Numerical Problems

1. Water at 20° C fall from a height of 854 m. If the whole energy is used in increasing the temperature. Find out the final temperature. Specific heat of water is 4200 J/kg.K.

Data:

Initial temperature of water T_1=20°C=293 K

Height from which the water falls = h= 854 m

Final temperature of water = T_2= ?

Specific heat of water = C = 4200J/Kg.K

Solution

Since

$$\Delta Q = mc\Delta T \quad \text{........ (1)}$$

But

$$mgh = \Delta Q \quad \text{........ (2)}$$

Comparing equation 1 and 2 we get

$$mc\Delta T = mgh$$

$$\Delta T = \frac{gh}{C}$$

$$T_2 - T_1 = \frac{gh}{C}$$

$$T_2 = \frac{gh}{c} + T_1 \quad \text{...........(3)}$$

Putting values

$$T_2 = \frac{9.8 \times 854}{420} + 293$$

$$T_2 = 295\ K$$

Or $\boxed{T_2 = 22^\circ C}$

2. 25200 J of heat is supplied to the system while the system does 6000J of work. Calculate the change in internal energy of the system.

Data:

The amount of heat supplied to the system = ΔQ = 25200 J

Work done by the system = W = 6000J

Change in internal energy = $\Delta \upsilon$=?

Solution

Using first law of thermodynamics

$$\Delta U = \Delta Q - \Delta W \qquad \text{..........} (1)$$

Putting values, we get

$$\Delta U = 25200 \text{ J} - 6000 \text{ J}$$

$$\boxed{\Delta U = 19200 \text{ J}}$$

3. A sample of gas is uniformly heated at constant pressure. If the amount of 180 J of heat is supplied to the gas. Calculate the change in internal energy of the gas and work done by the gas, take γ = 1.41

Data:

The amount of heat given to the gas ΔQ=180J

Change in internal energy ΔU = ?

Work done by the gas W =?

Solution

Since $W = P\Delta V$

$$W = nR\Delta T \quad ......(1) \quad \text{as } PV = nRT$$

1st we are going to find ΔT

As $\gamma = \frac{C_p}{C_v} = 1.41$

$\Rightarrow \quad \frac{C_p}{C_p - R} = 1.41$

$\Rightarrow \quad \frac{C_p - R}{C_p} = \frac{1}{1.41}$

$$1 - \frac{R}{C_p} = \frac{1}{1.41}$$

$$-\frac{R}{C_p} = \frac{1}{1.41} - 1$$

$$-\frac{R}{C_p} = 0.71 - 1$$

$$-\frac{R}{C_p} = -0.291$$

$$\frac{R}{C_p} = 0.291$$

$$C_p = \frac{1 \times 8.26}{0.291} \qquad \because R = 8.36\,Jmol^{-1}C^{\circ}$$

$$C_p = 2.73\,Jmol^{-1}C^{\circ}$$

Since $\Delta Q = nC_p\Delta T$

$$\Delta T = \frac{\Delta Q}{n\Delta T} = \frac{180}{1 \times 28.73}$$

$$\Delta T = 6.26\,^{\circ}C$$

Putting values in Equation (1)

$$W = 1 \times 8.36 \times 6.26$$

$$\boxed{W = 52.37\,J}$$

Now

$$\Delta U = \Delta Q - 1\Delta W$$

$$\Delta U = 180 - 52.37$$

$$\boxed{\Delta U = 127.62\,J}$$

4. 5 moles of oxygen is heated at constant volume from 10°C to 20°C. What will be the specific heat of oxygen at constant pressure is 8 Cal Mol^{-1} C^{-1} and R=8.36 J Mol^{-1} C^{-1}.

Data:

No. of moles of oxygen gas = n = 5

Change in temperature of oxygen at constant volume = ΔT=10°C = 10K

Change in internal energy = ΔU = ?

Specific heat at constant pressure of oxygen = C_p=8, Cal Mol^{-1} C^{-1}

R = 8.36 J Mol.C°

Solution

Since the oxygen is heated at constant volume so

$$\Delta U = nC_v\Delta T$$

$$C_p = C_p - R$$

$$C_p = 33.488 - 8.36$$

$$C_p = 25.128 \text{ J Mol}^{-1} \text{C}^{-1}$$

Put values in equation

$$\Delta U = 5 \times 25.128 \times 10$$

$$\Delta U = 1256.4 \text{ J}$$

$$\boxed{\Delta U = 300 \text{ Cal}}$$

5. Find the efficiency of a Carnot's Engine working between the seam and ice points.

Data:

Temperature of hot reservoir = T_1 = 100°C

$T_1 = 100°C + 273 = 373\ K$

Temperature of cold reservoir = $T_2 = 0°C + 273 = 273K$

Solution

As efficiency of Carnot Engine is given by

$$\%\eta = \left(1 - \frac{T_2}{T_1}\right) \times 100$$

$$\%\eta = \left(1 - \frac{273}{373}\right) \times 100$$

$$\boxed{\%\eta = 26.8\%}$$

6. A Carnot heat engine absorbs 200 J of heat from the source of heat engine at 227°C and rejects 1200 J of heat during each cycle to sink. Calculate efficiency of engine, temperature of sink and the amount of work done during each cycle.

Data:

i) Heat absorbs from hot temperature reservoir = Q_1 = 2000J

ii) Heat rejected to sink Q_2 = 1200 J

iii) Temperature of source = T_1 = 227°C + 273 = 500 K

iv) Temperature of the sink T_2 = ?

v) Efficiency of the engine = η = ?

vi) The amount of work done during each cycle = W = ?

Solution

Since

(i)

$$\eta = \left(1 - \frac{Q_2}{Q_1}\right) \times 100$$

$$\eta = \left(1 - \frac{1200}{2000}\right) \times 100$$

$$\boxed{\eta = 40\%}$$

(ii) As $W = Q_1 - Q_2$

$W = 2000J - 1200J$

$\boxed{W = 800\,J}$

(iii) Since $\eta = 1 - \frac{T_2}{T_1}$

$\frac{T_2}{T_1} = 1 - \eta$

$T_2 = (1 - \eta) \times T_1$

$T_2 = (1 - 0.4) \times 500$

$T_2 = 300\ K$

$\boxed{T_2 = 27\ ^\circ C}$

7. In a refrigerator, heat from inside at 227 K is transformed to a room at 300 K. How many joules of heat will be delivered to the room for each Joule of electric energy consumed ideally?

Data:

The temperature at which heat is transformed from refrigerator = T_1 = 277 K

Temperature of the room = T_2 = 300 K

Heat energy delivered to the room = E = ?

Solution

As $E_{cooling} = \frac{T_2}{T_1 - T_2}$

$$E = \frac{277}{300 - 277}$$

$\boxed{E = 12.04\,J}$

Since

$$E = \frac{Q_2}{W}$$

$$W = \frac{Q_2}{E}$$

Putting values

$$W = \frac{336\,J}{7.4}$$
$$W = 45.5\,J$$
$$W = Q_1 - Q_2$$
$$Q_1 = W + Q_2$$
$$Q_1 = 45.4\,J + 336\,J$$
$$\boxed{Q_1 = 381.4\,J}$$

8. What is the least amount of work that must be performed to freeze one gram of water at 0°C by means of refrigerator? Take the temperature of surrounding as 37°C. How much heat is passed on the surrounding during this process?

Data:

Mass of the water = m = 1g = 0.001 kg

Temperature of the surrounding T_1 = 37°+273 = 310K

Temperature of the water = T_2 = 0°C+273 = 273 K

Latent heat of fusion of water L_1 = 3.36 x10^5 J/kg

The amount of work to freeze the water = W = ?

Heat passed on the surrounding Q_1 =?

Solution

Since coefficient of performance of refrigerator is given as

$$E = \frac{T_2}{T_1 - T_2} \qquad \text{.........} (1)$$

Putting values we get

$$E = \frac{273K}{310K - 277K}$$

$$E = 7.4 \qquad \text{......} (2)$$

As $Q_2 = mL_1$

$Q_2 = 0.001 \times 3.36 \times 10^5$

$\boxed{Q_2 = 336\ J}$ .... (3)

9. Calculate the change in entropy when 10 kg of water is heated from 90°C to 100°C?

Data:

Mass of water = m = 10 kg

Initial temperature of water = T_1 = 90°C+273=363 K

Final temperature of water = T_2 = 100°C + 273 =373 K

Change in temperature = ΔT = 10°C = 10 K

Change in entropy ΔS = ?

Solution

Since entropy of irreversible process is given as

$$\Delta S = mc \ln\left(\frac{T_2}{T_1}\right) \quad \text{........ (1)}$$

Putting values in equation (1)

$$\Delta S = 10 \times 4180 \times \ln\left(\frac{373}{363}\right)$$

$$\boxed{\Delta S = 1135.9\ J/K}$$

10. A system absorbs 1176 J of heat and at the same time does 352.8 J of external work.

a) Find change in internal energy?

b) **Find change in internal energy when it absorbs 1050 J of heat while 84 J of work is done.**

c) **What will be change in internal energy of the gas of 210 J of heat is removed at constant volume.**

Data:

The heat absorbed by the system = Q_1=1176 J

Work done by the system = W = 352.8 J

Change in internal energy = ΔU = ?

Solution

a. As first law of thermodynamics

$$\Delta U = Q_1 - W$$

$$\Delta U = (1176 - 352.8)\ J$$

$$\boxed{\Delta U = 823.2\ J}$$

b. The heat absorbed by the system Q_2 =1050J

Work done by the system = W = 84 J

Change in internal energy = ΔU = ?

Since $\Delta U = Q_2 - W$

$$\Delta U = (1050 - 84) J$$

$$\boxed{\Delta U = 966\ J}$$

c. The heat rejected form the gas $Q_3 = -210$ J

Change in internal energy ΔU = ?

Since volume is constant so $\Delta U = 0$ and $W = 0$

Using 1st law of thermodynamics

$$\Delta U = Q_1 - W$$

$$\Delta U = (-210 - 0)\ J$$

$$\boxed{\Delta U = -210\ J}$$