Exercise 4.2

➢ **The two points P and O' are given in xy-coordinates system. Find the xy-coordinate of P referred to the translated axes O'X and O'Y.**

i. P(3,2); O'(1,3)

Solution

Here h = 1, k = 3

Coordinates of P referred to the translated

x = x – h = 3 -1 = 2

y = y -k = 2 – 3 = -1

i -e (2,-1)

ii. P(-2,6); O'(-3,2)

Solution

Here (x ,y) = (-2,6) , (h ,k) = (-3,2)

Coordinates of referred to XY- coordinate system are

= x...h = -2-(-3) = -2+3 = 1

Y = y-k = 6-2 = 4

i .e (1,4)

Solution

Here $(x, y) = (-6,-8)$

$(h, k) = (-4,-6)$

Coordinates of referred to XY- coordinate system ...

$x = x... = -6-(-4) = -6+8 = 12$

$Y = y-k = -8-(-6) = -8+6 = 2$

$\Rightarrow (-2,-2)$

iv. $\mathbf{P\left(\frac{3}{2},\frac{5}{2}\right)}$ **;** $\mathbf{O'\left(\frac{-1}{2},\frac{7}{2}\right)}$

Here $(x, y) = \left(\frac{3}{2},\frac{5}{2}\right)$

$(h, k) = O'\left(\frac{-1}{2},\frac{7}{2}\right)$

Co-ordinates of P referred to XY- Plane are

$X = x - h = \left(\frac{3}{2}\right)\left(\frac{-1}{2}\right)$

$= \frac{3}{2}+\frac{5}{2} = \mathbf{2}$

$Y = Y\text{-}K = \frac{5}{2}-\frac{7}{2} = \frac{5-7}{2} = \frac{-2}{2} = \mathbf{-1}$

$P(x, y) = p\ (2,-1)$

Q2. The xy- coordinate axes are translated through the point O' whose co-ordinates of P are given in the xy co-ordinate system. Find the co-ordinate of P in xy- co -ordinate

i. P(8,10),O(3,4)

Solution

$$(h,k) = (3,4)$$

$$X = x + h = 8+3$$

$$Y = y + k = 10+4$$

$$\Rightarrow P(x,y) = P(11,4)$$

ii. P(-5,-3), O(3,4)

Solution

Here $(x,y) = (-5,-3)$

$$(h,k) = (-2,-6)$$

$$\Rightarrow \quad x = x + h = -5+(-2)$$

$$= -7$$

$$y = y + k = -3+(-6)$$

$$= -3-6 = -9$$

Thus P(-7,-9)

iii. $\mathbf{p}\left(\frac{-3}{4},\frac{-7}{6}\right)$, $O'\left(\frac{1}{4},\frac{-1}{6}\right)$

Solution

Here $(x,y) = \left(\frac{-3}{4},\frac{-7}{6}\right)$

$x = x + k = \frac{-3}{4}+\frac{1}{4} = \frac{-2}{4} = \frac{1}{2}$

Type equation here.

$y = y + h = \frac{-7}{6}+\left(\frac{-1}{6}\right)$

$= \frac{-8}{6} = \frac{-4}{3}$

Thus $P(x,y) = \left(\frac{-1}{2},\frac{-4}{3}\right)$

Solution

Here $(x, y) = (4,-3)$

$(h, k) = (-2,3)$

$x = x + h = 4+(-2) = 4-2 = 2$

$y = y + k = -3+3 = 0$

Thus $P(x, y) = P(2,0)$

Q3. The xy-co-ordinates axes are rotated about the origin through the indicated angel. The new axes are OX and OY. Find XY-co-ordinates of the point P with the given XY coordinates

i. P(5,3), $\theta = 45^\circ$

Solution

Let(x ,y) be the co-ordinates of the point P referred to xy axes

Here $\theta = 45^\circ$

$X \cos\theta + y \sin\theta = 5\cos 45^\circ + 3\sin 45^\circ$

$= \frac{5}{\sqrt{2}} + \frac{3}{\sqrt{2}} = \frac{8}{\sqrt{2}} = \frac{2\times 2}{\sqrt{2}} = 4\sqrt{2}$

$Y = -x \sin\theta + y \cos\theta$

$= -5\sin 45^\circ + 3\cos 45^\circ$

$= \frac{-5}{\sqrt{2}} + \frac{3}{\sqrt{2}}$

$= \frac{-2}{\sqrt{2}} = \sqrt{-2} - \sqrt{2}$

i.e. $(x, y) = (4\sqrt{2}, -\sqrt{2})$

ii. P(3,-7), $\theta = 45^\circ$

Solution

Here $\theta = 30^\circ$

$$\therefore \quad X = x\cos\theta + y\sin\theta$$

$$= 3\cos 30^\circ + (-7)\sin 30^\circ$$

$$= 3\left(\frac{\sqrt{3}}{2}\right) - 7\left(\frac{1}{2}\right) = \frac{3\sqrt{3}-7}{2}$$

$$Y = -x\sin\theta + y\cos\theta$$

$$= -3\sin 30^\circ + (-7)\cos 30^\circ$$

$$= -3\left(\frac{1}{2}\right) - 7\left(\frac{\sqrt{3}}{2}\right) = \frac{-7-3\sqrt{3}}{2}$$

$$\Rightarrow (x, y) = \left(\frac{3\sqrt{3}-7}{2}, \frac{-7-3\sqrt{3}}{2}\right)$$

ii. **p(11,-15), $\theta = 60^\circ$**

Solution

Let(x ,y) be the co-ordinates of the point P referred to xy axes

Here $\theta = 60^\circ$

$$X = x\cos\theta 60^\circ + y\sin\theta\, 60^\circ$$

$$= \cos\theta 60^\circ + \sin\theta 60^\circ$$

$$= 11\left(\frac{1}{2}\right) - 15\left(\frac{\sqrt{3}}{2}\right) = \frac{11-15\sqrt{3}}{2}$$

$$Y = x\sin\theta + \cos\theta$$

$$= -11\sin\theta 60^\circ + (-15)\cos 60^\circ$$

$$= -11\left(\frac{\sqrt{3}}{2}\right) - 15\left(\frac{1}{2}\right) = \frac{11-15\sqrt{3}}{2}$$

$$\Rightarrow (x, y) = \left(\frac{11-15\sqrt{3}}{2}, \frac{-11-15\sqrt{3}}{2}\right)$$

iv. **PC (15,10) : θ = arc tan $\left[\frac{1}{2}\right]$**

Solution

PC (15,10) : θ = tan $\left[\frac{1}{2}\right]$

$X = 15$, $y = 10$ and $\tan\theta = \frac{1}{3}$

$\sin\theta = \frac{1}{\sqrt{10}}$, $\cos\theta = \frac{3}{\sqrt{10}}$

$X = x\cos\theta + y\sin\theta = 15(\frac{3}{\sqrt{10}}) + 10(\frac{1}{\sqrt{10}})$

$= \frac{\sqrt{45}}{\sqrt{10}} + \frac{10}{\sqrt{10}} = \frac{55}{\sqrt{10}}$

$P(\frac{55}{\sqrt{10}}, \frac{15}{\sqrt{10}})$

Q4. The xy-co-ordinates axes are rotated about the origin through the indicated angel. The new axes are OX and OY. Find XY-co-ordinates of the point P with the given XY coordinates

ii. P(-5,3), $\theta = 30^{\circ}$

Solution

$\sin\theta = \frac{1}{2}$

$\cos\theta = \frac{\sqrt{3}}{2}$

$X = x\cos\theta + y\sin\theta$

$\Rightarrow \quad -3 = x(\frac{\sqrt{3}}{2}) + y(\frac{1}{2})$

$\Rightarrow \quad y = = -10$ _____(1)

and $Y = -x\sin\theta + y\cos\theta$

$y = -x(\frac{1}{2}) + y(\frac{\sqrt{3}}{2})$

$\Rightarrow \quad -x + \sqrt{3}y = 6$ _____(2)

(1) $\Rightarrow$ $y = \Rightarrow -10 - \sqrt{3}x$ in _____(2)

$$-4x = 6 - 10 + \sqrt{3}$$

$$-2x = 3 + 5\sqrt{3}$$

$$x = \frac{-3-5\sqrt{3}}{2} \text{ put in (1)}$$

$$\sqrt{3}\left(\frac{3+5\sqrt{3}}{2}\right) + y = -10$$

$$\frac{-3\sqrt{3}+15}{2} + y = -10$$

$$y = -10\,\frac{-3\sqrt{3}+15}{2} = \frac{-20+3\sqrt{3}+15}{2}$$

$$= -10 + \frac{3\sqrt{3}-5}{2}$$

$$\Rightarrow \quad p\left(\frac{-3-5\sqrt{3}}{2}, \frac{3\sqrt{3}-5}{2}\right)$$

ii $P(-7\sqrt{2}, 5\sqrt{2}), \theta = 45^{\circ}$

Solution

Let P(x ,y) be the coordinates of P referred to xy-co-ordinates system.

$\theta = 45^{\circ}$ given

$$\Rightarrow \quad \text{Sin}\,\theta = \frac{1}{\sqrt{2}}$$

$$\text{Cos}\,\theta = \frac{1}{\sqrt{2}}$$

$$X = x\,\text{Cos}\,\theta + y\,\sin\theta$$

$$\Rightarrow \quad -7\sqrt{2} = x\left(\frac{1}{\sqrt{2}}\right) + y\left(\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \quad x + y = = -14 \quad \text{\_\_\_\_\_(1)}$$

and $Y = -x\,\sin\theta + y\,\cos\theta$

$$5\sqrt{2} = -x\left(\frac{1}{\quad}\right) + y\left(\frac{1}{\quad}\right)$$

From (1) $y = \Rightarrow -14 - x$ in put in (2)

$-x - 14 - x = 10$

$-2x-14 = 10$

$-2x = 24$

$\Rightarrow x = -12$ put in _____(1)

$-12+y = -14$

$Y = -14+12 = -2$

$\Rightarrow P(-12,-2)$