Exercise 7.8

1. The probability that a person A will be alive 15 years hence is $\frac{5}{7}$ and the probability that another person B will be alive 15 years hence is $\frac{7}{9}$. Find the probability that both will be live 15 years hence.

Solution

A = Event that A will be alive $P(A) = \frac{5}{7}$

B = Event that B will be alive $P(B) = \frac{7}{9}$

Required probability that both will be alive

$$P(A \cap B) = P(A).P(B)$$

$$= \frac{5}{7} \times \frac{7}{9} = \frac{5}{9}$$

Hence, the probability that A and B will alive $= \frac{5}{9}$

2. A die is rolled twice. Event E_1 is the appearance of even number of dots and event E_2 is the appearance of more than 4 dots. Prove that:

$P(E_1 \cap E_2) = P(E_1).P(E_2)$

Solution:

A dice is rolled twice, so that total possible outcomes = 36

E_1 = Appearance of even number of dots.

$[\{(2,2)(2,4)(2,6)(4,2)(4,4)(4,6)(6,2)(6,4)(6,6)\}$

$N(E_1) = 6$

$P(E_1) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

E_2 = Appearance of more than n dots.

$\{(5,5)(5,6)(6,5)(6,7)\}$

$N(E_2) = n$

$P(E_2) = \frac{n(E)}{n(S)} = \frac{4}{36} = \frac{1}{9}$

$(E_1 \cap E_2) = \{(6,6)\}$

$N(E_1 \cap E_2) = 1$

Thus

$P(E_1 \cap E_2) = \frac{n(E1 \cap E2)}{n(S)} = \frac{1}{36}$

And $P(E_1).P(E_2) = \frac{1}{4}.\frac{1}{9} = \frac{1}{36}$

Hence proved $P(E_1 \cap E_2) = P(E_1).P(E_2)$

3. Determine the probability of getting 2 heads in two successive tosses of a balanced coins.

Solution:

When a coin is tossed twice than the possible outcomes are.

S = {HH, HT, TH, TT}

$n(S) = 4$

H = Event of getting two heads

$n(H) = 1$

Thus, $P(H) = \frac{n(H)}{n(S)} = \frac{1}{4}$

Hence, probability $= \frac{1}{4}$

4. Two coins are tossed twice. Each find the probability that the head appears on the first toss and the same face appears in the two tosses.

Solution:

When a coin tossed than the possible outcomes.

S = {HH, HT, TH, TT}

$n(S) = 4$

Let A be the event that head appears in first toss.

$A = \{HT, TH\}$

$n(A) = ?$

$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

Let B be the event that same face appears in both sides.

$B = \{HH, TT\}$

$n(B) = 2$

$P(B) = \frac{n(B)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

Probability that head appear in first toss and same face appear on second time

$P(A \cap B) = P(A).P(B) = \frac{1}{2}.\frac{1}{2} = \frac{1}{4}$

Hence, the probability $= \frac{1}{4}$

5. **Two cards are drawn from a deck of 52 playing cards. If one card is drawn and replaced before drawing second card, find the probability that both the cards are aces.**

Solution:

Let A denote event, "first card is an ace" and B the event "second is also ace" Required

probability is

$$P(A \cap B) = P(A).P(B)$$

$$= \frac{4}{52} \times \frac{4}{52}$$

$$= \frac{1}{13} \times \frac{1}{13} = \frac{1}{69}$$

Hence, the probability of both cards are aces $= \frac{1}{169}$

6. **Two cards from a deck of 52 playing cards are drawn in such a way that the card is replaced after the first draw. Find the probability of following cases:**

i) **First card is king and second is a queen**

ii) **Both the cards are faced cards i.e. king, queen, jack.**

Solution:

i. **First card is king and second is a queen**

Let A denote the event first card is king and B denote the event second card is queen

$$P(A \cap B) = P(A).P(B)$$

$$= \frac{4}{52} \times \frac{4}{52} = \frac{1}{13} \times \frac{1}{13} = \frac{1}{169}$$

Hence; the probability $= \frac{1}{169}$

ii. Both the cards are faced cards i.e. king, queen, jack.

Let A be the first card is a face card

i.e. = 12 (4 king, 4 queen and 4 jack)

$n(P) = 12$

$$\boldsymbol{P(A) = \frac{n(P)}{n(S)} = \frac{12}{52}}$$

Let B be the second card is a face card

i.e. $P(B) = \frac{n(P)}{n(S)} = \frac{12}{52}$

Since A and B are independent

$$P(A \cap B) = P(A).P(B)$$

$$= \frac{12}{52} \times \frac{12}{52}$$

$$= \frac{3}{13} \times \frac{3}{13} = \frac{9}{169}$$

Hence, the probability $= \frac{9}{169}$

7. Two dice are thrown twice. What is the probability that sum of the dots shown in the first throw is 7 and that of the second row is 11?

Solution:

When two dice are thrown. Then the possible outcomes:

$$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)$$

$$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)$$

$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)$

$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)$

$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)$

$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

$n(S) \quad = 36$

Let A denote the total number of '7' on first throw

$A = \{(1,6)(2,5)(3,4)(4,3)(5,2)(6,1)\}$

$N(A) \quad = 6$

$P(A) \quad = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

Let B denote the total number of '11' on second throw

$B \ = \{(5,6)(6,5)\}$

$N(B) \quad = 2$

$P(B) \quad = \frac{n(B)}{n(S)} = \frac{2}{36} = \frac{1}{18}$

Required probability

$P(A \cap B) \qquad = P(A) - P(B)$

$= \frac{1}{6} \times \frac{1}{18} = \frac{1}{108}$

Hence, the probability $= \frac{1}{108}$

8. Find the probability that the sum of dots appearing in two successive throws of two dice is every time 7.

Solution:

$n(S) \quad = 36$

Let A denote the sum of two dice is '7'

$A = \{(1,6)(2,5)(3,4)(4,3)(5,2)(6,1)\}$

$N(A) \quad = 6$

$P(A) \quad = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

So, the probability that sum of dots is 7 in two successive throws.

$N(B) \quad = 36$

$P(B) \quad = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

Probability

$P(A \cap B) \qquad = P(A).P(B)$

$= \frac{1}{6}.\frac{1}{18} = \frac{1}{36}$

Hence, probability $= \frac{1}{36}$

9. A fair die is thrown twice. Find the probability that a prime number of dots appear in the First row and the number of dots in the second row is less than 5.

Solution:

A fair dice is thrown twice

$n(S) \quad = 6$

Suppose A be the event that the dice shows a prime number of dots in first throw

$A = \{2, 3, 5\}$

$N(A) \quad = 3$

$P(A) \quad = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

Suppose A be the event that the dice shows dots less than 7 in second throw

$\mathrm{B} = \{1, 2, 3, 5\}$

$N(B) \quad = 4$

$P(B) \quad = \frac{n(B)}{n(S)} = \frac{4}{6} = \frac{2}{3}$

Probability

$P(A \cap B) \qquad = P(A).P(B)$

$= \frac{1}{2}.\frac{2}{3} = \frac{1}{3}$

Hence, probability $= \frac{1}{3}$

10. A bag contains 8 red 5 white and 7 black balls. 3 balls are drawn from the bag. What is the probability that the first ball is red, the second ball is white and the third ball is black, when every time the ball is replaced?

$HINT;$ $\left(\frac{8}{20}\right)\left(\frac{5}{20}\right)\left(\frac{7}{20}\right)$ **is the probability.**

Solution:

Total balls $= 20$

$n(S) \quad = 20$

White balls $= 5$

$n(w) \quad = 5$

Red balls $= 8$

$n(R) \quad = 8$

And black balls $= 7$

$n(R) \quad = 7$

$P(w) \quad = \frac{n(w)}{n(S)} = \frac{5}{20} = \frac{1}{4}$

$P(R) \quad = \frac{n(R)}{n(S)} = \frac{8}{20} = \frac{2}{5}$

$P(B) \quad = \frac{n(B)}{n(S)} = \frac{7}{20} = \frac{7}{20}$

$P(w \cap R \cap B) = \frac{2}{5} \times \frac{1}{4} \times \frac{7}{20} = \frac{7}{200}$

Hence, probability (first red, second white, third black) $= \frac{7}{200}$