UNIT 2

VECTORS & EQUILIBRIUM

Q.1 How is a vector represented?

Answer

Vector Representation

A vector is represented in two ways

(i) Symbolic representation **(ii)** Graphical representation

Symbolic Representation

- It is represented by bold face letter such as **A, d, r** and **V** etc.
- It can also be represented by a letter with an arrow placed above or below letter such as $\bar{A}$ or $\underset{\rightarrow}{A}$

Graphical Representation

- It is represented by a straight line with an arrow head.
- The length of line represents magnitude of vector (according to suitable scale)
- Arrow head represents the direction of vector.

Representation magnitude of vector

The magnitude of vector is represented by:

- **Light face letter such as A, d, r and v.**

Modulus of a vector such as $|v|$

Q.2 What is rectangular coordinate system?

Answer

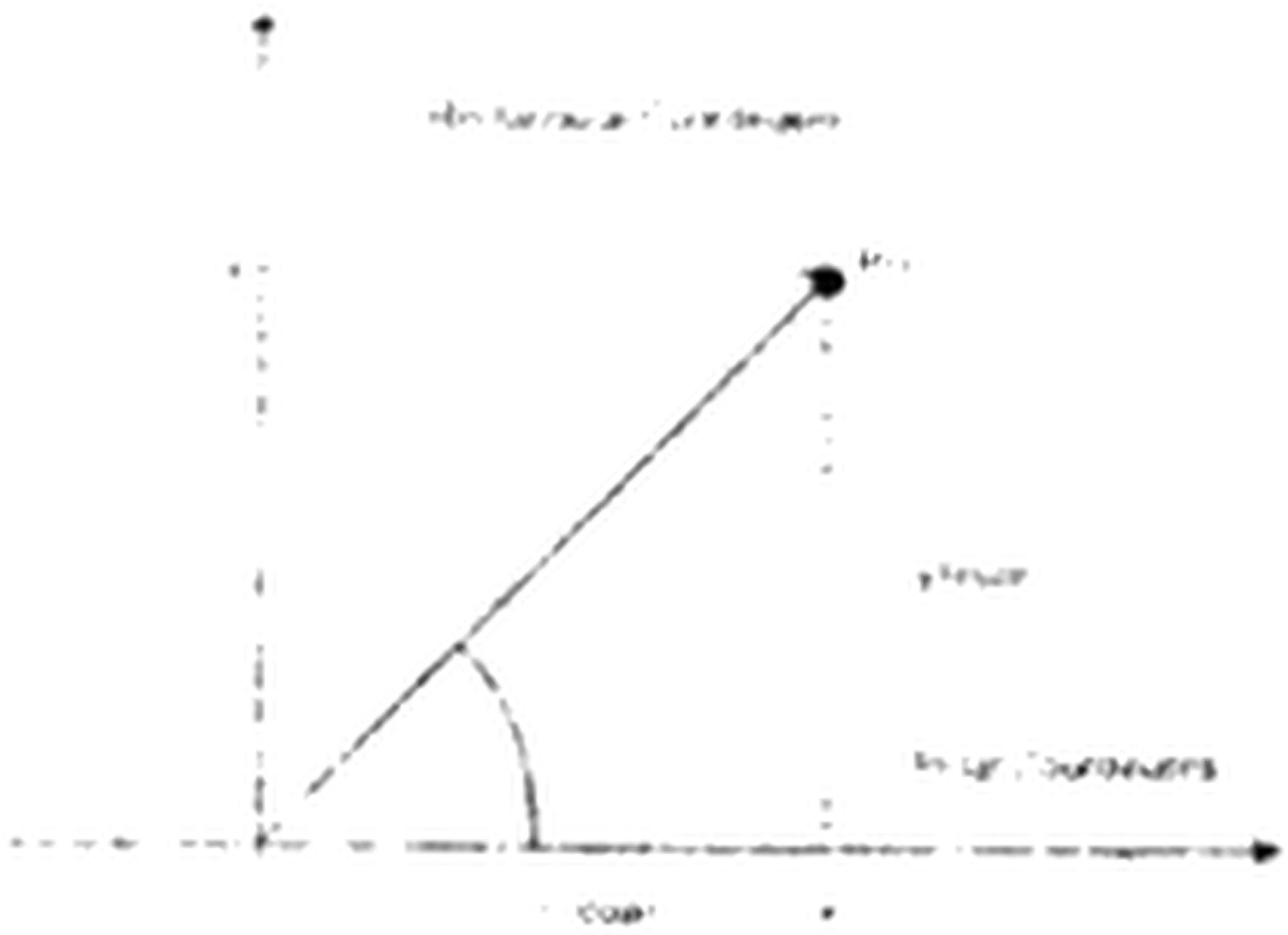
Rectangular Coordinate System (Cartesian Co-ordinate System)

The set of two or three mutually perpendicular lines intersecting at a point is called rectangular system.

- The lines are called coordinate axes.
- ✓ One of these lines is called x-axis (or horizontal axis)
- ✓ The other is called y-axis (or vertical axis)
- ✓ The line perpendicular to both x and y axes is called z-axis.
- The point of intersection is called origin.

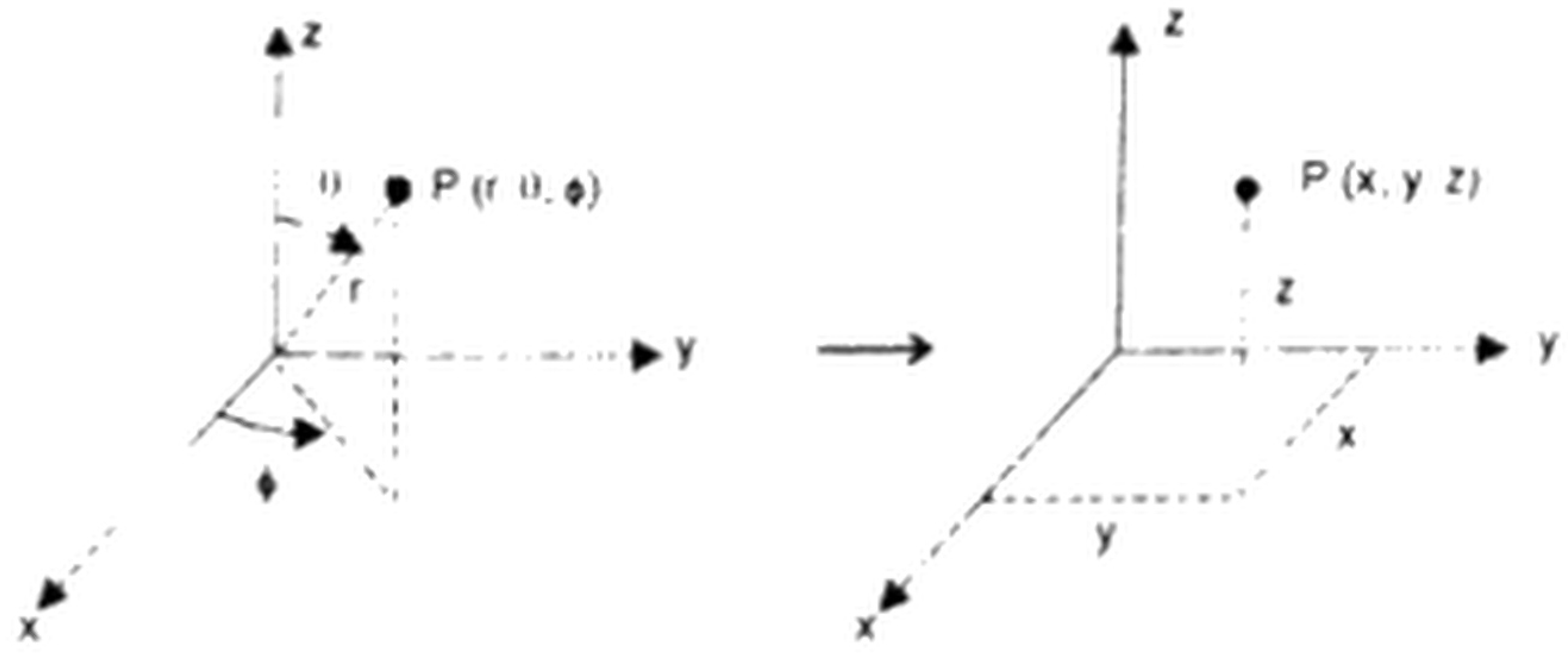
Two-dimensional coordinate system (Plane)

If the system consists of two perpendicular lines then it is called two-dimensional coordinate system.



Three-dimensional co-ordinate system (Space)

If the system consists of three perpendicular lines, then it is called three-dimensional co-ordinate system.



Q.3 How is the direction of vector represented in: (i) plane (ii) space?

Answer

Direction of a Vector in plane

It is represented by the angle which the vector makes with

- Positive x-axis
- In anti-clock wise direction

Direction of a vector in Space

It is represented by three angles which the vector makes with x, y and z axes.

Q.4 Describe the addition of vectors by head to tail rule. Is vector addition commutative?

Answer

Head to Tail Rule

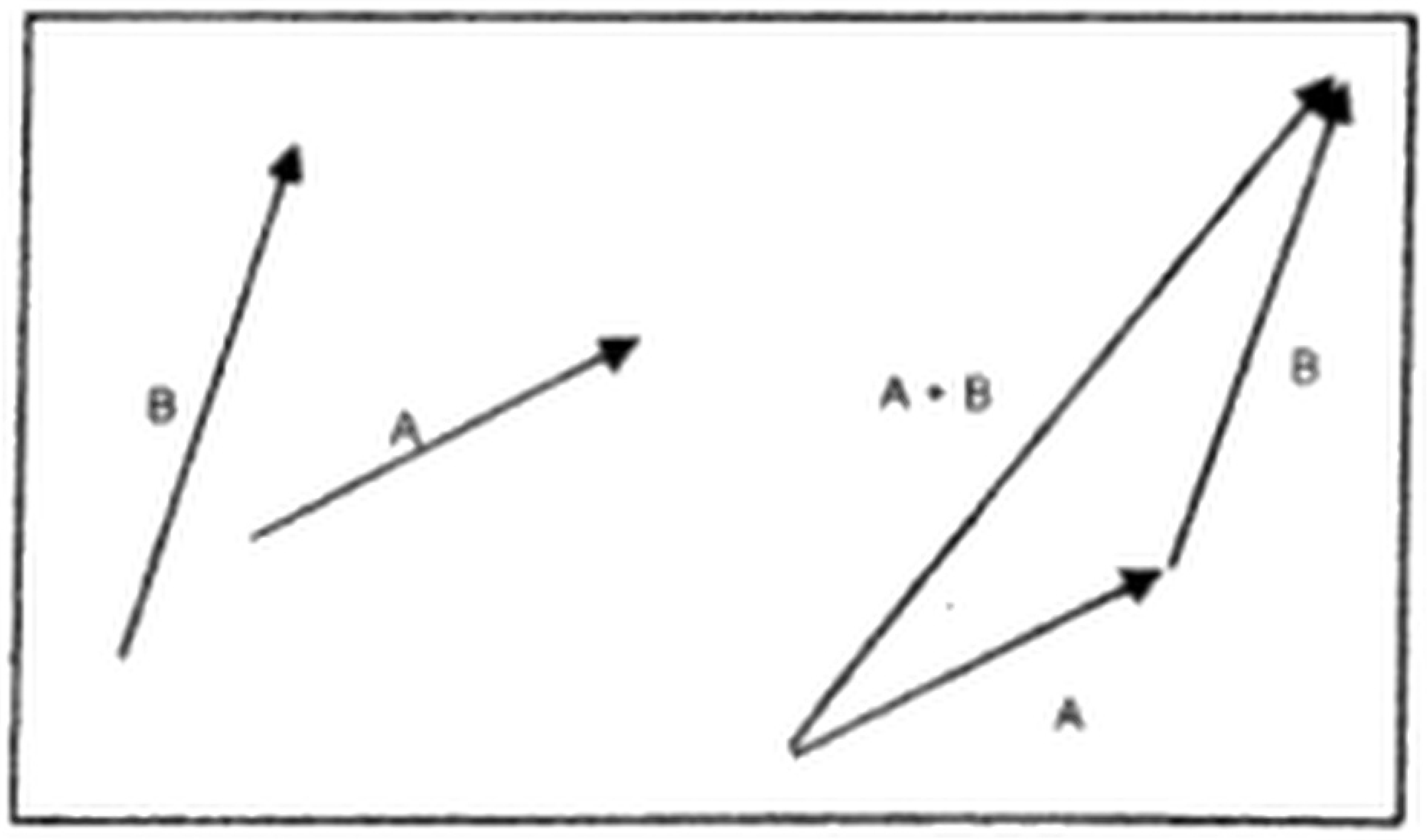
It is a graphical method to add of two or more vectors.

Explanation

- Draw the represented lines of vectors $\vec{A}$ and $\vec{B}$.

- Join the tail of vector $\vec{B}$, with the head of vector $\vec{A}$.
- Join the tail of vector $\vec{A}$ with head of vector $\vec{B}$.
- This line gives the magnitude of resultant vector $\vec{R}$.

The direction of resultant vector is from tail of vector $\vec{A}$ to the head of vector $\vec{B}$.



Vector addition is commutative or not:

Yes, it is commutative.

Explanation

From figure, it is clear that either we add $\vec{A}$ to $\vec{B}$ or $\vec{B}$ to $\vec{A}$, the resultant is same

i.e. $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

It means that when vectors are added, the result is the same for any order of addition.

Q.5 Explain the following terms:

(i) Resultant vector	**(ii) Vector subtraction**
(iii) Multiplication of vector by a scalar	**(iv) Unit vector**
(v) Null vector	**(vi) Equal vectors**

Answer

(i) Resultant vector

A vector which has the same effect as the combined effect of all the vectors to be added is called resultant vector.

(ii) Vector subtraction

The subtraction of a vector is equivalent to the addition of same vector with its direction reversed.

Explanation

Consider two vectors $\vec{A}$ and $\vec{B}$

To subtract $\vec{B}$ from $\vec{A}$

- First, take the negative of vector $\vec{B}$.
- Add $(-\vec{B})$ into vector $\vec{A}$ graphically as shown in fig. so

  $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

(iii) Multiplication of vector by a scalar

A vector can be multiplied by

(i) A positive number.

(2) A negative number.

(iii) A scalar with dimension.

(i) Multiplication with positive number

When a vector $\vec{A}$ is multiplied by a positive number n (i.e. n>0) then the product vector will have: -

- Magnitude equal to nA.
- Same direction as that of $\vec{A}$.

(i) Multiplication with negative number

When a vector $\vec{A}$ is multiplied by a negative number n (i.e. n<0), then the product vector will have:

- magnitude equal to nA.
- direction opposite to that of $\overline{A}$.

(i) Multiplication with scalar with dimension

When a vector $\overline{A}$ is multiplied by a scalar quantity n.

Then the product vector

- will be a new physical quantity.
- have dimensions equal to the product of dimensions of n and $\overline{A}$.

Examples

- Product of mass m and velocity $\vec{v}$ is momentum [$\vec{p} = m\vec{v}$]
- Product of mass m and acceleration $\overline{A}$ is force [$\vec{F} = m\vec{a}$]
- **Product of force $\vec{F}$ time t impulse [$\vec{I} = \vec{F} \times t$]**

(iv) Unit vector

A vector whose magnitude is equal to one with no units in a given direction is called unit vector. It is represented by a letter with a cap or hat on it.

If $\overline{A}$ is a vector with magnitude A . $\overline{A}$ = A . A **

OR $A = \frac{\vec{A}}{A}$ **

Examples

- Unit vectors *I, j. k* along x, y and z-axes respectively.

- Unit vector $\hat{r}$ represents the direction of $\vec{r}$.
- Unit vector $\hat{n}$ represents the direction of normal drawn on a certain surface.

(v) Null or Zero vector

A vector whose magnitude is zero and direction arbitrary is called a null vector. It is represented by $\vec{O}$.

Example

Sum of vector $\bar{A}$ and its negative vector $(-\bar{A})$ is a null vector i.e.

$$\bar{A} + (-\bar{A}) = \bar{O}$$

- Sum of vectors taken head to tail along the sides of a closed polygon is null vector.
- Vertical velocity of projectile at the highest point is null vector.
- Position vector of origin of rectangular co-ordinate system.

(vi) Equal vectors

The vectors are said to be equal vectors if they have:

- same magnitude.
- same direction.

(regardless of the position of their initial points)

Example

Two parallel vectors $\bar{A}$ and $\bar{B}$ of same magnitude and direction are equal vectors.

Q.6 Define component of vector? what are rectangular components of vector?

Answer

Component of a vector

The effective value of a vector in a given direction is called component of a vector. A vector may split up into two or more than two parts these parts are known as components of vector.

Rectangular Components of Vector

The components of a vector which are perpendicular to each other are called rectangular components.

Explanation

Consider a vector A as shown in figure below:

Let θ-angle which $\overline{A}$ makes with x-axis

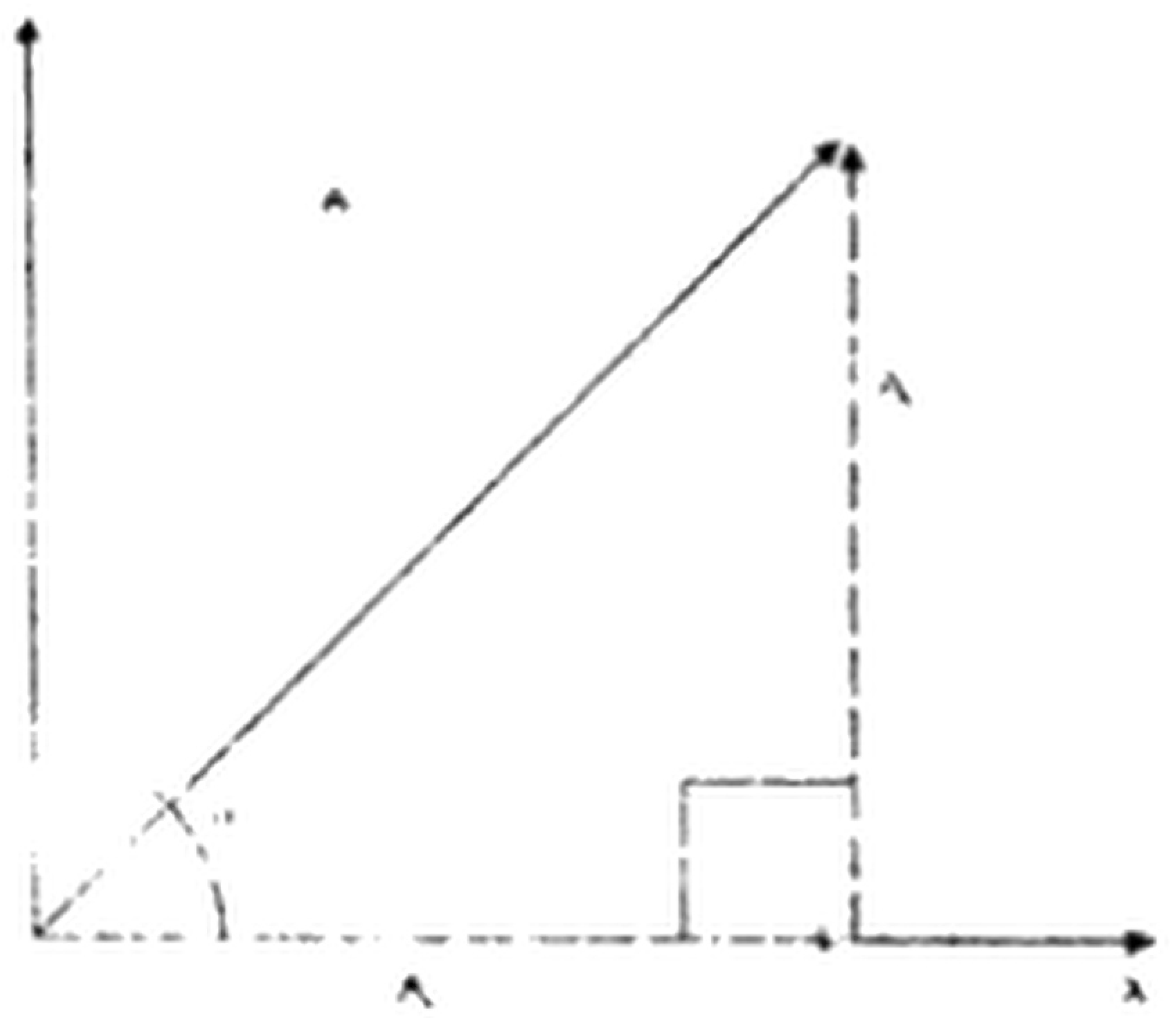
OM = projection of vector $\overline{A}$ on x-axis

ON = projection of vector $\overline{A}$ on y-axis.

As ON = MP

Thus, by head to tail rule

OR $\overline{A} = A_x i + A_y j$ ...... (1)



Thus, $A_x i$ and $A_y j$ are the components of vector $\vec{A}$. Since $A_x i$ and $A_y j$ are at right angle to each other, so they are called rectangular components.

X-component of $\vec{A}$

In right angled triangle OPM,

$$\frac{OM}{OP} = \cos\theta$$

OR $\frac{A_x}{A} = \cos\theta$

OR $A_x = A\cos\theta$ ....(2)

y-component of $\vec{A}$

In right angled triangle OPM,

$$\sin\theta = \frac{PM}{OP}$$

OR $\sin\theta = \frac{A_y}{A}$

$A_y = A\sin\theta$ ....(3)

(ii) Vector $\vec{A}$ from its rectangular components

Magnitude of $\vec{A}$

In right angled triangle OPM.

$OP^2 = OM^2 + MP^2$ (By Pythagorean Theorem)

OR $A^2 = A_x{}^2 + A_y{}^2$

$A = \sqrt{A_x{}^2 + A_y{}^2}$ ....(4)

Direction of $\vec{A}$

Again in right angled triangle OPM,

$$\tan\theta = \frac{MP}{OM}$$

OR $$\tan\theta = \frac{A_y}{A_x}$$

$$\theta = \tan^{-1}\frac{A_y}{A_x} \qquad ....(5)$$

Q.7. Define position vector.

Answer

Position Vector

The vectors which represents the position of a point or a particle with respect to fixed origin is called position vector. It is denoted by $\vec{r}$.

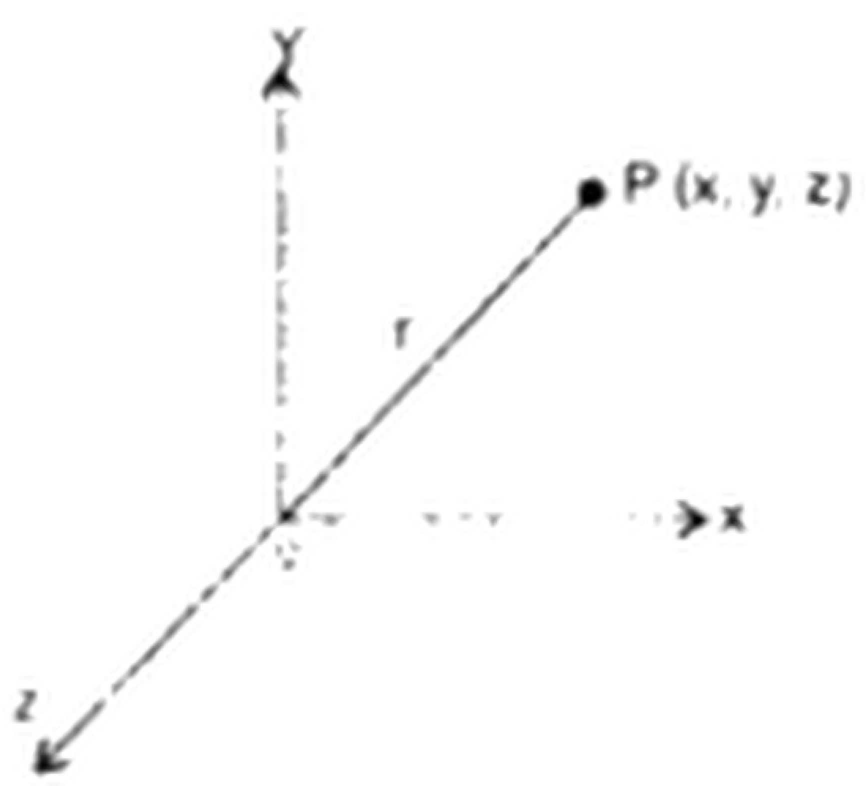
Explanation

In two-dimensional coordinate system (plane), the position of a point P (a, b) is represented by

$$\vec{r} = ai + bj$$

The magnitude of this position vector is

$$r = \sqrt{a^2 + b^2}$$



In three-dimensional coordinate system (space), the position of a point P (a, b, c) is represented by:

$$\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$$

And its magnitude is

$$r = \sqrt{a^2 + b^2 + c^2}$$

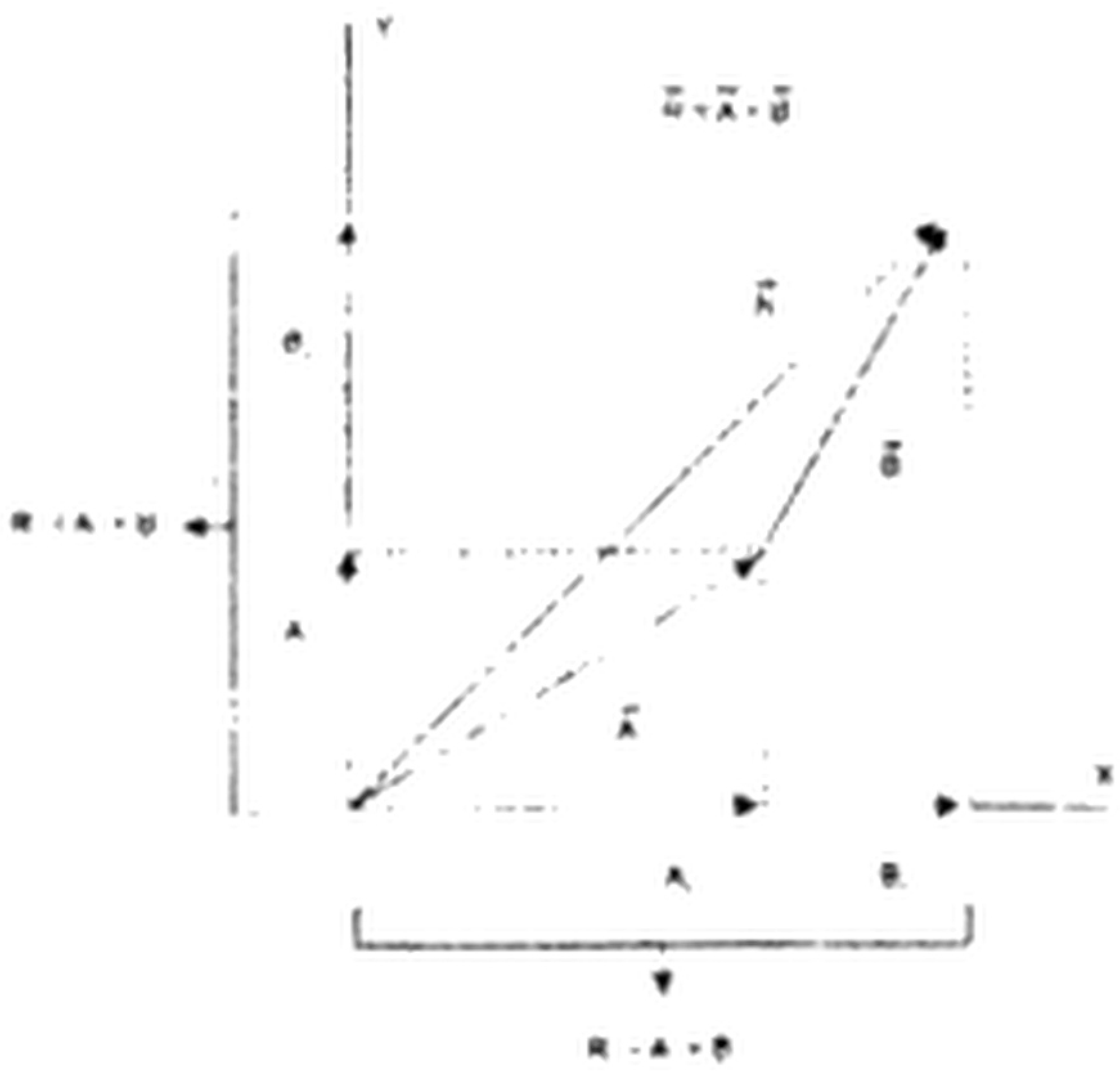
Q.8. Describe the method of addition of vectors by rectangular components.

Answer

Vector Addition by Rectangular Components

Consider two vectors $\vec{A}$ and $\vec{B}$ as shown in figure. The resultant of these two vectors is

$\vec{R} = \vec{A} + \vec{B}$ (by head to tail rule)



Resolve the vectors $\vec{A}$, $\vec{B}$ and $\vec{R}$ into rectangular components as shown in figure. Let

x- Component of Resultant

OQ = magnitude of x-component of vector $\vec{A}$

QR = MS = magnitude of x-component of vector $\vec{B}$

OR = magnitude of x-component of vector $\vec{R}$

From figure

$$RP = RS + SP$$

$$RP = QM + SP \text{ [since } RS = QM]$$

$$R_y = A_y + B_y \qquad \text{.........(2)}$$

This shows that the magnitudes of the sum of y-components of $\vec{A}$ and $\vec{B}$ is equal to the magnitude of y-component of resultant vector $\vec{R}$.

Since $R_x\hat{i}$ and $R_y\hat{j}$ are the rectangular component of resultant vector $\vec{R}$, hence

$$\vec{R} = R_x\hat{i} + R_y\hat{j}$$

Putting values of R_x and R_y from equation (1) and (2) we get,

$$\vec{R} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

Magnitude of Resultant Vector $\vec{R}$ is

$$R = \sqrt{R_x^2 + R_y^2}$$

$$R = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2} \qquad \text{.........(3)}$$

Direction of Resultant Vector $\vec{R}$

The direction of the resultant vector $\vec{R}$ is given by

$$\tan\theta = \left[\frac{R_y}{R_x}\right]$$

OR

$$\theta = \tan^{-1}\left[\frac{R_y}{R_x}\right]$$

$$\theta = \tan^{-1}\left[\frac{A_y + B_y}{A_x + B_x}\right] \qquad \text{........(4)}$$

In General

For any number of coplanar vectors, A, B, C, we can write

$$R = \sqrt{(A_x + B_x + C_x + ...)^2 + (A_y + B_y + C_y + ...)^2}$$

$$\theta = \tan^{-1}\left[\frac{A_y + B_y + C_y + ...}{A_x + B_x + C_x + ...}\right]$$

Q.9. Write summary of method of addition of vectors by rectangular components.

Answer

Summary

(i) Find the x and y-components of all given vectors.

(ii) Add x-components of all the vectors to find the x-components R_x of the resultant vector.

(iii) Add x-components of all the vector to find the y-component R_y of the resultant vector.

(iv) Find the magnitude of resultant vector $\vec{R}$ by using

$$R = \sqrt{R_x^2 + R_y^2}$$

(v) Find the direction of resultant vector $\vec{R}$ by using

$$\theta = \tan^{-1}\left[\frac{R_y}{R_x}\right]$$

Q.10. How can you determine the angel θ of the vector $\vec{R}$ by its rectangular components?

Answer

Determination of Angle (θ)

First find ϕ by the following relation.

$$\phi = \tan^{-1}\left[\frac{R_y}{R_x}\right]$$

Where ϕ = the angle which R makes with nearest x-axis.

(1) By the signs of R_x and R_y , find the quadrant in which $\vec{R}$ lies as follow:

- If both R_x and R_y are positive, the resultant lies in the 1st quadrant and its direction is $\theta = \phi$
- If R_x is -ve and R_y is +ve, the resultant lies in 2nd quadrant and its direction is
  $\theta = 180° - \phi$
- If both R_x and R_y are –ve, the resultant lies in 3rd quadrant and its direction is
  $\theta = 180° + \phi$
- If R_x is +ve and R_y is -ve, the resultant lies in **4th** quadrant and its direction is
  $\theta = 360° - \phi$

Multiplication of two vectors

Vectors can be multiplied in two ways:

(i) Scalar Product **(ii)** Vector Product

Q.11. Define scalar product of two vectors. Give examples.

Answer

Scalar Product (Dot Product)

If the product of two vectors is a scalar quantity then the product is called scalar product.

Definition

Scalar product of two vectors $\vec{A}$ and $\vec{B}$ is defined as

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

Where A and B are the magnitudes of vectors $\vec{A}$ and $\vec{B}$ and θ is the angle between them.

Physical meaning

From figure

$$\vec{A} \cdot \vec{B} = A \text{ (projection of } \vec{B} \text{ on } \vec{A}\text{)}$$

$$\vec{A} \cdot \vec{B} = A \text{ (magnitude of component of } \vec{B} \text{ along } \vec{A}\text{)}$$

$$\vec{A} \cdot \vec{B} = A\,(B\cos\theta)$$

Similarly

$$\vec{B} \cdot \vec{A} = B \text{ (Projection of } \vec{A} \text{ on } \vec{B}\text{)}$$

$$\vec{B} \cdot \vec{A} = B \text{ (magnitude of component of } \vec{A} \text{ along } \vec{B}\text{)}$$

$$\vec{B} \cdot \vec{A} = B\,(A\cos\theta)$$

Examples

(1) Work is scalar product of force and displacement [$W = \vec{F} . \vec{d}$]

(2) Power is scalar product of force and velocity [$P = \vec{F} . \vec{v}$]

(3) Electric flux is scalar product of electric intensity and vector area [$\phi = \vec{E} . \vec{A}$]

(4) Magnetic flux is scalar product of magnetic field strength and vector area

[$\phi = \vec{B} . \vec{A}$]

Q.12. Write down the characteristics of scalar product of two vectors.

Answer

Characteristics of Scalar Product

(i) Commutative property

Scalar product of two vectors is commutative i.e.

$$\vec{A} . \vec{B} = \vec{B} . \vec{A}$$

(ii) Perpendicular vectors

If two vectors are mutually perpendicular ($\theta = 90°$) to each other then, their scalar product is zero i.e.:

$$\vec{A} . \vec{B} = AB\cos 90°$$

$$\vec{A} . \vec{B} = AB(0)$$

$$\vec{A} . \vec{B} = 0$$

In case of units' vectors,

$$\hat{i} . \hat{j} = (1)(1) \cos 90° = (1)(1)(0) = 0$$

Similarly

$\hat{j}.\hat{k} = 0$ and $\hat{k}.\hat{i} = 0$

Thus

$\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

(iii) Parallel and anti-vectors

If two vectors are parallel (θ = 90°) to each other then their scalar product is equal to the product of their magnitudes i.e.

$$\bar{A}\ .\ \bar{B} = AB\cos\theta° = AB(1) = AB$$

This is the positive maximum value of scalar product.

If two vectors are anti-parallel (θ =180°) then their scalar product is negative.

$$\bar{A}\ .\ \bar{B} = AB\cos 180° = AB(-1) = -AB$$

This is the negative maximum value of scalar product.

(iv) Self Scalar Product

The self-product of a vector is equal to square of its magnitude

$$\bar{A}\ .\ \bar{A} = A\cos\theta° = A^2(1) = A^2$$

$$A^{**} = \sqrt{\vec{A}.\vec{A}}$$

In case of unit vectors,

$$\hat{i}.\hat{i} = (1)(1)(\cos 0°)=1$$

Similarly

$$\hat{j}.\hat{j} = 1 \text{ and } \hat{k}.\hat{k}=1$$

Thus $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k}=1$

Scalar product in terms of rectangular components

Consider two vectors $\overline{A}$ and $\overline{B}$ in space then

$$\overline{A} = A_x\hat{i}+A_y\hat{j}+A_z\hat{k} \text{ and } \overline{B}=B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$$

$$\overline{A}.\overline{B} = \left(A_x\hat{i}+A_y\hat{j}+A_z\hat{k}\right).\left(B_x\hat{i}+B_y\hat{j}+B_z\hat{k}\right)$$

$$= A_x\hat{i}.B_x\hat{i}+A_x\hat{i}.B_y\hat{j}+A_x\hat{i}.B_z\hat{k}+A_y\hat{j}.B_x\hat{i}+A_y\hat{j}.B_y\hat{j}+A_y\hat{j}.B_z\hat{k}$$

$$+A_z\hat{k}.B_x\hat{i}+A_z\hat{k}.B_y\hat{j}+A_z\hat{k}.B_z\hat{k}$$

$$= A_xB_x\hat{i}.\hat{i}+A_xB_y\hat{i}.\hat{j}+A_xB_z\hat{i}.\hat{k}$$

$$= A_zB_x\hat{k}.\hat{i}+A_zB_y\hat{k}.\hat{j}+A_zB_z\hat{k}.\hat{k}$$

$$\overline{A}.\overline{B} = A_xB_x+A_yB_y+A_zB_z$$

$$\overline{A}.\overline{B} = A_xB_x+A_yB_y+A_zB_z$$

(vi) Angle between two vectors

Since $\overline{A}.\overline{B} = AB\cos\theta$, so

$$AB\cos\theta = A_xB_x+A_yB_y+A_zB_z$$

$$\cos\theta = (A_xB_x+A_yB_y+A_zB_z)/AB$$

$$\theta = \cos\text{-}1(A_xB_x+A_yB_y+A_zB_z)/AB$$

Q.13. Define and explain vector product of two vectors? Give its examples.

Answer

If the product of two vectors is a vector quantity then the product is called vector product.

Definition

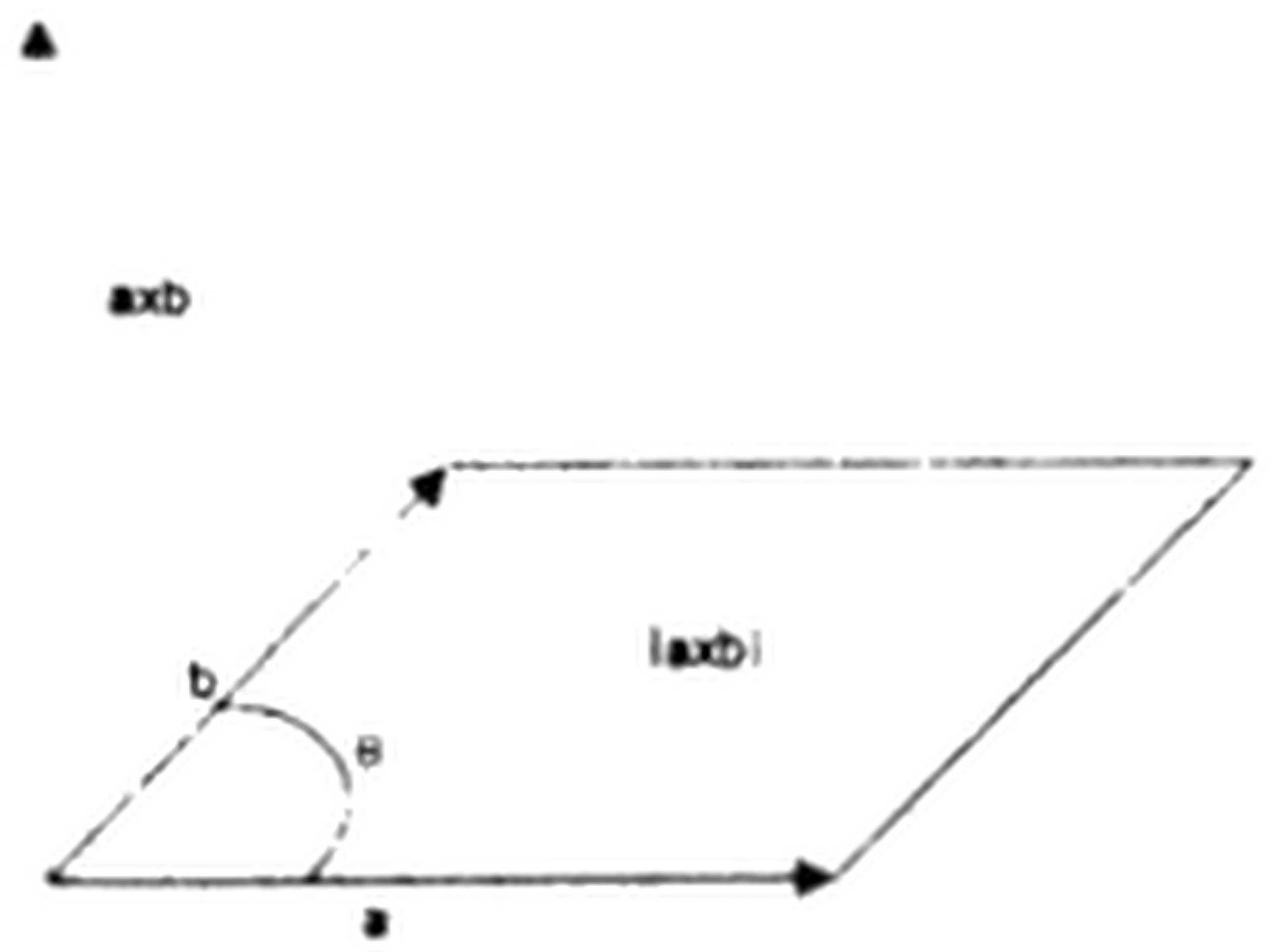
The vector product of two vectors $\vec{A}$ and $\vec{B}$ is defined as

$$\vec{A} \cdot \vec{B} = AB \sin\theta \, \vec{n}$$

Where

θ = angle between the two vectors $\vec{A}$ and $\vec{B}$

Unit vector perpendicular to the plane containing $\vec{A}$ and $\vec{B}$



Direction of vector product

The direction of vector product $\vec{A} \times \vec{B}$ can be found by right hand rule.

Right hand Rule

- Join the tails of the two vectors to define a plane of vectors.
- Rotate vector $\vec{A}$ and $\vec{B}$ through smaller of the two possible angles.
- Curl the fingers of the right hand in the direction of rotation.
- Erect thumb represents the direction of the $\vec{A} \times \vec{B}$.

Example

(1) Torque is the vector product of position vector $\vec{r}$ and force $\vec{F}$ i.e.

$$[\tau = \vec{r} \times \vec{F}]$$

Q.14. Write down the characteristics of vector product of two vectors. Is vector product of two vectors commutative?

Answer

Characteristics of Vector Product

(i) Violation of Commutative Law

The cross product of two vector $\vec{A}$ and $\vec{B}$ is not commutative i.e.

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

(ii) Perpendicular Vectors

The cross product of two perpendicular ($\theta = 90°$) vectors $\vec{A}$ and $\vec{B}$ has maximum magnitude i.e.

$$\vec{A} \times \vec{B} = AB \sin 90° n = AB\,(1)n = ABn$$

In case of units vectors

$$\hat{i} \times \hat{j} = (1)(1) \sin 90°\, k = (1)(1)\hat{k} = \hat{k}$$

Similarly

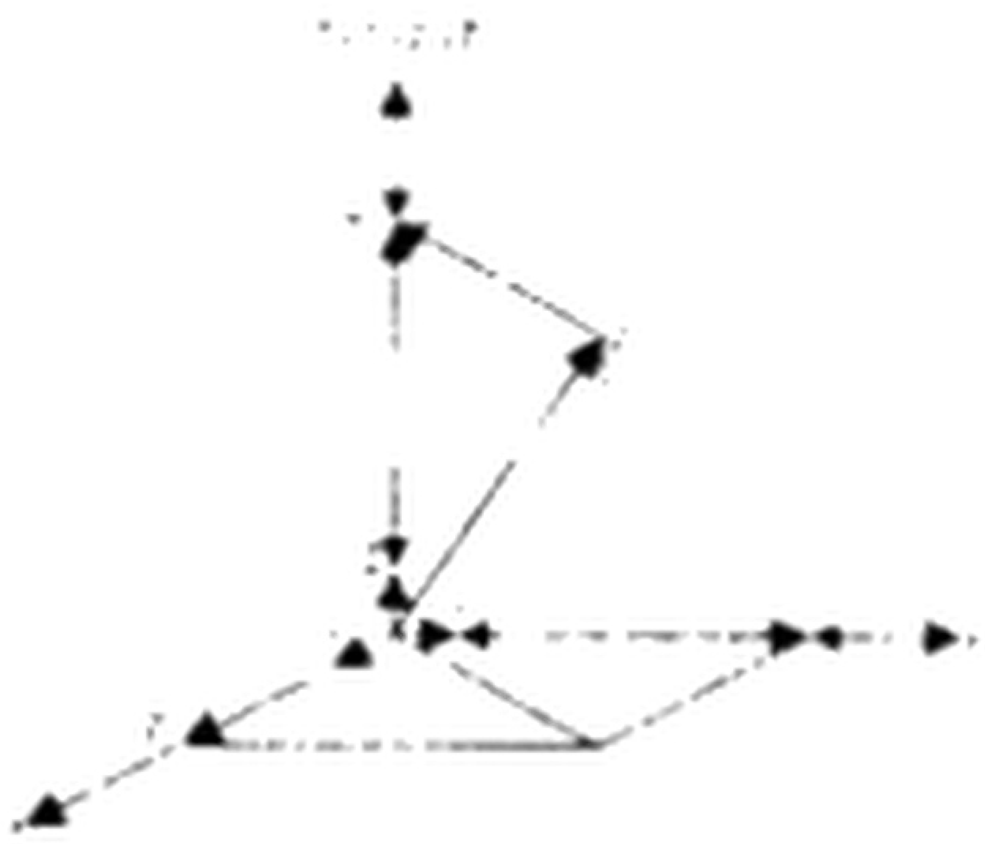
$$\hat{j} \times \hat{k} = \hat{i} \text{ and } \hat{k} \times \hat{i} = \hat{j}$$

Hence

$\hat{i} \times \hat{j} = \hat{k}$	$\hat{j} \times \hat{k} = \hat{i}$	$\hat{k} \times \hat{i} = \hat{j}$

Note:

$\hat{j} \times \hat{i} = -\hat{k}$	$\hat{k} \times \hat{j} = -\hat{i}$	$\hat{i} \times \hat{k} = -\hat{j}$



(iii) Parallel and Anti-parallel

The cross product of two parallel (θ = 0°) or two anti-parallel (θ = 180°) vectors is a null vector i.e.

In case of parallel vectors

$$\vec{A} \times \vec{B} = AB\sin 0° n = AB(0)n = (0)n = \overline{O}$$

In case of anti-parallel vectors

$$\vec{A} \times \vec{B} = AB\sin 180° \; n = AB(0)n = (0)n = \overline{O}$$

(iv) Self Vector Product

The self-product of a vector $\overline{A}$ is null vector

$$\overline{A} \times \overline{A} = AA\sin 180° \; n = AB(0)n = (0)n = \overline{O}$$

In case of unit vectors

$$\hat{i} \times \hat{i} = (1)(1)\sin n = (1)(1)(0)n = \overline{O}$$

Similarly

$\hat{i} \times \hat{j} = \hat{k} \times \hat{k} = \overline{O}$

$\hat{i} \times \hat{i} = \overline{O}$	$\hat{j} \times \hat{j} = \overline{O}$	$\hat{k} \times \hat{k} = \overline{O}$

(v) Area of Parallelogram

The magnitude of cross product of two vectors represents the area of parallelogram formed with these vectors as it two adjacent sides.

Area of parallelogram = (length)(height)

$= (A)(B \sin\theta)$

$= AB \sin\theta$

= Magnitude of $(\overline{A} \times \overline{B})$

Area of parallelogram = $[\overline{A} \times \overline{B}]$

(vi) Vector Product in Rectangular components form

Consider two vectors $\overline{A}$ and $\overline{B}$ in space, then

$$\overline{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \text{ and } \overline{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\overline{A} \times \overline{B} = \left(A_x \hat{i} + A_y \hat{j} + A_z \hat{k}\right).\left(B_x \hat{i} + B_y \hat{j} + B_z \hat{k}\right)$$

$$= A_x \hat{i} \times B_x \hat{i} + A_x \hat{i} \times B_y \hat{j} + A_x \hat{i} \times B_z \hat{k}$$

$$+ A_y \hat{j} \times B_x \hat{i} + A_y \hat{j} \times B_y \hat{j} + A_y \hat{j} \times B_z \hat{k}$$

$$+ A_z \hat{k} \times B_x \hat{i} + A_z \hat{k}.B_y \hat{j} + A_z \hat{k}.B_z \hat{k}$$

$$= A_x B_x \left(\hat{i} \times \hat{i}\right) + A_x B_y \left(\hat{i} \times \hat{j}\right) + A_x B_z \left(\hat{i} \times \hat{k}\right)$$

$$+ A_y B_x \left(\hat{j} \times \hat{i}\right) + A_y B_y \left(\hat{j} \times \hat{j}\right) + A_y B_z \left(\hat{j} \times \hat{k}\right)$$

$$+ A_z B_x \left(\hat{k} \times \hat{i}\right) + A_z B_y \left(\hat{k} \times \hat{j}\right) + A_z B_z \left(\hat{k} \times \hat{k}\right)$$

Evaluating the product of unit vectors and grouping, we find

$$\boxed{\vec{A}\times\vec{B} = \left(A_yB_z - A_zB_y\right)\hat{i} + \left(A_zB_x - A_xB_z\right)\hat{j} + \left(A_xB_y - A_yB_x\right)\hat{k}}$$

This result can be written in determinant form as

$$\vec{A}\times\vec{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

It does not obey commutative law

Reason

As the direction of $\vec{B}\times\vec{A}$ is opposite to the direction of $\vec{A}\times\vec{B}$, So,

$$\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$$

$$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$$

Q.15. Define and explain the term torque of moment of force.

Answer

Torque

Definition

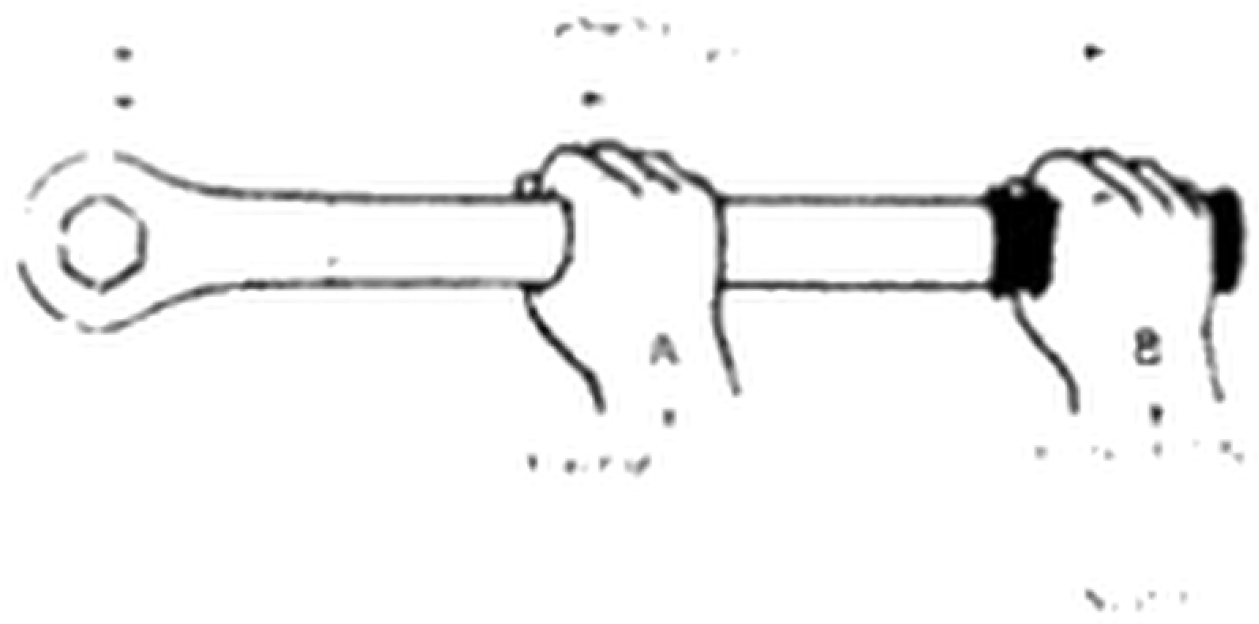
The turning effect of force produced in a body about an axis is called torque.

Other Definition

The product of magnitude force and the perpendicular distance from axis of rotation to line of action of force is called torque.

OR

The moment of a force can also be defined as the vector product of the radius vector from the axis of rotation to the point of application of the force and the force vector.



Q.16. What is value of torque when line of action of force passes through pivot point?

Answer

In this case torque will be zero

Reason

Because moment arm (l) becomes zero in this case. So

$$\tau = l F$$

$$\tau = (0)F = 0$$

Important note

- Torque is the counter part of force for rotational motion.
- Torque plays the same role in angular motion as force plays in linear motion.
- Torque determines angular acceleration as force determines linear acceleration.
- Anti-clock wise torque is taken positive and clock wise torque is taken negative.

Q.17. What is value of toque if the body is at rest or rotating with uniform angular velocity?

Answer

Torque acting on the body will be zero.

Reason

In this case angular acceleration is zero, so torque will be

$\tau = I\,\alpha$ (2nd law for rotational motion)

$\tau = I\,(0) = 0$

Q.18. What is equilibrium? Give its types. What are its different kinds? Also write down the conditions of equilibrium

Answer

Equilibrium

A body is said to be in equilibrium if it is at rest or moving with uniform velocity under the action a number of forces.

Types of equilibrium

There are two types of equilibrium

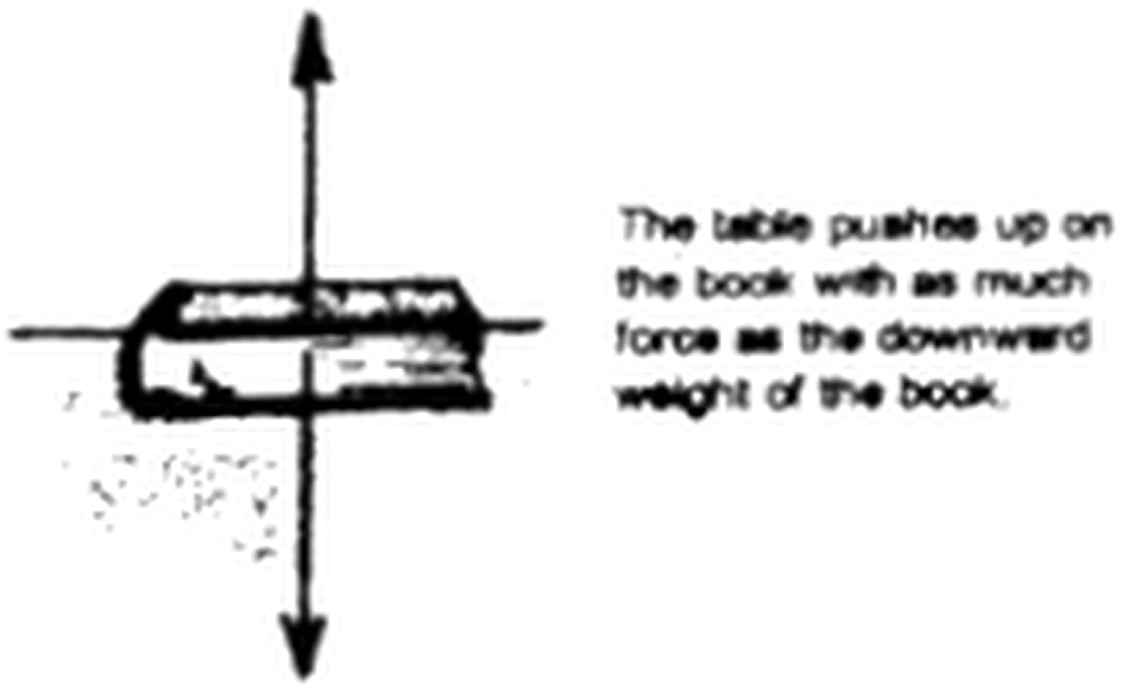
1. **Static equilibrium**

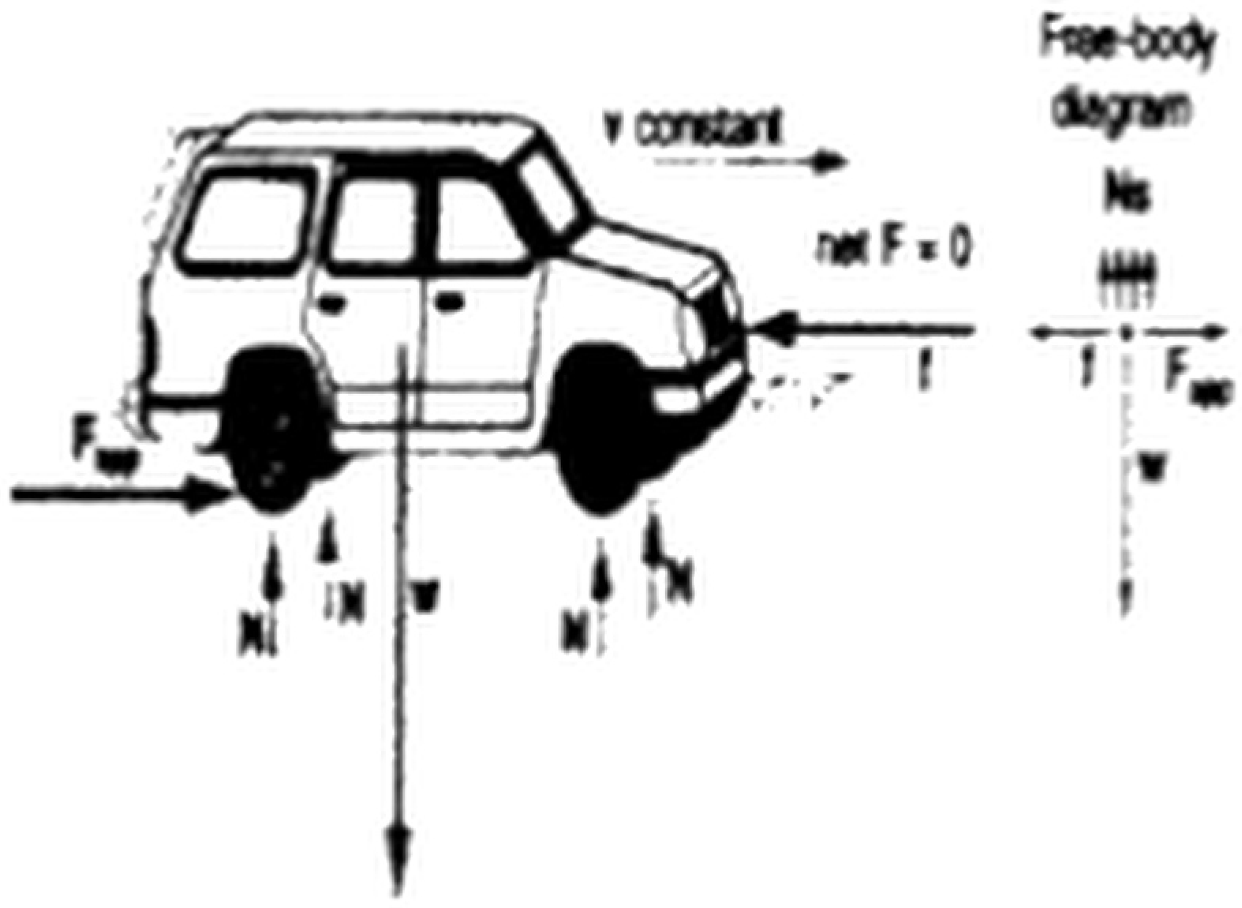
If a body is at rest. It is said to be in static equilibrium

Examples

- **Book tying on a table.**

2.3 Support Force





2. Dynamic equilibrium

If a body is moving with uniform velocity. It is said to be in dynamic equilibrium.

Example

- A car moving with uniform linear velocity
- A body is rotating with uniform angular velocity
- Motion of a paratrooper

Q.19. State the two conditions of equilibrium.

Answer

First condition of equilibrium (equilibrium of forces)

The vector sum of all the forces acting on a body must be null vector i.e.

$$\sum \vec{F} = \vec{O} \qquad ....(1)$$

In case of coplanar force, 1st condition can be expressed as:

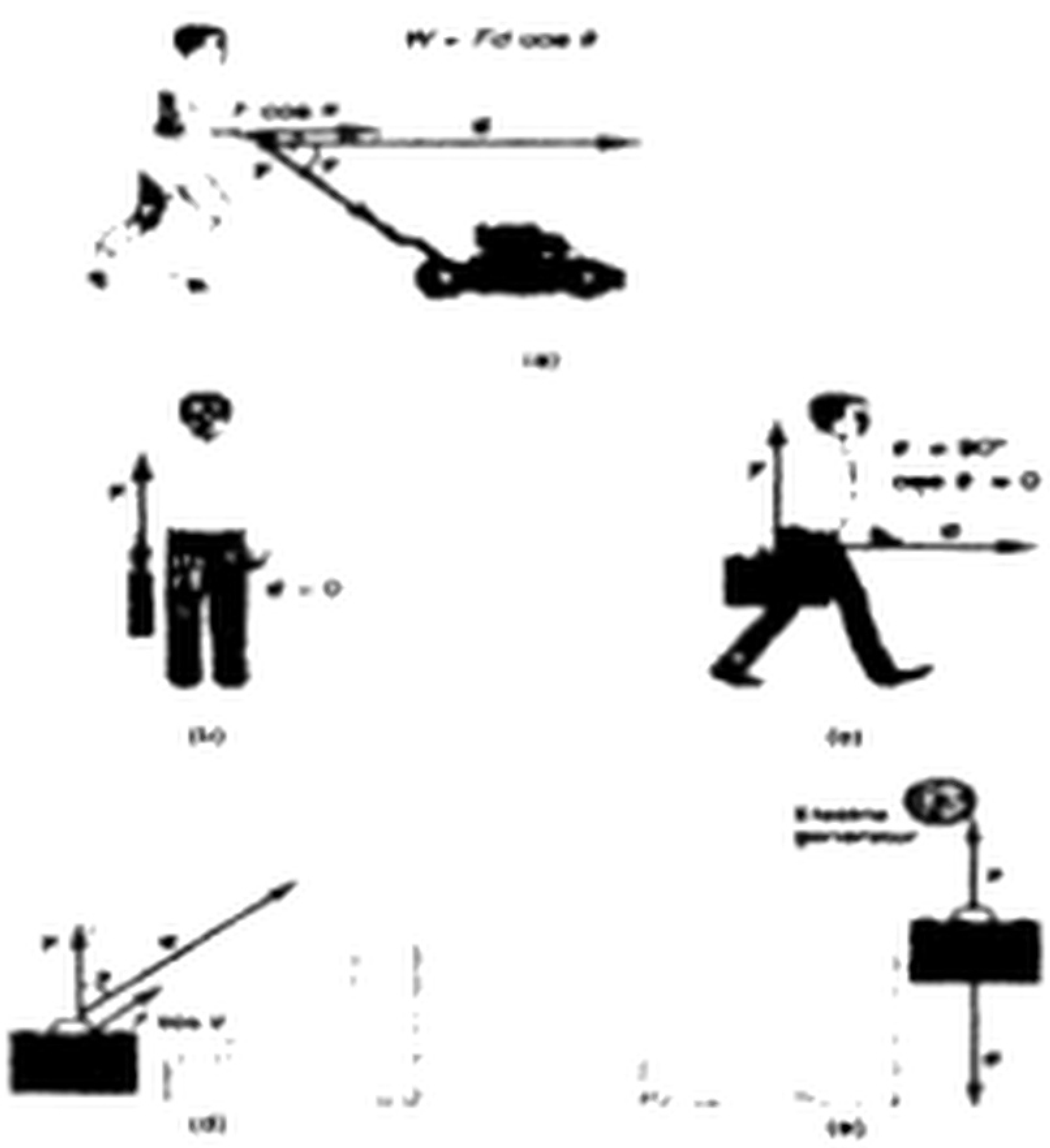
$$\sum \vec{F_x} = \vec{O} \qquad ....(2)$$

And = $$\sum \vec{F_y} = \vec{O} \qquad ....(3)$$

Where

$\sum \overline{F_x}$ = sum of x-directed forces

$\sum \overline{F_y}$ = sum of y-directed forces

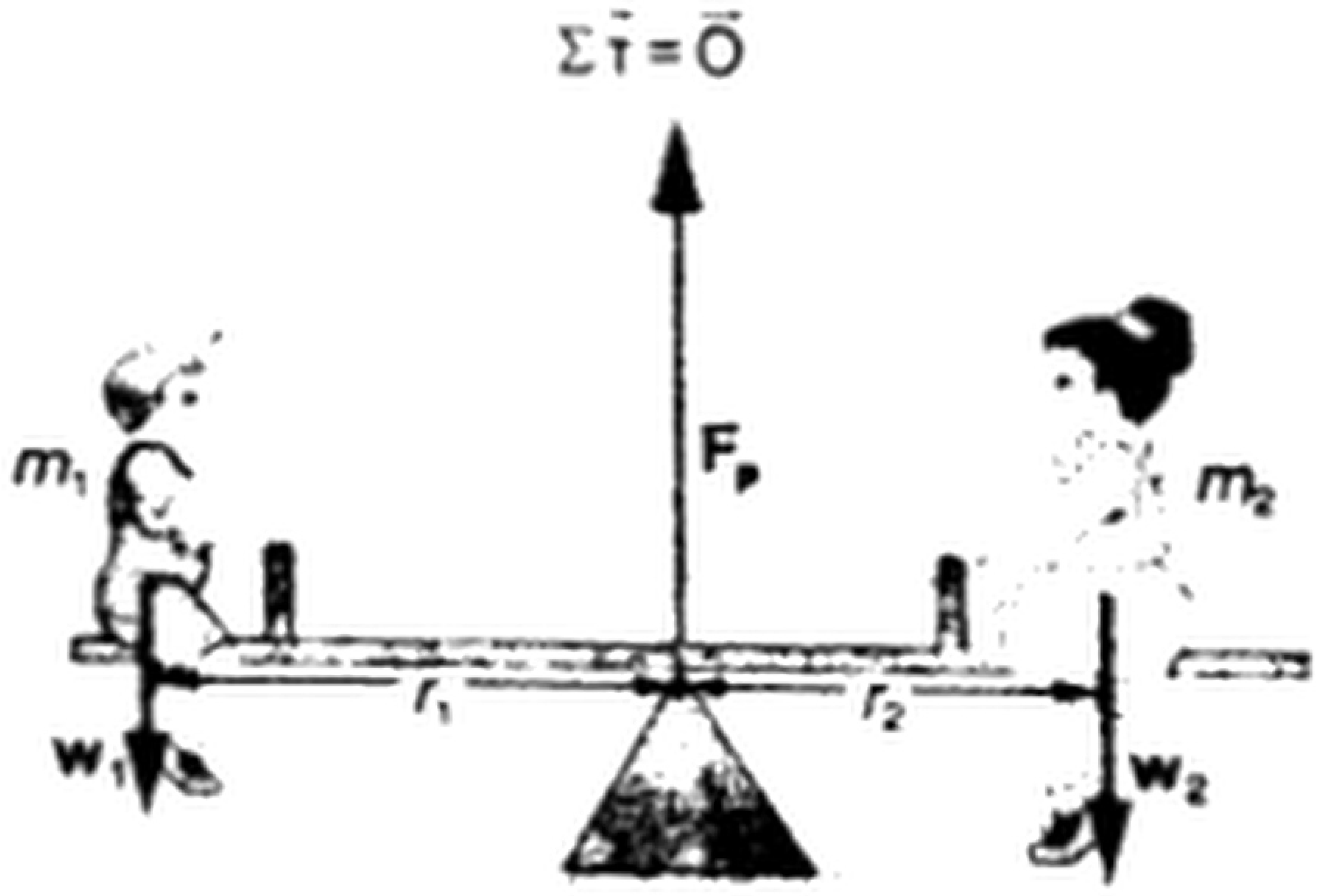


Note:

- If the rightward forces are taken as positive then leftward forces are taken as negative.
- If upward forces are taken as positive, then downward forces are taken as negative.
- Forces which lie in a common plane are said to be coplanar.

Second Condition of equilibrium (equilibrium of torques)

The vector sum of all the torques acting on the body about an axis must be null vector i.e.



Note:

Clockwise torque is taken as negative.

Anti-clockwise torque is taken as negative.

Q.20. Under what conditions the body is said to be in complete equilibrium?

Answer

Translational equilibrium

When first conditions satisfied, the linear acceleration of body is zero and the body is said to be in translational equilibrium.

Rotational equilibrium

When second condition is satisfied, angular acceleration of body is zero and the body is said to be in rotational equilibrium.

Thus, for a body to be in complete equilibrium, both conditions must be satisfied i.e. both linear acceleration and angular acceleration must be zero.

Note:

1. We will apply the conditions of equilibrium to situations in which all the forces are coplanar.
2. To calculate the torque, we choose an axis. The position of axis is arbitrary.
3. A most suitable place is one through which of action of many forces pass