Numerical Problems

1.   An object travels 60 m in first 6 sec. Then keeps uniform motion for next 90 sec. It decelerates in the last 10 sec of the journey and comes to rest.

Find   a) Maximum speed 'v'   b) Total distance travelled by the object d=?

Solution

a)   As the object moves with uniform speed up to 9 sec. So,

$$V = \frac{\text{Total distance}}{\text{Total time taken}}$$

$$V = \frac{S}{t} = \frac{(60+90)m}{(6+9)s}$$

$$V = \frac{150}{15}m/s$$

$$V = 10 m/s$$

b)   Total distance d = ?

i) Distance covered by the object 1ˢᵗ 9 sec.

$$d_i = 150 m \qquad\qquad (1)$$

ii) Since in last 10 sec it decelerates and come to rest. So

$$V_i = 10 m/s$$
$$V_f = 0 m/s$$
$$T = 10 sec$$
$$2ad_2 = V_f^2 - V_i^2$$
$$d_2 = \frac{V_f^2 - V_i^2}{2a} \qquad\qquad (2)$$

To find 'a' using 1st equation of motion

$$V_f = V_i + at$$
$$0 = 10 + a.10$$
$$a = -1 m/s^2$$

Putting values in equation (2)

$$d_2 = \frac{0 - 10^2}{2 \times (-1)}$$

$$d_2 = 50\,cm$$

2.   An object is travelling with constant acceleration of 10m/s². How much distance will it travelled in the 3ʳᵈ second of journey?

Data

Acceleration of the object $a = 10\,m/s^2$

Distance travelled by the object in 3ʳᵈ sec of journey d = ?

Solution

Let 'S_1' is the distance travelled by the object in 1ˢᵗ second i.e. $t_1 = 1$ sec.

Using 2ⁿᵈ equation of motion we get

$$S_1 = v_i t + \frac{1}{2} a t_1^2$$

Putting values

$$S_1 = 0 \times 1 + \frac{1}{2} \times 10(1)^2$$

$$S_1 = 5\,cm \qquad\qquad\qquad ....(1)$$

Now let "S_2" is the distance travelled by the object after time

$$T_2 = 2\,sec.$$

Using 2ⁿᵈ equation of motion we get

$$S_2 = 0 \times 2 + \frac{1}{2} \times 10 \times 4$$

$$S_2 = 20\,m \qquad\qquad\qquad .... (2)$$

Let S_3 is the distance travelled by the object after time $t_3 = 3$ sec i.e.

$$S_3 = v_i t_3 + \frac{1}{2} a t_3^2$$

$$S_3 = 0 \times 3 + \frac{1}{2} \times 10 \times (3)^2$$

$$S_3 = 45m \qquad\qquad \cdots\cdots (3)$$

The distance travelled by the object in 3rd second of journey will be

$$d = S_3 - S_2$$

$$= 30$$

$$d = 45\,m - 20\,m$$

$$\boxed{d = 25\,m}$$

3.   **A helicopter is ascending vertically at a speed of 19.6 m/s. When it is at a height of 156.8 m above the ground, a stone is dropped. How long does the stone take to reach the ground?**

Data

- Initial velocity of the helicopter v_h = 19.6 m/s
- Initial velocity of the stone relative to helicopter v_iS = -19.6m/s
- Height at which the stone is dropped h = 156.8m
- Time taken by the stone to reach the ground t = ?
- Acceleration due to gravity g = 9.8 m/s²

Solution

Using second equation of motion

$$H = v_i t + \frac{1}{2} t^2 \qquad\qquad \cdots\cdots (1)$$

Putting values we get

$$156.8 = -19.6 \times t + \tfrac{1}{2} \times 9.8t^2$$

or   R $4.9t^2 - 19.6t - 156.8 = 0$

Let   a = 4.9, b = -19.6 & c = -156 8

Using quadratic formula i.e. $t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Putting values in Equation (3)

$$t = \dfrac{-(-19.6) \pm \sqrt{(-19.6)^2 - 4(4.9)(-156.8)}}{2(4.9)}$$

$$t = \dfrac{19.6 + \sqrt{3457.44}}{9.8} \quad \& \quad t = \dfrac{19.6 - \sqrt{3457.44}}{9.8}$$

$$t = 8\,\text{sec} \quad \& \quad t = -4\,\text{sec}$$

Since time cannot be negative, so the required time taken by the stone to reach the ground is $\boxed{t = 8\,\text{sec}}$

**4.   Using the following date, draw velocity time graph for short journey on a straight road of a motor bike.**

Velocity	0	10	20	20	20	20	0
Time	0	30	60	90	120	150	180

Use the graph to calculate

a)   Initial acceleration a_i =?

b)   Final acceleration a_f =?

c)   Total distance travelled d = ?

Solution

Using the above data first draw velocity time graph. Time is taken along x-axis and velocity is taken along y-axis.
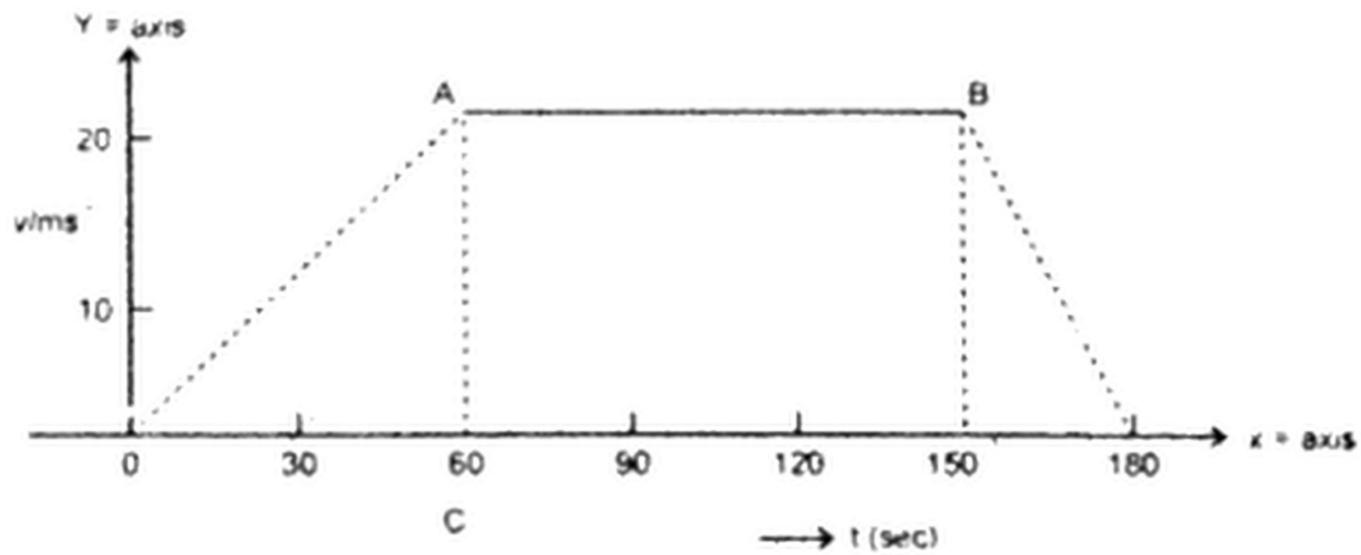
a)   From the graph velocity of motor bike increases from 0 m/s to 20 m/s in time interval 60 sec i.e. 0 -----A

Since acceleration is equal to the slope of the graph i.e.

$$\text{Slope} = \frac{AC}{OC}$$

or $$a_i = \frac{20m/s^2}{60s}$$

$$a_i = 0.33 m/s^2$$

i)   From point A to B velocity is constant and from B to C the magnitude of velocity decreases at point E final velocity is zero.

Using graph

$$a_r = \frac{v_f - v_i}{M}$$

$$a_r = \frac{0m/s - 20m/s}{30 sec}$$

$\boxed{a_r = -0.67 m/s^2}$ Negative sign shows retardation.

ii)   The total distance covered by motor bike is numerically equal to the area under the velocity time graph. From the graph,

d = area of ∆OAC + Area of rectangle

$$= ABCD + \text{area of } \Delta BDE$$

OR

$$d = \frac{1}{2} \times OC \times AC + CD \times BD + \frac{1}{2} \times DE \times BD$$

$\Rightarrow$

$$d = \frac{1}{2} \times 60 \times 20 + 90 \times 20 + \frac{1}{2} \times 30 \times 20$$

$$\boxed{d = 2700 \text{m}}$$

5. A proton moving with a speed of 1.0×10^7 ms passes through a 0.020 cm thick sheet of paper and emerges with a speed of 2.0×10^6 ms^{-1} assuming uniform declaration, find retardation and time taken to pass through the paper.

Data

Initial speed of the proton $v_i = 1.1 \times 10^7$ m/s

Final speed of the proton $v_f = 2.0 \times 10^6$ m/s

Thickness of the sheet $X = S = 0.020$ cm $= 2 \times 10^{-4}$

i) Retardation of Proton a=?

ii) Time taken by the proton to pass through the paper 't' = ?

Solution

i) Using 3rd equation of motion i.e

$$2aS = v_f^2 - v_i^2$$

$$2 \times a \times 2 \times 10^{-4} = (2 \times 10^6)^2 - (1 \times 10^7)^2$$

$$a = \frac{4 \times 10^{12} - 1 \times 10^{14}}{4 \times 10^1}$$

$$\boxed{a = -2.4 \times 10^{17} \text{m/s}}$$

ii) To find time taken by proton using first equation of motion i.e.

$$t = \frac{v_f - v_i}{a}$$

$$t = \frac{2 \times 10^6 - 1 \times 10^7}{-2.4 \times 10^{17}}$$

$$\boxed{t = 3.3 \times 10^{11} \, sec}$$

6. **A constant force "F" changes the speed of an 80 kg sprinter from 3m/s to 4 m/s in 0.5 sec. Calculate the acceleration of the sprinter?**

Data

Mass of the sprinter m = 80 kg

Initial speed of the sprinter v_i = 3 m/s

Final speed of the sprinter v_f = 4 m/s

Time taken by the sprinter t = 0.5 sec

Acceleration of the sprinter a = ? Solution

Solution

As

$$a = \frac{v_f - v_i}{t}$$

Putting values we get

$$a = \frac{4m/s - 3m/s}{0.5s}$$

$$\boxed{a = 2m/s}$$

7.   A 4kg ball travelling with the speed of 2m/s strikes a rigid wall and rebounds elastically. If the ball is in contact with the wall for 0.050 s. what is

a. Momentum imparted to the wall?

b. The average force exerted on the wall=?

Data

Mass of the ball m = 4kg

Initial speed of the ball v = 2m/s

Time of contact of ball with wall t = 0.05sec

Solution

a) Momentum imparted to the wall

$$\Delta P = 2 \text{ m/s}$$

$$= 2 \times 4 \times 2$$

$$\boxed{\Delta P = 16 \text{ kgm/s}}$$

b) The average force exerted on the wall

$$F = \frac{\Delta P}{\Delta t} = \frac{16}{0.05}$$

$$\boxed{F = 320 \text{N}}$$

8.   A projectile is thrown with a speed of 100 m/s at 30° with the horizontal.

Find

a) Time of flight

b) Maximum height it raised

c) The range travelled.

Data

Initial Speed V_i = 100 ms

Angle of project θ = 30°

Acceleration due to gravity g = 9.8 m/s²

a) As $\qquad T = \dfrac{2v_i \sin q}{g}$

$$T = \dfrac{2 \times 100 \times \sin 30°}{9.8}$$

$$\boxed{T = 10.20 \sec}$$

b) Maximum height H = ?

Since $\qquad H = \dfrac{v_i^2 \sin \theta^2}{2g}$

$$H = \dfrac{(100)^2 \times (\sin 30)^2}{2 \times 9.8}$$

$$\boxed{H = 127.55 m}$$

c) $\qquad$ Range R = ?

$$R = \dfrac{v_i^2 \sin^2 q}{g}$$

$$R = \dfrac{(100)^2 \times \sin 60°}{9.8}$$

$$\boxed{R = 883.67 m}$$

9. **The maximum height gained by a projectile is 300 m. If it travels a range of 800m. Find the displacement of summit point from the point of project.**
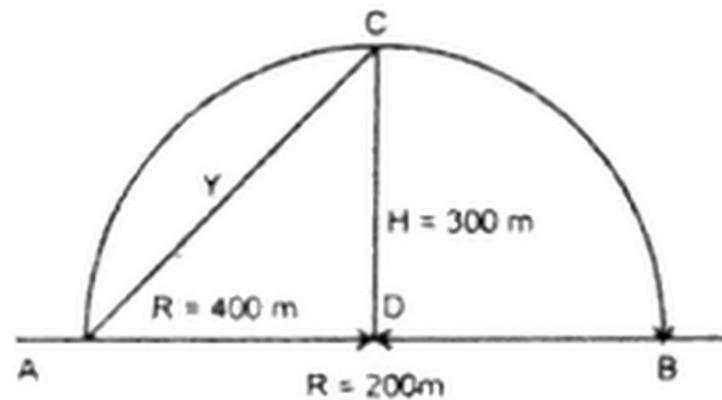
Data

Maximum height gained by the projectile H = 300m

Range of the projectile R = 800m

Displacement of summit point from the point of projection "y" = ?

Solution

Consider in the fig the summit point C.

Led AD = X = 400m

DC H = 300m

AC = y = ?

From triangle ADC

$$(AC)^2 = (AD)^2 + (DC)^2$$

$$Y^2 = x^2 + H^2$$

Putting values we get

$$Y = \sqrt{(400m)^2 + (300m)^2}$$

$$\boxed{Y = 500m}$$

10.   Calculate the angle of project of which K.E at the highest point of its trajectory is equal to one for its K.E at point of project.
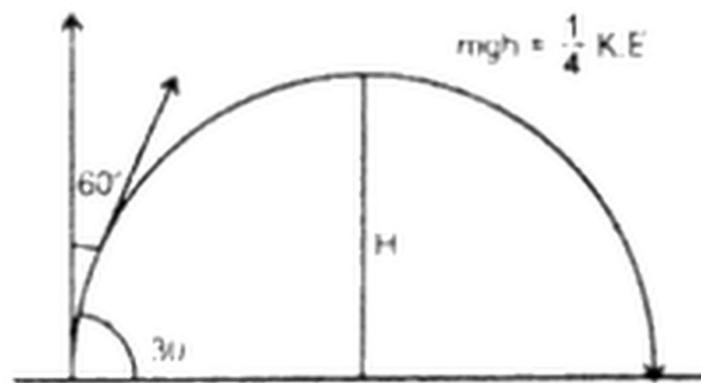
Data

Kinetic energy at the point of projection = ½ mv²

Kinetic energy at maximum height K.E = ¼ K.E

Angle of projection 'θ' =?

Solution

According to the given condition

$$P.E_{max} = \frac{1}{4} \times K.E$$

$$\Rightarrow mgH = \frac{1}{4} \times \frac{1}{2} mv_i^2$$

$$mgH = \frac{1}{8} mv_i^2$$

$$g \times \frac{v_i^2 \sin q}{2g} = \frac{1}{8} v_i^2$$

$$\sin^2 \theta = \frac{2}{8}$$

$$\sin^2 \theta = \frac{1}{4}$$

$$\sin \theta = \frac{1}{2}$$

$$\theta = \sin^{-1}\left(\frac{1}{2}\right)$$

$$\boxed{\theta = 30°}$$