Solved Exercise 2.3

- **Differential w.r.t x**

Q1. $x^4 + 2x^3 + x^2$

Solution

Let $Y = x^4 + 2x^3 + x^2$

Diff w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}[x^{4+}2x^3 + x^2]$$

$$= \frac{d}{dx}x^4 + 2\frac{d}{dx}x^3 + \frac{d}{dx}x^2$$

$$= 4x^3 + 6x^2 + 2x$$ **Ans**

Q2. $x^{-3} + 2x^{\frac{3}{2}} + 3$

Solution

Let Y $= x^{-3} + 2x^{\frac{3}{2}} + 3x^{-3} + 2x^{\frac{3}{2}} + 3$

Diff.w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx} \left[x^{-3} + 2x^{\frac{3}{2}} + 3\right]$$

$$= \frac{d}{dx}x^{-3} + 2\frac{d}{dx}x^{-3/2 \ 1} + \frac{d}{dx}(3)$$

$$= -3x^{-3-1} + 2\left(\frac{-3}{2}\right)x^{-5/2} + 0$$

$$= -3x^{-4} - 3x^{\frac{5}{2}}$$

$$= \frac{-3}{x^{\ 4}} - \frac{3}{x^{1/2}} = -3\left[\frac{1}{x^4} + \frac{1}{x^{5/2}}\right]$$ **Ans**

Q3. $\frac{a+x}{a-x}$

Solution

Let $Y = \frac{a+x}{a-x}$

Diff Both Sides w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{a+x}{a-x}\right)$$

$$= \frac{(1)(a-x)-(1)(a+x)}{(a-x)^2}$$

$$= \frac{a-x+a+x}{(a-x)^2}$$

$$= \frac{2a}{(a-x)^2}$$ **Ans**

Q4. $\frac{2x-3}{2x+1}$

Solution

Let $Y = \frac{2x-3}{2x-1}$

Diff.w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{2x-3}{2x+1}\right)$$

$$= \frac{(2+1)\frac{d}{dx}(2x-3)-(2x-3)\frac{d}{dx}(2z+1)}{(2x+1)^2}$$

$$= \frac{(2x+1)(2)-(2x-3)(2)}{(2x+1)^2}$$

$$= \frac{4x+2-4x+6}{(2x+1)^2}$$

$$= \frac{8}{(2x+1)^2}$$ **Ans**

Q5. (x-5)(3-x)

Solution

Let $Y = (x-5)(3-x)$

Diff.w.r.t. x

$$\frac{dy}{dx} = (x-5)\frac{d}{dx}(3-x) + (3-x)\frac{d}{dx}(x-5)$$

$$= (x-5)(-1) + (3-x)(1)$$

$$= -x + 5 + 3 - x \qquad = -2x + 8$$ **Ans**

Q6. $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2$

Solution

Let $Y = \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2$

$$Y = \left(\sqrt{x}\right)^2 + \left(\frac{1}{\sqrt{x}}\right)^2 - 2\sqrt{x}\frac{1}{\sqrt{x}}$$

$$= x + \frac{1}{\sqrt{x}-2}$$

Diff.w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx}(x) + \frac{d}{dx}\left(\frac{1}{x}\right) - \frac{d}{dx}(2)$$

$$= 1 - \frac{1}{x^2} - 0$$

$$= \frac{x^2-1}{x^2}$$ **Ans**

Q7. $\frac{(1+\sqrt{x})\left(x - x^{\frac{5}{2}}\right)}{\sqrt{x}}$

Solution

Let $y = \frac{(a+\sqrt{x})x(1-\sqrt{x})}{\sqrt{x}}$

$= \sqrt{x}(1+\sqrt{x})(1-\sqrt{x})$

$= \sqrt{x}(1-x)$

$= \left(\sqrt{x-x}\right)^{\frac{5}{2}} = \sqrt{x} - x\sqrt{x}$

$= x^{1}/2 - x^{3}/2$

Diff. w.r.t x

$\frac{dy}{dx} = \frac{d}{dx}\left(x^{1}/2 - x^{3}/2\right)$

$= \frac{1}{2\sqrt{x}} - \frac{3}{2}x^{\frac{1}{2}}$

$= \frac{1}{2}\left(\frac{1-3x}{\sqrt{x}}\right)$

$= \frac{1-3x}{2\sqrt{x}}$

Q8. $\frac{(x^2+1)^2}{x^2-1}$

Solution

Let $y = \frac{(x^2-1)^2}{x^2-1}$

Diff. w.r.t x

$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{(x^2+1)^2}{x^2-1}\right)$

$= \frac{(x^2-1)\frac{d}{dx}(x^2+1)^2 - (x^2+1)^2\frac{d}{dx}(x^2-1)}{(x^2-1)^2}$

$= \frac{(x^2-1)2(x^2+1)(2x) - (x^2+1)^2(2x)}{(x^2-1)^2}$

$$= \frac{4x(x^4-1)-2x(x^4+2x^2+1)}{(x^2-1)^2}$$

$$\frac{dy}{dx} = \frac{2x(x^2+1)(x^2+3)}{(x^2-1)^2}$$

Q9. $\frac{x^2+1}{x^2-3}$

Solution

Let $y = \frac{x^2+1}{x^2+3}$

Diff. w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2+1}{x^2+3}\right)$$

$$\frac{dy}{dx} = \frac{2x(x^2-3)-(2x)(x^2-1)}{(x^2-3)^2}$$

$$= \frac{2x\left[x^2-3-x^2-1\right]}{(x^2-3)^2}$$

$$= \frac{2x(-4)}{(x^2-3)^2}$$

$$= \frac{-8x}{(x^2-3)^2}$$ **Ans**

Q10. $\frac{\sqrt{1+x}}{\sqrt{1-x}}$

Solution

Let $Y = \frac{\sqrt{1+x}}{\sqrt{1-x}}$

Diff.w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)$$

$$= \frac{(\sqrt{1-x})\left[\frac{1}{2\sqrt{1+x}} - \sqrt{1+x}\,\frac{1}{2\sqrt{1-x}}\right]}{(1+x)}$$

$$= \frac{1}{1-x}\;\frac{(1-x)+(1+x)}{2\sqrt{(1+x)(1-x)}}$$

$$= \frac{\frac{(1-x)^{1/2}}{2(1+x)^{1/2}} + \frac{(1+x)^{1/2}}{2(1-x)^{1/2}}}{1-x}$$

$$= \frac{1-x+1+x}{2(1+x)^{1/2}(1-x)^{1/2}(1-x)}$$

$$= \frac{2}{2\sqrt{x+1}(1-x)^{1/2}}$$

$$= \frac{1}{\sqrt{1+x}(1-x)^{1/2}}$$ **Ans**

Q11. $\frac{2x-1}{\sqrt{x^2+1}}$

Solution

Let $Y = \frac{2x-1}{\sqrt{x^2+1}}$

Diff.w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{2x-1}{\sqrt{x^3+1}}\right)$$

$$\frac{dy}{dx} = \frac{2\left(\sqrt{x^2+1}\right) - (2x-1)\frac{2x}{2\sqrt{x^3+1}}}{\left(\sqrt{x^3}+1\right)^2}$$

$$= \frac{2x^2+1-\frac{(2x-1)}{\sqrt{x^2}+1}.x}{(x^2+1)}$$

$$\frac{dy}{dx} = \frac{2\left(x^2+1\right)-x(2x-1)}{\left(\sqrt{x^2}+1\right)(x^2+1)}$$

$$= \frac{2x^2+2-2x^2+x}{(x^2+1)^{1/2}(x^2+1)}$$

$$\frac{dy}{dx} = \frac{x+2}{(x^2+1)^{3/2}}$$ **Ans**

Q12. $\sqrt{\frac{a-x}{a+x}}$

Solution

Let $Y = \sqrt{\frac{a-x}{a+x}}$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{(a-x)^{1/2}}{(a+x)^{1/2}}\right]$$

$$= \frac{(a+x)^{1/2}\, d/dx(a-x)^{1/2} - (a-x)^{1/2} d/dx(a+x)^{1/2}}{\left[(a+x)^{1/2}\right]^2}$$

$$= \frac{(a+x)^{1/2}\ 1/2(a-x)^{1/2}\ ((0-1)-(a-x)^{1/2}\ 1/2(a+x)^{1/2}(0+1)}{a+x}$$

$$= \frac{\frac{(a+x)^{1/2}}{2(a\ \ x)^{1/2}} - \frac{(a\ \ x)^{1/2}}{2(a+x)^{1/2}}}{a+x}$$

$$= \frac{\frac{(a+x)^{1/2}\ \ (a+x)^{1/2}\ \ (a\ \ x)^{1/2}\ \ (a\ \ x)^{1/2}}{2(a\ \ x)^{1/2}\,(a+x)^{1/2}}}{a+x}$$

$$= \frac{(a+x)-(a-x)}{2(a-x)^{1/2}\,(a+x)^{1/2}(a+x)}$$

$$= \frac{a+x-a+x}{2\sqrt{a-x}(a+x)^{3/2}}$$

$$= \frac{x}{\sqrt{(a-x)}(a+x)^{3/2}}$$

Q13. $\frac{\sqrt{x^2+1}}{\sqrt{x^2-1}}$

Solution

Let $Y = \frac{\sqrt{x^2+1}}{\sqrt{x^2-1}}$

Diff. w.r.t x

$$\frac{dy}{dx} = \frac{\left(\frac{1}{2}\right)(x^2+1)^{-1/2} 2x(\sqrt{x^2-1}) - \frac{1}{2}(x^2-1)^{-1/2} 2x\sqrt{x^2+1}}{\left(\sqrt{x^2-1}\right)^2}$$

$$= \frac{\frac{x(x^2-1)-(x^2+1)}{\left(\sqrt{x^2+1}\right)\left(\sqrt{x^2-1}\right)}}{x^2-1}$$

$$= \frac{x^3-x^3-x-x}{(x^2-1)^{3/2}\sqrt{x^2-1}}$$

$$= \frac{-2x}{(x^2-1)^{3/2}\sqrt{x^2-1}} \quad \text{Ans}$$

Q14. $\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$

Solution

Let $Y = \frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$

$$Y = \frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}} \times \frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}$$

$$= \frac{\left(\sqrt{1+x}-\sqrt{1-x}\right)^2}{\left(\sqrt{1+x}\right)^2-\left(\sqrt{1-x}\right)^2}$$

$$= \frac{1+x+1-x-2\left(\sqrt{1+x}\right)\left(\sqrt{1-x}\right)}{1+x-1-x}$$

$$= \frac{2-2\sqrt{1-x^2}}{2x}$$

$$= \frac{1\sqrt{1-x^2}}{x}$$

Diff.w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1-\sqrt{1-x^2}}{x}\right)$$

$$= \frac{-\frac{1}{2}(1-x^2)^{-1/2}(-2x)-\left(1-\sqrt{1-x^2}\times 1\right)}{(x)^2}$$

$$= \frac{\frac{x^2}{\sqrt{1-x^2}} - \left(1-\sqrt{1-x^2}\right)}{(x)^2}$$

$$= \frac{\frac{x^2}{\sqrt{1-x^2}} - 1 + \sqrt{1-x^2}}{(x)^2}$$

$$= \frac{x^2 - \sqrt{1-x^2} + 1 - x^2}{x^2\sqrt{1+x^2}}$$

$$= \frac{1-\sqrt{1-x^2}}{x^2\sqrt{1-x^2}}$$ **Ans**

Q15. $\frac{x\sqrt{a+x}}{\sqrt{a-x}}$

Solution

Let $y = \frac{x\sqrt{a+x}}{\sqrt{a-x}}$

Diff. both sides w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x\sqrt{a+x}}{\sqrt{a-x}}\right)$$

$$= \frac{x\sqrt{a+x}}{\sqrt{a-x}}\frac{d}{dx}(x) + x\frac{d}{dx}\left(\frac{\sqrt{a+x}}{\sqrt{a-x}}\right)$$

$$= \frac{\sqrt{a+x}}{\sqrt{a-x}} \times 1 + x\left[\frac{a-x\frac{1}{2\sqrt{a+x}} - \frac{\sqrt{a+x}(-1)}{2\sqrt{a-x}}}{\left(\sqrt{a-x}\right)^2}\right]$$

$$= \frac{\sqrt{a+x}}{\sqrt{a-x}} + x\left[\frac{(a-x)(a+x)}{2\left(\sqrt{a-x}\right)\left(\sqrt{a+x}\right)(a-x)}\right]$$

$$= \frac{\sqrt{a+x}}{\sqrt{a-x}} + x\left[\frac{2a}{2\left(\sqrt{a-x}\right)\left(\sqrt{a+x}\right)(a-x)}\right]$$

$$= \frac{\sqrt{a+x}}{\sqrt{a-x}} + \frac{ax}{2\left(\sqrt{a-x}\right)\left(\sqrt{a+x}\right)(a-x)}$$

$$= \frac{(a+x)(a-x)+xa}{\left(\sqrt{a+x}\right)(a-x)^{\frac{3}{2}}}$$

$$= \frac{a^2-x^2+ax}{(\sqrt{a+x})(a-x)^{\frac{3}{2}}}$$

Q16. If $y = \sqrt{x} - \frac{1}{\sqrt{x}}$, which shows that $2x\frac{dy}{dx} + y = 2\sqrt{x}$

Solution

$$y = \sqrt{x} - \frac{1}{\sqrt{x}}$$

Diff. w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)$$

$$= \frac{1}{2}x^{\frac{1}{2}} - \frac{d}{dx}x^{\frac{1}{2}}$$

$$=\frac{1}{2}x^{\frac{1}{2}} - (-\frac{1}{2}x^{\frac{3}{2}})$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}} + \frac{1}{2}x^{\frac{3}{2}}$$

$$= \frac{1}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{\frac{3}{2}}$$

$$2\frac{dy}{dx} = \frac{1}{\sqrt{x}} + \frac{1}{\sqrt{x}.x}$$

$$2x\frac{dy}{dx} = \frac{x}{\sqrt{x}} + \frac{x}{\sqrt{x}.x}$$

$$2x\frac{dy}{dx} \quad = \sqrt{x} + \frac{1}{\sqrt{x}}$$

$$2x\frac{dy}{dx} + y = \sqrt{x} + \frac{1}{\sqrt{x}} + \sqrt{x} - \frac{1}{\sqrt{x}}$$

$2x\frac{dy}{dx} = 2\sqrt{x}$ Hence proved

Q17. If $y = x^4 + 2x^2 + 2$

Prove that $\frac{dy}{dx} = 4x\sqrt{y-1}$

Solution

$$y = x^4 + 2x^2 + 2$$

$$y = x^4 + 2x^2 + 1 + 1$$

$$\Rightarrow \quad y - 1 = x^4 + 2x^2 + 1$$

y-1=$(x^2 + 1)$

Taking square root Both sides

$$\sqrt{y-1} = x^2 + 1 \qquad (1)$$

Now $\quad y = x^4 + 2x^2 + 2 \qquad (2)$

Diff. both sides of w.r.t x $4x^3 + 4x^2 = \frac{dy}{dx}$

$$\frac{dy}{dx} = 4x(x^2 + 1) \qquad (3)$$

From (1) and (3)

$$\frac{dy}{dx} = 4x\sqrt{y-1}$$