UNIT 3

Integration

Exercise 3.1

Q1 Find δy and dy in the following cases

i. $y = x^2 - 1$ when x changes from 3 to 3.02

Solution

We have

$$x = 3$$

and change $= \delta = dx = 3.02 - 3 = 0.2$

when $x = 3$

$$\Rightarrow y = x^2 - 1$$

$$y = (3)^2 - 1 = 8$$

To find the δy(or change δx in x)

We have

$$y + \delta y = (x + \delta x)^2 - 1$$

put values from above

$$8 + \delta y = (3 + 0.2)^2 - 1$$

$$8 + \delta y = 9.1204 - 1$$

$$8 + \delta y = 8.1204 - 8$$

$$\delta y = 8.1204 - 8$$

$$\delta y = 0.1204$$

but $y = x^2 - 1$

$dy = 2xdx$ $\frac{d}{dx}(x^2 - 1) = 2x - 0$

put $x = 3$ $dx = 0.2 = 2x$

$= 2(3)(.02)$

$= 0.12$

ii $y = x^2+2$ when x changes from 2 to 1.8

Solution

We have

$x = 2$

change $\delta x = dx = 1.8-2 = -0.2$

To find δy we have the following procedure

$y+ \delta y = (x + \delta x)^2 +2(x+ \delta x)$

$\Rightarrow$ $y = x^2 +2x$ as $x = 2$

$y = (3)^2+2(2)$

$= 4+4 = 8$

Put in the above equation

$\delta y = (2 + 0.2)^2+2\{2+(-0.2)\}$

$\delta y = (2 - 0.2)^2+(2 -0.2)$

$\delta y = (1.8)^2+2(1.8)$

$\delta y = 3.24 +3.6$

$\delta y = 6.84$

As $y = x^2 +2x = 6.84-8 = -1.16$

$dy = (2x +2)dx$ $y = x^2 +2x$

$dy = (2 \times 2 + 2) \times (-0.2)$ $\quad \frac{dy}{dx} = 2x + 2$

$= (6)(-0.2)$ $\quad \Rightarrow dy = (2x+2)\,dx$

$= -1.2$

iii **$y = \sqrt{x}$ when x changes from 4 to 4.41**

Solution

We have

$x = 4$

change in x $\delta x = dx = 4.41 - 4 = -0.41$

To find δy we have

$y + \delta y = \sqrt{x + \delta x}$

As $y = \sqrt{x}$ but $x = 4$

$y = \sqrt{4} = 2$

Put values of y ,x and δx in above equation

$2 + \delta y = \sqrt{4 + 0.41}$

$2 + \delta y = \sqrt{4.41}$

$2 + \delta y = 2.1$

$\delta y = 2.1 - 2$ $\quad \therefore y = \sqrt{x} = x^{\frac{1}{2}}$

$\delta y = 0.1$ $\quad \frac{dy}{dx} = \frac{1}{2}(x)^{\frac{1}{2}-1}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$ $\quad = \frac{1}{2}(x)^{\frac{1}{2}}$

$= \frac{1}{2\sqrt{4}}(0.41)$ $\quad \frac{dy}{dx} = \frac{1}{2}\sqrt{x}$

$= \frac{1}{2\sqrt{4}}(0.41)$

$= 0.1025$

Q2 Using differential to find $\frac{dy}{dx}$ and $\frac{dx}{dy}$ in the following equations.

i. xy + x = 4

Solution

Talking differential

$d(xy + x) = d(4)$ $\quad d(f+g) = df + dg$

$d(xy) + d(x) = 0$

$x\,dy + ydx + dx = 0$ $\quad d(f.g) = fdg + gfd$

$xdy + ydy + dx = 0$

$xdy = -ydx - dx$

$xdy = -(1 + y)dx$

$\frac{dy}{dx} = -\frac{1+y}{x}$ but $x \neq 0$

$\Rightarrow \frac{dy}{dx} = -\frac{y+1}{x}$

Talking its reciprocal

$\frac{dx}{dy} = -\frac{x}{y+1}$ but $y \neq -1$

ii $x^2 + 2y^2 = 16$

Solution

Talking differential

$d(x^2 + 2y^2) = d(16)$ $\quad d(f+g) = df + dg$

$d(x^2) + d(2y^2) = 0$

$$2x\,dx + 2d(y^2) = 0$$

$$2xdy + 4ydy = 0$$

$$\Rightarrow \quad xdy + 2ydy = 0 \ldots..(1)$$

$$2ydy = -xdx$$

$$dy = -\frac{x}{2y}dx$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{x}{2y}$$

From (1)

$$xdx = -2ydy$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{2y}{x}$$

iii $\mathbf{x^4 + y^2 = xy^2}$

Solution

Talking differential

$$d(x^4 + y^2) = d(\boldsymbol{xy^2}) \qquad d(f+g) = df + dg$$

$$d(x^4) + d(y^2) = xd(\mathbf{y^2}) + \mathbf{y^2}(\boldsymbol{dx}) \qquad d(f.g) = fdg + gdf$$

$$4x\,d^3x + 2ydy = 2xydy + \mathbf{y^2}dx \ldots..(1)$$

$$2ydy - 2xydy = y^2dx - 4x^3\boldsymbol{dx}$$

$$2y(1-x)\,dy = (\mathbf{y^2} - 4x^3)dx$$

$$\Rightarrow \quad dy = \left(\frac{\mathbf{y^2} - 4x^3}{2y(1-x)}\right)dx$$

$$\frac{dy}{dx} = \frac{\mathbf{y^2} - 4x^3}{2y(1-x)}$$

Again from (1)

$$4x^3dx - y^2\,dx = 2xydy - 2ydy$$

$$(4x^3 - y^2)\,dx = 2y(x-1)dy$$

$$\Rightarrow \quad dx = \left(\frac{2y(x-1)}{y^2-4x^3}\right)dy$$

$$\frac{dx}{dy} = \frac{2y(x-1)}{4x^3-y^2}$$

iv **xy – ln x = c**

Solution

Talking differential

$$d(xy - \ln x) = d(c)$$

$$d(f+g) = df + dg$$

$$d(xy) + d(\ln x) = 0$$

$$d(f.g) = fdg + gdf$$

$$xdy + ydx - \frac{1}{x}dx = 0 \quad (1)$$

$$xdy = \frac{1}{x}dx - ydx$$

$$xdy = \left(\frac{1}{x} - y\right)dx$$

$$xdy = \left(\frac{1-xy}{x}\right)dx$$

$$\frac{dx}{dy} = \frac{1-xy}{x^2}$$

Again from (1)

$$ydx - \frac{1}{x}dx = -xdy$$

$$\left(y - \frac{1}{x}\right)dx = -xdy$$

$$\left(\frac{xy-1}{x}\right)dx = -xdy$$

$$\frac{dx}{dy} = \frac{-x.x}{xy-1}$$

$$\Rightarrow \qquad \frac{dx}{dy} = -\frac{x^2}{xy-1} = \frac{x^2}{1-xy}$$

Q3 Use differential to approximate the values of.

i. $\sqrt[4]{17}$

Solution

Let $y = x^{\frac{1}{4}}$.....(1)

Take x = 16

And $\delta x = 1 = dx$

From (1)

$$\frac{dy}{dx} = \frac{1}{4}x^{1-\frac{1}{4}} = \frac{1}{4}x^{-\frac{3}{4}}$$

$$dy = (\frac{1}{4}x^{-\frac{3}{4}})dx$$

$$dy = (\frac{1}{4} \times \frac{1}{x^{\frac{3}{4}}})dx$$

put x and dx

$$= (\frac{1}{4} \times \frac{1}{(16)^{\frac{3}{4}}})1$$

$$= (\frac{1}{4} \times \frac{1}{(24)^{\frac{3}{4}}})1$$

$$= (\frac{1}{4} \times \frac{1}{23})$$

$$= \frac{1}{8\times4} = \frac{1}{32} = 0.03125$$

Now $\sqrt[4]{17} = (16 + 1)^{\frac{1}{4}} \ldots (16)^{\frac{1}{4}} + dy$

$$(2^4)^{\frac{1}{4}} + 0.03125$$

$$= 2 + 0.01325$$

= 2.03125

ii. $(31)^{\frac{1}{5}}$

Solution

Let $y = x^{\frac{1}{5}}$.....(1)

Take x = 32

And $\delta x = -1 = dx$

From (1)

$$\frac{dy}{dx} = \frac{1}{5}x^{\frac{1}{5}-1} = \frac{1}{5}x^{-\frac{4}{5}}$$

$$dy = \frac{1}{5}x^{-\frac{4}{5}}$$

$$dy = \left(\frac{1}{x^{\frac{4}{5}}}\right)dx$$

put x and dx

$$= \left(\frac{1}{5(2^5)^{\frac{4}{5}}}\right)-1$$

$$= \left(\frac{-1}{5\times 16}\right) - 1$$

$$= \frac{-1}{80} = -0.0125$$

Now $31^{\frac{1}{5}} = (32-1)^{\frac{1}{5}} = (32)^{\frac{1}{5}} + dy$

$$= (32)^{\frac{1}{5}} - 0.0125$$

= 2 -0.0125

= 1.9875

iii. **Cos 29°**

Solution

Let y = Cos x .....(1)

Take $x = 30° = \frac{\pi}{6}$

$dx = 29° - 30°$

$= 1°$

$= 1 \times \frac{\pi}{180}$ rad

$= 0.0175$ rad

From (1)

$\frac{dy}{dx} = -\sin x$

$dy = (-sin\ x)dx$

put value x and dx

$dy = (-sin\ \frac{\pi}{6})(-\frac{\pi}{180})$

$= (sin\ \frac{\pi}{6})(\frac{\pi}{180})$

$= \frac{1}{2}(\frac{\pi}{180})$

$Cos\ 29° = \text{Cos}\ (29° - 1°)$

$= \text{Cos}\ 30° + dy$

$= \frac{\sqrt{3}}{2} + \frac{1}{2}(\frac{\pi}{180})$

$= 0.866 + 0.0087$

$Cos\ 29° = 0.8747$

iv Cos 29°

Solution

Let $y = \text{Cos } x$ .....(1)

Take $x = 60° = \frac{\pi}{3}$

$dx = 1° = \frac{\pi}{180}$

$= 1°$

$= 1 \times \frac{\pi}{180}$ rad

$= 0.0175$ rad

From (1)

$\frac{dy}{dx} = \text{Cos } x$

$dy = Cos\ x\ dx$

put value x and dx

$dy = Cos\ 60° \times \frac{\pi}{180}$

$= \frac{1}{2} \times \frac{\pi}{180}$

$Cos\ 29° = \text{Cos } (29° - 1°)$

$= \text{Cos } 30° + dy$

$= \frac{\sqrt{3}}{2} + \frac{1}{2}(\frac{\pi}{180})$

$= 0.866 + 0.0087$

$Cos\ 29° = 0.8747$

iv **Sin 61°**

Solution

Let $y = \sin x$ .....(1)

Take $x = 60° = \frac{\pi}{3}$

$dx = 1° = \frac{\pi}{180}$

From (1)

$\frac{dy}{dx} = \text{Cos } x$

$dy = Cos\ x\ dx$

put value x and dx

$dy = Cos\ 60° \times \frac{\pi}{180}$

$= \frac{1}{2} \times \frac{\pi}{180}$

$= \frac{\pi}{2 \times 180}$

$Sin\ 61° = \text{Cos } (60° + 1°)$

$= \text{Sin } 60° + dy$

$= \frac{\sqrt{3}}{2} + \frac{1}{2}(\frac{\pi}{180})$

$= 0.866 + 0.0087$

$sin\ 61° = 0.8747$

Q4. Find the approximation increase in the volume of a cube if the length of its each edge change from 5 to 5.02.

Solution

Let $v = x^3$ ....(1)

Take $x = 5$

And $dx = 5.02 - 5 = 0.02$

From (1)

$$\frac{dv}{dx} = 3x^2$$

$$dv = 3x^2\, dx$$

put value of x and dx

$$= 3\,(5)^2 \times (0.2)$$

$$= 75 \times 0.2$$

$$= 1.50$$

$$dv = 1.5 \text{ (cubic units)}$$

Q5 Find the approximation increase in the area of a circular disc if its diameter is increased from 44 to 44.4 cm.

Solution

Let A be the area of circular disc

$\Rightarrow$ $A = \pi r^2$ .....(1) (by def.)

$r = 22$ As radius $= \frac{diameter}{2}$

$dr = 22.2 - 22$ $\Rightarrow r = \frac{44}{2} = 2$

$= 0.2$ when diameter increase to 44.4

From (1)

$$\frac{dA}{dr} = 2\pi r \qquad r = = \frac{44}{2} = 2$$

$$dA = 2\pi r\, dr$$

put values we get

$dA = 2(\pi)(22)(0.2)$

$= 2\times 3.14 \times 22 \times 0.2$ $\pi = 3.14$

$= 27.646$ sq. units