Unit 8

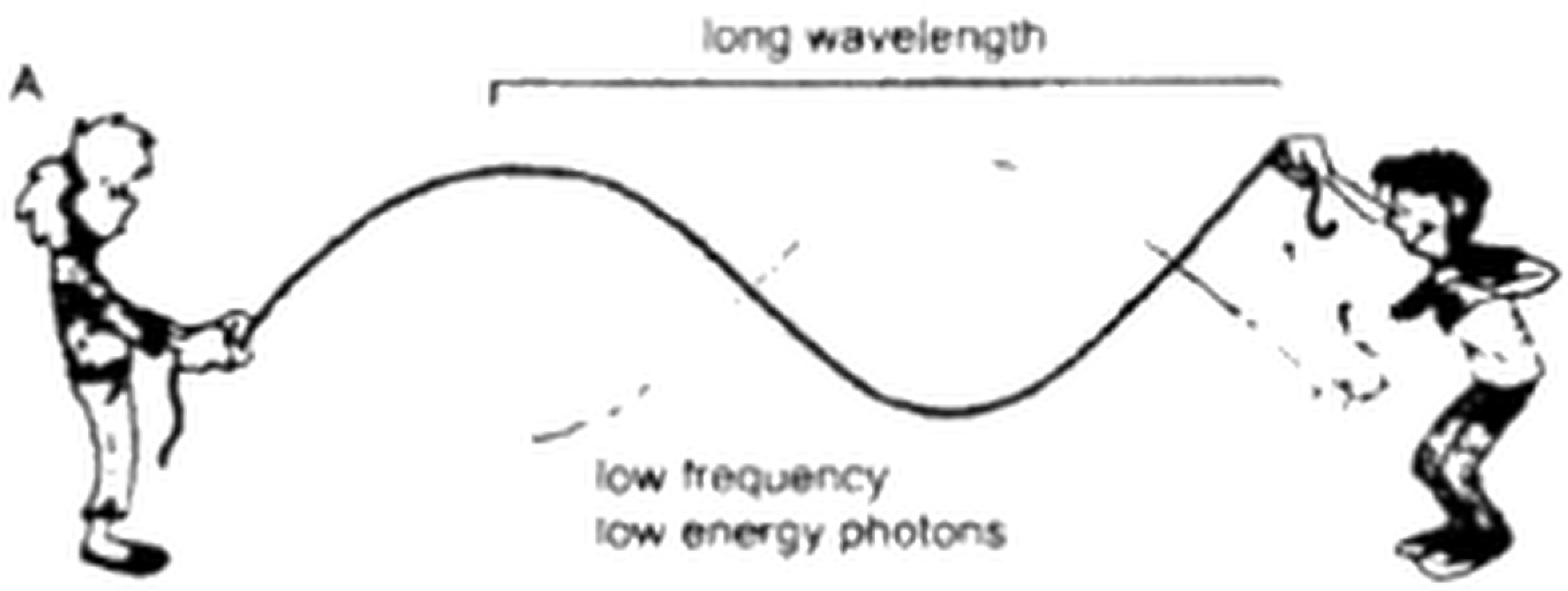
WAVES

Q.1 What are progressive waves? Give its types.

Answer

Progressive Waves

The waves which transfer energy by moving away from the source of disturbance are called progressive or travelling waves.



Example

Consider two persons holding the opposite ends of the rope. Suddenly one person gives a jerk to the rope. The disturbance in the rope produces a pulse which moves toward another person. When this reaches the other person, it pushes his hand upward. So, the energy and momentum transferred from one person towards the other person. This is an example of progressive wave.

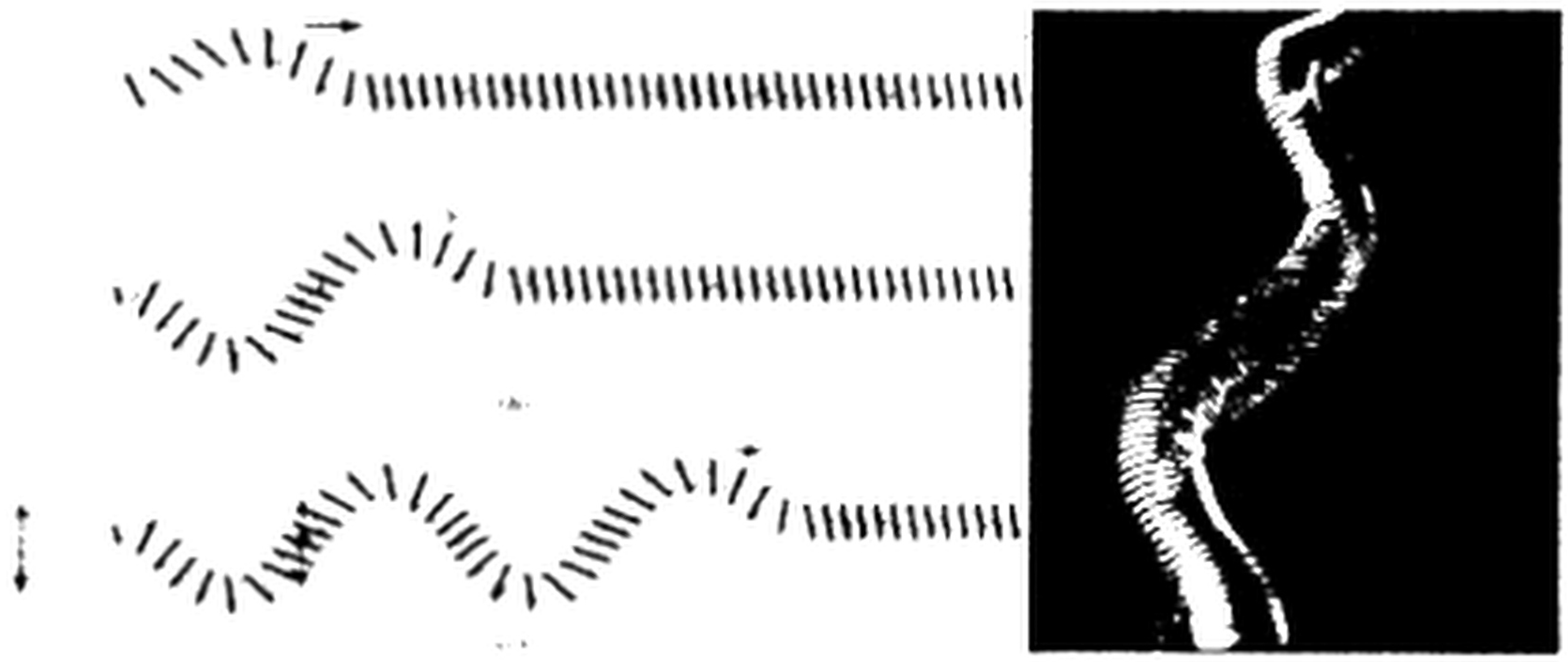
Kinds of waves

There are two kinds of progressive waves

i) Transverse waves **ii)** Longitudinal waves

Transverse Waves

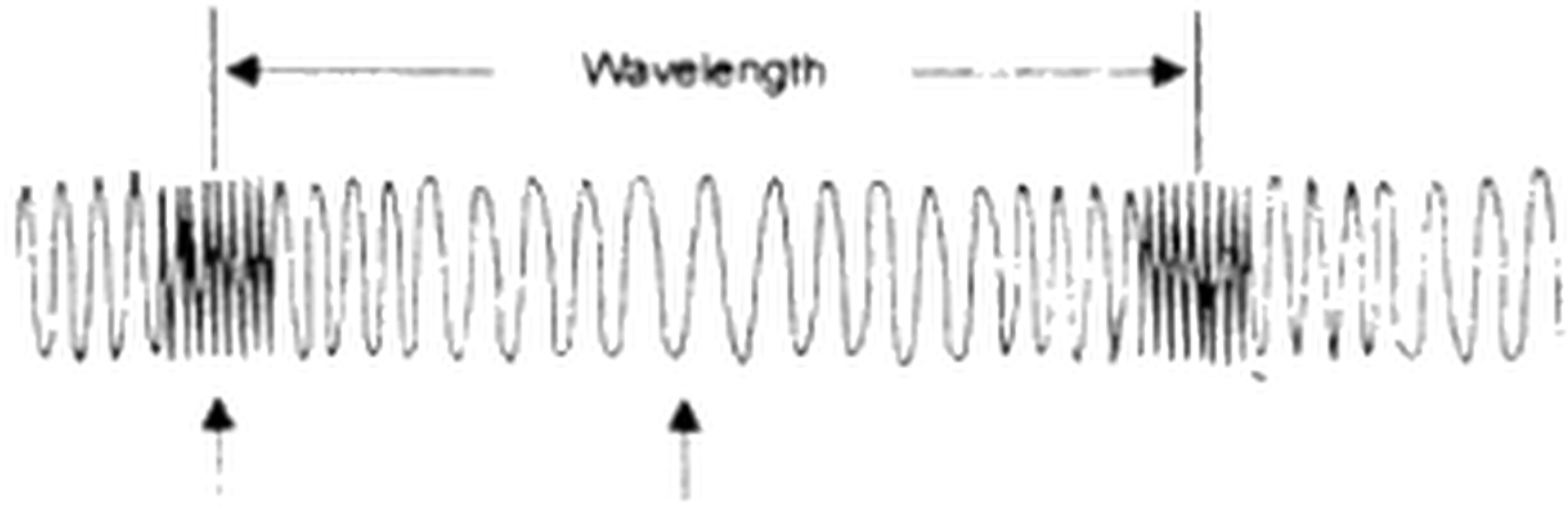
The waves, in which particles of the medium are displaced perpendicular to the direction of propagation of wave and such waves are called longitudinal waves, as shown in figure.



Now take a loose spring coil (slinky spring) for illustration of motion of source in generating waves in a medium. Slinky is the soft spring which has small initial length but relatively large extended figure. Consider a horizontal spring system with its one end fixed. When the free end is moved from side to side, a pulse of wave having a displacement pattern as shown in figure, which will move along the spring. This shows that displacement of particles is perpendicular to the direction of propagation of wave, hence transverse waves are produced.

Longitudinal Waves (Compressional waves)

The waves, in which particles of the medium are displaced along the direction of propagation of wave and such waves are called longitudinal waves, as shown in figure.



Note

(Why, sound waves in air are longitudinal in nature)

Both types of waves can be set up in solid. In fluids, however, transverse waves die out very quickly and usually cannot be produced at all. That is why, sound waves in air are longitudinal in nature.

Q.2 What are periodic waves? Also discuss its different types?

Answer

Period Waves

The waves which are produced by continuous and rhythmic disturbances in a medium are called periodic waves.

A graph of the displacement of one particle as a function of time

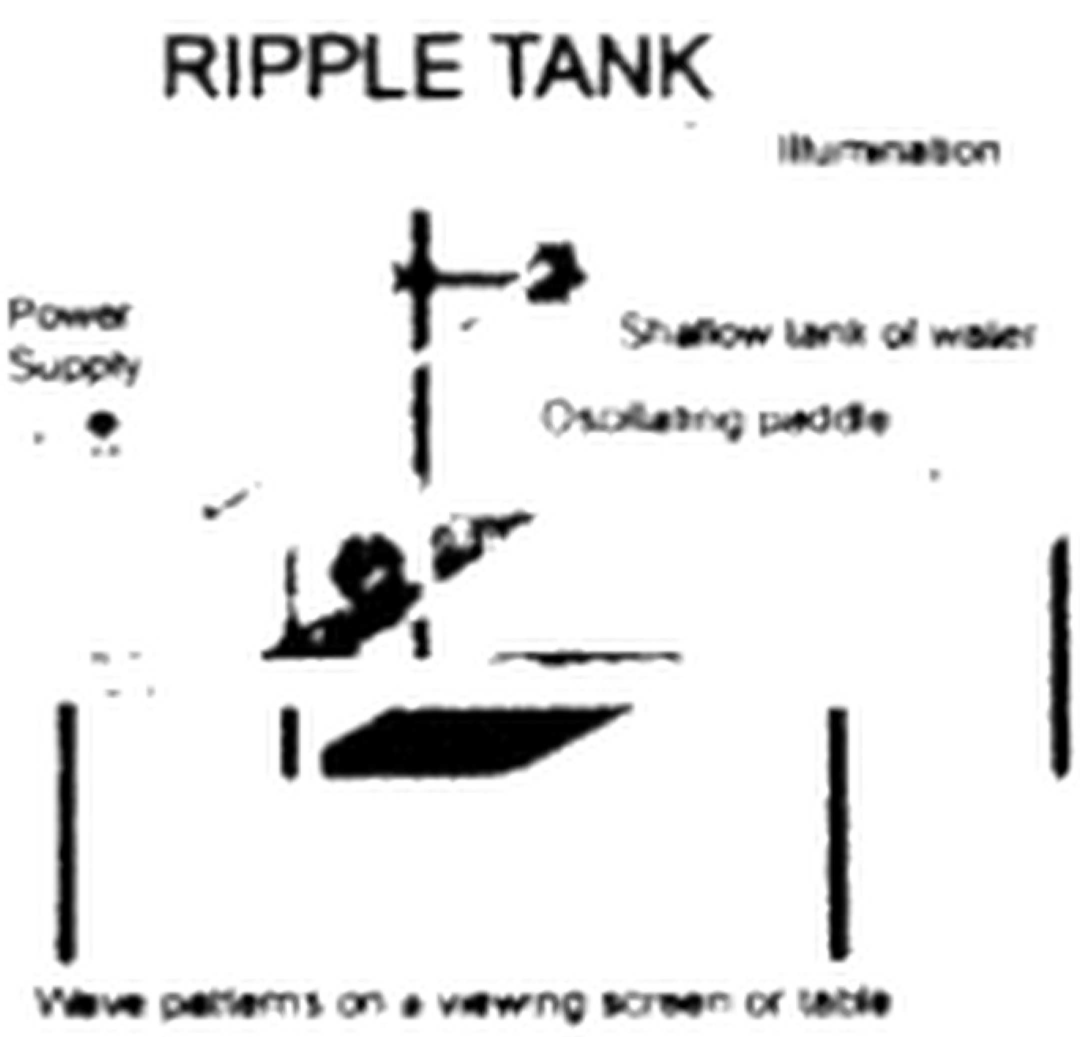
These may be transverse or longitudinal in nature. A good example of a periodic waves in an oscillating mass-spring system which executes SHM.

Transverse Periodic Waves

The periodic waves in which the displacement of particles of medium is perpendicular to the direction of motion of waves produced by continuous and rhythmic disturbances in a medium are called transverse periodic waves.

Experiment

Let us consider a mass-spring system which can vibrate horizontally as shown in figure. A long string of uniform thickness is stretched horizontally and its one end is attached with the oscillating mass m. Due to oscillation of mass-spring system a transverse wave is produced in the string.



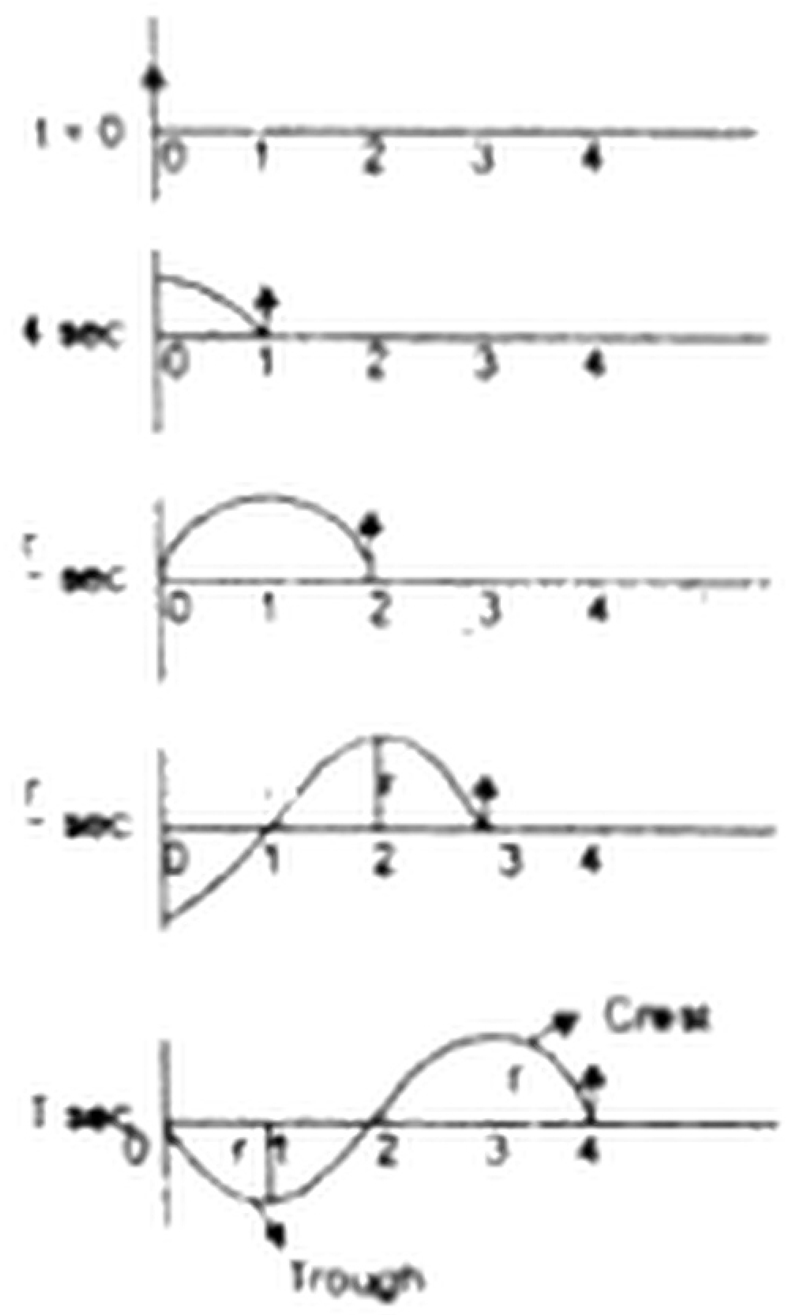


Figure 1

The wave appears to be travelling on the spring, from its own end to the other. In this case each part of string vibrates on the spring, from its own end to the other. In this case each part of string vibrates at right angle to the length of stretched string.

The crest and troughs are being replaced by one another periodically and waves appear to be travelling.

Crest

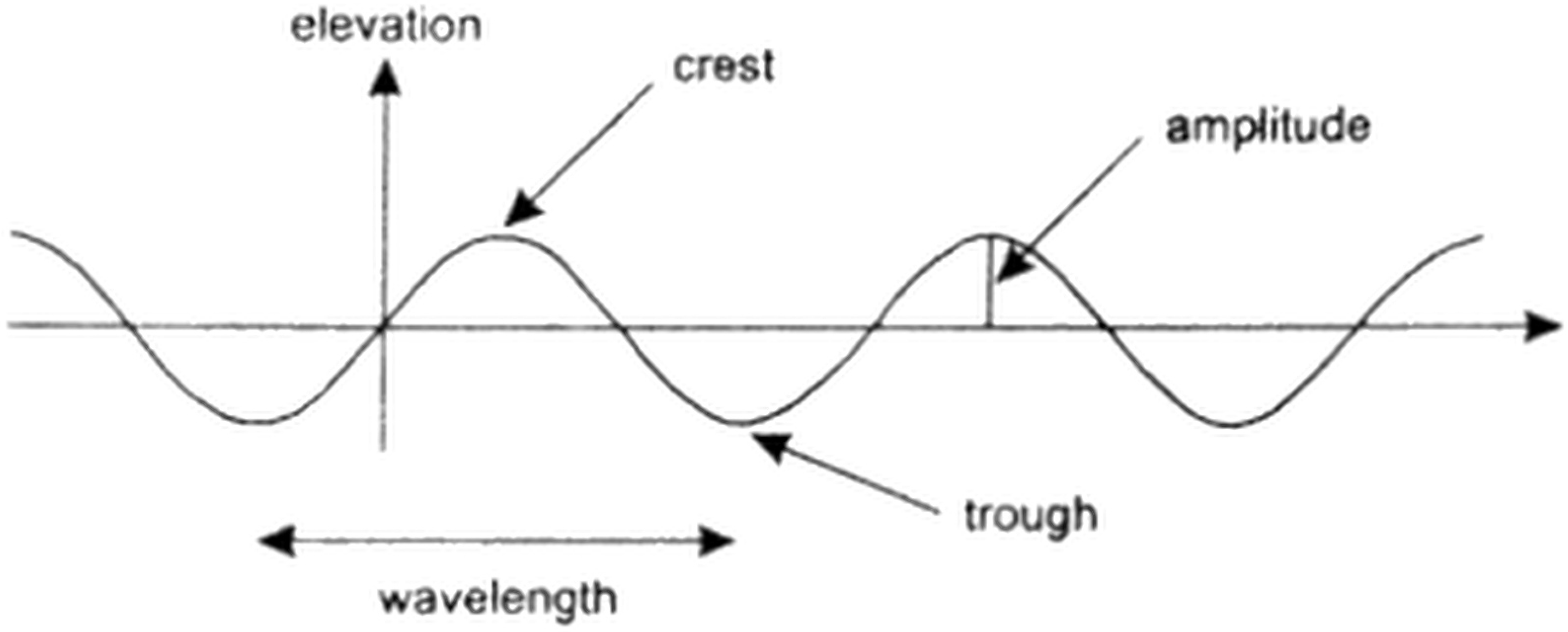
The portion of the wave above the mean level is called as crest.

Trough

The portion of the wave below the mean level is called trough.

Amplitude

The amplitude is the maximum displacement of point in a crest or a trough of the string.



Wave Length

The distance between two consecutive crests or two consecutive troughs is known as wave length, it is denoted by λ.

Time Period

The time for which the wave travels a distance of wave length is called time period. The time period of wave is equal to be the time period of the oscillator which produces it.

Speed of Wave

When a wave progresses, each particle in the medium performs SHM. The time that the crest required to move a distance, of one wave length is equal to the time required for a point in the medium to go through one complete oscillation. If 'v' be the speed of wave, then

$$V = \frac{\text{Distance covered}}{\text{Interval of time}}$$

$$V = \lambda / T$$

$$V = \lambda\left(\frac{1}{T}\right) \qquad \text{But } 1/T = f$$

Or $$V = f\lambda$$

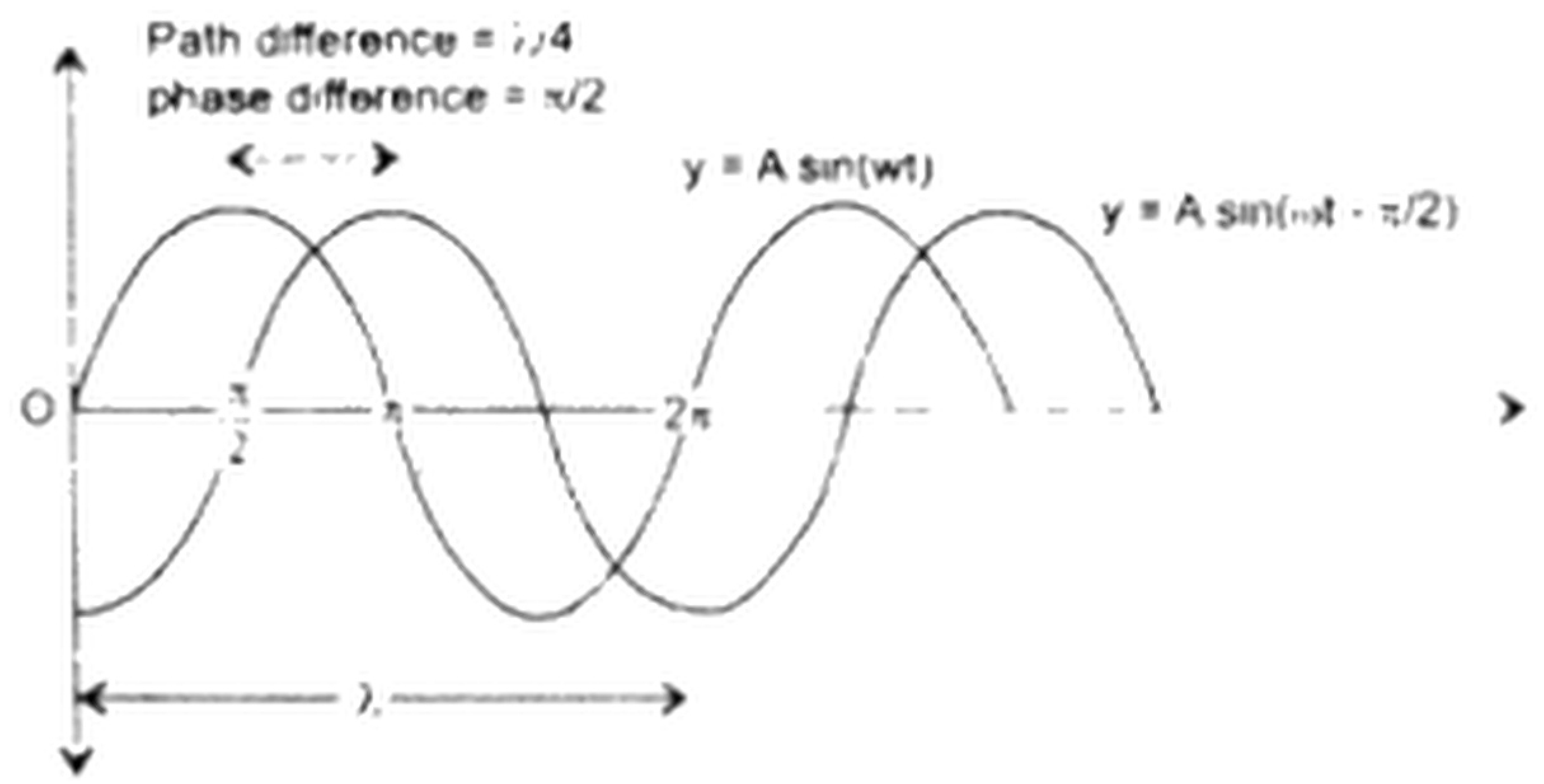
Where f is the frequency of the wave which is same as the frequency of oscillator (crest or trough) which produces it.

Wave Profile

Relation between path difference and phase difference

Consider the snapshot of the periodic waves moving through the medium. As any distance x from the reference point then phase difference can be described as

$$\Phi = \frac{2\pi}{\lambda}$$



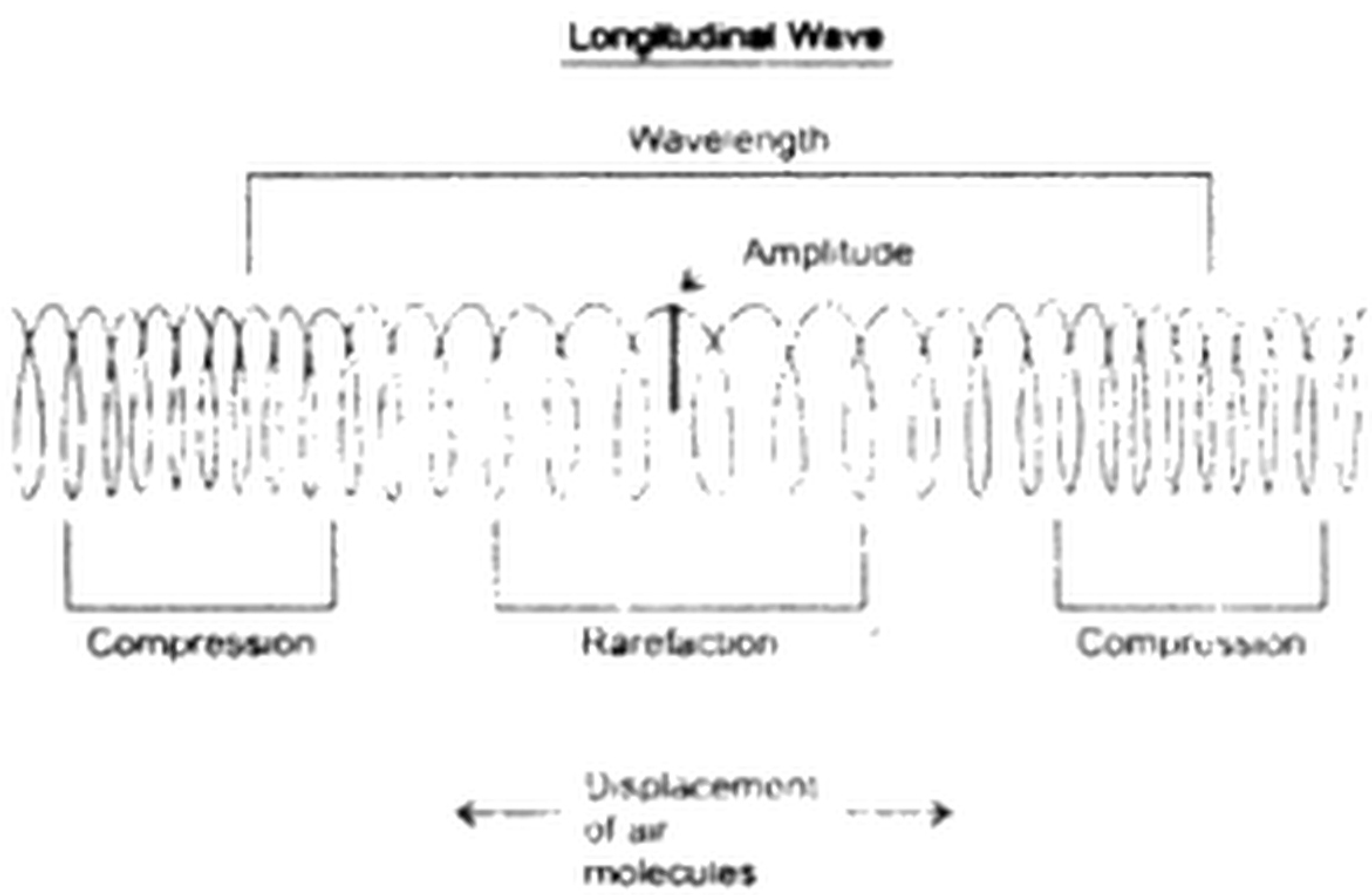
The points C and C', as they move up and down are always in the same state of vibrations (i.e. they have identical displacement and velocities). There are many points or particles along the medium which are vibrating in phase. The points separated from one another through distance λ, 2λ, 3λ........ are all in phase with each other.

Some points are exactly out of step, for example when point C reaches its maximum up wall displacement, at the same time reaches its down wall displacement.

The points separated from one another through a distance of λ/2, 3λ/2, 5λ/2………… are opposite in phase.

Longitudinal periodic waves

The periodic waves in which the displacement of particles of medium is along the direction of motion of waves produced by continuous and rhythmic disturbances in a medium are called transverse periodic waves.



Experiment

To explain longitudinal periodic waves, we take an example of a spring which is suspended by the help of threads. Longitudinal wave is produced in this spring by applying horizontal varying force at one end of the spring. This force produces compressions and rarefactions in the spring as shown in the figure.

In this case the various parts of the spring vibrate along the length of the spring (or along the direction of motion of the waves. When spring was disturbed than all the suspension threads were vertical. But when the longitudinal wave is produced in the thread then these suspension threads are displaced. Their displacement is same as the displacement of corresponding parts of the spring. The graph of displacement of various parts of spring and corresponding values of the distances of these various parts of spring, from its one end is shown in the figure.

Q.3 What are the factors on which the speed of sound depends upon?

Answer

Speed of sound in air

Sound waves are longitudinal waves and their speed depends upon

- Compressibility (i.e. elasticity) of the medium.
- Inertia (i.e. density) of the medium.

If E be the modules of elasticity and ρ be the density of the medium, then the speed v can be expressed as,

$$V = \sqrt{\frac{E}{\rho}}$$

Speed of sound in solids is much greater than in gases

Reason

- Since molecules are closer in solids than in the gases, so they respond more quickly to the disturbance.
- In other words, so the speed of sound in gases is smaller than in solids because the gases are more compressible and thus have smaller modulus of elasticity.

Q.4 What was Newton's formula for the speed of sound? What was drawback in it, how it was corrected by Laplace?

Answer

Newton's formula for the speed of sound in air

If E be the modules of elasticity and ρ be the density of the medium, then the speed v is

$$V = \sqrt{\frac{E}{\rho}} \qquad \text{......... (1)}$$

Calculation of modules of elasticity

Newton's Assumption

In order to calculate the elastic modulus for air, Newton assumed that the temperature of the air during a compression remains constant. (i.e. an isothermal change)

So, $PV = \text{constant}$

When the pressure increases from P to P+ΔP then the volume decreases from V to

V - ΔV.

According to Boyle's Law

$$PV = (P+\Delta P)(V-\Delta V)$$

$$PV = P\text{-}P\Delta + \Delta PV\text{-}\Delta P\Delta V \qquad (2)$$

Since changes ΔP and ΔV represent the small. So their product ΔPΔV can be neglected. Hence above equation becomes.

$$PV = PV-P\Delta V+V\Delta P$$

Or $$0 = -P\Delta V+V\Delta P$$

$$P\Delta V = V\Delta P$$

Or $$P = \frac{V\Delta P}{\Delta V}$$

Or $$P = \frac{\Delta P}{\Delta V/v} \qquad \left[\text{Where } \frac{\Delta P}{\Delta V/v} = \frac{\text{volumetric stress}}{\text{volumetric strain}} = E\right]$$

Or $$P = E$$

So, equation (1) becomes

$$v = \sqrt{\frac{P}{\rho}}$$

At S.T.P, for air P = 0.76 mHg = 1.01 = 1.01 x10^5 N/m^2

$$v = \sqrt{\frac{1.01\times 10^5}{1.29}}$$

$$v = 280 \text{ m/sec}$$

The experimental value of speed of sound is 332 m/sec. The theoretical value is about 16% less than the experiment value.

Drawback in Newton's Formula

During a compression the temperature of the air increases i.e. it is an adiabatic change.

Laplace Correction

Laplace assumed that compressions and rarefactions in air take place so rapidly that heat of compression does not able to transfer to the neighboring cooler regions.

Therefore, the temperature of the medium does not remain constant, i.e. it is an adiabatic change.

In this case, Boyle's law can take the form

$$PV^{\gamma} = \text{constant} \qquad \text{...........(4)}$$

When the pressure increases from P to P+ΔP then the volume decreases from V to (V-ΔV), so

$$PV^{\gamma} = (P + \Delta P)(V - \Delta V)^{\gamma}$$

Where

$$\gamma = \frac{C_p}{C_v} = \frac{\text{Molar heat capacity at constant pressure}}{\text{Molar heat capacity at constant volume}}$$

$$PV^{\gamma} = (P + \Delta P)V^{\gamma}\left(1 - \frac{\Delta V}{V}\right)^{\gamma}$$

Or $$P = (P + \Delta P)\left(1 - \frac{\Delta V}{V}\right)^{\gamma}$$

By Binomial expansion

$$(1 + x)^n = 1 + nx + n\frac{(n-1)x^2}{2!} + \text{..........}$$

So $$P = (P + \Delta P)\left(1 - \gamma\frac{\Delta V}{V} + \text{neglecting square and higher power of } \frac{\Delta V}{V}\right)$$

$$P = P - \gamma P\frac{\Delta V}{V} + \Delta P - \gamma\frac{\Delta P \Delta V}{V}$$

Since ΔP and ΔV both are small so neglecting the term $\gamma\frac{\Delta P \Delta V}{V}$

So, $$P = P - \gamma P\frac{\Delta V}{V} + \Delta P$$

Or $$\Delta P = \gamma P\frac{\Delta P}{\Delta V / V}$$

$$\gamma P = \frac{\Delta P}{\Delta V / V} \qquad \left[\text{Where } \frac{\Delta P}{\Delta V / V} = \frac{\text{stress}}{\text{strain}} = E\right]$$

$$\gamma P = E \qquad (4)$$

So equation (4) becomes

$$V = \sqrt{\frac{\gamma P}{\rho}}$$

At S.T.P

P =0.76 mHg = 1.01×10^5 N/m^2, ρ=1.29 kg/m^3

Value of ρ is different for different gases. For air, ρ=1.41 (for diatomic gas)

$$V = \sqrt{\gamma}\sqrt{\frac{P}{\rho}}$$
$$V = \sqrt{1.41} \times 280$$
$$V = 333 \text{ m/sec}$$

Which is close to the experimental value of 332 m/sec.

Q.5 How the variation of pressure, density and temperature effect the speed of sound in a gas?

Answer

Effect on Speed of Sound in Air

1) Effect of Pressure

As $$v = \sqrt{\frac{\gamma P}{\rho}}$$

Since density is directly proportional to the pressure. When pressure of gas is increased, density of gas also increases, so the speed of sound remains same.

2) Effect of Density

At constant temperature and pressure of gases having same value of γ, the velocity is inversely proportional to the square root of their densities, which shows, smaller the density, greater the speed i.e.

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

$$v = \sqrt{\gamma P}\sqrt{\frac{1}{\rho}}$$

$$v = \text{constant}\sqrt{\frac{1}{\rho}}$$

$$v \propto \sqrt{\frac{1}{\rho}}$$

Note

The speed of sound in hydrogen is four times to its speed in oxygen because density of oxygen is sixteen times as that of hydrogen.

3) Effect of temperature

When a gas is heated at constant pressure then its volume is increased and density is decreased.

As $$v = \sqrt{\frac{\gamma P}{\rho}}$$

So, the speed of sound is increased with the rise in temperature.

Q.6 Prove that $v_t = v_0 + 0.61\,t$

Answer

Let

v_0 = Speed of sound at 0°C

v_t = Speed of sound at t°C

ρ_0 = density of gas at 0°C

ρ_t = density of gas at t°C

So

$$v_0 = \sqrt{\frac{\gamma P}{\rho_0}} \qquad (1)$$

$$v_t = \sqrt{\frac{\gamma P}{\rho_t}} \qquad (2)$$

Dividing the equation (2) by (1)

$$\frac{v_t}{v_0} = \sqrt{\frac{\gamma P / \rho}{\gamma P / \rho_0}}$$

$$\frac{v_t}{v_0} = \sqrt{\frac{\rho}{\rho_0}} \qquad \text{.........(3)}$$

If v_0 is the volume of gas at 0°C and V_t is the volume at t°C. Then

$$V_t = V_0\, [1+\beta t]$$

Where β is the coefficient of volume expansion. For all gases, its value is about $\frac{1}{273}$.

$$V_t = v_0\left[1+\frac{t}{273}\right]$$

As $\qquad V_0 = \frac{m}{\rho_0}$ and $v_t = \frac{m}{\rho_t} \qquad \left[\because \rho = \frac{m}{v}\right]$

So $\qquad \frac{m}{\rho_t} = \frac{m}{\rho_0}\left[1+\frac{t}{273}\right]$

Or $\qquad \frac{\rho_0}{\rho_t} = \left[1+\frac{t}{273}\right] \qquad \text{........(4)}$

Using equation (4) in equation (3), we have

$$\frac{v_0}{v_t} = \left[1+\frac{t}{273}\right] \qquad \text{.......(5)}$$

Or $\qquad \frac{v_0}{v_t} = \sqrt{\frac{t^\circ + 273}{0^\circ C273}}$

Or $\qquad \frac{v_0}{v_t} = \sqrt{\frac{T}{T_0}} \qquad \text{......(6)}$

Where 273 + t°C = T = absolute temperature corresponding to T°C

And 273+0°C = T_0 = Absolute temperature corresponding to 0°C

Thus, the speed of sound varies directly as the square root of absolute temperature.

Speed of sound in air at t°C

As $$\frac{v_0}{v_t} = \sqrt{1 + \frac{t}{273}}$$

$$\frac{v_0}{v_t} = \left[1 + \frac{t}{273}\right]^{\frac{1}{2}}$$

By using the Binomial expansion and neglecting square and higher powers, we have

$$\frac{v_0}{v_t} = \left[1 + \left(\frac{1}{2}\right)\left(\frac{t}{273}\right) + \ldots\ldots\ldots\right] \qquad \left[\because (1+x)^n = 1 + nx + \ldots\ldots\ldots\ldots\right]$$

So $$v_t = v_0 + \frac{v_0}{546}t, \qquad v_t = v_0 + \frac{332}{546}t \qquad [\text{as } v_0 = 332 \text{ m/sec}]$$

$$v_t = v_0 + 61t$$

This equation shows that with one-degree Celsius rise in temperature, the speed of sound is increased by approximately 61 cm/sec.

Q.7 State and explain on principle of super position?

Answer

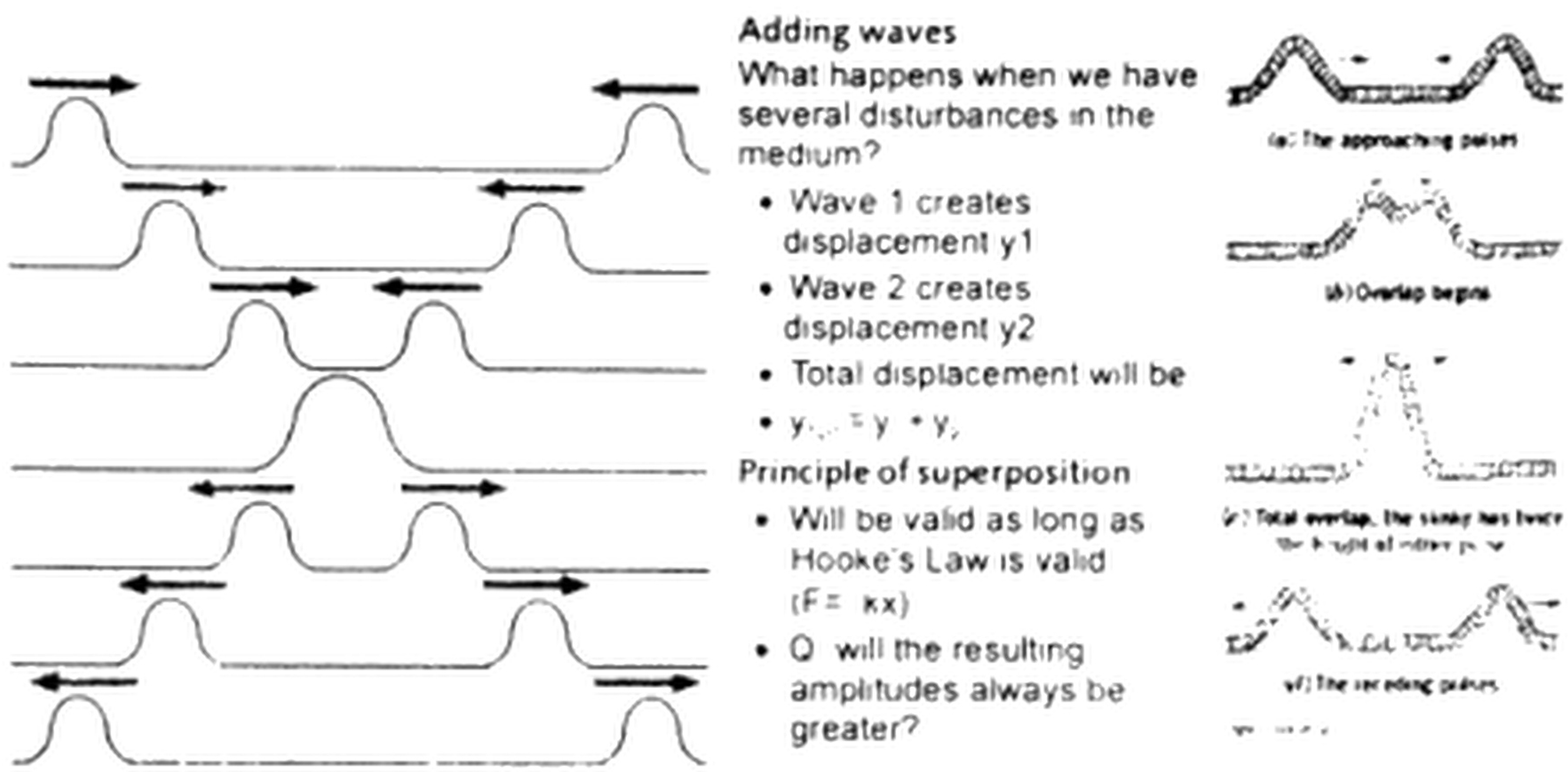
Superposition principle

If a particle of medium is simultaneously acted upon by the number of waves then the resultant displacement of the particle is the algebraic sum of their individual displacements. This is called superposition principle.

Let the displacement of the individual waves be $y_1, y_2, y_3 \ldots\ldots y_n$.

Then by super position principal the resultant displacement be

$$y = y_1 - y_2 - y_3 + \ldots\ldots\ldots + y_n.$$



Consider the two waves coming from opposite direction through a coil of spring as shown in figure. When these waves combine with each other than during the time of overlapping, the displacement of waves are added up as shown in figure(c). After having crossed each other they again adopt their original shapes and continue their motion along the spring in their respective directions as shown in figure (d), we can study three important cases of super position principal.

Cases of super position principal

i) When two waves having same frequency and travelling in the same direction produce the phenomena of interference.

ii) When two waves of slightly different frequencies and travelling in the same direction produce beats.

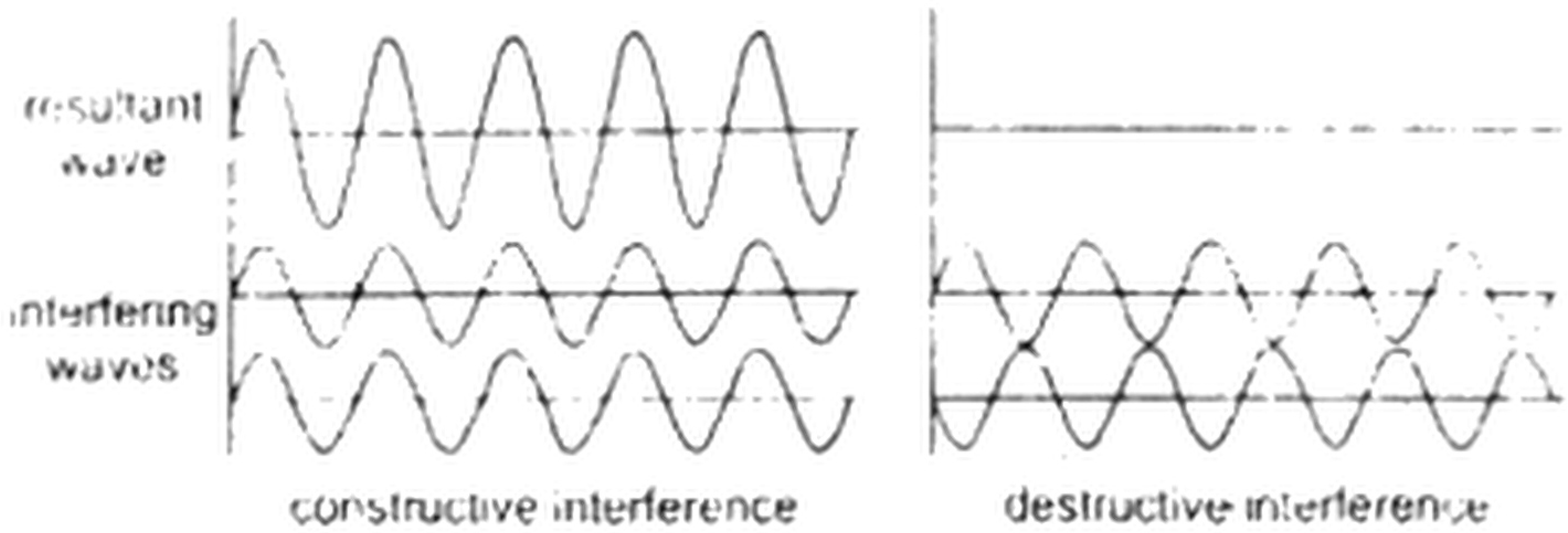
iii) When two waves of equal frequency travelling in opposite direction produce stationary waves.

Q.8 Define interference. Describe its types. Also write down the conditions for constructive and destructive interference?

Answer

Interference

When two identical waves meet each other in a medium then at some points they reinforce the effect of each other and at some points they cancel the effect of each other. This phenomenon is called interference.



Explanation

Consider an experiment arrangement shown in the figure. It consists of

- Two loud speakers S_1 and S_2 for the production of harmonic sound waves of fixed frequency.
- An audio generator.
- A microphone.

- A cathode ray oscilloscope (CRO) is attached to microphone to see the input signal wave form.

Types of interference

There are two types of interference

i) Constructive interference.

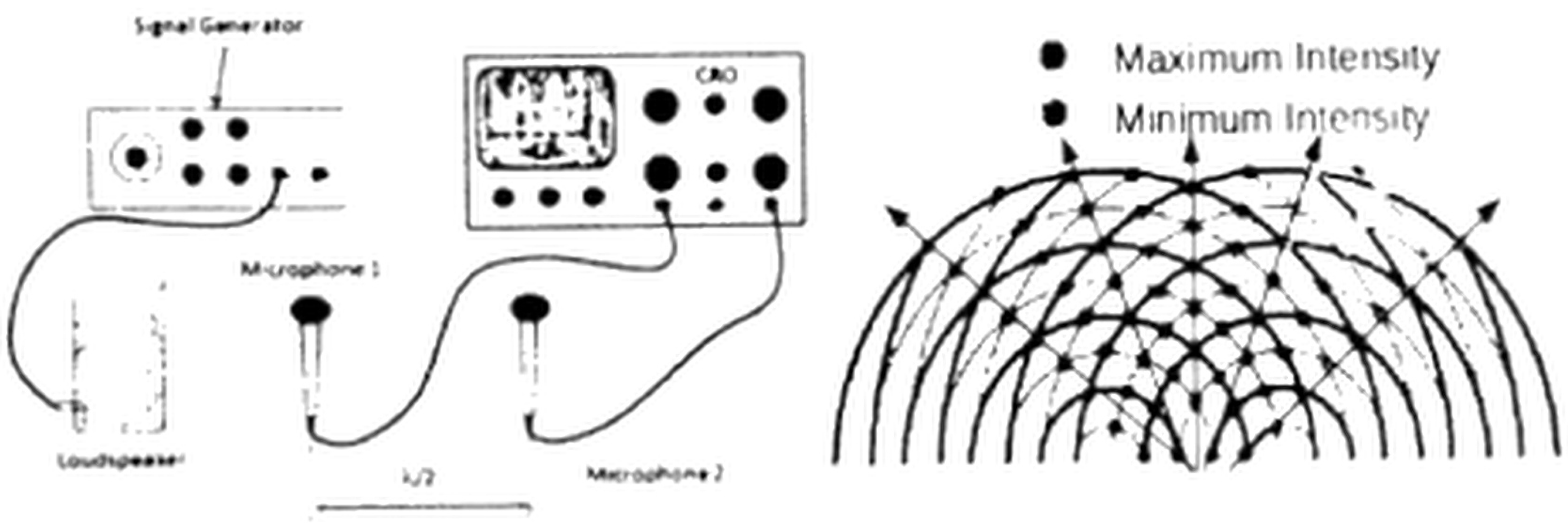
ii) Destructive interference.

Constructive interference

The microphone is placed at various points in front of loud speakers, as shown in figure.

At P_1, P_2 and P_3 a large signal is seen on CRO, as shown in figure. At these points' compression meets with compression and rarefaction meets with rarefaction.

So, the displacement of two wave are added up at these points and a large resultant displacement is produced.



We can find the path difference at point P_1 between two waves is

$$\Delta = S_2P_1 - S_1P_1$$

$$= 4\tfrac{1}{2}\lambda - 3\tfrac{1}{2}\lambda$$

$$= \lambda$$

Condition for constructive interference

Whenever path difference is an integral multiple of wave length displacements, the two waves are added up. This effect is called constructive interference.

$\Delta S = n\lambda$ Where $n = 0, \pm 1, \pm 2, \pm 3, \ldots\ldots$

Destructive Interference:

At P_2 and P_4, no signal is obtained on CRO, as shown in fig. At these points' compression meets with rarefaction and they cancel the effect of each other, so resultant displacement becomes zero. We can calculate the path difference between the waves.

At P_2

$$\Delta S = S_2P_2 - S_1P_2$$

$$= 4\lambda - 3\tfrac{1}{2}\lambda$$

$$= \tfrac{1}{2}\lambda$$

Similarly, at p_2

$$\Delta S = S_2P_4 - S_1P_4$$

$$= 3\tfrac{1}{2}\lambda - 4\lambda$$

$$= \tfrac{1}{2}\lambda$$

Condition for destructive interference

Whenever the path difference is odd integral multiple of half of wavelength, the displacements of two waves cancel the effect of each other. This effect is called destructive interference.

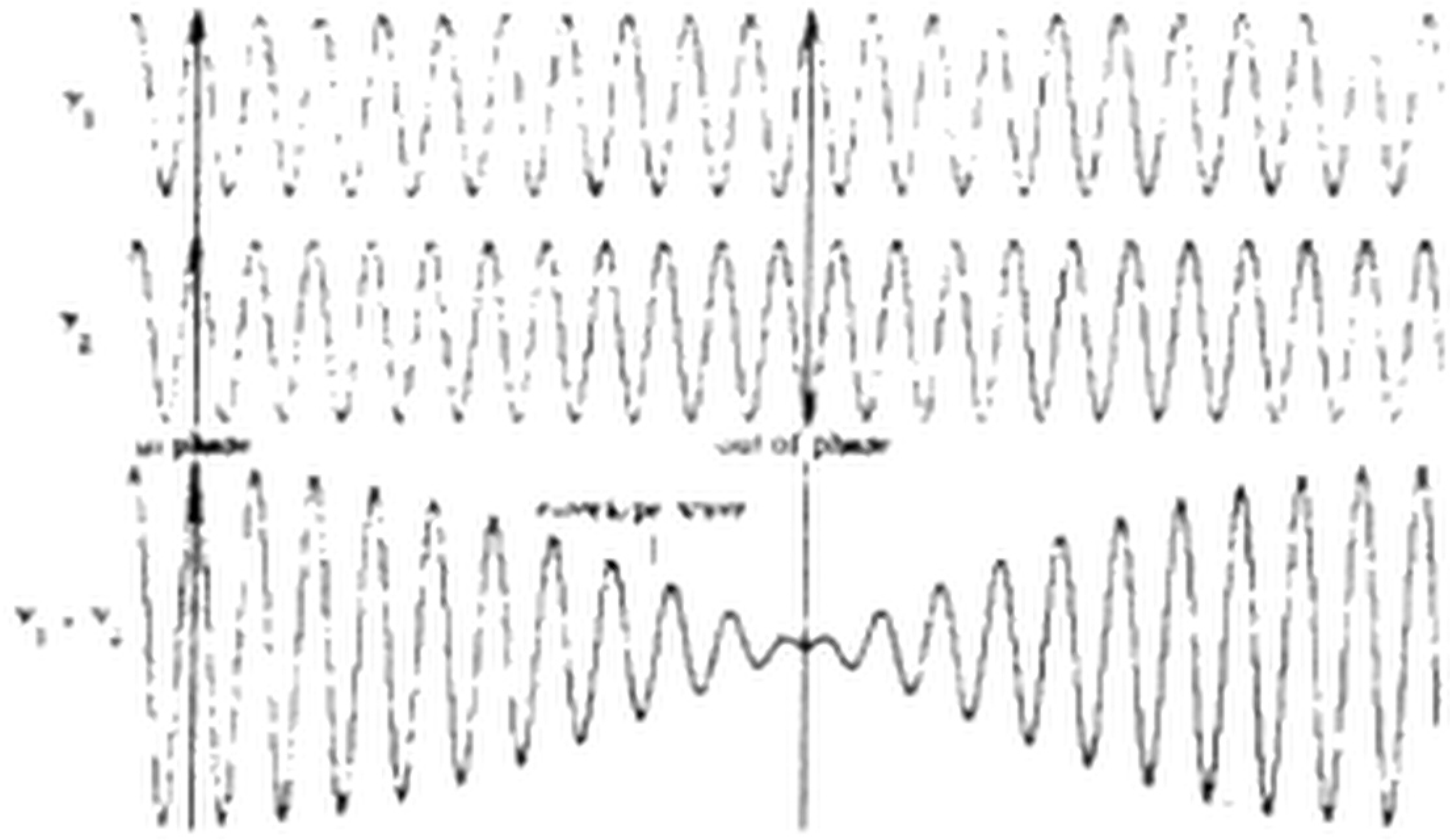
$\Delta S = (2n+1)\frac{\lambda}{2}$ were $n = 0, \pm 1, +2, \pm 3, \ldots\ldots\ldots$

Or $\Delta S = (n + \tfrac{1}{2})\lambda$

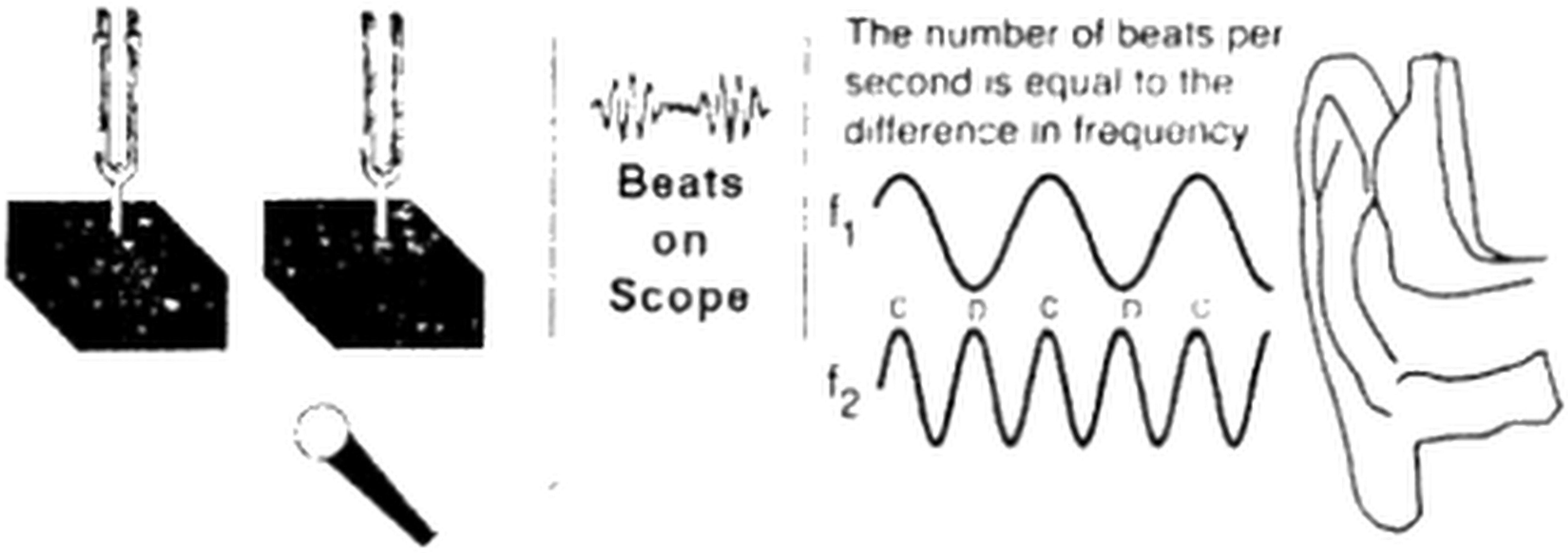
Q.9 What are beats? Explain it with the help of example as well as graphically? Also mention some uses of beats?

Answer

When two waves of slightly different frequencies and travelling in the same direction overlap to each other then there is a periodic variation of sound between maximum and minimum loudness are called beats.



Consider two tuning forks A and B of same frequency say 32 Hz are sounded separately then they will produce pure notes. But when they are sounded simultaneously then it is difficult to differentiate the notes. The sound waves of two will be superposed on each other and will be heard by the human ear as single pure notes. If the frequency of tuning fork B is lowered slightly by loading it with some wax, say it becomes 30 Hz.



Now if A and B sounded together, a sound of alternately increasing and decreasing intensity will be heard. Such a note is called beat, which is due to interference between the sound waves from A and B as shown in figure below. At some instant X the displacement of the two waves is in the same direction.

The resultant displacement is large and a loud sound is heard. After time $\frac{1}{4}$ sec, the displacement of wave due to one tuning fork is opposite to the displacement of waves due to the other tuning fork. As a result, a minimum displacement is produced at Y. So, a low or no sound is heard. After f14 sec, the displacements are again in the same direction and a loud sound is heard again at Z.

It represents a loud sound is heard two time in one second because the frequency of tuning fork.

Mathematically,

$$f_A - f_B = n \text{ (No. of beats)}$$

If the frequency difference is greater than 10 Hz, then it is difficult to recognize them. We can use beats to tune a string instrument or to find the unknown frequency.

Uses of beats:

1) Beats are used to tune a string instrument such as piano or violin, by beating a note against a note of known frequency. The string can then be adjusted to the desired frequency by tightening or loosening it until no beats are heard.

2) Beats are used to find unknown frequency of vibrating body.

3) Beats are used to produce variety in music.

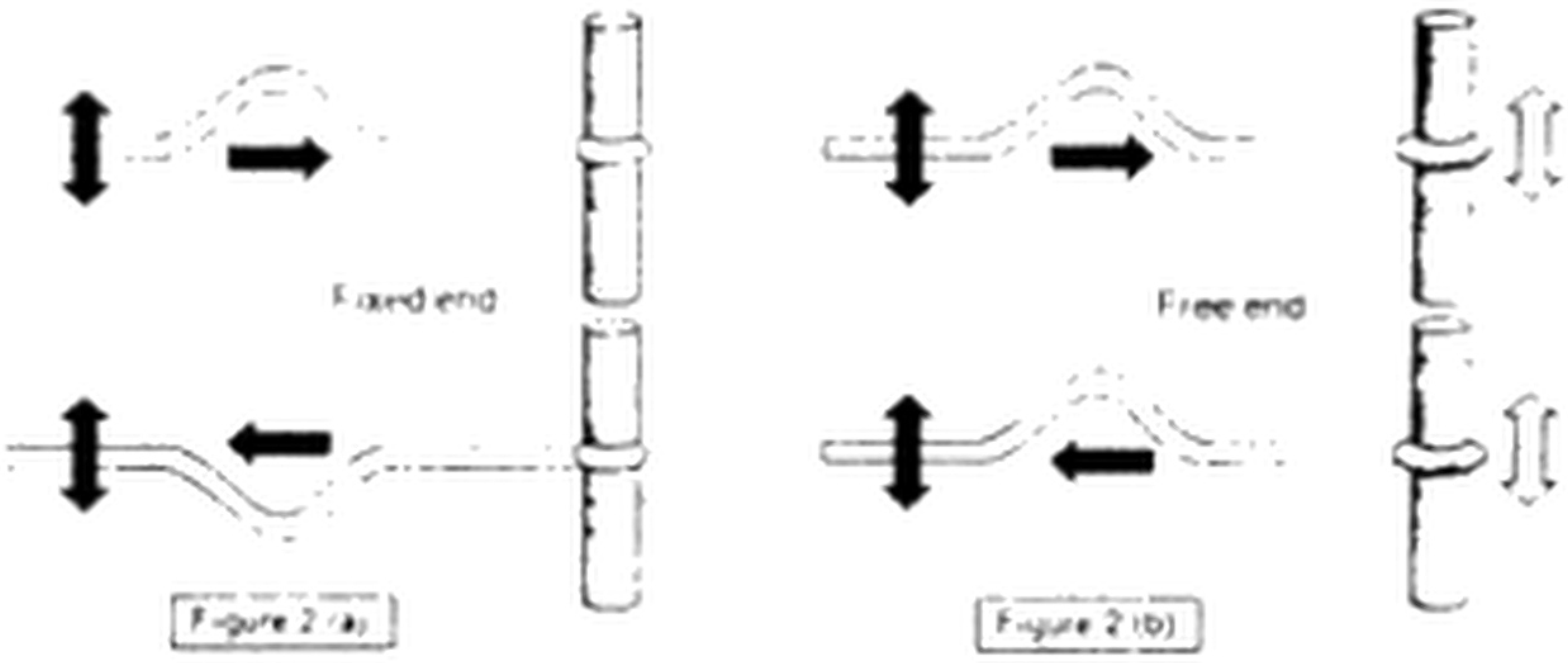
Q.10 Explain the reflection of waves from rare and denser mediums?

Answer

Reflection of Waves

"The bouncing back of wave from the boundary of a medium is called reflection of wave."

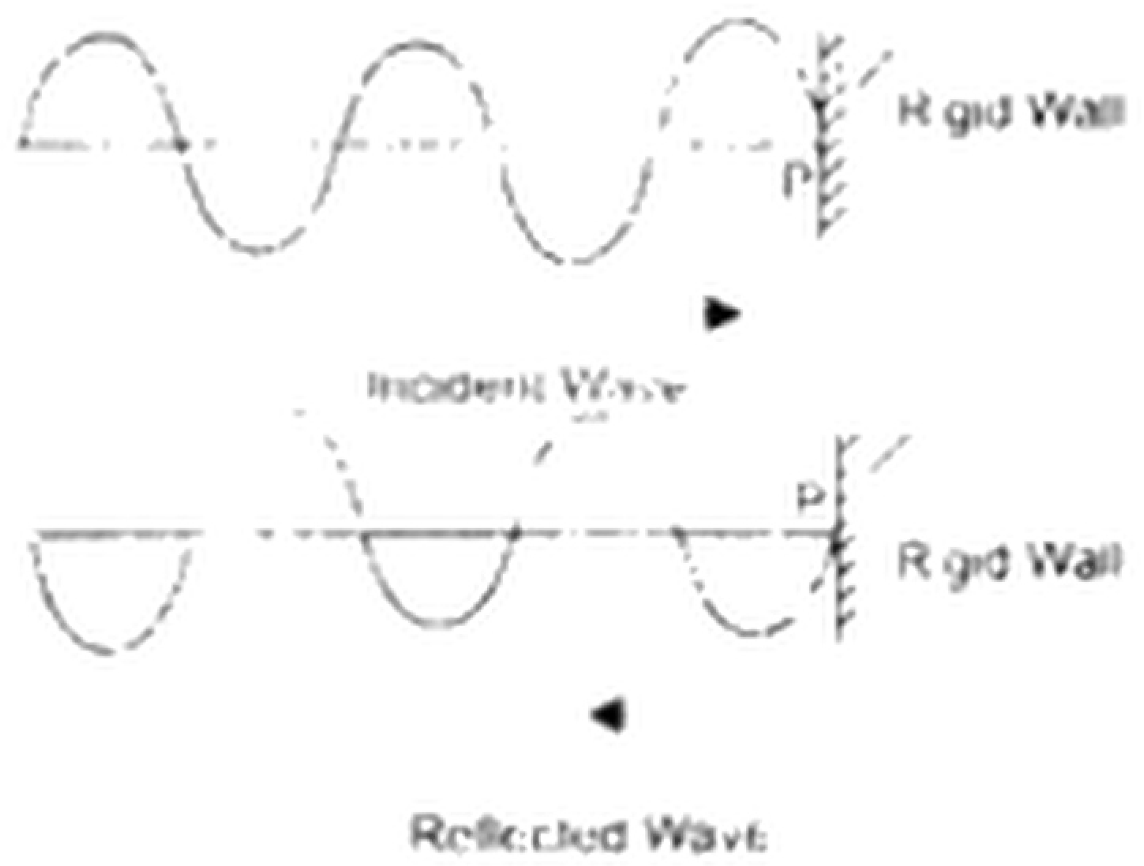
When a wave produced in one medium travel to the boundary to enter into another medium, then a part of incident wave is reflected from the boundary. This reflected part has same frequency and wavelength as the incident wave has.



But there may be the change of phase which depends upon the nature of boundary of medium.

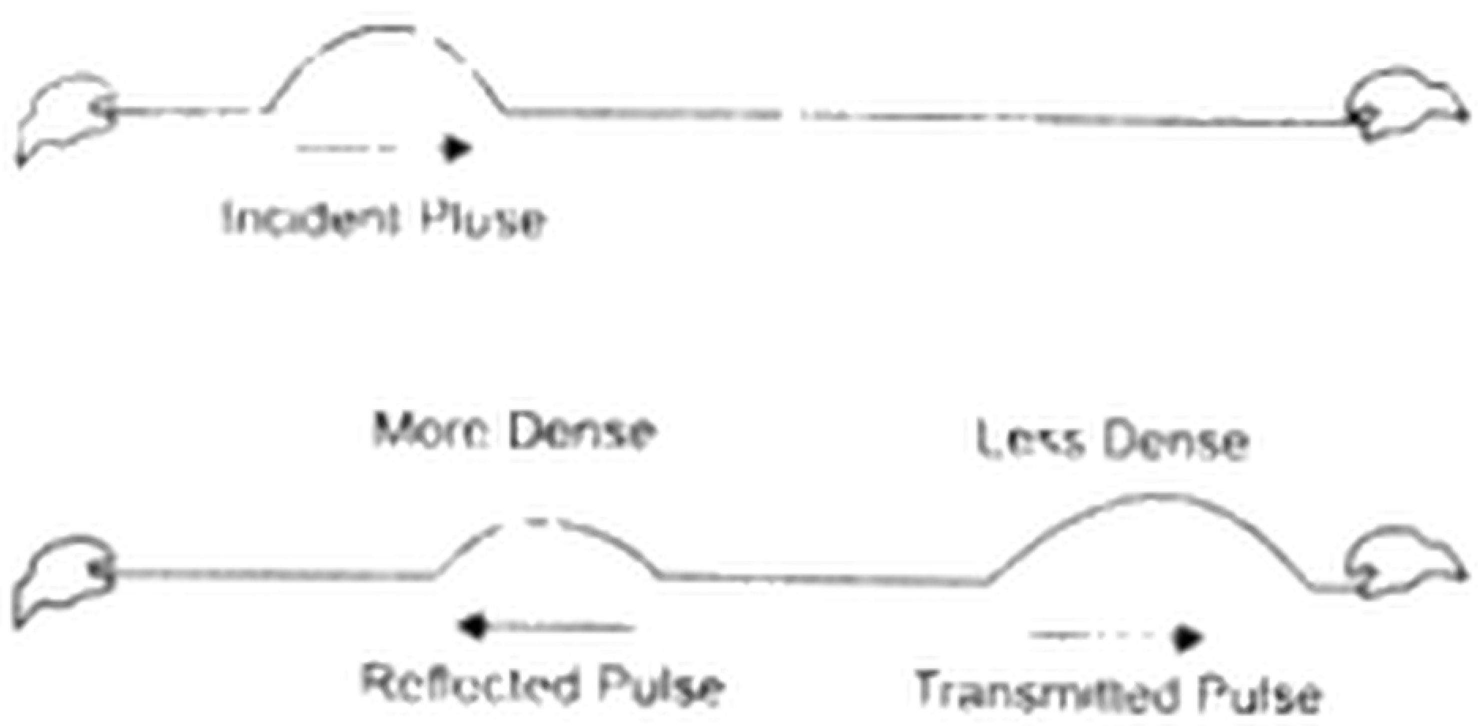
Reflection of waves from the boundary of denser medium:

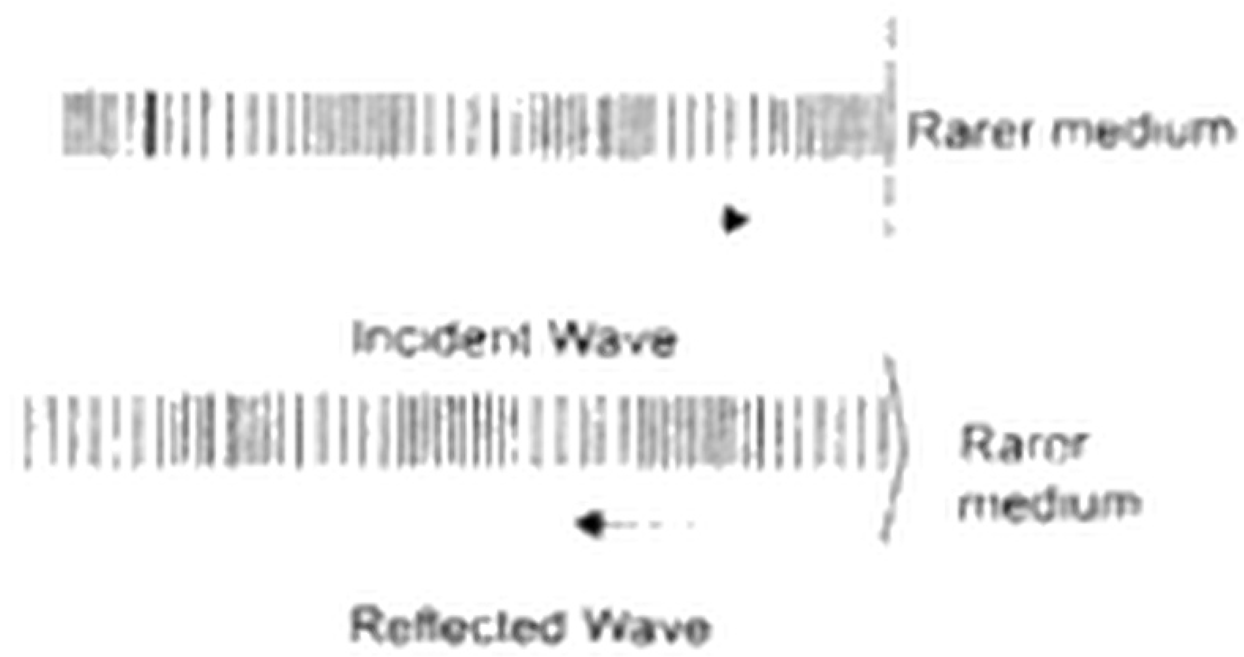
Let us take a long slinky spring whose one end is fixed to a rigid support on a horizontal surface of a table. The other end of this spring is free to oscillate. A sharp jerk is given to which crest or trough is produced on it. It travels on the spring from it send A towards the end B. On reaching at end B spring exerts a force on the rigid support to produce similar motion in it. But the rigid body has large density so it exerts equal and opposite reaction on the spring. Due to which crest is converted into trough and it travels back from end B to A.



Reflection of waves from the boundary of a rare medium:

If we attach end B of long slinky spring with a light string and keep its end A free, like before. Then giving a sudden jerk to end A, a crest is produced which travels on the spring from its end A, a crest is produced which travels on the spring from its end A to end B when the crest produced reaches the boundary of string. Then string being rare medium, do not give reaction to the spring. So that a crest is reflected back as a crest on the spring from end B to end A.





Result:

i) If a transverse wave travelling in a rarer medium is incident on a denser medium, it is reflected such that it undergoes a phase change of 180° (path difference of $\lambda/2$).

ii) If a transverse wave travelling in a denser medium in incident on a rarer medium, it is reflected without any change in phase (no path difference).

Q.11 Differentiate between echo and reverberation?

Answer

Difference between echo and reverberation

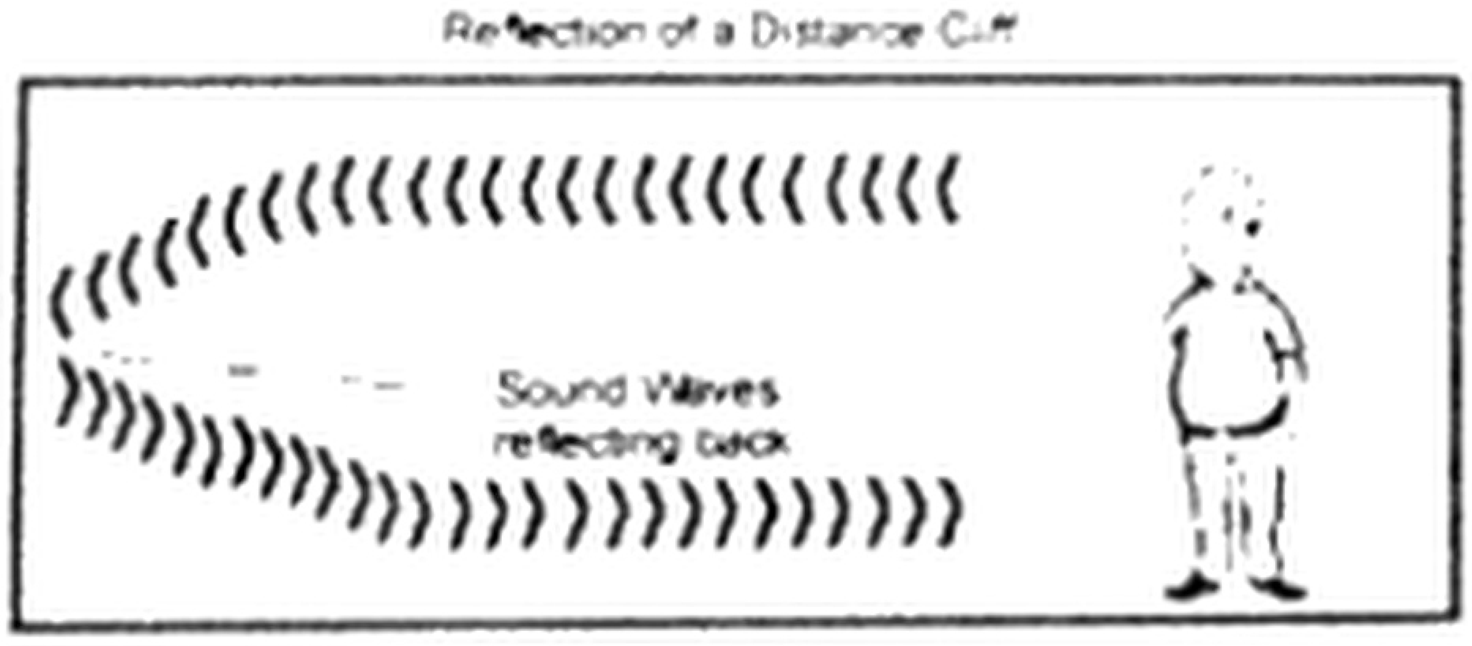
1) Echo is a single reflection of a sound wave off a surface. Reverberation is the sound or the pattern created by the superposition of such echoes.

2) An echo can be heard only when the distance between the source of sound and the reflecting body is at least 17 m. A reverberation can occur when sound wave is reflected by a nearby wall also.

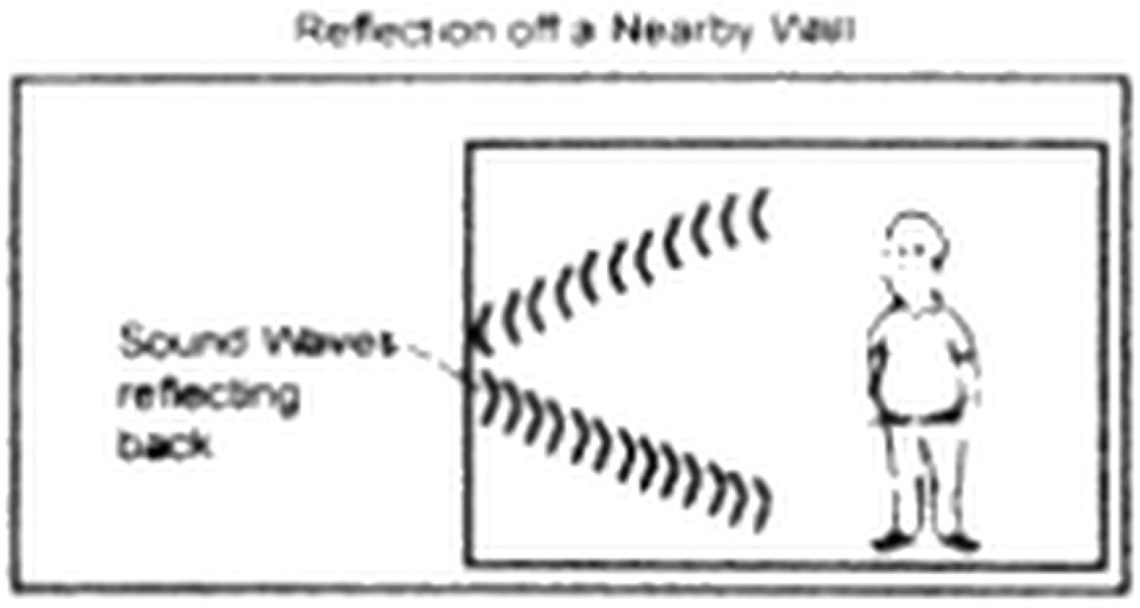
3) An echo is usually clear and can be clearly distinguished. A reverb is not a clear replica of the original sound sample.

4) Echo can be used to determine the distance of a reflecting object such as a large building or a mountain, if the ambient temperature is known. Reverberation cannot be utilized for distance measurement applications.

5) An echo can be heard both in open and closed spaces. Reverberation is usually experienced in closed spaces with multiple reflecting objects.



ECHO



Reverberation

Q.12 Define stationary waves and how they are produced? Give properties of stationary waves. Also define node and anti-node.

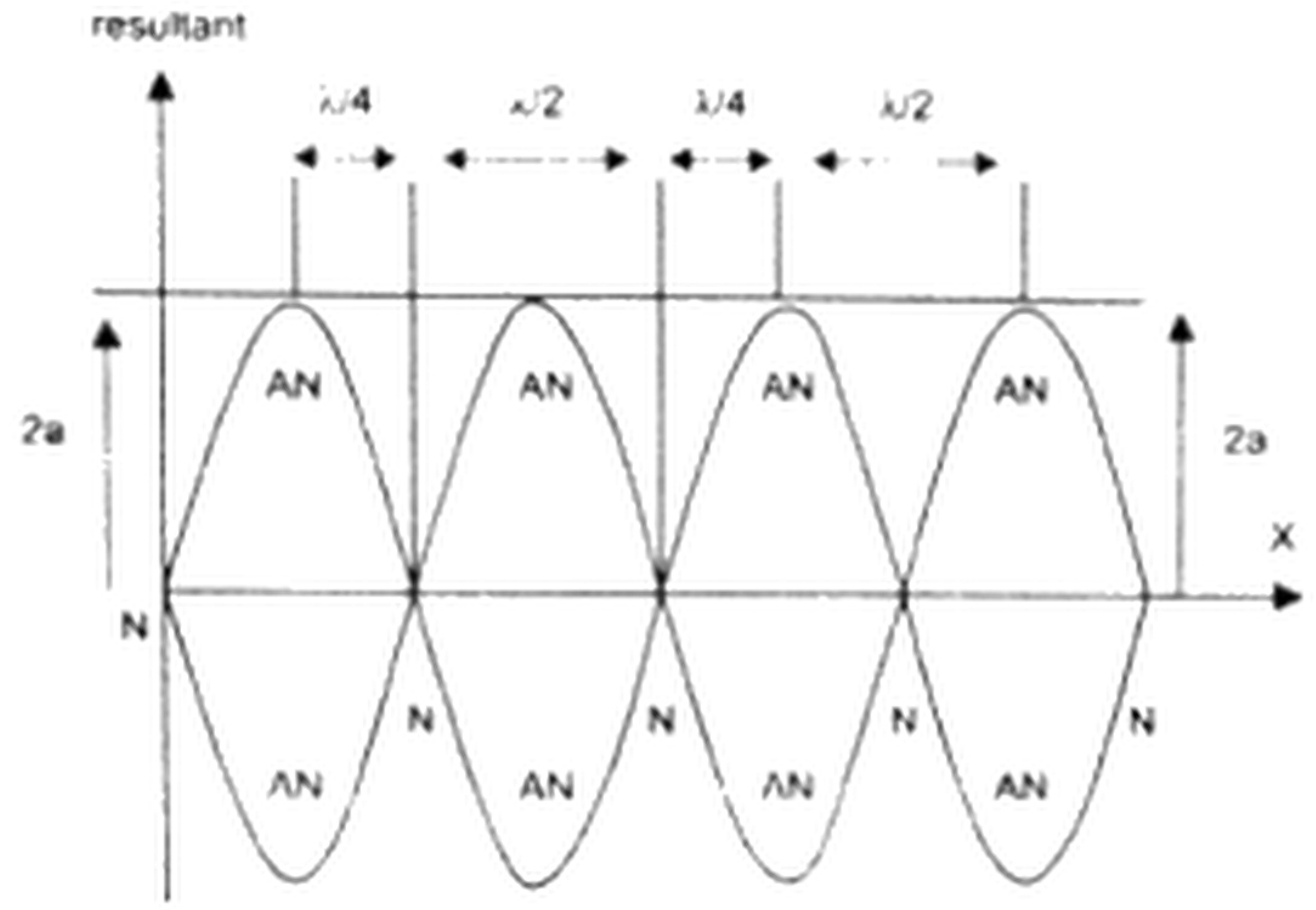
Answer

Stationary Wave

The resultant wave produced by the superposition two identical waves travelling along same line but in opposite direction is called stationary wave.

Production of Stationary Waves

Consider the superposition of two waves moving along a straight line along a string in opposite direction. The picture of such two waves at instants t=0, T/2, T/4, 3T/4 and T, as shown in figure (a) and (b).



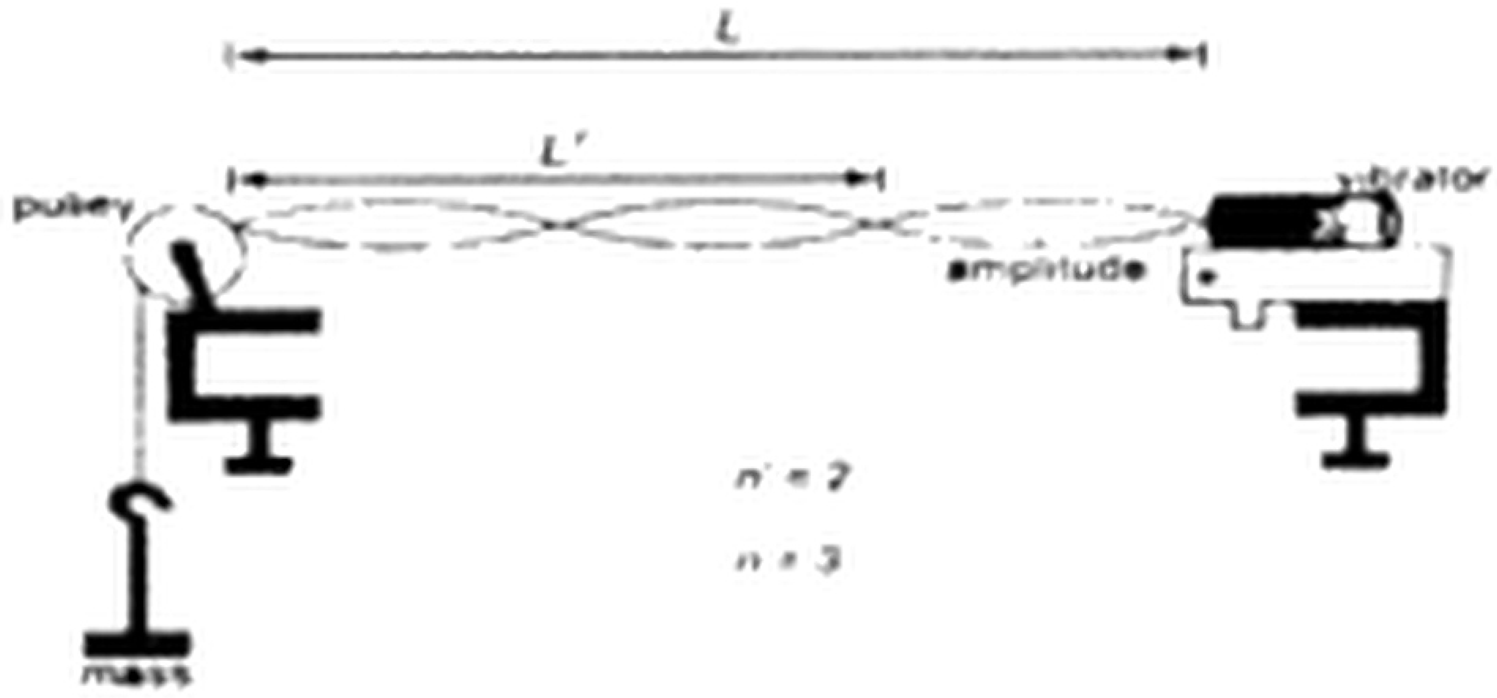
When two waves superpose to each other, we want to find out the displacements of the points 1, 2, 3, 4, 5, 6 and 7 shown in fig. It is clear that points 1, 2, 3 etc. are at a distance λ/4 apart from each other. The resultant displacements at these are calculated by applying the superposition principle.

Fig (c) shows that the resultant displacement of the point 1, 3, 5, 7 at the instants t =0, T/4, T/2, 3T/4 and T. It can be seen that the resultant displacement is always zero at all the instants.

Fig (d) shows the resultant displacement of points 2, 4 and 6 at instant t=0, T/4, T/2, 3T/4 and T. These points move with maximum displacement from mean positions.

Properties of Stationary waves

1) There are points of medium in stationary waves which permanently show zero displacement are called nodes.



2) The points between two successive nodes are in phase with each other.

3) Each point along the stationary waves vibrates with different amplitudes.

4) There are points of medium in stationary waves which have maximum amplitude are called antinodes.

5) The distance between two consecutive nodes is $\lambda/2$.

6) The distance between one node and next antinode is $\lambda/4$.

7) The energy remains standing in the medium between nodes because the nodes remain at rest, so energy cannot flow through these points. That is why stationary waves are also called standing waves.

8) Energy oscillates between P.E. and K.E. between nodes.

Note

- When antinodes are at their extreme positions the whole energy is P.E. while at passing through equilibrium position, the whole energy is K.E.
- Commonly the standing waves are produced due to superposition of waves traveling down a string with its reflection travelling in opposite direction.

Nodes

The points of zero displacement on stationary waves are called node points.

Antinodes

The points of maximum displacement on stationary waves are called antinodes.

Q.13 Show that frequencies of stationary waves in stretched string are quantized.

OR

Prove that for stationary waves in a stretched string $f_n=nf_1$.

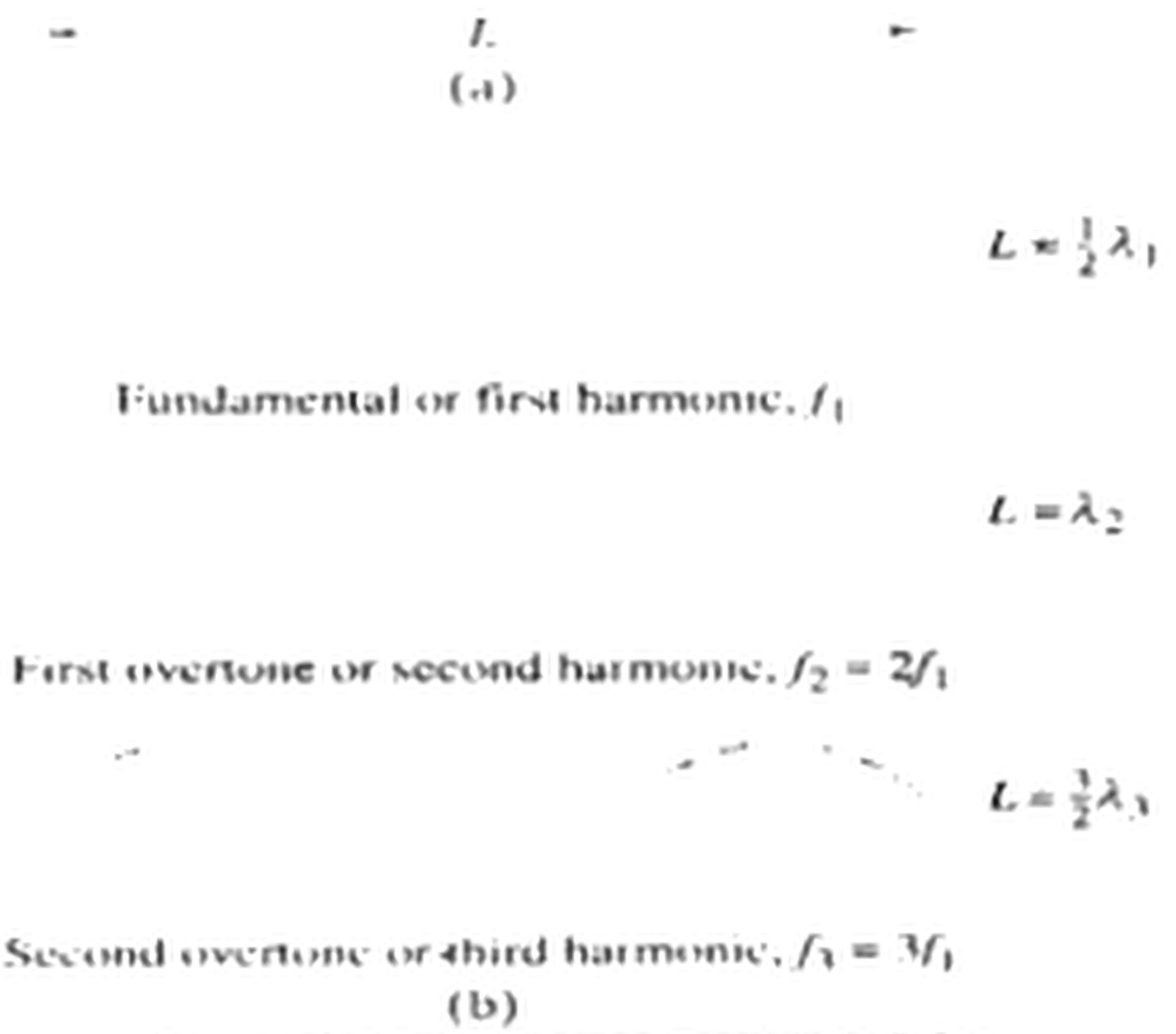
Answer

Stationary Waves in Stretched String

Let us consider a string of length l. It is stretched and is clamped at its two ends. The tension in the string is denoted by F.

When the string is plucked and then released, two waves are generated which moves in opposite direction along the string. Both of these are reflected back from the clamped ends of string with opposite phase to generate stationary waves on the string.

As the two ends are clamped with rigid support, so these do not vibrate and we get nodes at these ends.



Speed of waves on string

The speed of wave depends upon tension F in the string and mass per unit length m (i.e. thickness and nature of wire).

$$V = \sqrt{\frac{F}{m}} \qquad \text{..........(1)}$$

First mode of vibration

When the string is plucked at the middle of its length then the string vibrates in a single lop as shown in figure. Such a mode is called fundamental mode of vibration.

Distance between two consecutive nodes = $\frac{\lambda}{2}$

If λ_1, be the wave length and f_1 be the frequency of vibration in this mode, then

$$l = \frac{\lambda_1}{2} \qquad \text{or} \qquad \lambda = 2l$$

Thus, speed of wave v is

$$v = f_1\lambda_1 \qquad \text{or} \qquad f_1 = \frac{v}{\lambda_1}$$

Putting value of λ_1, we get

$$f_1 = \frac{v}{2l} \qquad \text{.........} (3)$$

Putting value of v from equation (1) in equation (3), we get

$$f_1 = \frac{1}{2l}\sqrt{\frac{F}{m}} \qquad \text{...........}(4)$$

Second Mode of Vibration

When the string is plucked from one quarter of its length then the string vibrates into two loops as shown in figure. If λ_2 be the wave length and f_2 be the frequency of vibration in this mode, then

$$l = \frac{\lambda_2}{2} + \frac{\lambda_2}{2}$$

$$l = \lambda_2$$

Or $$\lambda_2 = l$$

Thus, speed of wave v is

$$v = f_2\lambda_2$$

$$f_2 = \frac{v}{\lambda_2}$$

Putting value of λ_2, we get

$$f_2 = \frac{v}{l}$$

$$f_2 = \frac{2v}{2l}$$

$$f_2 = 2\left(\frac{v}{2l}\right)$$

So $\boxed{f_2 = 2f_1}$ $\left[\text{Since } \frac{v}{2l} = f_1\right]$

Thus, when the string vibrates in two loops, its frequency is double than when it vibrates in one loop. f_2 is called second harmonic.

Third mode of vibration

When the string is plucked from one sixth (1/6) of its length then the string vibrates into three loops as shown in figure. If λ_3 be the wave length and f_3 be the frequency of vibration in this mode, then

$$l = \frac{\lambda_3}{2} + \frac{\lambda_3}{2} + \frac{\lambda_3}{2}$$
$$l = \frac{3\lambda_3}{2}$$
$$\lambda_3 = \frac{2l}{3}$$

So, the speed becomes,

$$v = f_3\lambda_3$$
$$f_3 = \frac{v}{\lambda_3}$$
$$f_3 = \frac{3v}{2l}$$
$$f_3 = 3\frac{v}{2l}$$
$$f_3 = 3f_1 \quad \left[\text{since } \frac{v}{2l} = f_1\right]$$

The frequency f_3 is called third harmonic.

n_{th} mode of vibration

If string vibrates in n loops then,

$$f_n = n\left(\frac{v}{2l}\right) = nf_1$$

And wavelength is

$$\lambda_n = \frac{2l}{n} \qquad \text{where } n=1, 2, 3, 4, 5\ldots\ldots\ldots$$

So the stationary wave have a discrete set of frequencies f_1, $2f_1$, $3f_1$,..........nf_1, which is known as harmonic series. The frequency f_1 is known as fundamental frequency, and the other are called over tone.

Note

The stationary waves can be set up in the string only with the frequencies of harmonic series determined by the tension, length and mass per unit length of the string. Waves not in harmonic series are quickly damped out.

Q.14 How can we change the frequency of string on a musical instrument? Also discuss resonance of air column in resonance tube?

Answer

The frequency of a string on a musical instrument can be changed either by varying the

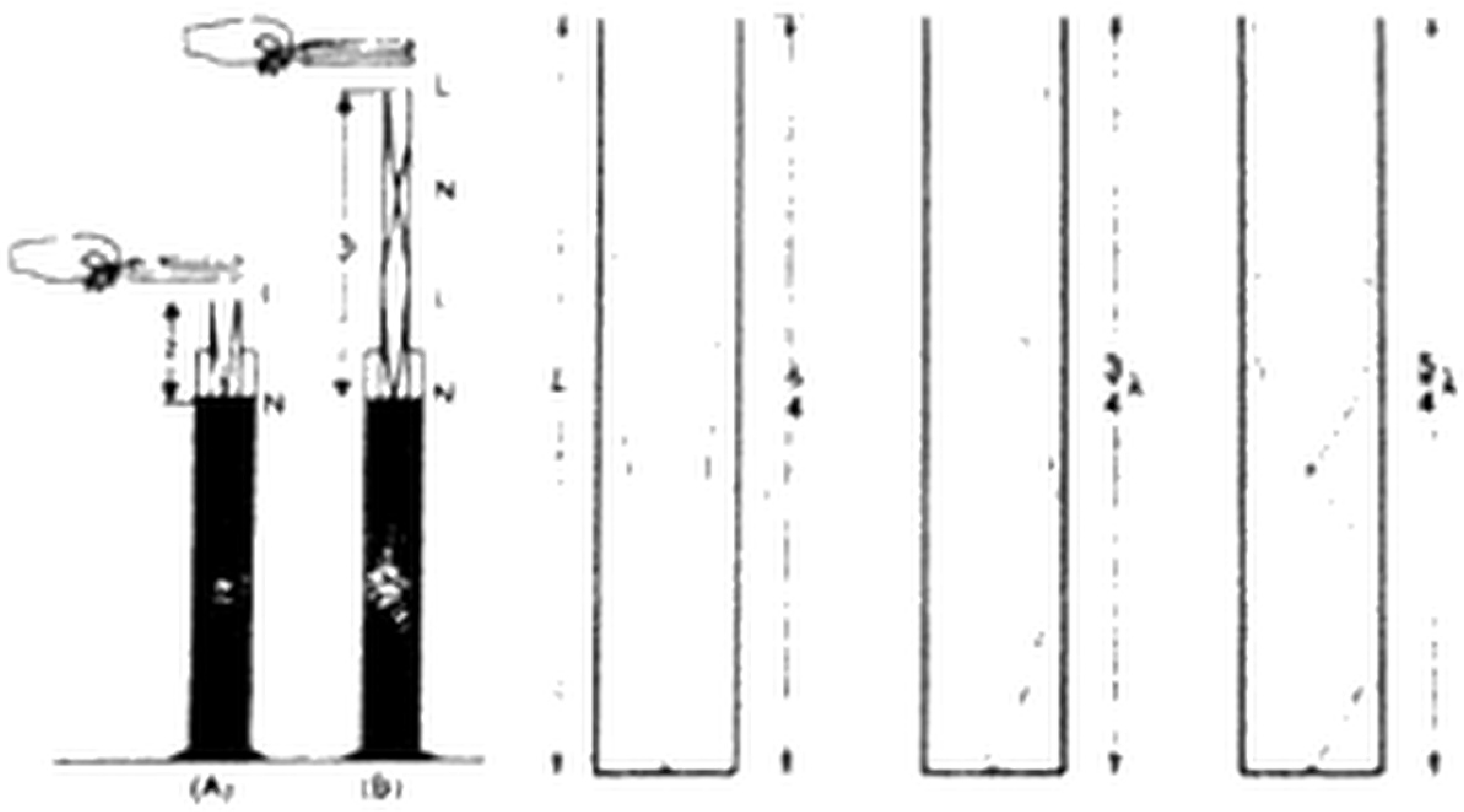
- Tension in string
- Length of string

For example

The tension in guitar and violin strings is varied by tightening the pegs on the neck of the instrument. Once the instrument is tuned, the musicians vary the frequency by moving their fingers along the neck. By doing so that changes the length of the vibrating portion of the string.

Resonance of air column in resonance tube:

Vibration of air column can be set up in a resonance column apparatus. It consists of a long metal tube held vertically in a tall jar containing the water. The tube can be fixed in vertical position. The length of the air column can be varied by raising or lowering the tube.



Here, the surface of water will act as the closed end. When a vibrated tuning fork is held above the open end, longitudinal waves are sent down the air column. These waves are reflected at the water surface and thus produce standing waves. Nodes are produced at the water surface and antinodes are produced at the open end.

When the frequency of waves in the air column becomes equal to the natural frequency of tuning fork, a loud sound is produced in the air column. It is the condition for resonance. It occurs only when the length of air column is proportional to one-fourth of the wavelength of sound waves having frequency equal to frequency of tuning fork.

Q.15 Find the frequencies produced in organ pipe when it

i) Open at both ends.

ii) Closed at one end.

Answer

Stationary Waves in Air Column

Stationary waves can be set up in air column inside a pipe or tube.

A common example vibrating air column is an organ pipe.

Organ Pipe

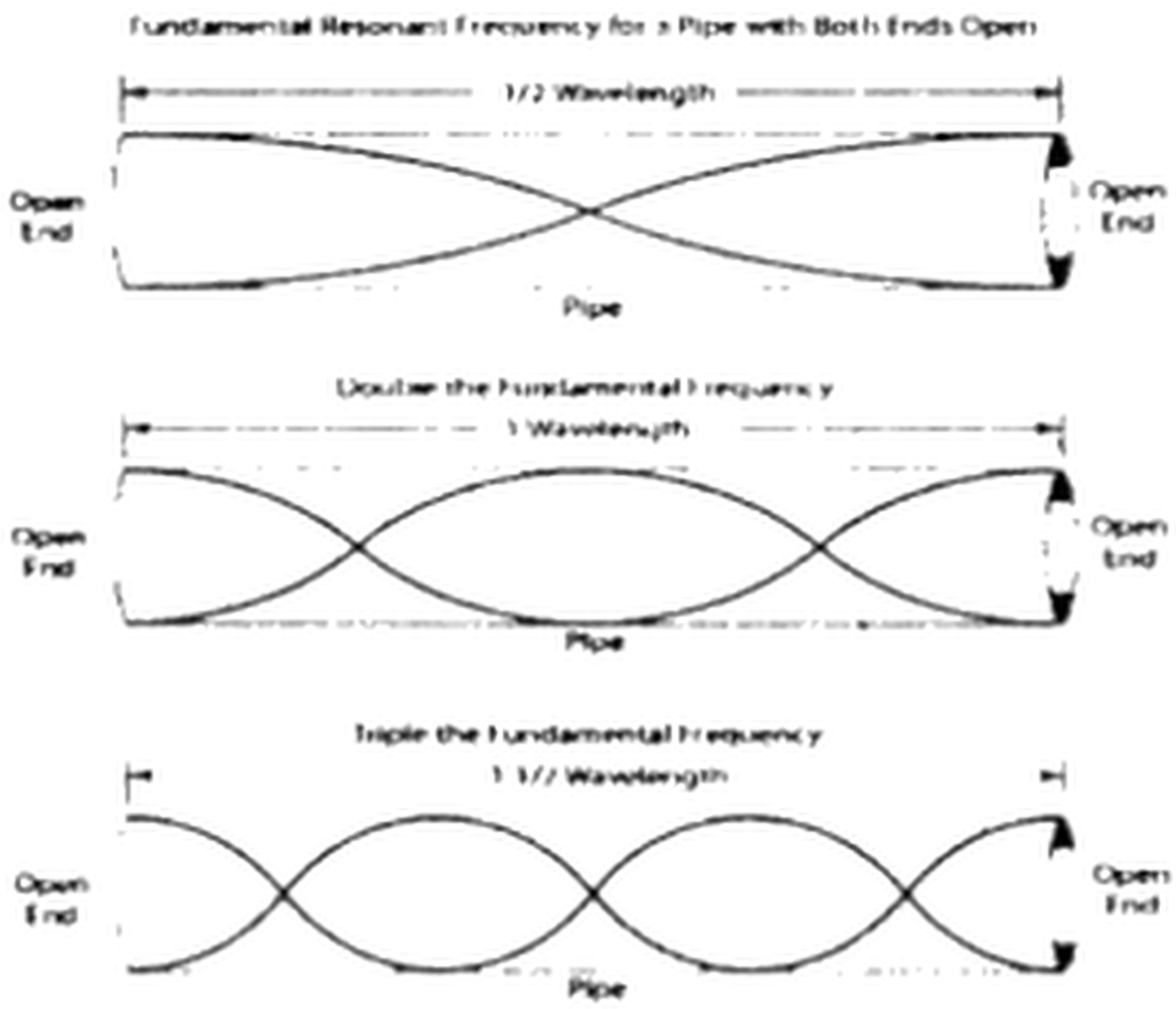
An organ pipe is a wind instrument in which sound is produced, due to setting up of stationary waves in air column. It consists of a hollow long tube with both end open or with one end open and the other closed. There are two types of organ pipes.

i) Open Organ pipe: It is that organ pipe whose both ends are open.

ii) Closed Organ pipe: It is that organ pipe whose one end is closed.

Modes of vibrations in organ pipe open at both ends

Let us consider an organ pipe of length *l* which is open at both ends. As the open ends air molecules have complete freedom of motion so it acts as antinode. Longitudinal waves set up inside, the pipe have been represented by transverse curves which represent the displacement and amplitude variations of air at various points.



Fundamental Mode of Vibration:

In this case there is only one node at the middle of the pipe. As both ends of pipe are pen, so there are two antinodes at both the ends. If λ is the wavelength of sound,

$$l = \frac{\lambda_1}{4} + \frac{\lambda_1}{4}$$

$$l = \frac{\lambda_1}{2}$$

If f_1 is the frequency of sound, then the velocity of sound is

$$v = f_1 \lambda_1$$

$$f_1 = \frac{v}{\lambda_1}$$

Putting value of λ_1 , we get

$$f_1 = \frac{v}{2l}$$

This frequency is called fundamental frequency for first harmonic.

Second mode of vibration:

In this case, there are three antinodes and two nodes.

If λ_2 is the wavelength of sound then

$$l = \frac{\lambda_2}{4} + \frac{\lambda_2}{2} + \frac{\lambda_2}{4}$$

$$l = \frac{(1+2+1)}{***}\lambda \quad ***$$

Or $$\lambda_2 = l$$

If f_2 is the frequency of sound, then speed becomes,

$$v = f_2\lambda_2$$

$$f_2 = \frac{v}{\lambda_2}$$

Putting value of λ_2, We get

$$f_2 = \frac{v}{l}$$

Or $$f_2 = \frac{2v}{2l}$$

Or $$f_2 = 2\left(\frac{v}{2l}\right)$$

Or $$f_2 = 2f_1 \qquad \left[\text{since } \frac{v}{2l} = f_1\right]$$

Third mode of vibration

For three loops, there are four antinodes and three nodes. If λ_3 is the wavelength of sound, the length of the pipe is

$$l = \frac{\lambda_3}{4} + \frac{\lambda_3}{2} + \frac{\lambda_3}{2} + \frac{\lambda_3}{4}$$

$$l = \left(\frac{1+2+2+1}{4}\right)\lambda_3$$

$$l = \frac{2\lambda_3}{3}$$

So the speed becomes

$$v = f_3\lambda_3$$

$$f_3 = \frac{v}{\lambda_3}$$

$$f_3 = \frac{3v}{2l}$$

$$f_3 = 3\left(\frac{v}{2l}\right)$$

$$f_3 = 3f_1 \qquad \left[\text{since } \frac{v}{2l} = f_1\right]$$

The frequency f_3 is called third harmonic.

nth mode of vibration

If air column vibrates in n loops then,

$$f_n = n\left(\frac{v}{2l}\right) = nf_1$$

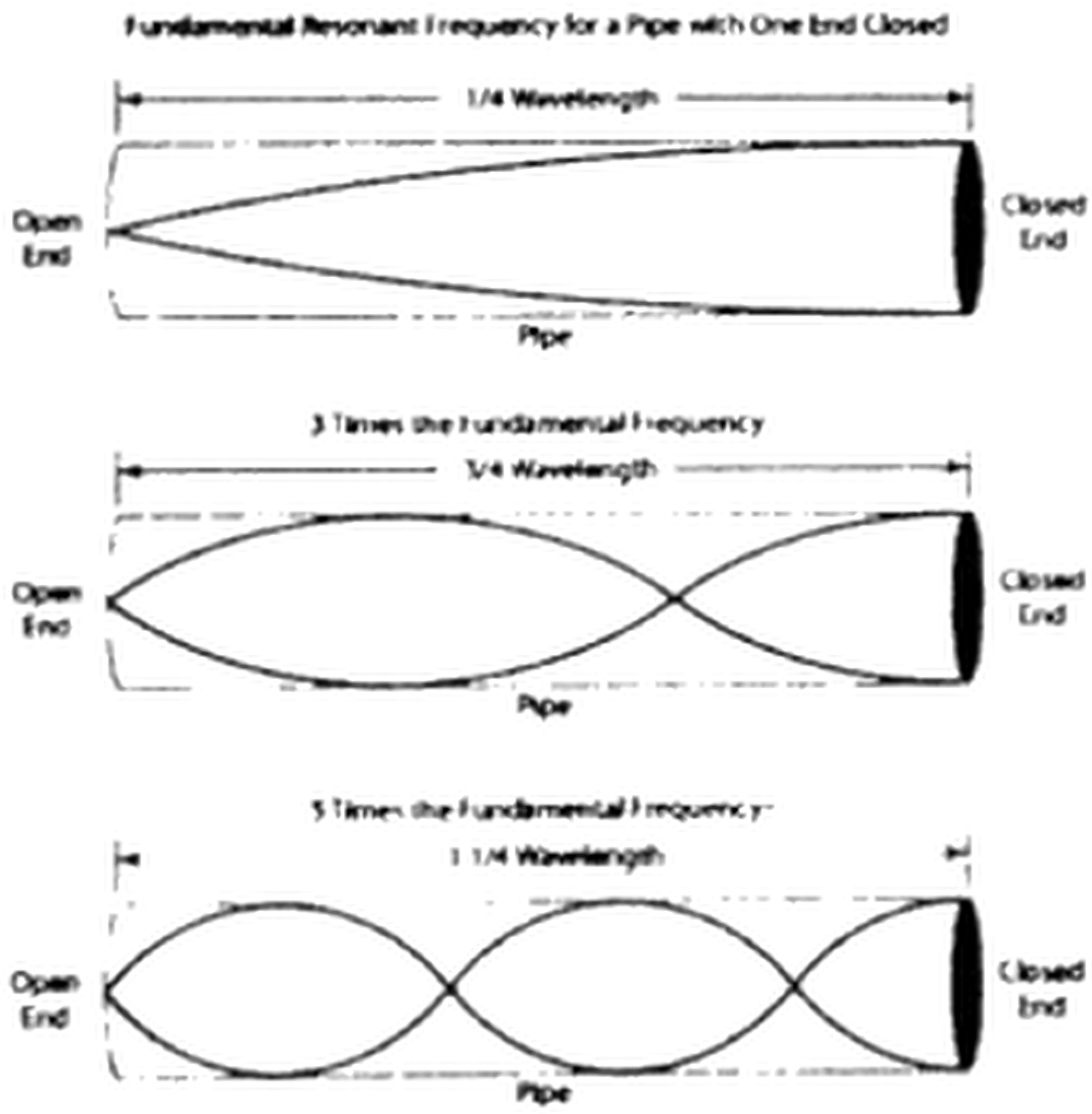
And wavelength is

$$\lambda_3 = \frac{2l}{n} \qquad \text{where } n = 1, 2, 3, 4,\ldots\ldots\ldots\ldots$$

So, the longitudinal stationary waves have a discrete setoff frequencies f_1, $2f_1$, $3f_1$..........nf_1, which is known as harmonic series. The frequency f_1 is known as fundamental frequency. And the other are called over tone.

Modes of vibration in organ pipe closed at one end

Let us consider an organ pipe of length *l* which is closed at one end. Then at the closed end we get node while at the open end we get anti-node.



Fundamental mode of vibration:

Fundamental mode of vibration has one node and one antipode. If λ_1 is the wavelength of fundamental mode then length of the pipe is:

$$l = \frac{\lambda_1}{4}$$

$$\boxed{\lambda_1 = 4l}$$

So the speed becomes,

$$v = f_1\lambda_1$$

Or $\quad f_1 = \frac{v}{\lambda_1} \quad$ as $\lambda_1 = 4l$

$$\boxed{f_1 = \frac{v}{4l}}$$

The frequency f_1 is called fundamental frequency.

Second Mode of Vibration

Second mode of vibration contains two nodes and two anti-nodes. If λ_3 is the wavelength, then length of the pipe is

$$l = \frac{\lambda_3}{4} + \frac{\lambda_3}{2}$$
$$l = \left(\frac{1+2}{4}\right)\lambda_3$$
$$l = \frac{3\lambda_3}{4}$$

Or $$\lambda_3 = \frac{4l}{3}$$

If f_3 is the frequency of sound, then speed becomes,

$$v = f_3\lambda_3$$

Or $$f_3 = \frac{v}{\lambda_3}$$

Putting value of λ_3, we get

$$f_3 = \frac{v}{4l/3}$$

$$f_3 = \frac{3v}{4l}$$

$$f_3 = 3\left(\frac{v}{4l}\right)$$

$$\boxed{f_3 = 3f_1} \qquad \left[\text{since } \frac{v}{4l} = f_1\right]$$

This is called second harmonic or first overtone.

Third Mode of vibration

Third mode of vibration contains three nodes and three anti-nodes. If λ_5 is the wavelength, the length of the pipe is

$$l = \frac{\lambda_5}{4} + \frac{\lambda_5}{2} + \frac{\lambda_5}{2}$$

$$l = \left(\frac{1+2+2}{4}\right)\lambda_5$$

$$l = \frac{5\lambda_5}{4}$$

$$\lambda_5 = \frac{4l}{5}$$

If λ_6 is the frequency of sound, then speed becomes,

$$v = f_5\lambda_5$$

or $$f_5 = \frac{v}{\lambda_5}$$

Putting value of, we get

$$f_5 = \frac{v}{4l/5}$$

Or $$f_5 = \frac{5v}{4l}$$

$$f_5 = 5\left(\frac{v}{4l}\right)$$

$$f_5 = 5f_1 \qquad \left[\text{since } \frac{v}{4l} = f_1\right]$$

Which is the frequency of third harmonic or second overtone.

nth mode of vibration

If air column vibrates in n loop then,

$$f_n = n\left(\frac{v}{4l}\right) = nf_1$$

And wavelength is

$$\lambda_n = \frac{4l}{n} \qquad \text{where } n = 1, 3, 5, 7, \ldots\ldots\ldots$$

So the longitudinal stationary waves have a discrete set of frequencies f_1, $3f_1$, $5f_1$,........ nf_1, which is known as harmonic series. The frequency f_1 is known as fundamental frequency and the other are called over tone.

Conclusion

By studying the both these cases, we conclude that the pipe which is open at both ends is richer in harmonics.

Q.16 What is Doppler Effect? Discuss its different cases.

Answer

Doppler Effect

The apparent change in the frequency (or pitch) of sound waves due to the relative motion between the source and observer is called Doppler's Effect.

Note

This effect was first observed by John Doppler while he was observing the frequency of light emitted from a star. In some cases, the frequency of emitted light was found to be slightly different from that emitted from a similar source on the Earth.

He found that the change of frequency of light depends upon motion of star relative to Earth.

Example

1) The pitch of whistle of an engine coming toward the platform appears to become higher to an observer standing on the platform.

2) The pitch of whistle of an engine going away from the platform appears to become lower to an observer standing on the platform.

Different cases

Consider a source of sound S at rest emits sound waves having wavelength λ. Let speed of the sound for a stationary observer is v. Then, the number of waves received by observer in one second is

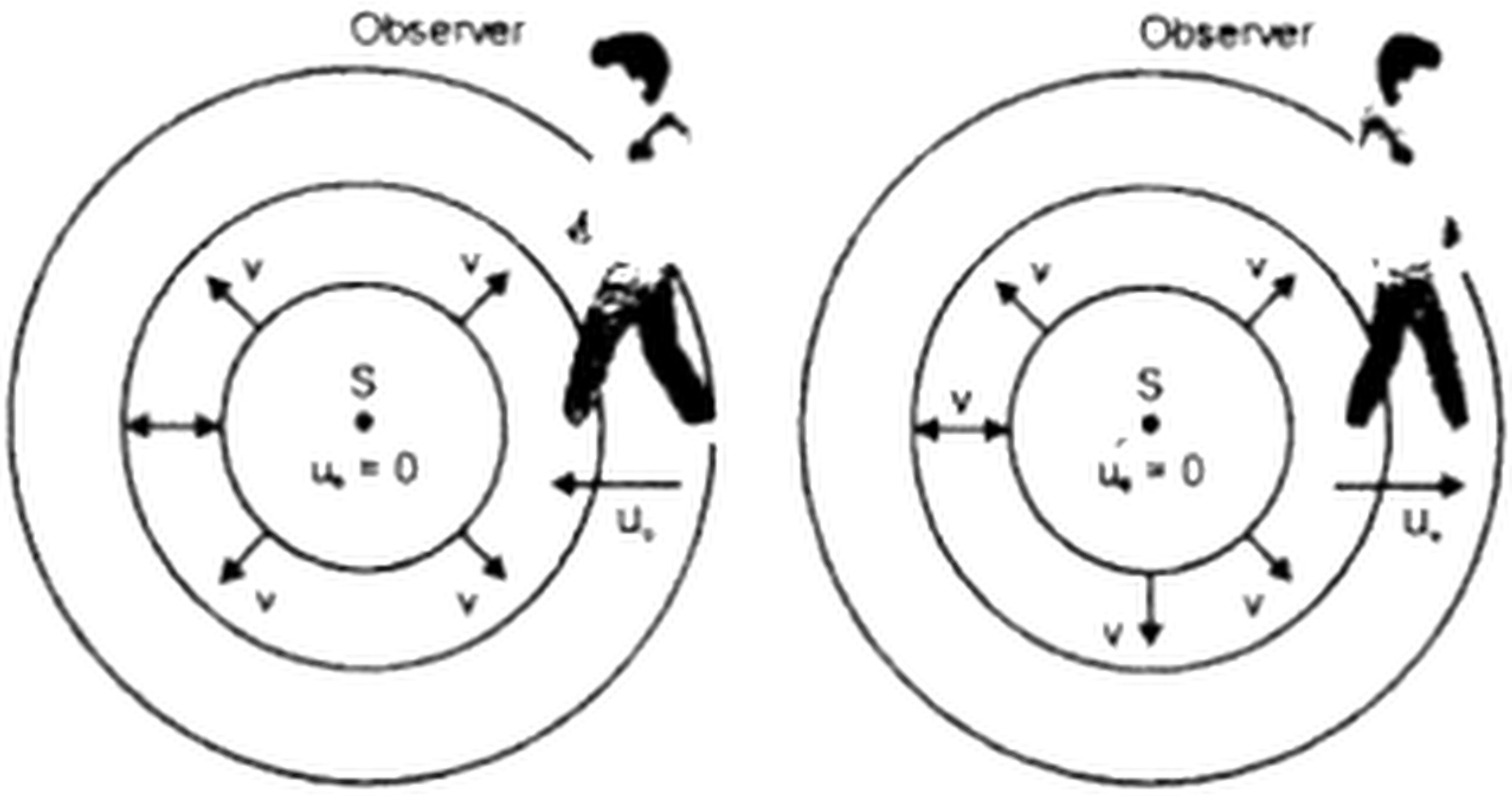
$$F = \frac{v}{\lambda} \qquad \text{........(1)}$$

Case I

[When observer moves towards stationary source]

Let observer A moves towards the source with velocity u_0. Then the relative velocity of the waves and the observer = $v+u_0$.

Now, the number of waves received by observer in one second is,



$$f_A = \left[\frac{v + u_0}{\lambda}\right]$$

$$f_A = \left[\frac{v + u_0}{v / f}\right] \qquad \text{(using equation 1)}$$

$$f_A = \left[\frac{v + u_0}{v}\right] f \qquad (2)$$

$$\because \left[\frac{v + u_0}{v}\right] > 1 \quad \text{so}$$

$$\Rightarrow \qquad f_A > f$$

Result

Thus the apparent frequency of sound heard by the observer will increase.

Case II

[Observer moves away from the stationary source]

Let observer B moves away from the source with velocity u_0, then the relative velocity of the sound and observer = $v - u_0$. Thus, the number of waves received by observer in one second is,

$$f_B = \left[\frac{v - u_0}{\lambda}\right]$$

$$f_B = \left[\frac{v - u_0}{v/f}\right] \qquad \text{(using equation 1)}$$

$$f_B = \left[\frac{v - u_0}{v}\right]f \qquad \text{..........(2)}$$

$$\because \left[\frac{v - u_0}{v}\right] < 1 \quad \text{so}$$

$$\Rightarrow \qquad f_B < f$$

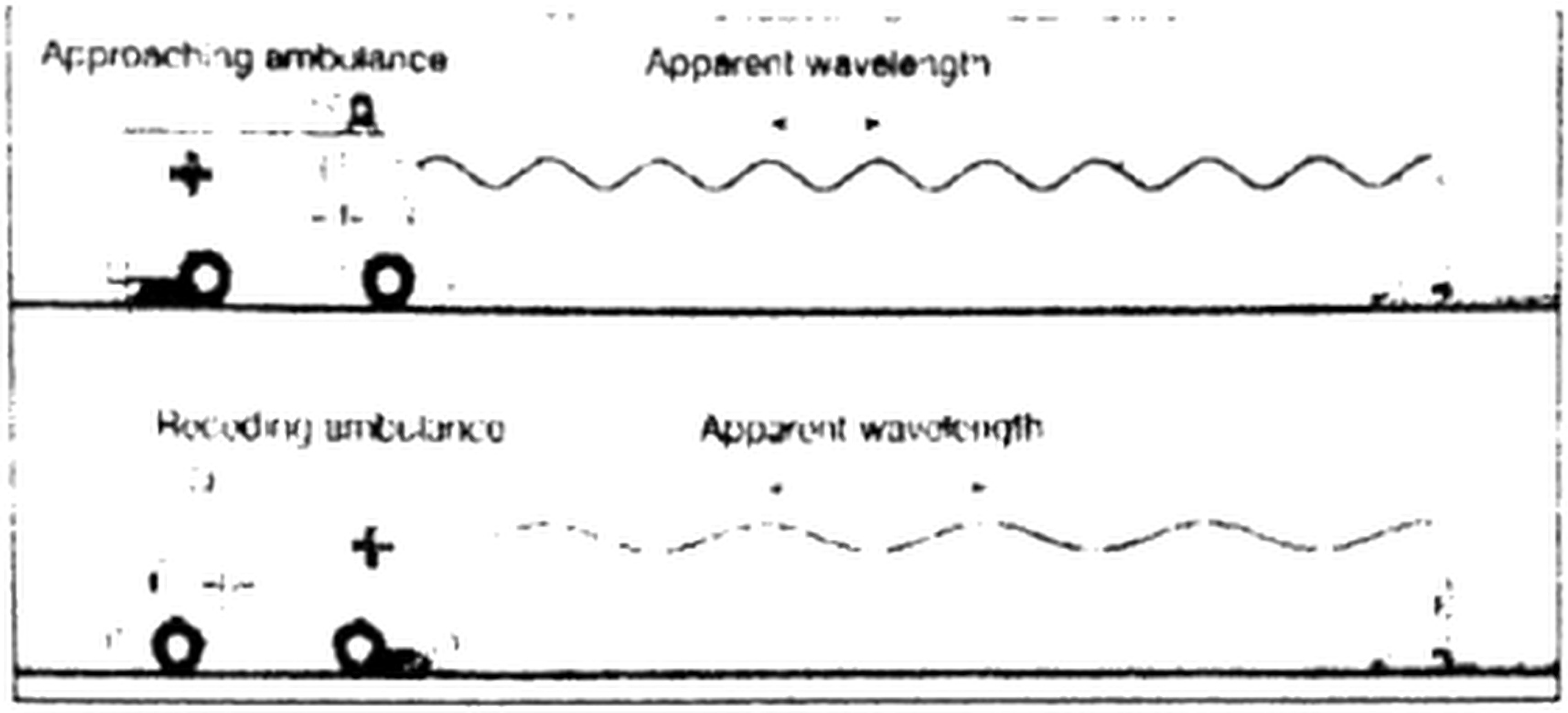
Result

Thus the apparent frequency of sound heard by the observer will decrease.

Case III

[When source moves towards the stationary observer]

When source moves towards the stationary observer C with velocity u_s then waves are compressed and their wavelength is decreased. In this case the waves are compressed a in distance equal to $v - u_s$ in one second.



Thus

The wavelength of sound waves for observer is

$$\lambda = \frac{v}{f}$$

$$\Delta\lambda = \frac{U_s}{f}$$

$$\lambda_0 = \lambda - \Delta\lambda$$

$$\lambda_0 = \frac{v}{f} - \frac{U_s}{f} \qquad \text{..........(3)}$$

New frequency observed by observer C is

$$f_c = \frac{v}{\lambda_c} \qquad \text{..........(4)}$$

Putting value of λ_c from equation (3) in (4)

$$f_c = \frac{v}{(v - U_s)/f}$$

$$f_c = \left(\frac{v}{v - U_s}\right)f \qquad \text{............(5)}$$

Since $\frac{v}{v - U_s} > 1$

So $f_C > f$

Result

Thus the apparent frequency of sound heard by the observer will increase.

Case IV

[Source move away from the stationary observer]

When source moves away from the stationary observer D with velocity u_s then waves are expanded and their wavelength is increased. In this case the waves expand in a distance equal to $v-u_s$ in one second.

Thus

The wavelength of sound waves for observer D is

$$\lambda = \frac{v}{f} \qquad \text{.......(6)}$$

Or $$\Delta\lambda = \frac{U_s}{f}$$

$$\lambda_D = \lambda + \Delta\lambda$$

$$\lambda_D = \frac{v}{f} + \frac{U_s}{f}$$

$$\lambda_D = \frac{v + U_s}{\lambda_D} \qquad \text{........(7)}$$

Putting the value of λ_D from equation (6), in equation (7)

$$f_D = \left[\frac{v}{(v + U_s)/f}\right]$$

$$f_D = \left[\frac{v}{v + U_s}\right]f \quad \text{..........(8)}$$

since $$\frac{v}{v + U_s} < 1$$

So $$f_D = f$$

Result

Thus the apparent frequency of sound heard by the observer will decrease.

Note

- When source and observer move towards each other with velocities u_s and u_0 respectively, then waves are compressed in a distance equal to $v - u_s$ in one second and the relative velocity of the sound and observer becomes $v+u_0$ .In this case both the relative velocity and the wavelength of wave changes . So, the apparent frequency

$$f' = \left[\frac{v + u_0}{\lambda'}\right] = \left[\frac{v + u_0}{(v - u_s)/f}\right] = \left[\frac{v + u_0}{v + u_s}\right] f \qquad \left[\because \lambda' = \frac{v - u_s}{f}\right]$$

- When source and observer move away each other with velocities u_s and u_0 respectively, then waves expand in a distance equal to $v+u_s$ in one second and the relative velocity of the sound and observer becomes $v - u_0$. In this case both the relative velocity and the wavelength of wave changes. So the apparent frequency is

$$f' = \left[\frac{v - u_0}{\lambda'}\right] = \left[\frac{v - u_0}{(v + u_s)/f}\right] = \left[\frac{v - u_0}{v + u_s}\right] f \qquad \left[\because \lambda' = \frac{v - u_s}{f}\right]$$

Q.17 Write down the uses of Doppler's Effect.

Answer

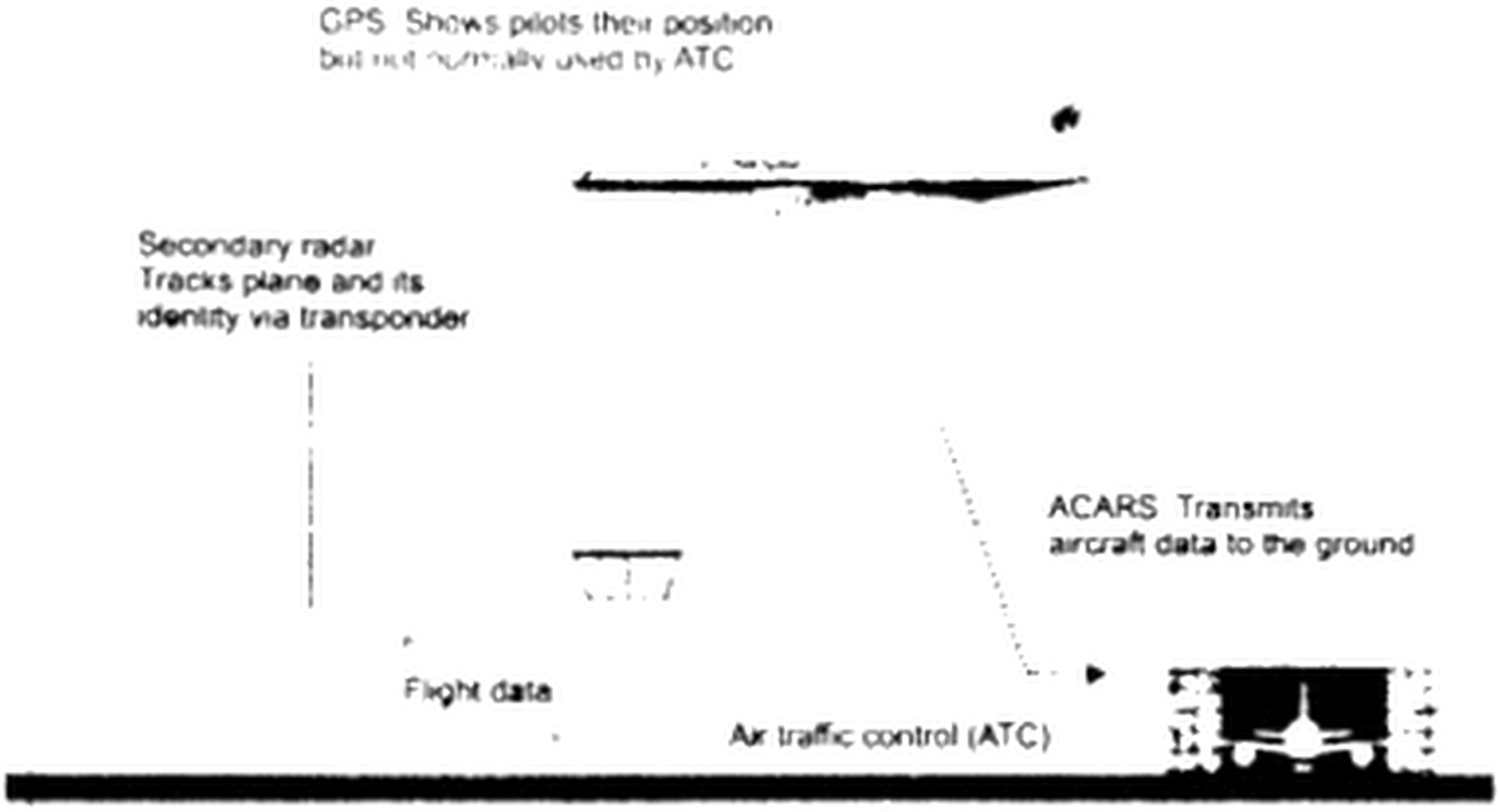
Application of Doppler Effect

Now we discuss some important application of Doppler's effect.

i) Radar System

- RADAR is an acronym.
- It is derived from Radio Detection and Ranging.
- Radar is a device which transmits and receives radio waves.
- The radar system uses to determine the height and speed of aeroplane.

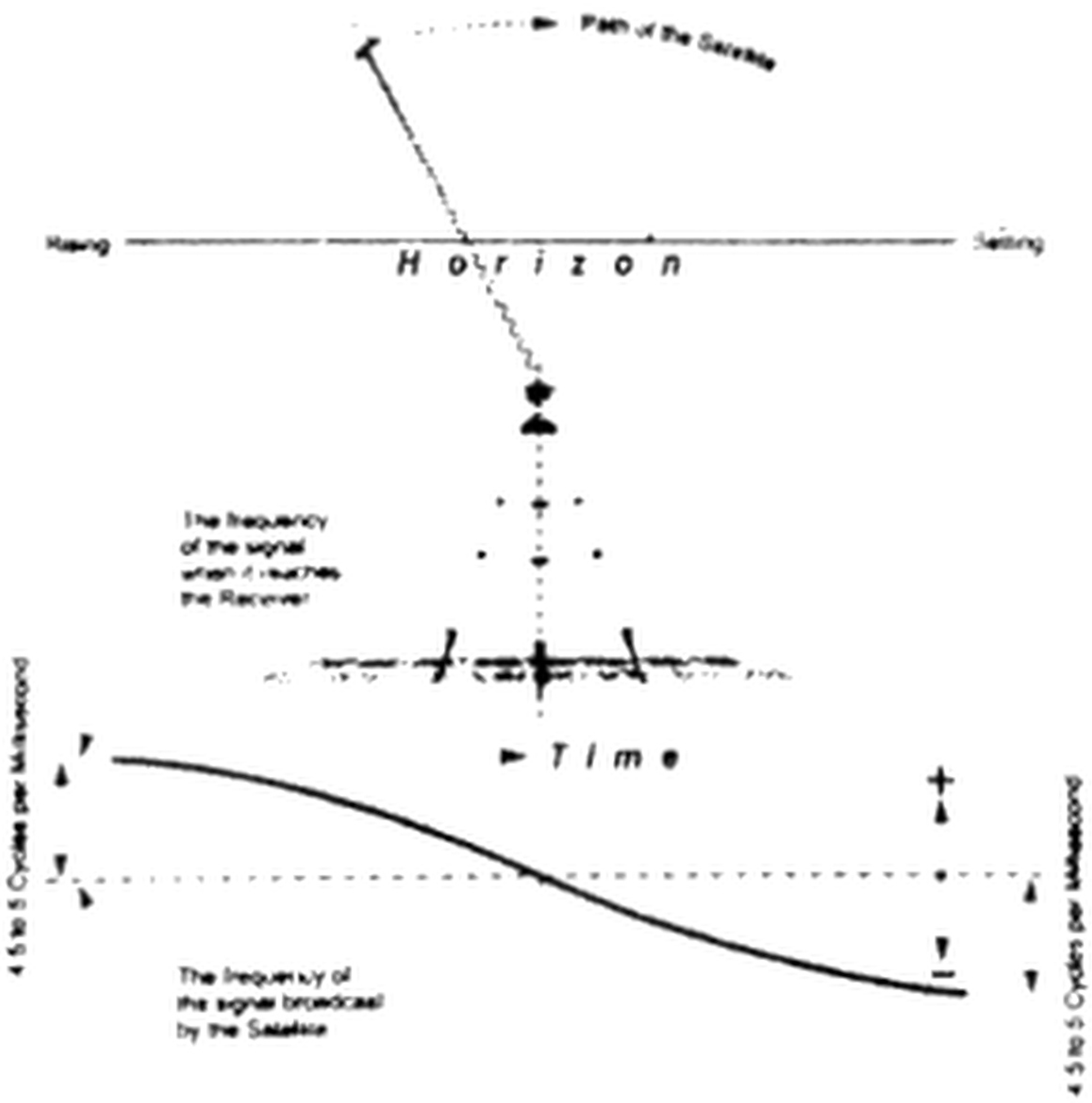
- This system emits radio waves which are reflected from aeroplane and received by the system.
- If reflected waves have longer wavelengths, then the object is moving towards a radar system.
- If reflected waves have longer wavelengths, then the object is moving away from the radar system.



Primary radar: Can only show approx position. No radar coverage 240km from land

ii) Speed of Satellite

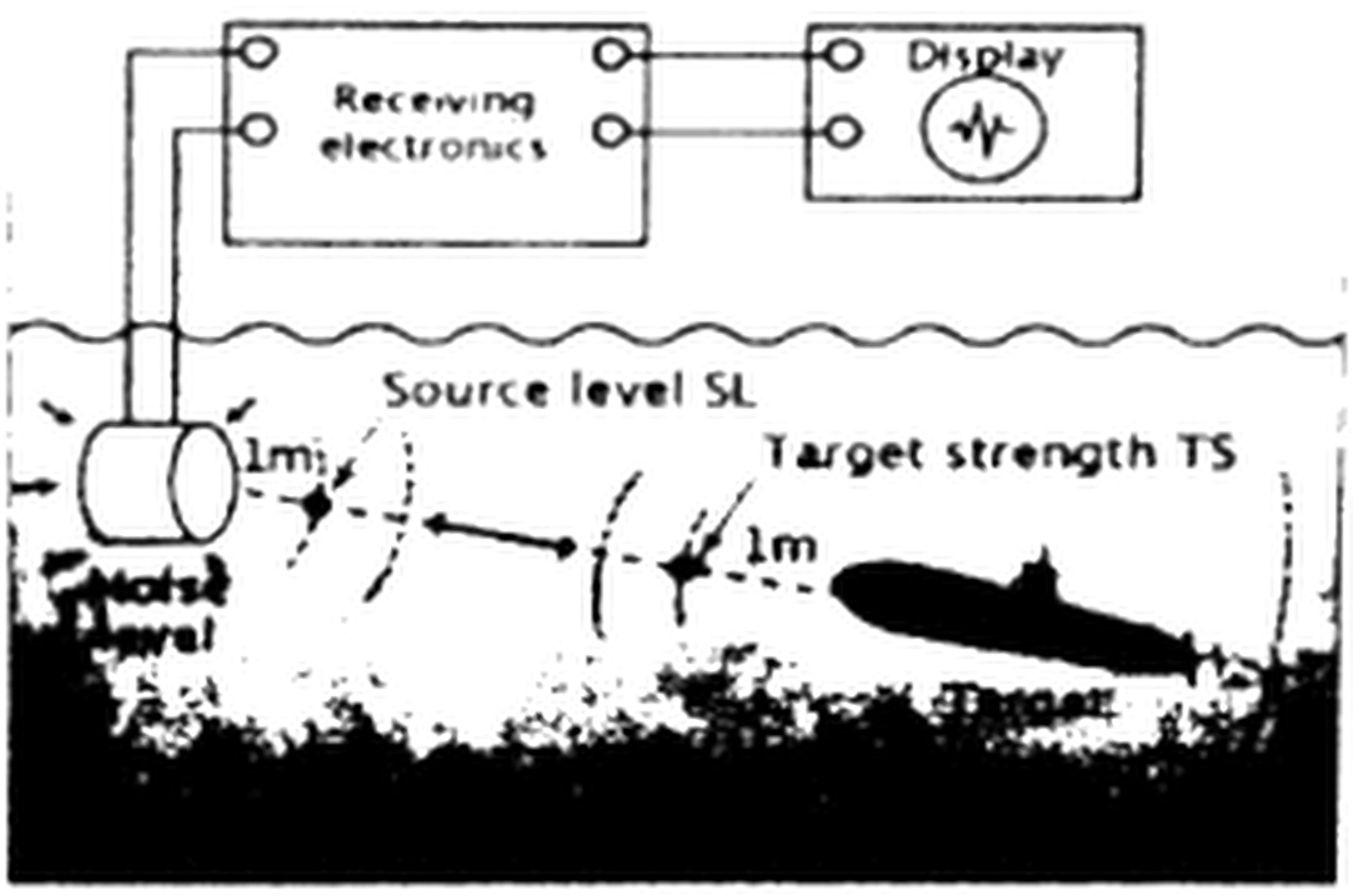
The speed of satellite can also be determined by sending electromagnetic radiations from earth. When these are reflected back after colliding with the satellite, then these are received on the earth. The value of Doppler's shift in wave length of these radiations given the estimation of speed of satellite.



iii) Sonar (Sound Navigation and Ranging)

- Sonar is a technique for detecting the presence of objects under water by an acoustic echo.
- This system uses the ultrasonic waves because they can travel longer distances in water.
- Doppler detection depends upon the relative speed of the target and the detector.
- The apparent change in frequency is observed, which is Doppler's shift.
- In this way we can locate and detect submarines, antisubmarine weapons and miens etc. Also, the depth of sea can be measured.

Echo Ranging and Sonar Parameters



iv) Speed of Star

By comparing the lien spectrum of light coming from a distant star and the light emitted from laboratory source, Doppler's shift can be measured to calculate the speed of star with respect to Earth.

Stars moving towards the earth show blue shift and away from source show red shift.

Blue Shift

The frequency of light emitted by the star increases (i.e. wavelength decreases) if it is moving towards the earth, as compared to the light emitted by stationary star. Thus, spectrum is shifted towards shorter wavelength i.e. to the Blue end of spectrum, which is called Blue Shift.

Red Shift

The frequency of light emitted by the star decreases i.e. wavelength increases) if it is moving away from earth. Thus, spectrum is shifted towards the longer wavelength i.e. towards the Red end of the spectrum, which is called red shift.

Note

As astronomers have also discovered that all the distant galaxies are moving away from us and by measuring their red shifts, they have estimated their speed.

v) Speed of Car

Microwaves are emitted from a source in form of short bursts. Each burst is reflected back by any moving car, in their way. The reflected bursts are detected in the detector by which speed of car is calculated. Microwaves are used to calculate the speed of car by computer program.