Exercise 9.4

Prove the following identities, state the domain of θ in each case:

1. $\tan\theta + \cot\theta = \operatorname{cosec}\theta \sec\theta$

2. $\sec\theta \operatorname{cosec}\theta \sin\theta \cos\theta = 1$

3. $\cos\theta + \tan\theta \sin\theta = \sec\theta$

4. $\operatorname{cosec}\theta + \tan\theta \sec\theta = \sec\theta \sec^2\theta$

5. $\sec^2\theta - \operatorname{cosec}^2\theta = \tan^2\theta - \cot^2\theta$

6. $\cot^2\theta - \cos^2\theta = \cot^2\theta \cos^2\theta$

7. $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

8. $2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

9. $\cos^2\theta - \sin^2\theta = 1 + \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

9. $\dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \dfrac{\cot\theta - 1}{\cot\theta + 1}$

11. $\dfrac{\sin\theta}{1+\cos\theta} + \cot\theta = \operatorname{cosec}\theta$

12. $\dfrac{\cot^2\theta - 1}{1+\cot^2\theta} = 2\cos^2\theta - 1$

13. $\dfrac{1+\cos\theta}{1-\cos\theta} = (\operatorname{cosec}\theta + \cot\theta)^2$

14. $(\sec\theta - \tan\theta)^2 = \dfrac{1-\sin\theta}{1+\sin\theta}$

15. $\dfrac{2\tan\theta}{1+\tan^2\theta} = 2\sin^2\theta \cos\theta$

16. $\dfrac{1-\sin\theta}{\cos\theta} = \dfrac{\cos\theta}{1+\sin\theta}$

17. $(\tan\theta + \cot\theta)^2 = \sec^2\theta \operatorname{cosec}^2\theta$

18. $\dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \tan\theta + \sec\theta$

19.

$$\frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}$$

20. $\sin^3\theta - \cos^3\theta = (\sin\theta - \cos\theta)$
$(1 + \sin\theta\cos\theta)$

21. $\sin^6\theta - \cos^6\theta = (\sin^2\theta - \cos^2\theta)$
$(1 - \sin^2\theta\cos^2\theta)$

22. $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta$
$\cos^2\theta$

23. $\dfrac{1}{1+\sin\theta} + \dfrac{1}{1-\sin\theta} = 2\sec^2\theta$

24. $\dfrac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} + \dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \dfrac{2}{1-2\sin^2\theta}$

1. $\tan\theta + \cot\theta = \cos ec\theta \sec\theta$

Solution:

L.H.S $= \tan\theta + \cot\theta$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$ (Taking L.C.M)

$= \dfrac{1}{\cos\theta\sin\theta}$

$= \dfrac{1}{\cos\theta}.\dfrac{1}{\sin\theta}$

$= \sec\theta\cos ec\theta$

= R.H.S

Hence proved L.H.S=R.H.S

$\tan\theta + \cot\theta = \cos ec\theta \sec\theta$

Domain of θ

$$\theta \in R \wedge \theta \neq n\frac{\pi}{2}, n \in z$$

2. $\sec\theta\cos ec\theta\sin\theta\cos\theta = 1$

Solution:

L.H.S $= \sec\theta\cos ec\theta\sin\theta\cos\theta$

$= \dfrac{1}{\cos\theta}.\dfrac{1}{\cos\theta}\sin\theta\cos\theta$

$= 1$

= R.H.S

Hence proved L.H.S=R.H.S

$$\sec\theta \operatorname{cosec}\theta \sin\theta \cos\theta = 1$$

Domain of θ

$$\theta \in R \wedge \theta \neq n\frac{\pi}{2}, n \in z$$

3. $\cos\theta + \tan\theta \sin\theta = \sec\theta$

Solution:

L.H.S = $\cos\theta + \tan\theta \sin\theta$

= $\cos\theta + \dfrac{\sin\theta}{\cos\theta}\sin\theta$

= $\cos\theta + \dfrac{\sin^2\theta}{\cos\theta}$

= $\dfrac{\cos^2\theta + \sin^2\theta}{\cos\theta}$ (taking L.C.M)

= $\dfrac{1}{\cos\theta} = \sec\theta = R.H.S$

Hence proved L.H.S=R.H.S

$$\cos\theta + \tan\theta \sin\theta = \sec\theta$$

Domain of θ

$$\theta \in R \wedge \theta \neq n(2n+1)\frac{\pi}{2}, n \in z$$

4. $\operatorname{cosec}\theta + \tan\theta \sec\theta = \sec\theta \sec^2\theta$

Solution:

L.H.S = $\cos ec\theta + \tan\theta \sec\theta$

= $\frac{1}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\frac{1}{\cos\theta} = \frac{1}{\sin\theta} + \frac{\sin\theta}{\cos^2\theta}$

= $\frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos^2\theta}$ (taking L.C.M)

= $\frac{1}{\sin\theta\cos^2\theta}$

= $\frac{1}{\sin\theta}\frac{1}{\cos^2\theta}$

= $\sec^2\theta \cos ec\theta$

= R.H.S

Hence proved L.H.S=R.H.S

$\cos ec\theta + \tan\theta\sec\theta = \sec\theta\sec^2\theta$

Domain of θ

$\theta \in R \wedge \theta \neq n\frac{\pi}{2}, n \in z$

5. $\sec^2\theta - \cos ec^2\theta = \tan^2\theta - \cot^2\theta$

Solution:

L.H.S = $\sec^2\theta - \cos ec^2\theta$

= $(1+\tan^2\theta) - (1+\cot^2\theta)$

= $1+\tan^2\theta - 1 - \cot^2\theta$

= $\tan^2\theta - \cot^2\theta$

= R.H.S

Hence proved L.H.S=R.H.S

$$\sec^2\theta - \operatorname{cosec}^2\theta = \tan^2\theta - \cot^2\theta$$

Domain of θ

$$\theta \in R \wedge \theta \neq n\frac{\pi}{2}, n \in z$$

6. $\cot^2\theta - \cos^2\theta = \cot^2\theta\cos^2\theta$

Solution:

L.H.S $= \cot^2\theta - \cos^2\theta$

$= \dfrac{\cos^2\theta}{\sin^2\theta} - \dfrac{\cos^2\theta}{1}$ $\qquad \cos^2\theta = \dfrac{\cos^2\theta}{\sin^2\theta}$

$= \dfrac{\cos^2\theta - \cos^2\theta\sin^2\theta}{\sin^2\theta}$

$= \dfrac{\cos^2\theta(1-\sin^2\theta)}{\sin^2\theta}$

$= \dfrac{\cos^2\theta}{\sin^2\theta}.\cos^2\theta$

$= \cot^2\theta\cos^2\theta$

$=$ R.H.S

Hence proved L.H.S=R.H.S

$$\cot^2\theta - \cos^2\theta = \cot^2\theta\cos^2\theta$$

Domain of θ

$$\theta \in R \wedge \theta \neq n\pi, n \in z$$

7. $(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)=1$

Solution:

L.H.S $= (\sec\theta+\tan\theta)(\sec\theta-\tan\theta)$

$= \sec^2\theta-\tan^2\theta$

$= \sec^2\theta-(\sec^2\theta-1)$

$= \sec^2\theta-\sec^2\theta+1$

$= 1$

= R.H.S

Hence proved L.H.S=R.H.S

$$\cot^2\theta-\cos^2\theta=\cot^2\theta\cos^2\theta$$

Domain of θ

$$\theta\in R\wedge\theta\neq(2n+1)\frac{\pi}{2},n\in z$$

8. $2\cos^2\theta-1=1-2\sin^2\theta$

Solution:

L.H.S $= 2\cos^2\theta-1$

$= 2(1-\sin^2\theta)-1$

$= 2-2\sin^2\theta-1$

$= 1-2\sin^2\theta$

= R.H.S

Hence proved L.H.S=R.H.S

$$2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

Domain of θ

$$\theta \in R$$

9. $\cos^2\theta - \sin^2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

Solution:

L.H.S $= \cos^2\theta - \sin^2\theta$

$= \cos^2\theta\left(1 - \dfrac{\sin^2\theta}{\cos^2\theta}\right)$

$= \dfrac{1-\tan^2\theta}{\sec^2\theta} = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

$=$ R.H.S

Hence proved L.H.S=R.H.S

$$\cos^2\theta - \sin^2\theta = 1 + \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

Domain of θ

$$\theta \in R \wedge \theta \neq (2n+1)\frac{n}{2}, n \in z$$

10. $\dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \dfrac{\cot\theta - 1}{\cot\theta + 1}$

Solution:

L.H.S $= \dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$

$$= \frac{\sin\theta\left(\frac{\cos\theta}{\sin\theta}-1\right)}{\sin\theta\left(\frac{\cos\theta}{\sin\theta}+1\right)}$$

$$= \frac{\cot\theta-1}{\cot\theta+1}$$

= R.H.S

Hence proved L.H.S=R.H.S

$$\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=\frac{\cot\theta-1}{\cot\theta+1}$$

Domain of θ

$$\theta\in R\wedge\theta\neq n\pi, n\in z$$

11. $\frac{\sin\theta}{1+\cos\theta}+\cot\theta=\cos ec\theta$

Solution:

L.H.S $= \frac{\sin\theta}{1+\cos\theta}+\cot\theta$

$$= \frac{\sin\theta(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}+\cot\theta$$

$$= \frac{\sin\theta(1-\cos\theta)}{1-\cos^2\theta}+\cot\theta$$

$$= \frac{\sin\theta(1-\cos\theta)}{\sin^2\theta}+\cot\theta$$

$$= \frac{1-\cos\theta}{\sin\theta}+\cot\theta$$

$$= \frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}+\cot\theta$$

$= \quad \operatorname{cosec}\theta - \cot\theta + \cot\theta$

$= \quad \operatorname{cosec}\theta$

$=$ R.H.S

Hence $\quad \dfrac{\sin\theta}{1+\cos\theta} + \cot\theta = \operatorname{cosec}\theta$

Domain of θ

$\theta \in R \wedge \theta \neq n\pi, n \in z$

12. $\dfrac{\cot^2\theta - 1}{1+\cot^2\theta} = 2\cos^2\theta - 1$

Solution:

L.H.S $= \quad \dfrac{\cot^2\theta - 1}{1+\cot^2\theta}$

$= \quad \dfrac{\cot^2\theta - 1}{\operatorname{cosec}^2\theta}$

$= \quad \sin^2\theta(\cot^2\theta - 1)$

$= \quad \sin^2\theta\left(\dfrac{\cos^2\theta}{\sin^2\theta} - 1\right)$

$= \quad \sin^2\theta\left[\dfrac{\cos^2\theta - \sin^2\theta}{\sin^2\theta}\right]$

$= \quad \cos^2\theta - \sin^2\theta$

$= \quad \cos^2\theta - \left(1 - \cos^2\theta\right)$

$= \quad \cos^2\theta - 1 + \cos^2\theta$

$= \quad 2\cos^2\theta - 1$

$= \quad$ R.H.S

Hence proved

$$\frac{\cot^2\theta - 1}{1+\cot^2\theta} = 2\cos^2 - 1$$

Domain of θ

$$\theta \in R \wedge \theta \neq n\pi, n \in z$$

13. $\frac{1+\cos\theta}{1-\cos\theta} = (\cos ec\theta + \cot\theta)$

Solution:

L.H.S $= \quad \frac{1+\cos\theta}{1-\cos\theta}$

$= \quad \frac{1+\cos\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}$

$= \quad \frac{(1+\cos\theta)^2}{1-\cos^2\theta}$

$= \quad \frac{(1+\cos\theta)^2}{\sin^2\theta} \qquad 1-\cos^2\theta = \sin^2\theta$

$= \quad \left(\frac{1+\cos\theta}{\sin\theta}\right)^2$

$= \quad \left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right)^2$

$= \quad (\cos ec\theta + \cot\theta)^2$

$= \quad$ R.H.S

Hence proved L.H.S=R.H.S

$$\frac{1+\cos\theta}{1-\cos\theta} = \left(\cos ec\theta + \cot\theta\right)^2$$

Domain of θ

$$\theta \in R \wedge \theta \neq 2n\pi, n \in z$$

14. $\left(\sec\theta - \tan\theta\right)^2 = \frac{1-\sin\theta}{1+\sin\theta}$

Solution:

L.H.S $=$ $\left(\sec\theta - \tan\theta\right)^2$

$=$ $\left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$

$=$ $\frac{\left(1-\sin\theta\right)^2}{\left(\cos\theta\right)^2}$

$=$ $\frac{\left(1-\sin\theta\right)^2}{1-\sin^2\theta}$ $\cos^2 = 1-\sin^2\theta$

$=$ $\frac{(1-\sin\theta)(1-\sin\theta)}{\left(1-\sin\theta\right)(1+\sin\theta)}$

$=$ $\frac{1-\sin\theta}{1+\sin\theta}$

$=$ R.H.S

Hence proved L.H.S=R.H.S

$$(\sec\theta - \tan\theta)^2 = \frac{1-\sin\theta}{1+\sin\theta}$$

Domain of θ

$$\theta \in R \wedge \theta \neq (2n+1)\frac{\pi}{2}, n \in z$$

15. $\frac{2\tan\theta}{1+\tan^2\theta} = 2\sin\theta\cos\theta$

Solution:

L.H.S $= \frac{2\tan\theta}{1+\tan^2\theta}$

$= \frac{2\tan\theta}{\sec^2\theta}$ $\qquad 1+\tan^2\theta = \sec^2\theta$

$= 2\tan\theta\cos^2\theta$

$= 2\frac{\sin\theta}{\cos\theta}\cos^2\theta$

$= 2\sin\theta\cos\theta$

= R.H.S

Hence proved L.H.S=R.H.S

$$\frac{2\tan\theta}{1+\tan^2\theta} = 2\sin\theta\cos\theta$$

Domain of θ

$$\theta \in R \wedge \theta \neq (2n+1)\frac{\pi}{2}, n \in z$$

16. $\frac{1-\sin\theta}{\cos\theta} = \frac{\cos\theta}{1+\sin\theta}$

Solution:

L.H.S $= \frac{1-\sin\theta}{\cos\theta}$

$= \frac{1-\sin\theta}{\cos\theta} \times \frac{\cos\theta}{\cos\theta}$

$= \dfrac{\cos\theta(1-\sin\theta)}{\cos^2\theta}$

$= \dfrac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta}$ $\cos^2\theta = 1-\sin^2\theta$

$= \dfrac{\cos\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$

$= \dfrac{\cos\theta}{1+\sin\theta}$

= R.H.S

Hence proved L.H.S=R.H.S

$$\frac{1-\sin\theta}{\cos\theta} = \frac{\cos\theta}{1+\sin\theta}$$

Domain of θ

$$\theta \in R \wedge \theta \neq (2n+1)\frac{\pi}{2}, n \in z$$

17. $(\tan\theta + \cot\theta)^2 = \sec^2\theta \operatorname{cosec}^2\theta$

Solution:

L.H.S $= (\tan\theta + \cot\theta)^2$

$= \left(\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}\right)^2$

$= \left(\dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right)^2$

$= \left(\dfrac{1}{\cos\theta}\dfrac{1}{\sin\theta}\right)^2$

$= (\sec\theta \operatorname{cosec}\theta)^2$

$= \sec^2\theta \operatorname{cosec}^2\theta$

$=$ R.H.S

Hence proved L.H.S=R.H.S

$$(\tan\theta + \cot\theta)^2 = \sec^2\theta \operatorname{cosec}^2\theta$$

Domain of θ

$$\theta \in R \wedge \theta \neq n\frac{\pi}{2}, n \in z$$

18. $\dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \tan\theta + \sec\theta$

Solution:

$$\text{L.H.S} = \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$$

$$= \frac{\tan\theta + \sec\theta - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\tan\theta + \sec\theta) - (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\tan\theta + \sec\theta)\left[1 - (\sec\theta - \tan\theta)\right]}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\tan\theta + \sec\theta)\left[1 - \sec\theta + \tan\theta\right]}{\tan\theta - \sec\theta + 1}$$

$$= \tan\theta + \sec\theta$$

$=$ R.H.S

Hence proved L.H.S=R.H.S

$$\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \tan\theta + \sec\theta$$

Domain of θ

$$\theta \in R \wedge \theta \neq (2n+1)\frac{\pi}{2}, n \in z$$

19. $\frac{1}{\cos ec\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\cos ec\theta + \cot\theta}$

Solution:

L.H.S $= \frac{1}{\cos ec\theta - \cot\theta} - \frac{1}{\sin\theta}$

$= \frac{1}{(\cos ec\theta - \cot\theta)}\left(\frac{\cos ec\theta - \cot\theta}{\cos ec\theta - \cot\theta}\right)\cos ec\theta$

$= \frac{\cos ec\theta - \cot\theta}{(\cos ec^2\theta - \cot^2\theta)} - \cos ec\theta$ $\left[\therefore \cos ec^2\theta - \cot^2\theta = 1\right]$

$= \cos ec\theta + \cot\theta - \cos ec\theta$

$= \cot\theta$

R.H.S $= \frac{1}{\sin\theta} - \frac{1}{\cos ec\theta - \cot\theta}$

$= \frac{1}{\sin\theta} - \frac{1}{\cos ec\theta - \cot\theta}\left(\frac{\cos ec\theta - \cot\theta}{\cos ec\theta - \cot\theta}\right)$

$= \cos ec\theta - \left(\frac{\cos ec\theta - \cot\theta}{\cos ec^2\theta - \cot^2\theta}\right)$

$= \cos ec\theta - \cos ec\theta + \cot\theta$

$= \cot\theta$

$=$ L.H.S

Hence proved L.H.S=R.H.S

$$\frac{1}{\cos ec\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\cos ec\theta - \cot\theta}$$

Domain of θ

$$\theta \in R \wedge \theta \neq n\pi, n \in z$$

20. $\sin^3\theta - \cos^3\theta = (\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta)$

Solution:

L.H.S $= \sin^3\theta - \cos^3\theta$

$= (\sin\theta)^3 - (\cos\theta)^3$

$= (\sin\theta - \cos\theta)\left[(\sin\theta)^2 - (\cos\theta)^2 + \sin\theta\cos\theta\right]$

$= (\sin\theta - \cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta)$

$= (\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta)$

$=$ R.H.S

Hence proved L.H.S=R.H.S

$$\sin^3\theta - \cos^3\theta = (\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta)$$

Domain of θ

$$\theta \in R$$

21. $\sin^6\theta - \cos^6\theta = (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$

Solution:

L.H.S $= \sin^6\theta - \cos^6\theta$

$= (\sin^2\theta)^3 - (\cos^2\theta)^3$

$= (\sin^2\theta - \cos^2\theta)(\sin^4\theta + \cos^4\theta + \sin^2\theta\cos^2\theta)$

$= \quad \left(\sin^2\theta-\cos^2\theta\right)\left[\sin^4\theta+\cos^4\theta+2\sin^2\theta\cos^2\theta-\sin^2\theta\cos^2\theta\right]$

$= \quad \left(\sin^2\theta-\cos^2\theta\right)\left[\left(\sin^2\theta+\cos^2\theta\right)^2-\sin^2\theta\cos^2\theta\right]$

$= \quad \left(\sin^2\theta-\cos^2\theta\right)\left[(1)^2-\sin^2\theta\cos^2\theta\right]$

$= \quad \left(\sin^2\theta-\cos^2\theta\right)\left[1-\sin^2\theta\cos^2\theta\right]$

= R.H.S

Hence proved L.H.S=R.H.S

$\sin^6\theta-\cos^6\theta=\left(\sin^2\theta-\cos^2\theta\right)\left(1-\sin^2\theta\cos^2\theta\right)$

Domain of θ

$\theta\in R$

22. $\sin^6\theta+\cos^6\theta=1-3\sin^2\theta\cos^2\theta$

Solution:

L.H.S $= \quad \sin^6\theta+\cos^6\theta$

$= \quad \left(\sin^2\theta\right)^3+\left(\cos^2\theta\right)^3$

$= \quad \left(\sin^2\theta+\cos^2\theta\right)\left[\sin^4\theta+\cos^4\theta-\sin^2\theta\cos^2\theta\right] \qquad \therefore \sin^2\theta+\cos^2\theta=1$

$= \quad (1)\left[\sin^4\theta+\cos^4\theta+2\sin^2\theta\cos^2\theta-3\sin^2\theta\cos^2\theta\right]$

$= \quad (1)\left[\left(\sin^2\theta+\cos^2\theta\right)^2-3\sin^2\theta\cos^2\theta\right]$

$= \quad \left[(1)^2-3\sin^2\theta\cos^2\theta\right]$

$= \quad 1-3\sin^2\theta\cos^2\theta$

= R.H.S

Hence proved L.H.S=R.H.S

$$\sin^6\theta+\cos^6\theta=1-3\sin^2\theta\cos^2\theta$$

Domain of θ

$$\theta\in R$$

23. $\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=2\sec^2\theta$

Solution:

L.H.S = $\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}$

= $\frac{1-\sin\theta+1+\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$

= $\frac{2}{1-\sin^2\theta}$ $(a+b)(a-b)=a^2+b^2$

= $\frac{2}{\cos^2\theta}$

= $2\sec^2\theta$

= R.H.S

Hence proved L.H.S=R.H.S

$$\frac{1}{1+\sin\theta}+\frac{1}{1-\sin\theta}=2\sec^2\theta$$

Domain of θ

$$\theta\in R\wedge\theta\neq(2n+1)\frac{\pi}{2},n\in z$$

24. $\dfrac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}+\dfrac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=\dfrac{2}{1-2\sin^2\theta}$

Solution:

L.H.S $= \dfrac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}+\dfrac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$

$= \dfrac{(\cos\theta+\sin\theta)^2+(\cos\theta-\sin\theta)^2}{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}$

$= \dfrac{2(\cos^2\theta+\sin^2\theta)}{\cos^2\theta+\sin^2\theta}$

$= \dfrac{2}{(1-\sin^2\theta)-\sin^2\theta}$ $\quad \cos^2\theta=1-\sin^2\theta$

$= \dfrac{2}{1-\sin^2\theta-\sin^2\theta}$

$= \dfrac{2}{1-2\sin^2\theta}$

= R.H.S

Hence proved L.H.S=R.H.S

$$\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}+\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=\frac{2}{1-2\sin^2\theta}$$

Domain of θ

$$\theta\in R\wedge\theta\neq(2n+1)\frac{\pi}{4}, n\in z$$