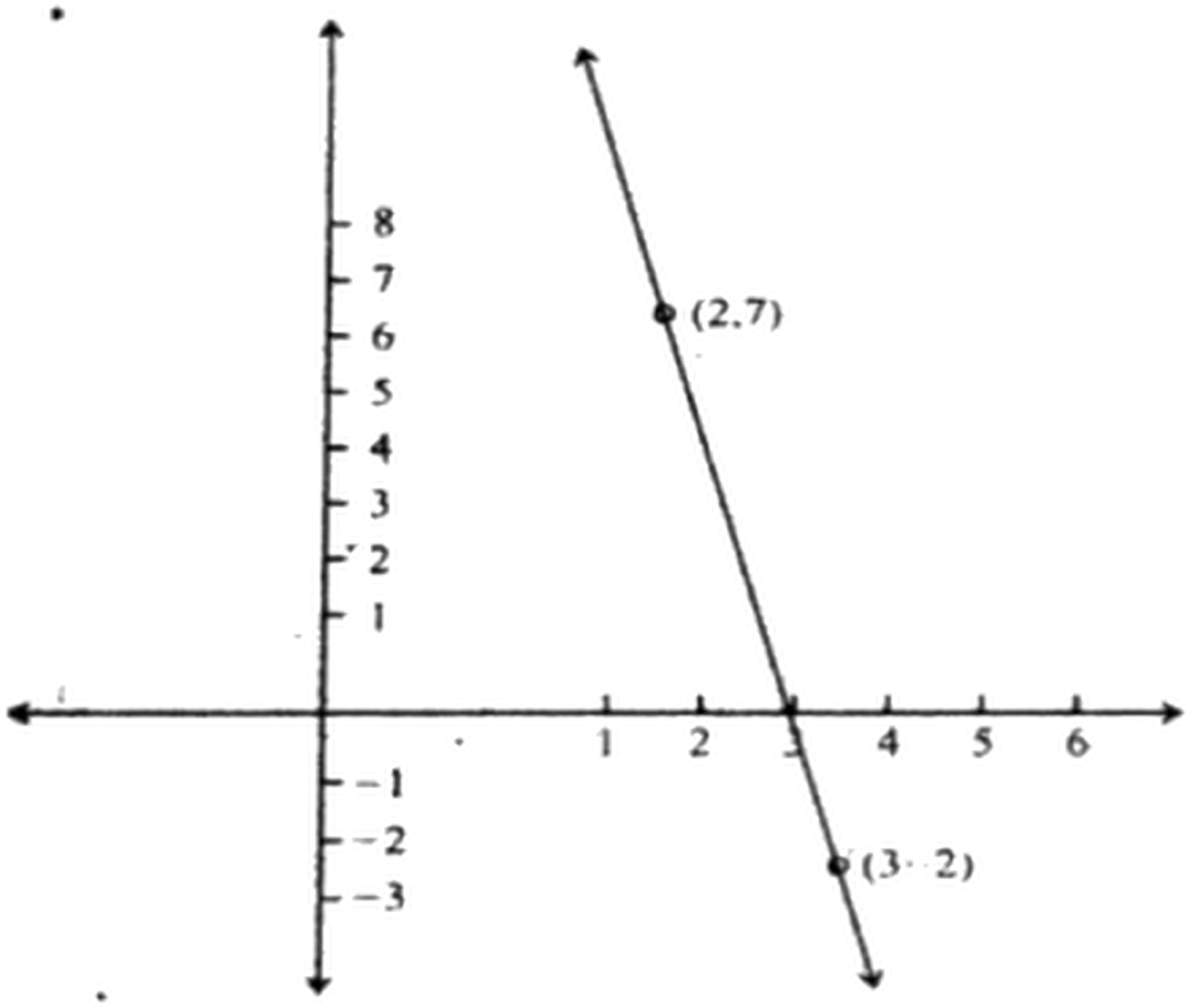
Slope of AB = $m = \frac{7+2}{2-3} = \frac{9}{-1} = -9$

m $= \tan\theta = -9$ ⇒ inclination = θ = $\tan^{-1}(-9)$

= -83.66

= 180- 83.66

= 96.34

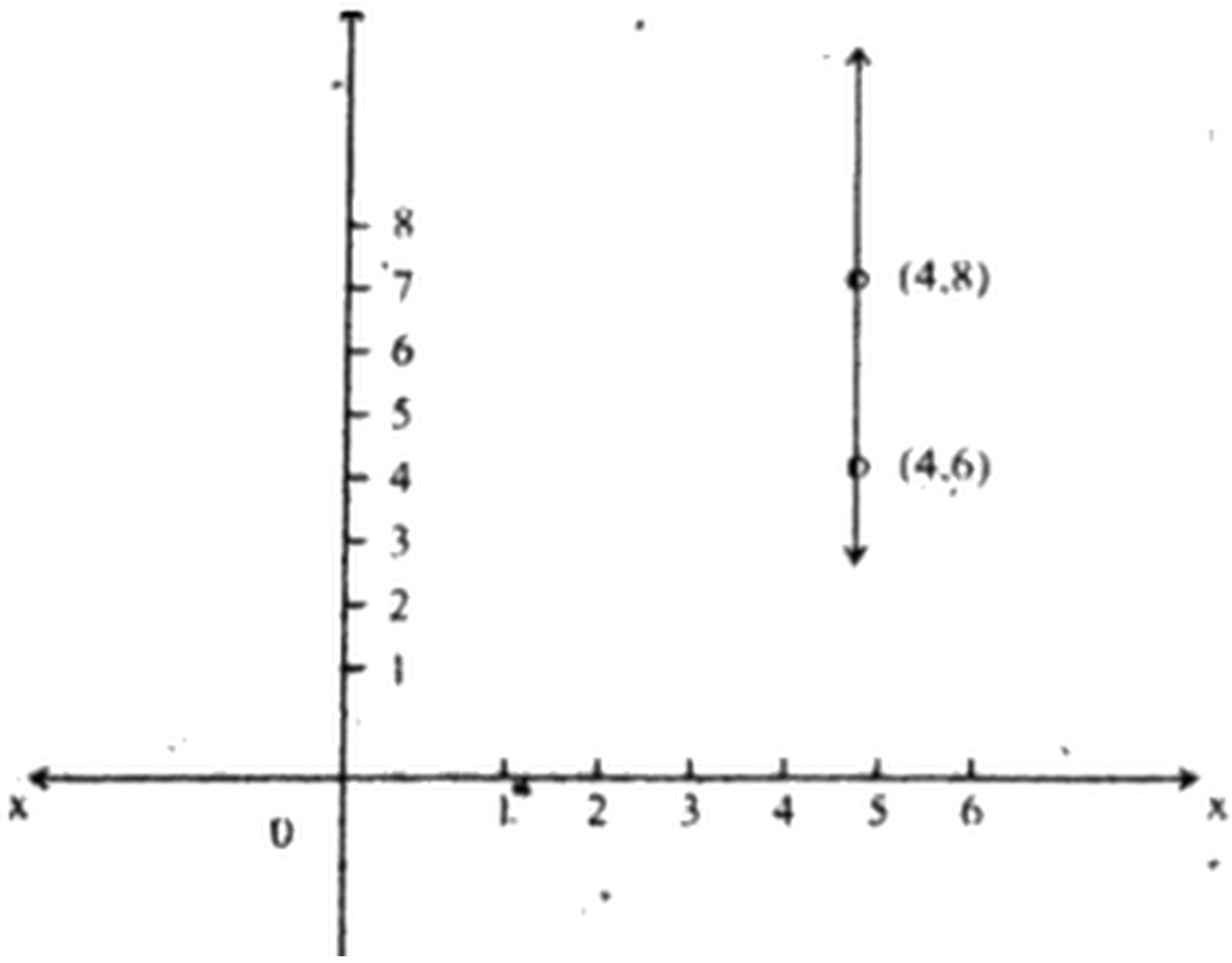


Let A(4,6), B(4,8) be the given point,

Now A(4,6) and B(4,8)

Slope of AB = $m = \frac{8-6}{4-4} = \frac{2}{0} = \infty$ (undefined)

$m = \tan\theta \Longrightarrow$ inclination $= \theta = \tan^{-1}(\infty) = 90^\circ$



Q2. In the triangle A(8,6), C(-2,-61) find the slope of

i. **Each side of the triangle**

ii. **Each median of the triangle**

iii. **Each altitude of D triangle**

Solution

Slope of $\overline{AB} = m_1 = \frac{y_2-y_1}{x_2-x_1} = \frac{2-6}{-4-8} = \frac{-4}{12} = \frac{1}{3}$

Slope of $\overline{BC} = m_2 = \frac{-6-2}{-2-(-4)} = \frac{-8}{2} = -4$

Slope of $\overline{AC} = m_3 = \frac{y_3-y_1}{x_3-x_1} = \frac{-6-6}{-2-8} = \frac{-1}{10} = \frac{6}{5}$

ii. **Let D,E,F be the mid-point of the sides $\overline{BC}$, $\overline{AC}$, $\overline{BC}$ respectively.**

$$D\left(\frac{-4-2}{2}, \frac{2-6}{2}\right) = (-3,-2)$$

Co-ordinates of $E\left(\frac{8-2}{2}, \frac{6-6}{2}\right) = (3,0)$

Co-ordinates of $F\left(\frac{8-4}{2}, \frac{6+2}{2}\right) = (2,4)$

Slope of median $\overline{AD} = \frac{-2-6}{-3-8} = \frac{-8}{-11} = \frac{8}{11}$

Slope of median $\overline{BE} = \frac{0-2}{3-(-4)} = \frac{-2}{3+4} = \frac{-2}{7}$

Slope of median $\overline{CF} = \frac{4-(-6)}{4-(-2)} = \frac{4+6}{2+2} = \frac{5}{2}$

iii. **Slope of altitude through vertex A**

$$= \frac{-1}{\text{Slope of BC}}$$

$$= \frac{-1}{\cdot}$$

Slope of Altitude of B $= \frac{-1}{\text{Slope of AC}} = \frac{-1}{\frac{6}{5}} = \frac{-5}{6}$

Slope of Altitude of vertex C $= \frac{-1}{\text{Slope of AB}} = \frac{-1}{\frac{1}{3}} = -3$

Q3. By means of slopes, show that the following point, lie on a line or collinear.

a. (-1,-3), (1,5), (2,9)

Solution

Let A(-1,-3), B(1,5), C(2,9)

Slope of the line through $m_1 = \overline{AB} = \frac{5-(-3)}{1-(-1)} = \frac{5+3}{2} = \frac{8}{2} = 4$

Slope of the line through $m_2 = \overline{BC} = \frac{9-5}{2-1} = \frac{4}{1} = 4$

Slope of the line through $m_3 = \overline{CA} = \frac{9-(-3)}{2-(-1)} = \frac{9+3}{3} = \frac{12}{3} = 4$

$\therefore$ All slopes are equal. i.e. $m_1 = m_2 = m_3$

b. Let A(4,-5), B(7,5),C(10,15)

m_1 = Slope of $\overline{AB} = \frac{5-(-5)}{7-4} = \frac{5+5}{3} = \frac{10}{3}$

m_2 = Slope of $\overline{BC} = \frac{15-5}{10-7} = \frac{10}{3}$

m_3 = Slope of $\overline{AC} = \frac{15-(-5)}{10-4} = \frac{15+5}{6} = \frac{10}{3}$

$\therefore \quad m_1 = m_2 = m_3$

Points, are collinear

c. (a,2b),(c , a+b),(2c-a,2a)

Solution

Slope of $\overline{AB}$ $= m_1 = \frac{(a+b)-(2b)}{c-a} = \frac{a-b}{c-a}$

Slope of $\overline{BC}$ $= m_2 = \frac{2a-(a+b)}{2c-a-c} = \frac{2a-a-b}{c-a} = \frac{a-b}{c-a}$

Slope of $\overline{AC}$ $= m_3 = \frac{2a-2b}{2c-a-a} = \frac{2(a-b)}{2c-2a}$

$= \frac{2(a-b)}{2c-2a} = \frac{a-b}{c-a}$

$m_1 = m_2 = m_3$

$\Rightarrow$ A,B,C are collinear

Q4. Find K so the line joining A(7,3),B(K,-6) and the line joining C(-4,5), D(-6,4) are (i) Parallel

(ii) Perpendicular.

Solution

Slope of $\overline{CB}$ $= m_2 = \frac{4-5}{-6-(-4)} = \frac{4-5}{-6+4}$

$\frac{-1}{-2} = \frac{1}{2}$

Slope of $\overline{AB}$ $= \frac{-6-3}{K-7} = \frac{-9}{K-7}$

i. $\overline{AB} \parallel \overline{CD}$

$m_1 = m_2$

$\Rightarrow \frac{-9}{K-7} = \frac{1}{2}$

$K = -18+7$

$\Rightarrow K = -11$

$\Rightarrow \quad 9 \quad = -2(K-7)$

2K-14= 9

$K = \frac{23}{2}$

Q5. Using Slope, show that the D with vertices A(6,1),B(2,7)and C(-6,-7) is a right triangle.

Solution

Slope of $\overline{AB} = m_1 = \frac{7-1}{2-6} = \frac{6}{-4} = \frac{-3}{2}$

Slope of $\overline{BC} = m_2 = \frac{-7-7}{-6-2} = \frac{-14}{-8} = \frac{7}{4}$

Slope of $\overline{AC} = m_3 = \frac{-7-7}{-6-6} = \frac{-18}{-12} = \frac{2}{3}$

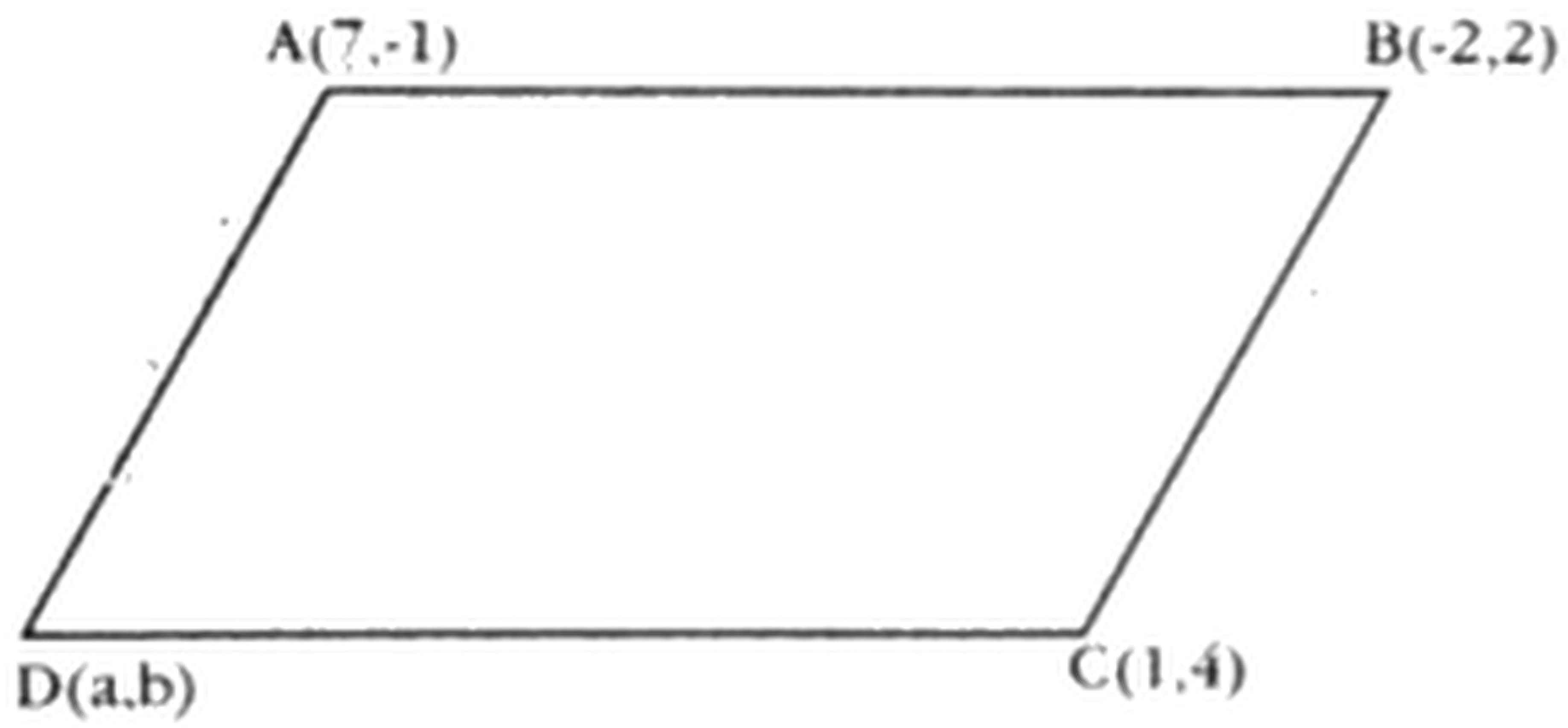
$(m_1)(m_2) = \frac{2}{3} \times \frac{3}{2} = -1$

$\Rightarrow$ DAB (i) right angle with $\angle A = 90^\circ$

$\Rightarrow \overline{AB} \perp \overline{CD}$

Q6. The points A(7,1), B(-2,2) and C (1,4) are the consecutive vertices of parallelogram. Find the fourth vertex.

Solution



Let D be the fourth vertex of parallelogram

m_1 = Slope of $\overline{AD} = \frac{2+1}{-2-7} = \frac{-1}{3}$

m_2 = Slope of $\overline{BC} = \frac{4-2}{1+2} = \frac{2}{3}$

m_3 = Slope of $\overline{AB} = \frac{2-(-1)}{-2-7} = \frac{-1}{3}$

m_4 = Slope of $\overline{CD} = \frac{4-b}{1-a}$

ABCD is ∥gram

$\Rightarrow \frac{b+1}{a-7} = \frac{2}{7}$

$\Rightarrow$ 2a-3b-17 = 0

$\Rightarrow$ Slope of $\overline{AD}$ = Slope of $\overline{CD}$

i.e. $m_1 = m_2$

$\frac{b+1}{a-7} = \frac{2}{3}$ $\Rightarrow$ 2a-3b-17 = 0 ________(1)

Also Slope of $\overline{AD}$ = Slope of $\overline{CD}$

$\Rightarrow$ $m_3 = m_4$

$\frac{-1}{3} = \frac{4-b}{1-9}$

$\Rightarrow$ a+3b-13 = 0 ___________(2)

Putting (1,2)

$\Rightarrow$ 3a=30 = 0

$\Rightarrow$ a = 10

Put a = 10 in ___________(1)

2(10)-3b-17 = 0

$\Rightarrow$ 20-3b-17 = 0

$\Rightarrow$ 3b = 3

Hence D(10,1)

Q7. The point A(-1,2),B(3,-1) and C(6,...) are consecutive vertices of a rhombus. Find the fourth vertex and show that the diagonal of the rhombus are perpendicular to each other.

Solution

Let D(a ,b) the IV vertex.

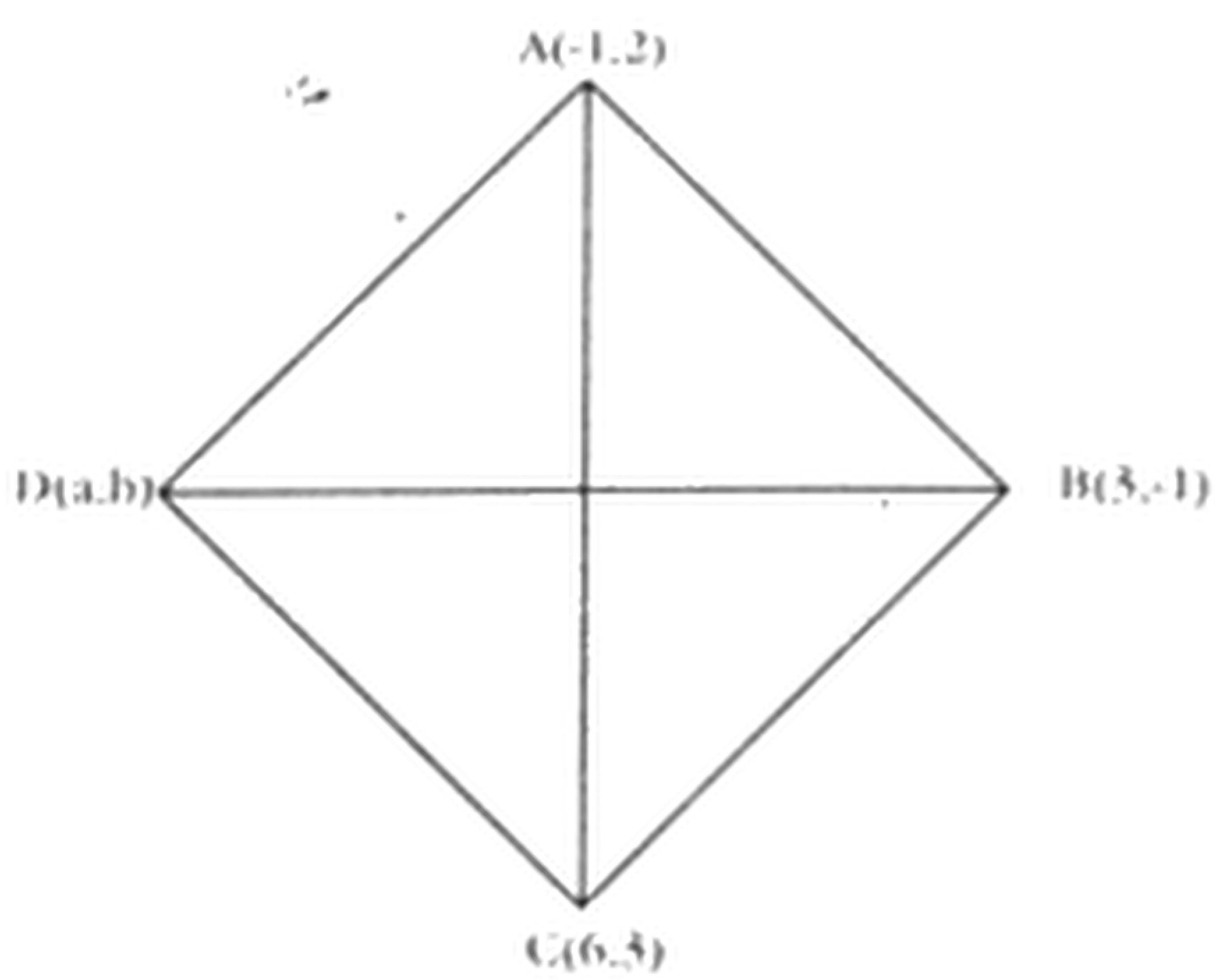
Slope of $\overline{AB}$ $= m_1 = \frac{-1-2}{...} = \frac{-3}{4}$

Slope of $\overline{CD}$ $= m_2 = \frac{b-3}{a-b}$

Slope of $\overline{BC}$ $= m_3 = \frac{3+1}{6-3} = \frac{4}{3}$

Slope of $\overline{AD}$ $= m_4 = \frac{b-2}{a-(-1)} = \frac{b-2}{a+1}$

Rhombus is a parallelogram



Slope of $\overline{AB}$ = Slope of $\overline{CD}$

$$\frac{-3}{4} = \frac{b-3}{a-6}$$

⇒ 3a+4b-30-30 = 0 ______(1)

$\Rightarrow \quad \frac{4}{3} = \frac{b-2}{b+1}$

$\Rightarrow \quad 4a+4 = 3b-6$

$\Rightarrow \quad 4a-3b+10 = 0$ ________(2)

By solving (1),(2) simultaneously

Multiplying (1) by -4 and (2) by (3)

$-12a-166+120 = 0$

$12a-9b+30 = 0$

Adding (3) and (4)

$-25b+150 = 0 \Rightarrow 25b = 150 \Rightarrow b = \frac{150}{25}$

$b = 6$ put in (1)

$3a+24-30 = 0$

$3a-6 = 0$

$3a = 6$

$\Rightarrow \quad a = 2$

This Co-ordinate of IV vertex are (1,6)

Slope of diagonal $\overline{AC} = \frac{3-2}{6+1} = \frac{1}{7}$

Slope of diagonal $\overline{BD} = \frac{-(-1)}{a-3} = \frac{6+1}{2-3} = \frac{7}{-1} = -7$

Slope of diagonal $\overline{AC}$ = Slope of diagonal $\overline{BD}$

Hence diagonal of given rhombus are ⊥ to each other.

Q8. Two Pairs of points are given. Find whether the two lines, determined by these points are (i) parallel (ii) perpendicular (iii) none

Let A(1,-2), B(2,4),C(4,1)

Slope of $\overline{AB}$ = $m_1 = \frac{4-(-2)}{2-1} = \frac{4+2}{1} = 6$

Slope of $\overline{CD}$ = $m_2 = \frac{2-1}{-8-4} = \frac{-1}{12}$

$\Rightarrow \quad m_1 \neq m_2$

$\Rightarrow$ neither AB ∥ CD or AB ⊥ CD

b. Let A(-3,4),B(6,2),C(4,5),D(-2,-7)

Solution

Slope of $\overline{AB}$ = $m_1 = \frac{2-4)}{6-(-3)} = \frac{-2}{9}$

Slope of $\overline{CD}$ = $m_2 = \frac{-7-5}{-2-4} = \frac{-12}{-6} = 2$

$\Rightarrow$ Lines are neither parallel nor perpendicular

Q9. Find an equation of

a. The horizontal through (7,-9)

Solution

Required line $y = -9$

Because $y - y_1 = m(x - x_1)$

$Y + 9 = 0(x-7)$

$Y = -9$

b. The vertical line through (-5,3)

Required line $x = -5$

c. The line bisecting the Ist and IIIrd quadrants.

Solution

Slope $= m = \tan 45^\circ = 1$

$Y = mx = (1)x \Rightarrow x = y$

d. The line bisecting the IInd and IV quadrants

Solution

Slope $= m = \tan 135^\circ = -1$

$Y = mx = (-1)x = -x \Rightarrow x = -y$

Q10. Find an equation of the line.

a. Through A(-6,5) having slope 7

Solution

$m = 7$, Pt. $(-6,5)$

$y - y_1 = m(x - x_1)$

$Y - 5 = 7(x-(-6))$

$Y - 5 = 7x+42$

$7x - y + 42 + 5 = 0$

$7x - y + 47 = 0$

Solution

$$m = 0, Pt. (8,-3)$$

$$y - y_1 = m(x - x_1)$$

$$Y - (-3) = 0(x-6)$$

$$Y + 3 = 0$$

$$\Rightarrow \quad y = -3$$

c. Through (-8,5) having slope undefined

Solution

$$m = x, Pt. (-8,5)$$

$$y - y_1 = m(x - x_1)$$

$$Y - 5 = \infty\,(x-(-8))$$

$$\frac{y-5}{\infty} = x + 8$$

$$\Rightarrow \quad x+8 = 0$$

$$\Rightarrow \quad x = -8$$

d. Through (-5,-3) and (9,-1)

Solution

$P_1(-5,-3)$, $P_2(9,-1)$

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$$\frac{y+3}{-1+3} = \frac{x+5}{9+5}$$

$Y+3 = \frac{x+5}{7}$

$7y+21 = x+5$

$x-7y-16 = 0$

e. Y-Intercept-7, Slope-5

Solution

We use slope intercept form

$Y = mx + c$

$= 5(x)+(-7)$

$Y+5x+7 = 0$

f. X-Intercept-9 and Slope 4

Solution

First we find C

At x-axis , y =0

$\Rightarrow \quad 0 = 4(-9)+c$

$\Rightarrow \quad C = 0$

Now we use slope-intercept form

$Y = mc$

$4x+y+36 = 0$

g. X-Intercept = a = -3 **y-intercept =b = 4**

Using intercept form

$\frac{x}{-3}+\frac{y}{4}=1$

Multiplying -12

4x-3y = -12

$\Rightarrow$ 4x-3y+12 = 0

Q11. Find an equation of the perpendicular bisector joining. The Points A(3,5) and B(9,6).

Solution

Mid – Point of $\overline{AB}$ I, $\left(6,\frac{13}{2}\right)$

Slope of $\overline{AB}$ = $\frac{1}{2}$

By Pt, Slope form

$Y-8=\frac{1}{2}(x,-9)$

$\Rightarrow$ 2y- 16 = x -9

$\Rightarrow$ x-2y+7 = 0

Let 2x+y+K = 0 _______(1)

If it passes through $\left(6,\frac{13}{2}\right)$then

$2(8)+\frac{13}{2}$ = K = 0

$12+\frac{13}{2}$ + K = 0

$\Rightarrow$ $\frac{24+13}{2}$ +K = 0

$\Rightarrow$ K = $\frac{-37}{2}$

Put in (1)

$2x + y - \frac{-37}{2} = 0$

$4x + 2y - 37 = 0$

Q12. Find an equation of the sides, altitudes and medians of a triangle whose vertices are A(-3,2),B(5,4) and C(3,-8)

Solution

Slope of $\overline{AB}$ $= m_1 = \frac{4-2}{5-(-3)} = \frac{2}{5+3} = \frac{1}{4}$

Slope of $\overline{BC}$ $= m_2 = \frac{-10}{3+3} = \frac{-10}{6} = \frac{-5}{3}$

Mid- point of $\overline{BC}$ = (1,3)

Mid- point of $\overline{BC}$ = (4,-2)

Mid- point of $\overline{AC}$ = (0,-3)

(i) Now we find equation of sides $\overline{AB}$ by point-slope form

y-2=1/4 (x-(-3))

4(y-2)=x+3

4y-8=x+3

x-4y+11=0

equation of sides $\overline{BC}$, Y-4 = 6(X-5)

y-4 = 6x-30

6x-y-26 = 0

Equation of sides $\overline{AC}$ i. y-2 = -5/3 (x-(3))

3(y-2) = -5(x+3)

5x+3y+9=0

Slope of line ⊥ to $\overline{AB}$ = -4

Slope of line ⊥ to $\overline{BC}$ = -1/6

Slope of line ⊥ to $\overline{AC}$ = -3/5

(ii) Eq. of altitude through the vertex A

y-2 = -1(x-(-3)

6(y-2) = -1(x+3)

6(y-12)= -x-3

x+6y-9=0

Eq. of altitude through vertex B

y-4 = 3/5 (x-5)

5y-20= 3x-15

3x-5y+5 = 0

Eq. of altitude through vertex

y-(-81) = -4(x-3)

y+81 = -4x+12

4x+y-4 = 0

(iii) Slope of median through vertex 1

$$\frac{-2-2}{4-(-3)} = \frac{-4}{4+3} = \frac{-4}{7}$$

Equation of median through vertex A

y-2 = -4/7 (x-(-3)

7(y-2) = -4x-12

Slope of median through vertex B

$$\frac{-3-4}{0-5} = \frac{7}{5}$$

Equation of median through vertex B

$y-4 = 7/5\ (x-5)$

$5y-20 = 7x-35$

$7x-5y-15= 0$

Slope of median through vertex C

$$\frac{-3-(-8)}{1-3} = \frac{-11}{2}$$

Equation of median through vertex $\overline{CB}$

$y-(-8) = -11/2\ (x-3)$

$22(y+8) = -11(x-3)$

$2y+16 = -11x+33$

$11x+2y-17 = 0$

Q13. Find an equation of the line through (-4,-6) and perpendicular to a line having slope $\frac{-3}{2}$.

Solution

We have given that $m = \frac{-3}{2}$

Any line $\perp$ to it will have Slope $= \frac{2}{3} = m_1$

The eq of line

$y- y_1 = m_1(x-x_1)$

$3(y + 6) = 2\ (x + 4)$

$\Rightarrow \quad 3y + 18 = 22\,x + 8$

$\Rightarrow \quad 2x - 3y - 10 = 0$

Q14. Find an equation of line through (11,-5) and parallel to a line with Slope- 14.

Solution

$m = -24$

$\Rightarrow \quad y - y_{1} = m(x - x_1)$

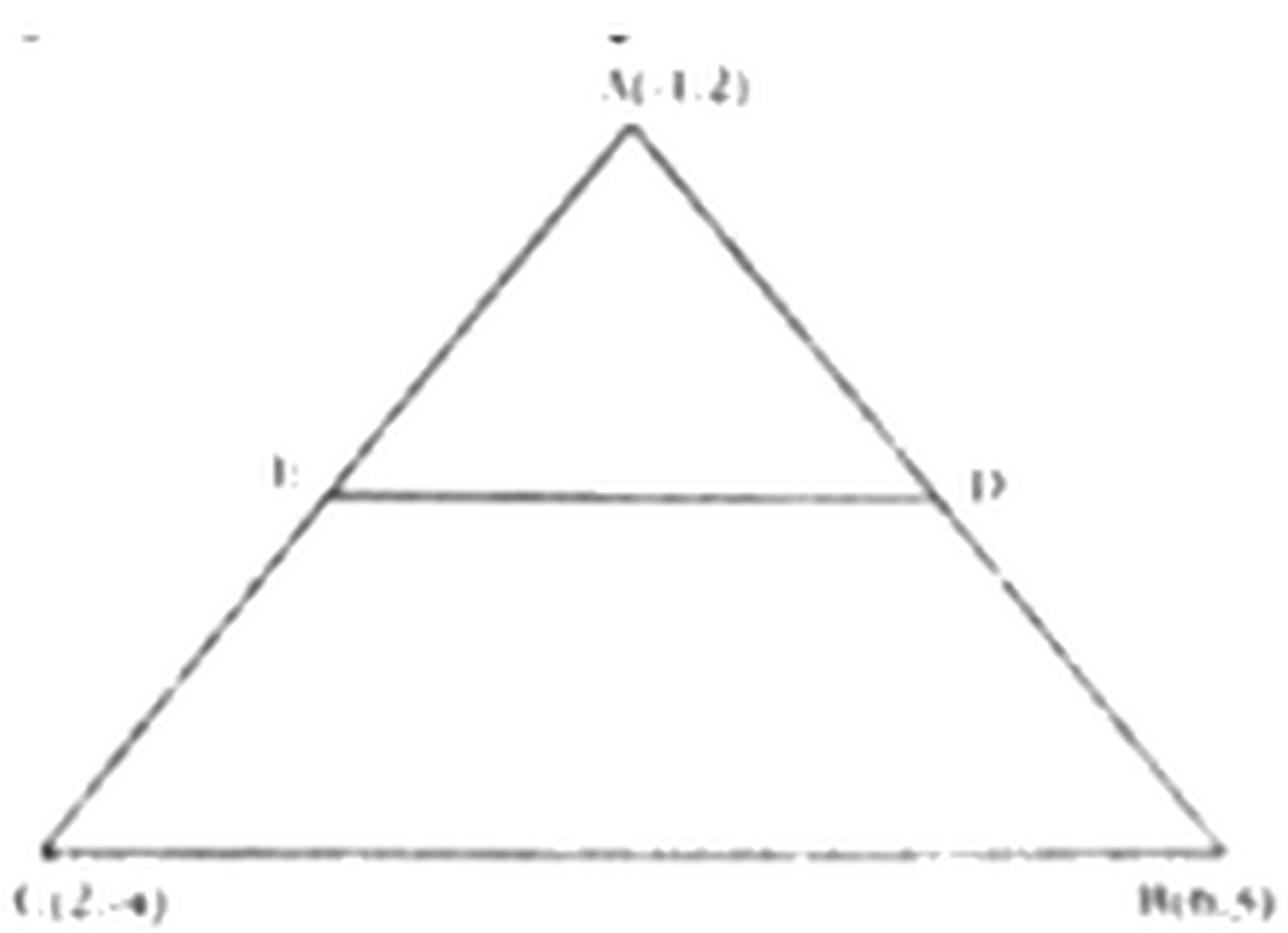
$Y + 5 = -24(x - 11)$

$Y + 5 = -24x + 264$

$24x + y - 259 = 0$

Q15. The point A(-1,2) , B(6,3) and C(2,-4) are vertices of a triangle. Show that the line joining and the mid-point D of AB and mid-point E of AC is parallel to BC and DE = $\frac{1}{2}$ BC

Solution



Co-ordinates of E are $\left(\frac{-1+2}{2}, \frac{2-4}{2}\right) = E\left(\frac{1}{2}, -1\right)$

Slope of $\overline{DE}$ = $\frac{-1-\frac{5}{2}}{\frac{1}{2}-\frac{5}{2}} = \frac{\frac{-2-5}{2}}{\frac{1-5}{2}} = \frac{-\frac{7}{2}}{-\frac{4}{2}} = \frac{7}{4}$

Slope of $\overline{BC}$ $= \frac{-4-3}{2-6} = \frac{7}{4}$

Slope of $\overline{DE}$ = Slope of $\overline{BC}$

$\Rightarrow \quad \overline{DE} \parallel \overline{BC}$

$$|\overline{BC}| = \sqrt{\left(\frac{1}{2}-\frac{5}{2}\right)^2 + \left(-1-\frac{5}{2}\right)^2}$$

$$= \sqrt{\left(\frac{-4}{2}\right)^2 + \left(\frac{-7}{2}\right)^2}$$

$$= \sqrt{\frac{16}{4} + \frac{49}{4}}$$

$$= \sqrt{\frac{16+49}{4}}$$

$$= \frac{\sqrt{65}}{2} \qquad \text{\_\_\_\_\_\_\_\_\_\_(1)}$$

$$|\overline{BC}| = \sqrt{(2-6)^2 + (-4-3)^2}$$

$$= \sqrt{16+49}$$

$$= \sqrt{65}$$

From above

$$DE = \frac{1}{2} BC$$

Q16. A milk man can sell 500 liters of milk at Rs. 12.50 per liter and 700 liters of milk at Rs. 12.00 per liter. Assuming the graph of the sale price and the milk

Solution

Let ℓ denoted the no. of litres of milk sold.

And P denoted the price

$\therefore (\ell_1, P_1) = (560.12.50)$

$(\ell_2, P_2) = (700,12.00)$

$m = \frac{P_1-P_2}{\ell_1-\ell_2} = \frac{12.50-12.00}{560-700} = \frac{50}{140} = \frac{-1}{280}$

Required eq. of line

$P - P_1 = m(\ell - \ell_1)$

$P - 12.50 = \frac{-1}{280}(\ell - 560)$

$-280(P- 12.50) = (\ell - 560)$

$\ell = 560\text{-}280(P\text{-}12.50)$

Also the sale price of milk is Rs.12.25 per liter.

$\ell = 560\text{-}280(12.25)\text{-}12.50)$

$= 560\text{-}280(-.25)$

$= 560+70$

$= 630$ liters

Q17. The Population of Pakistan to the nearest million was 60 million in 1961 and 95 million in 1981. Using t as the number of years after 1961, Find an equation of the line that gives the population in terms of t. Use this equation to find the population in (a) 1947 (b) 1997

Solution

$(t_2, P_2) = (1981,75)$

$m = \frac{P_1-P_2}{t_1-t_2} = \frac{60-65}{1961-1981} = \frac{-35}{-20} = \frac{7}{4}$

Equation of line through (t_1, P_1) (t_2, P_2)

$P - P_1 = m(t - t_1)$ $P - 60 = \frac{7}{4}(t - 1961)$

$P = 60+\frac{7}{4}t \frac{13727}{4}$

$= \frac{240-13727}{4}+\frac{7}{4}$

$P = \frac{13487}{4}+ \frac{7}{4}t$ ______(1)

(a.) Put t = 1947 in (1)

$P = \frac{13487}{4}+ \frac{7}{4}(1947)$

$= \frac{13487+13629}{4} = \frac{142}{4} =$ 35.5 million

(b.) Put t = 1997 in (1)

We get P = 123 million

Q18. A house was purchase for Rs. 1 million in 1980. It is worth Rs. 4 million in 1996. Assuming that the value increased by the same amount each year, Find an equation that gives the value of the house after t years of the date of purchase. What was its value in 1990?

Solution

Let P denote, Purchased cost of house, and t denotes purchase year after 1980.

Here $(P_1, t_1) = (1961,60)$

$(P_2, t_2) = (1981,75)$

$m = \frac{t_1-t_2}{P_1-P_2} = \frac{1980-1996}{1-4} = \frac{-16}{-3} = \frac{16}{3}$

$t - t_1 = m(P - P_1)$

$t - 1980 = \frac{16}{3}(P - 1)$

$P - 1 = \frac{3}{16}(4\text{-}1980)$

$P = 1 + \frac{3}{16}t \frac{5940}{16}$

$= \frac{1481}{4} + \frac{3}{16}t$ gives the value of the house outer t years

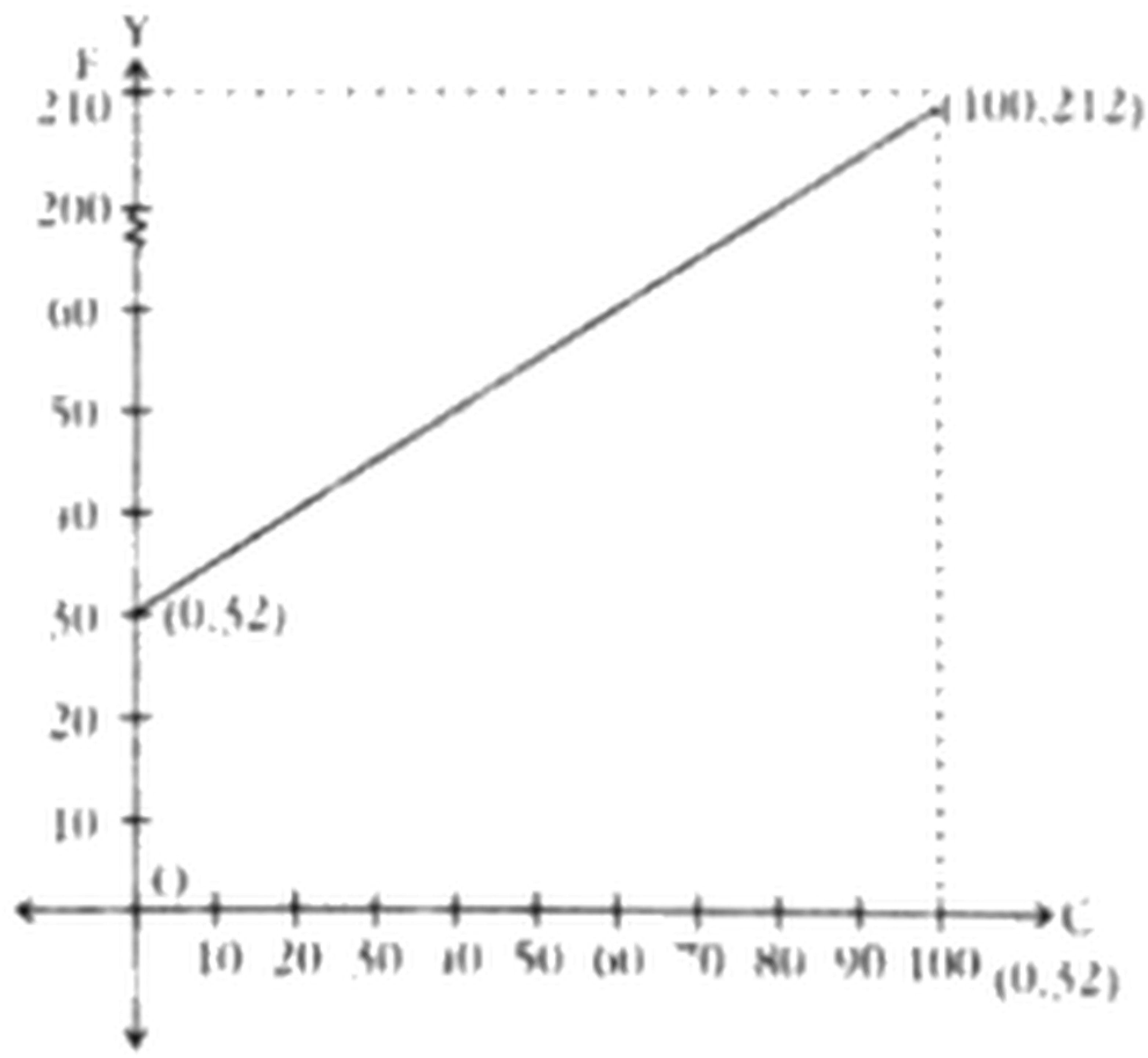
Put t = 1990

$P = \frac{-1481}{4} + \frac{3}{16}(1990) = \frac{5924+5970}{16}$

$= \frac{46}{16} = 2.8$ million

Q19. Plot the Celsius (C) and Fahrenheit (F) temperature scales on the horizontal axis and the vertical axis respectively. Draw the line joining the freezing point and the boiling point of water. Find an equation giving F temperature in terms of C.

Solution



One small along x-axis = 10C

One small along y-axis = 10 F

Freezing point = (0.32)

Boiling point = (100,212)

$$m = \frac{212-32}{100-0} = \frac{9}{5}$$

Using point slope form

$$F - F_1 = m\,(C - C_1)$$

$$F - 32 = \frac{9}{5}\ C - 0$$

$$F = 32 + \frac{9}{5}\ C$$

$$F = \frac{9}{5}\ C + 32$$

Q20. The average entry test score of engineering candidates was 592 in the year 1998 while the score was 564 in 2002. Assuming that the relationship between time and score is linear, find the average score for 2006.

Solution

Let S denotes the entry test score and y the year

$$(s_1, y_1) = (592,1998)$$

$$(s_2, y_2) = (564,2002)$$

$$m = \frac{y_1 - y_2}{s_1 - s_2} = \frac{2002-1998}{564-592} = \frac{4}{-28} = \frac{-1}{7}$$

Equation of line is

$$y - y_1 = m\,(s - s_1)$$

$$2006 - 1998 = \frac{-1}{7}(s-592)$$

$$8 = \frac{-1}{7}(s-592)$$

$$(s - 592) = -56$$

$$s = 592 - 56 = 536$$

Q21. Convert each of the following equation

i Slope intercept form ii. Intercept from iii. Normal form

a. $2x-4y+11=0$ b. $4x+7y-2=0$ c. $15y-8x+3=0$

Also find the length of the perpendicular from (0,0) to each line.

Solution

a. $2x-4y+11=0$

i. Slope intercept form

$$4y = 2x + 11$$

$$y = \frac{2}{4}x + \frac{11}{4}$$

$$\Rightarrow \quad y = \frac{x}{4} + \frac{11}{4}$$

$$y = \frac{1}{4}(x) + \frac{11}{4}$$

ii. Intercept form

$$2x - 4y = -11$$

$$\Rightarrow \quad \frac{2x}{-11} - \frac{4y}{-11} = 1$$

$$\frac{x}{\frac{11}{\,}} + \frac{y}{\frac{11}{\,}} = 1$$

iii. Normal form

$$2x - 4y + 11 = 0$$

$$3x - 4y = -11$$

$$\sqrt{(2)^2 + (-4)^2} = \sqrt{20}$$

$$\frac{2}{\sqrt{20}}x - \frac{4}{\sqrt{20}}y = \frac{11}{2\sqrt{5}}$$

$$\frac{2}{2\sqrt{5}}x - \frac{4}{2\sqrt{5}}y = \frac{11}{2\sqrt{5}}$$

$$\frac{1}{\sqrt{5}}x - \frac{2}{\sqrt{5}}y = \frac{11}{2\sqrt{5}}$$

$$= \frac{11}{2\sqrt{5}}$$

b. 4x +7y -2 = 0 ________(2)

i Slope intercept form

$$7y = -4x + 2$$

$$\Rightarrow \quad y = \frac{-4x+2}{7}$$

$$\Rightarrow \quad y = \frac{-4x}{7}x + \frac{2}{7}$$

Here $m = \frac{-4}{7}, c = \frac{2}{7}$

ii. Intercept form

$$4x+7y = 2$$

$$\Rightarrow \quad \frac{4x}{2} + \frac{7y}{2} = 1$$

$$\frac{x}{\frac{1}{2}} + \frac{y}{\frac{2}{7}} = 1$$

iii. Normal form 4x +7y = 2

Using both sides by $\sqrt{(4)^2 + (7)^2} = \sqrt{65}$

Length is $\frac{2}{\sqrt{65}}$

c. 15y – 8x +3 = 0

Solution

i Slope intercept form

$$15y = 8x - 3$$

$$\Rightarrow \quad y = \frac{8}{15}x - \frac{3}{15}$$

ii Intercept form

$$15y - 8x = -3$$

$$-8x + 15y = -3$$

$$\frac{-8}{-3}x + \frac{15}{-3}y = \frac{-3}{-3}$$

$$\frac{\frac{x}{3}}{8} + \frac{\frac{y}{3}}{5} = 1$$

iii Normal form

$$-8x + 15y = -3$$

Using both sides by $\sqrt{(8)^2 + (15)^2} = \sqrt{64 + 224} = \sqrt{289} = 17$

$$\Rightarrow \quad \frac{-8}{17}x + \frac{15}{17}y = \frac{-3}{17}$$

Using by -1

$$\frac{8}{17}x - \frac{15}{17}y = \frac{3}{17} = \frac{3}{17}$$

a. 2x +y – 3 = , 4x+4y +5 = 0

Solution

Slope of first line = -2 = m_1 and m_2 = -2

$$m_1 = m_2$$

$\Rightarrow$ lines are parallel

b. 3y = 2x +5, 3x +2y - 8 = 0

Slope of $\ell_1 = \frac{3}{2} = m_1$

Slope of $\ell_2 = \frac{3}{2} = m_2$

$$m_1 m_2 = -1$$

$\Rightarrow$ lines are perpendicular

c. 4y +2x -1 = 0 , x -2y – 7 = 0

Solution

Slope of $\ell_1 = -\frac{4}{-1} = 4 = m_1$

Slope of $\ell_2 = -m_2 = \frac{12}{-3} = 4$

$$m_1 = m_2$$

$\Rightarrow$ lines are parallel

d. 4x – y +2 =0 , 12x – 3y +1 = 0

Solution

Slope of first line $\ell_1 = \frac{-4}{-1} = 4$

Slope of $\ell_2 = \frac{12}{-3} = 4$

Slopes are equal, so gi given pair of lines are parallel.

e. $12x + 35y - 7 = 0 = \ell_1$

$105x - 36y + 11 = 0 = \ell_2$

Solution

Slope of $\ell_2 =\ m_1 = \frac{-12}{35}$

Slope of $\ell_2 =\ m_2 = \frac{105}{36} = \frac{35}{12}$

$\therefore \quad m_1 m_2 = -1$

$\Rightarrow$ lines are perpendicular

Q23. Find the distance between the given parallel lines sketch the lines. Also find an equation of the parallel line lying midway between them.

(a) $3x - ay + 3 = 0$; $3x - 4y + 7 = 0$

(b) $12x + 5y - 6 = 0$; $12x + 5y + 13 = 0$

(c) $x + 2y - 5 = 0$; $2x + 4y = 1$

(a). put $x = 0$

$0 - 4y = -3$

$y = \frac{3}{4}$

Hence $\left(0, \frac{3}{4}\right)$ is a point on

$3x - 4y + 7 = 0$

put $x = 0$

$0 - 4y = -7$

$y = \frac{7}{4}$

Hence $\left(0, \frac{7}{4}\right)$ is a

point on

Distance of $\left(0, \frac{3}{4}\right)$ from (ii) is

$$d = \frac{|ax_1 + by_1 + C|}{\sqrt{a^2 + b^2}} = \frac{|(3)(0) + (-4)\left(\frac{3}{4}\right) + 7|}{\sqrt{(3)^2 + (-4)^2}} = \frac{4}{5}$$

Mid Point of $\left(0, \frac{3}{4}\right)$ and $\left(0, \frac{7}{4}\right)$ is $\left[\frac{2+6}{2}, \frac{\frac{3}{4}+\frac{7}{4}}{2}\right]\left(0, \frac{5}{4}\right)$

$-4y = 3x - 3 \Rightarrow Y = \frac{3}{4}x + \frac{3}{4} \Rightarrow m = \frac{3}{4}$

Equation of line through $\left(0, \frac{5}{4}\right)$ with slope $\frac{3}{4}$ is

$Y - Y = m(x - x_1)$

$Y - \frac{5}{4} = \frac{3}{4}(x - 0)$

$\frac{3}{4}x + \frac{3}{4} = 0 \Rightarrow 3x - 7y + 5 = 0$

Put y = 0 in (1)

$3x - 0 = -3$

$x = -1$

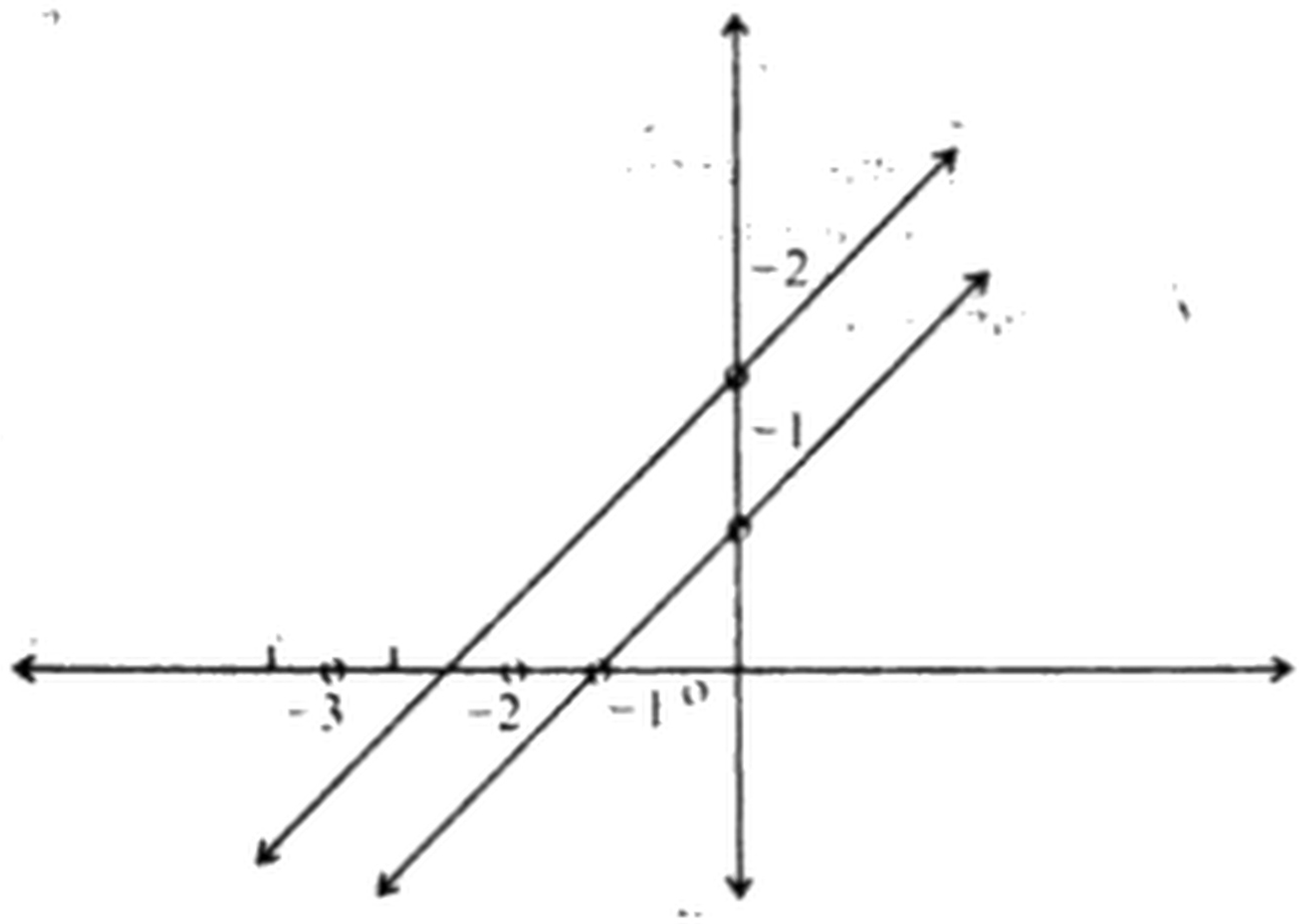
Hence (-1,0) is a another point on (i)

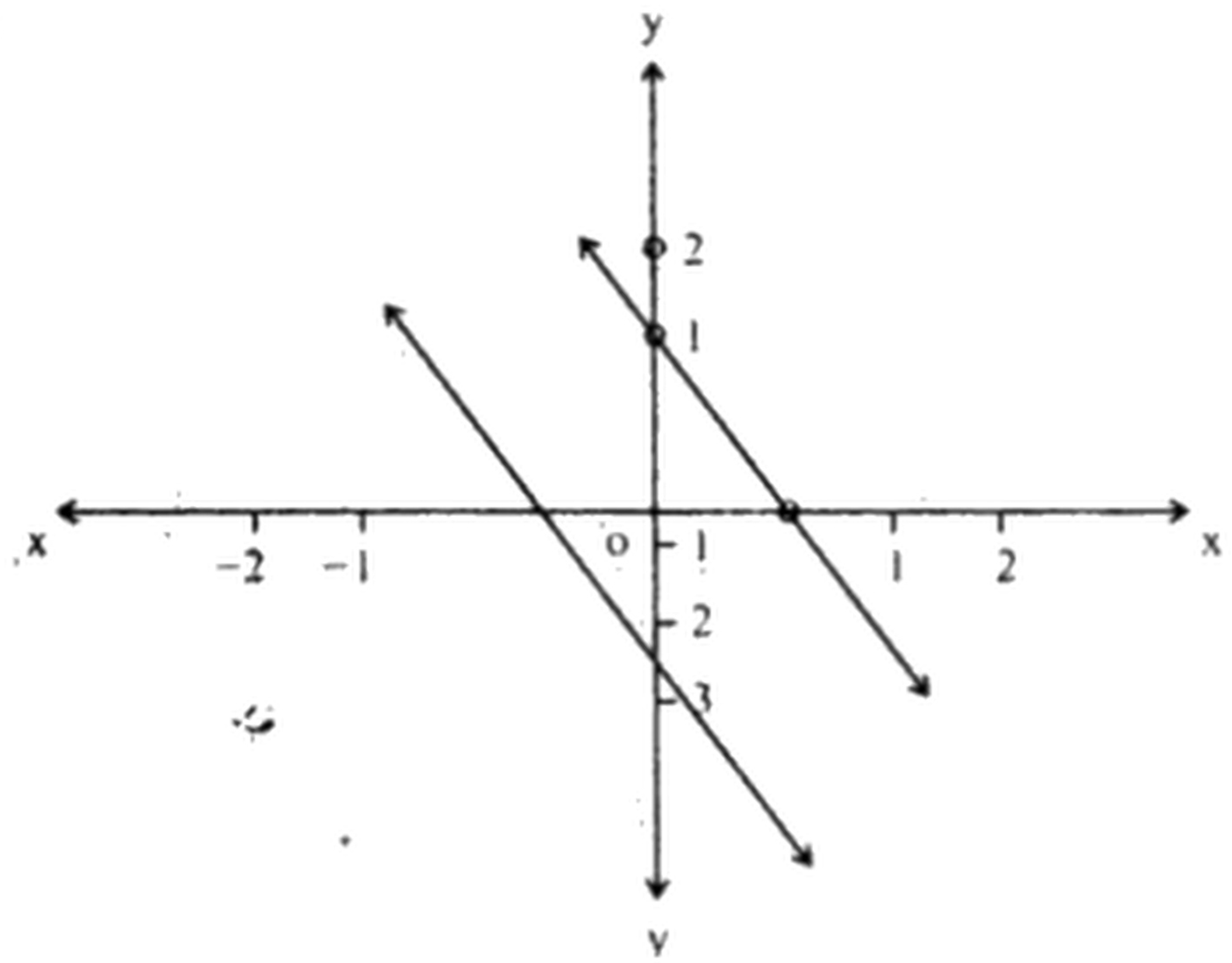
put y = 0 in (ii)

$3x - 0 = -7$

$x = \frac{-7}{3}$

Hence $\left(\frac{7}{-3}, 0\right)$ is another point on (ii)





x+ 2y- 5 = 0 | 2x +4y = 1

put x = 0 | put x = 0

0 + 2y = 5 | 0+4y = 1

y $= \frac{5}{2}$ | y $= \frac{1}{4}$

Hence $\left(0, \frac{5}{4}\right)$ is another point on Distance of | Hence $\left(0, \frac{1}{4}\right)$ is a point on (ii)

$\left(0, \frac{5}{4}\right)$ from ii is

$$d = \frac{|(2)^2(0)+(4)\left(\frac{3}{4}\right)+(-1)1|}{\sqrt{(2)^2+(4)^2}} = \frac{9}{2\sqrt{2}}$$

Mid Point of $\left(0, \frac{5}{4}\right)$ and $\left(0, \frac{1}{4}\right)$ is

$$\left[\frac{0+0}{2}, \frac{\frac{5}{2}+1}{2}\right] = \left(0, \frac{11}{8}\right)$$

12x+ 5y- 6 = 0 | 12x +5y +13 = 1

put x = 0

put x = 0

0 + 5y = 6

0+5 y = 1

y $= \frac{5}{6}$

y $= \frac{13}{5}$

Hence $\left(0, \frac{5}{6}\right)$ is another point on (i)

Distance of $\left(0, \frac{6}{5}\right)$ from ii is

Hence $\left(0 - \frac{13}{-5}\right)$ is a point on (ii)

$$d = \frac{|(12)^2(0)+(5)\left(\frac{6}{5}\right)+13|}{\sqrt{(2)^2+(4)^2}} = \frac{19}{13}$$

Mid Point of $\left(0, \frac{6}{5}\right)$ and $\left(0, \frac{13}{5}\right)$ is

$$\left[\frac{0+0}{2}, \frac{\frac{6}{5}+\frac{13}{5}}{2}\right] = \left(0, -\frac{7}{8}\right)$$

From 5Y = 12x +6 ⇒ Y = $\frac{12}{5}$X + $\frac{6}{5}$ ⇒ m = $\frac{12}{5}$

Equation of line through $\left(0, \frac{-7}{10}\right)$ with slope - $\frac{12}{5}$ is

$$Y + \frac{7}{10} = -\frac{12}{5}(x - 0)$$

$$\frac{12}{5}x + y + \frac{7}{10} = 0 \Rightarrow 24x + 10y + 7 = 0$$

SKETCH

Put y = 0 in (i)

12x = 6

x $= \frac{1}{2}$

y $= \frac{5}{}$

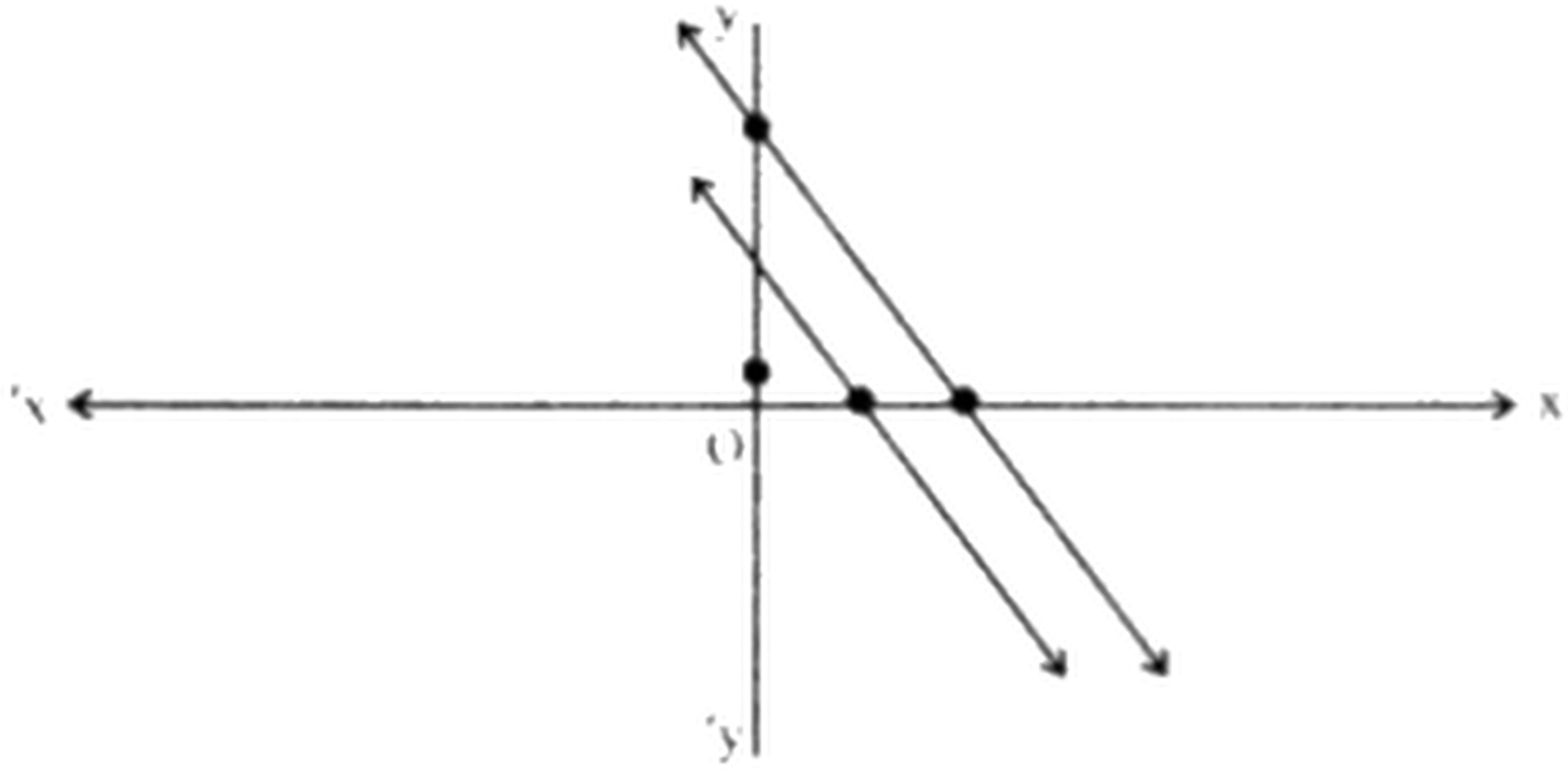
Put y = 0 us (ii)

12x 0 = -13

x $= \left(\frac{13}{-12}, 0\right)$

Hence $\left(\frac{13}{}, 0\right)$ is another point on (i)

Hence $\left(\frac{1}{0}, 0\right)$ is another point on (i)



Q24. Find an equation of line through (-4,7) and parallel to the 2x-7y+4 = 0

Solution

Slope of line 2x-7y+4 = 0 is $\left[\frac{-2}{-7}\right] = \frac{2}{7}$

We know that slope of parallel are equal hence equation of the line through (-4,7) and parallel to the line 2x-7y +4 = 0 is

$$y - y_1 = m(x - x_1)$$

$$y - 7 = \frac{2}{7}(x - (-4))$$

$$7(y-7) = 2\ (x + y)$$

$$7y - 49 = 2x + 8$$

$$2x - 7y + 57 = 0$$

Q25 Find an equation of the line through (5,-8) and perpendicular to the join of A(-15,-8), B(10,7)

$$= \frac{7-(-8)}{10-(-15)} = \frac{7+8}{10+15} = \frac{15}{25} = \frac{3}{5}$$

Slope of a line perpendicular to the $\overline{AB}$ is $\frac{-5}{3}$

Hence equation of required line is

$$y - y_1 = m(x - x_1)$$

$$y - (-8) = \frac{-5}{3}(x - (-5))$$

$$3(y+8) = -5 \ (x - 5)$$

$$3y + 24 = -5x + 25$$

$$5x + 3y - 1 = 0$$

Q26 Find an equation of two parallel line perpendicular to 2x -y+3 = 0 such that the product of the x- and y-intercept of each is 3

Solution

Any line perpendicular to 2x-y +3 = 0 is

$\Rightarrow$ $x + 2y + c = 0$ (1)

To find x-intercept put y = 0

$\Rightarrow$ $x + c = 0$

To find y-intercept put x = 0

$\Rightarrow$ $2y + c = 0$ $\Rightarrow$ $y = \frac{-c}{2}$

As product of the x-and y- intercept is 3, we have

$\Rightarrow$ $(-c)\left[\frac{-c}{2}\right] = 3$

$\Rightarrow$ $c^2 = 6 \Rightarrow c = \pm\sqrt{6}$

$$x + 2y - \sqrt{6} = 0$$

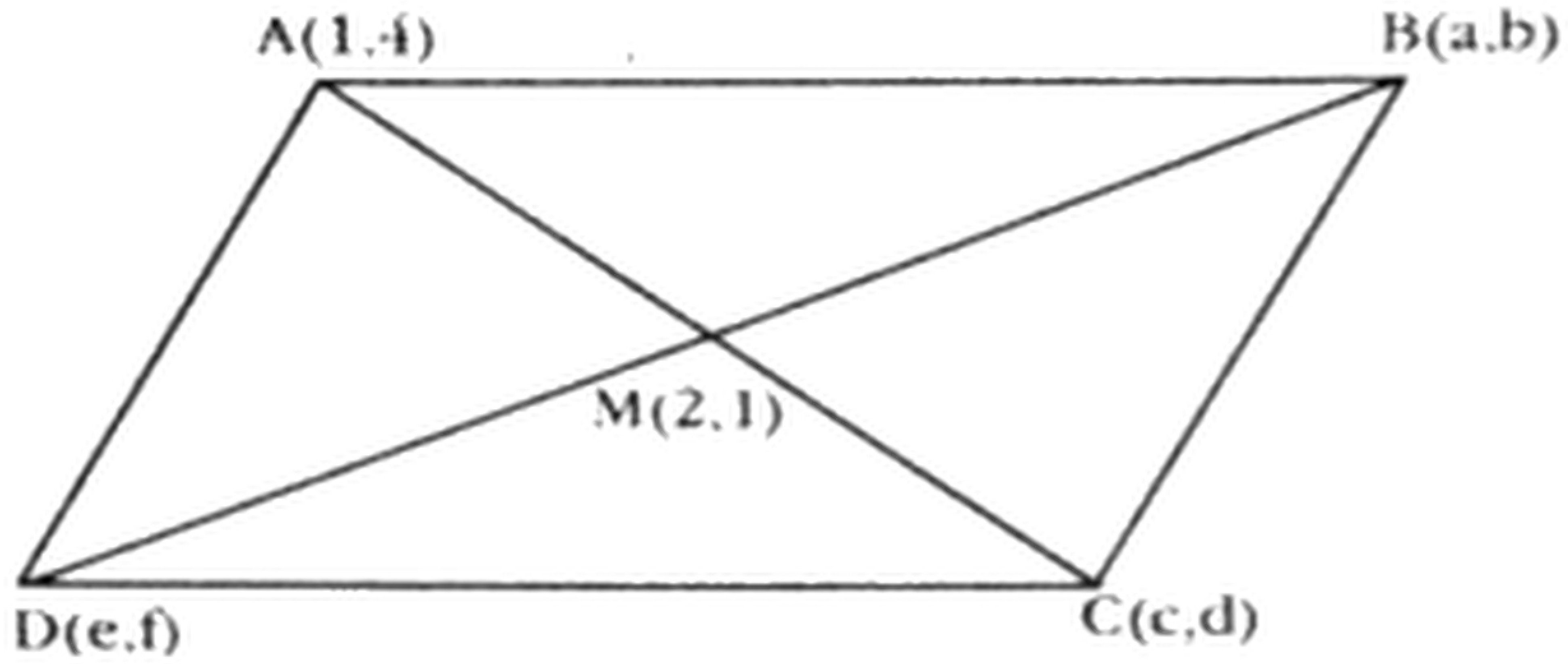
Thus required lines are

$$x + 2y + \sqrt{6} = 0, \quad x + 2y - \sqrt{6} = 0$$

Q27. One vertex of a parallelogram is (1,4); the diagonals intersect at (2,1) and the sides have slopes 1 and. Find the other three vertices.

Solution

Let A(1,4), B(a ,b), C(c, d), D(e ,f) be the vertices of the parallelogram as shown in figure



We know that diagonals of a parallelogram bisect each other at M(2,1)

So that $\frac{1+c}{2} = 2, \quad \frac{4+d}{2} = 1$

$\Rightarrow \qquad c = 3, d = -2$

Also slope of $\overline{AD} = 1 =$ slope of $\overline{BC}$

$$\frac{f-4}{e-1} = 1 = \frac{d-b}{c-a}$$

$$\frac{f-4}{e-1} = 1 = \frac{-2-b}{3-a} \quad (\because c=3, d=-2)$$

$$\frac{f-4}{e-1} = 1 \quad \Rightarrow f - 4 = e - 1$$

$e - f + 3 = 0 \qquad (i)$

$\Rightarrow \quad a - b - 5 = 0 \quad$ ______(ii)

Now slope of $\overline{AB} = \frac{-1}{7}$ = slope of $\overline{CD}$

$$\frac{-b-4}{a-1} = \frac{-1}{7} = \frac{f-d}{e-c}$$

$$\frac{-b-4}{a-1} = \frac{-1}{7} = \frac{f-(-2)}{e-3} \quad (\therefore c=3, d=-2)$$

$$\frac{-b-4}{a-1} = \frac{-1}{7} \Rightarrow 7b - 28 = -a + 1$$

$\Rightarrow \quad a+7b-29 = 0 \quad$ ______(i)

and $\frac{f+2}{e-3} = \frac{-1}{7} \Rightarrow 7f+14 = -e+3$

$\Rightarrow \quad e + 7f + 11 = 0$

Multiply equation (2) by 7 and add in (3)

$$a + 7b - 2a = 0$$

$$\frac{7a + 7b = 2a}{8a - 64 = 0}$$

$$8a = 64 \Rightarrow a = 8$$

Put a = 8 in (2)

$$8-b-5 = 0 \Rightarrow b = 3$$

Multiplying equation (1) by 7 and add in (3)

$$e + 7f + 11 = 0$$

$$\frac{7e + 7f + 21 \quad = 0}{82 - 32 \quad = 0}$$

$$8e - 32 \Rightarrow e = -4$$

Put 3 = -4 in (1)

$$-4-f+3 = 0 \Rightarrow f = -1$$

Thus required vertices are

Q28. Find whether the given point lies above or below the given line

a. (5,8); 2x -3y +6 = 0

Solution

$$2x-3y+6=0 \quad \_\_\_\_\_\_\_(1)$$

Substituting (5,8) in the L.H.S of (1) we have

$$2(5)-3(8)+6=0-24+6=-24 \quad \_\_\_\_\_\_\_\_\_\_(2)$$

Thus the coefficients of y in (1) and the expression (2) have the same sign(-) and so the point (5,8) lies above (1).

b. (-7,6);4x+3y-9 = 0

Solution

$$4x+3y-9=0 \quad \_\_\_\_\_\_\_(1)$$

Substituting (-7,6) in the L.H.S of (1) we have

$$4(-1)+3(6)-9=-28+18-9$$

$$=18-37=-19 \quad \_\_\_\_\_\_\_\_\_\_(2)$$

Thus the coefficients of y in (1) and the expression (2) have the opposite sign(+,-) and so the point (- 7,6) lies below (1).

Q29. Check whether the given points are on the same or opposite sides of the given line

a. (0,0) and (-4,7), 5x -7y +70 = 0 _______(1)

Solution

Substituting (0,0) in the L.H.S of (1) we have

Thus the coefficients of y in (1) and the expression (2) have the opposite signs and so the point (-0,0) lies below (1).

Substituting (-4,7) = -20 -49 +70

-69 +70 = 1 ________(3)

Thus the coefficients of y in (1) and the expression (2) have the opposite signs and so the point (-4,7) lies below (1).

Hence the points (0,0) and (-4,7) are on the same side of the given line

a. (2,3) and (-2,3) , 3x-5y+8 = 0

Solution

3x-5y + 8 = 0 ________(1)

Substituting (-2,3) in the L.H.S of (1) we have

3(2)-5(3) +8 = 6-15 +8=14-15 = 1 ______(2)

Thus the coefficients of y in (1) and the expression (2) have the same sign-) and so the point (-2,3) lies above (1).

Substituting (-2,3) in the L.H.S of (1) we have

3(-2)-5(3) +8 = 6-15 +8

= -21+8 = -13

Thus the coefficients of y in (1) and the expression his above (1)

Hence the points (2,3) and (-2,3) are on the same side of the line (1)

Q30.Find the distance from the point P(6,-1) to the line 6x – 4y +9 = 0

Solution

The required distance is

$$d = \frac{|6(6)-(4)(-1)+9|}{\sqrt{(6)^2+(-4)^2}} = \frac{49}{2\sqrt{13}}$$

Q31. Find the area of the triangular region whose vertices are A(5,3) B(-2,2)C(4,2)

Solution

Under given conditions

$A(x_1, y_{1\ }) = (5,3)$

$B(x_2, y_{2\ }) = (-2,2)$

$C(x_3, y_{3\ }) = (4,2)$

The required are C if Δ ABC

$$\Delta = \frac{1}{2}\begin{matrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{matrix}$$

$$= \frac{1}{2}\begin{matrix} 5 & 3 & 1 \\ -2 & 2 & 1 \\ 4 & 2 & 1 \end{matrix}$$

$$= \frac{1}{2}[5(2-2) - 3(-2-4) + 1(-4-8)]$$

$$= \frac{1}{2}[5(0) - 3(-6) + 1(-12)] = \frac{1}{2}[0 + 18 - 12]$$

$= \frac{1}{2}(6)$ = 3 square units

Q32. The coordinates of three points are A(2,3)B(-1,1) and (4,-5). By computing the area bounded by ΔABC check whether the points are collinear

$$\Delta = \frac{1}{2} \begin{matrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{matrix}$$

$$= \frac{1}{2} \begin{matrix} 3 & 3 & 1 \\ -1 & 1 & 1 \\ 4 & -5 & 1 \end{matrix}$$

$$= \frac{1}{2}[2(1+5) - 3(-1-4) + 1(5-4)]$$

$$= \frac{1}{2}[2(6) - 3(-5) + (1)] = \frac{1}{2}[12 + 15 - 1]$$

$= \frac{1}{2}[28] = 14 \neq 0 \Rightarrow$ A,B,C are non collinear