UNIT 4

Introduction to Analytic Geometry

Exercise 4.1

Q1. Describe the location in the plane of the point P(x ,y) for which

i. **$x > 0$**

The point lies in right half plane

ii. **$x > 0$ and $y > 0$**

Fourth quadrant

iii. **$x=0$**

The y axis

iv. **$y=0$**

The x-axis

v. **$x < 0$ and ≥ 0**

The 2nd quadrant

vi. **$x = y$**

The point lies in 1st and 3rd quadrant. With both coordinates equal.

vii. **$|x| = -|y|$**

A positive value cannot be equal to a negative value, so (0,0) is the only point which statistics $|x| = -|y|$ lies 1st, 3rd with same sign.

viii. **|x| >3 lies on x-axis, with opposite sign.**

Point on the x-axis less than or equal to -3 and greater than or equal to =3

ix. **x>2 and y=2**

Point in the 1st quadrant. With ordinate 2 and abscissa greater than 2

x. **x and y have opposite signs.**

2nd and 4th quadrant

Q2. Find in each of the following

i. **The distance between the two given points**

ii. **Mid point of the line segment joining the two point**

a. **A(3,1), B(2, -4)**

Solution

$$|AB| = \sqrt{(3+2)^2 + (1+4)^2}$$

$$= \sqrt{25+25} \qquad = \sqrt{50}$$

$$= 5\sqrt{2}$$

Mid-Point of AB = $\left(\frac{3+(-2)}{2}, \frac{1+(-4)}{2}\right)$

$$= \left(\frac{1}{2}, \frac{-3}{2}\right)$$

b. **A(-8,3), B(2,-1)**

Solution

$$|\mathbf{AB}| = \sqrt{(2+8)^2 + (-1-3)^2}$$

$$= \sqrt{100+16}|$$

$$= \sqrt{116}$$

$$= \sqrt{4 \times 29} \qquad = 2\sqrt{29}$$

Mid-Point of AB $= \left(\frac{8+2}{2}, \frac{3+(-1)}{2}\right)$

$$= (-3,1)$$

c. $\mathbf{A}\left(-\sqrt{5}, \frac{-1}{3}\right), \mathbf{B}\left(-3\sqrt{5},\right)$

Solution

$$|\mathbf{AB}| = \sqrt{\left(-3\sqrt{5} - (-\sqrt{5})\right)^2 + \left(5 + \frac{1}{3}\right)^2}$$

$$= \sqrt{\left(-3 + \sqrt{5}\right)^2 + \left(5 + \frac{1}{3}\right)^2}$$

$$= \sqrt{\left(-2\sqrt{5}\right)^2 \left(\frac{16}{3}\right)^2}$$

$$= \sqrt{20 + \frac{250}{9}} \quad = \frac{\sqrt{436}}{3}$$

$$= \sqrt{\frac{4 \times 109}{3}} \qquad = \frac{2\sqrt{109}}{3}$$

Mid-Point AB AB $= \left(\frac{-\sqrt{5}+(-3\sqrt{5})}{2}, \frac{\frac{1}{3}+5}{2}\right)$

$$= \left(\frac{-4\sqrt{5}}{2}, \frac{14}{6}\right)$$

$$= \left(-2\sqrt{5}, \frac{7}{3}\right)$$

Q3. Which of the following points are at a distance of 15 unit from the origin

a. $(\sqrt{176}, 7)$

Solution

Let $A\sqrt{176}, 7), O(0,0)$

$|AO = \sqrt{(\sqrt{176} - 0)^2 + (7 - 0)^2}$

$= \sqrt{176 + 49} = \sqrt{225} = 15$

Thus the point $(\sqrt{176}, 7)$ is not at distance of 15 from origin.

b. Let A(10,-10),O(0,0)

Solution

$|OA = \sqrt{(10 - 0)^2 + (-10 - 0)^2}$

$= \sqrt{200} = 10\sqrt{2}$

Thus the point (10,-10) is not at distance of 15 from origin

c. Let A(1,15),O(0,0)

Solution

$|OA = \sqrt{(1 - 0)^2 + (15 - 0)^2}$

$= \sqrt{1 + 225} = \sqrt{226}$

Thus the point (1,15) is not at distance of 15 from origin.

d. Let A$\left(\frac{15}{2},\frac{15}{2}\right)$,O(0,0)

Solution

$$OA = \sqrt{\left(\frac{15}{2},0\right)+\left(\frac{15}{2}-0\right)^2}$$

$$= \sqrt{\frac{225}{4}+\frac{225}{4}} = \sqrt{\frac{450}{2}} = 15\sqrt{\frac{2}{2}}$$

$$= \frac{\sqrt{15}}{2}$$

Thus the point $\left(\frac{15}{2},\frac{15}{2}\right)$is not at distance of 15 from origin.

Q4. Show that

i. The points A(0,2),B($\sqrt{3}$,1) and C(0,-2) are vertices of a right triangle

Solution

$$|AB| = \left(\sqrt{\left(\sqrt{5}-0\right)^2+(-1-2)^2}\right)$$

$$= \left(\sqrt{\sqrt{3}^2+(-3)^2}\right)$$

$$= \sqrt{3+9}$$

$$= \sqrt{12} \text{ \_\_\_\_\_\_\_\_\_\_(1)}$$

$$|BC| = \left(\sqrt{\left(0-\sqrt{3}\right)^2+(-2+1)^2}\right)$$

$$= \sqrt{3+1}$$

$$= \sqrt{4} \text{ \_\_\_\_\_\_\_\_\_\_(2)}$$

Now $|AC| = \left(\sqrt{(0-0)^2+(-2-2)^2}\right)$

$$= \sqrt{0+4}$$

$= \sqrt{(4)^2}$

$= \sqrt{16}$ ______(3)

Using (I,ii,iii)

$|AC| = |AB| + |BC|$

A ,B ,C are the vertices of a right triangle

ii. **The point A(3,1),B(-2,-3) and C(2,2) are the vertices of an isosceles Δ**

Solution $|AB| = \left(\sqrt{(-2-3)^2 + (-3-1)^2}\right)$

$= \left(\sqrt{(-5)^2 + (4)^2}\right)$

$= \sqrt{25 + 16}$

$= \sqrt{41}$ _________(1)

$|BC| = \left(\sqrt{(-2(-2))^2 + (-2(-3))^2}\right)$

$= \sqrt{16 + 25}$

$= \sqrt{41}$

Now $|AC| = \left(\sqrt{(2-3)^2 + (2-1)^2}\right)$

$= \sqrt{1 + 1}$

$= \sqrt{2}$ ______(2)

From (I,ii,)

$\Rightarrow |AB| = |BC|$

A ,B ,C are the vertices of an isosceles Δ

iii. The point A(5,2),B(-2,3),C(-3,-4) and D(4,-5) are vertices of a parallelogram. Is the parallelogram a square?

Solution

$|AB| = \left(\sqrt{(-2-5)^2 + (3-2)^2}\right)$

$= \left(\sqrt{(-7)^2 + (1)^2}\right)$

$= \sqrt{49+1}$

$= 5\sqrt{2}$

$|BC| = \left(\sqrt{(-3-(-2))^2 + (-4-3)^2}\right)$

$= \left(\sqrt{(-3+2)^2 + (-7)^2}\right)$

$= \left(\sqrt{(-1)^2 + (-7)^2}\right) \quad = \sqrt{1+49}$

$= \sqrt{50}$

$|AD| = \left(\sqrt{(4-5)^2 + (-5-2)^2}\right)$

$= \left(\sqrt{(-1)^2 + (-7)^2}\right) \quad = \sqrt{1+49}$

$=\sqrt{50}$

$= 5\sqrt{2}$

$|CD| = \left(\sqrt{(4-(-3))^2 + (-5-(-4))^2}\right)$

$= \left(\sqrt{(4+3)^2 + (-5+4)^2}\right) \quad = \sqrt{1+49}$

$=\sqrt{50}$

$= 5\sqrt{2}$

$\therefore \quad |AB| = |CD| = |BC| = |AD|$

⇒ ABCD is a parallelogram.

Also $|AD| = \left(\sqrt{(-3-5)^2 + (-4-2)^2}\right)$

$= \left(\sqrt{(-8)^2 + (-6)^2}\right) \quad = \sqrt{64+36}$

$= \sqrt{100}$

$= 10$

$|BD| = \left(\sqrt{(4+2)^2 + (-5-3)^2}\right)$

$= \sqrt{36+64}$

$= \sqrt{100}$

$= 10$

Diagonals are equal

$|AC| = |BD|$

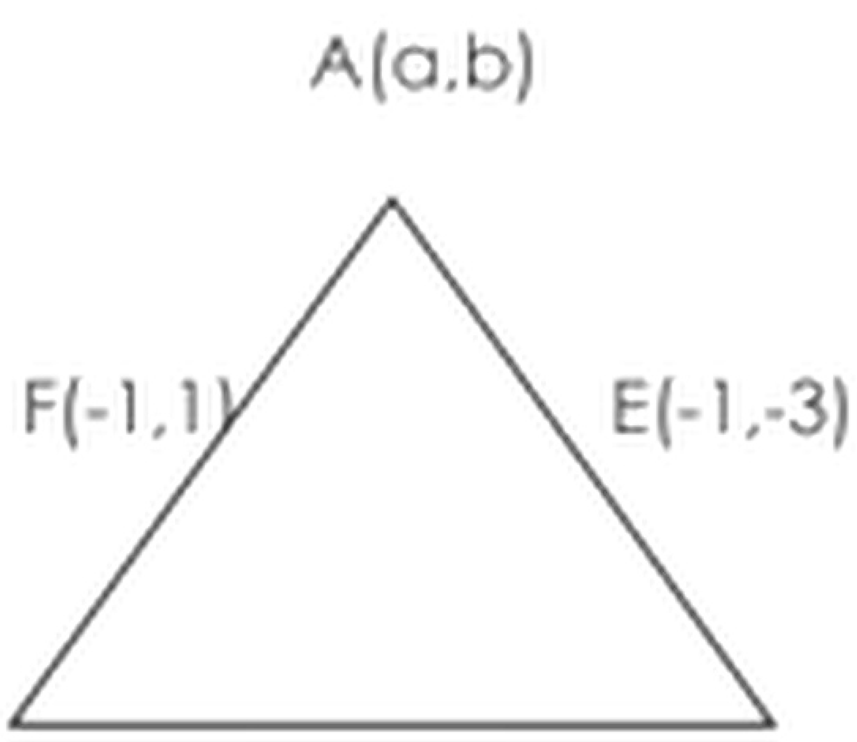
⇒ ABCD is a square

Q5. The mid-Point of the sides of a triangle are (1,-1),(-4,-3) and (-1,1) Find the co-ordinate of the vertices of the triangle.

Solution

Let A(a,b) , B(c,d),C(e,f) be the required vertices of the Δ

Let D(1,-1),E(-4,-3),F(-1,1) be the mid-Point of the B(c,d) d(1,-1) c(e,f) sides



B(c,d) D(1,-1) C(e,f)

$\frac{c+e}{2} = 1$ and $\frac{d+f}{2} = -1$

$\Rightarrow$ c +e =2 ________(1)

d +f =-2 ________(2)

Also $\frac{a+e}{2} = 4$ and $\frac{d+f}{2} = -3$

$\Rightarrow$ a +e = -8 ________(3)

b +f = -6 ________(4)

adding (1) and (3)

a +c +2e =2t (-8)

$\Rightarrow$ a +c+2f = -6 _______(5)

adding (2) and (4)

b +d +f +f = -2+(-6)

$\Rightarrow$ b +d +2f = -8 _____(6)

Also $\frac{a+c}{2} = -1$ and $\frac{d+d}{2} = 1$

$\Rightarrow$ a +c = -2 ______(2)

b +2 = -2 ______(8)

put a +c = -2 in ______(5)

-2 +2e = -6

2e = -6+2

$2e = -4$

$\Rightarrow \quad e = -2$ ______(3)

Put $e = -2$

$a+(-2)= -8$

$\Rightarrow \quad a = -8+2$

$\Rightarrow \quad a = -6$

Now put (8)in (6)

$2+2f = -8 \quad \Rightarrow \quad 2f = -8-2$

$\Rightarrow \quad 2f = -10$

$\Rightarrow \quad f = -5$

Put in __________(4)

$b+ (-5) = -6$

$\Rightarrow \quad b = -6+5$

$\Rightarrow \quad b = -1$

Put $e = -2$ in ___________(1)

$e-2 = 1$

$\Rightarrow \quad e = 2+2= 4$

Put $f = -5$ in ____________(2)

$d-5 = -2$

$d = -2+5$

$\Rightarrow \quad d = 3$

Thus required vertices are

A(-6,-1),B(4,3),C(-2,-5)

Q6. Find h such that the point A($\sqrt{3}$,-1),B(0,2) and C(h,-2) are vertices of a right triangle with right angle at the vertex A.

Solution

A($\sqrt{3}$,-1),B(0,2) , C(h,-2)

A,B,C are vertices of a right angle A, So by Pythagorean Theorem

$$AB^2 + AC^2 = BC^2$$

$$(0-\sqrt{3})^2 + (h\text{-}(\text{-}1))^2 + (h\text{-}\sqrt{3})^2 = (h\text{-}0)^2 + (-2-2)^2$$

$$\Rightarrow (-\sqrt{3})^2 + (2h)^2 + h^2 + (\sqrt{3})^2 - 2\sqrt{3}h + (-2+1)^2$$

$$= h^2 + (-4)^2$$

$$\Rightarrow 3+9 + h^2 + 3 - 2\sqrt{3}h + 1 = h^2 + 16$$

$$\Rightarrow 16 - 2\sqrt{3}h = 16$$

$$\Rightarrow -2\sqrt{3}h = 0$$

$$\Rightarrow h = 0$$

Q7. Find h such that A(-1,h), B(3,2) and C(7,3) are collinear.

Solution

The point A,B,C are collinear

$$\begin{matrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{matrix} = \begin{vmatrix} -1 & h & 1 \\ \dots & 2 & 1 \\ 7 & 3 & 1 \end{matrix}$$

Expand from R.

(-1) (2-3) – h(3-7) +1(9-14) = 0

(-1)(-1) – h(-4) +(-5)= 0

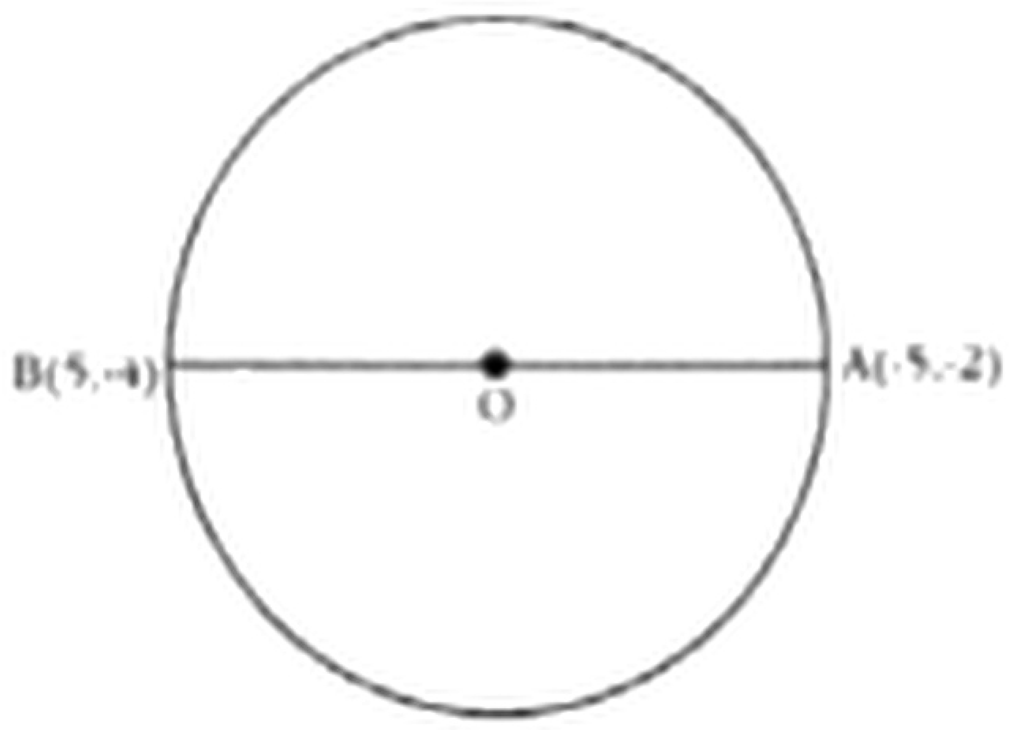
1+4h -5 = 0

⇒ 4h-4 = 0

⇒ 4h = 4

⇒ h = 1

Q8. The points (-5,-2) and B(5,-4) are ends of a diameter of a circle. Find the centre are radius of the circle.



Solution

Let 'O' be the of the circle.

'O' lies between A and B

'O' is the mid-Point of A and B

$\Rightarrow \left(\frac{-5+5}{2}, \frac{-2+(-4)}{2}\right) = (0,-3)$

Now find the radius

$\therefore$ Radius $= \frac{1}{2}$ of diameter

$\Rightarrow r = \frac{1}{2}$ AB

$= \frac{1}{2}\sqrt{(5-(-5)^2 + (-4-(-2))^2}$

$= \frac{1}{2}\sqrt{(5+5)^2 + (-4+2)^2}$

$= \frac{1}{2}\sqrt{100+4}$

$= \frac{1}{2}\sqrt{104}$

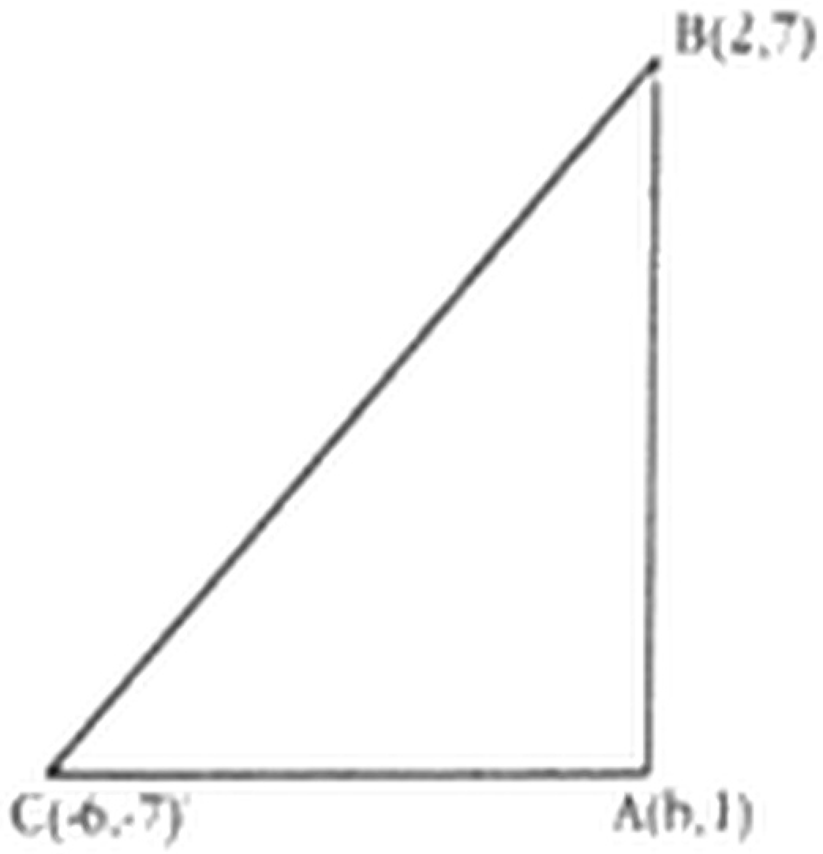
$= \frac{1}{2}\sqrt{4 \times 26}$

$= \frac{1}{2}2\sqrt{26}$

$= \sqrt{26}$

Q9. Find h such that the point A(h,1),B(2,7) and C(-6,-7) are vertices of a right triangle at the vertex A.

Solution



The point A(h,1),B(2,7) C(-6,-7) will form a light triangle

$\Rightarrow \quad (2-h)^2 + (7\text{-}1)^2 + (\text{-}6\text{-}h)^2(-7-1)^2 = (\text{-}6\text{-}2)^2 + (-7-7)^2$

$\Rightarrow \quad 4 + h^2 - 4h{+}36{+}h^2 + 12h{+}64 = 64 + 196$

$\Rightarrow \quad h^2 + 4h\text{-}120 = 0$

$\Rightarrow \quad h^2{+}10h\ \text{-}6h\text{-}60 = 0$

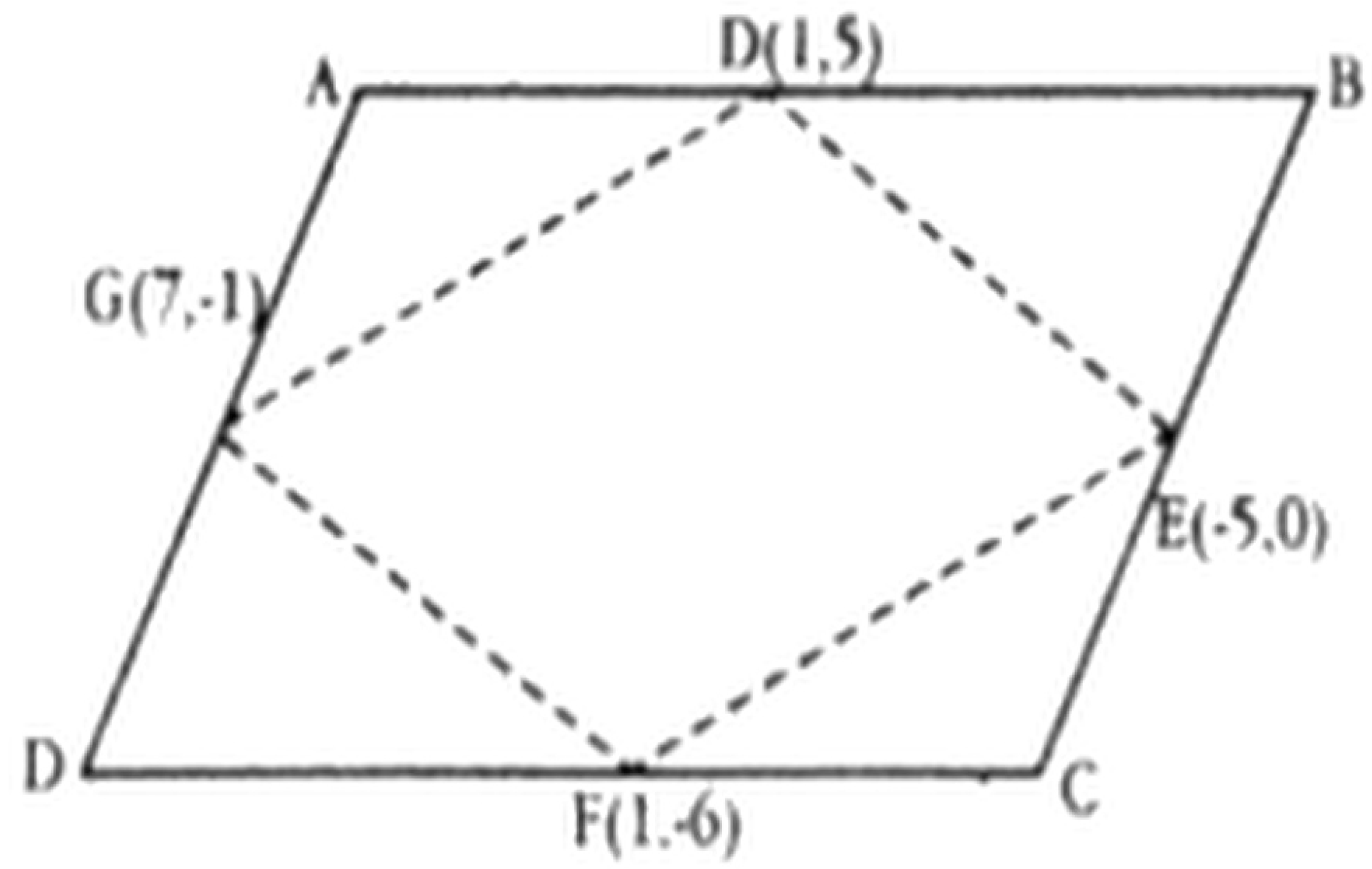
$\Rightarrow \quad h(h{+}10)\text{-}6(h{+}10) = 0$

$\Rightarrow \quad (h\text{-}6)(h{+}10) = 0$

$\Rightarrow$ h = -10, h = 6

Q10. A quadrilateral has the point A(9,3),B(-7,7),C(-3,-7) and D(5,-5) as its , vertices: Find the mid-point of its, sides show that the figure formed by joining the mid- point , consecutively is a parallelogram.

Solution



Mid-point of $\overline{AB}$ is D $\left(\frac{9-7}{7},\frac{3+7}{2}\right) = D(1,5)$

Mid-point of $\overline{BC}$ is E is $\left(\frac{-7-3}{2},\frac{-7-7}{2}\right) = E(-5,0)$

Mid-point of $\overline{DC}$ is F is $\left(\frac{-3+5}{2},\frac{-7-5}{2}\right) = F(1,-6)$

Mid-point of $\overline{AD}$ is G is $\left(\frac{9+5}{2},\frac{3-5}{2}\right) = G(7,-1)$

Now we will prove DEFG is a parallelogram

$|DE| = \sqrt{(-5-1)^2+(0-5)^2} = \sqrt{36+25} = \sqrt{61}$

$|FG| = \sqrt{(7-1)^2+(-1+6)^2} = \sqrt{36+25} = \sqrt{61}$

$|EF| = \sqrt{(1-5)^2+(-6-0)^2} = \sqrt{36+36} = \sqrt{72}$

$|GD| = \sqrt{(7-1)^2+(1+5)^2} = \sqrt{36+36} = \sqrt{72}$

$|DE| = |FG|$

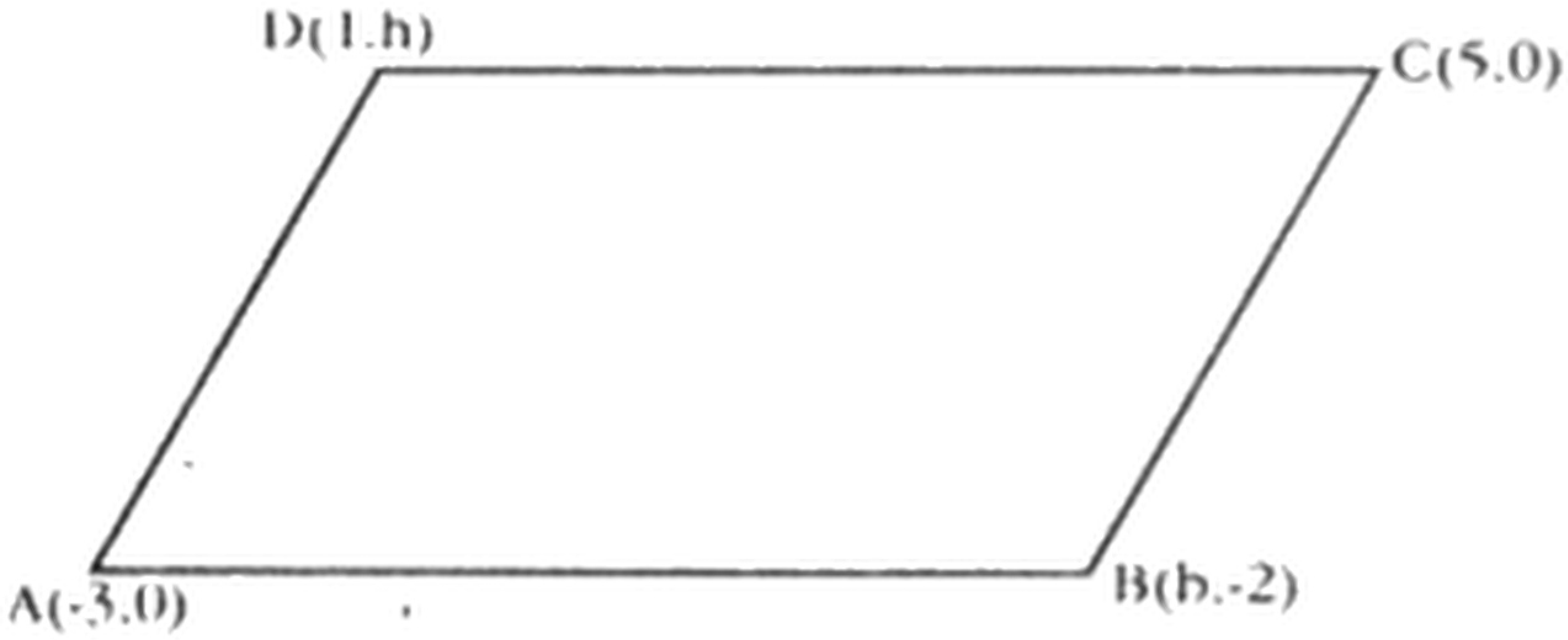
and $|EF| = |GD|$

Opposite sides are equal

$\Rightarrow$ DEFG is a parallelogram

Q11. Find h such that the quadrilateral with A(-3,0), B(1,-2),C(5,0) and D (1,h) is a parallelogram. Is its square

Solution



ABCD is parallelogram

If $|AB| = |CD|$

and $|BC| = |AD|$

$|AB| = \sqrt{(1+3)^2 + (-2-0)^2}$

$= \sqrt{16+4}$

$= \sqrt{20}$

Now $|CD| = \sqrt{(1-5)^2 + (h\text{-}0)^2}$

$= \sqrt{(4)^2 + (h)^2}$

$= 16+h^2$

Now As $|AB| = |CD|$

$\Rightarrow \quad h^2+16 = 20$

$\Rightarrow \quad h^2 = 20 - 16$

$\Rightarrow \quad h^2 = 4$

$\Rightarrow \quad h^2 = \pm 2$

When h = 2, vertices of a parallelogram are

$|AC| = \sqrt{(3-5)^2 + (0\text{-}0)^2}$

$= \sqrt{(-8)^2}$

$= 8$

$|BD| = \sqrt{(-1-1)^2 + (2\text{-}2)^2}$

$= \sqrt{(0)^2 + (4)^2}$

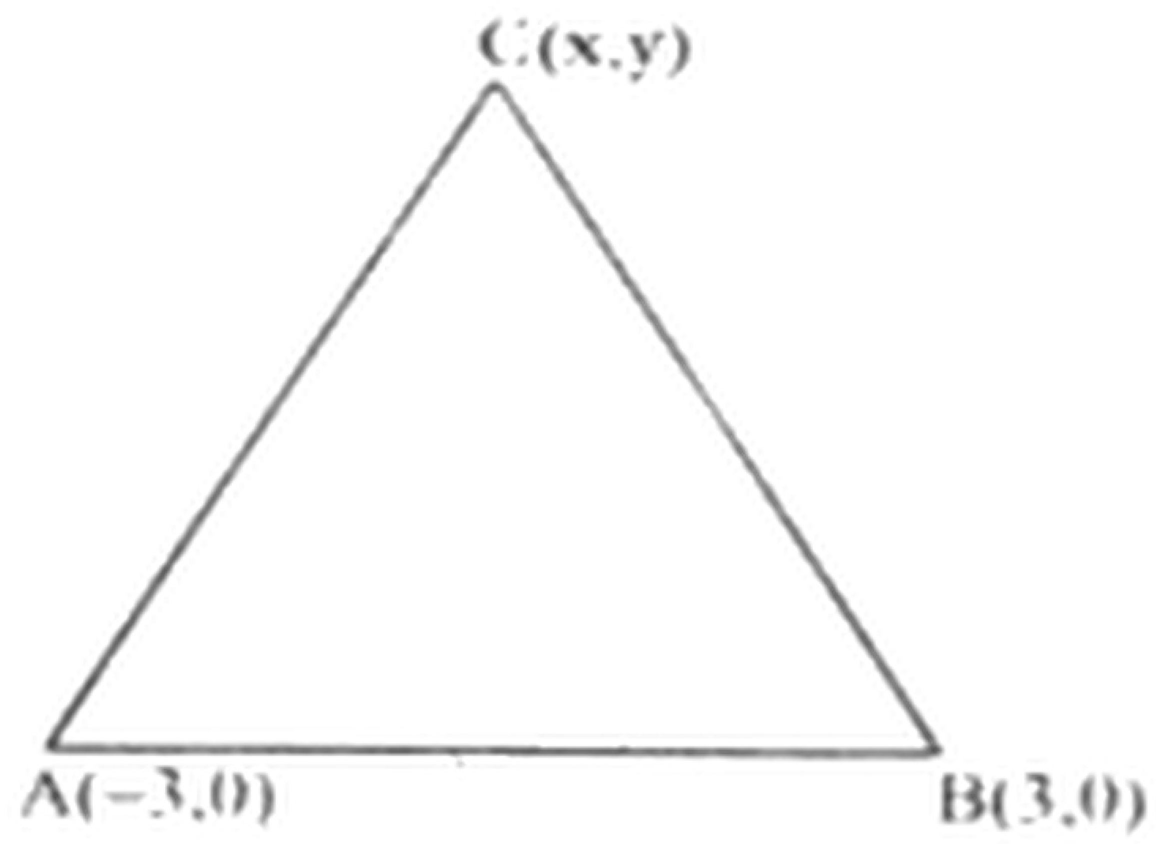
$= 4$

$\therefore \quad |AC| \neq |BD|$

Thus ABCD is not a square.

Q12. If two vertices of an equilateral triangle are A(-3,0) and B(3,0) Find the third vertex. How many of these triangles are possible.

Solution



Let C(x ,y) be the third vertex of the Δ ABC Such that

$$|AC| = |BC| = |AB|$$

$$\sqrt{(x+3)^2 + (y-0)^2} = \sqrt{(x-3)^2 + (y-0)^2}$$

$$= \sqrt{(3+3)^2 + (0-0)^2}$$

From first two we have

$$x^2 + 6x+9+y^2 = x^2 - 6x+9+y^2$$

$$\Rightarrow \quad 6x+6x = 0$$

$$12x = 0$$

$$x = 0 \quad \text{\_\_\_\_\_\_\_\_\_\_(1)}$$

Also $\quad x^2 - 6x+9+y^2 = 36$

Put $\quad x = 0$

$$9+y^2 = 36$$

$$y^2 = 36-9$$

$$y^2 = 27$$

$$y = \pm\sqrt{27}$$

$$= \pm 3\sqrt{3}$$

Thus the co-ordinate of third vertex are $(0, 3\sqrt{3})$, $(0,-3\sqrt{3})$

⇒ Two triangles are possible

Q13. Find the points trisecting join of A(-1,4) and B(6,2).

Solution

Let $C(x_1, y_1)$ and $D(x_2, y_2)$ be the point of intersection.

The point C divides internally in the ration 1=2

$$x_1 = \frac{1(6)+2(-1)}{1+2} = \frac{6-2}{3} = \frac{4}{3}$$

$$y_1 = \frac{1(2)+2(4)}{1+2} = \frac{2+8}{3} = \frac{10}{3}$$

⇒ Co-ordinates of C are $\left(\frac{4}{3}, \frac{10}{3}\right)$ the point D divides AB Internally in the ratio 2=1

$$x_2 = \frac{2(6)+1(-1)}{2+1} = \frac{12-1}{3} = \frac{11}{3}$$

$$y_2 = \frac{2(2)+1(4)}{2+1} = \frac{4+4}{3} = \frac{8}{3}$$

∴ Co-ordinates of D are $\left(\frac{11}{3}, \frac{8}{3}\right)$

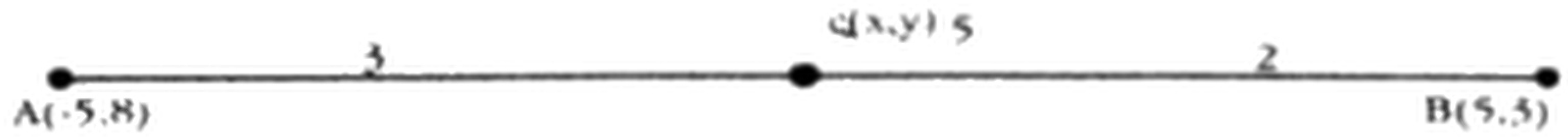
Other wise $D(x_2, y_2)$ being the mid-point of CB

$$\Rightarrow \quad x_2 = \frac{\frac{4}{3}+6}{2} = \frac{22}{6} = \frac{11}{3}$$

$$\Rightarrow \quad y_2 = \frac{\frac{10}{3}+2}{2} = \frac{16}{6} = \frac{8}{3}$$

Q14. Find the point threes fifth of the way along the line segment from A(-5,8) to B(5,3)

Solution



$$x = \frac{3(5)+2(-5)}{3+2}$$

$$= \frac{15-10}{3}$$

$$= \frac{5}{35}$$

$$= 1$$

And $y = \frac{3(3)+2(8)}{3+2}$

$$y = \frac{9+16}{5}$$

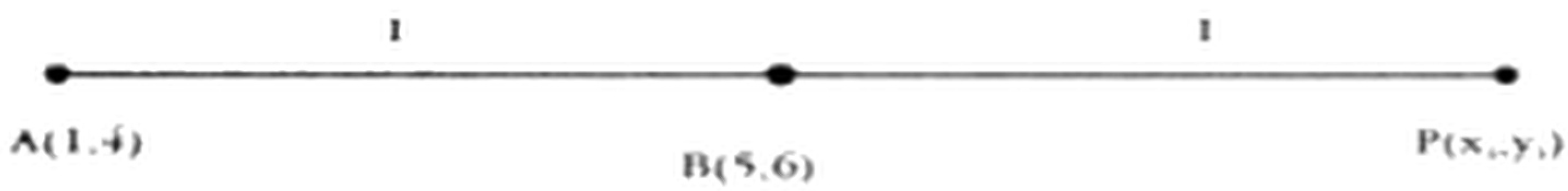
$$y = \frac{25}{5}$$

$$y = 5$$

so, C(1,5)

Q15. Find the point on the join of A(1,4) and B(5,6) that is twice an far from A as B is from A and 1. i-e.(i) on the same side of A as B does (ii) on the opposite of A as B does.

Solution



Let P (x_1, y_1) be the required point a which lies on the same side of A as B does

$|AB| : |BC| = 1:1$

$$\Rightarrow \frac{1+x_1}{2} = 5, \frac{4+y_1}{2} = 6$$

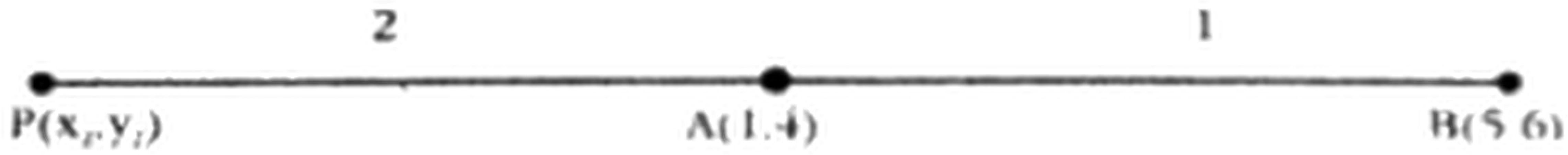
$\Rightarrow \quad 1 + x_1 = 10 \ , \ 4 + y_1 = 12$

$\Rightarrow \quad x_1 = 9 \ , \ y_1 = 8$

Thus the co-ordinates of C are (9,8)

Ii. Let P $(\mathbf{x_1}, \mathbf{y_1})$ be the required point which lies on the opposite side of A as B does, such that

$|PA| \quad : \quad |AB| \quad = \quad 2 \qquad 1$



$\Rightarrow \qquad 1 = \frac{2(5)+1(x_2)}{2+1}$

$\Rightarrow \qquad 1 = \frac{10+x^2}{3}$

$\Rightarrow \quad 10 + x_2 = 3$

$\Rightarrow \qquad x_2 = -7$

$\Rightarrow$ p(-7,0)

$\Rightarrow \qquad 4 = \frac{2(6)+1(y_2)}{2+1}$

$\Rightarrow \qquad 4 = \frac{12+(y_2)}{3}$ (Thus coordinates of 0 are [-7,0]

$\Rightarrow \quad 12 + (y_2) = 12$

$y_2 = 0$

Q16. Find the point which is equidistant from the point A(5,3), B(-2,2) and C(4,2).

What is the ratios of circum circle of the Δ ABC.

Solution

Let P (x_1, y_1) be the Point which is equidistant from the point

A(5,3), B(-2,2) and C(4,2).

$$|PA|^2 = |PB|^2 = |PC|^2$$

$$(x-5)^2 + (y-3)^2 = (x+2)^2 + (y-2)^2 = (x-4)^2 + (y-2)^2 \qquad (1)$$

$$(x-5)^2 + (y-3)^2 = (x+2)^2 + (y-2)^2$$

$$x^2 - 10x + 25 + y^2 - 6y + 9 = x^2 - 4x + 4 + y^2 - 4y + 4$$

$$\Rightarrow 7x + y - 13 = 0 \quad \_\_\_\_\_\_\_\_\_\_ (2)$$

Also from (1)

$$x^2 + 4x + 4 + y^2 - 4y + 4 = x^2 - 8x + 16 + y^2 - 4y + 4$$

$$\Rightarrow \quad 4x + 8x + 8 - 20 = 0$$

$$\Rightarrow \quad 12x = \ldots$$

$$x = 1 \quad \text{put in (2)}$$

$$7(1) + y - 13 = 0$$

$$y = 0$$

Thus P(x,y)= P(1,0) is the required point

Radius of circumcircle of A B C = R PA

$$= \sqrt{(x-5)^2 + (y-3)^2}$$

$$= \sqrt{(1-5)^2 + (0-3)^2}$$

$$= \sqrt{(-4)^2 + (3)^2}$$

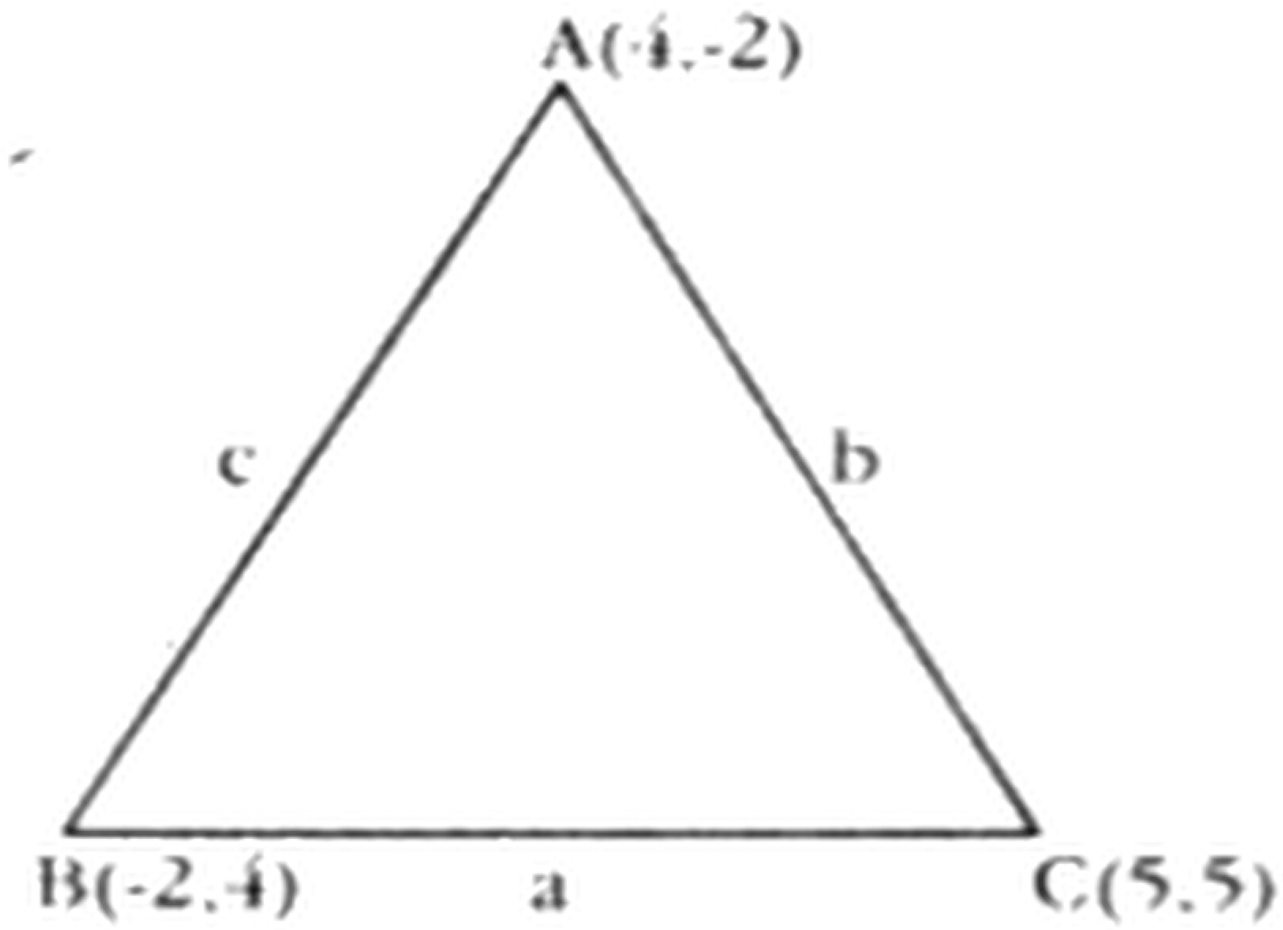
$$= \sqrt{(16)^2 + (9)^2}$$

$$= \sqrt{25}$$

$= 5$

Q17. The point (4,-2), (-2,4) and (5,5) are the vertices of a triangle . Find in circle of the triangle . Also find the centre of triangle ABC

Solution



Let A(4,-2), B (-2,4) and C (5,5)

$c = |AB| = \sqrt{(-2-4)^2 + (2+4)^2}$

$= \sqrt{36 + 36} = 6\sqrt{2}$

$a = |BC| = \sqrt{(5+2)^2 + (5-4)^2}$

$= \sqrt{49 + 1} = \sqrt{50}$

$= \sqrt{25 \times 2}$

$= 5\sqrt{2}$

$b = |AC| = \sqrt{(5+2)^2 + (5+2)^2}$

$= \sqrt{1 + 49} = \sqrt{50} = \sqrt{2}$

$x = \frac{ax_1 + bx_2 + cx_3}{a+b+c} = \frac{(5\sqrt{2})(4) + (5\sqrt{2})(-2) + (6\sqrt{2})(5)}{5\sqrt{2}) + (6\sqrt{2})(+(5\sqrt{2})}$

$$= \frac{20\sqrt{2} - 10\sqrt{2} + 30\sqrt{2}}{\ldots\sqrt{2}}$$

$$= \frac{40\sqrt{2}}{16\sqrt{2}}$$

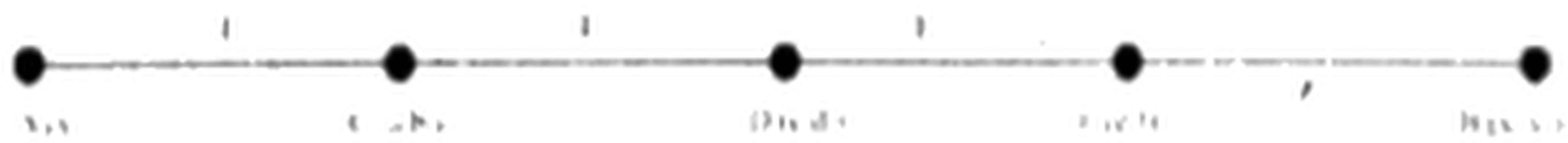
$$= \frac{\ldots}{16} = \frac{5}{2}$$

$$y = \frac{ay_1 + by_2 + cy_3}{a+b+c} = \frac{(5\sqrt{2})(2) + (5\sqrt{2})(4) + (6\sqrt{2})(5)}{16\sqrt{2})}$$

$= \frac{40\sqrt{2}}{16\sqrt{2}} = \frac{5}{2}$ they condinats in- centre are $\left[\frac{5}{2}, \frac{5}{2}\right]$

Q18. Find the point that divide the line segment joining A(x_1, y_1) and B(x_2, y_2) into four equal parts

Solution



The point c(a ,b) divide $\overline{AB}$ in the ratio 1:3

$$a = \frac{(1)(x_2) + (3)(x_1)}{1+3} = \frac{x_2 + 3x_1}{4}$$

$$b = \frac{(1)(y_2) + (3)(y_1)}{1+3} = \frac{y_2 + 3y_1}{4}$$

$\therefore$ c(a ,b) = $C\left[\frac{x_2 + 3x_1}{4}, \frac{y_2 + 3y_1}{4}\right]$

The point D(c ,d) divide $\overline{AB}$ in the ratio 1:1

$$C = \frac{x_1 + x_2}{2} = \quad d = \frac{y_1 + y_2}{2}$$

$\therefore$ D(a ,b) = $D\left[\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right]$

The point F(e ,f) divide $\overline{AB}$ in the ratio 3:1

$$e = \frac{3x_2 + 1x_2}{3+1} = \frac{3x_2 + x_2}{4}$$

$$f = \frac{3y_1 + 1y_1}{3+1} = \frac{3y_2 + y_1}{4}$$

$$F(e, f) = E\left[\frac{3x_2 + x_2}{4}, \frac{3y_2 + y_1}{4}\right]$$