Unit 6

FLUID DYNAMICS

Q.1. What is meant by the term viscosity? Explain fluid friction or drag force and state Stock's Law.

Answer

Viscosity

- Viscosity of a fluid is the measure of its resistance to flow.
- It is the frictional effect between different layers of a flowing fluid.
- It measures that how much force is required to slide one layer of the liquid over another layer.
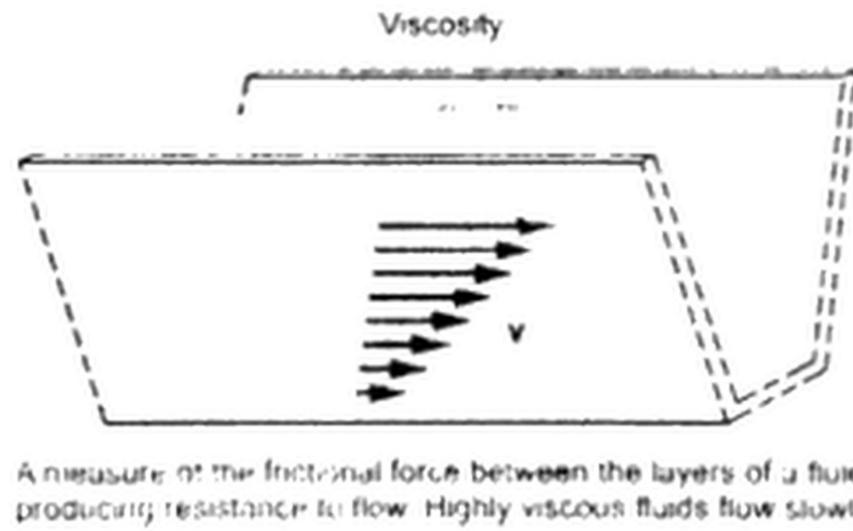
Examples

- Substances like honey and thick tar have large coefficient of viscosity. So, they cannot flow easily.
- Substance like water and milk has small coefficient of viscosity η. So, they can flow easily.

Unit

- The SI unit of coefficient of viscosity is $kgm^{-1}s^{-1}$ or $Nnm^{-2}s$.
- Its dimension is $[ML^{-1}T^{-1}]$.

Viscosity

A measure of the frictional force between the layers of a fluid producing resistance to flow. Highly viscous fluids flow slowly

Viscosity of liquids and gases.

- Liquid and gases have non-zero viscosity.
- Viscosity of gases increases with increase in temperature while that of solids and liquids decreases.

Drag Force

An object moving through a fluid experiences a retarding force called drag force.

<div align="center">OR</div>

The force which resists the motion of a body moving through the fluid is called drag force or fluid friction.
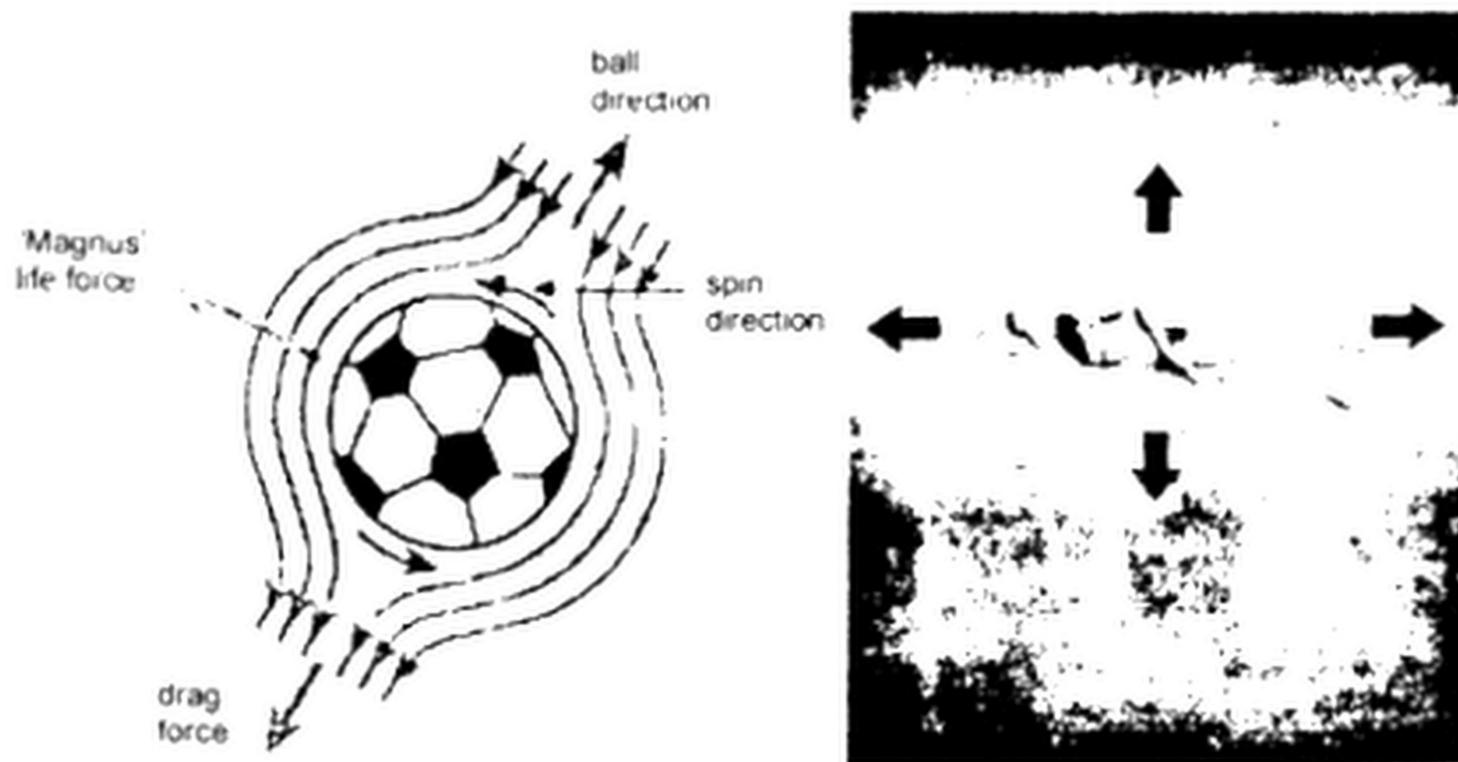
For example

When we stick out our hand out of the window of a fast-moving car, we feel a force opposite to our motion.

Factors on which depends upon

According to Stroke's law, drag force is given by
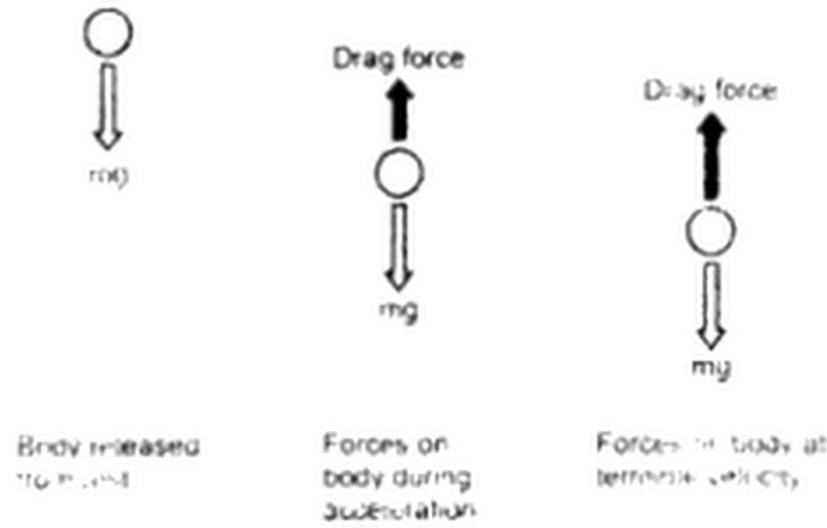
$$F = 6\pi\eta\, r\, v$$

This equation shows that drag force depends upon

i)   Speed of sphere (v)

ii)   Size of sphere (r)

Stroke's Law

This law states that the drag force F acting on a sphere of radius r moving slowly with velocity v in a fluid of viscosity η is given by

$$F = 6\pi\eta\, r\, v$$

Body released   Forces on   Forces on body at
from rest   body during   terminal velocity
   acceleration

Limitation

This law is valid only for spherical bodies moving slowly.

Q2.   What is terminal velocity? Show that terminal velocity of fog droplet is directly proportional to the square its radius?

Answer

Terminal Velocity

The maximum constant velocity of an object falling vertically downward for which drag force become equal to its weight is called terminal velocity.
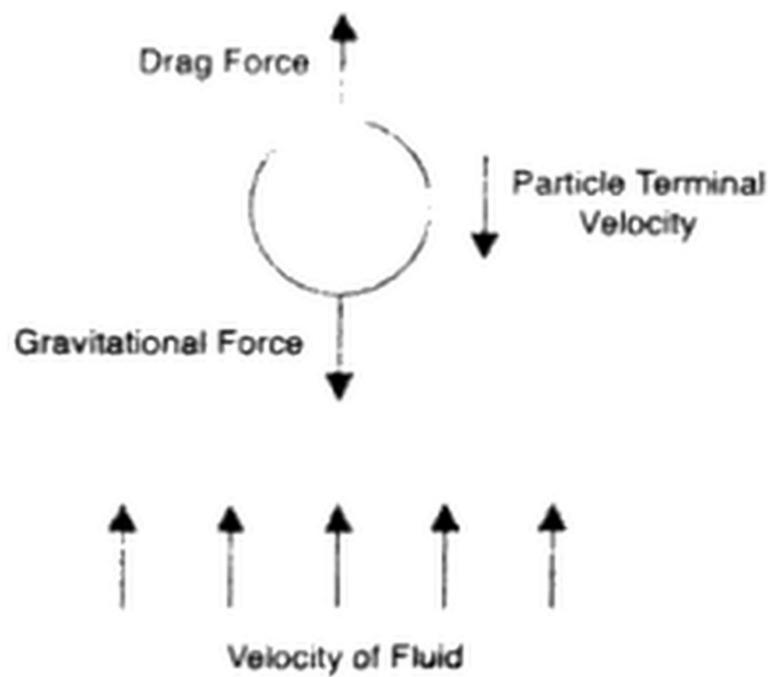
Explanation

Consider a fog droplet falling vertically downward. When drag force of air increases, the velocity of droplet increases. The net force on the droplet is,

Net force = weight - drag force

$$F = mg - 6\pi\eta\, r\, v$$

$$Ma = mg - 6\pi\eta\, r\, v$$

When drag force becomes equal to the weight of droplet then it will start to move with uniform velocity, called terminal velocity (v_t). So its acceleration becomes zero (i.e. a=0)

Thus, the above equation becomes.

$$m(0) = mg - 6\pi\eta\, r\, v_t$$

Or $6\pi\eta\, r\, v_t = mg$

Or $v_t = \dfrac{mg}{6\pi\eta\, r\, v}$ ......(1)

Where v_t = the terminal velocity and η =coefficient of viscosity

Relation between terminal velocity and radius of droplet

Since   density $= \dfrac{mass}{volume}$

Or $\rho = \dfrac{m}{v}$

Or $m = \rho v$

Or $m = \left(\dfrac{4}{3}\pi r^3\right)\rho$ $\left[Qv = \left(\dfrac{4}{3}\pi r^3\right) \text{for sphere}\right]$

Hence equation (1) becomes

$$v_t = \dfrac{\left(\dfrac{4}{3}\pi r^3\right)\rho g}{3 \times 6 \pi \eta r}$$

$$\boxed{v_t = \dfrac{2gr^2\rho}{9\eta}}$$ (2)

Or $v_t = \left[\dfrac{2g\rho}{9\eta}\right]r^3$ $\left[Q\dfrac{2g\rho}{9\eta} = constant\right]$

Or $\boxed{v_t \propto r^2}$

Thus, terminal velocity is directly proportional to the square of the radius of droplet.
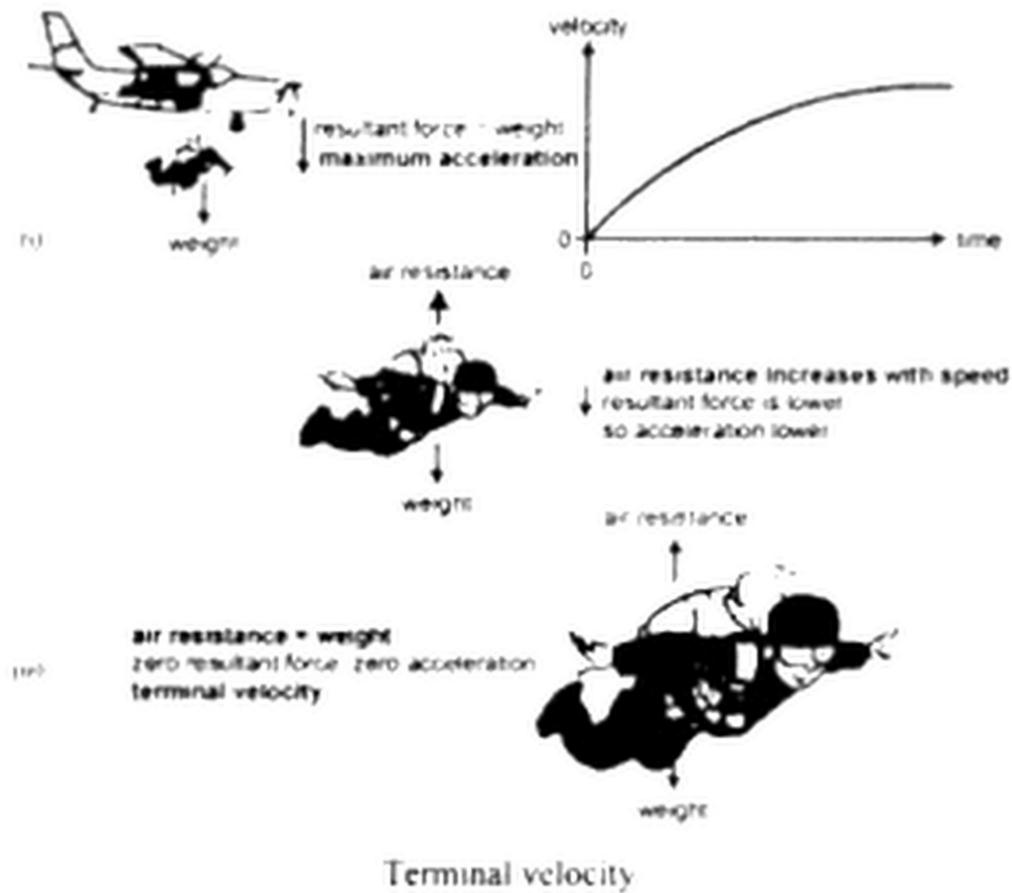
Q3. Discuss the terminal velocity of a paratrooper? Show its graph.

Answer

The terminal velocity of a falling body occurs during free fall when a falling body experiences zero acceleration.

This is because of the retarding force known as air resistance. Air resistance exists because air molecules collide into a falling body creating an upward force opposite gravity. This upward force will eventually balance the falling body's weight. It will continue to fall at constant velocity known as the terminal velocity.

The magnitude of terminal velocity depends on the weight of the falling body. For a heavy object, the terminal velocity is generally greater than a light object.

velocity

resultant force = weight
maximum acceleration

weight

air resistance

air resistance increases with speed
resultant force is lower
so acceleration lower

weight

air resistance

air resistance = weight
zero resultant force zero acceleration
terminal velocity

weight

Terminal velocity

This is because air resistance is proportional to the falling body's velocity squared. For an object to experience terminal velocity, air resistance must balance weight. An example that shows this phenomenon was the classic illustration of a rock and a feather being dropped simultaneously. In a vacuum with zero air resistance, these two objects will experience the same acceleration. But on the earth, this is not true. Air resistance will equal weight more quickly for the feather than it would for the rock. Thus, the rock would accelerate longer and experience a terminal velocity greater than the feather.

Sky Driver

The graph shows how the vertical velocity of a parachutist changes from the moment the parachutist jumps from the aircraft until landing on the ground. Using the idea of forces, explain why the parachutist reaches a terminal velocity and why opening the parachute reduce the terminal velocity.
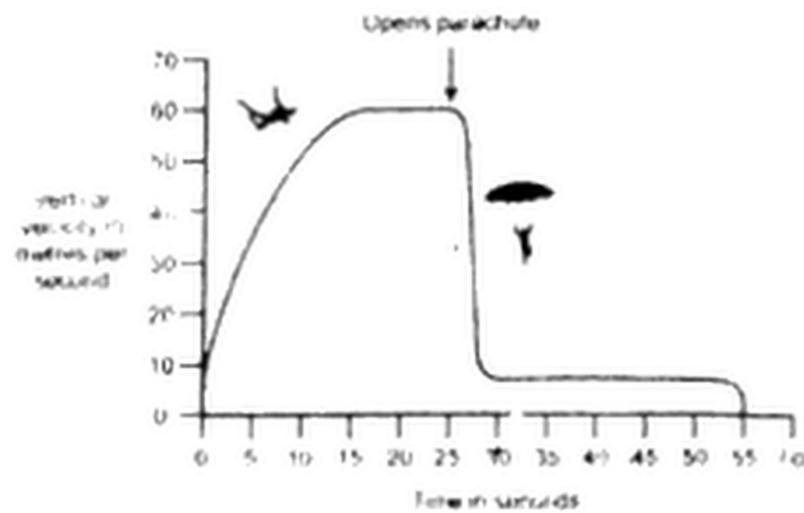
Another factor that affects terminal velocity is the orientation at which a body fall. If an object falls with a larger surface area perpendicular to the direction of motion it will experience a greater force and a smaller terminal velocity. On the other hand, if the object fell with a smaller surface area perpendicular to the direction of motion. It will experience a smaller force and a greater terminal velocity.

Q.4   What is the difference between steady and turbulent flow? What is an ideal fluid? Answer

Fluid Flow

Let us consider the flow of the fluid through the pipes. It may be either streamline or turbulent.
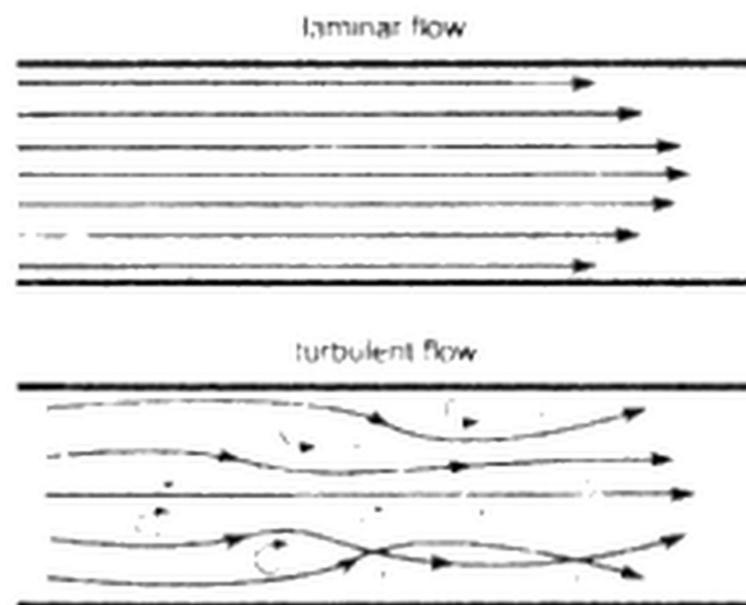
Laminar Flow

The flow of a fluid in which every particle of the fluid moves along a smooth path is called laminar flow.

Turbulent Flow

When the speed of flowing fluid exceeds a certain critical value, the flow becomes extremely irregular and complex and it changes continuously with time.

Thus.

The irregular or unsteady flow of the fluid is called turbulent flow.

Difference between Laminar Flow and Turbulent Flow

- In laminar flow, each particle of fluid moves along a smooth path which does not change with time.
- In turbulent flow, the flow pattern is not smooth but continuously changes with time.
- In laminar flow, stream lines do not cross each other while in turbulent flow it does not happen.
- In turbulent flow, the velocity of fluid changes abruptly.

Not (Steady flow condition)

For steady flow, different streamlines can never intersect each other. This condition is called steady flow condition.

Ideal Fluid

A fluid which satisfies the following condition is called an ideal fluid.

1)   The fluid is non-viscous i.e., there is no internal frictional force between adjacent layers of fluid.

2)   The fluid is incompressible i.e. its density is constant.

3) The fluid motion is steady.

4) Irrational flow.

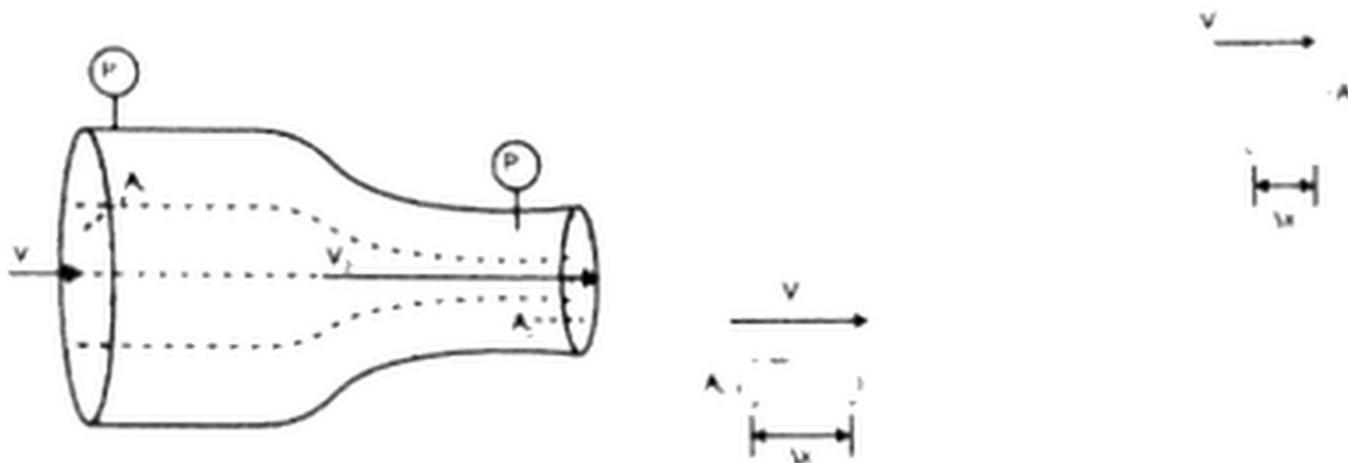## Q.5   State and explain equation of continuity.

Answer

Equation of Continuity

For an ideal fluid, the product of cross-sectional area of the pipe and the fluid speed at any point along the pipe is a constant.

<div align="center">OR</div>

For an ideal fluid, the volume flow per second of the fluid (or simply flow rate) always remains constant.

Explanation

Consider, a fluid is flowing through the non-uniform pipe, the particles in the fluid move along the streamlines in a steady state flow as shown in figure.

At lower end of pipe

Let

Area of cross-section of pipe   = A_1

Distance moved by fluid in time Δt = Δx_1

Velocity of fluid $= v_1$

Density of fluid $= \rho_1$

Volume of the fluid contained at lower end $= V_1$

Mass of the fluid contained at lower end $= \Delta m_1$

Then

$\Delta m_1 = \rho_1 V_1$ **Or** $\Delta m_1 = \rho_1 A_1 \Delta x_1$ **Or** $\Delta m_1 = \rho_1 A_1 \Delta t$ $[\because \Delta x_1 = v_1 \Delta t]$

At upper end of pipe

Let

Area of cross-section of pipe $= A_2$

Distance moved by fluid in time Δt $= \Delta x_2$

Velocity of fluid $= v_2$

Density of fluid $= \rho_2$

Volume of the fluid contained at upper end $= V_2$

Mass of the fluid contained at lower end $= \Delta m_2$

Then $\Delta m_2 = \rho_2 A_2 v_2 \Delta t$

If the fluid is incompressible and flow is steady, the mass of fluid is conserved. i.e.

The mass that flows into the bottom of the pipe through A_1 in time Δt must be equal to mass of the liquid that flows out through A_2 in the same time. Therefore,

$\Delta m_1 = \Delta m_2$

$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

This equation is called the equation of continuity.

Since the fluid is incompressible, so the density is constant i.e.

$\rho_1 = \rho_2 = \rho$ (say)
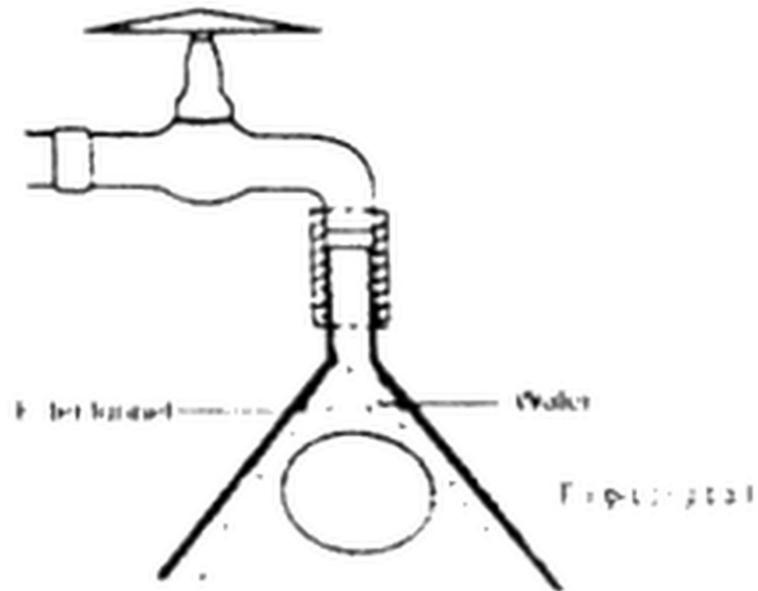
Thus, the equation of continuity becomes

$$\rho A_1 v_1 = \rho A_2 v_2$$

$$A_1 v_1 = A_2 v_2$$

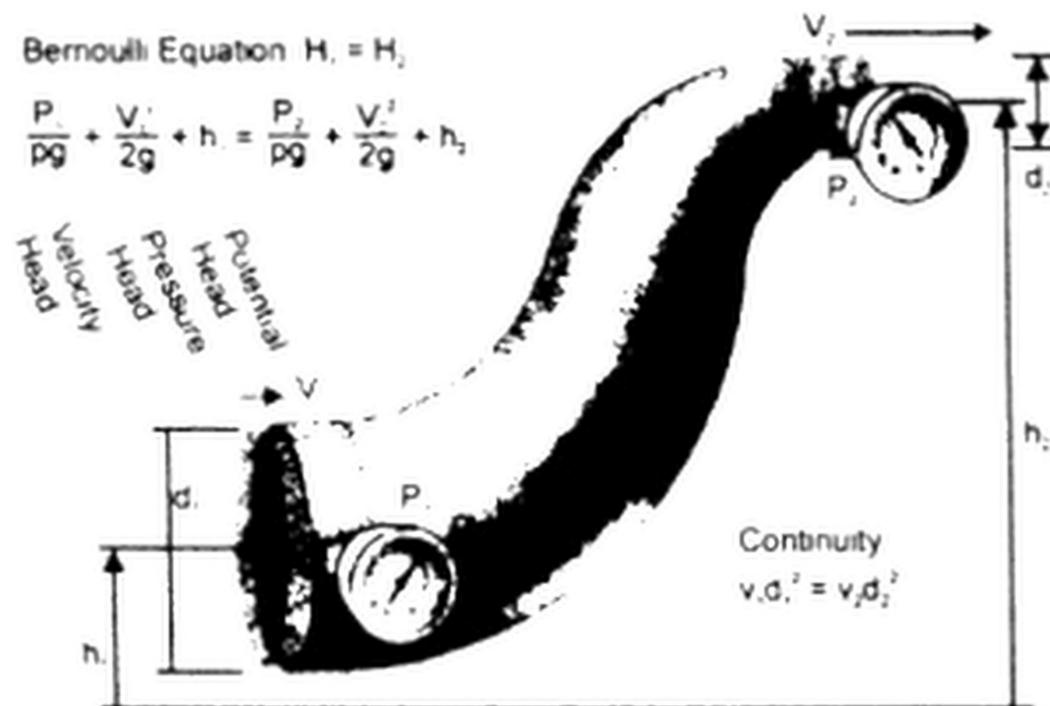$$\boxed{Av = constant}$$

The product Av is called flow rate.

Q.6   State and explain Bernoulli's Equation.

Answer

Bernoulli's Equation

Bernoulli's equation is the fundamental equation in fluid dynamics which relates the pressure to fluid speed and height.

Bernoulli Equation $H_1 = H_2$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + h_2$$

Continuity
$$v_1 d_1^2 = v_2 d_2^2$$

Statement

The sum of pressure, K.E. per unit volume and P.E. per unit volume of an incompressible, non-viscous fluid flowing in steady state is constant at each point along a stream line in a pipe.

Mathematically

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$$

Proof

Let us consider the steady flow of an incompressible, non-viscous fluid through a pipe in time Δt, as shown in figure below.

Work done at upper end

Let, at upper end

       Area of cross section of pipe = A_1

       The pressure of fluid = P_1

       Force exerted on the fluid = F_1

Distance moved by fluid in time $\Delta t = \Delta x_1$

Velocity of fluid = v_1

Density of fluid = ρ

Volume of the fluid contained at upper end = V

The work done on the fluid by the fluid behind it is given by

$$W_1 = F_1 \Delta x_1$$

$$W_1 = P_1 A_1 \Delta x_1 \left[\because P_1 = \frac{F_1}{A_1} \Rightarrow F_1 = P_1 A_1 \right]$$

Or
$$W_1 = P_1 A_1 v_1 \Delta t \qquad (1)$$
$$\left[\text{As } \Delta x_1 = v_1 \Delta t \right]$$

Work done at lower end

Let, at lower end

Area of cross section of pipe = A_2

The pressure of fluid = P_2

Force exerted on the fluid = F_2

Distance moved by fluid in time $\Delta t = \Delta x_2$

Velocity of fluid = v_2

Density of fluid = ρ

Volume of the fluid contained at lower end = V

Similarly, the work done on the fluid by the fluid ahead of it is given by

$$W_2 = -F_2 \Delta x_2$$

$$W_2 = -P_2 A_2 \Delta x_2 \left[\because P_2 = \frac{F_2}{A_2} \Rightarrow F_2 = P_2 A_2 \right]$$

Or
$$W_2 = -P_2 A_2 v_2 \Delta t \qquad (2)$$
$$\left[\text{As } \Delta x_2 = v_2 \Delta t \right]$$

The negative sign shows that this work is done against the fluid force.

Net work done

The net work done both at upper and lower end is

$$W = W_1 + W_2$$
$$W = P_1 A_1 \Delta x_1 \Delta t - P_2 A_2 \Delta x_2 \Delta t$$
$$W = P_1 A_1 v_1 \Delta t - P_2 A_2 v_2 \Delta t \qquad (3)$$

According to equation of continuity

$$A_1 v_1 = A_2 v_2$$

Or $$A_1 v_1 \Delta t = A_2 v_2 \Delta t = V \, (\text{volume})$$

Hence, equation (3) becomes

$$W = P_1 V - P_2 V$$
$$W = (P_1 P_2) V \qquad (4)$$
$$W = (P_1 P_2) \frac{m}{\rho} \qquad (5)$$
$$\left[\because \rho = \frac{m}{V} \Rightarrow V = \frac{m}{\rho} \right]$$

Change in K.E and change in P.E

A part of this work done changes K.E. of fluid and a part changes its gravitational P.E. so,

Change in K.E. = $\Delta(\text{K.E}) = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$

Change in P.E = $\Delta(\text{P.E}) = mgh_2 - mgh_1$

Where h_2 and h_1 are the heights of the upper and lower end respectively.

By law of conservation of energy,

Work done = change in K.E. + change in P.E

$$\left(P_1 - P_2\right)\frac{m}{\rho} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgh_2 - mgh_1$$

Multiplying both sides by $\frac{\rho}{m}$, we get,

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho gh_2 - \rho gh_1$$

$$\boxed{P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2}$$

This is the Bernoulli's equation and it can also be expressed as

$$\boxed{P + \frac{1}{2}\rho v^2 + \rho gh = constant} \qquad\qquad (6)$$

Q.7 State and explain Torricelli's Theorem.

Answer

Application of Bernoulli's Equation

Torricelli's Theorem

Torricelli's Theorem Bernoulli's equation with certain assumption made.

Statement

The speed of efflux is equal to the velocity gained by the fluid in falling through the distance (h_1-h_2) under the action of gravity.
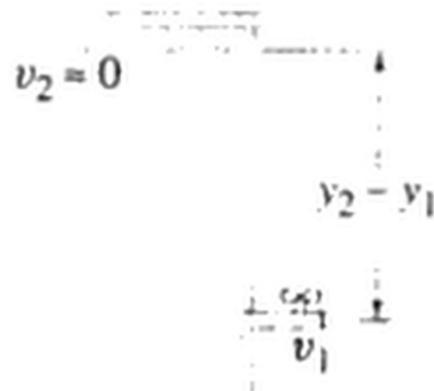
<p align="center">OR</p>

The speed of efflux of a liquid from an opening in a reservoir is equal to the speed that the liquid would acquire if allowed to fall from rest from the surface of the reservoir to the opening.

Proof

Consider a large tank of fluid having small orifice (hole) A on it, as shown in figure.

Let

$v_2 \approx 0$

$y_2 - y_1$

v_1

Area of cross-section of upper end of tank = A_1

Area of cross-section of lower small hole = A_2

Speed of the fluid at upper surface of tank = v_1

Speed of the efflux = v_2

Pressure at the upper end of tank = P_1

Pressure at the lower small hole of tank = P_2

Height of fluid at the upper end = h_1

Height of fluid at the lower end = h_2

Density of fluid = ρ

According to Bernoulli's equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 \qquad (1)$$

By equation of continuity

$$A_1 v_1 = A_2 v_2$$

Or $\qquad v_1 = \frac{A_2}{A_1} v_2$

Also, the pressure will be the same because they are open to the atmosphere.

So

$$P_1 = P_2 = P$$

Hence, equation (1) becomes

$$P + \frac{1}{2}\rho(0)^2 + \rho gh_1 = \frac{1}{2}\rho v_2^2 + \rho gh_2 \rho$$

Or

$$\rho gh_1 = \frac{1}{2}\rho v_2^2 + \rho gh_2$$

Dividing both sides by ρ, we get
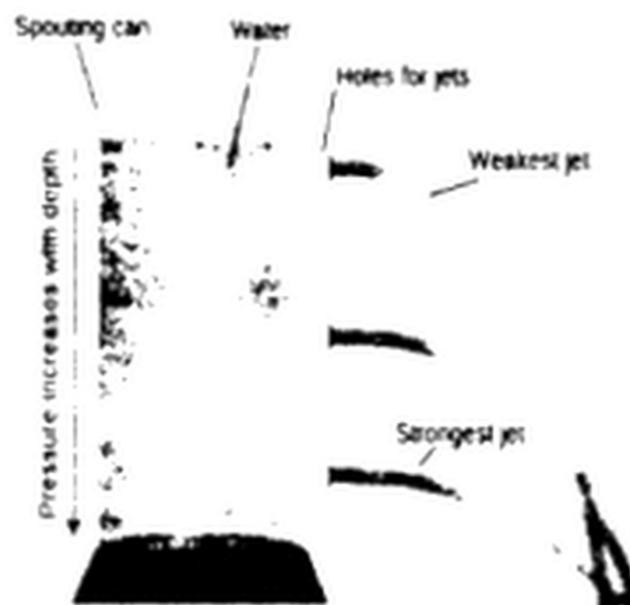
$$gh_1 = \frac{1}{2}\rho v_2^2 + gh_2$$

Or

$$\frac{1}{2}v_2^2 = gh_1 - gh_2$$

Or

$$\frac{1}{2}v_2^2 = g(h_1 - h_2)$$

Or

$$v_2^2 = 2g(h_1 - h_2)$$

Or

$$v_2 = \sqrt{2g(h_1 - h_2)} \qquad \ldots\ldots\ldots(3)$$

This is Torricelli's theorem.

Note:

* The speed of efflux is independent of the direction of flow whether the opening is directed upward, downward, or horizontally.

- Speed of efflux of liquid is the same as the speed of the ball that falls freely under the force of gravity through a height $(h_1 - h_2)$.

- If the hole is pointed upward as a **B** shown in figure, this K.W. would allow the liquid to rise to the level of the water tank.

- In practice, viscous energy losses would change the result to some extent.

Q.8   What is relation between pressure and speed of fluid in a horizontal pipe system?
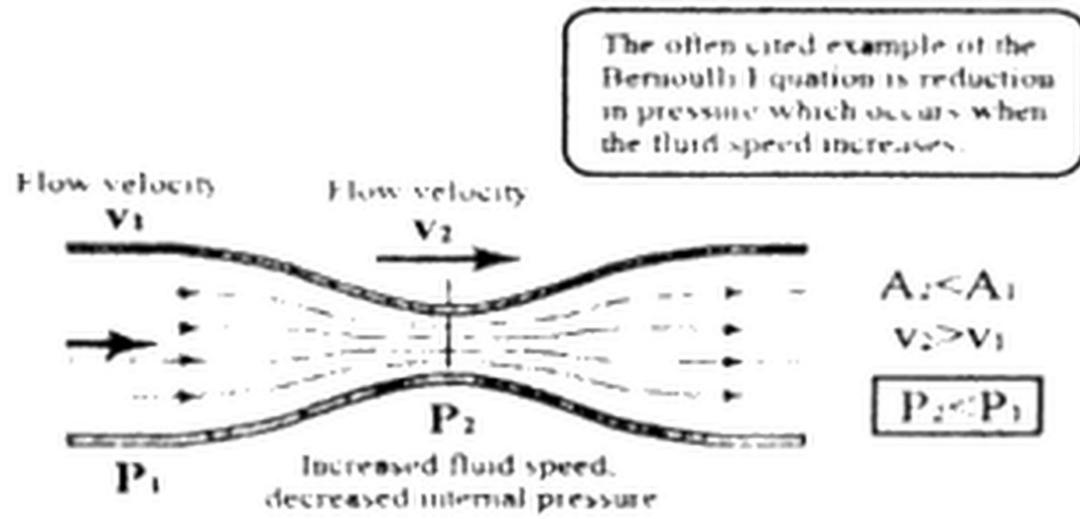
Answer

Relation between speed and pressure of the Fluid

Where the speed is high, the pressure will be low.

Proof

- Suppose that water flows through a horizontal pipe system as shown in figure.

- The area of cross-section of the pipe at A is greater than the area of cross-section at B.

- The speed of fluid at B is greater than at A. (By equation of continuity).

- Applying Bernoulli's equation at points A and B, we have

Flow velocity
v_1

Flow velocity
v_2

$A_2 < A_1$

$v_2 > v_1$

$P_2 < P_1$

P_2

Increased fluid speed, decreased internal pressure

P_1

$$P_A + \frac{1}{2}\rho v^2_{A} + \rho gh^2_{A} = P_B = \frac{1}{2}\rho v^2_{A} + \rho gh_B \qquad \dots\dots\dots\dots(1)$$

Where P_A = pressure of water at A

P_B = pressure of water at B

v_A = speed of water at A

v_B = speed of water at B

ρ = density of water = 1000 kg/m³

As the pipe system is horizontal, therefore $h_A = h_B$. Hence average P.E. is same at both ends. So equation (1) becomes

$$P_A + \frac{1}{2}\rho v^2_{A} = P_B = \frac{1}{2}\rho v^2_{A} \qquad \dots\dots\dots(2)$$
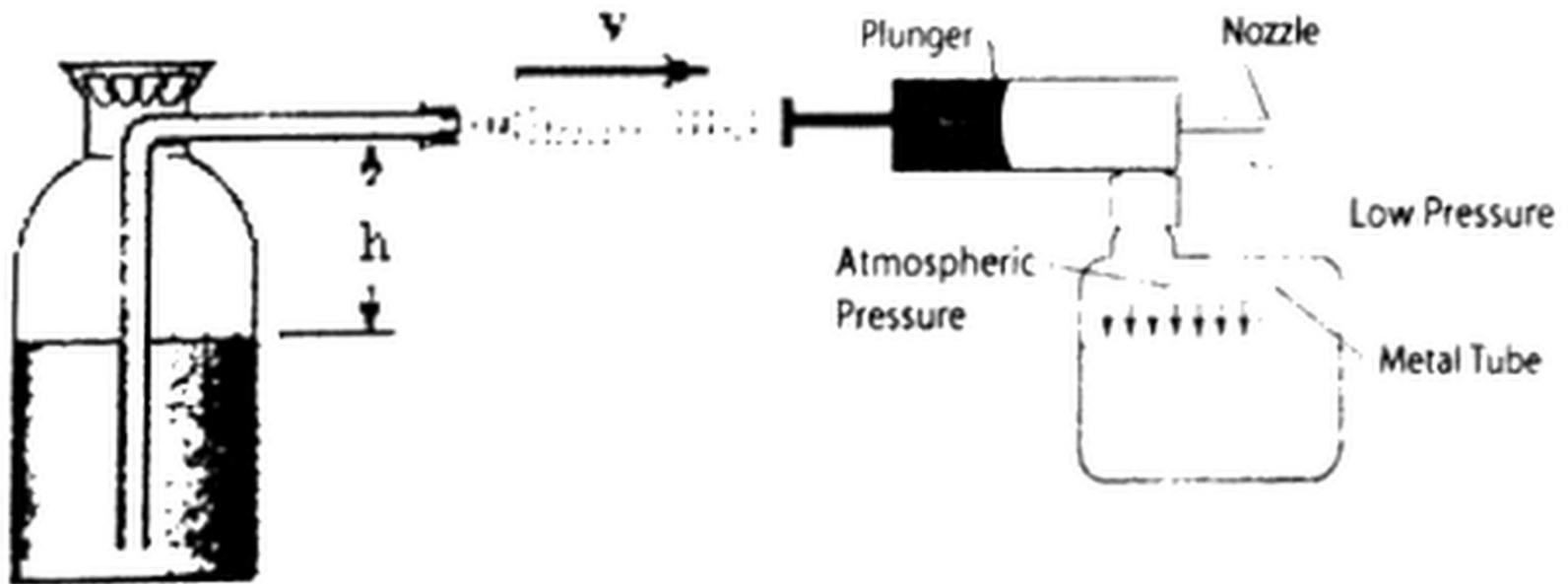
Also, let v_A = 0.20 m/s and v_B = 2.0 m/s

Using these values in equation (2), we have

$$P_A + \frac{1}{2}(1000)(0.20)^2 = P_B = \frac{1}{2}(1000)(2)^2$$

$$P_A + \frac{1}{2}(20) = P_B = \frac{1}{2}(2000)$$

$$P_A - P_B = 2000 - 20$$

$$P_A - P_B = 1980 N/m^2$$

This shows that the pressure in narrow pipe where stream lines are closer together is smaller than wider pipe. Thus
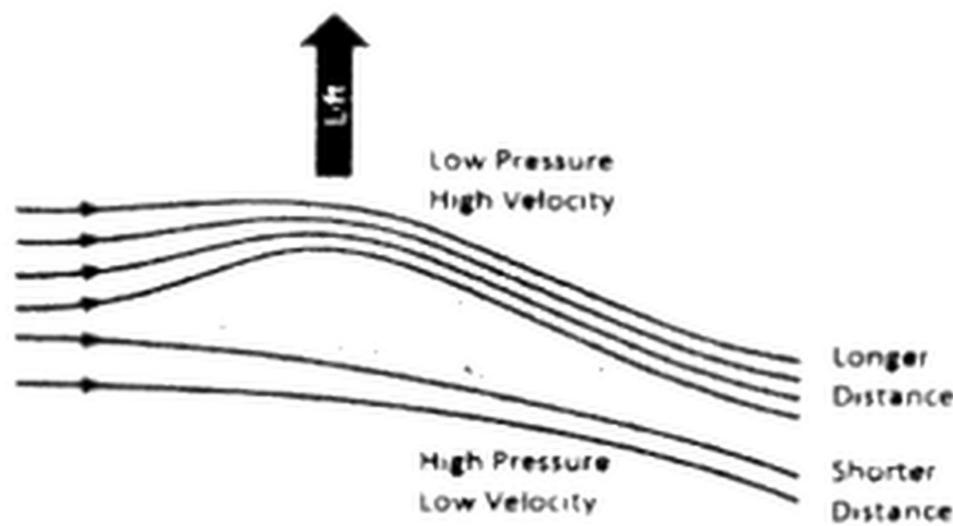
Where speed is high, the pressure will be low.

Q.9   How a dynamic lift is produced in an aeroplane?

Answer

Lift on an aeroplane

The lift on an aeroplane is due to the effect, where speed of fluid is high, its pressure will be low.

Aerodynamic Lift – Explained by Bernoulli's Conservation of Energy Law

Also known as the "Longer Path" or "Equal Transit" Theory

Explanation

The design of wing deflects the air in such a way that

- Stream lines are closer together above the wing than lower side.

- Air flows faster at the upper side of the wing than the lower side.

- Pressure is lower at the top of the wing and pressure is high below the wing.

- Hence, the wing experiences a net upward force.

Q.10   How swing produced in a tennis ball?

Answer

Swinging of ball

Consider a tennis ball moving through air from left to right (shown in figure) in such a way that it spins as well as move forward. The steam lines of air appear to move from right to left.

The layers of air near the surface of ball are pulled around in the direction of spin due to friction between the ball and the air.
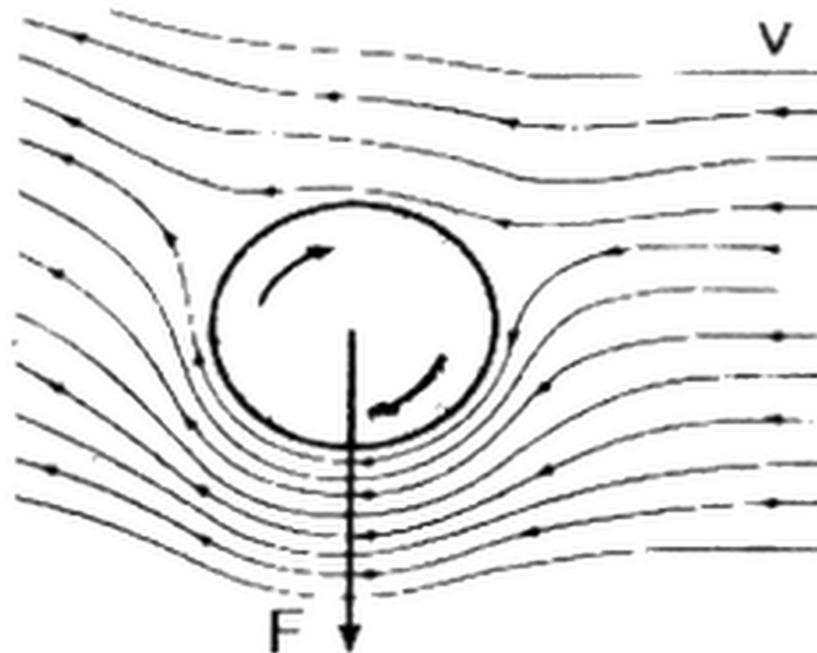
The speed of air at top of the ball increases than at bottom. In this case, the average pressure at the top of ball becomes smaller than that at the bottom. This gives an extra curvature to the ball known as swing, which deceives an opponent player.

Q.11 What is venture relation? Explain.

Answer

Venture Relation

The effect of decrease of pressure with increase of velocity of the fluid in horizontal pipe is called venture effect.

Suppose that an ideal fluid flows through a horizontal pipe system as shown in figure.

The area of cross-section of the pipe at A is greater than the area of cross-section at B.

The speed of fluid at B is greater than at A. [By equation of continuity]

Applying Bernoulli's equation at points A and B, we have

Working principle

Venturi meter works under the principle of Bernoulli's equation and continuity equation.

Increased fluid speed.
Decreased internal pressure.
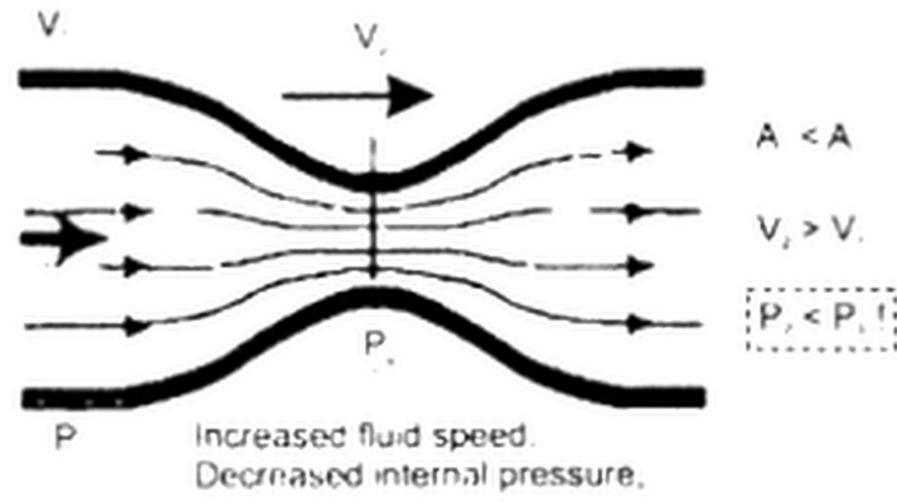
Working Principle

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2 \qquad \dots(1)$$

Where

P₁ = pressure of fluid at wide part of the pipe

P₂ = pressure of fluid at narrow part (called throat) of the pipe

v₁ = speed of fluid at wide part

v₂ = speed of fluid at narrow part

ρ = density of fluid

As the pipe system is horizontal, there h₁ = h₂

Hence average P.E. is same at both ends. So equation (1) becomes

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$$

Or $\qquad P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) \qquad \dots(2)$

This is called Venturi relation which used in Venture-meter to find the speed of fluid.

Special case

If A₁ >> A₂ therefore, v₁ << v₂.

Then, according to equation of continuity

$$A_1 v_1 = A_2 v_2$$

$$v_1 = \left(\frac{A_2}{A_1}\right) v_2 \approx 0$$

Hence equation (2) becomes,

$$P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - 0\right)$$

$$P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2\right) \qquad \ldots\ldots(3)$$

Venturi-meter

The device which measures the fluid speed is called Venturi meter. The working principle of Venture meter is Venturi relation.

Q.12   What do you know about blood pressure? How it can be measured dynamically? Answer

Blood Flow

Blood

- Blood is an incompressible fluid.

- Density of blood is nearly equal to that of water.

- Viscosity of blood increased three to five times that of water due to high concentration of red blood cells (~50%)

Blood Pressure (BP)

The pressure exerted by the circulating blood on the walls of the blood vessels is called blood pressure.

Blood vessels are not rigid.

- Under normal conditions, the volume of the blood is sufficient to keep the vessels inflated at all times.

- So, the pressure of the blood inside the vessels is greater than atmospheric pressure.

- During each heartbeat, BP varies between a maximum (systolic) and a minimum (diastolic) pressure.

- A person's BP is usually expressed in terms of the systolic pressure over diastolic pressure (mmHg), for example 120/80.
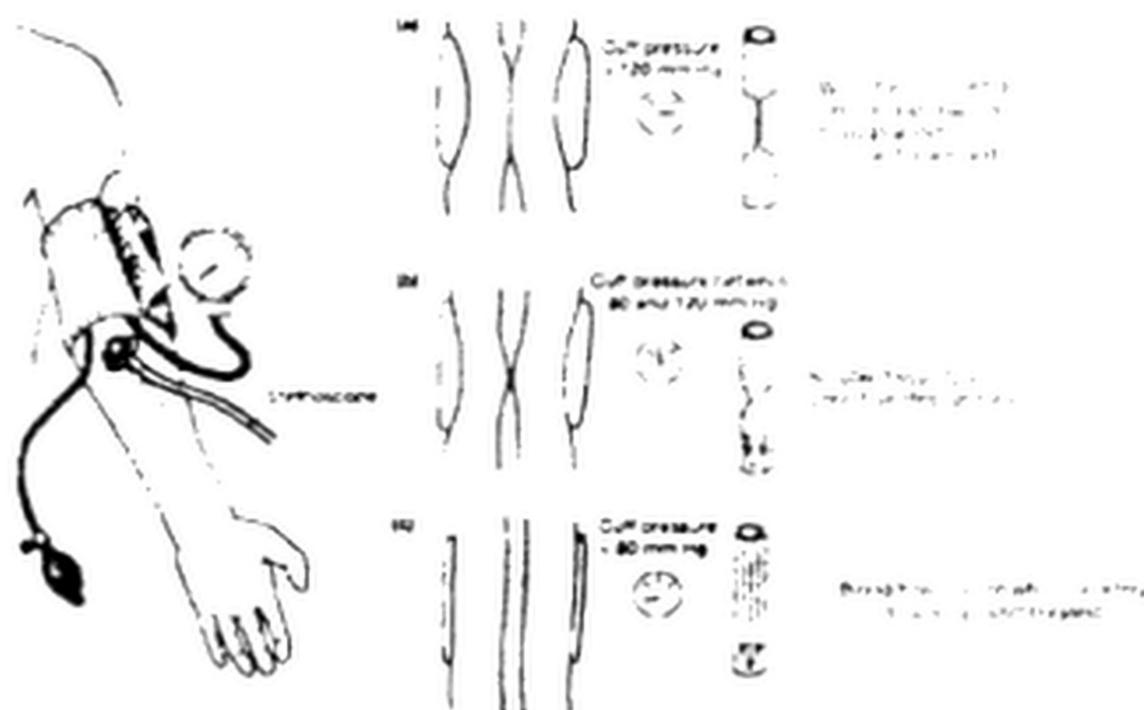
Systolic Pressure

- It represents the maximum pressure exerted when the heart contracts.
- The value of high blood pressure (systolic pressure) is 120 Torr.

Diastolic Pressure

- It represents the minimum pressure in the arteries when the heart is at rest.

- The value of low blood pressure (diastolic pressure) is about 75-80 Torr.

Note

The value of blood pressure increased with age due to decrease in the flexibility of vessels.

Unit of blood pressure

The blood pressure is measured in Torr or mm of Hg.

Relation between Torr and Pascal

1 Torr = 133.3 Pa = 133.3 N/m²

Measurement of Blood Pressure

Sphygmomanometer is used to measure the pressure of blood dynamically in the vessels.

Steps to measure blood pressure

- An inflatable bag is bound around the arm.

- The bag is inflated to increase the external pressure on the arm to compress the blood vessels inside.

- When the external pressure becomes larger than systolic pressure, the vessels fall down.

- The flow of blood is cut off.

- Head of a stethoscope is placed over the artery.

Systolic Pressure

- Open the release valve to decrease the external pressure gradually.

- When external pressure becomes equal to systolic pressure, the vessel opens a little bit.

- A first surge of blood flows out of the narrow opening of vessel with high speed.

- As the flow speed is high, so the flow is turbulent.

- At this instant, we hear a gurgle in the stethoscope.

- This is signal to record systolic pressure.

Diastolic Pressure

- Now decrease the external pressure further till it becomes equal to diastolic pressure.

- The vessel gets normal.

- The blood flow changes from turbulent to laminar

- The gurgle in the stethoscope disappear.

- This is time to record diastolic pressure.