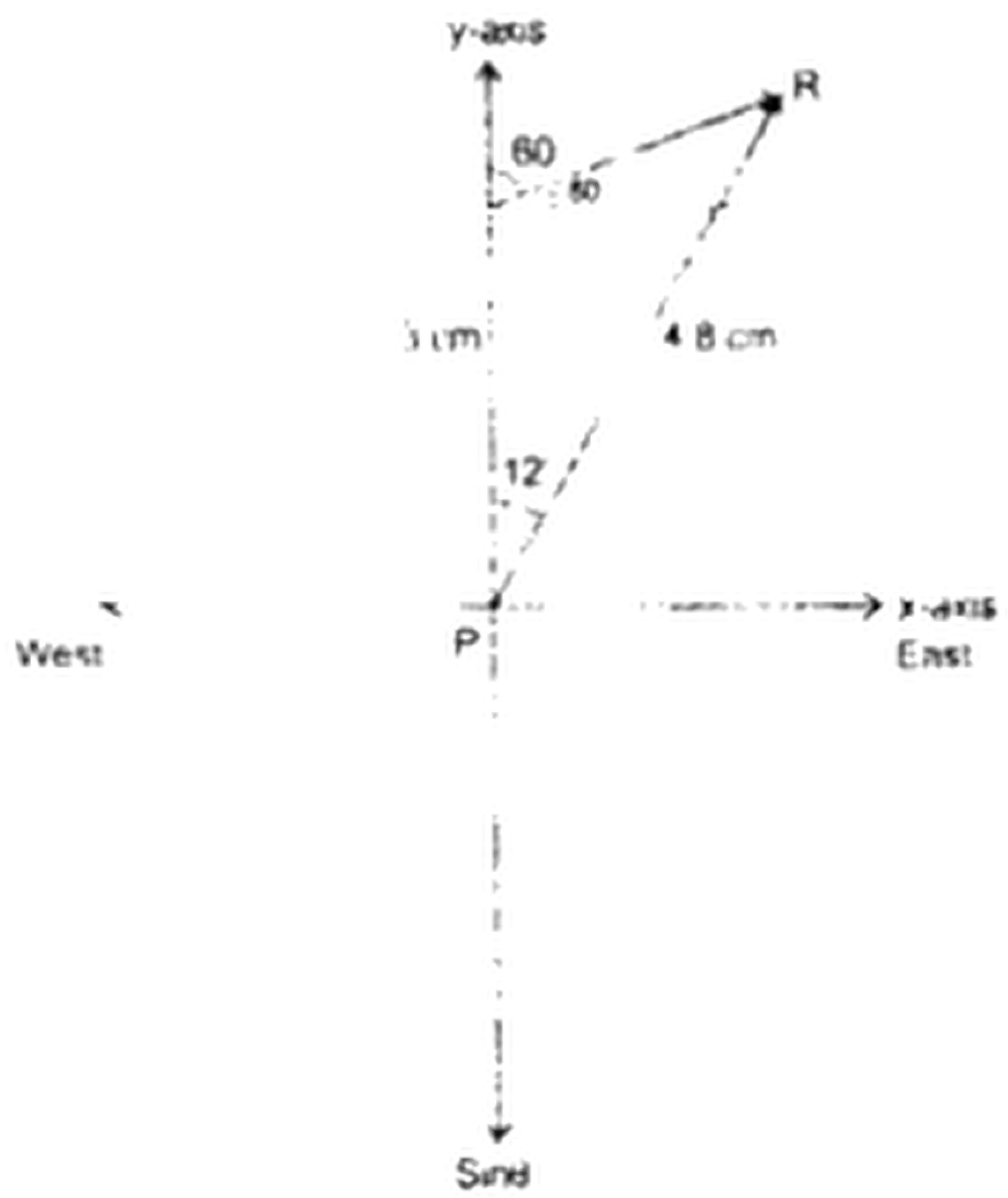
Numerical Problems

1. A ship leaves a part "P" & travels 30 km due north. Then it travels 20 km in a direction 30° east of north to reach fort "R". Calculate the distance from P to R.

Data

- The displacement covered by ship from part "P" due north $\vec{d_1}$ =31km.
- The displacement covered by the ship from that point in a direction 30° east of north to part R, $\vec{d_2}$ = 20 km.
- Tout displacement from P to R, $\vec{d}$ = ?

Solution



- 1st draw rectangular coordinate system.
- 2nd taking suitable scale let 10km = 1 cm,

⇒ 20km = 2 cm & 30 km = 3 cm

- Now drawing the given vectors according to the scale in rectangular coordinate system.

From the fig. $P\vec{R} = \vec{d} = 4.8$ cm

Since 1 cm = 10 km

$\Rightarrow$ 4.8 cm = 48 km, approx.

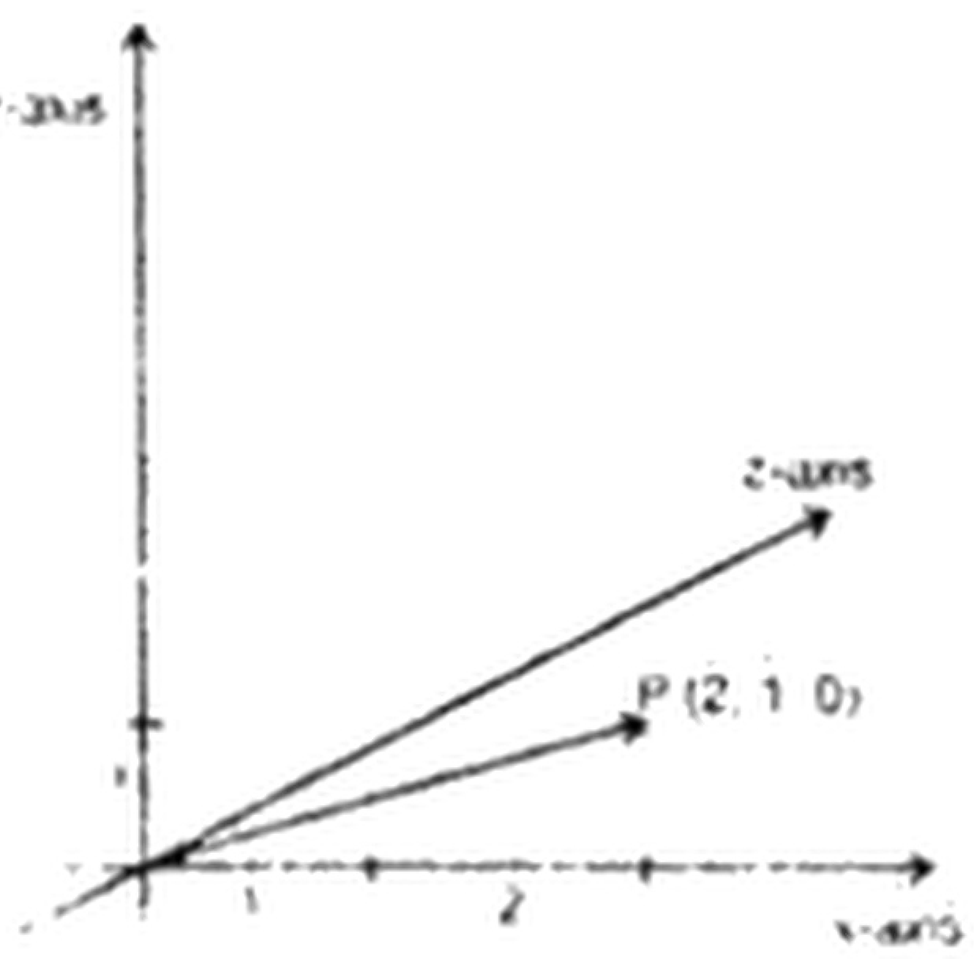
So total displacement from P to R

$\vec{d}$ = 48km, 12° east of north.

2. A certain corner of the room is selected as origin of rectangular coordinate system if an insect is sitting on an adjacent wall at a point whose coordinates arc (2,1, 0) in units of meter, what is the distance of the insect from this corner of the room.

Solution

The coordinates of point P (2, 1, 0) in position vector can be written as



$$\hat{r} = \left(2\hat{i}\right)^2 + \left(1\hat{j}\right)^2 + 0\hat{k}$$

$$\hat{r} = \sqrt{\left(2\hat{i}\right)^2 + \left(1\hat{j}\right)^2 + 0\hat{k}}$$

$$r = \sqrt{4-1-0}$$

$$r = \sqrt{5}m$$

$$\boxed{r = 2.2m}$$

3. The magnitude of dot & cross product of two vector are $6\sqrt{3}$ & 6 respectively. Find the angle between the vectors.

Data

Given that

$AB\cos\theta = 6\sqrt{3}$ ....(1)

& $AB\sin\theta = 6$ ....(2)

Angle between A & B "θ" = ?

Dividing equation (2) by (1)

$\Rightarrow \quad \frac{AB\sin\theta}{AB\cos\theta} = \frac{6}{6\sqrt{3}}$

$\Rightarrow \quad \tan\theta = 1/\sqrt{3}$

$\theta = \tan^{-1} 1/\sqrt{3}$

$\theta = 30^\circ$

Which is the required angle between vector A & B.

4. A load of 10.0 N is suspended from a clothes line. This distorts the line so that it makes an angle of 15° with the horizontal at each end. Find the tension in the clothes line.

Data

Given that

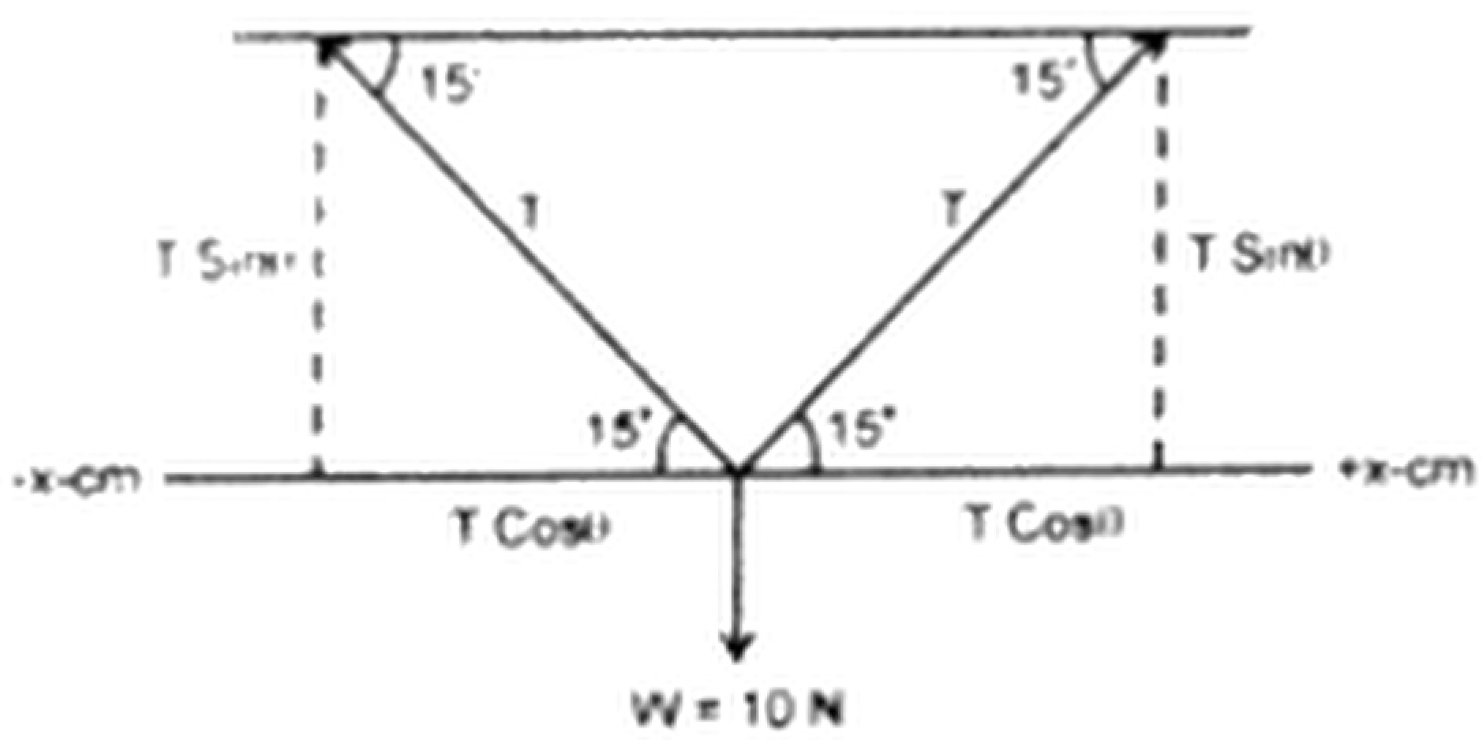
Load suspended to clothes line "W" = 10.0 N

Angle with horizontal "θ" = 15°

Tension in the clothes line T = ?

Solution

Consider the fig.



Applying 1st condition of Equilibrium

$$\sum \vec{F_x} = 0$$

⇒ $T\cos\theta - T\cos\theta = 0$ ...... (1)

& $\sum \vec{F_y} = 0$

⇒ $T\sin\theta + T\sin\theta - w = 0$ ...... (2)

Using equation (2)

⇒ $2T\sin\theta = w$

⇒ $T = W/2\sin\theta$ ...... (3)

Putting values in equation (3)

$$T = \frac{10N}{2 \times **}$$

$\boxed{T = 19.3N}$

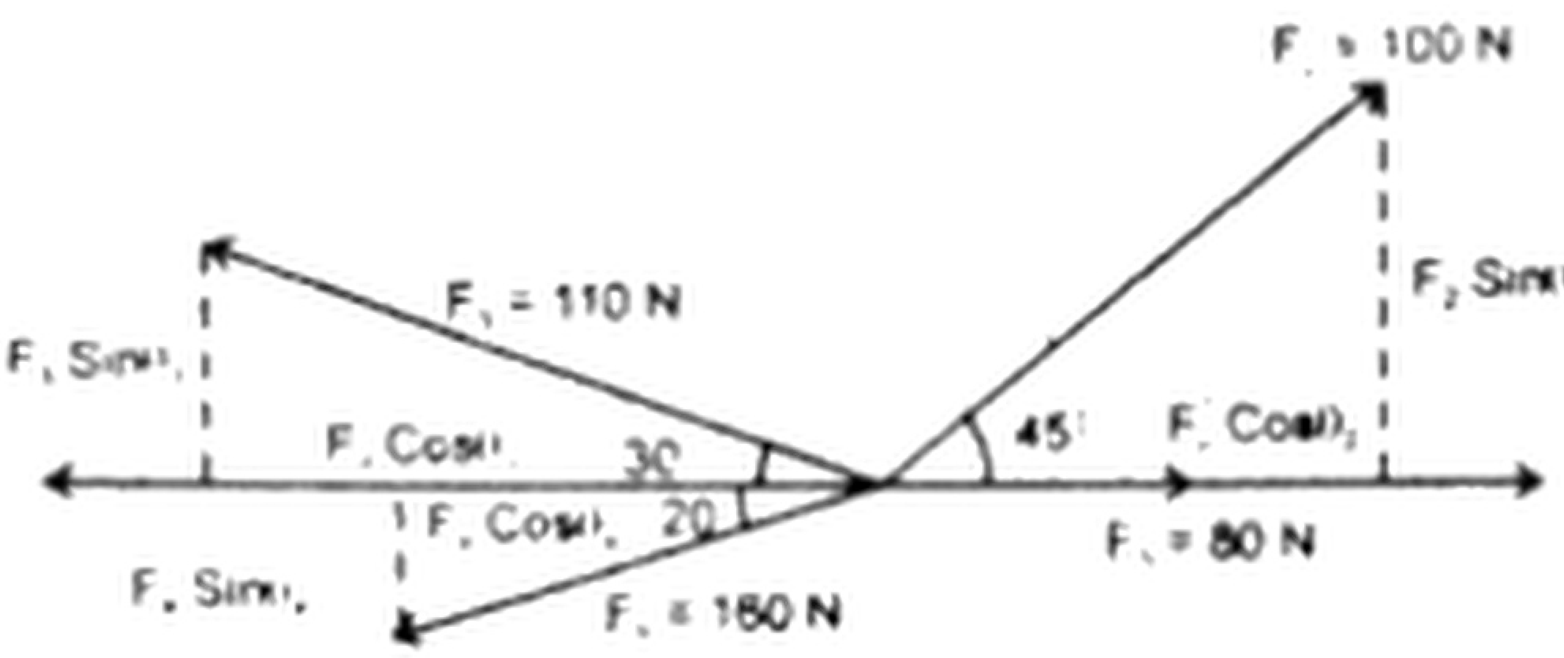
5. Four coplanar forces act on a body at point Q as shown in fig. Find their resultant.

Data

- **Force F_1 making angel "O" with x-axis = 20 N**
- **Force F_2 making angle 45° with x-axis = 100 N**
- **Force F_3 making angle 30° with x-axis = 110 N**
- **Force F4 making angle 20° with x-axis = 160 N**

Resolving into components:

- Forces along x-axis



$$\vec{F}_x = \vec{F}_{1x} + \vec{F}_{2x} + \vec{F}_{3x} + \vec{F}_{4x}$$

$\Rightarrow \quad F_x = F_1\cos\theta_1 + F_2\cos\theta_2 + F_3\cos\theta_2 - F_4\cos\theta_4$ ....(1)

$F_x = 80\cos\theta + 100 \times \cos45° - 110\cos30° - 160\cos0°$

$= 80 \times 1 + 100 \times 0.707 - 110 \times 0.866 - 160 \times 0.94°$

$= 80 + 707 - 95.26 - 150.4$

$F_x = 541.34N$ ....(2)

- **Forces acting along y-axis**

$$\vec{F}_y = \vec{F}_{1y} + \vec{F}_{2y} + \vec{F}_{3y} + \vec{F}_{4y}$$

$\Rightarrow \quad F_y = F_1\sin\theta_1 + F_2\sin\theta_2 + F_3\sin\theta_2 - F_4\sin\theta_4 \qquad$ ....(3)

$F_y = 0 + 100 \times \sin45° - 110 \times \sin 30° - 160 \sin 0°$

$F_y = 100 \times 0.707 + 110 \times 0.5 - 160 \times 0.34$

$F_y = 707 + 55 - 54.4$

$F_y = 707.5\ N \qquad$ ....(4)

- **Now resultant force**

$F = \sqrt{F_x^2 + F_y^2} \qquad$ ....(5)

$F = \sqrt{(541.34)^2 + (707.5)^2}$

$\boxed{F = 890.8\ N} \qquad$ ....(6)

6. A force of 5N is applied perpendicular to the plane of a uniform door 2m high and 0.6 m wide. Find the torque about the line joining the hinges.

Data:

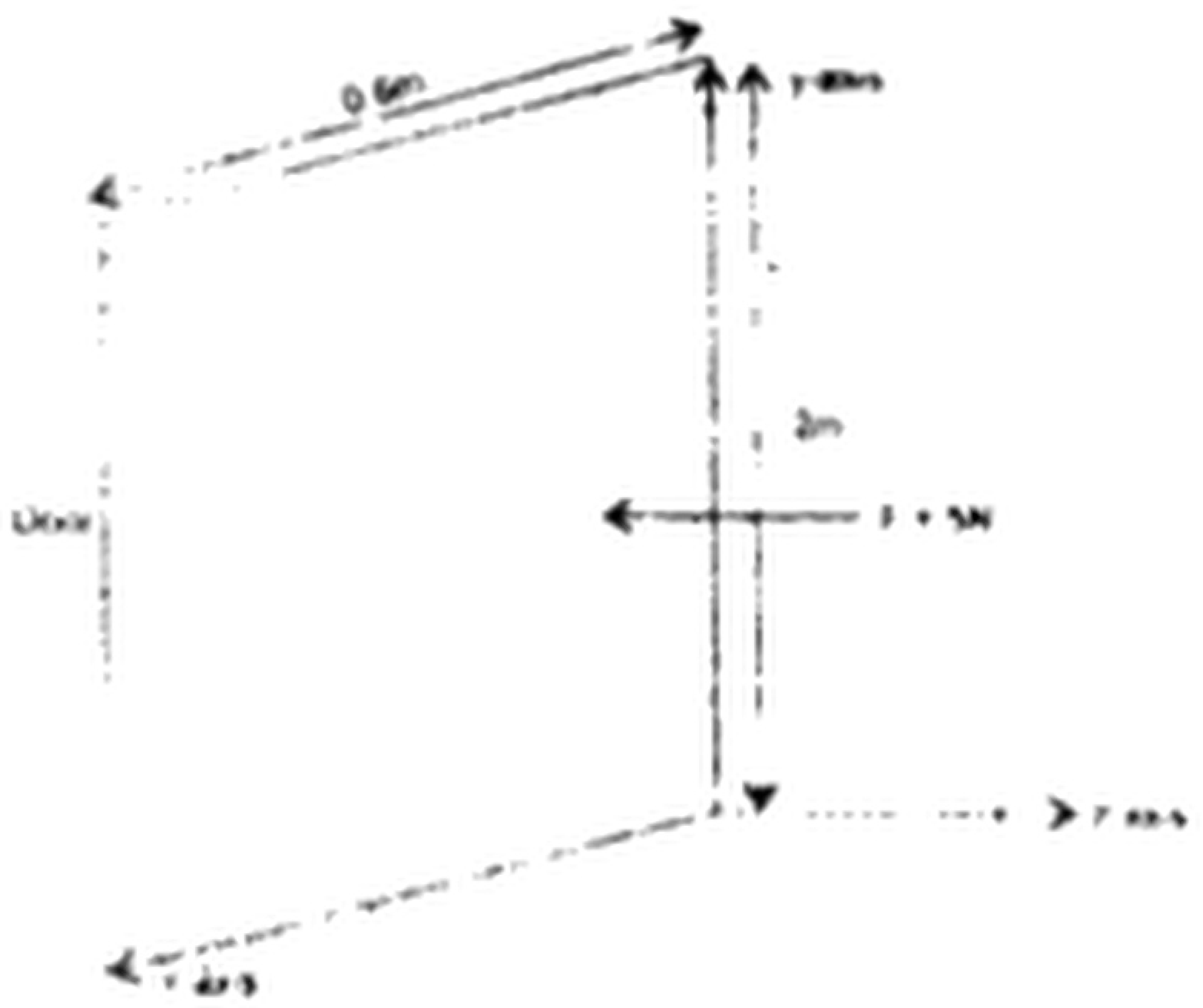
Applied force F = 5 N

Moment arm $\gamma = 0.6$ m

Angle between γ & F "0" = 90°

Applied torque τ = ?

Solution



As $\tau = \lambda F \sin\theta$ ....(1)

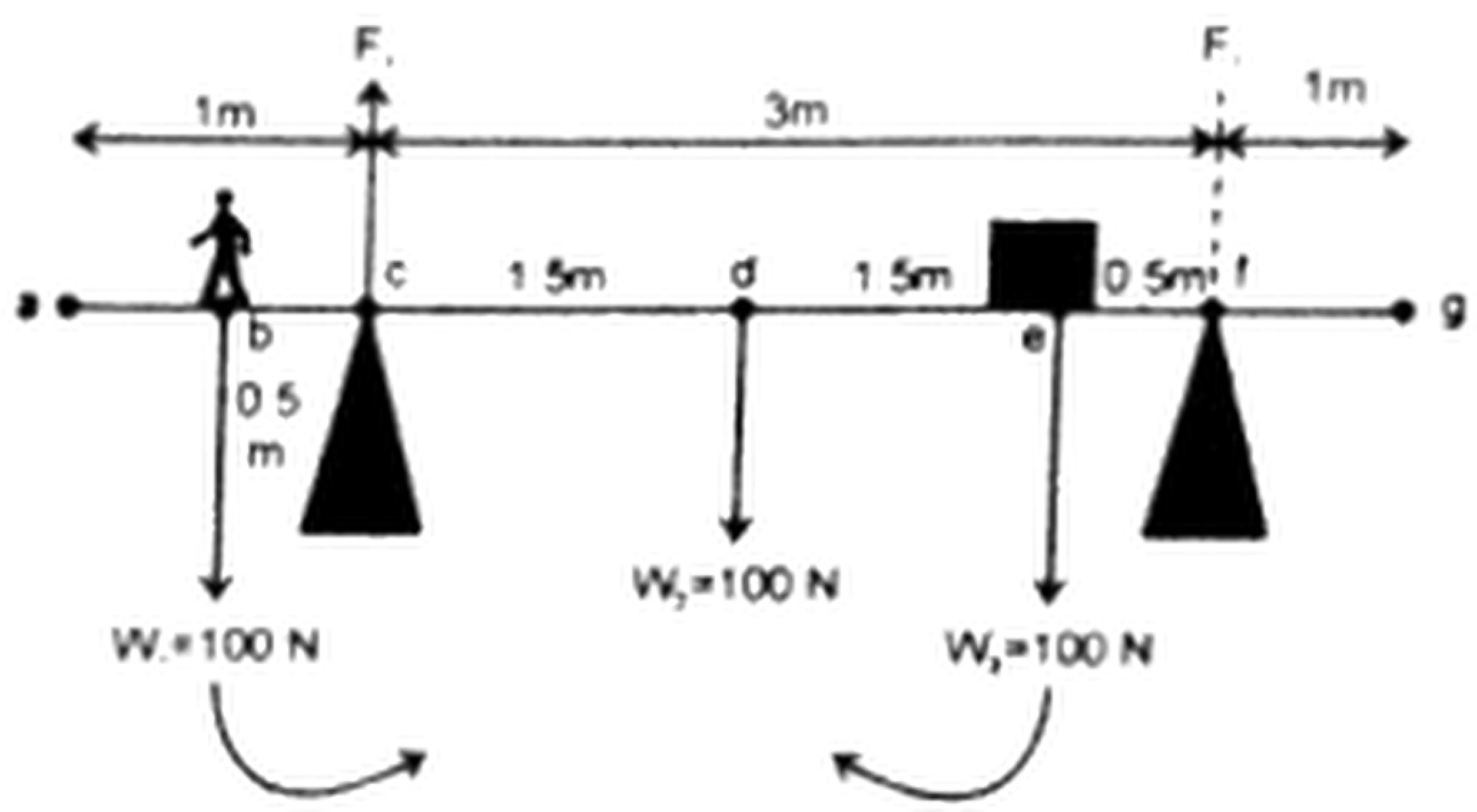
Putting values in equation (1)

$\tau = (0.6m)(5N) \sin 90°$

$\tau = 30N\ m$

7. Find the magnitudes of the forces provided by the supports A and B if shown a balanced condition. Weight of the plant is 500 N and it is uniform in shape; weights of the block & student are 100 N and 500 N respectively.

Data:



Weight of the student W_1 = 500 N

Weight of the plank W_2 = 500 N

Weight of the block W_3 =100 N

Solution

Magnitude of forces by the supports A & B$F_1 + F_2$ = ?

Since upward forces = downward forces:

$F_1 + F_2 = W_1 + W_2 + W_3$

$F_1 + F_2$ = 500 N + 500 N + 100 N

$\boxed{F_1 + F_2 = 1100\ N}$

8. Three forces are acting on a body as shown. Find the magnitude of their resultant.

Data:

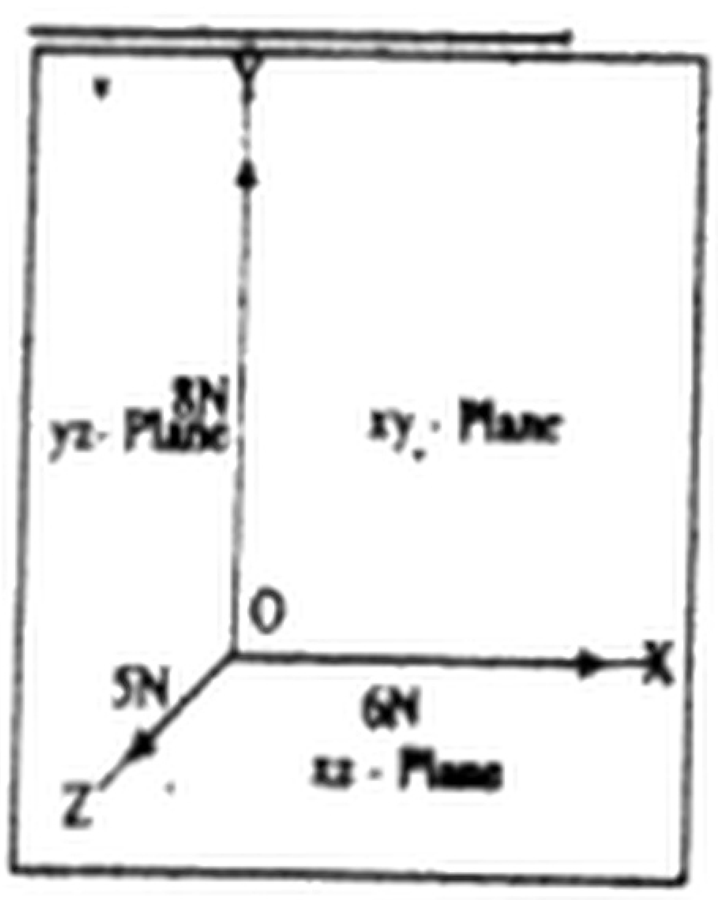
Force along x-axis F_x = 6 N

Force along y-axis F_y = 8 N

Force along z-axis F_z = 5 N

Magnitude of resultant 'F' = ?

Solution



$$F = \sqrt{F_x^{\ 2} + F_y^{\ 2} + F_z^{\ 2}}$$

Putting values

$$F = \sqrt{(6N)^2 + (8N)^2 + (5N)^2}$$

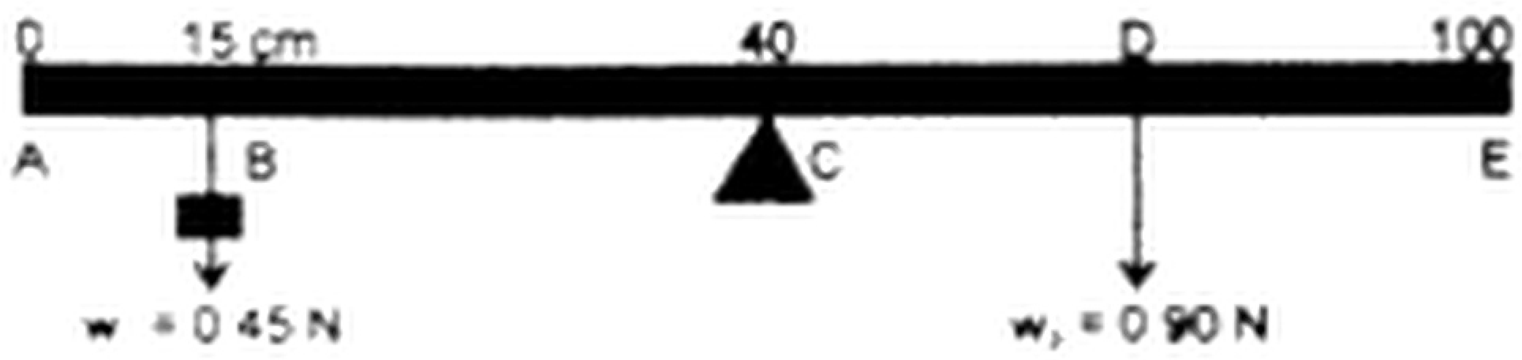
$$F = 11.2N$$

9. A meter rule is supported on a knife edge placed at the 40 cm. It is found that the meter rule balances horizontally when a mass which has weight 0.45 N is suspended at the 15 cm graduation as shown in fig.

a. Calculate the moment, about the knife edge in this balanced condition, of the force due to the mass of the rule.

b. If the weight of the rule is 0.90 N, calculate the position of its centre of gravity. Solution

a) From the fig.



Weight of the object $W_1 = 0.45$ N

$BC = 25$ cm $= 0.25$ m

Moment or torque about the edge "C" = τ

$$\tau = rF$$

$$\tau = (BC)(W_1)$$

$\tau = (0.25)(0.45)$, $\boxed{\tau = 0.1125 \text{ Nm}}$

b) Applying second condition of Equilibrium i.e. clockwise torque = anti clock wise torque

$$(CD)(W_2) = (BC)(W_1)$$

Putting values

$$\Rightarrow \quad CD = \frac{0.25m \times 0.45N}{0.90N}$$

$$\Rightarrow \quad CD = 0.125 \text{ m}$$

$$\Rightarrow \quad CD = 12.5 \text{ cm}$$

Position of centre of gravity is OD

$$OD = OC + CD$$

$$OD = 40 \text{ cm} + 12.5 \text{ cm}$$

$$\boxed{OD = 52.5cm}$$

10. A uniform plank AB of length 4.0m and weight 500 N is suspended by vertical rope at each end. A girl of weight 300 N is standing at a distance of 1.2m from end A. By taking moment about A, calculate the tension in the rope supporting the end B.

Data:

Length of the plank AB = 4.0 m

Weight of the plank W = 500 N

Weight of the girl W_1 = 300 N

Distance of the girl from end A = 1.2m

Tension in the rope supporting the end BT =?

Solution

Taking moment about A

$T \times AB - W \times CB - W_1 \times AD$ ....=0

$T \times 4 - 500 \times 2 - 300 \times 1.2$ ....=0

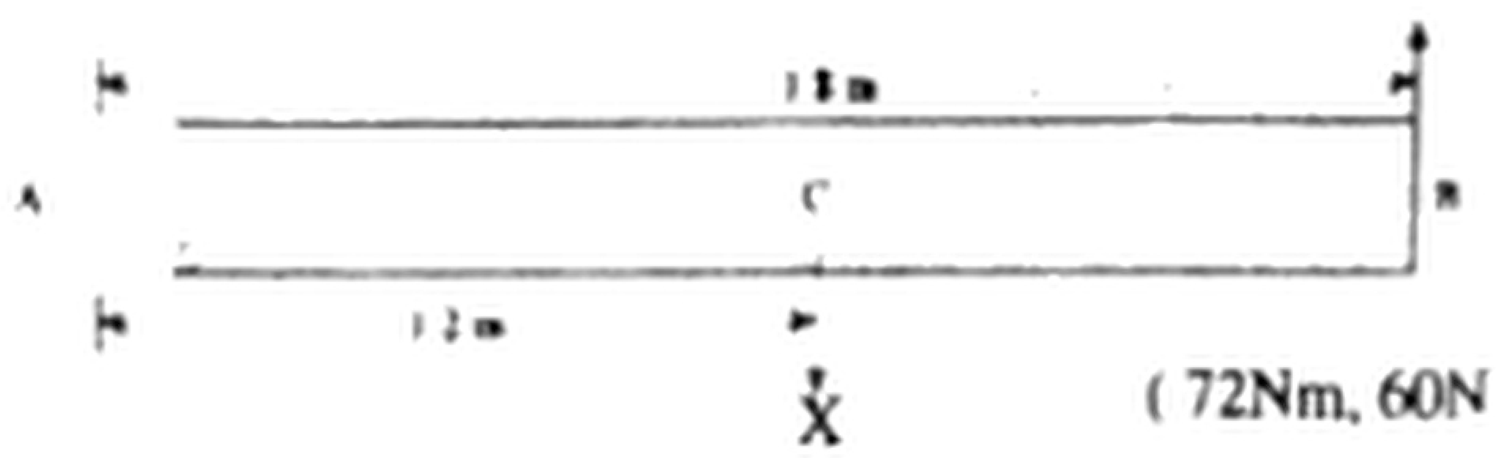
$T \times 4 = 1360$

$T = 1360/4$

$T = 340\ N$

11. The diagram below shows the plane a view of a door hinged at A. If a man applied a force F of 40N at the end marked B, calculate the moment of this force at A. What is the minimum force 'x' must be applied at A in order to stop the door from turning?

Data:



(72Nm, 60N)

Applied force F = 40N

Moment arm λ_1 = 1.8m

a) Moment of force f, τ = ?

Since $\tau_1 = \lambda_1 F$

$\tau = 1.8m \times 40N$

$\tau = 72Nm$

(b) Minimum force applied at C in order to stop the door x = ?

Moment arm λ_2 = 1.2 m

Since the door is in Equilibrium so clock wise torque = anti-clock wise torque.

⇒ $\tau_1 = \tau_2$

⇒ $\lambda_2 \times X = \tau_1$

⇒ $1.2m \times x = 72Nm$

⇒ $x = 72Nm / 1.2m = 60$ m

12. Consider a ladder weighing 200 N resting against a smooth wall such that it makes an angle of 60° with horizontal. Find the reactions on the ladder due to wall and ground.

Data:

Weight of the ladder w = 200 N

The angle which makes the ladder with wall θ = 60°

(a) Reaction of the wall on the ladder R_1 = ?

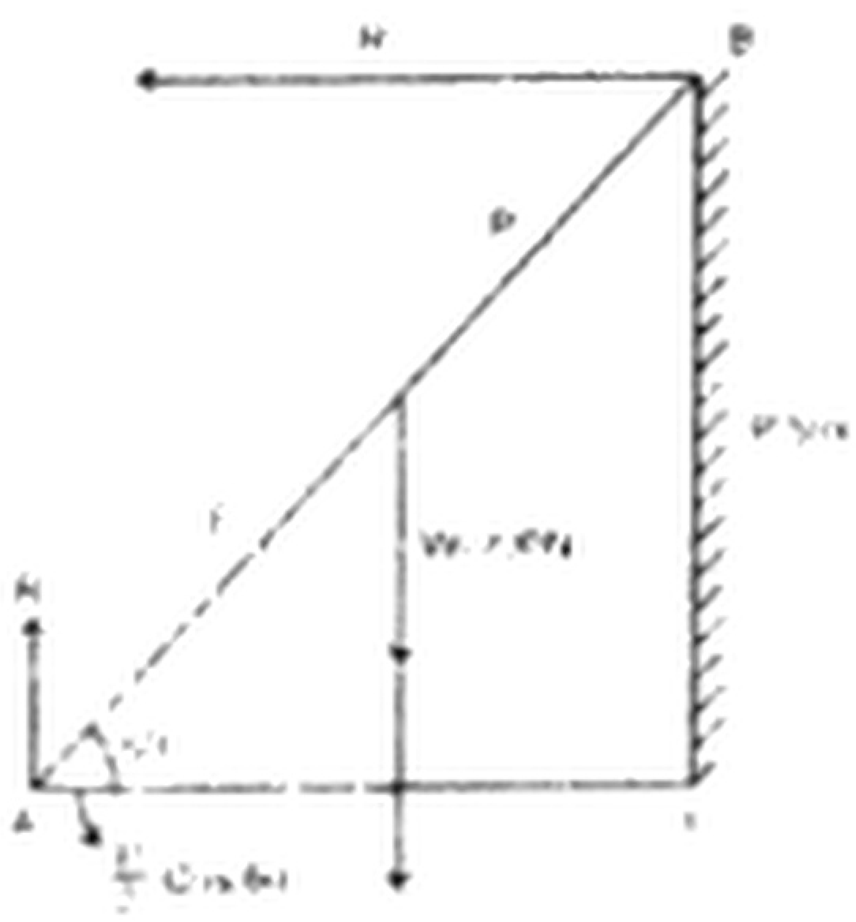
(b) Reaction of the ground on the ladder R_2 = ?

(c) Resultant reaction of wall and ground on ladder R = ?

Solution

a) Reaction of the wall on the ladder R_1 = ?

Using second condition of equilibrium



$R_1 \times 1 \sin\theta - W \times 1/2 \cos\theta = 0$

$$R_1 \times 1 \sin 60° = W \times 1/2 \cos 60°$$

$$R_1 = \frac{200 \times \cos 60^\circ}{2 \times \sin 60^\circ}$$

$$R_1 = 11.5N/2$$

$$\boxed{R_1 = 57.75N} \qquad (1)$$

b) Reaction of the ground on the ladder R_2 = ?

Since reaction of the ground is acting in upward direction, so

$$W = R_2 = 200\ N$$

$$R_2 = 200\ N \qquad (2)$$

Resultant reaction of ground & wall on the ladder

$$R = \sqrt{R_1^2 - R_2^2}$$

$$R = \sqrt{(57.75)^2 - (200)^2}$$

$$R = \sqrt{3335 - 40000}$$

$$\boxed{R = 208N}$$