Exercise 9.2

1. Find the signs of the following:

i. $\sin 160^0$ **ii.** $\cos 190^0$ **iii.** $\tan 115^0$

iv. $\sec 245^0$ **v.** $\cot 80^0$ **vi.** $\operatorname{cosec} 297^0$

i. $\sin 160^0$

The terminal side of angle 160^0 lie in II quadrant.

Therefore, sign of $\sin 160^0$ is positive.

Hence, value of $\sin 160^0$ = positive

ii. $\cos 190^0$

The terminal side of angle 190^0 lie in III quadrant.

Therefore, sign of $\cos 190^0$ is negative.

Hence, value of $\cos 190^0$ = negative

iii. $\tan 115^0$

The terminal side of angle 115^0 lie in II quadrant.

Therefore, sign of $\tan 115^0$ is negative.

Hence, value of $\tan 115^0$ = negative.

iv. $\sec 245^0$

The terminal side of angle 245^0 lie in III quadrant.

Therefore, sign of $\sec 245^0$ is negative.

Hence, value of sec 245^0 = negative

v. cot80^0

The terminal side of angle 80^0 lie in I quadrant.

Therefore, sign of cot 80^0 is positive.

Hence, value of cot 80^0 = positive

vi. cosec297^0

The terminal side of angle 297^0 lie in IV quadrant.

Therefore, sign of cosec 297^0 is negative.

Hence, value of cosec297^0 = negative

Q2. Fill in the blanks:

i.	**sin(-310^0) = … sin 310^0**	**ii.**	**cos(-75^0) = … cos75^0**
iii.	**tan(-182^0) = … tan 182^0**	**iv.**	**cot(-137^0) = … cot137^0**
v.	**sec(-216^0) = …sec 216^0**	**vi.**	**cosec(-15^0) = …cosec15**

Solution:

i. sin(-310^0) = ___–___ sin310^0

ii. cos(-75^0) = ___+___ cos75^0

iii. tan(-182^0) = ___–___ tan 182^0

iv. cot(-137^0) = ___–___ cot137^0

v. sec(-216^0) = ___+___ sec216^0

vi. cosec(-15^0)= ___–___ cosec15^0

Q3. In which quadrant are the terminal arms of the angle lie when

i.	**$\sin\theta<0$ and $\cos\theta>0$**	**ii.**	**$\cot\theta>0$ and cosec>0**
iii.	**$\tan\theta<0$ and $\cos\theta>0$**	**iv.**	**$\sec\theta<0$ and $\sin\theta<0$**
v.	**$\cot\theta>0$ and $\sin\theta<0$**	**vi.**	**$\cos\theta<0$ and $\tan\theta<0$**

i. $\sin\theta<0$ and $\cos\theta>0$

$\sin\theta$ = -ve

and $\cos\theta$ = +ve

The terminal arms lie in IV quadrant.

ii. $\cot\theta>0$ and $\text{cosec}\,\theta>0$

$\cot\theta$ = +ve

and $\text{cosec}\,\theta$ = +ve

The terminal arms lie in I quadrant.

iii. $\tan\theta$**<0 and $\cos\theta>0$**

$\tan\theta$ = -ve

$\cos\theta$ = +ve

The terminal arms lie in IV quadrant.

iv. $\sec\theta<0$ and $\sin\theta<0$

$\sec\theta$ = -ve

$\sin\theta$ = -ve

The terminal arms lie in III quadrant.

v. cotθ>0 and sinθ<0

$\cot\theta$ = +ve

$\sin\theta$ = -ve

The terminal arms lie in III quadrant.

vi. cosθ<0 and tanθ<0

$\cos\theta$ = -ve

$\tan\theta$ = -ve

The terminal arms lie in II quadrant.

4. Find the values of remaining trigonometric functions.

i. **$\sin\theta = \frac{12}{13}$ and the terminal arm of the angle is in quad. I.**

ii. **$\cos\theta = \frac{9}{41}$ and the terminal arm of the angle is in quad. IV.**

iii. **$\cos\theta = -\frac{\sqrt{3}}{2}$ and the terminal arm of the angle is in quad. III.**

iv. **$\tan\theta = -\frac{1}{3}$ and the terminal arm of the angle is in quad. II.**

v. **$\sin\theta = -\frac{1}{\sqrt{2}}$ and the terminal arm of the angle is not in quad. III.**

Solution:

i. **a)** $\sin\theta = \frac{12}{13}$

b) $\cos ec\theta = \frac{13}{12}$

c) $\cos\theta = \pm\sqrt{1-\sin^2\theta}$ $\qquad Q\cos^2\theta + \sin^2\theta = 1$

$= \pm\sqrt{1-\left(\frac{12}{13}\right)^2}$ $\qquad \cos^2\theta = 1-\sin^2\theta$

$= \pm\sqrt{1-\frac{144}{169}}$ $\qquad \cos\theta = \sqrt{1-\sin^2\theta}$

$= \pm\sqrt{\frac{169-144}{169}} = \pm\sqrt{\frac{25}{169}} = \pm\frac{5}{13}$

$\Rightarrow$ $\cos\theta = \frac{5}{13}$ [Terminal arms lie in I quadrant]

d) $\sec\theta = \frac{13}{5}$

e) $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{12/13}{5/13} = \frac{12}{5}$

f) $\cot\theta = \frac{1}{\tan\theta} = 5/12$

Hence;

$\sin\theta = \frac{12}{13}$; $\cos ec\theta = \frac{13}{12}$

$\cos\theta = \frac{5}{12}$; $\sec\theta = \frac{13}{5}$

$\tan\theta = \frac{12}{5}$; $\cot\theta = \frac{5}{12}$

ii. $\cos\theta = \frac{9}{41}$ **and the terminal arm of the angle is in the quadrant IV.**

a. $\cos\theta = \frac{9}{41}$ $\qquad$ $Q\sin^2\theta + \cos^2\theta = 1$

b. $\sec\theta = \frac{1}{\cos\theta} = \frac{1}{9/41} = \frac{41}{9}$ $\qquad$ $\sin^2\theta = 1 - \cos^2\theta$

c. $\sin\theta = \pm\sqrt{1-\cos^2\theta}$ $\qquad$ $\sin\theta = \sqrt{1-\cos^2\theta}$

$$= \pm\sqrt{1-\left(\frac{9}{41}\right)^2}$$

$$= \pm\sqrt{1-\frac{81}{1681}}$$

$$= \pm\sqrt{\frac{1681-81}{1681}} = \pm\sqrt{\frac{1600}{1681}} = \pm\frac{40}{41}$$

$\Rightarrow \sin\theta = -\frac{40}{41}$ [the terminal arm of the angle is in IV quadrant]

d. $\cos ec\theta = \frac{1}{\sin\theta} = \frac{1}{-41/40} = -\frac{41}{40}$

e. $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{40/41}{-9/41} = -\frac{40}{9}$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-40/9} = -\frac{9}{40}$$

Hence;

$\cos\theta = \frac{9}{40}$; $\sec\theta = \frac{41}{9}$

$\sin\theta = -\frac{40}{41}$; $\cos ec\theta = -\frac{41}{40}$

$\tan\theta = -\dfrac{40}{9}$; $\cot\theta = -\dfrac{9}{40}$

iii. $\cos = -\dfrac{\sqrt{3}}{2}$ **and the terminal arm of the angle is in quadrant** III**.**

a. $\cos\theta = -\dfrac{\sqrt{3}}{2}$

$Q\sin^2\theta + \cos^2\theta = 1$

b. $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{-\sqrt{3}} = -\dfrac{2}{\sqrt{3}}$

$\sin^2\theta = 1 - \cos^2\theta$

c. $\sin\theta = \pm\sqrt{1-\cos^2\theta}$

$\sin\theta = \sqrt{1-\cos^2\theta}$

$$= \pm\sqrt{1-\left(-\frac{\sqrt{3}}{2}\right)^2}$$

$$= \pm\sqrt{1-\frac{3}{4}}$$

$$= \pm\sqrt{\frac{4-3}{4}} = \pm\sqrt{\frac{1}{4}} = -\frac{1}{2}$$

[The terminal arm of angle lies in III quadrant]

$\Rightarrow \sin\theta = -\dfrac{1}{2}$

d. $co\sec\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{-1/2} = -2$

e. $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{-1/2}{-\sqrt{3}/2} = \dfrac{1}{\sqrt{3}}$

f. $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{1/\sqrt{3}} = \sqrt{3}$

Hence;

$\sin\theta = -\frac{1}{2}$; $\operatorname{cosec}\theta = -2$

$\cos\theta = -\frac{\sqrt{3}}{2}$; $\sec\theta = -\frac{2}{\sqrt{3}}$

$\tan\theta = \frac{1}{\sqrt{3}}$; $\cot\theta = \sqrt{3}$

iv. $\tan\theta = -\frac{1}{3}$ **and the terminal arm of the angle is in quadrant** II **.**

a. $\tan\theta = -\frac{1}{3}$

b. $\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-1/3} = -3$

c. $1 + \tan^2\theta = \sec^2\theta$

$\sec^2\theta = 1 + \left(-\frac{1}{3}\right)^2$

$= 1 + \frac{1}{9} = \frac{9+1}{9} = \frac{10}{9}$

$\Rightarrow$ $\sec\theta = \sqrt{\frac{10}{9}} = \pm\frac{\sqrt{10}}{3}$

$\sec\theta = -\frac{\sqrt{10}}{3}$ [The terminal arm lie in II quadrant]

d. $\cos\theta = \frac{1}{\sec\theta} = \frac{1}{-\sqrt{10}/3} = \frac{-3}{\sqrt{10}}$

e. $\sin\theta = \cos\theta . \tan\theta$

$$= \frac{-3}{\sqrt{10}} \times \left(-\frac{1}{3}\right) = \frac{1}{\sqrt{10}}$$

f. $\cos ec\theta = \frac{1}{\sin\theta} = \frac{1}{1/\sqrt{10}} = \sqrt{10}$

Hence;

$\sin\theta = \frac{1}{\sqrt{10}}$; $\cos ec\theta = \sqrt{10}$

$\cos\theta = -\frac{3}{\sqrt{10}}$; $\sec\theta = -\frac{\sqrt{10}}{3}$

$\tan\theta = -\frac{1}{3}$; $\cot\theta = -3$

v. $\sin\theta = -\frac{1}{\sqrt{2}}$ **and the terminal arm of the angel is not in quad.** III**.**

a. $\sin\theta = -\frac{1}{\sqrt{2}}$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\cos\theta = \sqrt{1 - \sin^2\theta}$$

b. $\cos ec\theta = \frac{1}{\sin\theta} = \frac{1}{-1/2} = -\sqrt{2}$

c. $\cos\theta = \pm\sqrt{1 - \sin^2\theta}$

$$= \pm\sqrt{1 - \left(\frac{1}{2}\right)^2}$$

$$= \pm\sqrt{1 - \frac{1}{2}}$$

$$= \pm\sqrt{\frac{2-1}{2}} = \pm\sqrt{\frac{1}{2}} = +\frac{1}{\sqrt{2}}$$

[Terminal arms of the angle lies in IV quad.]

d. $\sec\theta = \frac{1}{\cos\theta} = \frac{1}{1/\sqrt{2}} = \sqrt{2}$

e. $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\Rightarrow$ $\tan\theta = \frac{-1/\sqrt{2}}{1/\sqrt{2}} = -1$

f. $\cot\theta = \frac{1}{\tan\theta} = -\frac{1}{1} = -1$

$\Rightarrow \cot\theta = -1$

Hence;

$\sin\theta = -\frac{1}{\sqrt{2}}$; $\cos ec = -\sqrt{2}$

$\cos\theta = +\frac{1}{\sqrt{2}}$; $\sec\theta = +\sqrt{2}$

$\tan\theta = -1$; $\cot\theta = -1$

Q5. If $\cot\theta = \frac{15}{8}$ and the terminal arm of the angle is not in quad. 1. Find values of $\cos\theta$ and $\cos ec\theta$

Solution:

$$\cot\theta = \frac{15}{8}$$

We know that $1 + \cot^2\theta = \cos ec^2\theta$

$$\operatorname{cosec}^2\theta = 1 + \left(\frac{15}{8}\right)^2$$

$$= 1 + \frac{225}{64} = \frac{64 + 225}{64} = \frac{289}{64}$$

$\Rightarrow$ $$\operatorname{cosec}\theta = \pm\sqrt{\frac{289}{64}} = \pm\frac{17}{8}$$

$\Rightarrow$ $\operatorname{cosec}\theta = -\frac{17}{8}$ [terminal arm lie in III quadrant]

$\Rightarrow$ $$\sin\theta = \frac{1}{\operatorname{cosec}\theta} = \frac{1}{-\frac{17}{8}} = -\frac{8}{17}$$

We know that $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\Rightarrow$ $$\cos\theta = \cot\theta \times \sin\theta$$

$$= \frac{15}{8} \times \left(-\frac{8}{17}\right)$$

$\Rightarrow$ $$\cos\theta = -\frac{15}{17}$$

Hence; $\operatorname{cosec}\theta = -\frac{17}{8}$

And $\cos\theta = -\frac{15}{17}$

Q6. If $\operatorname{cosec}\theta = \frac{m^2+1}{2m}$ and $\left(0 < \theta < \frac{\pi}{2}\right)$, find the values of the remaining trigonometric ratios.

Solution:

$0<\theta<\frac{\pi}{2}$; shows that the terminal arm lies in I quadrant. All values are positive.

$\Rightarrow \quad \cos ec\theta=\frac{m^2+1}{2m}$

And

$$\sin\theta=\frac{1}{\cos ec\theta}=\frac{1}{\frac{m^2+1}{2m}}=\frac{2m}{m^2+1}$$

And

$$\cos\theta=\pm\sqrt{1-\sin^2\theta}$$

$$=\pm\sqrt{1-\frac{(2m)^2}{(m^2+1)^2}}$$

$$=\pm\sqrt{\frac{(m^2+1)^2-(2m)^2}{(m^2+1)^2}}=\pm\sqrt{\frac{(m^2-1)^2}{(m^2+1)^2}}=\frac{m^2-1}{m^2+1}$$

So, $\quad \cos\theta=\frac{m^2-1}{m^2+1}$

And $\quad \sec\theta=\frac{1}{\cos\theta}=\frac{1}{\frac{m^2-1}{m^2+1}}=\frac{m^2+1}{m^2-1}$

And $\quad \tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{2m/m^2+1}{m^2-1/m^2+1}=\frac{2m}{m^2-1}$

And $\quad \cot\theta=\frac{1}{\tan\theta}=\frac{1}{\frac{2m}{m^2-1}}=\frac{m^2-1}{2m}$

Hence; $\quad \sin\theta=\frac{2m}{m^2+1} \quad ; \quad \cos ec\theta=\frac{m^2+1}{2m}$

$$\cos\theta = \frac{m^2-1}{m^2+1} \quad ; \quad \sec\theta = \frac{m^2+1}{m^2-1}$$

$$\tan\theta = \frac{2m}{m^2-1} \quad ; \quad \cot\theta = \frac{m^2-1}{2m}$$

7. If $\tan\theta = \frac{1}{\sqrt{7}}$ and the terminal arm of the angle is not in the III quad, find the values of $\frac{\csc^2\theta - \sec^2\theta}{\csc^2\theta + \sec^2\theta}$

Solution:

$\tan\theta = \frac{1}{\sqrt{7}}$; the terminal arm of the angle is in I quadrant; where all values are positive.

$$\tan\theta = \frac{1}{\sqrt{7}}$$

and we know that

$$1 + \tan^2\theta = \sec^2\theta$$

$$\Rightarrow \quad \sec^2 = 1 + \left(\frac{1}{\sqrt{7}}\right)^2$$

$$= 1 + \frac{1}{7} = \frac{7+1}{7} = \frac{8}{7}$$

$$\sec\theta = \sqrt{\frac{8}{7}} = \frac{2\sqrt{2}}{\sqrt{7}}$$

So, $$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\frac{2\sqrt{2}}{\sqrt{7}}} = \frac{\sqrt{7}}{2\sqrt{2}}$$

And $$\sin\theta = \cos\theta . \tan\theta$$

$$= \frac{\sqrt{7}}{2\sqrt{2}} \times \frac{1}{\sqrt{7}} = \frac{1}{2\sqrt{2}}$$

$\Rightarrow \quad \sin\theta = \frac{1}{2\sqrt{2}}$

And $\quad \csc\theta = \frac{1}{\sin\theta} = \frac{1}{2\sqrt{2}} = 2\sqrt{2}$

Now $\quad = \frac{\csc^2\theta - \sec^2\theta}{\csc^2\theta + \sec^2\theta}$

$$= \frac{\left(2\sqrt{2}\right)^2 - \left(\frac{2\sqrt{2}}{7}\right)^2}{\left(2\sqrt{2}\right)^2 + \left(\frac{2\sqrt{2}}{7}\right)^2}$$

$$= \frac{8 - \frac{8}{7}}{8 + \frac{8}{7}}$$

$$= \frac{\frac{56-8}{7}}{\frac{56+8}{7}} = \frac{48}{64} = \frac{3}{4}$$

Hence,

$$\frac{\cos e^2\theta - \sec^2\theta}{\cos e^2\theta + \sec^2\theta} = \frac{3}{4}$$

Q8. If $\cot\theta = \frac{5}{2}$ and the terminal arm of the angle is in the I quadrant find the value of $\frac{3\sin\theta + 4\cos\theta}{\cos\theta - \sin\theta}$

Solution:

$$\cot\theta = 5/2$$

$$\Rightarrow \quad \operatorname{cosec}^2\theta = 1 + \cot^2\theta$$

$$= 1 + \left(\frac{5}{2}\right)^2 = 1 + \frac{25}{4} = \frac{4+25}{4} = \frac{29}{4}$$

$$\Rightarrow \quad \operatorname{cosec}\theta = \sqrt{\frac{29}{4}} = \frac{\sqrt{29}}{2}$$

So, $\sin\theta = \dfrac{1}{\operatorname{cosec}\theta} = \dfrac{1}{\frac{\sqrt{29}}{2}} = \dfrac{2}{29}$

And $\cos\theta = \sin\theta \times \cot\theta$

$$= \frac{2}{\sqrt{29}} \times \frac{5}{2} = \frac{5}{\sqrt{29}}$$

Now,

$$= \frac{3\sin\theta + 4\cos\theta}{\cos\theta - \sin\theta}$$

$$= \frac{3\left(\frac{2}{\sqrt{29}}\right) + 4\left(\frac{5}{\sqrt{29}}\right)}{\frac{5}{\sqrt{29}} - \frac{2}{\sqrt{29}}} = \frac{\frac{6}{\sqrt{29}} + \frac{20}{\sqrt{29}}}{\frac{5}{\sqrt{29}} - \frac{2}{\sqrt{29}}}$$

$$= \frac{\frac{6+20}{\sqrt{29}}}{\frac{5-2}{\sqrt{29}}}$$

$$= \frac{6+20}{5-2} = \frac{26}{3}$$

Hence,

$$\frac{3\sin\theta + 4\cos\theta}{\cos\theta - \sin\theta} = \frac{26}{3}$$