Chapter 9

Fundamentals of Trigonometry

Exercise 9.1

1. Express the following sexagesimal measures of angles in radians:

i) 30° **ii)** 45° **iii)** 60° **iv)** 75°

v) 90° **vi)** 105° **vii)** 120° **viii)** 135°

ix) 150° **x)** $10^\circ 15''$ **xi)** $35^\circ 20''$ **xii)** $75^\circ 6' 30''$

xiii) $120' 40''$ **xiv)** $154^\circ 20''$ **xv)** $0''$ **xvi)** $3''$

Solution:

We know that

$$1^\circ = \frac{\pi}{180^\circ} radians$$

Or $$1^\circ = 0.01745 radians$$

i. 30°

$$\Rightarrow 30^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{6} radians$$

ii. 45°

$$\Rightarrow 45° \times \frac{\pi}{180°} = \frac{\pi}{4} radians$$

iii. $60°$

$$\Rightarrow 60° \times \frac{\pi}{180°} = \frac{\pi}{3} radians$$

iv. $75°$

$$\Rightarrow 75° \times \frac{\pi}{180°} = \frac{5\pi}{12} radians$$

v. $90°$

$$\Rightarrow 90° \times \frac{\pi}{180°} = \frac{\pi}{2} radians$$

vi. $105°$

$$\Rightarrow 105° \times \frac{\pi}{180°} = \frac{7}{12} \pi radians$$

vii. $120°$

$$\Rightarrow 120° \times \frac{\pi}{180°} = \frac{2}{3} \pi radians$$

viii. $135°$

$$\Rightarrow 135° \times \frac{\pi}{180°} = \frac{3}{4} \pi radians$$

ix. $150°$

$$\Rightarrow 150° \times \frac{\pi}{180°} = \frac{5}{6}\pi radians$$

x. $10°15''$

$$10°15'' = 10° + \left(\frac{1}{60} \times 15\right)^{o} \qquad [60' = 1°]$$

$$= 10° + \frac{1°}{4} = 10.25°$$

$$= 10.25 \times \frac{\pi}{180°} = 10.25 \times 0.01745 = 0.1789 radians$$

xi. $35°20'$

$$35° + 20' = 35° + \left[\frac{1}{60} \times 20\right]^{o}$$

$$= 35° + \left[\frac{1}{3}\right]^{o} = \left[\frac{106}{3}\right]^{o}$$

$$= \frac{106°}{3} \times 0.01745 = 0.6166 radians$$

xii. $75°6'30''$

$$75° + 6'30'' = 75° + \left[\frac{1}{60} \times 6\right]^{o} + \left[\frac{1}{3600} \times 30\right]^{o}$$

$$= 75° + \frac{1}{10} + \frac{1}{120} = \frac{9000 + 12 + 1}{120} = \left[\frac{9013}{120}\right]^{o}$$

$$= \frac{9013°}{120} \times 0.01745 = 1.3106 radians$$

xiii. 120'40"

$$= 120' + \left[40 \times \frac{1}{60}\right]'$$

$$= 120' + \frac{2}{3} = \frac{360+2}{3} = \left[\frac{362}{3}\right]'$$

$$= \left[\frac{362}{3} \times \frac{1}{60}\right]^{o} = \left[\frac{362}{180}\right]^{o}$$

$$= \frac{362}{180°} \times 0.01745 = 0.03541 radians$$

xiv. 154°20"

$$154° + 20" = [154]^{o} + \left[\frac{20}{3600}\right]^{o}$$

$$= 154° + \left[\frac{1}{180}\right]^{o}$$

$$= \left[\frac{27720+1}{180}\right]^{o} = \left[\frac{27721}{180}\right]^{o}$$

$$= \left[\frac{27721}{180}\right]^{o} \times 0.0175 = 2.6874 radians$$

xv. 0°

$$= 0° \times \frac{\pi}{180°} = 0 radians$$

xvi. $3''$

$$= \frac{3}{3600} = \left[\frac{1}{120}\right]^{o}$$

$$= \left[\frac{1}{120}\right]^{o} \times 0.01745 = 0.0000145 radians$$

2. Convert the following radian measures of angles into the measures of system:

i)	$\frac{\pi}{8}$	**ii)**	$\frac{\pi}{6}$	**iii)**	$\frac{\pi}{4}$	**iv)**	$\frac{\pi}{3}$	**v)**	$\frac{\pi}{2}$
vi)	$\frac{2\pi}{3}$	**vii)**	$\frac{3\pi}{4}$	**viii)**	$\frac{5\pi}{6}$	**ix)**	$\frac{7\pi}{12}$	**x)**	$\frac{9\pi}{5}$
xi)	$\frac{11\pi}{27}$	**xii)**	$\frac{13\pi}{16}$	**xiii)**	$\frac{17\pi}{24}$	**xiv)**	$\frac{25\pi}{36}$	**xv)**	$\frac{19\pi}{32}$

Solution:

We know that

$$1 \text{ radians } = \frac{180^0}{\pi} \deg ree$$

Or 1 radians = 57.296 degree

i. $\frac{\pi}{8}$

$$= \frac{\pi}{8} \times \frac{180^0}{\pi} = \frac{45}{2} = (22.5)^0$$

ii. $\frac{\pi}{6}$

$$= \frac{\pi}{6} \times \frac{180^0}{\pi} = 30^0$$

iii. $\frac{\pi}{4}$

$$= \frac{\pi}{4} \times \frac{180^0}{\pi} = 45^0$$

iv. $\frac{\pi}{3}$

$$= \frac{\pi}{3} \times \frac{180^0}{\pi} = 60^0$$

v. $\frac{\pi}{2}$

$$= \frac{\pi}{2} \times \frac{180^0}{\pi} = 90^0$$

vi. $\frac{2\pi}{3}$

$$= \frac{2\pi}{3} \times \frac{180^0}{\pi} = 120^0$$

vii. $\frac{3\pi}{4}$

$$= \frac{3\pi}{4} \times \frac{180^0}{\pi} = 135^0$$

viii. $\frac{5\pi}{6}$

$$= \frac{5\pi}{6} \times \frac{180^0}{\pi} = 150^0$$

ix. $\frac{7\pi}{12}$

$$= \frac{7\pi}{12} \times \frac{180^0}{\pi} = 105^0$$

x. $\frac{9\pi}{5}$

$$= \frac{9\pi}{5} \times \frac{180^0}{\pi} = 324^0$$

xi. $\frac{11\pi}{27}$

$$= \frac{11\pi}{27} \times \frac{180^0}{\pi} = \left[\frac{1980}{27}\right]^0 = \left[73\frac{1}{3}\right]^0 = 73^0 20'$$

xii. $\frac{13\pi}{16}$

$$= \frac{13\pi}{16} \times \frac{180^0}{\pi}$$

$$= \frac{2340}{16} = \left[146\frac{1}{4}\right]^0 = 146^0 30'$$

xiii. $\frac{17\pi}{24}$

$$= \frac{17\pi}{24} \times \frac{180^0}{\pi}$$

$$= \frac{3060}{24} = \left[127\frac{1}{2}\right]^0 = 127^0 30'$$

xiv. $\frac{25\pi}{36}$

$$= \frac{25\pi}{36} \times \frac{180^0}{\pi} = 125^0$$

xv. $\frac{19\pi}{32}$

$$= \frac{19\pi}{32} \times \frac{180^0}{\pi}$$

$$= \frac{3420}{32} = \left[106\frac{7}{8}\right]^0 = 106^0 52' 30''$$

3. What is the circular measure of the angle between the hands of a watch at 4 O' clock?

Solution

As total angle inscribed in a circle is 360^0 and we know. That one hour is $\frac{1}{12}th$

part

of the circle Therefore 1 hour $= 360^0 \times \frac{1}{12} = 30^0$ In radians $= 30^0 \times \frac{\pi}{180} = \frac{\pi}{6}$ radian So,

the angle between the hand of a watch at 4O'clock. $= \frac{\pi}{6} \times 4 = \frac{2}{3}\pi$ radian.

Hence, the angle is $\frac{2}{3}\pi$ radian.

4. Find θ , when:

i) $l = 1.5cm, r = 2.5cm$ **ii)** $l = 3.2m, r = 2m$

Solution:

We know that

$$l = r.\theta$$
$$\Rightarrow \theta = \frac{l}{r}$$

i) $l = 1.5cm, r = 2.5cm$

$$\Rightarrow \theta = \frac{l}{r} = \frac{1.5}{2.5} = 0.6 \text{ radians}$$

Hence, $\theta = 0.6$ radians

ii) $l = 3.2cm, r = 2m$

$$\Rightarrow \theta = \frac{l}{r} = \frac{3.2}{2} = 1.6 \text{ radians}$$

Hence, $\theta = 1.6$ radians

5. Find l, when:

i) $\theta = \pi$ **radians, r = 6cm** **ii)** $\theta = 65^{0}20'$**,r = 18mm**

Solution:

We know that $l = r.\theta$

i. $\theta = \pi$ **radians, r = 6cm**

$$\Rightarrow l = r.\theta$$

$$= 6\pi = 6\pi cm$$

Hence, $l = 6\pi cm$

ii. $\theta = 65^{0}20'$**, r=18mm**

$$\theta = 65^{0}25' \times \frac{\pi}{180}$$

$$= \left[65^{0} + \frac{25}{60}\right]^{0} \times \frac{\pi}{180^{0}} = 1.1417 \text{ radians}$$

$$\Rightarrow l = r\theta$$

$$= 1.1417 \times 18 = 20.55mm$$

Hence, $l = 20.55mm$

6. Find r, when:

i) $l = 5cm, \theta = \frac{1}{2}$ **radian** **ii)** $l = 56cm, \theta = 45^0$

Solution:

We know that

$$l = r.\theta$$

$$\Rightarrow r = \frac{l}{2}$$

i. $l = 5cm, \theta = \frac{1}{2}$ **radian**

$$\Rightarrow r = \frac{l}{\theta} = \frac{5}{1/2} = 5 \times 2 = 10cm$$

Hence, $l = 10cm$

ii. $l = 56cm, \theta = 45^0$

$$\Rightarrow \theta = 45^0 \times \frac{\pi}{180^0} = \frac{\pi}{4} \text{ radians}$$

$$r = \frac{l}{\theta} = \frac{56}{\pi/4} = \frac{56 \times 4}{\pi} = 71.30cm$$

Hence, r = 71.30cm

7. What is the length of the arc intercepted on a circle of radius 14 cms by the arms of a central angle of 45^0?

Solution:

$l = ?$

r radius = radius = 14cm

$\theta = 45^0$

$= 45 \times \frac{\pi}{180^0} = \frac{\pi}{4}$radian

we know that:

$l = r\theta$

$l = 14 \times \frac{\pi}{4} = \frac{7\pi}{2} = 10.99 \cong 11cm$

Hence, length of the arc intercepted = 11cm

8. Find the radius of the circle, in which the arms of a central angle of measure 1 radian cut off an arc of length 35cm.

Solution:

Radius = ?

Measure of central angle = 1 radian

arc of length = 35cm

we know that

$l = r\theta$

$\Rightarrow r = \frac{l}{\theta}$

$= \frac{35}{1} = 35cm$

Hence, radius of the circle = 35cm

9. A railway train is running on a circular track of radius 500 meters at the rate of 30km per hour. Through what angle will it turn in 10 sec?

Solution:

Radius = 500meters

Speed of train = 30km per hour

$$= \frac{30 \times 1000}{3600} = \frac{300}{30} = \frac{35}{3} m/s$$

= 8.33 m/s

Distance covered by train = v x t

= 8.33 x 10

= 83.33 m

We know that $l = r\theta$

$$\Rightarrow \theta = \frac{l}{r}$$

$$= \frac{83.33}{500} = 0.166 rad$$

Hence, angle of turning of train = 0.166 rad or $\frac{1}{6}$ rad

10. A horse is tethered to a peg by a rope of 9 meters length and it can move in a circle with the peg as centre. If the horse moves along the circumference of the circle, keeping the rope tight, how far will it have gone when the rope has turned through an angle of 70^0?

Solution:

Radius = 9 meter

Angle = 70^0 = 70 x $\frac{\pi}{180^0} = \frac{7}{18}\pi rad$

$l = ?$

We know that

$l = r\theta$

$l = 9 \times \frac{7}{18}\pi$

$= \frac{7}{2}\pi = \frac{7}{2} \times \frac{22}{7} = 11 meter$

Hence, circumference of the circle = 11m

11. The pendulum of a clock is 20 cm long and it swings through an angle of 20^0 each second. How far does the tip of the pendulum move in 1 second?

Solution:

Radius = 20cm

Angle = 20^0 = 20 x $\frac{\pi}{180^0} = \frac{\pi}{9} rad$

Length =?

We know that,

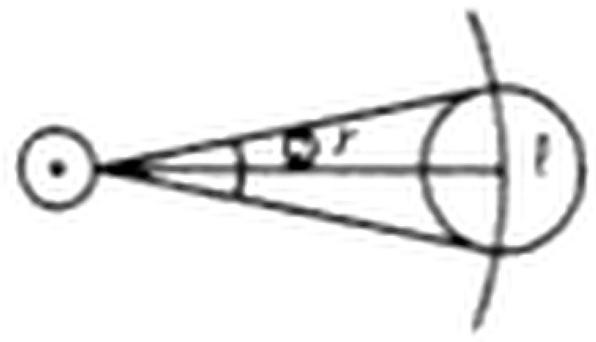
$l = r\theta$

$= 20 \times \frac{\pi}{9} = \frac{20}{9}\pi cm$

Hence, the pendulum covered distance in one second is 6.98 cm

12. Assuming the average distance of the earth from the sun to be 148 x 10^6 km and the angle subtended by the sun at the eye of a person on the earth of measure 9.3x10^{-3} radian. Find the diameter of the sun.

Solution:



r = 148 x 10^6 km

$\theta = 9.3 \times 10^{-3}\, radian$

We know that;

$l = r.\theta$

$= 148 \times 10^6 \times 9.3 \times 10^{-3}$

$= 1376.4 \times 10^3\, km$

$\Rightarrow l = 1.376 \times 10^6\, km$

Hence, the diameter of sun = 1.376 x 10^6 km

13. A circular wire of radius 6cm is cut straightened and then bent so as to lie along the circumference of a hoop of radius 24cm. Find the measure of the angle which it subtends at the centre of the hoop.

Solution:

Radius of circular wire = 148 x 10^6km

$\Rightarrow$ Length of circular wire = $2\pi \times 6$

$= 12\pi$ cm

Radius = 24cm

$\theta = ?$

We know that: $l = r\theta$

$\Rightarrow$ $\theta = \frac{l}{r}$

$= \frac{12\pi}{24} = \frac{1\pi}{2} = \frac{\pi}{2} radians$

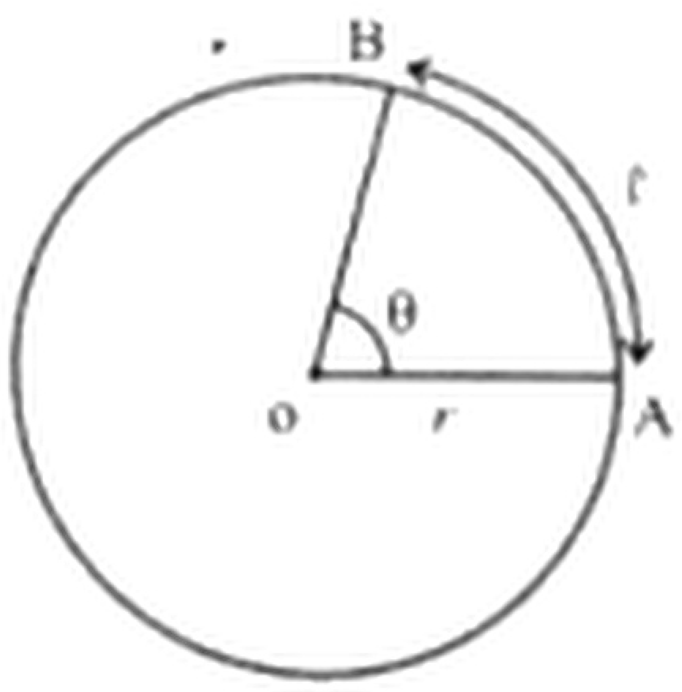
Hence, angle of the centre of the hoop $= \frac{\pi}{2} radians$

14. Show that the area of a sector of circular region radius r is $\frac{1}{2}r^2\theta$, where θ is the circular measure of the central angle of the sector.

Solution:

Consider the circular region having radius = r

And central angle = θ rad



So,

$$\frac{\text{Angle in a sector}}{\text{Area of a sector}} = \frac{\text{Angle of circle}}{\text{Area of circle}}$$

$$\frac{\theta}{A} = \frac{2\pi}{\pi r^2}$$

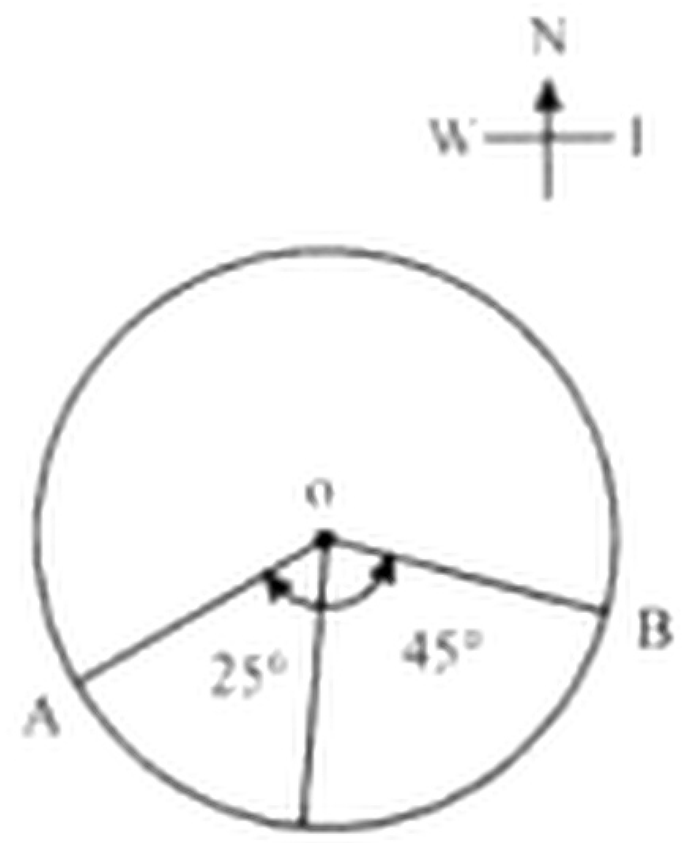
$\Rightarrow \qquad \dfrac{A}{\theta} = \dfrac{\pi r^2}{2\pi}$

$\Rightarrow \qquad A = \dfrac{r^2}{2}\theta$

Hence, Area of a sector $= \dfrac{1}{2}r^2\theta$

15. Two cities A and B lie on the equator such that their longitudes are 45^0 E and 25^0 W respectively. Find the distance between the two cities, taking radius of the each as 6400 kms.

Solution:



Total angle of two cities = $45^0 + 25^0$

$= 70^0 = 70 \times \dfrac{\pi}{180} = \dfrac{7}{18}\pi$ radians

Radius = 6400 km

$l = ?$

We know that; $l = r\theta$

$= 6400 \times \dfrac{7}{18}\pi$

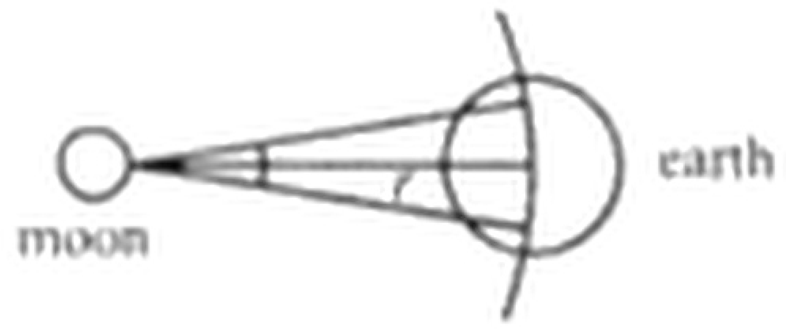
$l = 7822.22km$

Hence; the distance between the two cities = 7822.22km

16. The moon subtends an angle of 0.5⁰ at the eye of an observer on earth. The distance of the moon from the earth is 3.844 x 10⁵ km approx. What is the length of the diameter of the moon?

Solution:

$$\theta = 0.5^0 = 0.5 \times \frac{\pi}{180} = 8.72 \times 10^{-3} rad$$



$r = 3.844 \times 10^5$ km

$l = ?$

We know that:

$$l = r\theta$$

$= 8.72 \times 10^{-3} \times 3.844 \times 10^5$

$= 33.5196 \times 10^2$

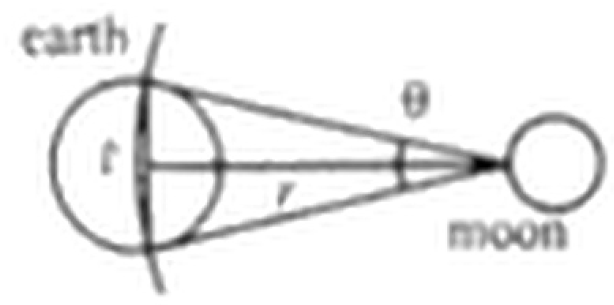
$l = 3351.96$ km

Hence, the diameter of the moon

= 3351.96 km

17. The angle subtended by the earth at the eye of a spaceman, landed on the moon, is $1^0 54'$. The radius of the earth is 6400 km. Find the approximate distance between the moon and the earth.

Solution:



$\theta = 1^0 54' = 1.9^0$

$= 1.9 \times \frac{\pi}{180} = 0.033 rad$

Radius = 6400km

$\Rightarrow$ Diameter = 6400 x 2 = 12800km

$\Rightarrow$ $l = 12800 km$

We know that $l = r\theta$

$r = \frac{l}{\theta}$

$= \frac{12800}{0.033} = 3.86 \times 10^5 km$

Hence, the distance, between the moon and the earth = 3.86 x 10^5km