Exercise 3.5

Evaluate the following integrals.

Q1. $\int \frac{3x+1}{x^2-x-6}\,dx$

Solution

$$\frac{3x+1}{(x+2)(x-3)} = \frac{A}{x+2} + \frac{B}{x-3}$$

$$3x+1 = A(x-3) + B(x+2)$$

For A, $x + 2 = 0$, $x = -2$

Put in above

$$-5A = -5$$

$$A = 1$$

And $B = 2$

$$= \frac{1}{x+2} + \frac{2}{x-3} \text{ (By the method of partial fractions)}$$

Now $\int \frac{3x+1}{x^2-x-6}\,dx = \int \left(\frac{1}{x+2} + \frac{2}{x-3}\right)dx$

$$= \int \frac{1}{x+2}dx + 2\int \frac{1}{x-3}dx$$

$$= \int (x+2)^{-1} \times 1\,dx + 2\int (x-3)^{-1}.1\,dx$$

$$= \ln|x+2| + 2\ln|x-3| + C$$

Q2. $\int \frac{5x+8}{(x+3)(2x-1)}\,dx$

Solution

Consider $\frac{5x+8}{\ldots} = \frac{A}{\ldots} + \frac{B}{\ldots}$

$$5x+8 = A(2x-1) + B(x+3) \qquad ....(1)$$

Put x =-3 in (1) we get

$$5(-3) +8 = A(2(-3) -1 +B(-3+3)$$

$$-15+7 = A(-6-1) +0$$

$$-7 = -7 A \qquad \Rightarrow A = 1$$

Put x = $\frac{1}{2}$ in (1) we get

$$= 5(\tfrac{1}{2}) +8 = A\ (2\times \tfrac{1}{2}) + B(\tfrac{1}{2} + 3)$$

$$\frac{5}{2} + 8 \ = B(\tfrac{1+6}{2})$$

$$\frac{21}{2} = \frac{7}{2} B \qquad \Rightarrow B = 3$$

$$\text{Thus } \int \frac{5x+8}{(x+3)(2x-1)}\, dx = \int [\ \frac{1}{x+3} + \frac{3}{2x-1}]\ dx$$

$$= \int \frac{1}{x+3} dx + \int \frac{3}{2x-1} dx$$

$$= \int (x+3)^{-1}\times 1\, dx + \frac{3}{2} \int (2x-1)^{-1}.\, 2\, dx$$

$$= \ln|x+3| + \frac{3}{2}\ln|2x-1| + C$$

Q3. $\int \frac{x^2+3x-34}{x^2+2x-15}\, dx$

Solution

$$= \int [\ 1 + \frac{x-19}{x^2+2x-15}]\ dx$$

$$= \int [\ 1\ dx + \frac{x-19}{x^2+2x-15}]\ dx$$

$$= x\int [\ x + \frac{x-19}{x^2+2x-15}]\ dx \qquad \_\_\_\_\_\_\_\_\_\_(a)$$

Consider

$$= \frac{x-19}{x^2+2x-15} = \frac{x-19}{(x+5)(2x-3)}$$

Multiplying (i) by (x+5)(x-3) we have

$x - 19 = A(x - 3) + B(x+5)$ ....(ii)

Put $x = 3$ in (ii) we get

$3 - 19 = A(3-3)+B(3+5)$

$-16 = 8B$ $\Rightarrow$ $-2 = B$

$B = -2$

Put $x = -5$ in above we get

$-5 - 19 = A(-5-3) + B(3+5)$

$-24 = -8A$ $\Rightarrow$ $3 = A$

Thus $\int \frac{x^2+3x-34}{x^2+2x-15}\, dx = x + \int [\frac{3}{x+5} + \frac{-2}{x-3}]\, dx$

$= (x+3) \int (x+5)^{-1}\, dx - 2\int (x-3)^{-1}.1\, dx$

$= x + 3\ln|x+5| - 2\ln|x-3| + C$

Q4. $\int \frac{(a-b)x}{(x-a)(x-b)}$ **dx** **dx,(a>b)**

Solution

Consider $\frac{(a-b)x}{(x-a)(x-b))} = \frac{A}{x-a} + \frac{B}{x-b}$ ....(i)

Multiplying both sides of (i) by (x-a)(x-b) we have

$(a-b)x = A(x-b) + B(x-a)$ ....(ii)

Put $x = b$ in (1) we get

$(a-b)b = B(b-a)$ $\Rightarrow$ $B = -b$

Put $x = 9$ in (ii) we get

$(a-b)a = A(a-b)) + (a-a)$

$\Rightarrow A = a$

$= a\int (x-a)^{-1}\times 1\,dx - b\int (x-b)^{-1}\,dx$

$= a \ln |x-a| - b \ln |x-b| + C$

Q5. $\int \frac{3-x}{1-x-6x^2}\,dx$

Solution

$\frac{3-x}{(1+2x)(1-3x)} = \frac{A}{1+2x} + \frac{B}{1-3x}$

$3 - x = A(1-3x) + B(1+2x)$ $\Rightarrow x = \frac{-1}{2}$

Put in above we get $A = \frac{7}{5}$

$1-3x = 0 \Rightarrow x = \frac{1}{3}$ put in above we get

SO, $B = \frac{8}{5}$

$= \frac{\frac{7}{5}}{1+2x} + \frac{\frac{8}{5}}{1-3x}$

Thus $\int \frac{3-x}{1-x-6x^2}$

$= \int \frac{7}{5}\cdot\frac{1}{1+2x}\,dx + \int \frac{8}{5}\cdot\frac{1}{1+2x}\,dx$

$= \frac{7}{5}\cdot\frac{1}{2}\int (1+2x)^{-1}\times 2\,dx - \frac{8}{5}\int (\frac{1}{3})(1-3x)^{-1}\,(-3)\,dx$.

$= \frac{7}{10} \ln |1+2x| \frac{8}{15} \ln |1-3x| + C$

Q6. $\int \frac{2x}{x^2-a^2}\,dx$

Solution

$\int \frac{2x}{x^2-a^2}\,dx = \frac{A}{x-a} + \frac{B}{x+a}$

$\Rightarrow x + a = 0$ $\Rightarrow x = -a$

Then $A = 1$

Then B =1

$$= \int \frac{2x}{(x-a)(x+a)} dx$$

$$= \int (\frac{1}{(x-a)} + \frac{1}{(x+a)})dx$$

$$= \int (x-a)^{-1} + \int (x+a)^{-1}\, dx$$

$$= \ln|x\text{-}a| + \ln|x+ a| + C$$

Q7. $\int \frac{1}{6x^2+5x-4} dx$

Solution

$$\int \frac{1}{6x^2+5x-4} dx$$

$$= \int \frac{1}{6x^2+8x-3x-4} dx$$

$$= \int \frac{1}{2x(3x+4)-(3x+4)} dx$$

$$= \int \frac{1}{(2x-1)(3x+4)} dx \quad ......(a)$$

Consider

$$\frac{1}{(3x+4)(2x-1)} = \frac{A}{3x+4} + \frac{B}{2x-1} \quad ......(i)$$

Multiplying both sides of (i) by (3x+4)(2x -1) we have

$$1 = A(2x-1) + B(3x+4) \qquad ....(ii)$$

Put $x = \frac{1}{2}$ in (ii) we get

$$1 = A(3 \times \frac{1}{2} + 4)+B\ 2\ (\frac{1}{2}) - 1$$

$$1 = B(\frac{3}{2} + 4) \qquad \Rightarrow \frac{11}{2}B = 1$$

$$A = \frac{2}{11}$$

$$1 = A\ (3(\frac{-4}{3}) + 4) + B(2(\frac{-4}{3}) - 1)$$

$$\Rightarrow\ B = \frac{-3}{11}$$

Hence (i) can be written as

$$\text{Thus} \int \frac{1}{6x^2+5x-4}\,dx \quad = \quad \int [\,\frac{-3}{11}\frac{1}{3x+4} + \frac{2}{11}\frac{1}{2x-1}]\,dx$$

$$= \frac{2}{2(11)}\int \frac{2}{2x-1}\,dx + \frac{-3}{3(11)}\int \frac{3}{3x+4}$$

$$= \frac{1}{11}\ln|2x-1| - \frac{1}{(11)}\ln|3x+4| + C$$

$$= \frac{1}{11}\ln\left|\frac{2x-1}{3x+4}\right| + c$$

Q8. $\int \frac{2x^3-3x^2-x-7}{2x^2-3x-2}\,dx$

Solution

$$\frac{2x^3 3x^2-x-7}{2x^2-3x-2}\,dx$$

$$= \int [x + \frac{x-7}{2x^2-3x-2}]$$

$$= \int x\,dx + \int \frac{x-7}{2x^2-3x-2}\,dx$$

$$= \int \frac{2x^2-3x^2-x-7}{2x^2-3x-2} = \frac{x^2}{2} + \int \frac{x-7}{2x^2-3x-2}\,dx$$

$$= \frac{x^2}{2} + \frac{x-7}{2x^2-4+x-2}\,dx$$

$$\int \frac{2x^2-3x^2-x-7}{2x^2-3x-2} = \frac{x^2}{2} + \int \frac{x-7}{(x-2)(2x+1)}\ (a)$$

Consider

$$\frac{x-7}{(x-2)(2x+1)} = \frac{A}{x-2} + \frac{B}{2x+1} \quad \ldots\ldots(i)$$

Multiplying both sides of (i) by (x-2)(2x +1) we have

$$x-7 = A(2x+1) + B(x-2) \qquad \ldots\ldots\ldots(ii)$$

$-1 = A$

Put $x = \frac{-1}{2}$ in above we get

$$\frac{-1}{2} - 7 = A\,(2(\frac{-1}{2}) + 1) + B((\frac{-1}{2}) - 2)$$

$$\frac{-15}{2} = B(\frac{-5}{2}) \qquad \Rightarrow B = 3$$

Hence (A) can be written as

Thus $\int \frac{2x^3-3x^2-x-7}{2x^2-3x-2}\,dx$

$$= \frac{x^2}{2} + \int \frac{-1}{x-2}\,dx + \int \frac{3}{2x+1}dx$$

$$= \frac{x^2}{2} - \int (x-2)^{-1} + \frac{3}{2}\int (2x+1)^{-1} - 1(2dx)\,dx$$

$$= \frac{x^2}{2} - \ln|x-2| + \frac{3}{2}\ln|2x+1| + C$$

Q9. $\int \frac{3x^2-12x+11}{(x-1)(x-2)(x-3)}\,dx$

Solution

$\int \frac{3x^2-12x+11}{(x-1)(x-2)(x-3)}\,dx$ ……………..(A)

Consider

$\frac{3x^2-12-x+11}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$ ……(i)

Multiplying both sides of (i) by (x-1)(x-2) (x-3)we have

$3x^2 - 12x + 11 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$ ……….(ii)

Put x = 1 in (ii)

$3-12+11 = A(-1)(-2)$

$2 = 2A$

$\Rightarrow A = 1$

$3(2)^2 - 12(2) + 11 = B(2-1)(2-3)$

$3.4-24+11 = B(1)(-1)$

$12-24+11 = -B$

$-1 = -B \Rightarrow B = 1$

Put x = 3 in above we get

$3(3)^2 - 12(3) + 11 = C(3-2)(3-1)$

$3.9-36+11 = C(1)(2)$

$27-36+11 = 2C$

$2 = 2C \Rightarrow C = 1$

Hence (A) can be written as

Thus $\int \frac{3x^2-12x+11}{(x-1)(x-2)(x-3)}\,dx$

$= \int [\frac{1}{x-1} + \frac{1}{x-2} + \frac{1}{x-3}]dx$

$= \int \frac{1}{x-1}\,dx + \frac{1}{x-2}dx + \frac{1}{x-3}dx$

$= \ln|x-1| + \ln|x-2| + \ln|x-3| + C$

Q10. $\int \frac{2x-1}{x(x-1)(x-3)}\,dx$ **.........(A)**

Solution

Consider

$\frac{2x-1}{x(x-1)(x-3)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-3}$ ......(i)

Multiplying both sides of (i) by x (x-1)(x-3)we have

$2x-1 = A(x-1)(x-3) + B(x)(x-3) + C(x)(x-1)$ ....(ii)

$-1 = A(-1)(-3)$

$\Rightarrow \quad A = \frac{-1}{3}$

Put x = 1 in above we get

$2(1)-1 = B(1)(-3)$

$2-1 = B(-2)$

$1 = -2B \qquad \Rightarrow B = \frac{-1}{2}$

Put x = 3 in above we get

$2(3)-1 = C(3)(3-1)$

$6-1 = C(3)(2)$

$5 = 6C$

$\Rightarrow C = \frac{5}{6}$

Hence (A) can be written as

Thus $\int \frac{2x-1}{x(x-1)(x-3)}\,dx$

$= \int [\frac{-1}{3x} - \frac{1}{2(x-1)} + \frac{5}{6(x-3)}]dx$

$= \frac{-1}{3}\int (x)^{-1}dx + \frac{-1}{2}\int (x-1)^{-1}dx + \frac{5}{6}\int (x-3)^{-1}.1dx$

$= \frac{-1}{3}\ln|x| \frac{1}{2}\ln|x-1| + \frac{5}{6}\ln|x-3| + C$

Q11. $\int \frac{5x^2+9x+6}{(x^2-1)(2x+3)}dx$

Solution

Consider

$$\frac{5x^2+9x+6}{(x1)(x+1)(2x+3)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{2x+3} \quad ......(i)$$

Multiplying both sides of (i) by (x+1) (x-1)(2x+3)we have

$$5x^2 + 9x + 6 = A(x+1)(2x+3) + B(x-1)(2x+3) + C(x-1)(x+1) \quad ....(ii)$$

Put x =1 in (ii)

$$5(1)^2 + 9(1) + 6 = A(1+1)\{2(1)+3\}$$

$$5+9+6 = A(2)(5)$$

$$20 = 10A$$

$$\Rightarrow A = 2$$

Put x = -1 in above we get

$$5(-1)^2 + 9(-1) + 6 = B(-1-1)\{2(-1)+3\}$$

$$2 = B(-2)$$

$$\Rightarrow B = -1$$

Put $x = \frac{-3}{2}$ in above we get

$$5\left(\frac{-3}{2}\right)^2 + 9\left(\frac{-3}{2}\right) + 6 = C\left(\frac{-5}{2}\right)\left(\frac{-1}{2}\right)$$

$$5.\frac{9}{4} - \frac{27}{2} + 6 = \frac{5}{4}C$$

$$\frac{45-54+24}{4} = \frac{5}{4}C$$

$$\frac{15}{4} = \frac{5}{4}C$$

$$\Rightarrow C = 3$$

Hence (A) can be written as

Thus $\int \frac{5x^2+9x+6}{(x^2-1)(2x+3)}\,dx$

$=2\int (x-1)^{-1}dx - \int (x+1)^{-1}.1dx + \frac{3}{2}\int (2x+3)^{-1}(2)dx$

$= 2\ln|x-1| - \ln|x+1| + \frac{3}{2}\ln|2x+3| + C$

Q12. $\int \frac{4+7x}{(1+x)^2(2+3x)}$ **dx**

Solution

$\int \frac{4+7x}{(1+x)^2(2+3x)}$ dx ....(a)

Consider

$\frac{4+7x}{(1+x)^2(2+3x)} = \frac{A}{1+x} + \frac{B}{(1+x)^2} + \frac{C}{2+3x}$ ......(i)

Multiplying both sides of (i) by $(1+x)^2(2+3x)$ we have

$4+7x = A(1+x)(2+3x) + B(2+3x) + C(1+x)^2$ ....(ii)

Put x =-1 in (ii)

$4+7(-1) = B(2+3)(-1)$

$4 - 7 = B(2-3)$

$-3 = -B$

$\Rightarrow B = 3$

Put $x = \frac{-2}{3}$ in above we get

$4+7(\frac{-2}{3}) = C(1-\frac{-2}{3})^2$

$\frac{12-14}{3} = C(\frac{3-2}{3})^2 = C(\frac{1}{9})$

$\frac{-2}{3} = \frac{1}{9}C$

$C = \frac{-2}{3} \times \frac{9}{1} = -6$

Equating Co-efficient terms of both sides of (ii). We have

$4 = 2A+2B+C$

$4 = 2A+2(3) -6$

$4 = 2A +6-6$

$A = 2$

Hence (a) can be written as

Thus $\int \frac{4+7x}{(1+x)^2(2+3x)}\,dx$

$$= \int \left[\frac{2}{1+x} + \frac{3}{(1+x)^2} + \frac{-6}{2+3x} \right] dx$$

$$= 2\ln|x+1| + 3\frac{(x+1)^{-2+1}}{2+1} - 2\ln|2+3x|$$

$$= 2ln|x+1| - \frac{3}{(x+1)} - 2ln|2+3x| + C$$

$$= \ln(1+x)^2 - \ln(2+3x)^2 - \frac{3}{x+1} + C$$

Q13. $\int \frac{2x^2}{(x-1)^2(x+1)}$ **dx**

Solution

$$\frac{2x^2}{(x-1)^2(x+1)} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2} \quad ......(i)$$

Multiplying both sides of (i) by $(x-1)^2(x+1)$ we have

$$2x^2 = A(x-1) + B(x+1) + C(x-1)^2 \quad ....(ii)$$

Put x = 1 in (ii)

$$2(1)^2 = B(1+1) \quad \Rightarrow \quad B = 1$$

Put x = -1 in above we get

$$2(-1)^2 = C(-1-1)^2$$

$2 = C(-2)^2$ OR $4C = (^1)$

$\Rightarrow$ C=$\frac{1}{2}$

Equating Co-efficient terms of x^2 of (ii), We have

2 = A+C

2 = A+$\frac{1}{2}$

A = $\frac{3}{2}$

Hence (a) can be written as

Thus $\int \frac{2x^2}{(x-1)^2(x+1)}$ dx

= $\int [\ \frac{\frac{3}{2}}{x-1} + \frac{1}{(x-2)^2} + \frac{\frac{1}{2}}{x+1}]$dx

=$\frac{3}{2}$ln | x-1 | +$\frac{(x-1)^{-2+1}}{-2+1}$+$\frac{1}{2}$ln | x+1 | +C

$= \frac{3}{2}\,ln|x+1| + \frac{1}{2}\,ln|x+1| - \frac{1}{x-1} + C$

Q14. $\int \frac{1}{(x+1)^2(x-1)}$ **dx**

Solution

Consider $\frac{1}{(x+1)^2(x-1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$ ……(i)

Multiplying both sides of (i) by $(x+1)^2$(x-1)we have

1 = A$(x+1)^2$+ B(x+1)(x-1)+C$(x-1)$ ……….(ii)

Put x = -1 in above we get

1= A$(1+1)^2$

4A= 1

$\Rightarrow$ A=$\frac{1}{4}$

1 = C (-1-1) $\Rightarrow$ -2C = 1

$\Rightarrow$ C=$\frac{-1}{2}$

Equating Co-efficients terms of x^2 of (ii), We have

0 = A+B $\Rightarrow$ B= -A

B = $\frac{-1}{4}$

Hence (a) can be written as

Thus $\int \frac{1}{(x+1)^2(x-1)}$ dx

$$= \int [\frac{1}{4(x-1)} - \frac{1}{4(x+1)} - \frac{1}{2(x+1)^2}]dx$$

$$=\frac{1}{4}\int (x-1)^{-1}\,dx - \frac{1}{4}\int (x+1)^{-1}\,dx - \frac{1}{2}\int (x+1)^{-2}\,dx$$

$$=\frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x+1| - \frac{1}{2}\ln|\frac{x+1}{-2+1}|+C$$

$$= \frac{1}{4}\ln|\frac{x-1}{x+1}| + \frac{1}{2(x+1)} + C$$

Q15. $\int \frac{x+4}{x^3-3x^2+4}\,dx$

Solution

$\int \frac{x+4}{(x+1)(x-2)^2}$

Let p(x) = $x^3 - 3x^2 + 4$

P(-1) = $(-1)^3 - 3(-1)^2 + 4$

= -4+4 = 0

x +1 = 0

$$\frac{x+4}{(x+1)(x-2)^2} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{(x-2)^2} \quad ......(i)$$

Multiplying both sides of (i) by $(x-2)^2(x+1)$ we have

$x+1 = 0 \quad \Rightarrow x = -1 \quad \Rightarrow A = \frac{1}{3}$

$C = 2 \qquad$ and $\quad B = \frac{-1}{3}$

Thus $\int \frac{x+4}{(x-2)^2(x+1)}\,dx$

$= \int [\frac{1}{3(x+1)} - \frac{1}{3(x-2)} + \frac{2}{(x-2)^2}]dx$

$= \frac{1}{3}[\ln|x+1| - \ln|x-2|] - \frac{2}{x-2}$

$= \frac{1}{3} ln\frac{x+1}{x-1} - \frac{2}{x-2} + C$

Q16. $\int \frac{x^2-6x^2+25}{(x+1)^2(x-2)^2}\,dx$

Solution

$\int \frac{x^2-6x^2+25}{(x+1)^2(x-2)^2}\,dx \qquad$ ……….(a)

$\frac{x^2-6x^2+25}{(x+1)^2(x-2)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-2} + \frac{D}{(x-2)^2}$ ……………(i)

Multiplying both sides of (i) by $(x-2)^2(x+1)^2$ we have

$x^2 - 6x^2 + 25 = A(x+1)(x-2)^2 + B(x-2)^2 + C(x-2)(x+1)^2$

$+D(x+1)^2$…….(ii)

Put x = -1 in (ii) we have

$(-1)^3 - 6(-1)2 + 25 = B(-1-2)^2 + C(x-2)(x+1)^2$

$+D(x+1)^2$…………..(iii)

Put x =-1 in (ii) we have

$(-1)^3 - 6(-1)2 + 25 = B(-1-2)^2$

$-1 -6 +25 = B(-3)2 \quad \Rightarrow \quad 9B = 18 \Rightarrow B = 2$

Put x = 2 in (ii) we have

$(2)^3 - 6(2)^2 + 25 = D(2+1)^2$

8-24 +25 = 9D ⇒ 9D = 9 ⇒ D= 1

Comparing coefficients of x^3 constant term both sides of(ii) we have

1 = A+C……………(a)

A = 1-C

25 = 4A +4B-2C+D

25 = 4(1-C) +8-2C+1

25 = 13 -6C ⇒ -6C = 25 - 13

-6C = 12 ⇒ C = -2

A = 1-C

= 1-(-2) = 1+2 = 3 A = 3

Hence (a) can be written as

$$\int \frac{x^2-6x^2+25}{(x+1)^2(x-2)^2}\, dx = \int [\frac{3}{x+1} + \frac{2}{(x+1)^2} + \frac{-2}{x-2} + \frac{1}{(x-2)^2}]dx$$

$$= 3\int (x+1)^{-1}(1)dx + 2\int (2+1)^{-1}\, dx - 2\int (x-2)^{-1}\, dx + \int (x-2)^{-1}\, dx$$

$$= 3\ln|x+1 - \frac{2x}{x+1} - 2ln|x-2| - \frac{1}{x-2} + C$$

Q17. $\int \frac{x^3+22x^2+14x-17}{(x-3)(x+2)^3}$ **dx…………(a)**

Solution

$$\frac{x^3+22x^2+14x-17}{(x-3)(x+2)^3} = \frac{A}{x-3} + \frac{B}{x+2} + \frac{C}{(x+2)^2} + \frac{D}{(x+2)^3} \text{ ……………(i)}$$

Multiplying both sides of (i) by $(x+2)^2(x-3)$we have

$x^3 + 22x^2 + 14x - 17 = A(x+2)^3 + B(x-3)(x+2)^2 + C(x-3)(x+2) + D(x-3)$….(ii)

Put x = 3 in (ii) we have

27 +22.9+47-17 = A(5)^3

250.4 = 125 A

A = 2

Put x = -2 in (ii) we have

$(-2)^3 + 22(-2)^2 + 14(-2) - 17$ = D(-2-3)

-8+88-28-17= -5D ⇒ 35= -5D ⇒ D = -7

Comparing coefficients of x^3 constant term both sides of(ii) we have

1 = A+B B = 1-A

= 1-2 = -1

B = -1

17 = 8A -1B-6C-3D

17 = 8(2)-12(-1)-6C-3(-7)

17 = 16+12-6C+21 ⇒ -6C = -49-17

-6C = -66 ⇒ C = 11

Hence (a) can be written as

$$\int \frac{x^3+22x^2+14x-17}{(x-3)(x+2)^3}\,dx = \int \left[\frac{2}{x-3} - \frac{1}{x+2} + \frac{11}{(x+2)^2} - \frac{7}{(x+2)^3}\right]dx$$

$$= 2\ln|x-3| - ln|x+2| + 11\frac{(x+2)^{\ 2+1}}{-2+1} - 7\frac{(x+2)^{\ 3+1}}{-3+1}$$

$$= 2\ln|x-3| - ln|x+2| - \frac{11}{x+2} + \frac{7}{2(x+2)^2} + C$$

Q18. $\int \frac{x-2}{(x+1)(x^2+1)}\,dx$

Solution

$$\frac{x-2}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{(x^2+1)} \quad \ldots\ldots\ldots\ldots\ldots(i)$$

$x-2 = A(x^2 + 1+(Bx +C)(x+1)$ ....(ii)

Put x = -1 in (ii) we have

$-1-2 = A\{(-1)^2+1\}$ $\Rightarrow$ $3 = A(2)$

$A = \frac{-3}{2}$

(ii) can be written as

$x - 2 = Ax^2+A+Bx^2Bx+Cx+C$

$x -2 = (A+B)\, x^2+(B+C)X+(A+C)$.........(iii)

Comparing co-efficients of x^2constant terms on both sides of (iii) we have

(i) $= A+B$ $\Rightarrow B = -A = B = -(\frac{-3}{2})$

$B = \frac{3}{2}$

$-2 = A+C$ $\Rightarrow C= -2-A$

$= -2 -(-\frac{3}{2}) = -2+\frac{3}{2}$

$= \frac{-4+3}{2}$ $C = \frac{-1}{2}$

Hence (a) can be written as

$$\int \frac{x-2}{(x+1)(x^2+1)}\,dx = \int \left[\frac{-3}{2(x+1)} + \frac{\frac{3}{2}x-\frac{1}{2}}{x^2+1}\right]dx$$

$$= \frac{-3}{2}\int (x+1)^{-1}(1)dx + \int \frac{3x-1}{2(x^2+1)}dx$$

$$= \frac{-3}{2}\ln|x+1| + \frac{3}{2}\int \frac{x}{(x^2+1)}dx - \frac{1}{2}\int \frac{1}{(x^2+1)}dx$$

$$= \frac{-3}{2}\ln|x+1| + \frac{3}{4}\int \frac{2x}{(x^2+1)}dx - \frac{1}{2}\tan^{-1}x + C$$

$$= \frac{-3}{2}\ln|x+1| + \frac{3}{4}\ln(x^2+1) - \frac{1}{2}\tan^{-1}x + C$$

Q19. $\int \frac{x}{(x-1)(x^2+1)}$ **dx**

Solution

Multiplying both sides of (i) by $(x^2+1)(x-1)$we have

x-= A(x^2+1)+(Bx +C)(x-1) ....(ii)

Put x =-1 in (ii) we have

$1 = A\{(1)^2+1\}$ $\Rightarrow$ 1 = A(2)

$A = \frac{1}{2}$

Comparing co-efficients of x^2 *and* x on both sides of (ii) we have

0 = A+B $\Rightarrow$B = -A = = $-(\frac{1}{2})$

$B = \frac{-1}{2}$

1= -B+C $\Rightarrow$ C= 1+B

$C = 1 - (\frac{1}{2}) = \frac{1}{2}$

$C = \frac{1}{2}$

Hence (a) can be written as

$$\int \frac{x}{(x-1)(x^2+1)}\,dx = \int \left[\frac{\frac{1}{2}}{(x-1)} + \frac{\frac{-1}{2}x+\frac{1}{2}}{x^2+1}\right]dx$$

$$= \frac{1}{2}\int (x-1)^{-1}(1)dx - \frac{1}{4}\int \frac{2x}{(x^2+1)}dx + \frac{1}{2}\int \frac{1}{(x^2+1)}dx$$

$$= \frac{1}{2}\ln|x-1| - \frac{1}{4}\ln(x^2+1) + \frac{1}{2}\tan^{-1}x + C$$

Q20. $\int \frac{9x-7}{(x+3)(x^2+1)}\,dx$

Solution

$$\frac{9x-7}{(x+3)(x^2+1)} = \frac{A}{x+3} + \frac{Bx+C}{(x^2+1)} \;\;\ldots\ldots\ldots\ldots\ldots(i)$$

Multiplying both sides of (i) by $(x^2+1)(x+3)$we have

9x-7-= A(x^2+1)+(Bx +C)(x+3) ....(ii)

Put x =-3 in (ii) we have

-27-7 = A(9+1) $\Rightarrow$ -34 = A(10)

$A = \frac{-32}{10}$ $\Rightarrow A = \frac{-17}{5}$

Comparing co-efficients of x^2 *and x* on both sides of (ii) we have

0 = A+B $\Rightarrow$B = -A = = -$(\frac{-17}{5})$

$B = \frac{17}{5}$

9= 3B+C $\Rightarrow$ C= 9-3B

C = -3 .$(\frac{17}{5})$ +9= $\frac{-6}{5}$

$C = \frac{-6}{5}$

Hence (a) can be written as

$$\int \frac{9x-1}{(x+3)(x^2+1)}\ dx = \int \left[\frac{-17}{5(x-1)} + \frac{17x-6}{5(x^2+1)}\right]dx$$

$$= \frac{-17}{5}\int \frac{1}{x+3}dx + \frac{1}{5}\int \frac{17x-6}{(x^2+1)}dx$$

$$= \frac{-17}{5}\ln|x+3| + \frac{17}{10}\ln(x^2+1) - \frac{6}{5}\tan^{-1}x + C$$

Q21. $\int \frac{1+4x}{(x-3)(x^2+4)}\ dx$

Solution

$$\frac{1+4x}{(x-3)(x^2+4)} = \frac{A}{x-3} + \frac{Bx+C}{(x^2+4)} \ ....(i)$$

Multiplying both sides of (i) by $(x^2+4)(x-3)$we have

1+4x-= A(x^2+4)+(Bx +C)(x-3) ....(ii)

Put x = 3 in (ii) we have

1+4(3) = A$\{(3)^2+1\}$ +0

13= A(9+4) $\Rightarrow$ 13 = A(13) $\Rightarrow A = 1$

Comparing co-efficients of x^2 *and x* on both sides of (ii) we have

Hence (a) can be written as

$\int \frac{1+4x}{(x-3)(x^2+4)}$ dx $= \int [\frac{1}{(x-3)} + \frac{(-1)x+1}{x^2+4}]$dx

$= \int (x-3)^{-1}(1)dx - \frac{1}{2}\int (x^2+4)^{-1}(2x)\,dx + \int \frac{1}{(x^2+4)}dx$

$= \ln|x-3| - \frac{1}{2}\ln(x^2+4) + \frac{1}{2}\tan^{-1}x + C$

Q22. $\int \frac{12}{x^3+8}$ **dx**

Solution

$\int \frac{12}{x^3+8}$ dx

$=\int \frac{12}{(x)^3+(2)^3}$ dx $= \int \frac{12}{(x+2)(x^2-2x+4)}$ dx

Now $\int \frac{12}{(x+2)(x^2-2x+4)}$ dx

Consider $\frac{12}{(x+2)(x^2-2x+4)} = \frac{A}{x+2} + \frac{Bx+C}{(x^2-2x+4)}$ ……………(i)

Multiplying both sides of (i) by $(x^2-2x+4)(x+2)$we have

12-= A($x^2 - 2x + 4$)+(Bx +C)(x+) …….(ii)

Put x = -2 in (ii) we have

12 = A{$(-2)^2$-2(-2)+4} = A(4+4+4)

12= A(12) $\Rightarrow$ $A = 1$

Comparing co-efficients of x^2 *and* x on both sides of (ii) we have

0 = A+B $\Rightarrow$B = -1

12= 4A+2C $\Rightarrow$ 2C= 12-4A

=12-4(1)

C = 4

Hence (a) can be written as

$=\ln|x+2| -\frac{1}{2}\ln\int \frac{2x-2}{x^2+-2x+4)}dx \quad +3\int \frac{1}{(x^2+-2x+4)}dx$

$= \ln|x+2| \; -\frac{1}{2}\ln(x^2+-2x+4)+3[\frac{1}{\sqrt{3}}\tan^{-1}(\frac{x-1}{\sqrt{3}})+C$

$= \ln|x+2| \; -\frac{1}{2}\ln(x^2+-2x+4)+\sqrt{3}\tan^{-1}(\frac{x-1}{\sqrt{3}})+C$

Q23. $\int \frac{9x+6}{(x^2-8)}\,dx$

$\frac{9x+6}{(x^2-8)}= \quad =\int \frac{9x+6}{(x)^3-(2)^3}\;dx = \int \frac{9x+6}{(x-2)(x^2+2x+4)}\;dx$

Now $\int \frac{9x+6}{(x-2)(x^2+2x+4)}\;dx$

Consider $\frac{9x+6}{(x-2)(x^2+2x+4)} = \frac{A}{x-2}+\frac{Bx+C}{(x^2+2x+4)}$ ……………(i)

Multiplying both sides of (i) by $(x^2+2x+4)(x-2)$we have

$9x+6= A(x^2+2x+4)+(Bx+C)(x-2)$ …….(ii)

Put x = 2 in (ii) we have

24 = 12A +0

$\Rightarrow \quad A=2$

Comparing co-efficients of x^2 *and* x on both sides of (ii) we have

0 = A+B ⇒B = -2

9 = 2A-2B

9 =2(2)-2(-2)-C

C = 2

Hence (a) can be written as

$\int \frac{9x+6}{(x-2)(x^2+2x+4)}\,dx = 2\int \frac{dx}{(x-2)} - \int \frac{2x-1}{x^2+2x+4}]dx$

$=2\ln|x-2|-\int \frac{2x+2-3}{x^2+2x+4)}dx \quad +3\int \frac{1}{(x^2+2x+4)}dx$

$=2\ln|x-2|-\int \frac{2x+2}{x^2+2x+4)}dx \quad +3\int \frac{1}{(x^2+2x+4)}dx$

$= \ln|x-2| - \ln(x^2+2x+4) + \sqrt{3}\tan^{-1}(\frac{x-1}{\sqrt{3}}) + C$

Q24. $\int \frac{2x^2+5x+3}{(x-1)^2(x^2+4)} dx$

Solution

Consider $\frac{2x^2+5x+3}{(x-1)^2(x^2+4)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{(x^2+4)}$ ....(i)

Multiplying both sides of (i) by $(x^2+4)(x-1)$ we have

$2x^2 + 5x + 3 = A(x-1)+B(x^2+4) +(Cx+D)(x-1)^2$ ....(ii)

Put x = 1 in (ii) we have

$2(1)^2 + 5(1) + 3 = B((1)^2+4)$

$\Rightarrow B = 2$

Comparing co-efficients of x^3, x^2 *and* x on both sides of (ii) we have

$0 = A + C$ ...(i)

$2 = -A + B - 2C + D$ .....(ii)

$5 = 4A + C - 2D$ ....(iii)

$3 = -4A + 4B + D$ ....(iv)

$2 = -A + 2 - 2C + D \Rightarrow A + 2C - D = 0$

$\Rightarrow (A+C) + C - D = 0 \quad \Rightarrow 0 + C - D = 0$

$C = D$

Putting values of B =2,D=C, A=-C IN (iv)

$3 = 4C+8+C = 5C = -5 \quad \Rightarrow C = -1$

$3 = 5C+8 \quad \Rightarrow 5C = -8+3$

$5C = -5 \quad \Rightarrow C = -1 = D$

$A = -C \quad \Rightarrow A = -(-1) = 1$

C =D $\Rightarrow$ D = -1

Hence (a) can be written as

$$\int \frac{2x^2+5x+3}{(x-1)^2(x^2+4)}\,dx = \int \left[\frac{1}{(x-1)} + \frac{2}{(x-1)^2} + \frac{-x-1}{x^2+4}\right]dx$$

$$= \int (x-1)^{-1}\,dx + 2\int (x-1)^{-2}\,dx - \frac{1}{2}\int (x^2+4)^{-1}(2x)\,dx - \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$$

$$= \ln|x-1| + 2\frac{(x-1)^{-2+1}}{-2+1} - \frac{1}{2}\ln(x^2+4) - \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$$

$$= \ln|x-1| - \frac{2}{(x-1)} - \frac{1}{2}\ln(x^2+4) - \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$$

Q25. $\int \frac{2x^2-x-7}{(x+2)^2(x^2+x+1)}\,dx$

Solution

$$\frac{2x^2-x-7}{(x+2)^2(x^2+x+1)} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{Cx+D}{(x^2+x+1)} \quad \ldots\ldots\ldots\ldots\ldots(i)$$

Multiplying both sides of (i) by $(x^2+x+1)(x+2)^2$ we have

$$2x^2 - x - 7 = A(x+2)(x^2+x+1) + B(x^2+x+1) + (Cx+D)(x+2)^2 \quad \ldots\ldots(ii)$$

Put x = -2 in (ii) we have

$$2(-2)^2 - (-2) - 7 = B((-2)^2 - 2 + 1)$$

$$\Rightarrow \quad B = 1$$

Comparing co-efficients of x^3, x^2 *and* x on both sides of (ii) we have

0 = A+ C $\Rightarrow$ C =2 So, A =-2

2 = 3A +B+4C+D .....(iii) put values

-1 = 3A+B+4C+4D ....(iv) we get

-7 = 2A+B+4D ....(v) D = -1

Hence (a) can be written as

$$\int \frac{2x^2-x-7}{(x+2)^2(x^2+x+1)}\,dx = \int \left[\frac{-2}{(x+2)} + \frac{1}{(x+2)^2} + \frac{2X-1}{x^2+x+1}\right]dx$$

$$=-2\int (x+2)^{-1}\,dx + \int (x+2)^{-2}\,dx + \int \frac{2x+1}{x^2+x+1}\,dx - \int \frac{2}{(x+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}$$

$$= 2\ln|x+2| - \frac{1}{(x+2)} + \ln(x^2+x+1) - 2\times\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + C$$

$$= 2\ln|x+2| - \frac{1}{(x+2)} + \ln(x^2+x+1) - \frac{4}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + C$$

Q26. $\int \frac{3x+1}{(4x^2+1)(x^2-x+1)}\,dx$

Solution

Consider $\frac{3x+1}{(4x^2+1)(x^2-x+1)} = \frac{Ax+B}{(4x^2+1)} + \frac{Cx+D}{(x^2-x+1)}$ ……………(i)

Multiplying both sides of (i) by $(x^2-x+1)(4x+1)$ we have

$3x+1 = (Ax+B)(x^2-x+1) + (Cx+D)(4x^2+1)$

$= A(x^3-x^2+x) + B(x^2-x+1) + C(4x^2+x) + D(4x^2+1)$

$= (A+4C)x^3 + (4D+B-A)x^2 + (A-B+C)x + (B+D)$

Comparing co-efficients of x^3, x^2 *and x and constant* on both sides of (ii) we have

$0 = A+4C$ …(i)

$0 = -A+B+4D$ …..(ii)

$3 = A-B+C$ ….(iii)

$1 = B+D$ ….(iv)

Adding (ii) and (iii) we get

$4D+C = 3 \quad \Rightarrow C = 3-4D$

From (i)

$A = -4C = -4(3)-4D = -12+16D$, From (iv)

$B = 1-D$

Putting values of C ,B and A in (iii)

- 12+16D-(1-D)+3-3D =3

-12+16D-1+D-3-4D = 3

=13D = 13 ⇒D = 1

A = -12 +16D = -12 +16(1) = 4

A = 4

C = 3-4D = 3-4(1) = -1, B = 1-D = 1-1 = 0= B =0

C = -1

Hence (a) can be written as

$$\int \frac{3x+1}{(4x^2+1)(x^2-x+1)}\,dx = \frac{1}{2}\int \frac{8x}{(4x^2+1)} - \frac{1}{2}\int \frac{2x-1-1}{(x^2-x+1)}dx$$

$$= \frac{1}{2}\int (4x^2+1)^{-1}(8x)\,dx - \frac{1}{2}\int \frac{2x-1-}{(x^2-x+1)}dx + \frac{1}{2}\int \frac{1}{(x^2-x+1)}dx$$

$$= \ln(\sqrt{4x^2+1}) - ln\sqrt{x^2-x+1} + \frac{1}{\sqrt{3}}\tan^{-1}(\frac{2x-1}{\sqrt{3}}) + C$$

$$= \ln(\sqrt{\frac{4x^2+1}{x^2-x+1}}) + \frac{1}{\sqrt{3}}\tan^{-1}(\frac{2x-1}{\sqrt{3}}) + C$$

Q27. $\int \frac{4x+1}{(x^2+4)(x^2+4x+5)}\,dx$

Solution

Consider $\frac{4x+1}{(x^2+4)(x^2+4x+5)} = \frac{Ax+B}{(x^2+4)} + \frac{Cx+D}{(x^2+4x+5)}$ ....(i)

Multiplying both sides of (i) by $(x^2+4x+5)(x^2+4)$we have

4x+1 = (Ax +B)(x^2+4x+4) +(Cx +D) (x^2+4)

From above C = 4B -4

Using values of A and D in (b) we have

-5B = 0

$\Rightarrow$ B = 1

NOW A = 4 - 4(B) = 4 - 4(1) = 0

C = 4B - 4 = 0

$D = \frac{1-5(1)}{4} = \frac{1-5}{4} = \frac{-4}{4} = -1 \Rightarrow D = -1$

Putting C = 0

So A = 0

4B = 64C = 4

$\pm$4B $\pm$64C = 4

65C = 0

C = 0

Hence (a) can be written as

$$\int \frac{4x+1}{(x^2+4)(x^2+4x+5)}\,dx = \int \frac{dx}{(2)^2+x^2} - \int \frac{dx}{(x+2)^2+1}$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) - \int \frac{dx}{(x+2)^2+1} + C$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) - \tan^{-1}(x+2) + C$$

Q28. $\int \frac{6a^2}{(x^2+a^2)(x^2+4a^2)}\,dx$

Solution

Consider $\frac{6a^2}{(x^2+a^2)(x^2+4a^2)} = \frac{Ax+B}{(x^2+a^2)} + \frac{Cx+D}{(x^2+4a^2)}$ ....(i)

Multiplying both sides of (i) by $(x^2+4a^2)(x^2+a^2)$ we have

$6a^2 = (Ax+B)(x^2+4a^2) + (Cx+D)(x^2+a^2)$

$6a^2 = A(x^3+4a^2x) + B(x^2+4a^2) + C(x^3+xa^2) + D(x^2+a^2)$

Equating Co-efficients of x^3, x^2, x and constant terms ,we have

$$A+C = 0$$

$$B+D = 0$$

$$4x^2A+a^2C = 0$$

$$4A+C = 0$$

$$4a^2B+a^2D = 6a^2 \quad \Rightarrow (d)$$

Multiplying (b) by $4a^2$and subtracting from (d) , we have

$$-3a^2D = 6a^2$$

$$D = -2$$

$$B+D = 0 \qquad \Rightarrow B = -D = -(-2) = 2$$

From (a) putting C = -A in (C) , we have

$$4x^2A+a^2(-A) = 0 \qquad \Rightarrow A = 0$$

$$\text{And } C = -A = 0 \quad \Rightarrow C = 0$$

Hence (1) can be written as

$$\int \frac{6a^2}{(x^2+a^2)(x^2+4a^2)} dx = \int [\frac{2}{(x^2+a^2)} - \frac{2}{(x^2+4a^2)}]dx$$

$$= \frac{2}{a}\tan^{-1}(\frac{x}{2}) - \frac{2}{2a}\tan^{-1}(\frac{x}{2a}) + C$$

$$= \frac{2}{a}\tan^{-1}(\frac{x}{2}) - \frac{1}{a}\tan^{-1}(\frac{x}{2a}) + C$$

Q29. $\int \frac{2x^2-2}{(x^4+x^2+1)} dx$

Solution

$$x^4 + x^2 + 1 = (x^2)^2 + 2(x)^2(1) + (1)^2 - x^2$$

$$= (x^2 + 1)^2 - x^2$$

$$= (x^2 + x + 1)(x^2 - x + 1)$$

$$\frac{2x^2-2}{(x^4+x^2+1)} = \frac{2x^2-2}{(x^2+x+1)(x^2-x+1)}$$

Consider $\frac{2x^2-2}{(x^2+x+1)(x^2-x+1)} = \frac{Ax+B}{(x^2+x+1)} + \frac{Cx+D}{(x^2-x+1)}$ ....(i)

$$2x^2 - 2 = (Ax + B)(x^2 - x + 1) + (Cx + D)\,(x^2 - x + 1)$$

$$= x^3 - Ax^2 + Ax + Bx^2 - BX + B + Cx^2 + Cx + Dx^2 + Dx +$$

D

$$2x^2 - 2 = (A+C)\,x^3 + (-A+B+C+D)\,x^2 + (A-B+C+D)x + (B + D)$$

Equating Co-efficients of x^3, x^2, x and constant terms ,we have

A+C = 0	-A+B+C+D = 2 (iii)	A-B+C+D = 0 (iv)
A = -C (ii)	C-2-D+C+D = 2	B+D = 2
-A+B+C+D-2	2C = 4	B =-2-D
A-B+C+D = 0	C = 2	2+ D = 1
2(C+2D) =2		D =-1
2(C+D) = 2		PUT C = in(ii)
C+ D = 1		A = -2

D = -1 in(v)

B = -1

Hence (1) can be written as

$$\int \frac{2x^2-2}{(x^2+x+1)(x^2-x+1)}\,dx = -\int \frac{2x+1}{(x^2+x+1)}dx + \int \frac{2x+1}{(x^2-x+1)}]dx$$

$$= \ln|x^2 + x + 1| + \ln|x^2 - x + 1| + C$$

$$= \ln\left|\frac{x^2-x+1}{x^2+x+1}\right| + C$$

Q30. $\int \frac{3x-8}{(x^2+x+2)(x^2-x+2)}dx$

Solution

Consider $\frac{3x-8}{(x^2+x+2)(x^2-x+2)} = \frac{Ax+B}{(x^2+x+2)} + \frac{Cx+D}{(x^2-x+2)}$ ....(i)

Multiplying both sides of(i) by$(x^2-x+2)(x^2+x+2)$ *we hav*

$3x\text{-}8 = (Ax+B)(x^2+x+2) + (Cx+D)(x^2-x+2)$

$= A(x^3+x^2+2x) + B(x^2+X+2) + C(x^3-x^2+2x) +$
$D(x^2-x+2)$

$2x^2-2 = (A+C)x^3 + (A+B-C+D)x^2 + (2A+B+2C-D)x + (2B+2D)$

Equating Co-efficients of x^3, x^2, x and constant terms ,we have

$A + C = 0$ (a)

$A = -C$

$A+B-C+D = 0$ (b)

$2A+B+2C-D = 3$ (c)

$2B + 2D = -8$

$B + D = -4$ (d)

Putting B+D = -4 in(b) we get

$A-C-4 = 0$ $\Rightarrow A - C = 4$

$A = 2$ $\Rightarrow C = -2$

NOW

$2(2)+B+2(-2)-D = 3$ $\Rightarrow B - D = 3$ (e)

Adding (d) and (e) we get

$2B = -1$

$B = -\frac{1}{2}$

$$-\frac{1}{2} + D = -4 \qquad \Rightarrow D = -4+\frac{1}{2} = -\frac{7}{2} \quad \Rightarrow D = -\frac{7}{2}$$

Hence (i) can be written as

$$\int \frac{3x-8}{(x^2+x+2)(x^2-x+2)}dx = \int \frac{2x-\frac{1}{2}}{(x^2+x+2)} + \frac{-2x-\frac{7}{2}}{(x^2-x+2)}dx$$

$$= \ln|x^2 - x + 2| + \frac{1}{2}\int \frac{dx}{x^2-2(x)(\frac{1}{2})+(\frac{1}{2})^2-\frac{1}{4}+2}$$

$$=\ln|\frac{x^2-x+2}{x^2+x+2}| + \frac{1}{2}\int \frac{dx}{(\frac{\sqrt{7}}{2})^2+(x-\frac{1}{2})^2} + \frac{5}{2}\int \frac{dx}{(\frac{\sqrt{7}}{2})^2+(x+\frac{1}{2})^2}$$

$$= \ln|\frac{x^2-x+2}{x^2+x+2}| + \frac{1}{\sqrt{7}}[\tan^{-1}\frac{2x-1}{\sqrt{7}} + 5\tan^{-1}\frac{2x+1}{\sqrt{7}} + C]$$

Q31. $\int \frac{3x^3+4x^2+9x+5}{(x^2+x+1)(x^2+2x+3)}\, dx$

Solution

Consider $\frac{3x^3+4x^2+9x+5}{(x^2+x+1)(x^2+2x+3)} = \frac{Ax+B}{(x^2+x+1)} + \frac{Cx+D}{(x^2+2x+3)}$ ....(i)

Multiplying both sides of(i) by$(x^2 + x + 1)(x^2 + 2x + 3)$ we h

$3x^3 + 4x^2 + 9x + 5 = (Ax + B)(x^2 + 2x + 3) + (Cx + D)(x^2 + x + 1)$

$= A(x^3 + 2x^2 + 3x) + B(x^2 + 2x + 3) + C(x^3 + x^2 + x) +$

$D(x^2 + x + 1$ $\quad 3x^3 + 4x^2 + 9x + 5 = (A+C)x^3 + (2A+B-$

$C+D)x^2 + (3A+2B+C+D)x + (3B + D)$

Equating Co-efficients of x^3, x^2, x and constant terms ,we have

A+C = 3 (a)

2A+B+C+D = 4 (b)

3A+2B+C+D = 9 (c)

3B +D = 5 (d)

Putting C = 3-A in(b) and (C) we get

2A+B+3-A+D= 4 $\Rightarrow$ A+B+D =1 (e)

$3A+2B+3-A+D=9 \qquad \Rightarrow \quad 2A+2B+D=6$

(b)

Multiplying (e)by 2 and subtract from(f)

$2A+2B+D=6$

$2A+2B+2D=2$

$-D=4 \qquad \Rightarrow D=-4$

Since $3B+D=5 \qquad \Rightarrow 3B-4=5$

$3B=9 \qquad \Rightarrow B=3$

Putting values of B and D in(e) we have

$A+3-4=1 \qquad \Rightarrow A-1=1 \Rightarrow A=2$

$C=3-A=3-2=1$

$C=1$

Hence (i) can be written as

$$\int \frac{3x^3+4x^2+9x+5}{(x^2+x+1)(x^2+2x+3)} \quad dx = \int \frac{2x+3}{(x^2+x+1)} + \frac{x-4}{(x^2+2x+3)} dx$$

$$= \ln|x^2+x+1| + ln(x^2+2x+3)^{\frac{1}{2}2} + 2\int \frac{dx}{(\frac{\sqrt{4}}{2})^2+(x+\frac{1}{2})^2} \; 5\int \frac{dx}{(\sqrt{2})^2+(x+1)^2}$$

$$= \ln|x^2+x+1\sqrt{x^2+2x+3}| + \frac{4}{\sqrt{3}}[\tan^{-1}\frac{2x+1}{\sqrt{3}} - \frac{5}{\sqrt{2}}\tan^{-1}\frac{x+1}{\sqrt{2}} + C]$$