Numerical Problems

1. Express the following quantities by using prefixes:

(a) 4.0×10^{-4} m (b) 15.0×10^{-8} s (c) 7.5×10^{-7} g

Solution

(a) 4.0×10^{-4} m

$=4.0 \times 10^{-4}$m

$=0.4 \times 10^{-3}$ m 10^{-3} = milli

$=0.4$ mm

(b) 15.0×10^{-8} s

$=15.0 \times 10 \times 10^{-1} \times 10^{-8}$ s

$=150 \times 10^{-9}$ s 10^{-9} = nano

$=150$ ns

(c) 7.5×10^{-7} g

$=7.5 \times 10^{-1} \times 10^{-8}$ g

$=0.75 \times 10^{-3}$ g 10^{-6} = micro = μ

$= 0.75$ μg

2. The length and width of a rectangular plate are (15.6 ± 0.1) cm and (10.80 ± 0.01) cm respectively. Calculate area of the plate and uncertainty in it.

Data: Length of the plate l = (15.6 ± 0.1) cm, width of the plate w (10.80 ± 0.01) cm,

area of the plate with uncertainty = ?

Solution

- **%age uncertainty in length**

  $= \frac{0.1}{15.6} \times 100$

  = 0.64%

- **%age uncertainty in width**

  $= \frac{0.01}{10.80} \times 100$

  = 0.093%

Since in multiplication uncertainty is added so total uncertainty (%age)

= (0.64 + 0.093) %

= 0.732%

- **As area of rectangular plate, A = l × w**

A = 15.6 × 10.80 cm^2 with %age

Uncertainty 0.732%

A = 168.4 cm^2 with %age uncertainty of 0.732%

OR = (168.4 ± 1.22) cm^2

3. The length of a pendulum is (100 ± 0.1) cm. If acceleration of a free fall is (9.8 ± 0.1) ms^{-2}, calculate the percentage uncertainty in time period of the pendulum.

Solution

%age uncertainty in length

= 0.1/100 × 100

= 0.1%

%age uncertainty in acceleration due to gravity

= 0.1/9.8 × 100

= 1.02%

Total %age uncertainty

= 0.1% + 1.02%

= 1.12%

%age uncertainty in time period of the pendulum

= 1/2 (1.12%) $T = \text{const}\left(\frac{1}{g}\right)^{h}$

= 0.6%

Note:

For power factor multiply the percentage uncertainty by that power. Since the power of $\frac{l}{g} = \frac{1}{2}$. That is why total %age uncertainty is multiplied by $\frac{1}{2}$.

4. Theory suggests that drag force depends upon the viscosity of the medium, average radius of the object and velocity of the object moving through the fluid. Derive a formula for dragging force of fluid by using dimensional analysis.

Answer

The dragging force depends on the following factors.

(a) Viscosity of the medium 'η'

(b) Radius of the object 'r'

(c) Velocity of the object 'v'

So the relation for the dragging force 'F' will be of the form

$F = Const\ \eta^{a} r^{b} v^{c}$ .... (1)

Write the dimensions of Equation (1) on both sides we get

$[MLT^{-2}] = Const\ [ML^{-1}T^{-1}]^{a}\ [L]^{b}\ [LT^{-1}]^{c}$

$(M^{1}L^{1}T^{-2}] = Const\ [M]^{a}[L]^{-a+b+c}\ [T]^{-a-c}$ ....(2)

Comparing the dimension of both sides we get

$[M]^{1} = [M]^{a}$ ....(3)

$[L]^{1} = [L]^{-a+b+c}$ ....(4)

$[T]^{-2} = [T]^{-a-c}$ ....(5)

Equating the powers on both sides we get

$\boxed{a = 1}$ .... (A)

$-a+b+c = 1$ .... (B)

$-a-c = 2$

$a+c = 2$ .... (C)

Put Equation (A) in (C) we get

$1+c = 2$

$\boxed{c = 1}$ .... (D)

Equation (D) => $-1 + b + 1 = 1$

$\boxed{b = 1}$

Substituting the values of a, b & c in equation (1)

$F = Const\ \eta^a r^b v^c$

$\boxed{F = Const\ \eta r v}$

5. (a) Suppose that the displacement of an object is related to time according to the expression $x = Bt^2$, what are the dimensions of 'B'.

(b) A displacement is related to time as $x = A \sin(2\pi ft)$, where 'A' and 'f' are constants. Find the dimension of 'A'.

Solution:

(a) Given that

$x = Bt^2$

$B = x/t^2$ ....(1)

As dimension of 'x' is [L] and t^2 is $[T^2]$, so Equation becomes

$B = \left[\frac{**}{**}\right]$

$B = [LT^{-2}]$

(b) Given that

$X = A \sin(2\pi ft)$

$A = X/\sin 0$ ....$2\pi ft = 0$

$A = [L]/1$

$\boxed{A = [L]}$

6. Carry out the following conversions.

(a) Calculate the density 1.33 x 10^{-7} gcm^{-3} into kgm^{-3}

(b)Calculate a speed 20ms^{-1} into kmh^{-h}

Solution

a. *= 1.33 × 10^{-7} gm^{-3}

*= 1.33 × 10^{-7} × 10^{-3} kg × $(10^{-2})^{-3}$$m^{-3}$

*= 1.33 × $10^{-7\ 3+6}$ ka/m^{-3}

*= 1.33 ×10^{-4} kgm^{-3}

b. v = 20 m/s

$$v = \frac{20 \times 3600}{1000} km/h$$

v =72 kmh^{-1}

7. If there are No. = 6.02 x 10^{23} atoms in 4.0 g of helium what is the mass of helium atom?

Solution

Mass of helium atom

$$= \frac{\text{mass of He}}{\text{No}}$$

Putting values, we get

$$= \frac{4.0\ g}{6.02x\ 10^{23}}$$

Mass of the He atom = 6.6 x 10^{-22} g

8. Compute the following to correct significant digits.

(a) 3.85 m x 3.9 m

= 15.015m^2

= 15 m^2

(b) 1023 kg + 8.5489 kg

= 1031.5489 g

=1032 kg

(c) =22/7

=3.14285

(d) $= \frac{1.67 \times 10^{-27}\ kg}{9.1096 \times 10^{-31} kg}$

$\boxed{1.83 \times 10^3}$

9. A rectangular metallic piece is (3.70 ± 0.01) cm we and (7.20 ± 0.01) cm long.

(a) Find the area of the rectangular metallic piece and uncertainty in area.

Length 'l'= (7.20 ± 0.01) cm

Width 'w' = (3.70 ±0.01) cm

%age uncertainty in length

'l'= 0.01 /7.20 × 100

= 0.139%

%age uncertainty in width

'w' = 0.01 / 3.70 × 100

'w' = 0.27%

Total %age uncertainty in area

$= 0.4\%$

Area = 26.64 cm^2 with %age uncertainty 0.4%

$A = (26.64 \pm 0.1)\ cm^2$

10. Calculate the answer up to appropriate numbers of significant

(a) 168.99×9

$= 1520.91$

$= 1521$

(b) $23.5 + 234$

$=257.59$

$=257.6$

(c) $\frac{984.25}{80}$

$= 12.303$

$= 12.3$