Numerical Problems

1. A force of 0.4 N is required to displace a body attached to spring through 10.1 m from its mean position. Calculate the spring constant.

Data:

Applied force F = 0.4 N

Displacement of the body from mean position x=0.1 m

Spring constant k = ?

Solution

According to Hook's Law

$$F = kx$$

$$\Rightarrow \qquad k = \frac{F}{x}$$

Putting values we get

$$K = \frac{0.4N}{0.1m}$$

$$\boxed{K = 4N/m}$$

2. A body of mass 0.025 kg attached to a spring is displaced through 0.10 m to right of mean position. If the spring constant is 0.4 N/m and its velocity at the end of this displacement be 0.4 m/s. Calculate (i) Time Period (ii) frequency (iii) angular frequency to total energy (v) amplitude (vi) maximum velocity (viii) maximum acceleration.

Data:

Mass of the body = 0.025 kg

Displacement from mean position x = 0.1 m

Spring constant k = 0.4 N/m

Linear velocity v = 0.4 m/s

i) Time period T=?

ii) Frequency f=?

iii) Angular frequency ω=?

iv) Total energy E=?

v) Amplitude x_0=?

vi) Maximum velocity v_0=?

vii) Maximum acceleration a_0=?

Solution

(i)   Since $T = 2\pi\sqrt{\dfrac{m}{k}}$

$$T = 2 \times 3.14\sqrt{\dfrac{0.025}{0.4}}$$

$$\boxed{T = 1.57 \text{ s}}$$

(ii)   As $f = 1/T$

$f = 1/1.57$

$$\boxed{f = 0.638 \text{ Hz}}$$

(iii) $\omega = 2\pi f$

$\omega = 2 \times 3.14 \times 0.638$

$\omega = 4$ rad/s

(iv)   Since $E_T = \frac{1}{2} mv^2 + \frac{1}{2} kx^2$

$E_T = \frac{1}{2} \times 0.025 \times (0.4)^2 + \frac{1}{2} \times 0.4 (0.1)^2$

$E_T = 4 \times 10^{-3} \, J$

$E_T = \frac{1}{2} k x_0^2$

(v) $x_0^2 = \dfrac{2E_T}{k}$ $\qquad\qquad$ $x_0 = \sqrt{\dfrac{2E_T}{k}}$

$$\boxed{x_0 = 0.1414 \, m}$$

(vi) Since maximum velocity is given as

$$v = x_0 \sqrt{\frac{k}{m}}$$

$$V_0 = 0.1414 \times \sqrt{\frac{0.4}{0.025}}$$

(vii) Maximum acceleration 'a_0' is given as

$$a_0 = kx_0$$

$$a_0 = \frac{0.4}{0.025} \times 0.1414$$

$$\boxed{a_0 = -2.26 \, m/s}$$

3. A simple pendulum completes on vibration in one second. Calculate its length when g=9.81 m/s².

Data:

Frequency of simple pendulum f=1 Hz.

Length of the simple pendulum l=?

Acceleration due to gravity g = 9.81 m/s²

Time period T = 1 sec

Since $\qquad T = 2\pi\sqrt{\dfrac{l}{g}}$

$\Rightarrow \qquad l = \dfrac{T^2 g}{4\pi^2}$

$\Rightarrow \qquad l = \dfrac{1 \times 9.81}{4 \times (3.13)^2}$

$\Rightarrow \qquad \boxed{l = 24.8 \text{ cm}}$

4. Calculate the length of a second pendulum having time period of 2 seconds when g=9.81 m/s.

Data:

Time period of pendulum T=2 sec.

Acceleration due to gravity g=9.81 m/s²

Solution

As $\qquad T = 2\pi\sqrt{\dfrac{l}{g}}$

$\Rightarrow \qquad l = \dfrac{T^2 g}{4\pi^2}$

$\Rightarrow \qquad l = \dfrac{4 \times 9.81}{4 \times (3.13)^2}$

$\Rightarrow \qquad \boxed{l = 0.992 \text{ m}}$

5. A body of mass 'm' suspended from a spring with spring content 'k' vibrates with 'f₁'. When its length is cut into half and suspended from one of the halves, the frequency is 'f₂'. Find out $f_1 f_2^{-1}$.

Solution

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad \cdots\cdots (1)$$

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{2k}{m}} \qquad \cdots\cdots (2)$$

(Spring constant will be doubled if the spring is cut in two equal parts).

$$\frac{f_1}{f_2} = \sqrt{\frac{1}{2}}$$

$$\boxed{f_1 f_2^{-1} = 0.707}$$

6. A man at the end of spring describes S.H.M. with T=.40 sec. Find out acceleration when the displacement is 0.04 m.

Data:

Time period T = 0.40 sec

Displacement x = 0.04 m

Acceleration a=?

Solution

Since

$$a = \frac{k}{m}x \qquad \cdots\cdots\cdots (1)$$

But $\qquad\qquad \frac{k}{m} = ?$

As

$$T = 2\pi \sqrt{\frac{m}{k}}$$

$\Rightarrow \qquad \frac{m}{k} = 2\pi \frac{T^2}{3\pi^2}$

$\Rightarrow \qquad \frac{m}{k} = \frac{4\pi^2}{T^2} = \frac{3 \times (3.14)^2}{(0.4)^2}$

$\Rightarrow \qquad \frac{k}{m} = 246.49$

Put the above value in equation (1) we get

$$a = -246.49 \times 0.04$$

$\boxed{a = 9.865 \ m/s^2}$

7. A block weighing 4 kg extends a spring by 0.16 m from its unstretched position. The block is removed and 0.5 kg body is hung from the same spring. If the spring is now stretched and then released. What is its period of vibration?

Data:

For 4 kg extension in the spring x = 0.16 m

Spring constant k = $\frac{mg}{x} = \frac{4 \times 9.8}{0.16}$

$\qquad$ K = 245 N/m

Time period of vibration when 0.5 kg mass is attached to the spring.

$$T = 2\pi \sqrt{\frac{m}{k}}$$

$\Rightarrow \qquad T = 2 \times 3.14 \sqrt{\frac{0.5}{245}}$

$$\boxed{T = 0.28 \ sec}$$

8. What should be the length of simple pendulum whose period is one second? What is its frequency?

Data:

Time period T = 1 sec

Frequency f = ?

Length l = ?

Solution

As

$$T = 2\pi \sqrt{\frac{l}{g}}$$

$$\Rightarrow \qquad l = \frac{T \times g}{4p^2}$$

$$\Rightarrow \qquad l = \frac{1 \times 9.8}{4 \times (3.14)^7}$$

$$l = 0.25 \ m$$

As

$$T = \frac{1}{f}$$

$$\Rightarrow \qquad f = \frac{1}{T}$$

$$\Rightarrow \qquad f = \frac{1}{1}$$

$$\Rightarrow \qquad \boxed{f = 1 \ Hz}$$

9. A spring whose spring constant is 80 N/m vertically supports a mass of 1.0 kg is in the rest position. Find the distance by which the mass must be pulled down, so that on being released. It may pass the mean position with velocity of 1m/s.

Data:

Spring constant of the spring k=80 N/m

Mass attached to the spring m=1.0 kg

Velocity of mass at mean position v_0= 1m/s

Distance traveled by mass 'x_0' =?

Solution

As

$$v = x_0 \sqrt{\frac{k}{m}}$$

$$x_0 = v_0 \times \sqrt{\frac{1}{80}}$$

$$\boxed{x_0 = 0.11\,m}$$

10. A 800g body vibrates S.H.M with amplitude 0.30 m. The restoring force is 60N. Find out (a) T, (b) a, (c) v, (d) K.E., (e) P.E. when the displacement is 12 cm.

Data

Mass of vibrating body m=8 kg

Amplitude of vibrating body x_0=0.30 m

Restoring force F_r = 60 N

Displacement x = 12 cm = 0.12 m

Spring constant = 200 N/m

a) Time period T = ?

b) Acceleration $\bar{a}$ = ?

c) Velocity $\bar{v}$ = ?

d) K.E = ?

e) P.E. = ?

Solution

a)
$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2 \times 3.14\sqrt{\frac{8}{200}}$$

$$T = 1.3 \text{ sec}$$

b) Since
$$\bar{a} = -\frac{k}{m}\vec{x}$$

$$\bar{a} = -\frac{800}{8} \times 0.12$$

$$\bar{a} = -3 \text{ m/s}^2$$

c)   As

$$v = \sqrt{\frac{k}{m}\left(x_0^2 - x^2\right)}$$

$$v = \sqrt{\frac{200}{8}(0.3)^2 - (0.12)^2}$$

$$v = 1.4 \text{ m/s}$$

d)   As

K.E. = ½ mv²

K.E. = ½ × 8 × (1.4)²

K.E. = 7.6 J

e)   As

P.E. = ½ kx²

P.E. = ½ × 200 × (0.12)²

$$\boxed{P.E. \ = \ 1.44 \ J}$$

11. Find the amplitude, frequency and time period of an object oscillating at the end of a spring, the equation for its position at any instant 't' is given by x = 25 Cos $\frac{1}{16}$ t. Find the displacement of the object after 2.05.

Data

i) x_0 = ?

f = ?

∵ We know that x = x cos (2πf) t   ....(1)

Whereas we have,

$$x \ = \ 25 \cos\left(\frac{\pi}{8}\right) t \qquad .... (2)$$

Comparing (1) & (2) we have:

$$\boxed{x_0 = 25m}, \qquad \boxed{f = \frac{1}{16} Hz}$$

ii)   x = ?

t = 2 sec

⇒ $x \ = \ 25 \cos\left(\frac{\pi}{8}\right) t$

$$= 25 \cos\left(\frac{\pi}{8}\right) \times 2$$

$$\boxed{x = 0.18 \text{ m}}$$