Exercise 3.6

Evaluate the following definite integrals

Q1. $= \int_1^2 (x^2 + 1)dx$

Solution

$$= \int_1^2 (x^2 + 1)dx$$

$$= \int_1^2 x^2 dx + \int_1^2 1dx = \frac{x^3}{3} \Big|_1^2 + x \Big|_1^2$$

$$= \left(\frac{(2)^3 - (1)^3}{3}\right) + \{(2) - (1)\}$$

$$= \left(\frac{8-1}{3}\right) + 1 = \frac{7}{3} + 1$$

$$= \frac{7+3}{3} = \frac{10}{3}$$

Q2. $= \int_1^1 \left(x^{1/3} + 1\right) dx$

Solution

$$= \int_1^1 \left(x^{1/3} + 1\right) dx$$

$$= \int_1^1 x^{1/3} dx + \int_1^1 1\, dx$$

$$= \frac{x^{1/3}}{1/3} \Big|_{-1}^{1} + x \Big|_{-1}^{1}$$

$$= \frac{3}{4} x^{4/3} \Big|_{-1}^{1} + \{(1)-(-1)\}$$

$$= \frac{3}{4} [(1)^{4/3} - (-1)^{3/4}] + 1 + 1$$

$$= \frac{3}{4} [1 - (-1)^{2/3}] + 2$$

Q3. $= \int_{-2}^{0} \frac{1}{(2x-1)^2} dx$

$= \int_{-2}^{0} (2x-1)^{-2} dx$

$= \frac{1}{2} \int_{-2}^{0} (2x-1)^{-2} dx$

$= \frac{1}{2} \left[\frac{(2x-1)^{-2+1}}{-2+1}\right]_{-2}^{0} = \frac{-1}{2} \left[(2x-1)^{-1}\right]_{-2}^{0}$

$= \frac{-1}{2} \left[\frac{1}{2x-1}\right]_{-2}^{0} = \frac{-1}{2} \left[\frac{1}{2(0)-1} - \frac{1}{2(-2)-1}\right]$

$= \frac{-1}{2}\left[-1 - \frac{1}{5}\right] = \frac{-1}{2}\left[\frac{-5+1}{5}\right]$

$= \left(\frac{-1}{2} x\right)\left(\frac{-4}{5}\right) = \frac{2}{5}$

Q4. $= \int_{-6}^{2} \sqrt{3-x} dx$

Solution

$=(-1) \int_{-6}^{2} (3-x)^{1/2} (-1) dx$

$= (-1) \left[\frac{(3-x)^{1/2+1}}{1/2+1}\right]_{-6}^{2}$

$=(-1) \, 2/3 \left[(3-x)^{3/2}\right]_{-6}^{2}$

$= \frac{-2}{3}[(3-2)^{3/2} - (3-(-6))^{3/2}]$

$=\frac{-2}{3}[(1)^{3/2} - (9)^{3/2}]$

$=\frac{-2}{3}[1 - (3)^{2\times 3/2}]$

$= \frac{-2}{3}[1 - (3)^3]$

$= \frac{-2}{3}[1 - 27] = \frac{-2}{3}(-26)$

$= \frac{52}{3}$

Solution

$$= \int_1^{\sqrt{5}} \sqrt{(2t-1)^3}dx$$

$$= \frac{1}{2}\int_1^{\sqrt{5}} (2t-1)^{3/2} \times (2)dx$$

$$= \frac{1}{2} \quad \frac{(2t-1)^{3/2+1}}{2/2+1} \quad {}_1^{\sqrt{5}}$$

$$= \frac{1}{2} \quad \frac{2}{5}|(2t-1)^{5/2}| \quad {}_1^{\sqrt{5}}$$

$$= \frac{1}{5} \quad (2t-1)^{5/2} \quad {}_1^{\sqrt{5}} = \frac{1}{5}[(2\sqrt{5})-1]^{5/2} - \{2(1)-1\}^{5/2}$$

$$= \frac{1}{5}[(2\sqrt{5}-1)^{5/2} - (1)^{5/2}]$$

$$= \frac{1}{5}[(2\sqrt{5}-1)^{5/2} - 1]$$

Q6. $\int_2^{\sqrt{5}} x\sqrt{x^2-1}dx$

Solution

$$= \int_2^{\frac{2}{\sqrt{5}}} (x^2-1)^{\frac{1}{2}} x\, dx$$

$$= \frac{1}{2}\int_2^{\frac{2}{\sqrt{5}}} (x^2-1)^{\frac{1}{2}} x\, dx$$

$$= \frac{1}{2} \quad \frac{(x^2-1)^{1/2+1}}{1/2+1} \quad {}_2^{\sqrt{5}}$$

$$= \frac{1}{2} \times \frac{2}{3} \quad (x^2-1)^{3/2} \quad {}_2^{\sqrt{5}}$$

$$= \frac{1}{3}\left[[(\sqrt{5})^2-1]^{\frac{3}{2}} - [(2)^2-1]^{\frac{3}{2}}\right]$$

$$= \frac{1}{3}\left[[5-1]^{\frac{3}{2}} - [4-1]^{\frac{3}{2}}\right]$$

$$= \frac{1}{3}\left[[4]^{\frac{3}{2}} - [3]^{\frac{3}{2}}\right]$$

$$= \frac{1}{3}[[2]^3 - 3\sqrt{3}] = \frac{1}{3}[8 - 3\sqrt{3}]$$

Q7. $\int_1^2 \frac{x}{x^2+2}\,dx$

Solution

$$\int_1^2 \frac{x}{x^2+2}\,dx = \frac{1}{2}\int_1^2 \frac{2x}{x^2+2}\,dx$$

$$= \frac{1}{2}|ln\ x^2 + 2\ |_1^2 = \frac{1}{2}[ln\{(2)^2 + 2\} - \ln\{(1)^2 + 2\}]$$

$$= \frac{1}{2}[ln(4+2) - \ln(1+2)] = \frac{1}{2}[ln6 - \ln 3]$$

$$= \frac{1}{2} ln\frac{6}{3} = \frac{1}{2} ln\ 2$$

Q8. $\int_2^3 (x - \frac{1}{x})^2\,dx$

Solution

$$\int_2^3 (x - \frac{1}{x})^2\,dx = \int_2^3 [x^2 - 2x(\frac{1}{x}) + \frac{1}{x^2}]\,dx$$

$$= \int_2^3 [x^2 - 2 + x^{-2}]\,dx = \int_2^3 x^2\,dx - 2\int_2^3 1\,dx + \int_2^3 x^{-2}\,dx$$

$$= \frac{x^3}{3}\ {}_2^3 - 2\ x\ {}_2^3 + \frac{x^{-2+1}}{-2+1}\ {}_2^3 = \frac{(3)^3-(2)^3}{3} - 2[(3)-(2)] - [\frac{1}{x}]_2^3$$

$$= \frac{27-8}{3} - 2(1) - [\frac{1}{3} - \frac{1}{2}] = \frac{19}{3} - 2 - [\frac{2-3}{3}]$$

$$= \frac{19}{3} - 2 - \frac{1}{6}$$

$$= \frac{38-12+1}{6} = \frac{27}{6} = \frac{9}{2}$$

Q9. $\int_{-1}^1 (x+\frac{1}{2})\sqrt{x^2+x+1}\,dx$

Solution

$$\int_{-1}^1 (x+\frac{1}{2})\sqrt{x^2+x+1}\,dx$$

$$= \frac{1}{2}\int_{-1}^1 (2x+1)(x^2+x+1)^{\frac{1}{2}}\,dx$$

$= \frac{1}{2} \left[\frac{(x^2+x+1)^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right]_{-1}^{1} = \frac{1}{2} \times \frac{2}{3} \left[(x^2+x+1)^{\frac{3}{2}} \right]_{-1}^{1}$

$= \frac{1}{3}[\{(1)^2 + (1) + 1)^{\frac{3}{2}} - \{(-1)^2 - 1 + 1)^{\frac{3}{2}}]$

$= \frac{1}{3}[(3)^{\frac{3}{2}} - (1)^{\frac{3}{2}}] = \sqrt{3} - \frac{1}{3}$

Q10. $\int_0^3 \frac{dx}{x^2+9}$

Solution

$\int_0^3 \frac{dx}{x^2+9} = \int_0^3 \frac{1}{(x)^2+(3)^2} dx$

$= \frac{1}{3} \left[\tan^{-1} \frac{1}{3} \right]_0^3 = \frac{1}{3} [\tan^{-1}(\frac{3}{3}) - \tan^{-1} 0]$

$= \frac{1}{3} [\tan^{-1}(1) - \tan^{-1} 0] = \frac{1}{3}(\frac{\pi}{4} - 0) = \frac{\pi}{12}$

Q11 $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}$ **Cos t dt**

Solution

$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \text{Cos t dt} = \left[Sin\ t \right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} \quad [\because \frac{d}{dt}(Sin\ t) = Cos\ t]$

$= \text{Sin} \frac{\pi}{3} - Sin \frac{\pi}{6} = \frac{\sqrt{3}}{2} - \frac{1}{2}$

$\Rightarrow \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \text{Cos t dt} = \frac{\sqrt{3}-1}{2}$

Q12. $\int_1^2 (x + \frac{1}{x})^{\frac{1}{2}}(1 - \frac{1}{x^2})$ **dx**

Solution

$\int_1^2 (x + \frac{1}{x})^{\frac{1}{2}}(1 - \frac{1}{x^2}) \ dx$

$dy = (1 - \frac{1}{x^2})dx$

when x = 2, $y = \frac{5}{2}$

when x =1 , y =2

$= \int_2^{\frac{5}{2}} y^{\frac{1}{2}} \; dy = \frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1} \; {}_2^{\frac{5}{2}} = \frac{2}{3} \; y^{\frac{1}{2}+1} \; {}_2^{\frac{5}{2}} = \frac{2}{3}[\,(5/2)^{\frac{3}{2}} - (2)^{\frac{3}{2}}] = \frac{2}{3}[\frac{5^{3/2}}{2^{3/2}} - 2^{3/2}]$

$= \frac{2}{3}[\frac{5\sqrt{5} - 2^3}{2\sqrt{2}} = \frac{2}{3\sqrt{2}}[5\sqrt{5} \; -8]$

Q13. $\int_1^2 \ln x \, dx$

Solution

$\int_1^2 \ln x \, dx = \; x \ln x \; {}_1^2 - \int_1^2 x \frac{d}{dx}(\ln x) \, dx$

$= \; x \ln x \; {}_1^2 - \int_1^2 x \times \frac{1}{x} \, dx = 2\ln(2) - 1\ln(1) - \int_1^2 1 \, dx$

$= 2 \ln 2 - (0) - \; x \; {}_1^2 \quad (\therefore \ln 1 = 0)$

$= 2\ln 2 - (2-1)$

$= 2 \ln 2 - 1$

Q14. $\int_0^2 (e^{x/2} - e^{-x/2}) dx$

Solution

$\int_0^2 (e^{x/2} - e^{-x/2}) dx = \int_0^2 e^{x/2} \, dx - \int_0^2 e^{-x/2} dx$

$= 2 \int_0^2 e^{x/2} \times \frac{1}{2} dx + 2\int_0^2 e^{-x/2} \times (-\frac{1}{2}) dx$

$= 2 \; e^{x/2} \; {}_0^2 + 2 \; e^{-x/2} \; {}_0^2 = 2[e^{2/2} - e^0] + 2[e^{-2/2} - e^0]$

$= 2[e-1] + [e^{-1} - 1] = 2[e-1+\frac{1}{e} - 1]$

$= 2[e+\frac{1}{e} - 2] = \frac{2}{e}[e^2 + 1 - 2e]$

Q15. $\int_0^{\frac{\pi}{4}} \frac{\text{Cos }\theta+\text{Sin}\theta}{\text{Cos }2\theta}d\theta$

Solution

$$\int_0^{\frac{\pi}{4}} \frac{\text{Cos }\theta+\text{Sin}\theta}{2Cos^2\theta}d\theta$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{\text{Cos }\theta}{Cos^2\theta}d\theta+\frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{\text{Sin }\theta}{\text{Cos}\theta} \times \frac{1}{Cos\theta}d\theta$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{4}} \text{Sec }\theta d\theta+\frac{1}{2}\int_0^{\frac{\pi}{4}} Sec\ \theta\ tan\theta\ d\theta$$

$$= \frac{1}{2}\ \ln(Sec\ \theta\ tan\theta\ )\ \Big|_0^{\frac{\pi}{4}} + \frac{1}{2}\ \ Sec\ \theta\ \Big|_0^{\frac{\pi}{4}}$$

$$=\frac{1}{2}[\ ln(Sec\frac{\pi}{4}+\ \ tan\frac{\pi}{4}) - \ ln(Sec\ 0\ +\ tan\ 0)]$$

$$+\frac{1}{2}[Sec\frac{\pi}{4}\ -\ Sec\ 0]$$

$$= \frac{1}{2}[ln|\sqrt{2}+1| - ln|1+0| + \frac{1}{2}[(\sqrt{2}) - (1)]$$

$$= \frac{1}{2}[ln|\sqrt{2}+1| - 0 + \frac{1}{2}[(\sqrt{2}) - (1)]$$

$$=\frac{1}{2}[ln|\sqrt{2}+1| - 0 + \sqrt{2} - 1]$$

Q16. $\int_0^{\frac{\pi}{6}} \text{Cos}^3\theta d\theta$

Solution

$$\int_0^{\frac{\pi}{6}} \text{Cos}^3\theta d\theta = \int_0^{\frac{\pi}{6}} \text{Cos}^2\theta\ \times \text{Cos}\theta d\theta$$

$$= \int_0^{\frac{\pi}{6}} (1 - \text{Sin}^2\theta\ )\text{Cos}\theta d\theta = \ \ Sin\ \theta\ \Big|_0^{\frac{\pi}{6}} - \ \frac{\text{Sin}^3\theta}{3}\ \Big|_0^{\frac{\pi}{6}}$$

$$= (\text{Sin}\frac{\pi}{6}\ - Sin\ 0)\ - [\frac{\text{Sin}^3(\frac{\pi}{6}) - \text{Sin}^3(0)}{3}]$$

$$= (\frac{1}{2} - 0) - \frac{1}{3}[(\frac{1}{2})^3\ - (0)^3$$

$$= \frac{1}{2} - \frac{1}{3} \times \frac{1}{8} = \frac{1}{2} - \frac{1}{24}$$

Q17. $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \mathbf{Cos^2\theta Cot^2\theta d\theta}$

Solution

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} Cos^2\theta(Cosec^2\theta - 1)d\theta$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} Cos^2\theta \times Cosec^2\theta\, d\theta - \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} Cos^2\theta\, d\theta$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} Cot^2\theta\, d\theta - \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (\frac{1+Cos^2\theta}{2})\, d\theta$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (Cot^2\theta - 1)\, d\theta - \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 1\, d\theta - \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} Cos\, 2\theta\, d\theta$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (Cosec^2\theta\, d\theta - \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 1\, d\theta - \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 1\, d\theta - \frac{1}{2}\times\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} Cos\, 2\theta \times 2d\theta$$

$$= \quad -Cot\,\theta \Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} - \frac{3}{2}\ \theta \Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} - \frac{1}{4}\ Sin2\theta \Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}}$$

$$= -[(Cot\frac{\pi}{4} - Cot\ \frac{\pi}{6})] - \frac{3}{2}(\frac{\pi}{4} - \frac{\pi}{6})\ - \frac{1}{4}(Sin\frac{\pi}{2} - Sin\ \frac{\pi}{3})$$

$$= (1-\sqrt{3}) - \frac{3}{2}(\frac{3-2}{12})\pi\ - \frac{1}{4}(-\frac{\sqrt{3}}{2})$$

$$= \sqrt{3} - 1 - \frac{\pi}{8} - \frac{1}{4} + \frac{\sqrt{3}}{8} = \frac{3\sqrt{3}-8-\pi-2+\sqrt{3}}{8}$$

$$= \frac{9\sqrt{3}-10-\pi}{8}$$

Q18. $\int_{0}^{\frac{\pi}{4}} \mathbf{Cos^4 t dt}$

Solution

$$= \frac{1}{4}\int_{0}^{\frac{\pi}{4}} (2Cos^2t)(2Cos^2t)dt\ = \frac{1}{4}\int_{0}^{\frac{\pi}{4}} (1 + Cos\, 2t)^2dt$$

$$= \frac{1}{4}[\int_{0}^{\frac{\pi}{4}} 1 + Cos^2 2t + 2Cos\, 2t)dt\,]$$

$$= \frac{1}{4}[\int_{0}^{\frac{\pi}{4}} 1 + \frac{1}{2}\int_{0}^{\frac{\pi}{4}}(1 + Cos\, 4t)\, dt + \int_{0}^{\frac{\pi}{4}}(Cos2t)2.\, dt\,]$$

$= \frac{1}{4}[(\frac{\pi}{4} - 0) + \frac{1}{2}(\frac{\pi}{4} - 0) + \frac{1}{8}(\text{Sin } 4\frac{\pi}{4} - Sin\ 0) + (\text{Sin } 2\frac{\pi}{4} - Sin\ 0)]$

$= \frac{1}{4}[\frac{\pi}{4} + \frac{\pi}{8} + \frac{1}{8}(Sin\ \pi - Sin\ 0) + (Sin\ \frac{\pi}{4} - Sin\ 0)]$

$= \frac{1}{4}[\frac{\pi}{4} + \frac{\pi}{8} + \frac{1}{8}(0 - 0) + 1 - 0]$

$= \frac{1}{4}[\frac{2\pi + \pi}{8} + 1] = \frac{3\pi + 8}{32}$

Q19. $\int_0^{\frac{\pi}{3}} \text{Cos}^2\ \text{Sin}\theta d\theta$

Solution

$= -\int_0^{\frac{\pi}{3}} \text{Cos}^2\theta \times (-\text{Sin}\theta)d\theta = -\ \frac{\text{Cos}^3\theta}{3} \Big|_0^{\frac{\pi}{3}}$

$= -[\frac{\text{Cos}^3\frac{\pi}{3} - \text{Cos}^3\theta}{3}] = -[\frac{(1/2)^3 - (1)^3}{3}]$

$= -\frac{1}{3}[\frac{1}{8} - 1] = -\frac{1}{3}[\frac{1-8}{3}]$

$= -\frac{1}{3} \times \frac{-7}{8} = \frac{7}{24}$

Q20. $\int_0^{\frac{\pi}{4}} (1 + \text{Cos}^2\theta)\tan^2\theta d\theta$

Solution

$= \int_0^{\frac{\pi}{4}} \tan^2\theta d\theta + \int_0^{\frac{\pi}{4}} \text{Cos}^2\theta \times \frac{\text{Sin}^2\theta}{\text{Cos}^2\theta} d\theta$

$= \int_0^{\frac{\pi}{4}} (\text{Sec}^2\theta - 1)d\theta + \int_0^{\frac{\pi}{4}} \text{Sin}^2\theta d\theta$

$= \int_0^{\frac{\pi}{4}} \text{Sec}^2\theta\ d\theta - \int_0^{\frac{\pi}{4}} 1\ d\theta + \frac{1}{2}\int_0^{\frac{\pi}{4}} 2\text{Sin}^2\theta d\theta$

$= \tan\theta \Big|_0^{\frac{\pi}{4}} - \theta \Big|_0^{\frac{\pi}{4}} + \frac{1}{2}\int_0^{\frac{\pi}{4}} (1 - \text{Cos } 2\theta)d\theta$

$= [(\tan\frac{\pi}{4} - tan\ 0)] - (\frac{\pi}{4} - 0) + \frac{1}{2}\int_0^{\frac{\pi}{4}} 1\ d\theta - \frac{1}{2}\int_0^{\frac{\pi}{4}} 2Cos\ 2\theta d\theta$

$= 1\frac{\pi}{4} + \frac{1}{2}(\frac{\pi}{4} - 0) - \frac{1}{4}(Sin\ 2 \times \frac{\pi}{4} - Sin\ 0)$

$= 1 - \frac{\pi}{4} + \frac{\pi}{8} - \frac{\pi}{4} = \frac{3}{4} + \frac{\pi - 2\pi}{4} = \frac{3}{4} - \frac{\pi}{8}$

$= \frac{3}{4} - \frac{\pi}{8} = \frac{6-\pi}{8}$

Q21. $\int_0^{\frac{\pi}{4}} \frac{Sec\theta}{Sin\theta + Cos\,\theta} d\theta$

Solution

$= \int_0^{\frac{\pi}{4}} \frac{Sec\theta}{Cos\,\theta(\frac{Sin\,\theta}{Cos\,\theta}+1)} d\theta = \int_0^{\frac{\pi}{4}} \frac{Sec^2\theta}{(\tan\theta+1)} d\theta$

$= \int_0^{\frac{\pi}{4}} (1 + \tan\theta)^{-1} Sec^2\theta\, d\theta = \ln(1 + tan\,\theta\ {}_0^{\frac{\pi}{4}}$

$= \ln|1 + \tan\frac{\pi}{4}| - \ln|1 + \tan 0|$

$= \ln|1+1| - \ln|1+0| = \ln 2 - \ln 1 = \ln\frac{2}{1} \ln 2$

Q22. $\int_{-1}^{5} |x - 3|$

Solution

$= \int_{-1}^{3} |x-3|\, dx + \int_{3}^{5} |x-3|\, dx$

$= \int_{-1}^{3} (x-3)\, dx + \int_{3}^{5} (x-3)\, dx$

$= \frac{(x-3)^2}{2}\ {}_{-1}^{3} + \frac{(x-3)^2}{2}\ {}_{3}^{5}$

$= \frac{1}{2}[(3-3)^2 - (-1-3)^2] + \frac{1}{2}[(5-3)^2 - (3-3)^2]$

$= -\frac{1}{2}[0 - 16] + \frac{1}{2}[4 - 0] = 8+2 = 10$

Q23. $\int_{1/8}^{1} \frac{(x^{1/3}+2)^2}{x^{2/3}} dx$

$$= \int_{1/8}^{1} (x^{1/3} + 2)^2 \frac{1}{3} \times \frac{-2}{3} dx$$

$$= 3 \left[\frac{(x^{1/3}+2)^{2+1}}{2+1}\right]_{1/8}^{1} = \left[(x^{1/3} + 2)^3\right]_{1/8}^{1}$$

$$= ((1)^{1/3} + 2)^3 - ((\tfrac{1}{8})^{1/3} + 2)^3 = (1 + 2)^3 - ((\tfrac{1}{8})^{1/3} + 2)^3$$

$$= 27 - (\tfrac{1}{2} + 2)^{1/3} = 27 - \frac{125}{8} = \frac{91}{8}$$

Q24. $\int_1^3 \frac{x^2-2}{x+1} dx$

Solution

$$= \int_1^3 \frac{x^2-1-1}{x+1} dx = \int_1^3 (x-1)\, dx - \int_1^3 \frac{1}{x+1} dx$$

$$= \left[\frac{(x-1)}{2}\right]_1^3 - \left[ln(x+1)\right]_1^3$$

$$= \frac{(3-1)^2-(1-1)^2}{2} - [\ln(3+1) - \ln(1+1)]$$

$$= \frac{2^2-0}{2} - [\ln 4 - \ln 2]$$

$$= \frac{4}{2} \ \ln(\tfrac{4}{2}) = 2 - \ln 2$$

Q25. $\int_2^3 \frac{3x^2-2x+1}{(x-1)(x^2+1)} dx$

Solution

Consider $\frac{3x^2-2x+1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{(x^2+1)}$.........(i)

Multiplying on both sides by $(x-1)(x^2+1)$ we have

$3x^2 - 2x + 1 = A(x^2 + 1) + (Bx + C)(x - 1)$.........(ii)

Put x =1 in(ii) we have

$(1)^2 - 2(1) + 1 = A((1)^2 + 1)$

Comparing Co-efficients of $(x)^2$,x and constant terms of (ii) , we have

A+B = 3 $\Rightarrow$ 1+B = 3 $\Rightarrow$ B =2

-B +C = -2

-2+C = -2 $\Rightarrow$ C = -2+2 = 0 $\Rightarrow$ C =0

Hence (1) can be written as:

$=\int_2^3 \frac{3x^2-2x+1}{(x-1)(x^2+1)}dx \int_2^3 \frac{1}{(x-1)}dx + \int_2^3 \frac{2x}{(x^2+1)}dx$

$= \quad ln(x-1) \;{}_2^3 + \; ln(x^2+1) \;{}_2^3$

= ln|3-1| -ln|2-1| +ln| $(3)^2+1| - ln\,|(2)^2+1|$

= ln 2 -ln 1+ln 10 -ln 5 = ln 2 -ln 5+ln 10

= ln $(\frac{2}{5})$ + $ln\ 10$ = l n $(\frac{2}{5} \times 10)$

= ln 4

Q26. $\int_0^{\pi/4} \frac{1}{1+\text{Sin}\,x}\,dx$

Solution

$=\int_0^{\pi/4} \frac{1}{1+\text{Sin}\,x} \times \frac{(1-Sin\,x)}{(1-Sin\,x)}dx = \int_0^{\pi/4} \frac{(1-Sin\,x)}{(1-Sin^2\,x)}dx$

$= \int_0^{\pi/4} \frac{(1-Sin\,x)}{Cos^2\,x}dx = \int_0^{\pi/4} \frac{1}{Cos^2\,x} - \frac{Sin\,x}{Cos^2\,x}]\,dx$

$= \int_0^{\pi/4} Sec^2 x\,dx - \int_0^{\pi/4} Sec\,x\,tanx\,dx$

$= \quad tan\,x \;{}_0^{\pi/4} - \; Sec\,x \;{}_0^{\pi/4}$

= tan $^{\pi}/_4$ -tan 0 –[sec $^{\pi}/_4 -$ $Sex\,(0)$] = 1 -0 –[$\sqrt{2}$ -1]

= 1- $\sqrt{2}$ +1 = 2 -$\sqrt{2}$

$= -\int_0^1 \frac{-3x}{\sqrt{4-3x}}\,dx = -\int_0^1 \frac{4-3x-4}{\sqrt{4-3x}}\,dx$

$= -\int_0^1 \frac{4-3x}{\sqrt{4-3x}}\,dx + 4 - \int_0^1 \frac{1}{\sqrt{4-3x}}\,dx$

$= -\int_0^1 \sqrt{4-3x}\,dx + 4 \times (\frac{-1}{3}) \int_0^1 (4-3x)^{-\frac{1}{2}}\,(-3)\,dx$

$= \frac{1}{3}\int_0^1 (4-3x)^{\frac{1}{2}}\,(-3)dx - \frac{4}{3}\int_0^1 (4-3x)^{-\frac{1}{2}}\,(-3)\,dx$

$= \frac{1}{3} \quad \frac{(4-3x)^{\frac{1}{2}+1}}{\frac{1}{2}+1} \quad {}_0^1 - \frac{4}{3} \quad \frac{(4-3x)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \quad {}_0^1$

$= \frac{2}{9}[(4-3(1))^{\frac{3}{2}} - (4-0)^{\frac{3}{2}}] \ \frac{8}{3}[(4-3(1))^{\frac{1}{2}} - (4-0)^{\frac{1}{2}}]$

$= \frac{2}{9}[(1))^{\frac{3}{2}} - (4)^{\frac{3}{2}}] \ \frac{8}{3}[(1))^{\frac{1}{2}} - \sqrt{4}]$

$= \frac{2}{9}[1-(2)^3] \ \frac{8}{3}[1-2] \quad = \frac{2}{9}[1-8] \ \frac{8}{3}[-1]$

$= \frac{2}{9}[-7] + \frac{8}{3} = \frac{-14}{9} + \frac{8}{3} = \frac{-14+24}{9} = \frac{10}{9}$

Q 29. $\int_{\pi/6}^{\pi/2} \frac{\text{Cos x}}{\text{Sin x}(2+\text{Sin x})}\,dx$

Solution

$= \int_{\pi/6}^{\pi/2} (\frac{2}{Sin\,x} + 1)^{-1}(-2Cosec\,x\,Cot\,x)\,dx$

$= \int_{\pi/6}^{\pi/2} (2Cosec\,x + 1)^{-1}(-2Cosec\,x\,Cot\,x)\,dx$

$= -\frac{1}{6}\int_{\pi/6}^{\pi/2} (2Cosec\,x + 1)^{-1}(-2Cosec\,x\,Cot\,x)\,dx$

$= -\frac{1}{2} \quad \ln(2\text{Cosec x} + 1) \quad {}_{\pi/6}^{\pi/2} = -\frac{1}{2} \quad \ln(\frac{2}{Sin\,x} + 1) \quad {}_{\pi/6}^{\pi/2}$

$= -\frac{1}{2}\left[ln(\frac{2}{1} + 1) - ln(\frac{2}{1/2+1})\right] = -\frac{1}{2}[\ln(2+1)\ \ln(4+1)]$

$= \frac{1}{2}[\ln 3 - \ln 5] = \frac{1}{2}[\ln 5 - \ln 3]$

$= \frac{1}{2} \ln \frac{5}{}$

Q30. $\int_0^2 \frac{\text{Sin x}}{(1+\text{Cosx})(2+\text{Cosx})}\,dx$

Solution

Let u = 1+Cos x

When x = 0, u = 2

When $x = \frac{\pi}{2}$, u =1

du = - Sin x dx

-du = Sin x dx

Now $\int_0^{\pi/2} \frac{\text{Sin x}}{(1+\text{Cosx})(2+\text{Cosx})}\,dx = \int_2^1 [\frac{-du}{u(1+u+1)}]du$

$= -\int_2^1 [\frac{1}{u} - \frac{1}{u+1}]du = \int_2^1 \frac{1}{u}\,du + \int_2^1 \frac{1}{u+1}\,du$

$= -\ \ln u\ \Big|_1^2 + \ln(u+1)\ \Big|_1^2$

= -(ln 2 – ln 1) + (ln (2+1) – ln (1+1))

= ln 2+ 0 + ln 3 – ln 2 = -2 ln 2 + ln 3

= -ln 4 + ln 3 = ln 3 – ln 4 = ln $\frac{3}{4}$