Solved Exercise 2.6

Q1. Find $f'(x)$ if

i. $f(x) = e^{\sqrt{x}-1}$

Solution

Diff. w.r.t x

$$f'(x) = e^{\sqrt{x}-1}.\frac{d}{dx}(\sqrt{x}-1)$$

$$= e^{\sqrt{x}-1}\frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}e^{\sqrt{x}-1}$$

ii. $f(x) = x^3 e^{\frac{1}{x}}$

Solution

$$\Rightarrow \quad f'(x) = e^{\frac{1}{x}}(3x^2) + x^3 e^{\frac{1}{x}}\left(-\frac{1}{x^2}\right)$$

$$= 3x^2 e^{\frac{1}{x}} - xe^{\frac{1}{x}}$$

$$= xe^{\frac{1}{x}}(3x-1)$$

iii. $f(x) = e^x(1 + lnx)$

Solution

$$\Rightarrow \quad f'(x) = \frac{d}{dx}[e^x(1+lnx)]$$

$$= (1+lnx)e^x + e^x\frac{1}{x}$$

$$= \frac{(x(1+lnx)+1)e^x}{x}$$

iv. $\boldsymbol{f(x) = \frac{e^x}{e^{\ \ x}+1}}$

Solution

$$f^{'}(x) = \frac{d}{dx}\left(\frac{e^x}{e^{\ \ x}+1}\right)$$

$$= \frac{(e^{-x}+1)\frac{d}{dx}(e^x) - e^x\frac{d}{dx}(e^{-x}+1)}{(e^{-x}+1)^2}$$

$$= \frac{(e^{-x}+1)(e^x-e^x(-e^{-x}+1)}{(e^{-x}+1)^2}$$

$$= \frac{e^{-x}e^x+e^x+e^xe^{-x}}{(e^{-x}+1)^2}$$

$$= \frac{1+e^x+1}{(e^{-x}+1)^2} = \frac{e^x+2}{(e^{-x}+1)^2}$$

v. $\boldsymbol{f(x) = \ln(e^x + e^{-x})}$

Solution

$$f(x) = \frac{1}{e^x+e^{-x}}[e^x + e^{-x}(\ \ 1)]$$

$$= \frac{e^x-e^{-x}}{e^x+e^{-x}}$$

$$= \frac{e^x-\frac{1}{e^x}}{e^x+\frac{1}{e^x}}$$

$$= \frac{\frac{e^{2x}-1}{e^x}}{\frac{e^{2x}+1}{e^x}}$$

$$f(x) = \frac{e^{2x}-1}{e^{2x}+1}$$

vi. $\boldsymbol{f(x) = \frac{e^{ax} - e^{-ax}}{e^{ax} + e^{-ax}}}$

Solution

$$\Rightarrow \ f(x) = \frac{(e^{ax}+e^{-ax})(e^{ax}.a+e^{-ax}.a)-(e^{ax}-e^{-ax})(e^{ax}.a)-(e^{ax}.a)}{(e^{ax}+e^{-ax})^2}$$

$$= \frac{a[(e^{ax}+e^{-ax})(e^{ax}+e^{-ax})-(e^{ax}-e^{-ax})(e^{ax}-e^{-ax})}{(e^{ax}+e^{-ax})^2}$$

$$= a\left[\frac{(e^{2ax}+e^{-2ax}+2)-(e^{2ax}+e^{-2ax}-2)}{(e^{ax}+e^{-ax})^2}\right]$$

$$= a\left[\frac{4}{(e^{ax}+e^{-ax})^2}\right]$$

$$= \frac{4}{(e^{ax}+e^{-ax})^2}$$

vi. $\boldsymbol{f(x) = \sqrt{\ln(e^{2x} + e^{-2x})} = [\ln(e^{2x} + e^{-2x})]^{1/2}}$

Solution

$$f(x) = \frac{d}{dx}(\ln(e^{-2x} + e^{-2x}))^{-1/2}\frac{d}{dx}[ln(e^{2x} + e^{-2x})]$$

$$= \frac{1}{2\sqrt{\ln(e^{2x}+e^{-2x})}}\ \frac{1}{e^{2x}+e^{-2x}}\ \frac{d}{dx}(e^{2x} + e^{-2x})$$

$$= \frac{1}{2\sqrt{\ln(e^{2x}+e^{-2x})}}\ \frac{1}{e^{2x}+e^{-2x}}\ (2e^{2x} \quad 2e^{-2x})$$

$$= \frac{2(e^{2x}+e^{-2x})}{2(e^{2x}+e^{-2x})\sqrt{\ln(e^{2x}+e^{-2x})}}$$

$$= \frac{e^{2x}-e^{-2x}}{(e^{2x}+e^{-2x})\sqrt{\ln(e^{2x}+e^{-2x})}}$$

viii. $\boldsymbol{f(x) = ln\left(\sqrt{e^{2x} + e^{-2x}}\right)}$

Solution

$$\Rightarrow \; f(x) = \frac{1}{\sqrt{e^{2x}+e^{\ \ 2x}}}$$

$$= \frac{1}{\sqrt{e^{2x}+e^{-2x}}}\ \frac{d}{dx}\ \frac{1}{2\sqrt{e^{2x}+e^{-2x}}}(e^{2x}.2 \quad e^{-2x}.2)$$

$$= \frac{1}{2(e^{2x}+e^{-2x})} \times 2e^{2x} + e^{-2x}(\ \ 2)$$

$$= \frac{2(e^{2x}-e^{-2x})}{2(e^{2x}+e^{-2x})}$$

$$= \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}} = Tanh2x$$

Q2. $\boldsymbol{Find\ \frac{dy}{dx}\ if}$

i. $\boldsymbol{y = x^2 ln\sqrt{x}}$

Solution

$$\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}(x^2 lnx\ )$$

$$= \frac{1}{2}\left(x^2.\frac{d}{dx} + lnx(2x)\right)$$

$$= \frac{x}{2}(1 + 2lnx)$$ **Ans**

ii. $\boldsymbol{y = x\sqrt{lnx}}$

Solution

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{dx}\left(x\sqrt{lnx}\right)$$

$$= x\frac{d}{dx}\left(\sqrt{lnx}\right) + \sqrt{lnx}\ \frac{dy}{dx}(x)$$

$$= x\frac{1}{2\sqrt{lnx}}\ \frac{1}{x} + \sqrt{lnx}(1)$$

$$= \frac{1}{2\sqrt{lnx}} + \sqrt{lnx}$$

$$= \frac{1+2lnx}{2\sqrt{lnx}}$$ **Ans**

iii. $y = \frac{x}{lnx}$

Solution

$$\Rightarrow \frac{dy}{dx} = \frac{lnx.1 - x.\frac{1}{x}}{(lnx)^2}$$

$$= \frac{lnx - 1}{(lnx)^2}$$ **Ans**

iv. $y = x^2 . ln\left(\frac{1}{x}\right)$

Solution

$$\Rightarrow \frac{dy}{dx} = ln\frac{1}{x}2x + x^2 \; \frac{1}{\frac{1}{x}}\left(-\frac{1}{x^2}\right)$$

$$= 2x\, ln\left(\frac{1}{x}\right) - x$$

$$= x\left[2ln\left(\frac{1}{x}\right) - 1\right]$$ **Ans**

v. $y = ln\sqrt{\frac{x^2-1}{x^2+1}}$

Solution

$$\Rightarrow \qquad y = ln\left(\frac{x^2-1}{x^2+1}\right)^{1/2}$$

$$= \frac{1}{2}ln\left(\frac{x^2-1}{x^2+1}\right)$$

$$\Rightarrow \qquad \frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{\frac{x^2-1}{x^2+1}} \frac{d}{dx}\left(\frac{x^2-1}{x^2+1}\right)\right]$$

$$= \frac{1}{2}\left[\frac{x^2+1}{x^2-1}\left(\frac{2x(x^2+1)-(2x)(x^2-1)}{(x^2+1)^2}\right)\right]$$

$$= \frac{1}{2}\left[\frac{x^2+1}{x^2-1}\,\frac{2x^3+2x-2x^3+2x}{(x^2+1)^2}\right]$$

$$= \frac{1}{2}\left[\frac{4x}{(x^2-1)(x^2+1)}\right]$$

$$= \frac{2x}{x^4-1}$$ **Ans**

vi. $y = ln\left(x + \sqrt{x^2+1}\right)$

Solution

$$\frac{dy}{dx} = \frac{1}{x+\sqrt{x^2+1}}\left(1 + \frac{1}{2\sqrt{x^2+1}}(2x)\right)$$

$$= \frac{1}{x+\sqrt{x^2+1}}\left(1 + \frac{x}{\sqrt{x^2+1}}\right)$$

$$= \frac{1}{x+\sqrt{x^2+1}}\left[\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}\right]$$

$$= \frac{1}{\sqrt{x^2+1}}$$ **Ans**

vii. $y = \ln(9 - x^2)$

Solution

$$\frac{dy}{dx} = \frac{1}{9-x^2}(-2x)$$

$$= \frac{-2x}{9-x^2}$$ **Ans**

viii. $y = e^{-2x}\,Sin2x$

Solution

$$\frac{dy}{dx} = e^{-2x}(cos2x.2) + sin2x(e^{-2x} - 2)$$

$$= 2e^{-2x}(cos2x - sin2x)$$ **Ans**

ix. $y = e^{-x}(x^3 + 2x^2 + 1)$

Solution

$$\frac{dy}{dx} = e^{-x}\frac{d}{dx}(x^3 + 2x^2 + 1) + (x^3 + 2x^2 + 1)\frac{d}{dx}e^{-x}$$

$$= e^{-x}(3x^2 + 4x) + (x^3 + 2x^2 + 1)(-e^{-x})$$

$$= e^{-x}[3x^2 + 4x - x^3 - 2x^2 - 1]$$

$$= e^{=x}(-x^3 + x^3 + 4x - 1)$$

$$= e^{-x}(x^3 - x^2 - 4x + 1)$$ **Ans**

x. $y = xe^{sinx}$

Solution

$$\frac{dy}{dx} = x\frac{d}{dx}\left(e^{sinx}\right) + e^{sinx}\frac{d}{dx}(x)$$

$$= xe^{sinx}.cosx + e^{sin} - 1$$

$$= (x\cos x + 1)e^{sinx}$$ **Ans**

xi. $y = 5e^{3x-4}$

Solution

$$\frac{dy}{dx} = \frac{d}{dx}(5e^{3x-4})$$

$$= 5\frac{d}{dx}(e^{3x-4})$$

$$= 5e^{3x-4}\frac{d}{dx}(3x - 4)$$

$$= 5e^{3x-4}(3)$$

$$\frac{dy}{dx} = 15e^{3x-4}$$ **Ans**

xii. $\boldsymbol{y = (x - 1)^2}$

Solution

$$lny = ln(x+1)^2$$

$$lny = xln(x+1)$$

Diff.w.r.t. x

$$\frac{1}{y}9lny) = \frac{d}{dx}(x.\ln(x+1))$$

$$\frac{1}{y}\ \frac{dy}{dx} \quad = x.\frac{1}{x+1} + \ln(x+1).1$$

$$\frac{1}{y}\ \frac{dy}{dx} \quad = \frac{x}{x+1} + \ln(x+1).1$$

$$\frac{dy}{dx} \quad = y\left[\frac{x}{x+1} + \ln(x+1)\right]$$

$$As\ y\ = (x+1)^{`}$$

So, $\quad = (x+1)^{`}\left[\left(\frac{x}{x+1}\right) + \ln(x+1)\right]$ **Ans**

xiii. $\boldsymbol{y = (lnx)^{lnx}}$

Solution

Taking in both sides

$$lny = ln(lnx)^{lnx}$$

$$lnx = lnx.\ln(lnx)$$

Diff.w.r.t. x

$$\frac{1}{y}\ \frac{dy}{dx} \quad = lnx\ \frac{1}{lnx} + \frac{1}{x} + \frac{1}{x}(\ln(lnx))$$

$$= \frac{1}{x} + \frac{\ln(lnx)}{x}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{\ln(lnx)+1}{x}$$

$$\frac{dy}{dx} = y\left[\frac{\ln(lnx)+1}{x}\right]$$

$$= (lnx)^{lnx}\left[\frac{\ln(lnx)+1}{x}\right]$$ **Ans**

xiv. $y = \frac{\left(\sqrt{x^2-1}\right)(x+1)}{(x^3+1)^{1/2}}$

Solution

$$y = \frac{\sqrt{(x+1)}\left(\sqrt{x-1}\right)(x+1)}{(x+1)^{3/2}(x^2-x+1)^{3/2}}$$

$$= \frac{(x-1)^{1/2}(x+1)^{3/2}}{(x+1)^{3/2}(x^2-x+1)^{3/2}}$$

$$y = \frac{(x-1)^{1/2}}{(x^2-x+1)^{3/2}} \ldots\ldots\ldots\ldots(1)$$

Taking in both sides

$$lny = ln\left[\frac{(x-1)^{1/2}}{(x^2-x+1)^{3/2}}\right]$$

$$= ln(x-1)^{1/2} - \ln(x^2-x+1)^{3/2}$$

$$= \frac{1}{2}\ln(x-1) - \frac{3}{2}\ln(x^2-x+1)$$

Diff w.r.t x

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2} - \frac{3}{2}\frac{(2x-1)}{x^2-x+1}$$

$$= \frac{1}{2}\left[\frac{1}{x-1} - \frac{3(2x-1)}{x^2-x+1}\right]$$

$$= \frac{1}{2}\left[\frac{-x^2-x+1-3(2x-1)(x-1)}{(x-1)(x^2-x+1)}\right]$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left[\frac{-x^2-x+1-3(2x^2-3x+1)}{(x-1)(x^2-x+1)}\right]$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{y}{2}\left[\frac{x^2 - x + 1 - 6x^2 - 9x - 3}{(x-1)(x^2 - x + 1)}\right]$$

From above

$$= \left(\frac{\sqrt{x-1} - 5x^2 + 8x - 2}{2(x^2 - x + 1)^{\frac{3}{2}}(x-1)(x^2 - x + 1)}\right)$$

$$= -\frac{+5x^2 - 8x + 2}{2(x^2 - x + 1)^{3/2}\sqrt{x-1}}$$ **Ans**

Q3. $\boldsymbol{Find\ \frac{dy}{dx}\ if}$

1. $\boldsymbol{y = (\cos h2x)}$

Solution

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(\cos h2x)$$

$$= \sin h2x.2$$

$$= 2\sin h2x$$ ***Ans***

ii. $\boldsymbol{y = \sin h3x}$

Solution

$$\Rightarrow \quad \frac{dy}{dx} = \cos h3x\ \ 3$$

$$= 3\cos h3x$$ **Ans**

iii. $\boldsymbol{y = \tan h^{-1}(sinx)}$

Solution

$\Rightarrow \tan hy = \sin x$

Diff w.r.t x we have

$\frac{d}{dx}(Tan\ hy) = \frac{d}{dx}\sin x$

$\sec h^2\, y \frac{dy}{dx} = \cos x$

$\frac{dy}{dx} = \frac{\cos x}{\sec h^2 y}$

$= \frac{\cos x}{1-\tan h^2\, y} = \frac{\cos x}{1-sin^2 x}$

$= \frac{\cos x}{cos^2\, x} = \frac{1}{\cos x}$

$= \sec x$ **Ans**

iv. $\boldsymbol{y = \sin h^{-1}(x^3)}$

Solution

$\Rightarrow \sin hy = x^3$...............(1)

$\frac{d}{dx}(\sin hy) = \frac{d}{dx}(x^3)$

$\cos hy\, \frac{dy}{dx} = 3x^2$

$\frac{dy}{dx} = 3x^2$

$\frac{dy}{dx} = 3x^2 \times \frac{1}{\cos hy}$

$= \frac{3x^2}{\sqrt{1+\sin h^2 y}} = \frac{3x^2}{\sqrt{1+(x^3)^2}}$

$= \frac{3x^2}{\sqrt{1+x^6}}$ **Ans**

v. $\boldsymbol{y - ln(Tan\ hx)}$

Solution

$\frac{dy}{dx} = \frac{1}{Tan\ hx}\ \frac{d}{dx}(Tan\ hx)$

$= \frac{1}{Tan\ hx} sec^2 hx$

$= \frac{\cos hx}{\sin hx}\ \frac{1}{cos^2\ hx}$

$= \frac{1}{sinx.\cos hx}$

$= \frac{2}{2sinx.\cos hx}$h

$= \frac{2}{sinh2x}$

$= 2cosec\ h2x$ **Ans**

vi. $\boldsymbol{y = sin\ h^{-1}\left(\frac{x}{2}\right)}$

Solution

$y = sinh^{-1}\left(\frac{x}{2}\right)$

$\Rightarrow\ \sin hy = \frac{x}{2}$ (1)

Diff w.r.t x both sides

$\frac{d}{dx}(\sin hy) = \frac{d}{dx}\left(\frac{x}{2}\right)$

$\cos hy \frac{dy}{dx} = \frac{1}{2}$ (2)

$\Rightarrow\ = \sqrt{1 + sin^2 hy}\ \frac{dy}{dx} = \frac{1}{2}$

$\Rightarrow\ = \sqrt{1 + \left(\frac{x}{2}\right)^2}\ \frac{dy}{dx} = \frac{1}{2}$

$$= \sqrt{\frac{4+x^2}{4}} \qquad \frac{dy}{dx} = \frac{1}{2}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\frac{4+x^2}{2}}}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{4+x^2}}$$ **Ans**