Numerical Problems

1. Water flows through a 1 cm diameter pipe with a speed of 1 m/s. What should be the diameter of the nozzle if the water is to emerge at 2.1 m/s.

Data:

Diameter of the pipe $d_1 = 1$ cm = 0.01 m

Speed of water at this point $v_1 = 1$ m/s

Diameter of the nozzle $d_2 = ?$

Speed of water at this point $v_2 = 2.1$ m/s

Solution

From equation of continuity

$$A_1 v_1 = A_2 v_2$$

$$\Rightarrow \quad \pi \gamma_1^2 v_1 = \pi \gamma_2^2 v_2$$

$$\Rightarrow \quad \left(\frac{d_1}{2}\right)^2 v_1 = \left(\frac{d_1}{2}\right)^2 v_2$$

$$\Rightarrow \quad d_2 = \sqrt{d_1^2 \times \frac{v_1}{v_2}}$$

$$\Rightarrow \quad d_2 = \sqrt{(0.01)^2 \times \frac{1}{2.1}}$$

$$\boxed{d_2 = 2.2 \times 10^{-3}}$$

2. Water if flowing smoothly through a closed pipe system. At one-point speed of water is 3 m/s, while at another point 3m high, the speed is 4m/s. At lower point the pressure is 80k pa. Find the pressure at the upper point.

Data:

Speed of water at lower end v_1=3 m/s

Speed of water at higher end v_2=4m/s

Height of the pipe at higher end h_2=3 cm

Height of the pipe at lower end h_1=0 cm

Pressure of water at lower end ρ =80k 10^3 pa

Pressure of water at lower end ρ_2 =?

Solution

From Bernoulli Equation

$$P_2 + \varsigma v_2^2 + \varsigma g h_2^2 = P_1 + \varsigma v_1^2 + \varsigma g h_1$$

$\Rightarrow$ $P_2 + \times 1000 \times 16 + 1000 \times 9.8 \times 3$

$\Rightarrow$ $80 \times 10^3 + \times 1000 \times 9 + 0$

$\Rightarrow$ $P_2 + 8000 + 29400 = 80 \times 1000 + 4500$

$\Rightarrow$ $P_2 + 37400 = 84500$

$\Rightarrow$ $\boxed{P_2 = 47.1 Kp}$

3. An airplane wing is designed to that when the speed of the air across the top of the wing is 450 m/s, the speed of air below the wing is 410 m/s. what is the pressure difference between top and bottom of the wings?

Data:

Speed of the air at the top wing v_1=450 m/s

Speed of the air at the bottom wings v_2=410 m/s

Pressure difference between bottom & top

$$P1 - P2 = ?$$

Density of the air δ = 1.29 kg/m^3

Solution

$$P_2 - P_1 = \frac{1}{2}\varsigma v_1^2 - \frac{1}{2}\varsigma v_2^2$$

$\Rightarrow$ $$P_2 - P_1 = \frac{1}{2}\varsigma\left(v_1^2 - v_2^2\right)$$

$\Rightarrow$ $$P_2 - P_1 = \frac{1}{2}\times 1.29\left[(450)^2 - (410)^2\right]$$

$\Rightarrow$ $$\boxed{P_2 - P_1 = 22.18\text{KP}}$$

4. Water flows through pipe whose internal diameter is 2cm at a speed of 1 m/s. What should be the diameter of the nozzle if the water is to emerge at a speed of 4m/s.

Data:

Internal diameter of the pipe at one end d_1=2 cm = 0.02 m

Speed of water at this point v_1=1 m/s

Diameter of the nozzle d_2=?

Speed of emerging water v_2=4 m/s

Solution

From equation of continuity

$$A_1v_1 = A_2v_2$$

$\Rightarrow$ $$\pi\gamma_1^2 v_1 = \pi\gamma_2^2 v_2$$

$\Rightarrow$ $$\left(\frac{d_1}{2}\right)^2 v_1 = \left(\frac{d_1}{2}\right)^2 v_2$$

$\Rightarrow$ $$d_2 = \sqrt{\frac{d_1^2 \times v_1}{v_2}}$$

$$\boxed{d_2 = 0.01\,\text{m}}$$

5. Eight equal drops of oil are falling through air with a steady velocity of 0.1 m/s. If the drops recombine to form a single drop, what should be the new terminal velocity?

Data:

Terminal velocity of each drop $v_T = 0.1$ m/s

Terminal velocity of drops when they combine to form a single drop $v_T' = ?$

Solution:

When drops combine to form a single drop then

$$\gamma_2 = 2ri$$

Since

$$v_T = \frac{2g\varsigma r_1^2}{9\eta} \quad \text{........(1)}$$

&

$$v_T = \frac{2g\varsigma r_2^2}{9\eta} \quad \text{........(2)}$$

Dividing equation (2) by (1)

$$\frac{v_T}{v_T} = \frac{\frac{2g\varsigma r_1^2}{9\eta}}{\frac{2g\varsigma r_2^2}{9\eta}}$$

$$\frac{v_T}{v_T} = \frac{r_2^2}{r_1^2}$$

$$\Rightarrow \quad v_T = v_T \times \frac{r_2^2}{r_1^2} \quad \text{.........(3)}$$

Putting the value in Equation (3)

$$v_T = (0.1) \times \left(\frac{2\gamma_1}{\gamma_1}\right)$$

$$\boxed{v_T = 0.4 m/s}$$

6. Calculate the speed of efflux of kerosene oil from narrow hole of a tank, in which pressure is 4 atm. Density of kerosene oil is 0.72 kg/m³.

Data

Density of kerosene oil δ=0.72 kg/m³

Pressure P = 4 atm = 4 ×1.03 × 10⁵ pa

Speed of efflux v = ?

Solution

Since

$$P = \frac{1}{2}\varsigma v^2$$

v $$= \sqrt{\frac{2\times 4\times 1.03\times 10^5}{0.72}}$$

$$\boxed{V = 338\,m/s}$$

7. A small sphere of volume v falling in a viscous medium acquires a terminal velocity υ. What will be the terminal velocity of a sphere of same material and volume 8v falling through the same medium?

Solution

Since

$$\upsilon_t = \frac{2g\varsigma\gamma_1^{\,2}}{9\eta} \qquad \text{......(1)}$$

As $$v = \frac{4}{3}\pi\gamma^3$$

$$\gamma = \left(\frac{3v}{4\pi}\right)^{\frac{4}{3}}$$

Putting the value of γ in Equation (1)

$$\upsilon_t = \frac{2g\varsigma}{9\eta}\left(\frac{3v}{4\pi}\right)^{\frac{2}{3}} \qquad \text{........(2)}$$

Since $\upsilon' = 8V$ so new terminal velocity of the sphere is

$$\upsilon_t = \frac{2g\varsigma}{9\eta}\left(\frac{3\times 8v}{4\pi}\right)^{\frac{2}{3}}$$

$$\upsilon_t = \frac{2g\varsigma}{9\eta}\left(\frac{6}{\pi}\right)^{\frac{2}{3}} \times V^{\frac{2}{3}}$$

$$\upsilon_T = \frac{2g\varsigma}{9\eta} \times 1.54\left(\frac{4}{3}\pi\gamma^3\right)^{\frac{2}{3}}$$

$$\upsilon_T = \frac{2g\varsigma}{9\eta} \times 1.54\left(\frac{4\times\pi^3}{3}\right)^{\frac{2}{3}} \times \gamma^2$$

$$\upsilon_T = \frac{2g\varsigma}{9\eta} \times 1.54 \times 2.6 \times \gamma^2$$

$$\upsilon_T = \frac{2g\varsigma\gamma^2}{9\eta} \times 4$$

$$\boxed{\upsilon_T = 4\upsilon_1} \quad \text{........(3)}$$

From Equation (3), it is clear that if volume becomes 8 times terminal velocity becomes 4 times of initial terminal velocity.

8. Determine the radius of a water drop falling through air with a terminal velocity of 0.012 m/s.

Data

Terminal velocity of the drop υ=0.012 m/s

Viscosity of air η = 1.9 ×10-5 NSm-2

Density of air δ =1.2 kg/m^3

Density of water δ_{water}= l×10^3 kg/m^3

Solution

Since

$$\upsilon_T = \left(\frac{2g\varsigma r^2}{9\eta}\right)$$

$$\gamma = \sqrt{\frac{9\eta V_t}{2\varsigma g}}$$

$$\gamma = \sqrt{\frac{9\times 0.019\times 10-3\times 0.012}{2\times 1\times 10^3\times 9.8}}$$

$$\boxed{\gamma = 0.3\text{mm}}$$